The parity-odd structure function of the nucleon from the Compton amplitude

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Motivation

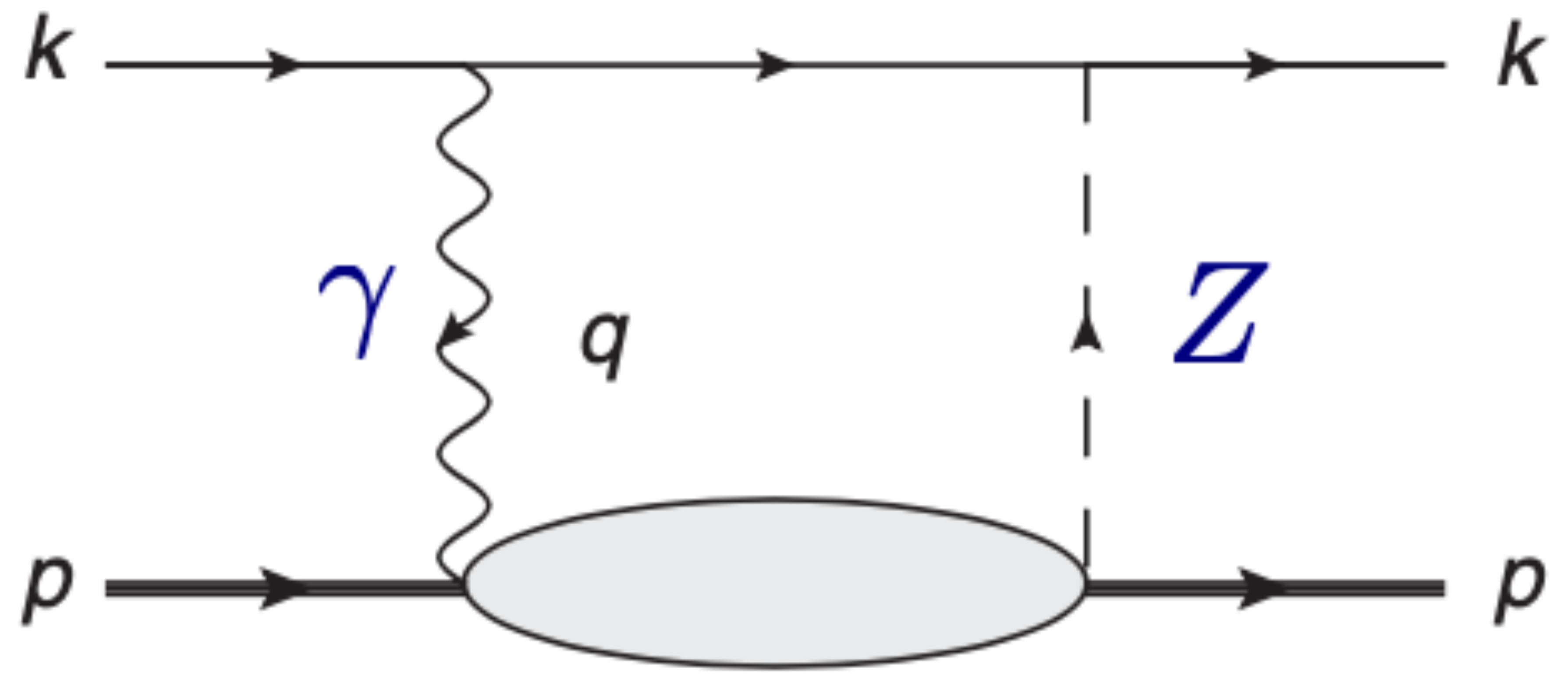
- Leading theoretical uncertainty in:
- Weak charge of the proton,

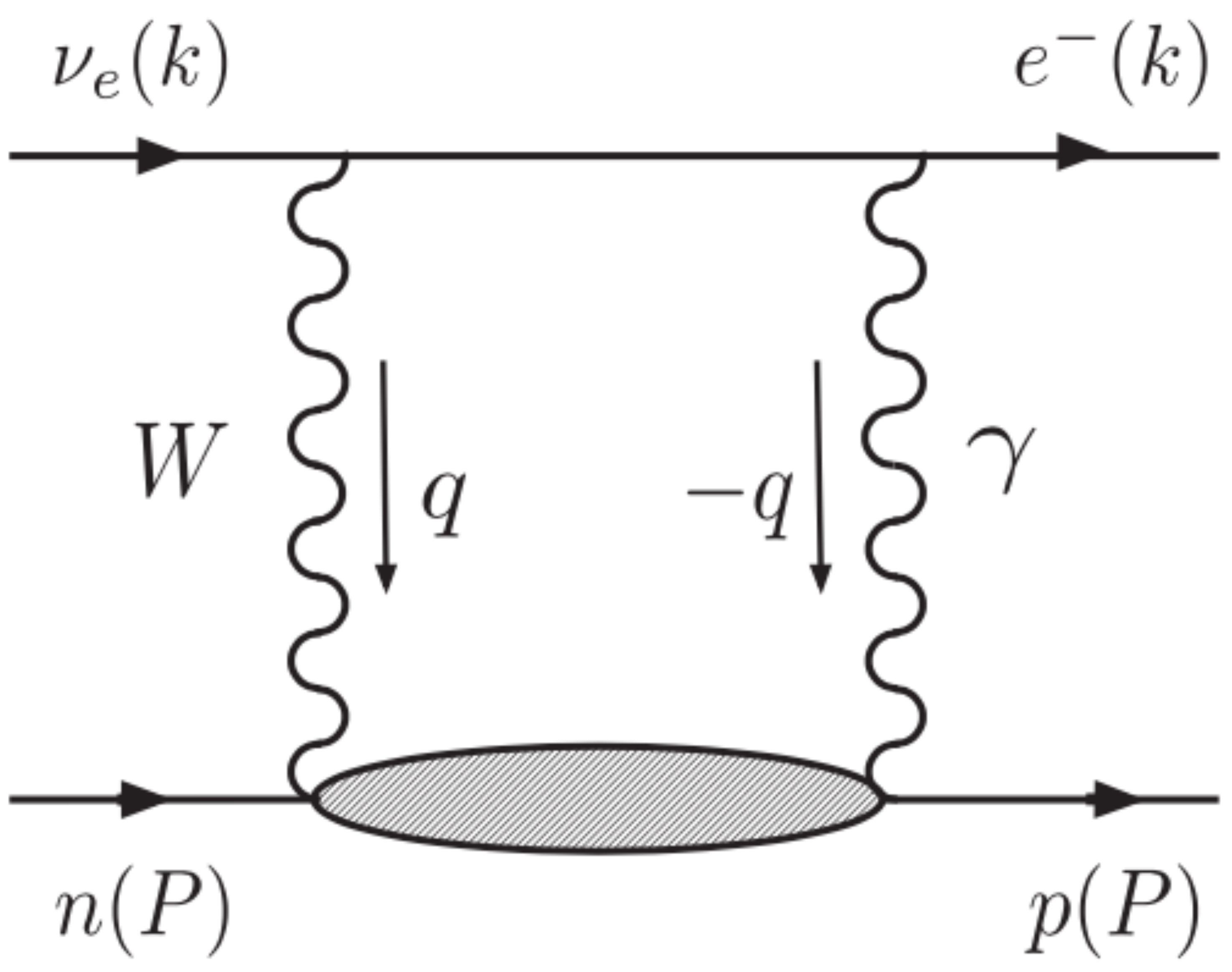
$$Q_W = (1 + \Delta_\rho + \Delta_e)(1 - 4\sin^2\theta_W(0) + \Delta_e') + \Box^{WW}_{AA} + \Box^{ZZ}_{AA} + \Box^{\gamma Z}_{VA}$$

- CKM matrix element extracted from superallowed neutron β decays,

$$|V_{ud}|^2 = \frac{0.97148(20)}{1 + \Delta^V_R}$$

$\Delta^V_R \rightarrow 0.01691 + 2\Box^{\gamma W}_{VA}$

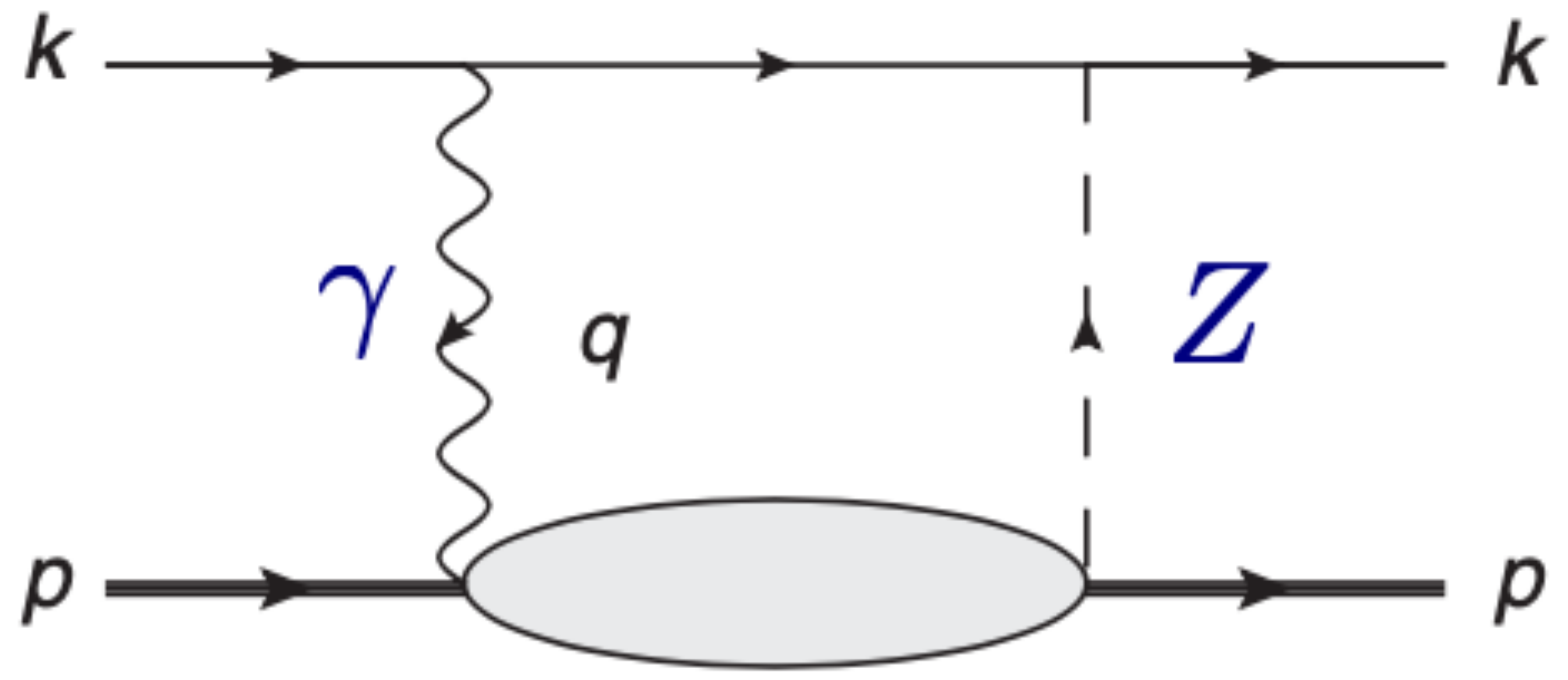




K. Shiells, P.G. Blunden, W. Melnitchouk, PRD104, 033003 (2021) [2012.01580]

Motivation

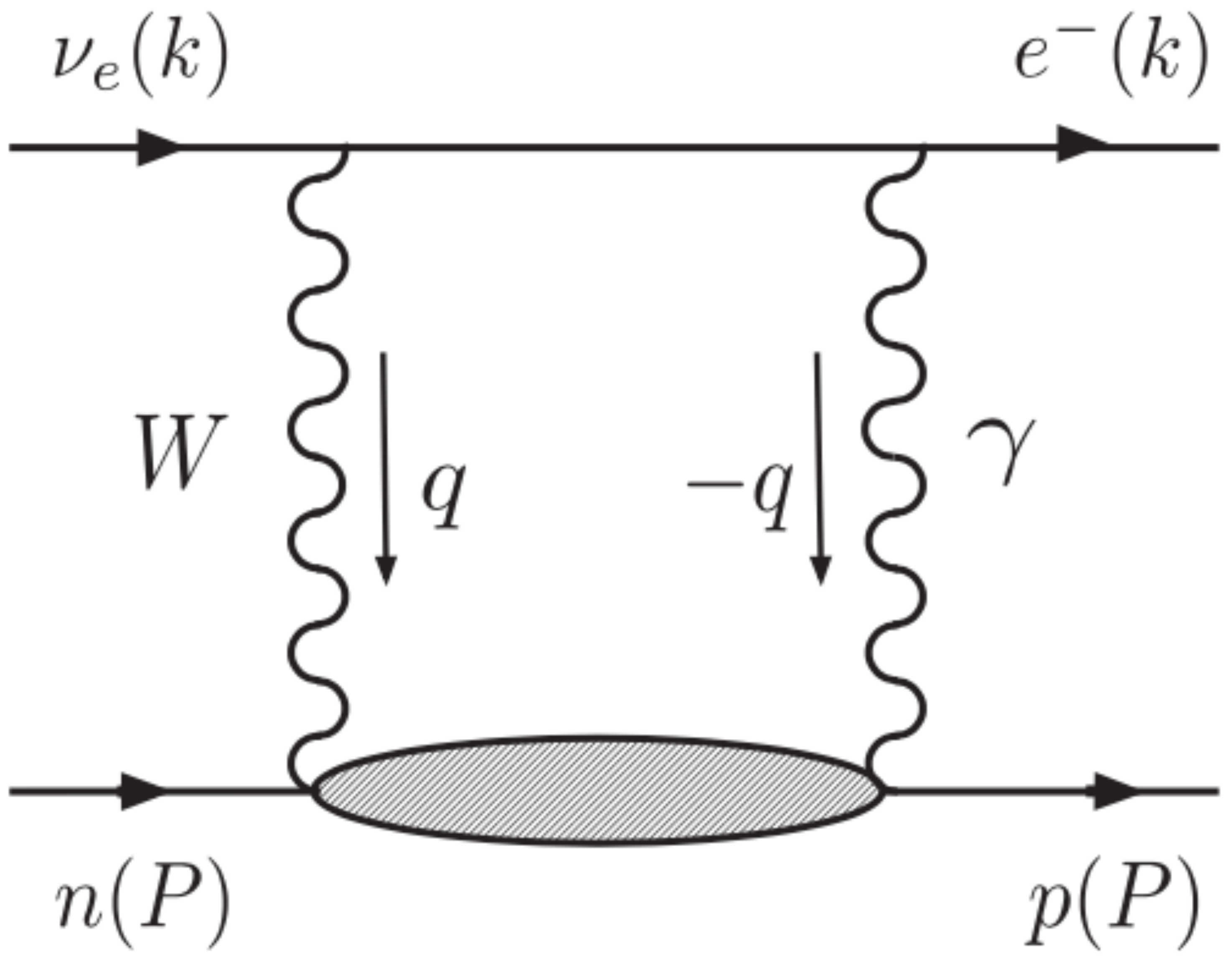
$$\Box_A^{\gamma Z} = \nu_e \frac{3\alpha_{EM}}{2\pi} \int_0^\infty \frac{dQ^2}{Q^2} \frac{M_Z^2}{M_Z^2 + Q^2} \underbrace{\int_0^1 dx F_3^{\gamma Z}(x, Q^2)}$$



First moment of F_3

$$\Box_{VA}^{\gamma W} = \frac{3\alpha_{EM}}{2\pi} \int_0^\infty \frac{dQ^2}{Q^2} \frac{M_W^2}{M_W^2 + Q^2} \overbrace{\int_0^1 dx F_3^{(0)}(x, Q^2)}$$

$$F_3^{(0)} = F_3^{\gamma Z,p} - F_3^{\gamma Z,n}$$

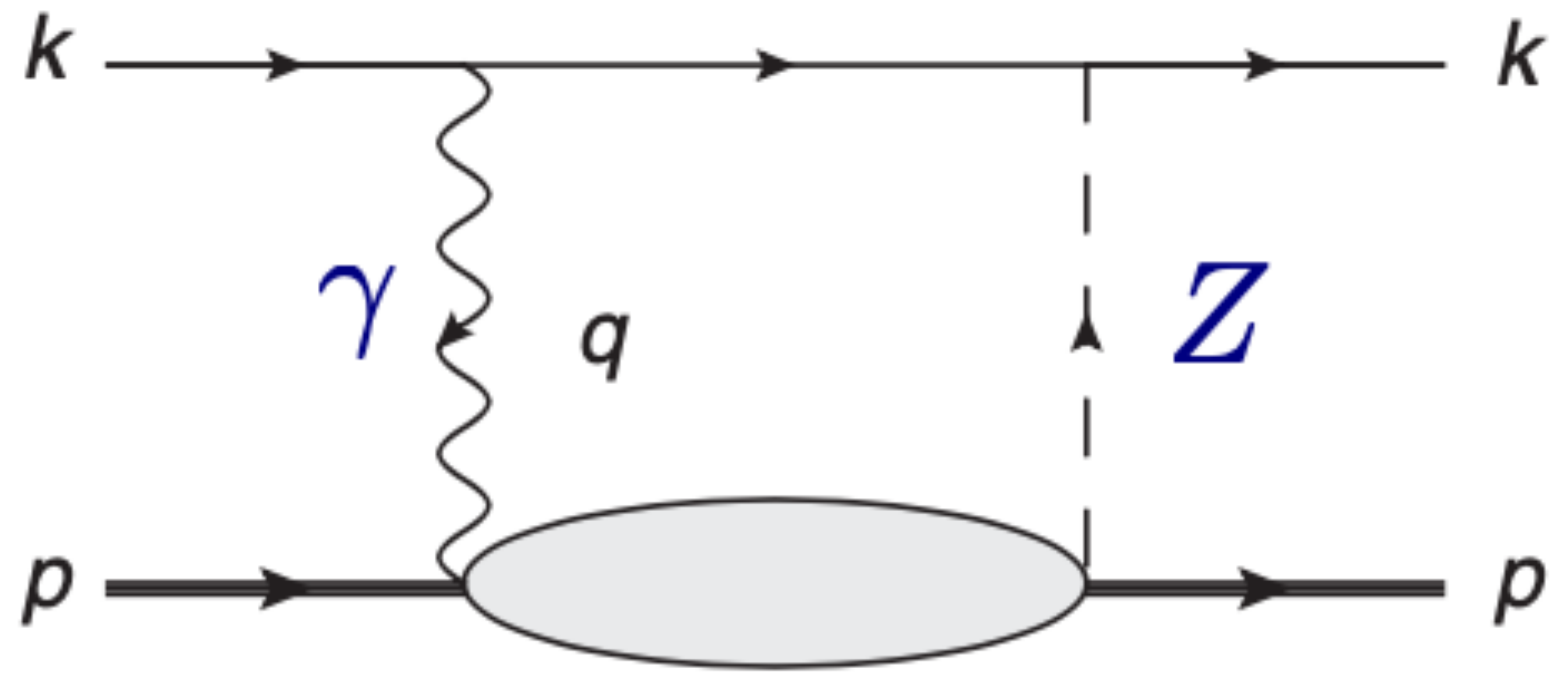


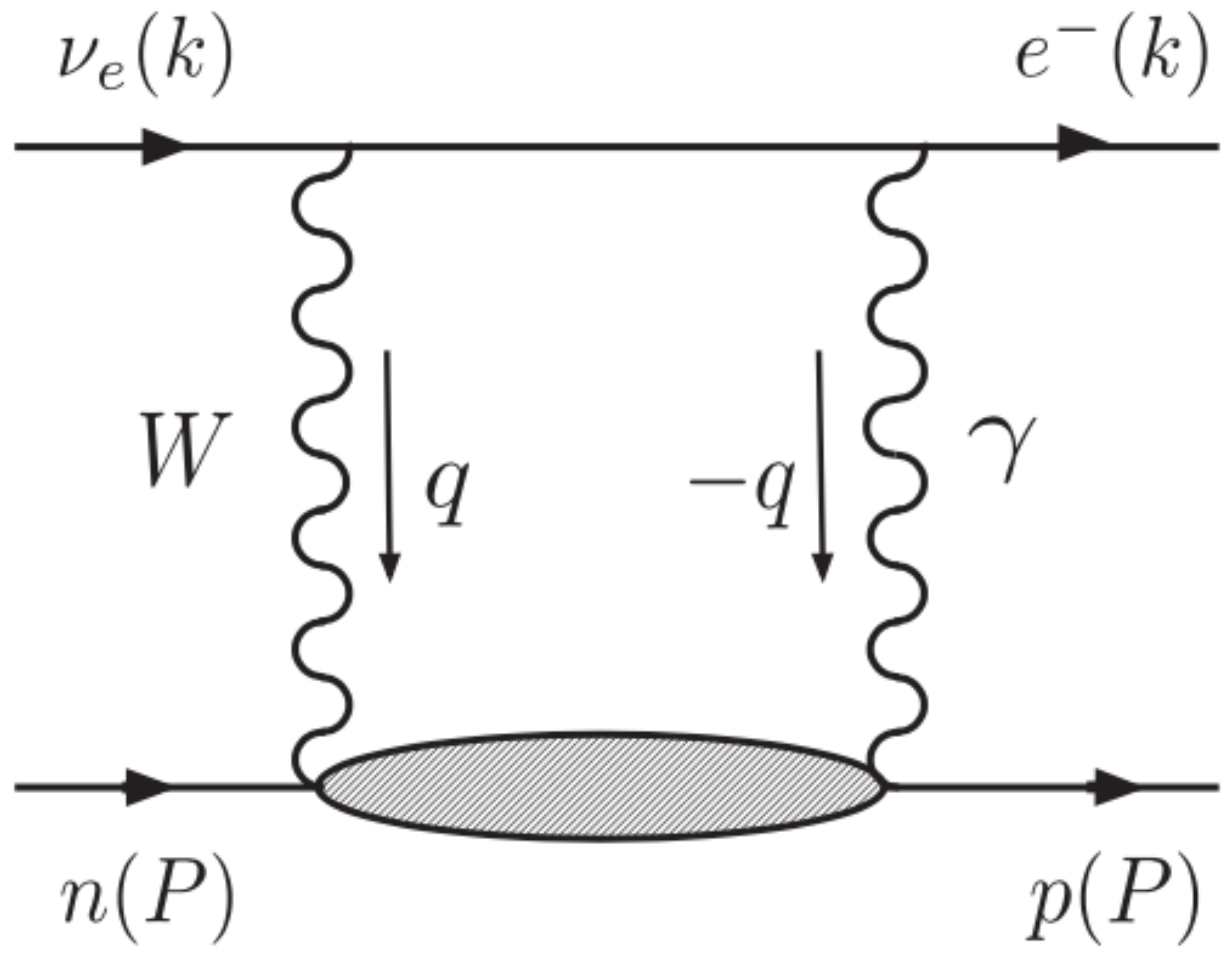
Motivation

- Box diagrams proportional to an integral over the whole Q^2 range

$$\Box_A^{\gamma Z/W} \propto \int_0^\infty \frac{dQ^2}{Q^2} M_1^{(3)}(Q^2)\,(\ldots)$$

- Low-Q^2 (non-perturbative) regime dominates the integral
- F_3 is experimentally poorly determined in low Q^2
- Lattice approach is ideal for a high-precision determination of $M_1^{(3)}(Q^2)$



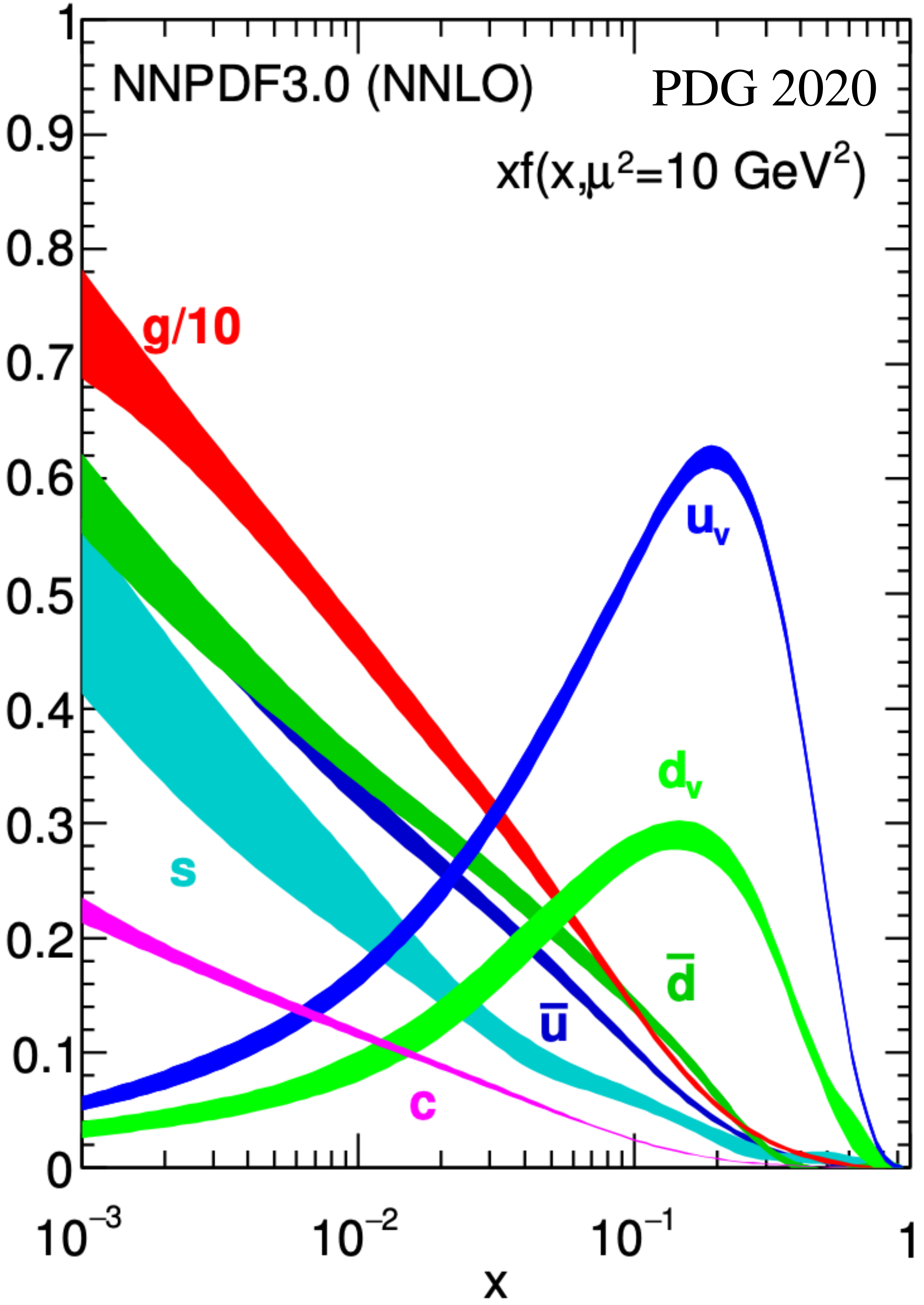


Motivation

- Nucleon structure (leading twist)
  - Structure functions from first principles
- In the parton model

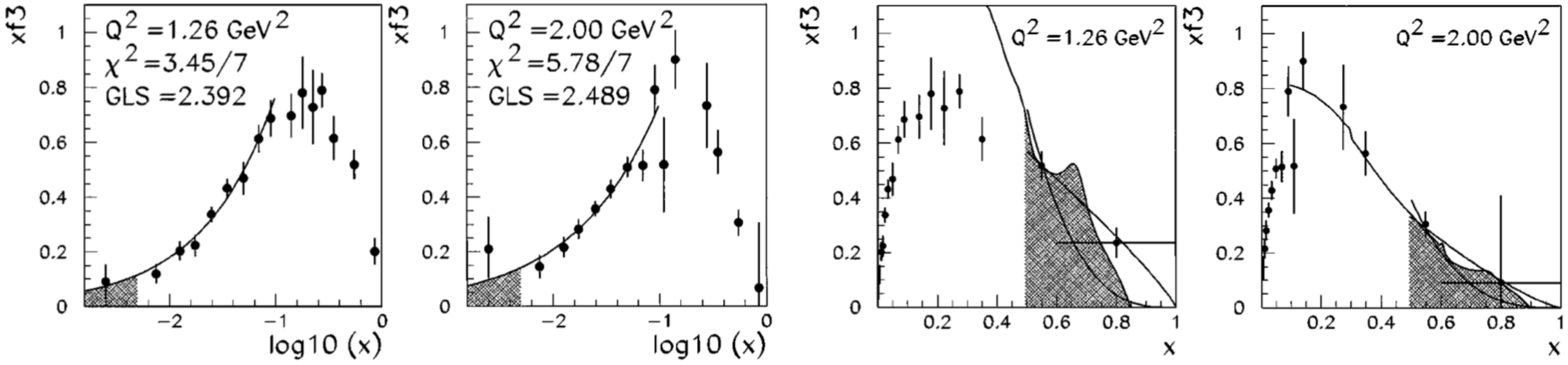
$$F_2 \propto (q + \bar{q})$$

$$F_3^{\gamma Z} \propto (q - \bar{q})$$



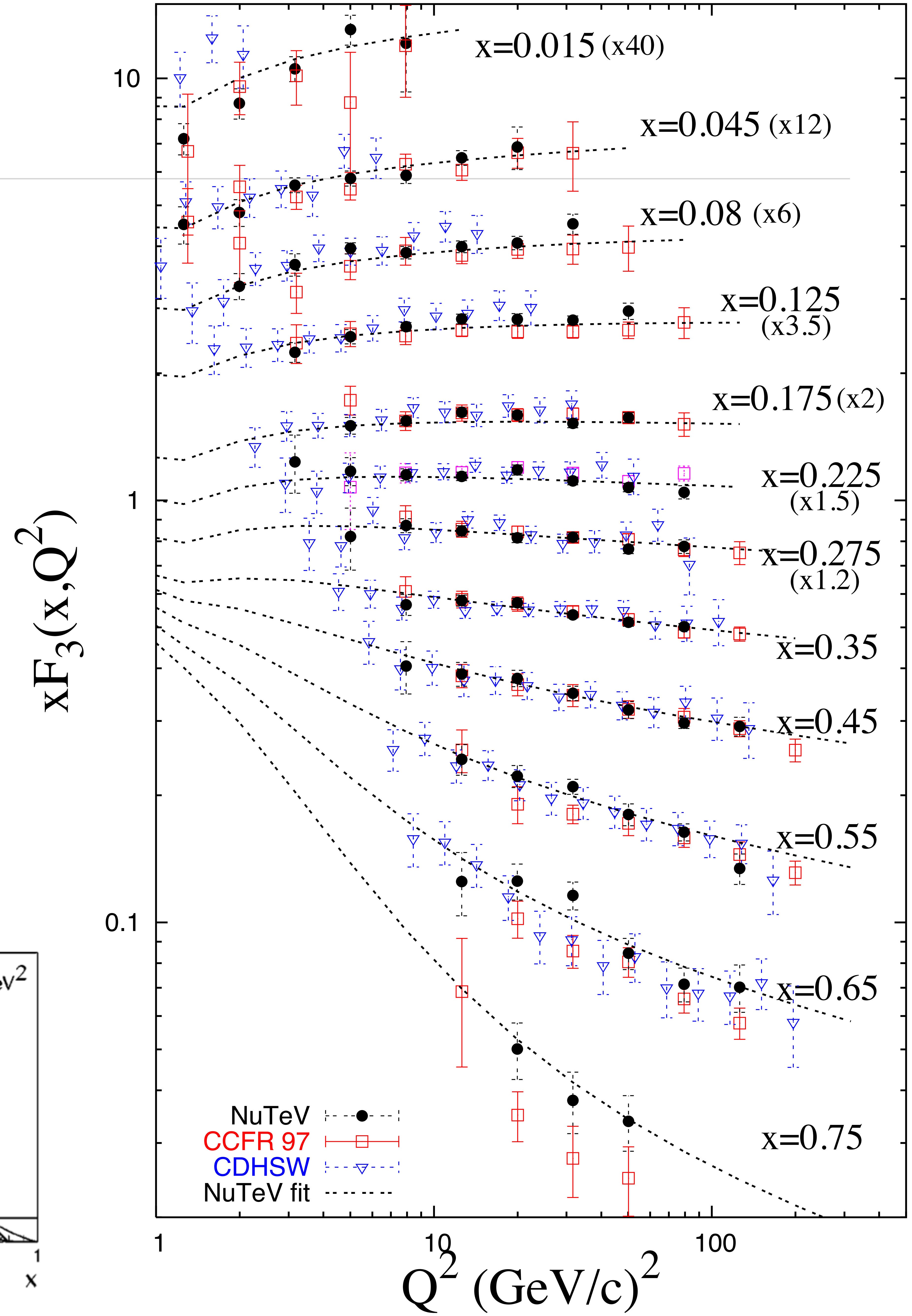
Motivation

- <u>Nucleon structure</u> (leading twist)
  - Understanding the behaviour in the high- and low-x regions
- World ν-N data:
  - **NuTev (Fermilab)**
  - **CHORUS (CERN)**
  - **CCFR (Fermilab) E744, E770, and older E180**
  - **BEBC (CERN) Gargamelle, WA25, and WA59**
  - **SKAT (Zeuthen)**



J. H. Kim et al., PRL81 (1998) 3595-3598 [hep-ex/9808015]

M. Tzanov et al. (NuTeV Collab.), PRD74 012008 (2006) [hep-ex/0509010]



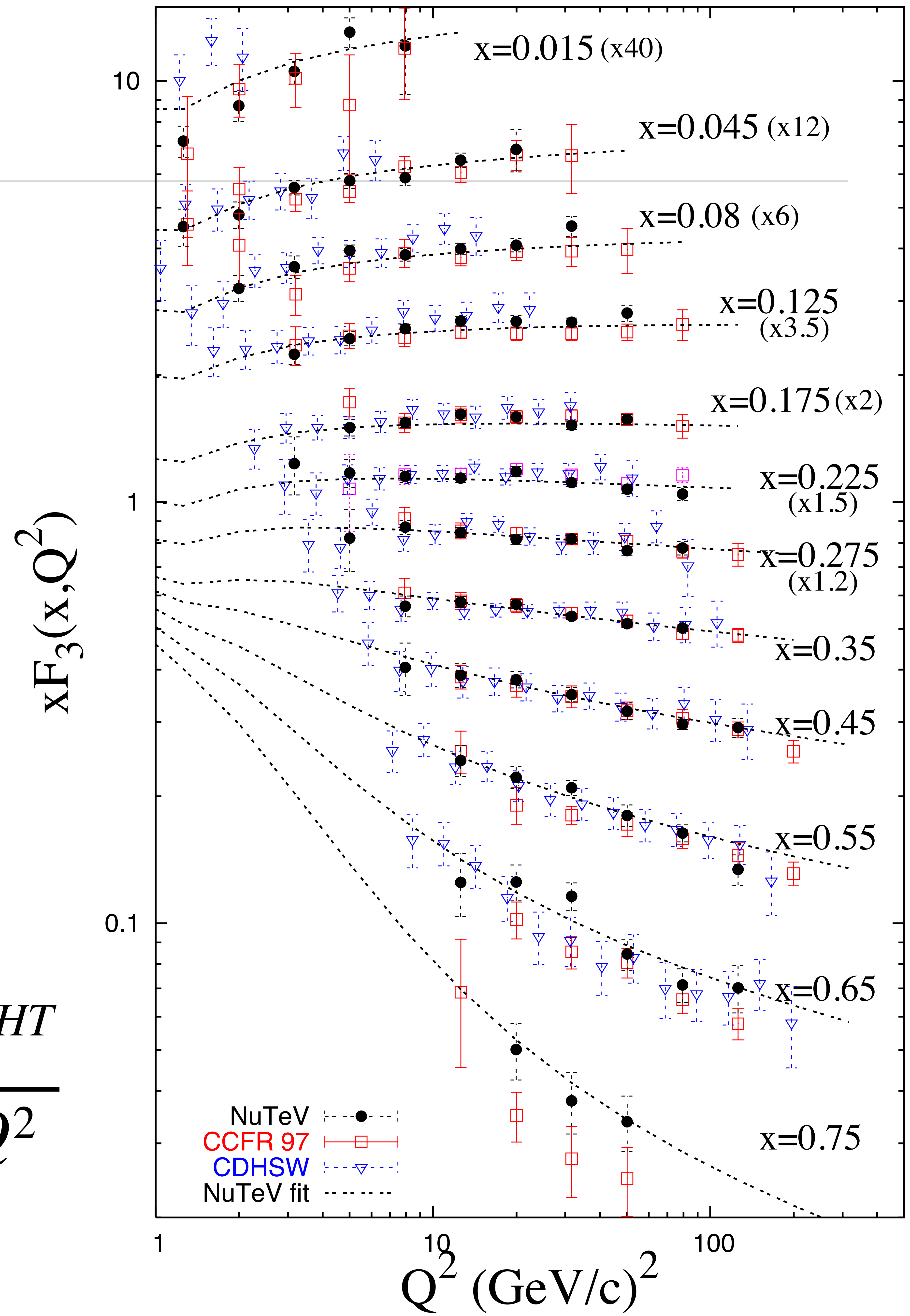
Motivation

- Scaling
  - Q^2 cuts of global QCD analyses
- Power corrections / Higher twist effects
  - Target mass corrections
  - Twist-4 contributions
  - GLS sum rule:

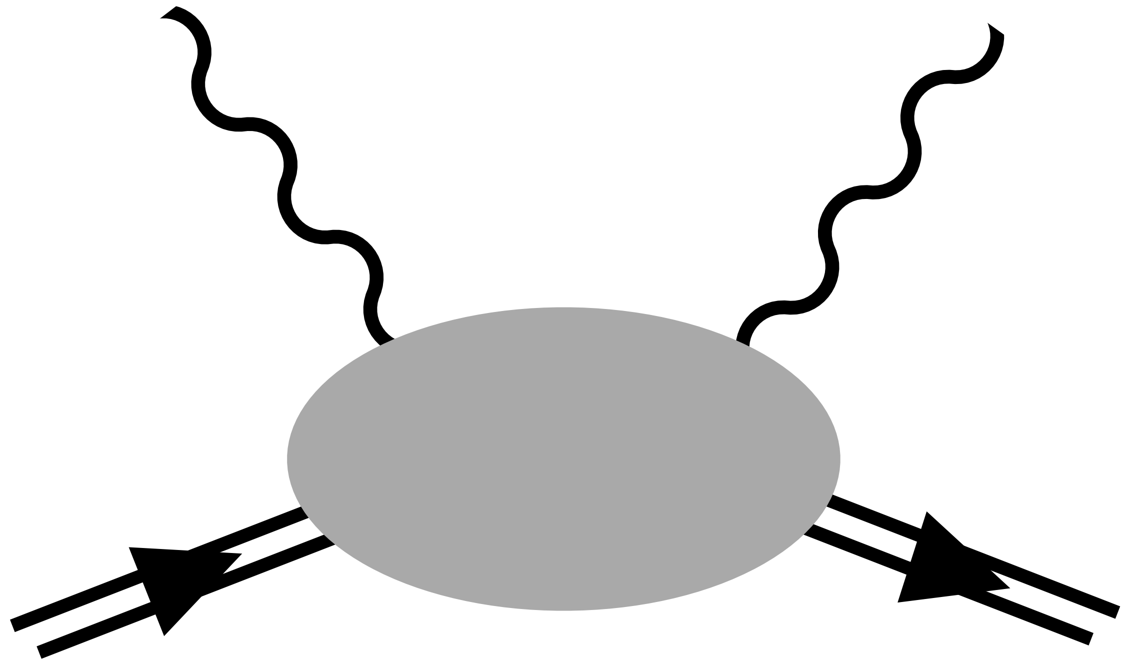
$$S^{GLS} = \int_0^{1^-} dx F_3^{(\nu p+\bar{\nu}p)}(x, Q^2) = 3\left[1 + \frac{\alpha_s(Q^2)}{\pi} + \ldots\right] - \frac{\Delta^{HT}}{Q^2}$$

  - $\Delta^{HT} \sim 0.15 - 0.5$ see X.-D. Huang et al., NPB969 (2021) 115466 [2101.10922]

M. Tzanov et al. (NuTeV Collab.), PRD74 012008 (2006) [hep-ex/0509010]



Forward Compton Amplitude



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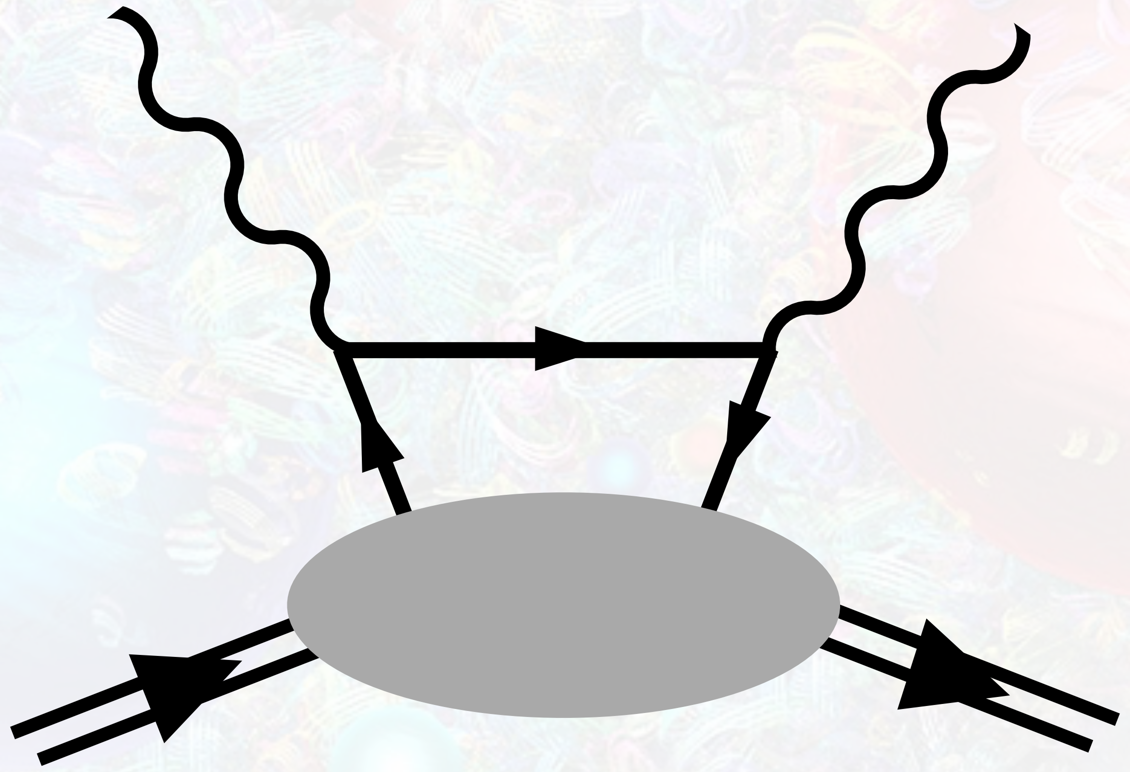


$$+ \quad \mathcal{O}\left(\frac{M_N}{Q^2}, \frac{1}{Q^2}\right)$$

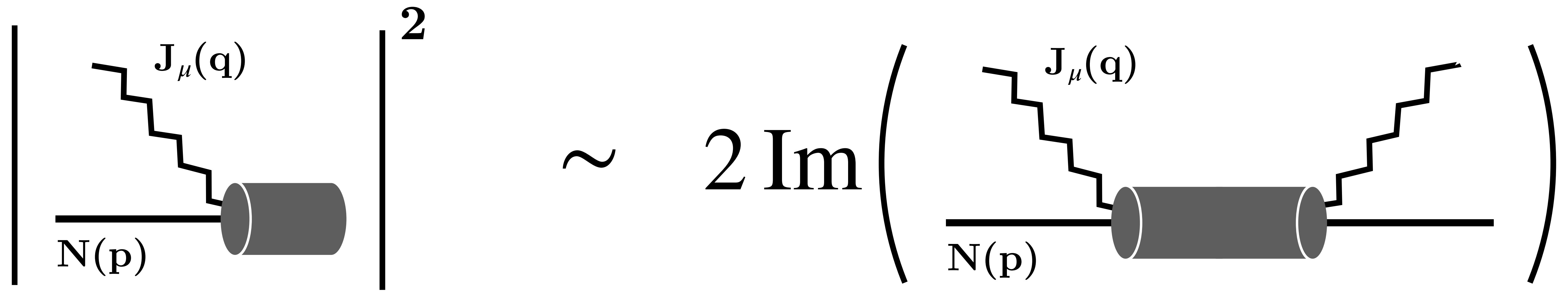
Parity Violating Forward Compton Amplitude

$$T_{\mu\nu}(p,q) = i\int d^4z\, e^{iq\cdot z}\rho_{ss'}\langle p,s'|\mathscr{T}\{J^V_\mu(z)J^A_\nu(0)\}|p,s\rangle \text{ , spin avg. } \rho_{ss'} = \frac{1}{2}\delta_{ss'}$$

$$= -g_{\mu\nu}\mathscr{F}_1(\omega,Q^2) + \frac{p_\mu p_\nu}{p\cdot q}\mathscr{F}_2(\omega,Q^2) + i\,\varepsilon^{\mu\nu\alpha\beta}\frac{p_\alpha q_\beta}{2p\cdot q}\mathscr{F}_3(\omega,Q^2)$$

$$+\frac{q_\mu q_\nu}{p\cdot q}\mathscr{F}_4(\omega,Q^2) + \frac{p_{\{\mu}q_{\nu\}}}{p\cdot q}\mathscr{F}_5(\omega,Q^2) + \frac{p_{[\mu}q_{\nu]}}{p\cdot q}\mathscr{F}_6(\omega,Q^2)$$

allowed terms because parity is violated



DIS Cross Section ~ Hadronic Tensor

Forward Compton Amplitude ~ Compton Tensor

$$\omega = \frac{2p\cdot q}{Q^2}$$

$$\varepsilon^{0123} = 1$$

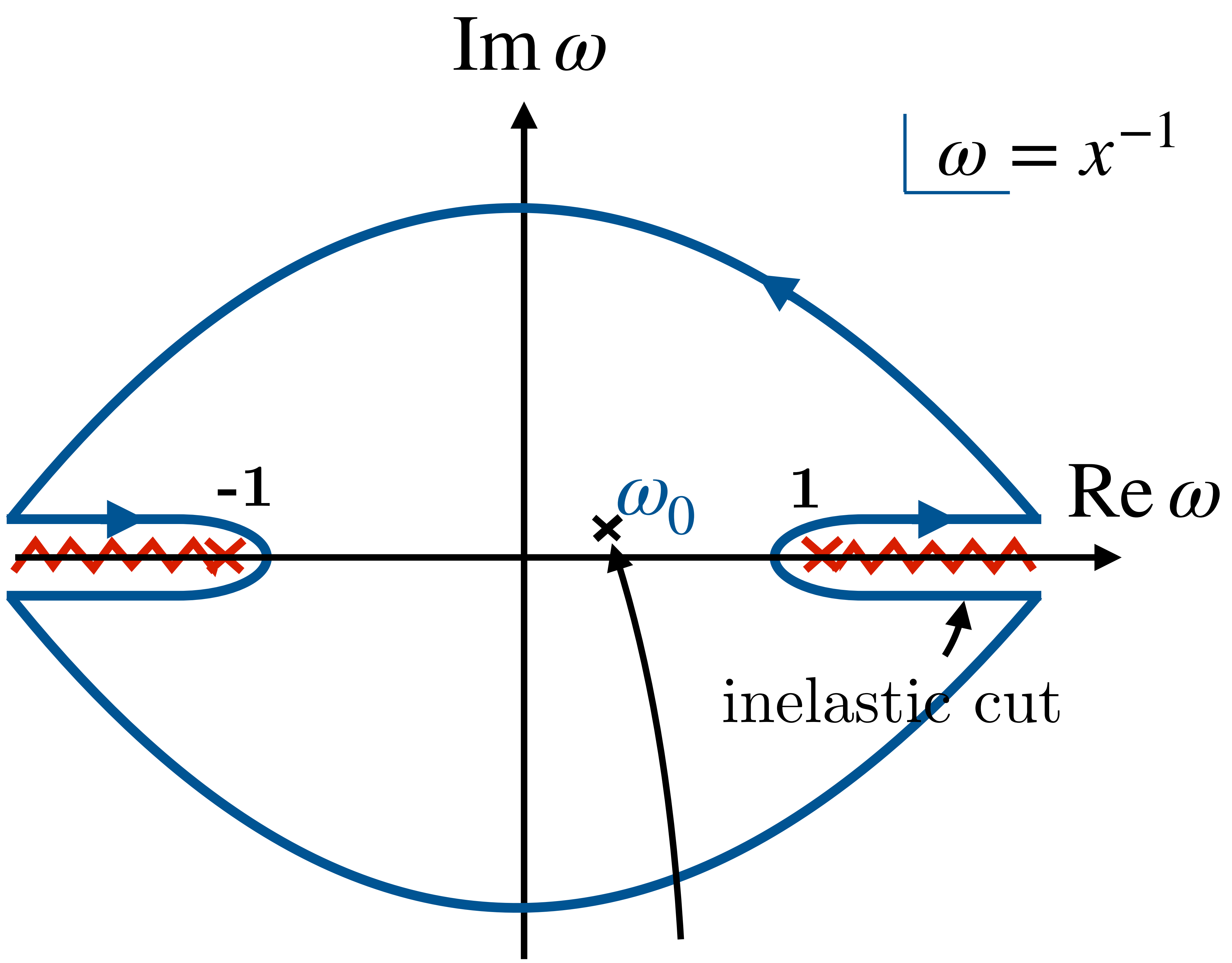
Nucleon Structure Functions

- for $\mu \neq \nu$ and $p_\mu = q_\mu = 0$, and $\beta \neq 0$, we isolate,

$$T_{\mu\nu}(p,q) = i\,\varepsilon^{\mu\nu\alpha\beta}\frac{p_\alpha q_\beta}{2p\cdot q}\mathcal{F}_3(\omega, Q^2)$$

- we can write down dispersion relations and connect Compton SFs to DIS SFs:

$$\mathcal{F}_3(\omega, Q^2) = 4\omega \int dx \frac{F_3(x,Q^2)}{1-x^2\omega^2}$$



Compton Amplitude is an analytic function in the unphysical region $|\omega_0| < 1$

Nucleon Structure Functions

$\omega = \frac{2p\cdot q}{Q^2} \equiv x^{-1}$

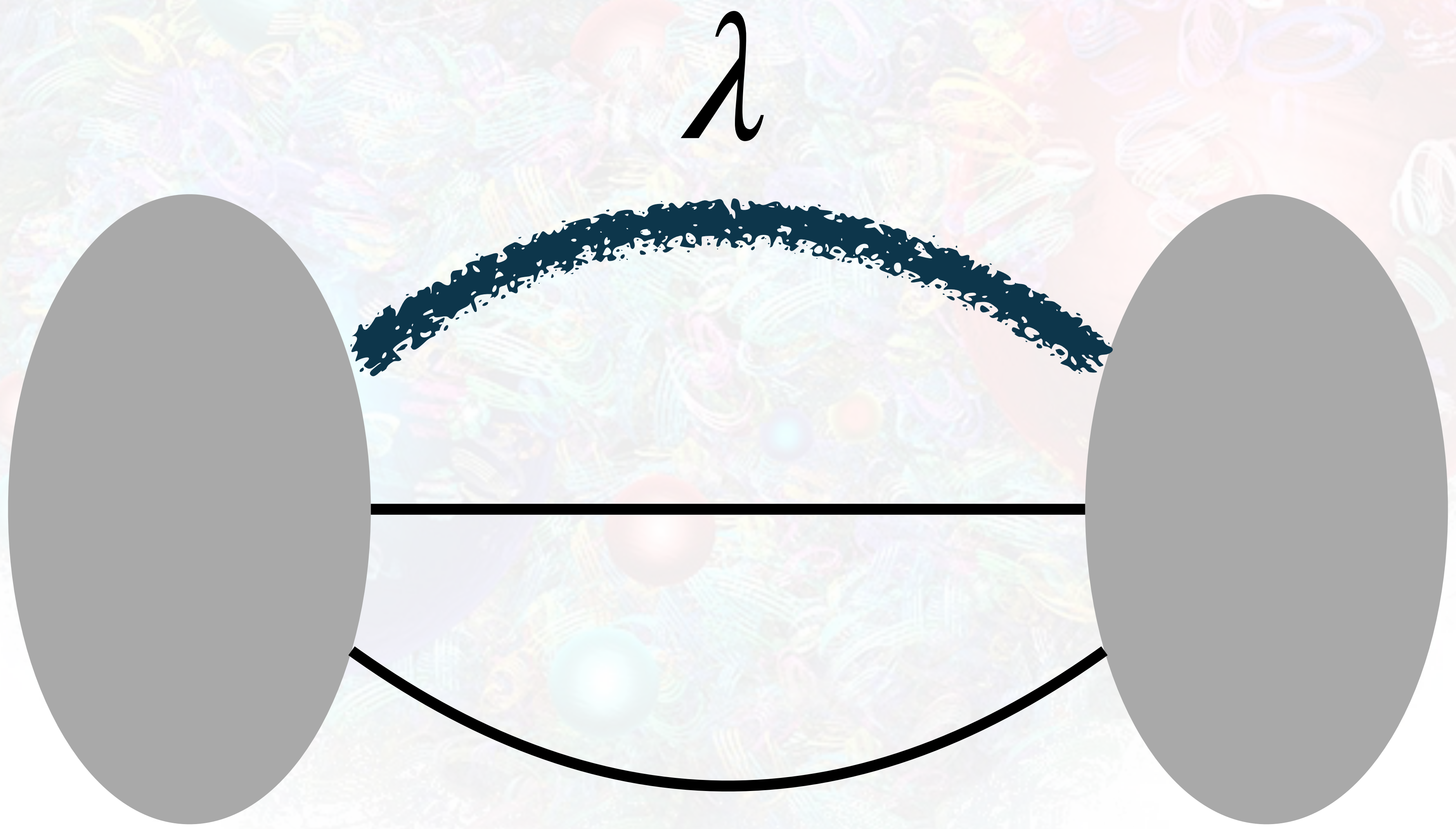
- **using the Taylor expansion,** $\frac{1}{1-(x\omega)^2} = \sum_{n=1}^{\infty} (x\omega)^{2n-2}$

$$\mathcal{F}_3(\omega, Q^2) = 4 \sum_{n=1,2,\ldots} \omega^{2n-1} M^{(3)}_{2n-1}(Q^2)$$

Mellin moments

$$M^{(3)}_{2n-1}(Q^2) = \int_0^1 dx\, x^{2n-2} F_3(x, Q^2), \quad \text{for } \; n = 1,2,3,\ldots$$

Feynman-Hellmann Theorem



FH Theorem at 1st order

in Quantum Mechanics:

$$\frac{\partial E_\lambda}{\partial \lambda} = \langle \phi_\lambda | \frac{\partial H_\lambda}{\partial \lambda} | \phi_\lambda \rangle$$

H_λ: perturbed Hamiltonian of the system

E_λ: energy eigenvalue of the perturbed system

ϕ_λ: eigenfunction of the perturbed system

- **expectation value of the perturbed system is related to the shift in the energy eigenvalue**

in Lattice QCD: energy shifts in the presence of a weak external field

$$S \to S(\lambda) = S + \lambda \int d^4x\, \mathcal{O}(x) \longrightarrow \bar{q}(x)\Gamma_\mu q(x) \quad , \Gamma_\mu \in \{\mathbf{1}, \gamma_\mu, \gamma_5\gamma_\mu, \ldots\}$$

e.g. local bilinear operator

real parameter (λ)

@ 1st order

$$\frac{\partial E_\lambda}{\partial \lambda} = \frac{1}{2E_\lambda} \langle 0 | \mathcal{O} | 0 \rangle$$

$E_\lambda \to$ spectroscopy, 2-pt function

$\langle 0 | \mathcal{O} | 0 \rangle \to$ determine 3-pt

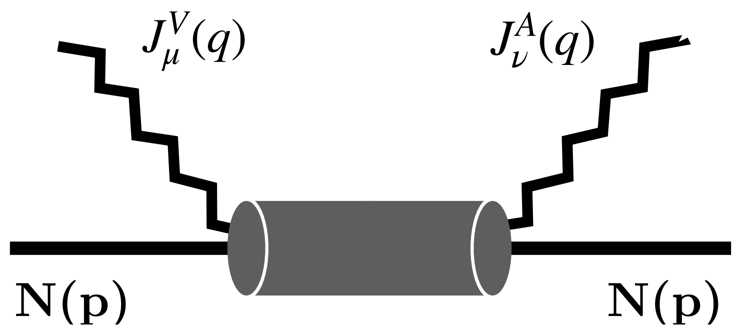
Applications:

- σ **- terms**
- **Form factors**

Compton amplitude via the FH relation at 2nd order

- **unpolarised Compton Amplitude**

$$T_{\mu\nu}(p,q) = \int d^4z e^{iq\cdot z}\langle N(p)\,|\,\mathscr{T}\{J^V_\mu(z)J^A_\nu(0)\}\,|\,N(p)\rangle$$



- **Action modification**

$$S \to S(\lambda) = S + \lambda_1 \int d^4z\, \cos(q\cdot z)\, J^V_\mu(z) + \lambda_2 \int d^4y\, \sin(q\cdot y)\, J^A_\nu(z)$$

local V, A currents

$$J^V_\mu(z) = Z_V \sum_q e_q \bar{q}(z)\gamma_\mu q(z)$$

$$J^A_\nu(z) = Z_A \sum_q \bar{q}(z)\gamma_\nu\gamma_5 q(z)$$

- **2nd order mixed derivatives of the 2-pt correlator, $G^{(2)}_\lambda(\mathbf{p};t)$, in the presence of the external field**

$$\left.\frac{\partial^2 G^{(2)}_\lambda(p;t)}{\partial\lambda_1\partial\lambda_2}\right|_{\lambda=0} = \left[\frac{\partial^2 A_\lambda(p)}{\partial\lambda_1\partial\lambda_2} - tA(p)\frac{\partial^2 E_{N_\lambda}(p)}{\partial\lambda_1\partial\lambda_2}\right] e^{-E_N(p)t}$$

from spectral decomposition

from path integral

$$\left.\frac{\partial^2 G^{(2)}_\lambda(p;t)}{\partial\lambda_1\partial\lambda_2}\right|_{\lambda=0} = -t\sum_{s,s'} iA_{ss'}(p)\frac{e^{-E_N(p)t}}{E_N(p)}\left[\int d^4z\, e^{iq\cdot z}\left\langle N_s(p)\,|\,J_\mu(z)J_\nu(0)\,|\,N_{s'}(p)\right\rangle - (q\to -q) + \ldots\right]$$

- **equate the time-enhanced terms:**

$$\left.\frac{\partial^2 E^\lambda_N(p)}{\partial\lambda_1\partial\lambda_2}\right|_{\lambda=0} = \frac{i}{2E_N(p)}\left[\overbrace{\int d^4z\, e^{iq\cdot z}\left\langle N_s(p)\,|\,J_\mu(z)J_\nu(0)\,|\,N_{s'}(p)\right\rangle}^{T_{\mu\nu}(p,q)} - (q\to -q)\right]$$

Compton amplitude is related to the second-order energy shift

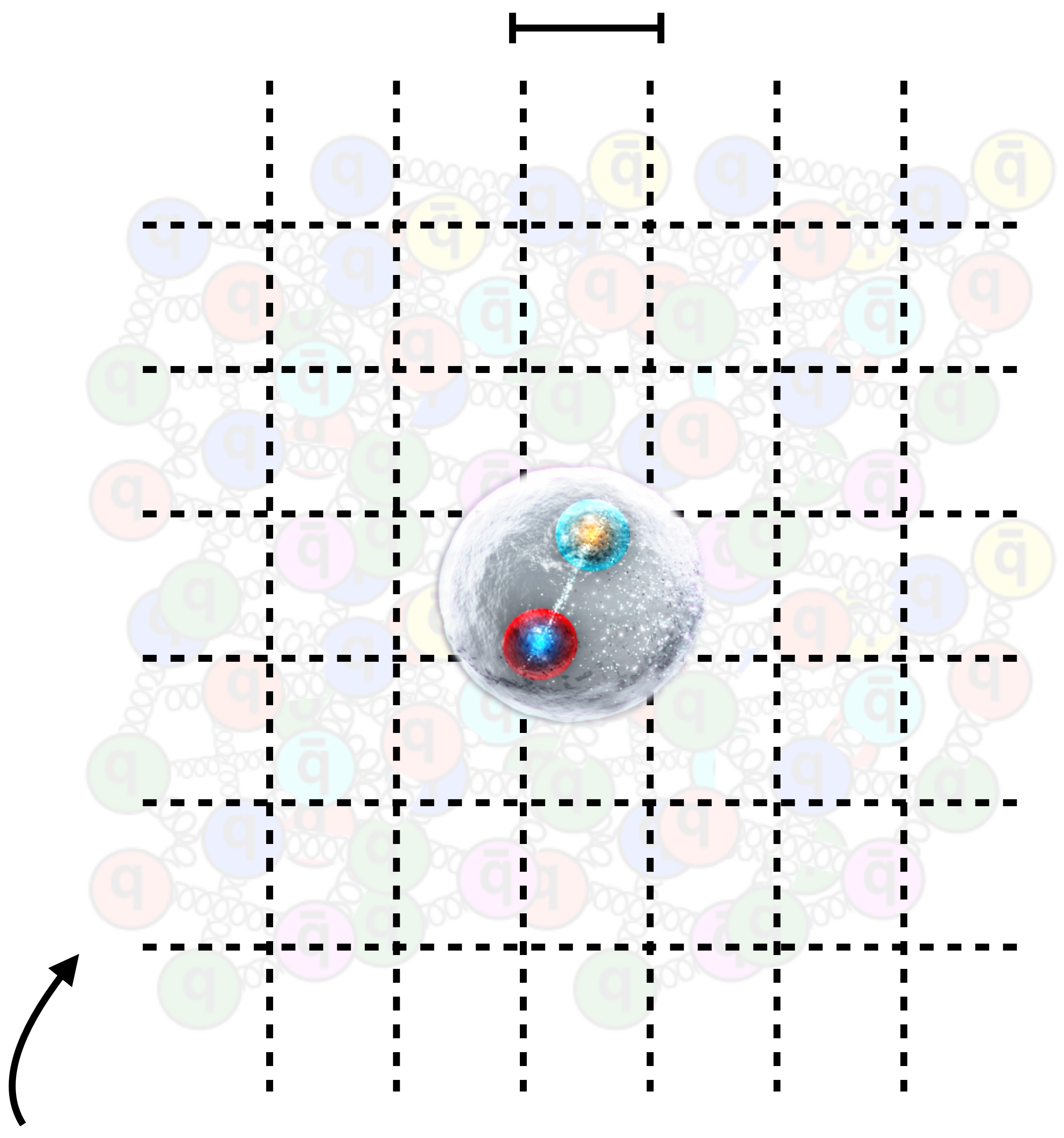
Calculating the Compton Amplitude

Simulation Details

QCDSF/UKQCD configurations
$48^3 \times 96$, 2+1 flavor (u/d+s)
$\beta = 5.65$ Symanzik improved gauge
NP-improved Clover action
Phys. Rev. D 79, 094507 (2009), arXiv:0901.3302 [hep-lat]

$m_\pi \sim 420$ MeV, SU(3) sym.

$m_\pi L \sim 6.9$ $a = 0.068\,\text{fm}$



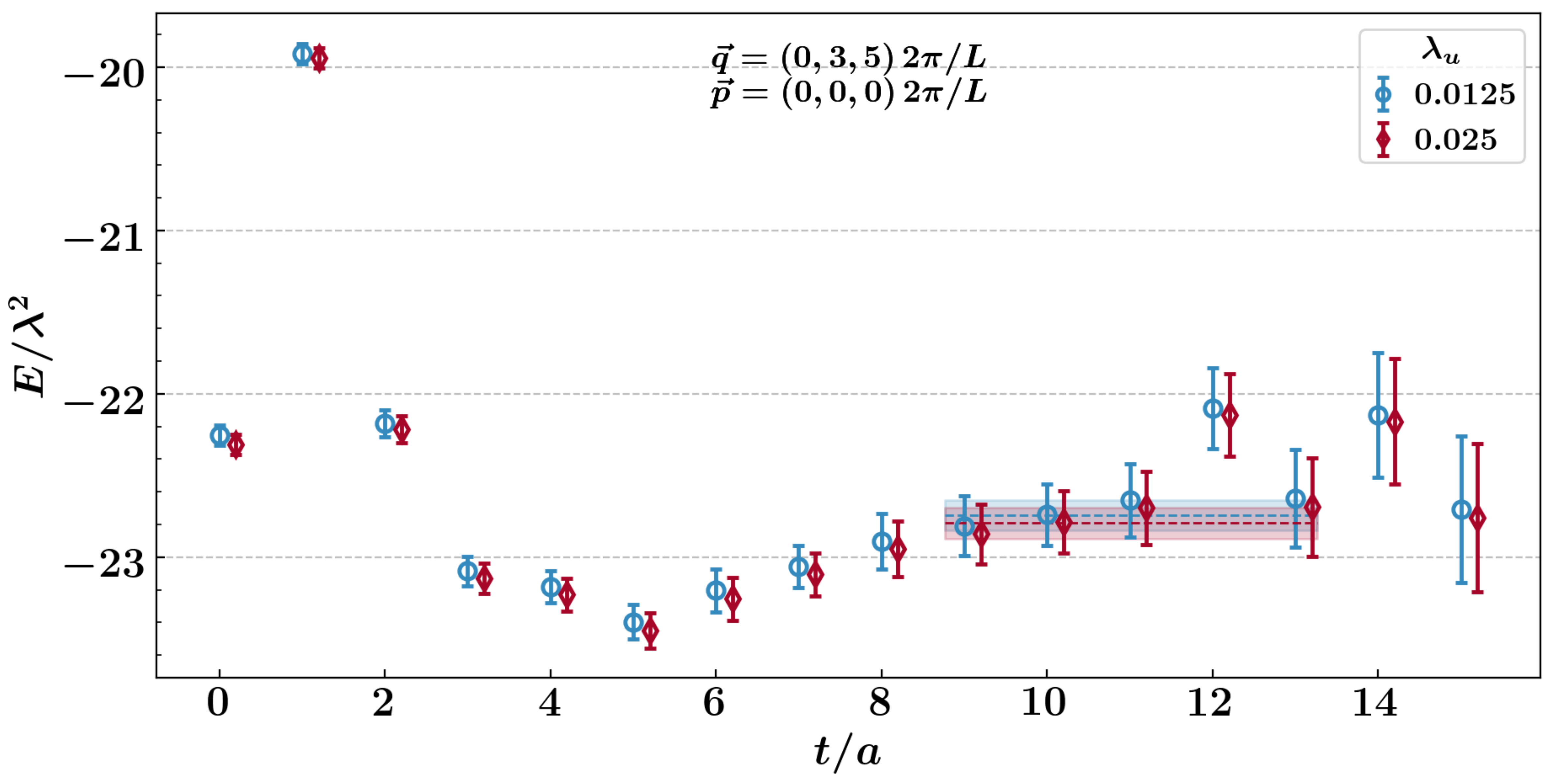
- FH implementation at the valence quark level
  - Valence u/d quark props with modified action, $S(\lambda)$
  - Local V, A current insertions, $J_\mu^{V[A]}(x) = Z_{V[A]}\bar{q}(x)\gamma_\mu[\gamma_5]q(x)$
- 4 Distinct field strengths, $\lambda = [\pm 0.0125, \pm 0.025]$
- Presently, 1 current momenta $Q^2 \sim 5\,GeV^2$
- Roughly 500 measurements
- Access to a range of $\omega = 2\,p \cdot q/Q^2$ values for several (p, q) pairs
  - An inversion for each q and λ, varying p is relatively cheap
- Connected 2-pt correlators calculated only, no disconnected

Strategy | Energy shifts

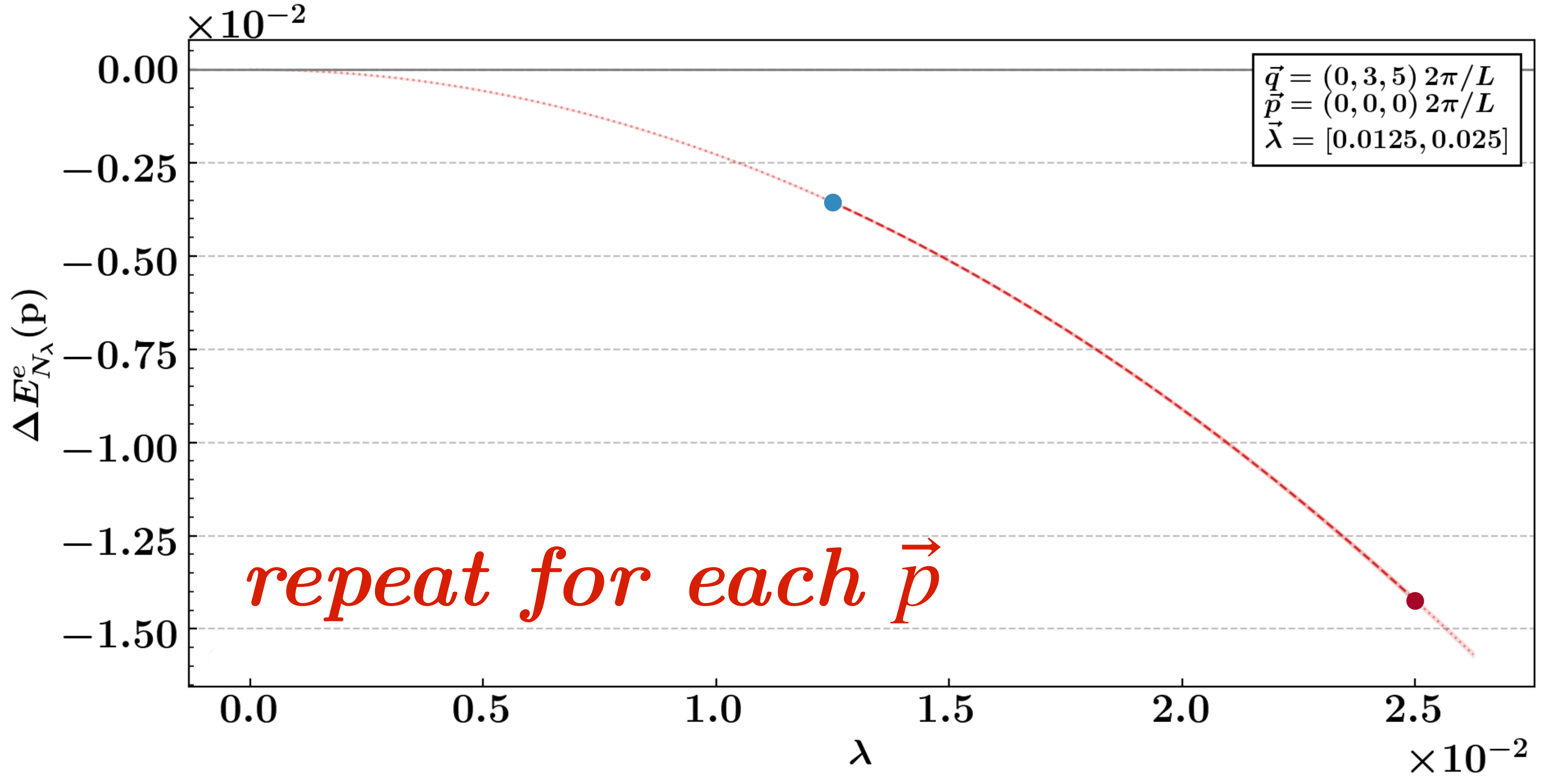
- **Ratio of perturbed to unperturbed 2-pt functions**

$$\mathcal{R}^{qq}_{\lambda}(p,t) \equiv \frac{G^{(2)}_{+\lambda_1^q,+\lambda_2^q}(p,t)\, G^{(2)}_{-\lambda_1^q,-\lambda_2^q}(p,t)}{G^{(2)}_{+\lambda_1^q,-\lambda_2^q}(p,t)\, G^{(2)}_{-\lambda_1^q,+\lambda_2^q}(p,t)} \to A_\lambda e^{-4\Delta E_{N_\lambda}(p)\,t}$$

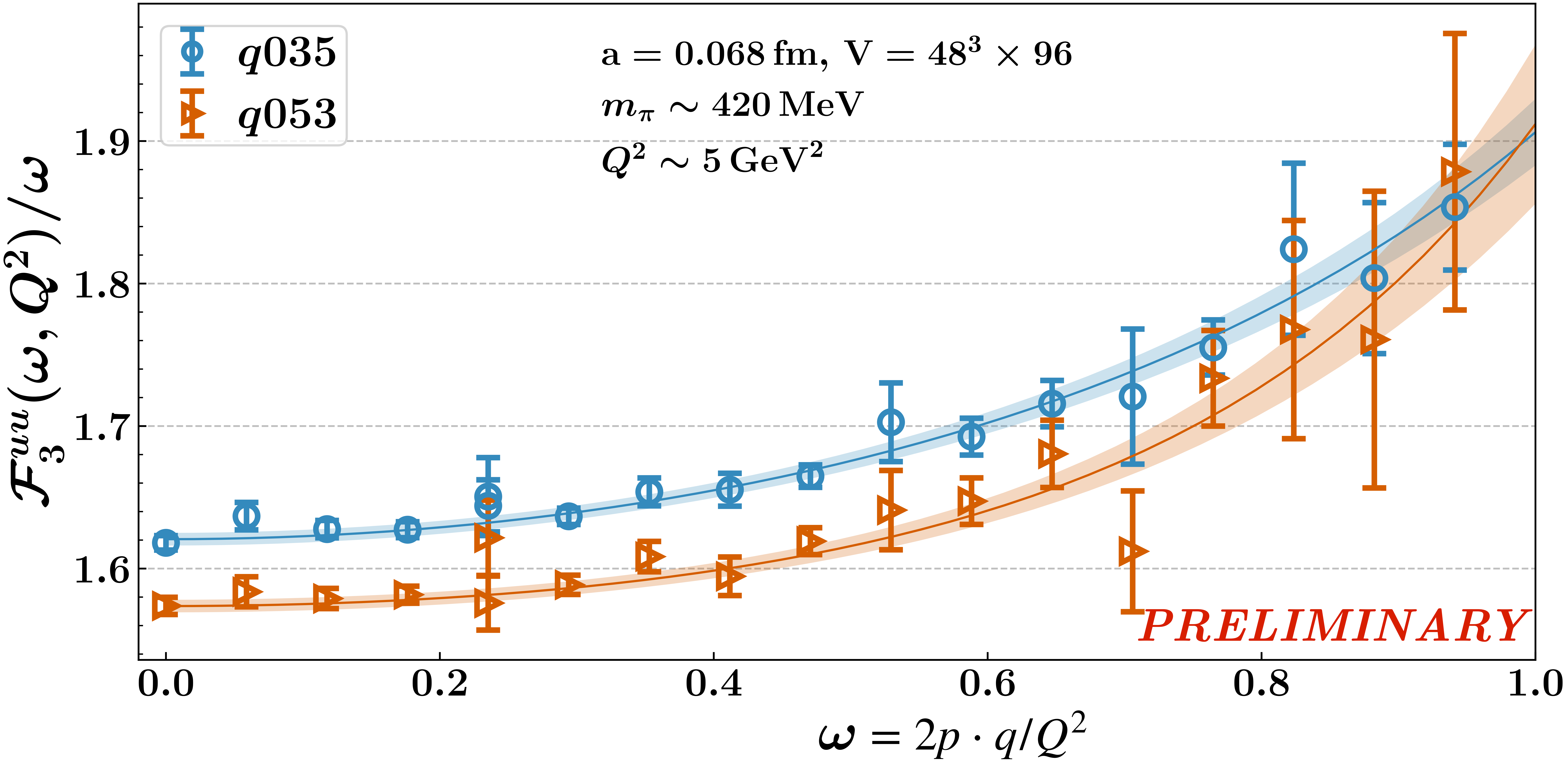
- **Extract energy shifts for each $|\lambda|$**



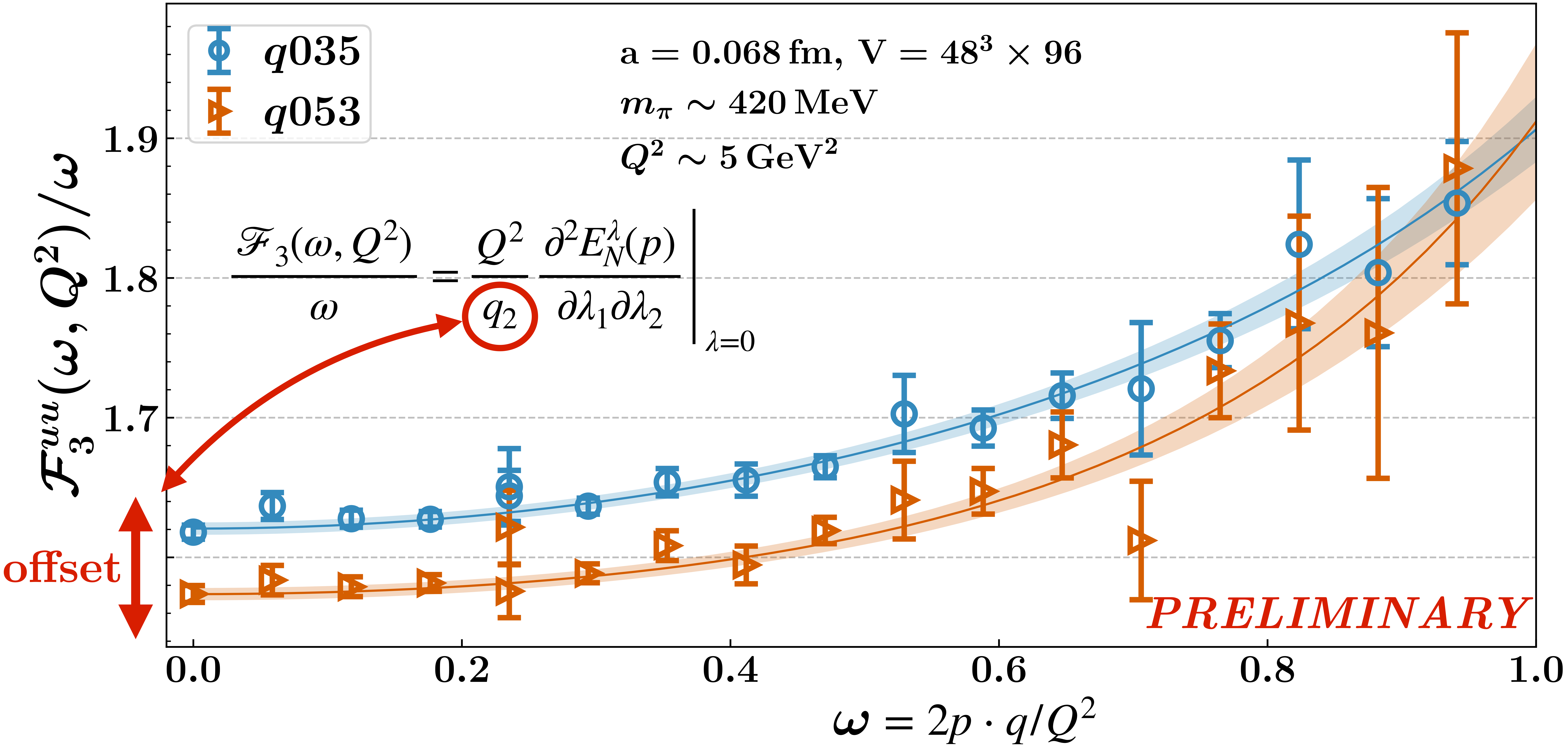
- **Get the 2nd order derivative**



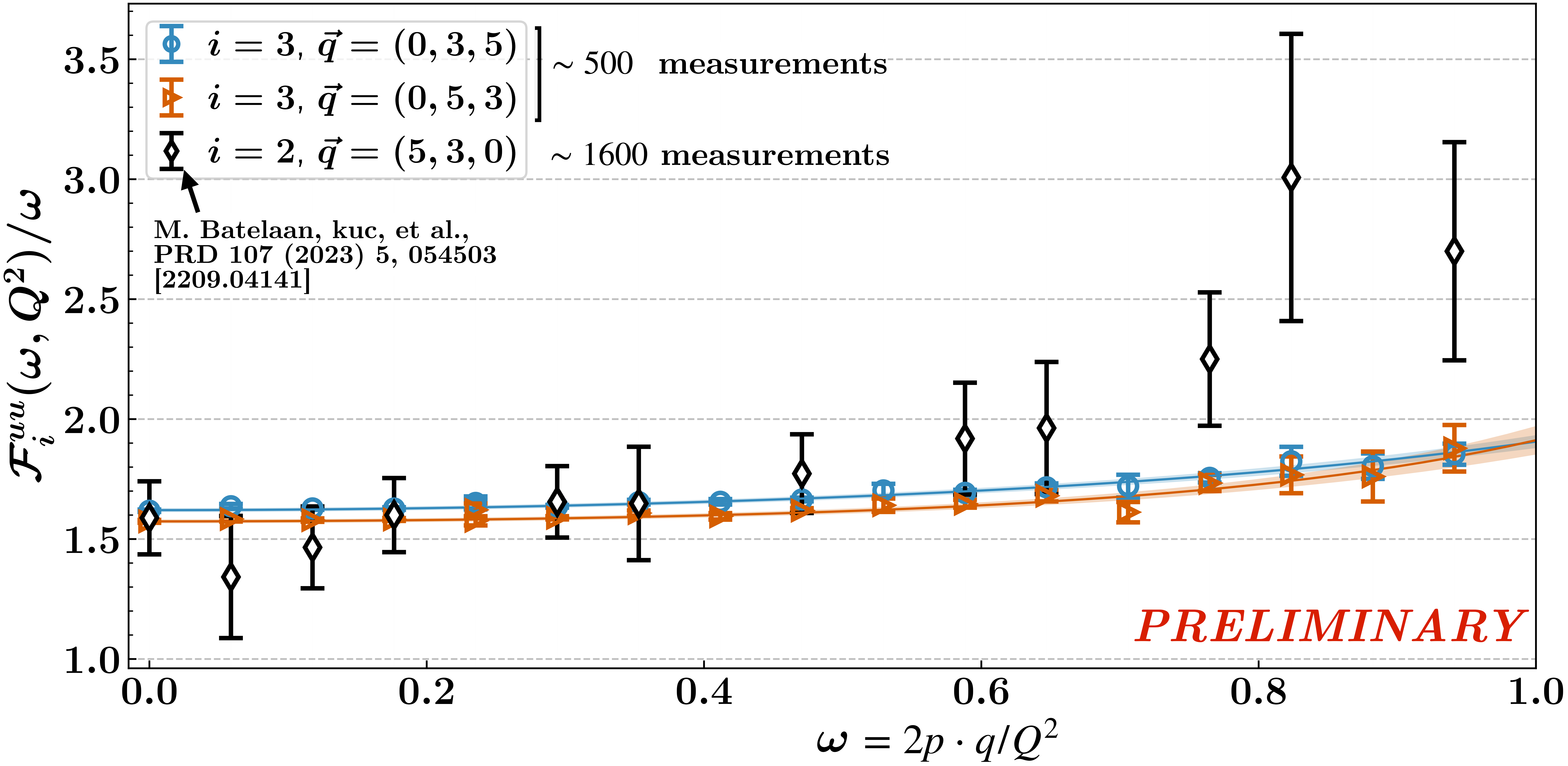
Compton Structure Functions



Compton Structure Functions

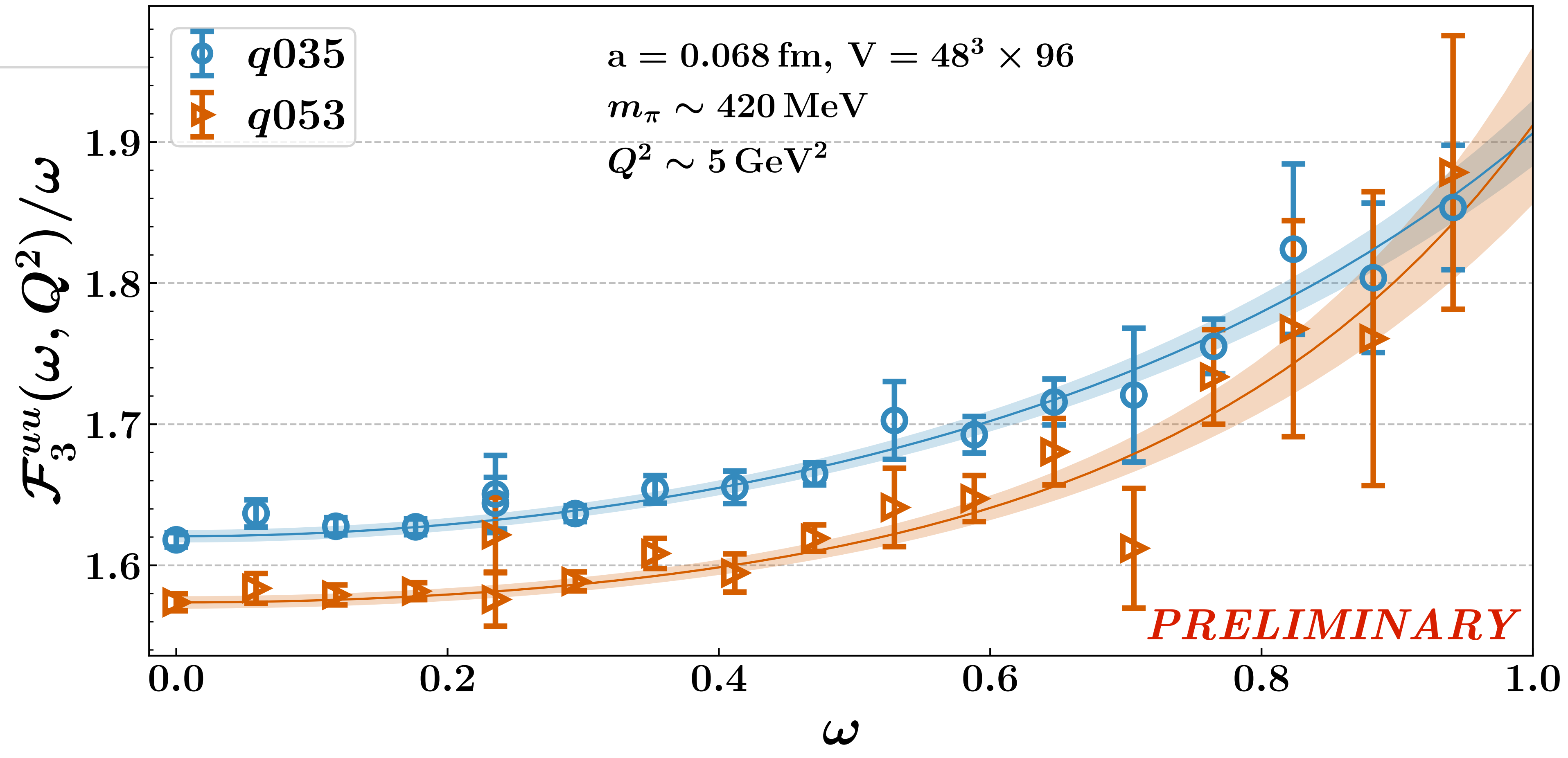


Compton Structure Functions



Summary & Outlook

- Exploratory calculation of ~~$\mathcal{F}_3(\omega, Q^2)$~~ Parity Violating $\mathcal{F}_3(\omega, Q^2)$



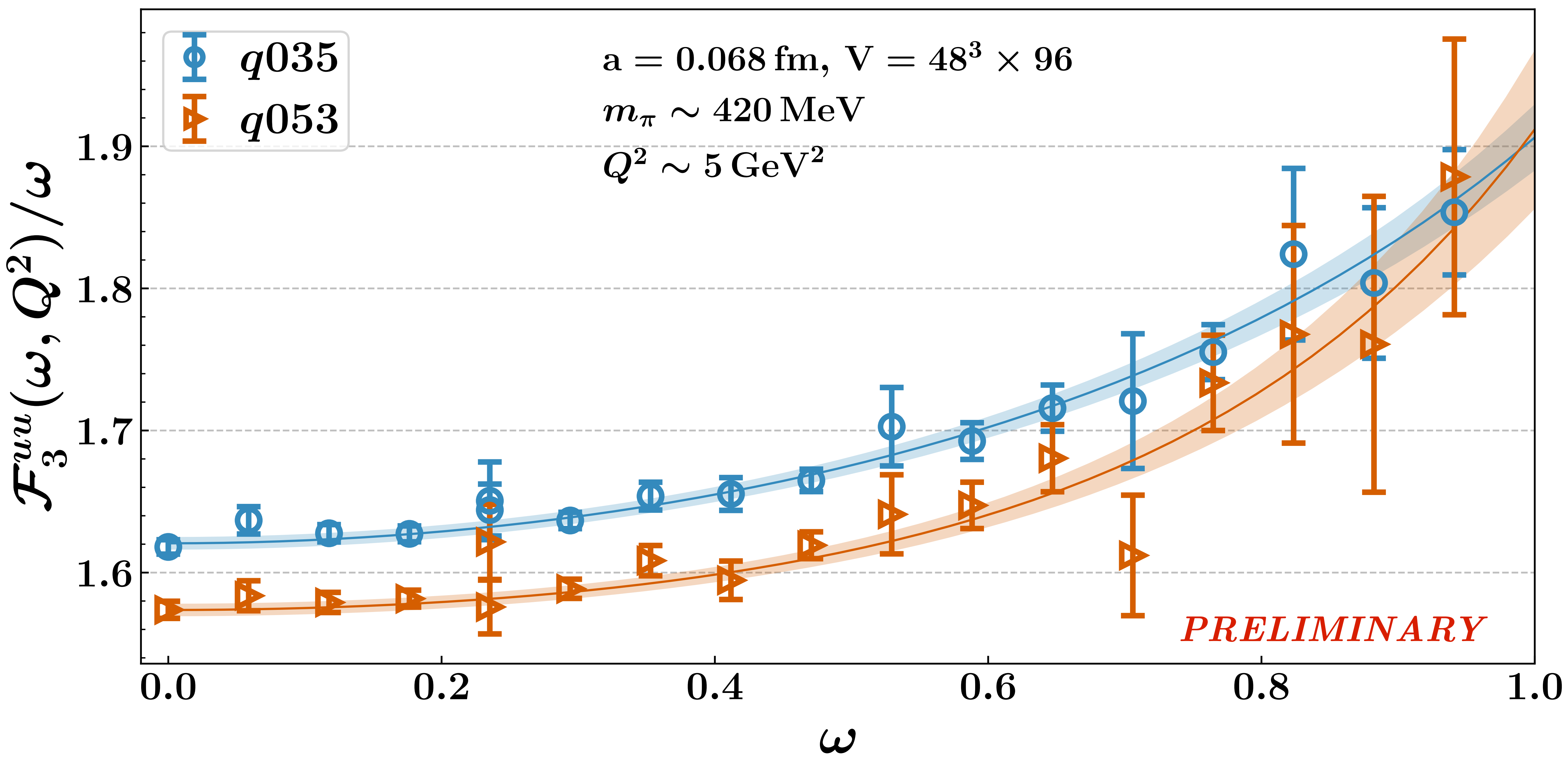
- We achieve a good statistical precision
- A good chance to study the discretisation errors

<u>Outlook</u>

- Understand the discretisation errors better
  - Need calculations on finer lattices
- Simulate new Q^2 covering low- and high-Q^2

Backup

Moments | Fit details



$$\frac{\mathcal{F}_3(\omega, Q^2)}{\omega} = \sum_{n=1,2,\dots} 4\omega^{2n-2} M_{2n-1}^{(3)}(Q^2)$$

- **Enforce monotonic decreasing of moments for uu and dd only, $|ud|^2 \leq 4uu * dd$**

$$M_1(Q^2) \geq M_3(Q^2) \geq \cdots \geq M_{2n-1}(Q^2) \geq \cdots \geq 0$$

We truncate at $n = 6$

- **Bayesian approach by MCMC method**

Sample the moments from Uniform priors
individually for u- and d-quark

$M_1(Q^2) \sim |\mathcal{N}(0,5)|$ positive half, long tail uninformative prior

$M_{2n+1}(Q^2) \sim \mathcal{U}\left(0, M_{2n-1}(Q^2)\right)$ positive, bounded from above by the previous moment

Maximise the multivariate Likelihood function, $exp(-\chi^2/2)$

$$\chi^2 = \sum_{i,j} \left[\mathcal{F}_{3,i} - \mathcal{F}_3^{obs}(\omega_i)\right] C_{ij}^{-1} \left[\mathcal{F}_{3,j} - \mathcal{F}_3^{obs}(\omega_j)\right]$$

i, j **runs through all the** ω **values of all flavour contributions**