Quark Masses and Low Energy Constants in the Continuum from the Tadpole Improved Clover Ensembles

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with

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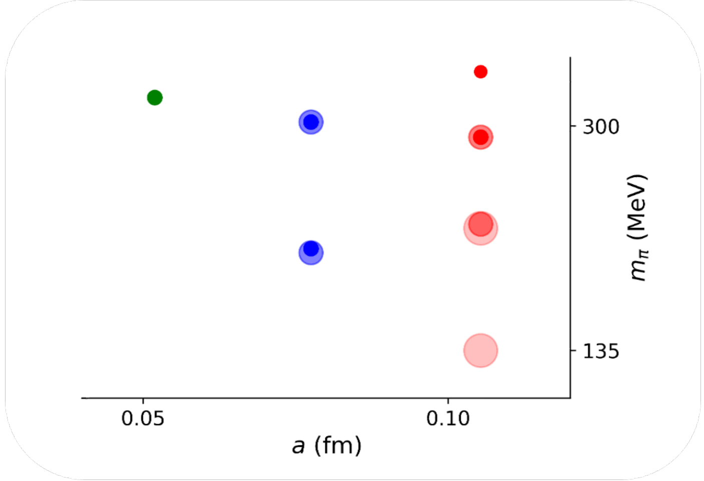
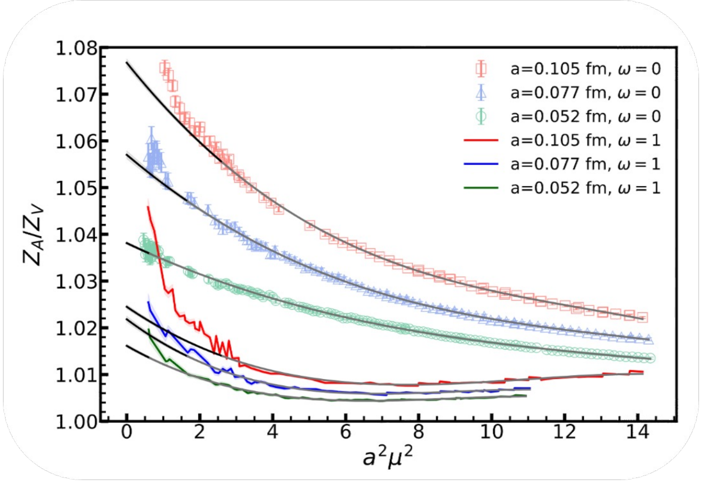
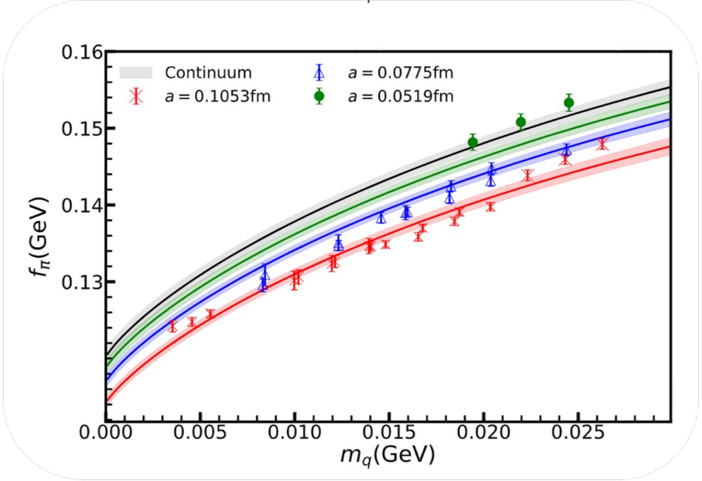
Lattice 2023 July/31/2023



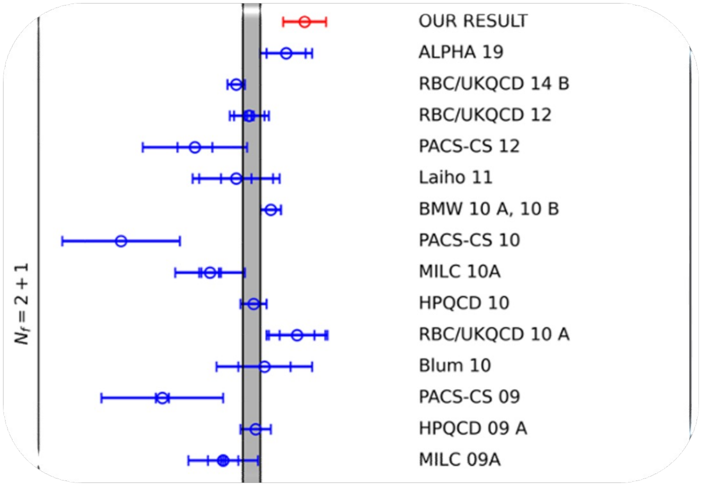
1.Simulation setup



2.Renormalization (RI/MOM and SMOM)



3.Global fit using chiral perturbation theory (χPT)

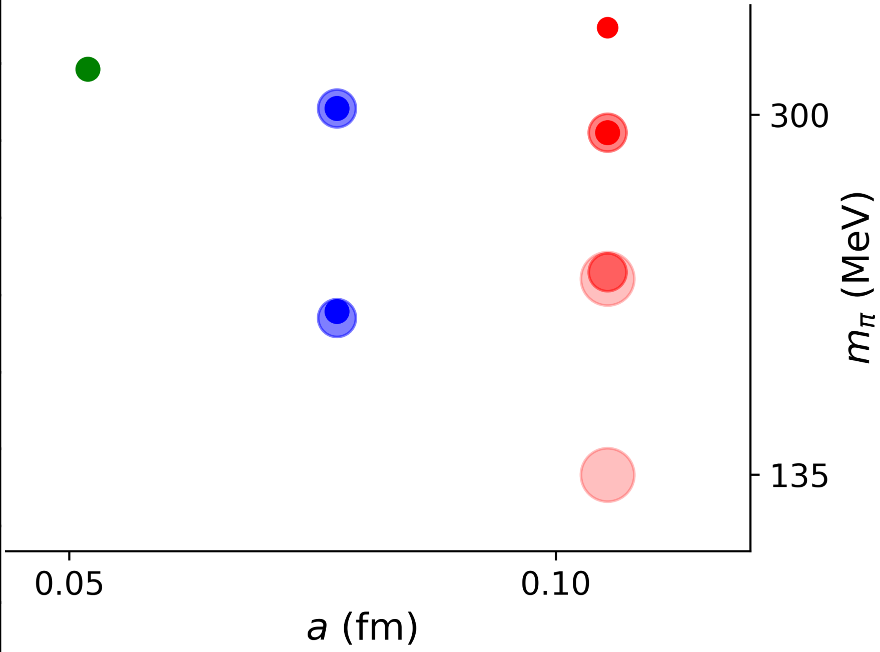


4.Summery

Ensembles used in this work

name	β	Lattice spacing a	Volume	L	m_πL	π mass	η_s mass	n_{conf}
C24P34	6.20	0.1053fm	$24^3\times64$	2.6fm	4.38	340MeV	748MeV	200
C24P29	6.20	0.1053fm	$24^3\times72$	2.6fm	3.75	292MeV	658MeV	476
C32P29	6.20	0.1053fm	$32^3\times64$	3.5fm	5.01	292MeV	658MeV	198
C32P23	6.20	0.1053fm	$32^3\times64$	3.5fm	3.91	228MeV	643MeV	400
C48P23	6.20	0.1053fm	$48^3\times96$	5.4fm	5.79	225MeV	643MeV	62
C48P14	6.20	0.1053fm	$48^3\times96$	5.4fm	3.56	135MeV	706MeV	203
F32P30	6.41	0.0775fm	$32^3\times96$	2.6fm	3.81	303MeV	681MeV	206
F48P30	6.41	0.0775fm	$48^3\times96$	3.8fm	5.72	303MeV	679MeV	99
F32P21	6.41	0.0775fm	$32^3\times64$	2.6fm	2.67	210MeV	665MeV	194
F48P21	6.41	0.0775fm	$48^3\times96$	3.8fm	3.91	207MeV	667MeV	98
H48P32	6.72	0.0519fm	$48^3\times144$	2.6fm	4.06	321MeV	709MeV	98

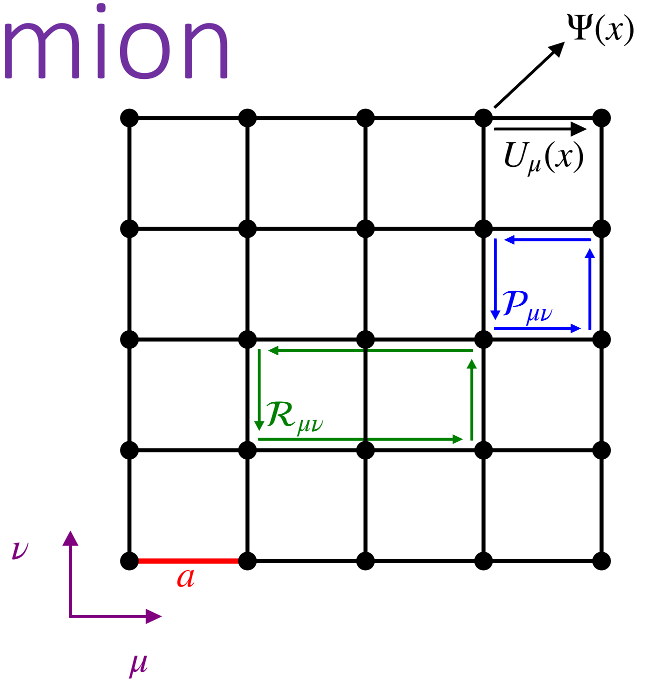
We now have 11 ensembles:
3 lattice spacings $a \in [0.05,0.11]$ fm,
7 pion masses $m_\pi \in [135,350]$ MeV,
3 spatial sizes $L \in [2.5,5.1]$ fm



Article used these ensembles:
[Chin.Phys.C 46 (2022) 1, 011002,
arXiv: 2207.00183,arXiv: 2207.14132,
Phys.Lett.B 841 (2023) 137941]

CLQCD

Lattice action (tadpole improved fermion action with stout smearing)



In the generation of our **2+1 flavor full QCD** ensembles, we employ the **tadpole improved** Symanzik gauge action, and Clover fermion action

$$S_g = \frac{1}{N_c}\sum_x \mathrm{Re}\sum_{x,\mu<\nu}\mathrm{Tr}\left[1-\hat{\beta}\left(\mathcal{P}^U_{\mu,\nu}(x) - \frac{c_1\mathcal{R}^U_{\mu,\nu}(x)}{1+8c_1^0}\right)\right]$$

$$S_q = \bar{\psi}(x)\psi(x) - \frac{\kappa}{v_0}\sum_\mu\left[\bar{\psi}(x)(1+\gamma_\mu)V_\mu\psi(x) + \bar{\psi}(x)(1-\gamma_\mu)V_\mu^\dagger\psi(x)\right] - \frac{1}{2}\frac{\kappa}{v_0}c_{SW}\bar{\psi}(x)\sigma_{\mu\nu}F_{\mu\nu}\psi(x)$$

where $\hat{\beta} = 10/(g_0^2 u_0^4)$ with $c_1^0 = -\frac{1}{12}, c_1 = \frac{c_1^0}{u_0^2}$

$2\kappa = 1/(m+4)$ tree level $c_{SW} = \frac{1}{v_0^3}$

- Similar to the JLAB setup but using different beta
- Computationally cheap and simpler to implement
- Explicitly breaks chiral symmetry at non-zero a, which leads to additive renormalization of the quark masses.

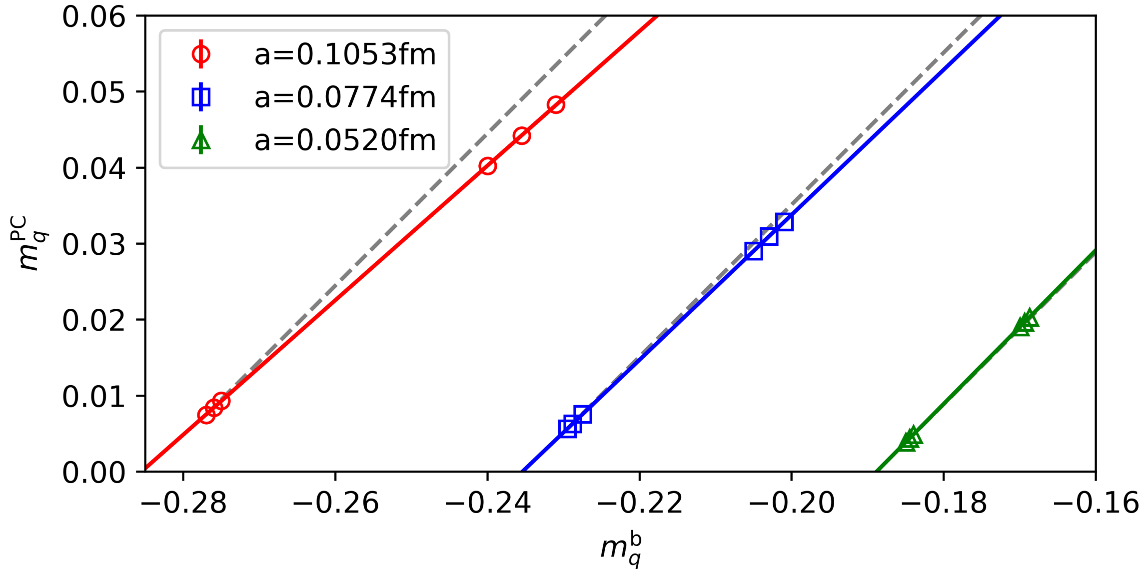
- $\mathcal{R}^U_{\mu,\nu}$ reduces the discretization error from $O(a^2)$ to $O(a^4)$
- tree level Clover term is expected to reduces the discretization error from $O(a)$ to $O(a^2)$, but may still have residual $O(a)$ effect

- Stout smeared link V with smearing parameter $\rho = 0.125$
- u_0 is the tadpole improvement factor
- v_0 is similar to u_0, but with the smeard link variable
- Both are determined self-consistently

Quark mass

The bare quark mass may be negative due to explicit chiral symmetry breaking.

one solution is to use the **PCAC** mass $2\tilde{m}_q^{\mathrm{PC}} = \frac{\langle 0|\nabla_4 A_4|\mathrm{PS}\rangle}{\langle 0|P|\mathrm{PS}\rangle}$,



$$\tilde{m}^{\mathrm{PC}} = k_m\left(\tilde{m}^b - \tilde{m}_{\mathrm{crti}}\right)$$

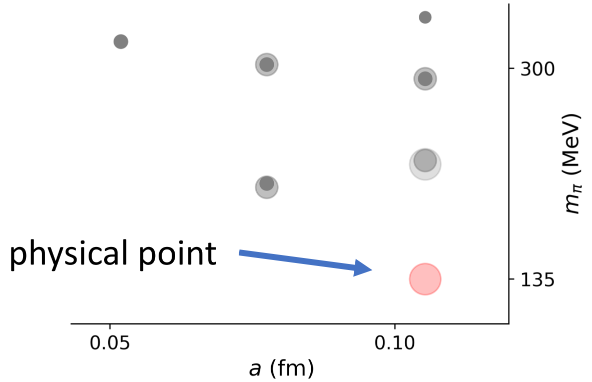
$\tilde{m}_q^{\mathrm{PC}}$ is positive and appears to be linear in $\tilde{m}_q^b$

Joint fit of 2-Point functions

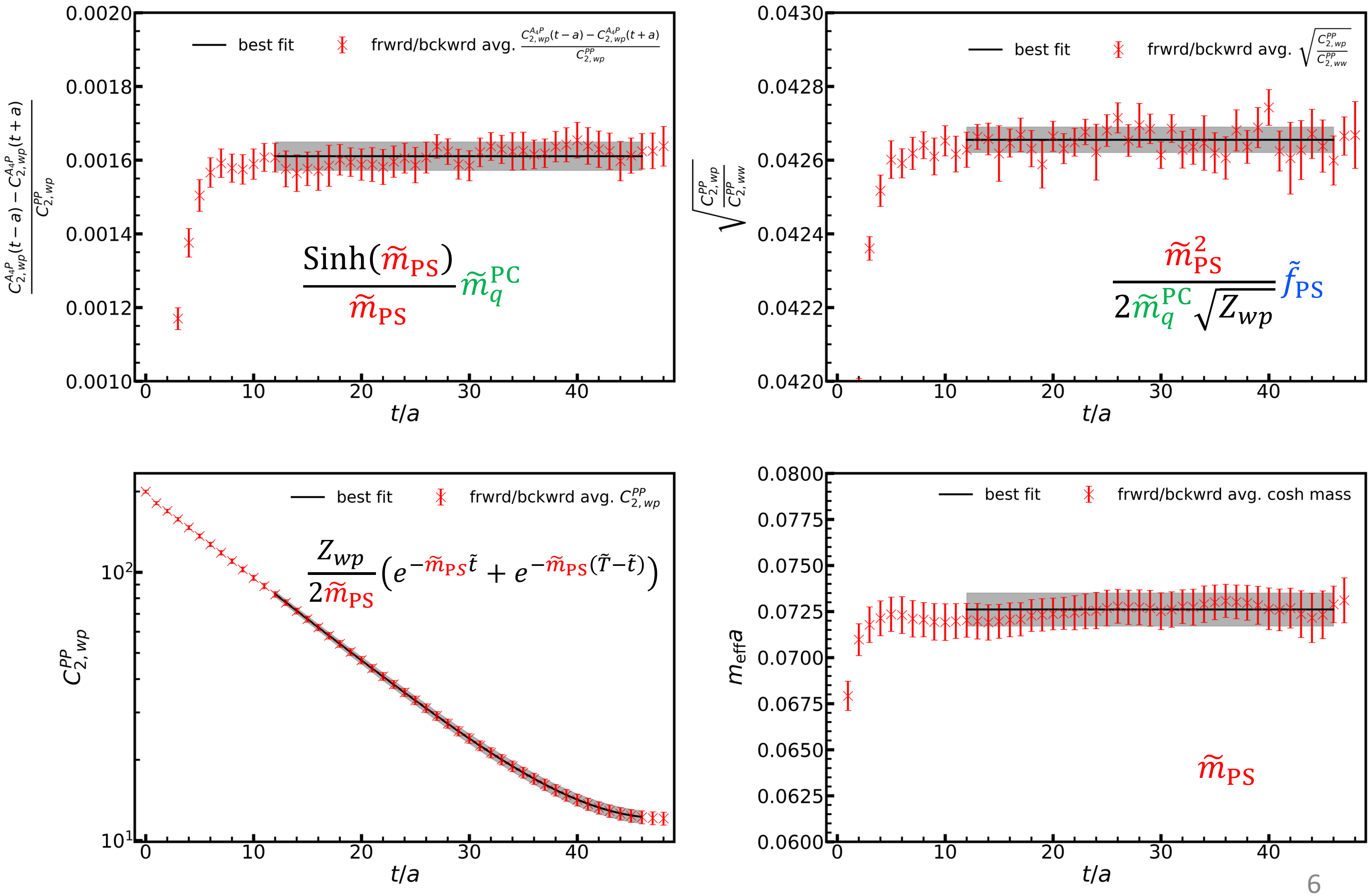
Based on the **PCAC** relation, the definition of decay constant

$$\langle 0|A_4|\mathrm{PS}\rangle = f_{\mathrm{PS}} m_{\mathrm{PS}}$$

and the form of 2pt, we do joint fit of the data obtained by evaluating 2pt in simulations and obtain dimensionless $\tilde{m}_q^{\mathrm{PC}}$, $\tilde{f}_{\mathrm{PS}}$ and $\tilde{m}_{\mathrm{PS}}$



n_{src}=48 of 96 times slices on n_{cfg}=203 configurations



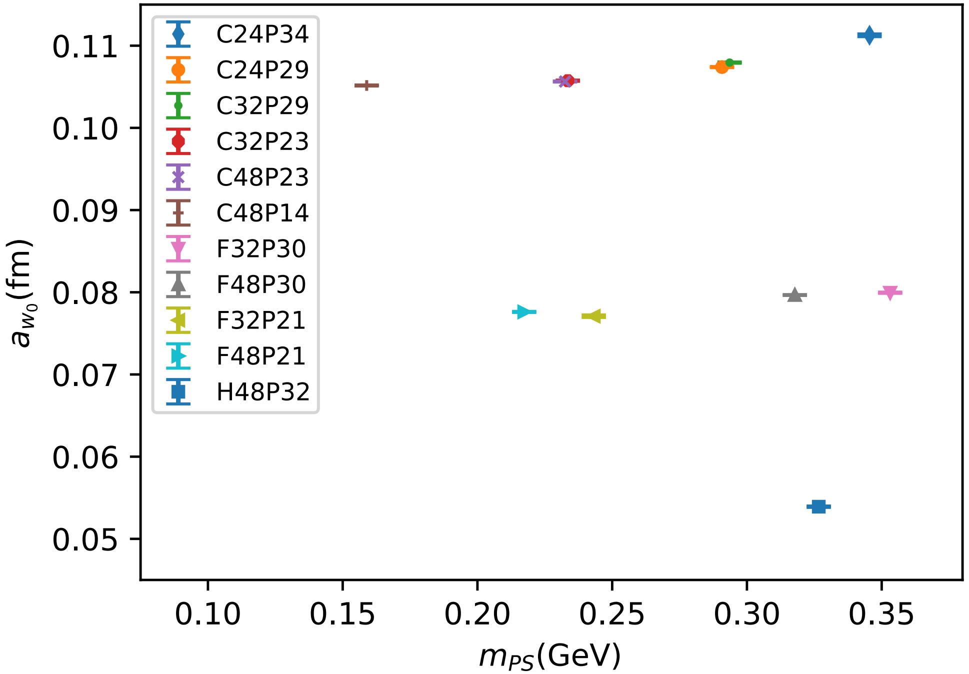
Scale setting by gradient flow

The Wilson flow scale w_0 is a quantity with the dimension of length :

$$t\frac{d}{dt}\left(t^2\langle E(t)\rangle\right)\Big|_{t=w_0^2} = 0.3$$

We use $w_0 = 0.1736(9)$ fm

E is the discretized Yang-Mills action density $E = \frac{1}{2}\mathrm{tr}\left(F_{\mu\nu}F_{\mu\nu}\right)$



[JHEP08(2010)071,
JHEP09(2012)010,]

RI/MOM and RI/SMOM scheme renormalization

$$Z_V = \lim_{m_R \to 0} \frac{\langle \pi | \pi \rangle}{\langle \pi | V_4 | \pi \rangle}$$ current conservation

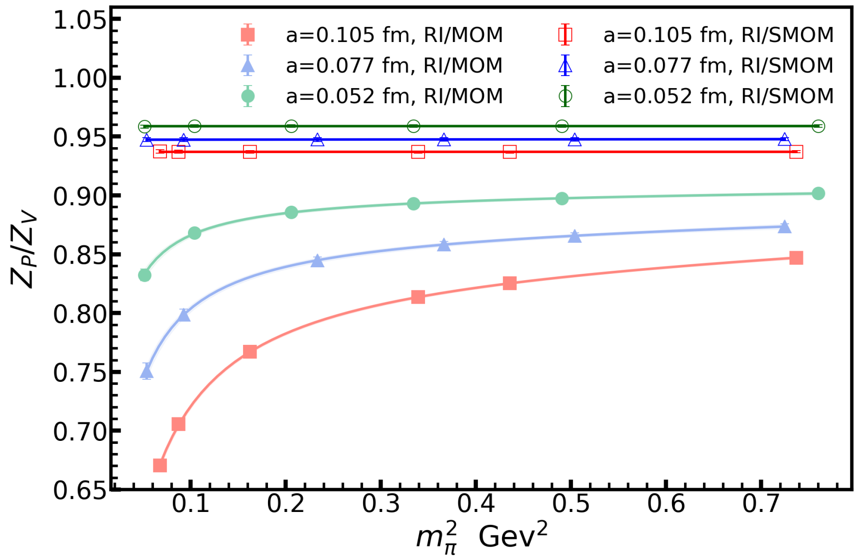
RI/MOM : $\omega = 0$
RI/SMOM : $\omega = 1$

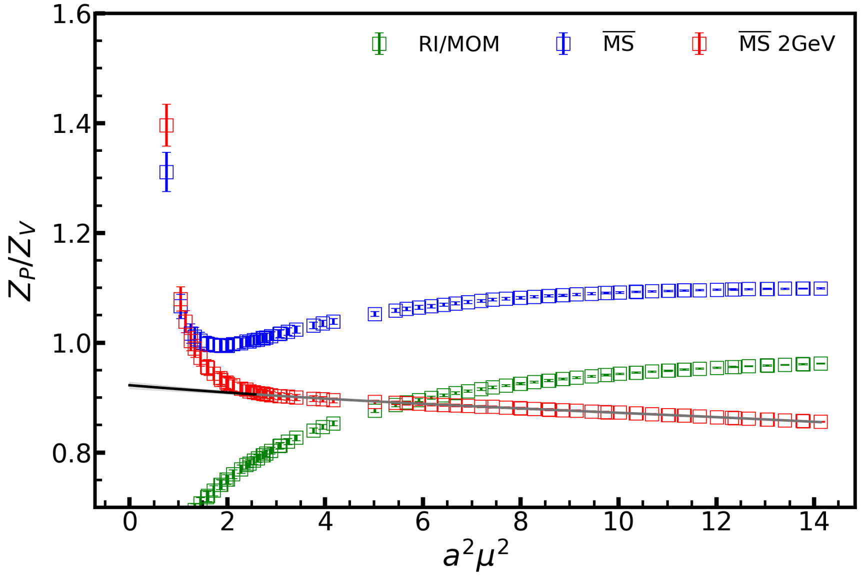
$$Z_{\mathcal{O}}^{\omega} = \lim_{m_R \to 0} \frac{Z_V \mathrm{Tr}[\Lambda_V^{\mu}(p_1, p_2)\gamma_\mu]}{\mathrm{Tr}[\Lambda_{\mathcal{O}}(p_1, p_2)\Lambda_{\mathcal{O}}^{tree}(p_1, p_2)]} \Bigg|_{\substack{p_1^2 = p_2^2 = \mu^2 \\ (p_1 - p_2)^2 = \omega\mu^2}}$$

$\Lambda_{\mathcal{O}}(p_1, p_2)$ is the amputated Green function can be calculated by

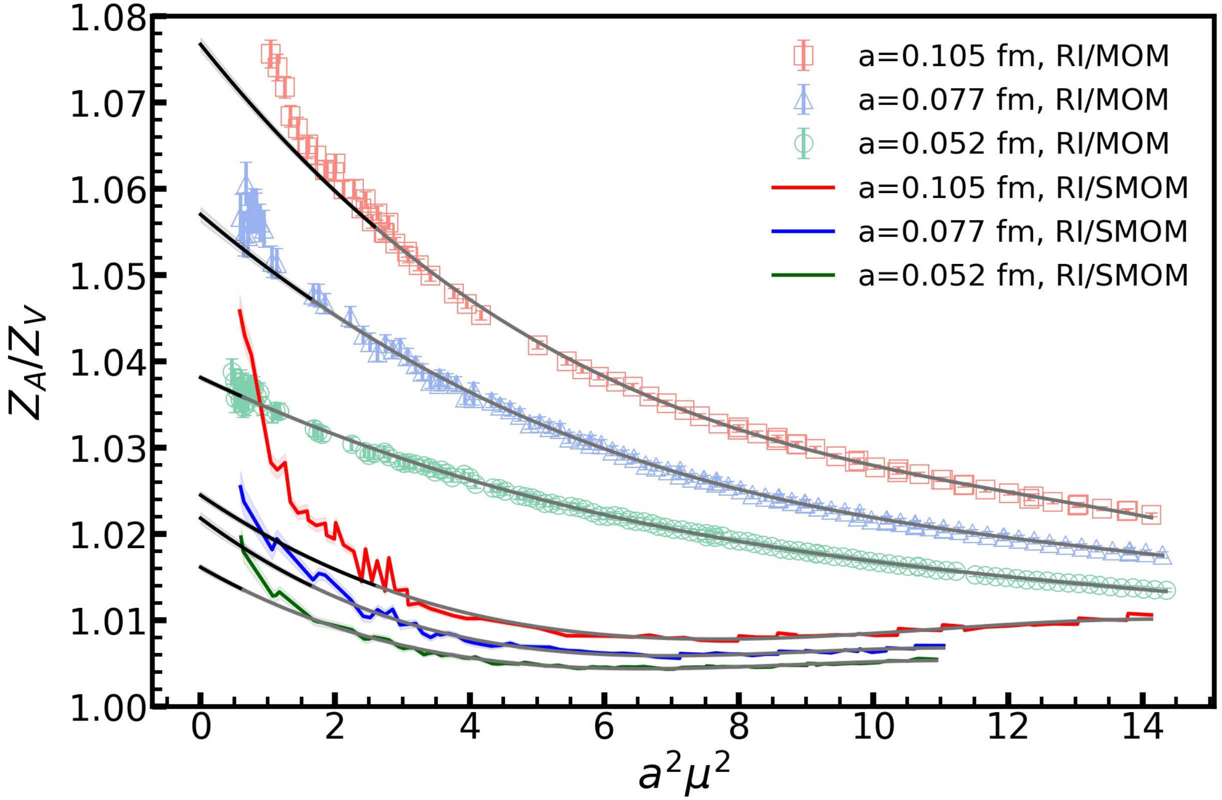
$$\Lambda_{\mathcal{O}}(p_1, p_2) = S(p_1)^{-1} \sum_{x,y} e^{-i(p_1 \cdot x - p_2 \cdot y)} \langle \bar{\psi}(x)\, \mathcal{O}(0) \psi(y) \rangle\, S(p_2)^{-1}$$

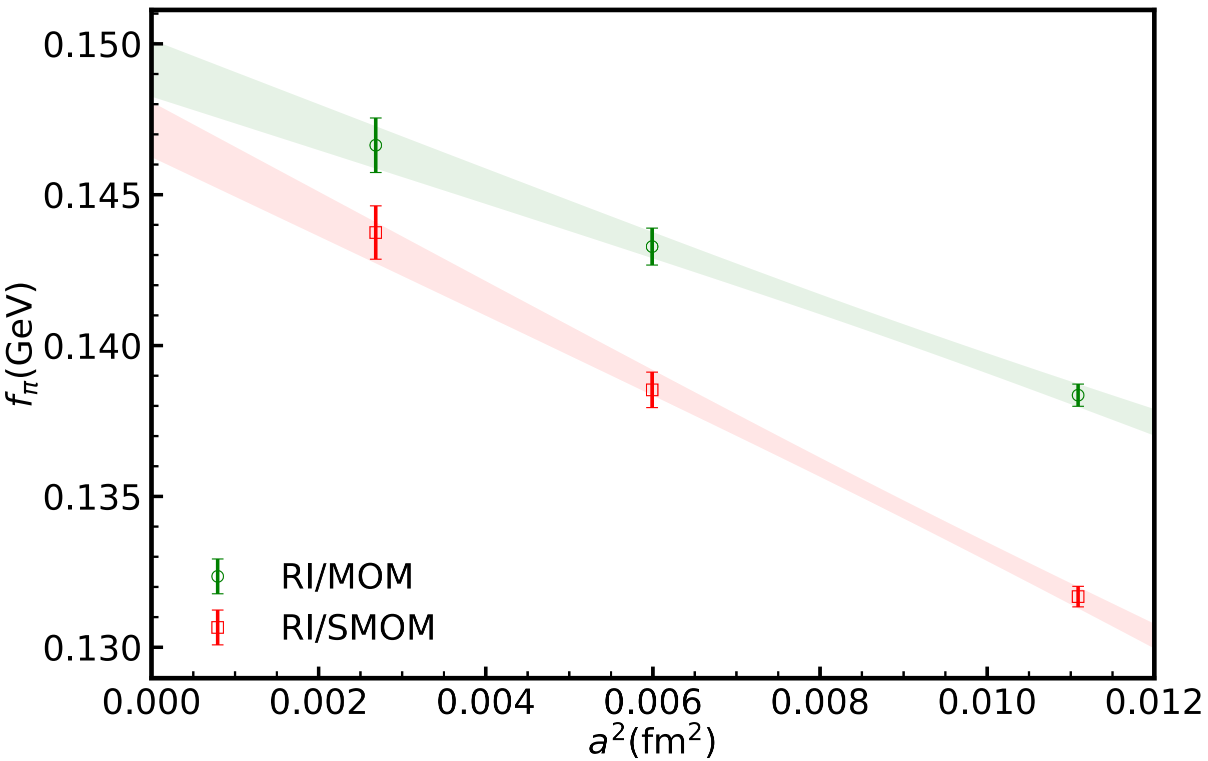
1. Chrial extrapolation
2. Convert to $\overline{\mathrm{MS}}$ and run to 2 GeV
3. $a^2\mu^2$ extrapolation to suppress discretization error





RI/MOM vs. RI/SMOM



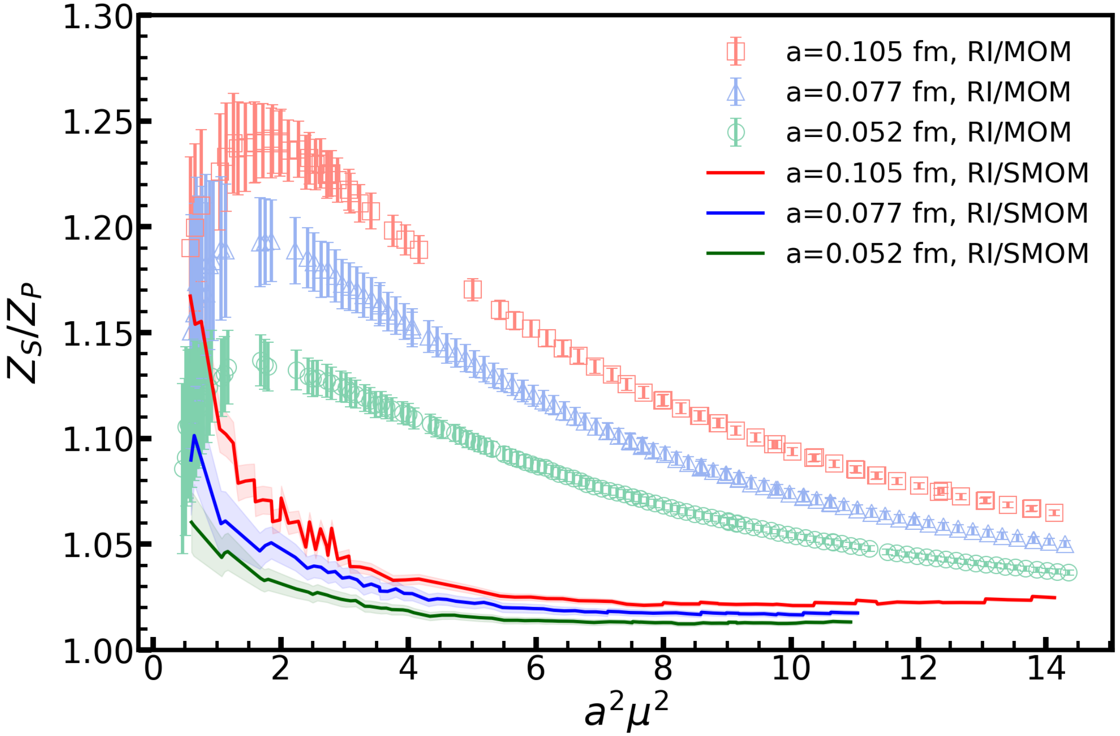


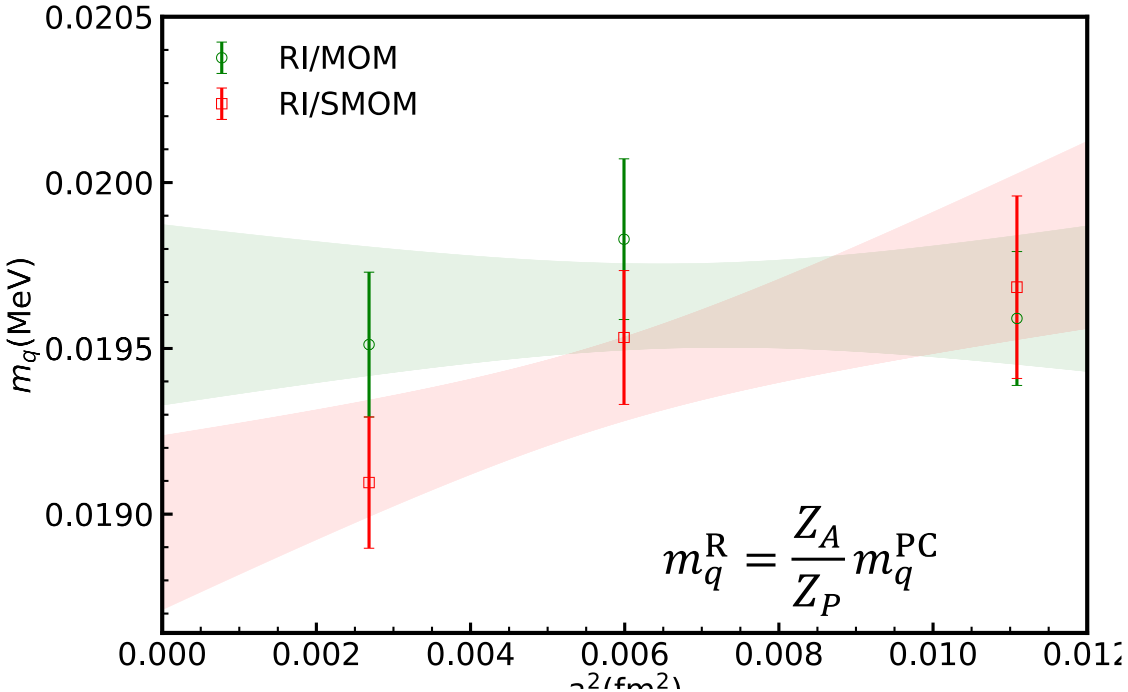
- In SMOM the chiral symmetry breaking effects are smaller
- At smaller lattice spacings breakings are suppressed
- $O(\alpha s)$ term is necessary to restore chiral symmetry

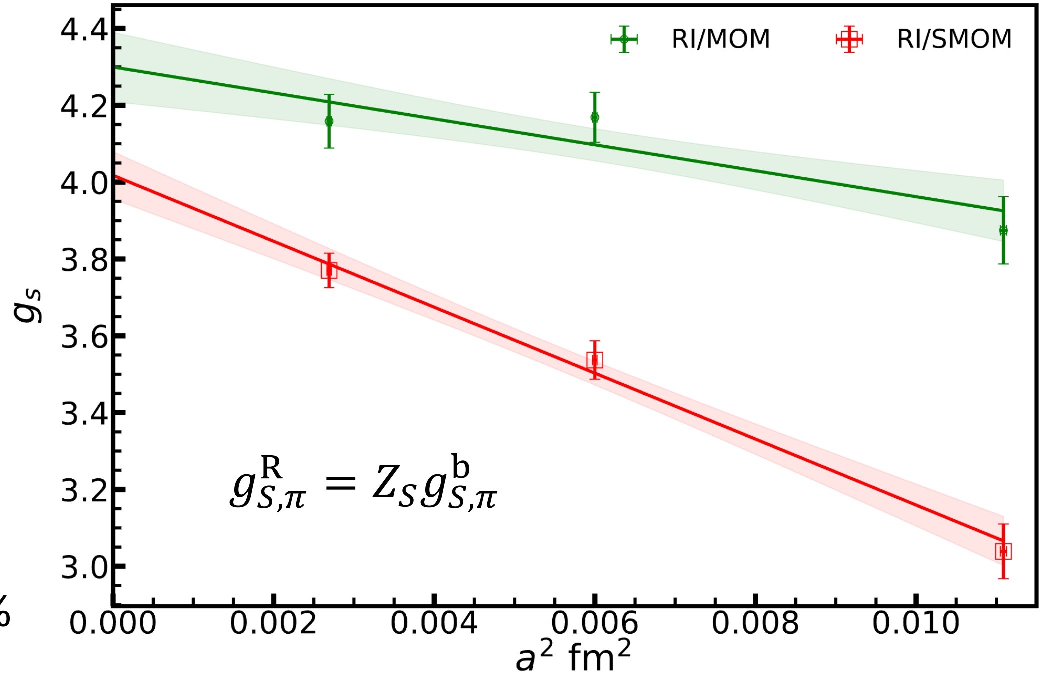
- Differ by ~1% after a linear a^2 continuum extrapolation
- MOM discretization 25% smaller

We choose the RI/MOM scheme in our work, for more reliable after continuum extrapolation.

RI/MOM vs. RI/SMOM







$$g_{S,\pi}^{\rm FH} = \frac{1}{2}\frac{\partial m_\pi}{\partial m_q^R} \simeq 4.06(4)$$

Breaking of the Feynman-Hellman theorem $g_{S,\pi}^{\rm FH} = g_{S,\pi}^{R}$ is about 7(3)%

Global fit

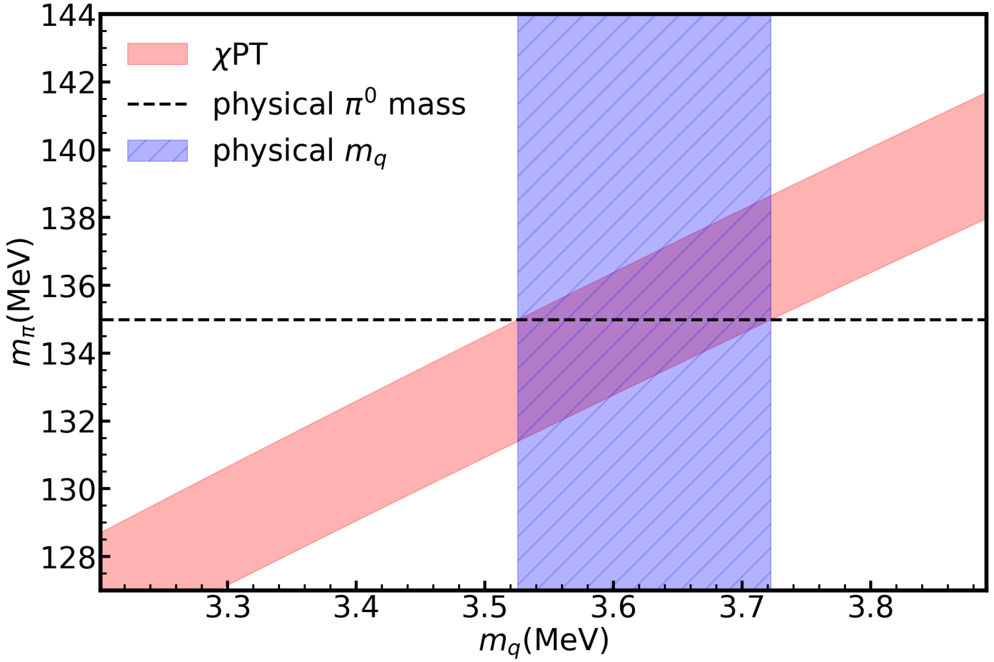
After the procedures of joint fit, scale setting and renormalization, we obtain m_q , m_π and f_π across 11 ensembles, we can conduct a joint fit using χPT:

$$m^2_{\pi,\mathrm{vv}} = \Lambda_\chi^2 2y_\mathrm{v}\left\{1+\frac{2}{N_f}[(2y_\mathrm{v}-y_\mathrm{s})\ln(2y_\mathrm{v})+(y_\mathrm{v}-y_\mathrm{s})] +2y_\mathrm{v}(2\alpha_8-\alpha_5)+2y_\mathrm{s}N_f(2\alpha_6-\alpha_4)\right\} \left(1+c_{m,a}a^2+c_{m,l}e^{-m_\pi L}\right)$$

$$F_{\pi,\mathrm{vv}} = F\left(1-\frac{N_f}{2}(y_\mathrm{v}+y_\mathrm{s})\ln(y_\mathrm{v}+y_\mathrm{s})+y_\mathrm{v}\alpha_5+y_\mathrm{s}N_f\alpha_4\right) \left(1+c_{f,a}a^2+c_{f,l}e^{-m_\pi L}\right)$$

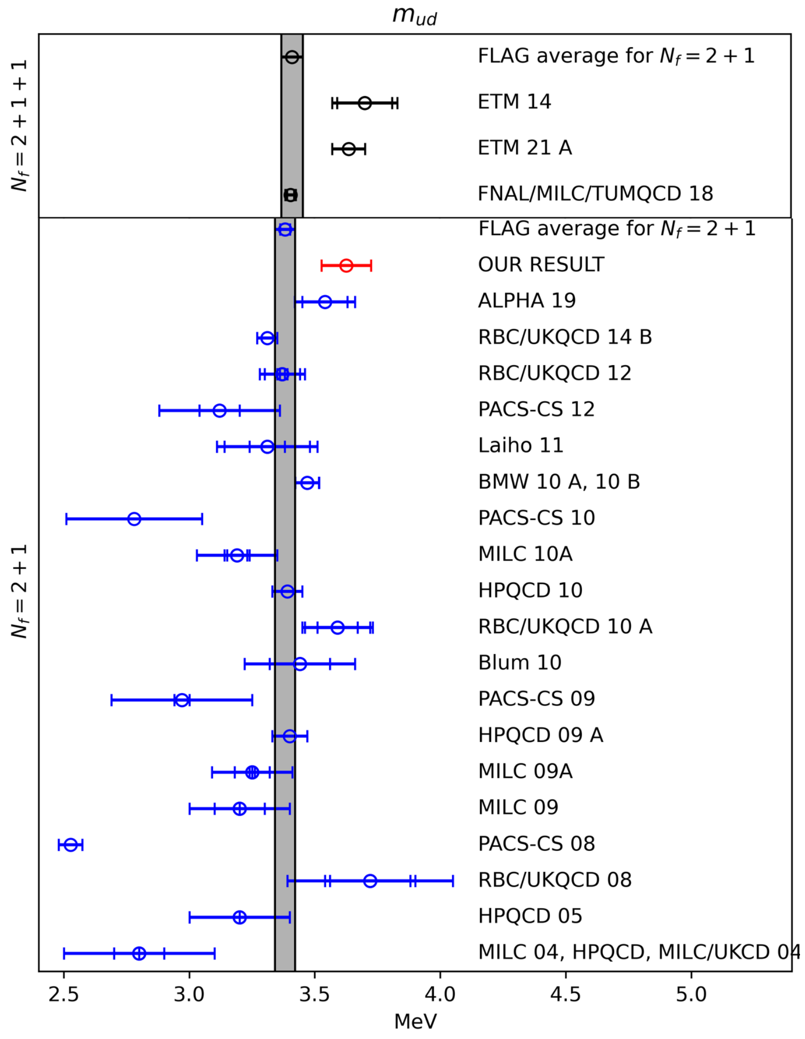
- y_v and y_s are variables defined by m_q
- F is pion decay constant in the chiral limit
- α_i are NLO low energy constants

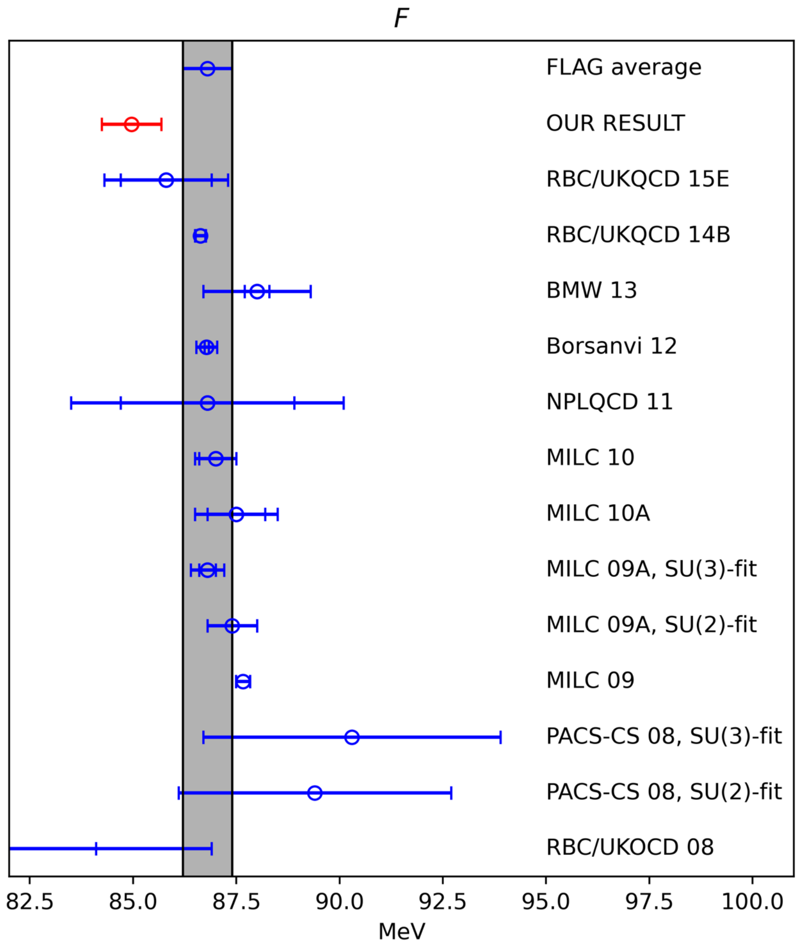
requiring $y_\mathrm{s}=y_\mathrm{v}$,
$m_{\pi,\mathrm{vv}}=m_{\pi,\mathrm{phys}}=134.98\mathrm{MeV}$,
$a\to 0$, and $L\to\infty$

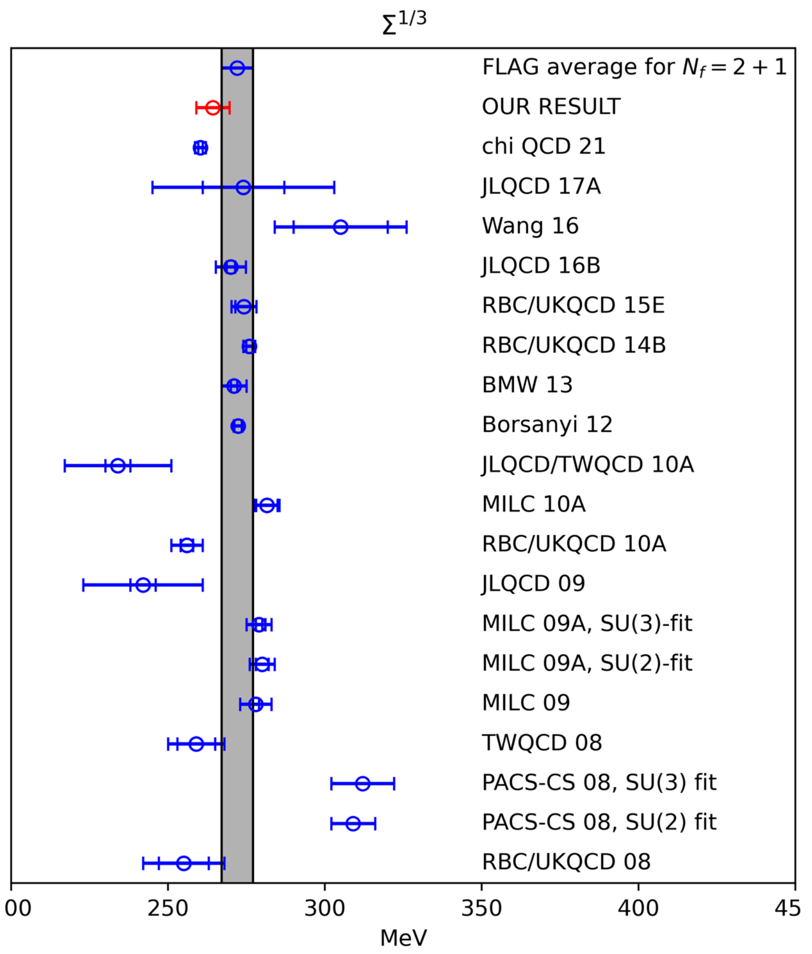


δZ_P included	N/A	δ_np	δ_p	$\delta_\mathrm{np}+\delta_\mathrm{p}$
$m_{l,\,\mathrm{phys}}$ (MeV)	3.628(29)	3.625(95)	3.628(34)	3.625(99)
$f_{\pi,\,\mathrm{phys}}$ (MeV)	128.55(90)	128.5(1.1)	128.55(92)	128.6(1.1)

Global fit results



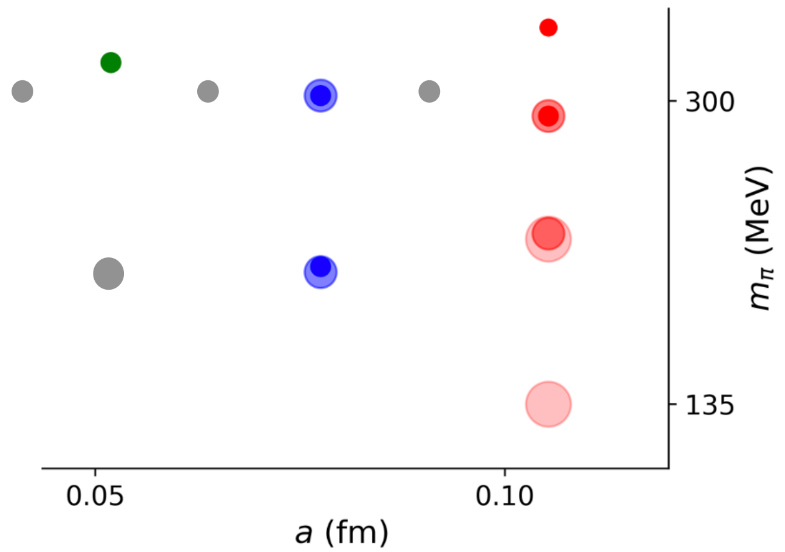




2 σ away from FLAG average

Summery and outlook

- Results agree with the FLAG average within 2 σ, and the non-perturbative renormalization is the main source of the quark mass error.
- Further investigation of renormalization, plan to use the volume source to suppress the uncertainty.
- More ensembles at more lattice spacing and pion masses to improve the reliability of our chiral and continuum global fit.



Thank you for listening!

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