Quark Masses and Low Energy Constants in the Continuum from the Tadpole Improved Clover Ensembles

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with
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1. Simulation setup

2. Renormalization (RI/MOM and SMOM)

3. Global fit using chiral perturbation theory ($\chi$PT)

4. Summery
We now have 11 ensembles:
3 lattice spacings \( a \in [0.05, 0.11] \) fm,
7 pion masses \( m_\pi \in [135, 350] \) MeV,
3 spatial sizes \( L \in [2.5, 5.1] \) fm

<table>
<thead>
<tr>
<th>name</th>
<th>( \beta )</th>
<th>Lattice spacing ( a )</th>
<th>Volume</th>
<th>( L )</th>
<th>( m_\pi L )</th>
<th>( \pi ) mass</th>
<th>( \eta_s ) mass</th>
<th>( \eta_{\text{conf}} )</th>
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</thead>
<tbody>
<tr>
<td>C24P34</td>
<td>6.20</td>
<td>0.1053fm</td>
<td>( 24^3 \times 64 )</td>
<td>2.6fm</td>
<td>4.38</td>
<td>340MeV</td>
<td>748MeV</td>
<td>200</td>
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<td>C24P29</td>
<td>6.20</td>
<td>0.1053fm</td>
<td>( 24^3 \times 72 )</td>
<td>2.6fm</td>
<td>3.75</td>
<td>292MeV</td>
<td>658MeV</td>
<td>476</td>
</tr>
<tr>
<td>C32P29</td>
<td>6.20</td>
<td>0.1053fm</td>
<td>( 32^3 \times 64 )</td>
<td>3.5fm</td>
<td>5.01</td>
<td>292MeV</td>
<td>658MeV</td>
<td>198</td>
</tr>
<tr>
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<td>( 32^3 \times 64 )</td>
<td>3.5fm</td>
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<td>228MeV</td>
<td>643MeV</td>
<td>400</td>
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<td>( 48^3 \times 64 )</td>
<td>5.4fm</td>
<td>5.79</td>
<td>225MeV</td>
<td>643MeV</td>
<td>62</td>
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<td>( 48^3 \times 96 )</td>
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<td>135MeV</td>
<td>706MeV</td>
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<td>0.0775fm</td>
<td>( 32^3 \times 96 )</td>
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<td>3.81</td>
<td>303MeV</td>
<td>681MeV</td>
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<tr>
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<td>( 48^3 \times 96 )</td>
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<td>210MeV</td>
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<td>( 48^3 \times 96 )</td>
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<td>3.91</td>
<td>207MeV</td>
<td>667MeV</td>
<td>98</td>
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<tr>
<td>H48P32</td>
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<td>0.0519fm</td>
<td>( 48^3 \times 144 )</td>
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<td>4.06</td>
<td>321MeV</td>
<td>709MeV</td>
<td>98</td>
</tr>
</tbody>
</table>

We also used these ensembles:
Lattice action (tadpole improved fermion action with stout smearing)

In the generation of our 2+1 flavor full QCD ensembles, we employ the **tadpole improved** Symanzik gauge action, and **Clover** fermion action

\[
S_g = \frac{1}{N_c} \sum_x \text{Re} \sum_{x, \mu < \nu} \text{Tr} \left[ 1 - \hat{\beta} \left( P_{\mu, \nu}^U(x) - \frac{c_1 R_{\mu, \nu}^U(x)}{1 + 8c_1^0} \right) \right]
\]

\[
S_q = \bar{\psi}(x)\psi(x) - \frac{\kappa}{v_0} \sum_\mu [\bar{\psi}(x)(1 + \gamma_\mu) V_\mu \psi(x) + \bar{\psi}(x)(1 - \gamma_\mu) V_\mu^+ \psi(x)]
\]

\[\text{where } \hat{\beta} = 10/(g_0^2 u_0^4) \quad \text{with } c_1^0 = -\frac{1}{12}, c_1 = \frac{c_1^0}{u_0^2}\]

\[2\kappa = 1/(m + 4) \quad \text{tree level } c_{SW} = \frac{1}{v_0^3}\]

- Similar to the JLAB setup but using different beta
- Computationally cheap and simpler to implement
- Explicitly breaks chiral symmetry at non-zero \(a\), which leads to additive renormalization of the quark masses.

\(R_{\mu, \nu}^U\) reduces the discretization error from \(O(a^2)\) to \(O(a^4)\)

- tree level **Clover** term is expected to reduces the discretization error from \(O(a)\) to \(O(a^2)\), but may still have residual \(O(a)\) effect

- Stout smeared link \(V\) with smearing parameter \(\rho = 0.125\)
- \(u_0\) is the tadpole improvement factor
- \(v_0\) is similar to \(u_0\), but with the smeared link variable
- Both are determined self-consistently
Quark mass

The bare quark mass may be negative due to explicit chiral symmetry breaking. One solution is to use the PCAC mass:

\[ 2 \tilde{m}_q^{\text{PC}} = \frac{\langle 0|\nabla V_4 A_4|\text{PS} \rangle}{\langle 0|P|\text{PS} \rangle}, \]

where \( \tilde{m}_q \) is positive and appears to be linear in \( \tilde{m}_q^b \):

\[ \tilde{m}_q^{\text{PC}} = k_m \left( \tilde{m}_q^b - \tilde{m}_{crti} \right) \]
Joint fit of 2-Point functions

Based on the **PCAC** relation, the definition of decay constant
\[ \langle 0|A_4|PS \rangle = f_{PS} m_{PS} \]
and the form of 2pt, we do joint fit of the data obtained by evaluating 2pt in simulations and obtain dimensionless \( \tilde{m}_q^{PC}, \tilde{f}_{PS} \) and \( \tilde{m}_{PS} \)

\[ n_{\text{src}}=48 \text{ of } 96 \text{ times slices on } n_{\text{cfg}}=203 \text{ configurations} \]
Scale setting by gradient flow

The Wilson flow scale $w_0$ is a quantity with the dimension of length:

$$ t \frac{d}{dt} \left( t^2 \langle E(t) \rangle \right) \bigg|_{t=w_0^2} = 0.3 $$

We use $w_0 = 0.1736(9)$ fm

$E$ is the discretized Yang-Mills action density

$$ E = \frac{1}{2} \text{tr}(F_{\mu\nu}F_{\mu\nu}) $$

$F_{\mu\nu}$ is the field strength tensor.

[\text{JHEP08(2010)071}, \text{JHEP09(2012)010},]
RI/MOM and RI/SMOM scheme renormalization

\[ Z_V = \lim_{m_R \to 0} \frac{\langle \pi | \pi \rangle}{\langle \pi | V_4 | \pi \rangle} \]  
\[ Z_\omega^\omega = \lim_{m_R \to 0} \frac{Z_V \text{Tr}[\Lambda^\mu_V(p_1, p_2) \gamma_\mu]}{\text{Tr}[\Lambda_0(p_1, p_2) \Lambda^{\text{tree}}_0(p_1, p_2)]} \]  
\[ p_1^2 = p_2^2 = \mu^2 \]  
\[ (p_1 - p_2)^2 = \omega \mu^2 \]

\( \Lambda_0(p_1, p_2) \) is the amputated Green function can be calculated by

\[ \Lambda_0(p_1, p_2) = S(p_1)^{-1} \sum_{x, y} e^{-i(p_1 \cdot x - p_2 \cdot y)} \langle \bar{\psi}(x) O(0) \psi(y) \rangle S(p_2)^{-1} \]

1. Chiral extrapolation
2. Convert to \( \overline{\text{MS}} \) and run to 2 GeV
3. \( a^2 \mu^2 \) extrapolation to suppress discretization error
RI/MOM vs. RI/SMOM

- In SMOM the chiral symmetry breaking effects are smaller
- At smaller lattice spacings breakings are suppressed
- $O(\alpha s)$ term is necessary to restore chiral symmetry

- Differ by ~1% after a linear $a^2$ continuum extrapolation
- MOM discretization 25% smaller

We choose the RI/MOM scheme in our work, for more reliable after continuum extrapolation.
RI/MOM vs. RI/SMOM

\[ g_{S,\pi}^{FH} = \frac{1}{2} \frac{\partial m_\pi}{\partial m_q^R} \approx 4.06(4) \]

Breaking of the Feynman-Hellman theorem \( g_{S,\pi}^{FH} = g_{S,\pi}^R \) is about 7(3)%
Global fit

After the procedures of joint fit, scale setting and renormalization, we obtain $m_q$, $m_\pi$, and $f_\pi$ across 11 ensembles, we can conduct a joint fit using $\chi$PT:

$$m_{\pi, vv}^2 = \Lambda^2 \chi \frac{1}{2} 2y_v \left[ 1 + \frac{2}{N_f} \left[ (2y_v - y_s)\ln(2y_v) + (y_v - y_s) \right] + 2y_v(2\alpha_8 - \alpha_5) + 2y_sN_f(2\alpha_6 - \alpha_4) \right] (1 + c_m a^2 + c_m L e^{-m\pi L})$$

$$F_{\pi, vv} = F \left( 1 - \frac{N_f}{2} \left[ (y_v + y_s)\ln(y_v + y_s) + y_v\alpha_5 + y_sN_f\alpha_4 \right] \right) (1 + c_f a^2 + c_f L e^{-m\pi L})$$

- $y_v$ and $y_s$ are variables defined by $m_q$
- $F$ is pion decay constant in the chiral limit
- $\alpha_i$ are NLO low energy constants

requiring $y_s = y_v$, $m_{\pi, vv} = m_{\pi, phys} = 134.98\text{MeV}$, $a \to 0$, and $L \to \infty$
Global fit results

2 $\sigma$ away from FLAG average
Summery and outlook

- Results agree with the FLAG average within $2 \sigma$, and the non-perturbative renormalization is the main source of the quark mass error.
- Further investigation of renormalization, plan to use the volume source to suppress the uncertainty.
- More ensembles at more lattice spacing and pion masses to improve the reliability of our chiral and continuum global fit.

Thank you for listening!

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