Digital Quantum Simulation for the Spectroscopy of Schwinger Model

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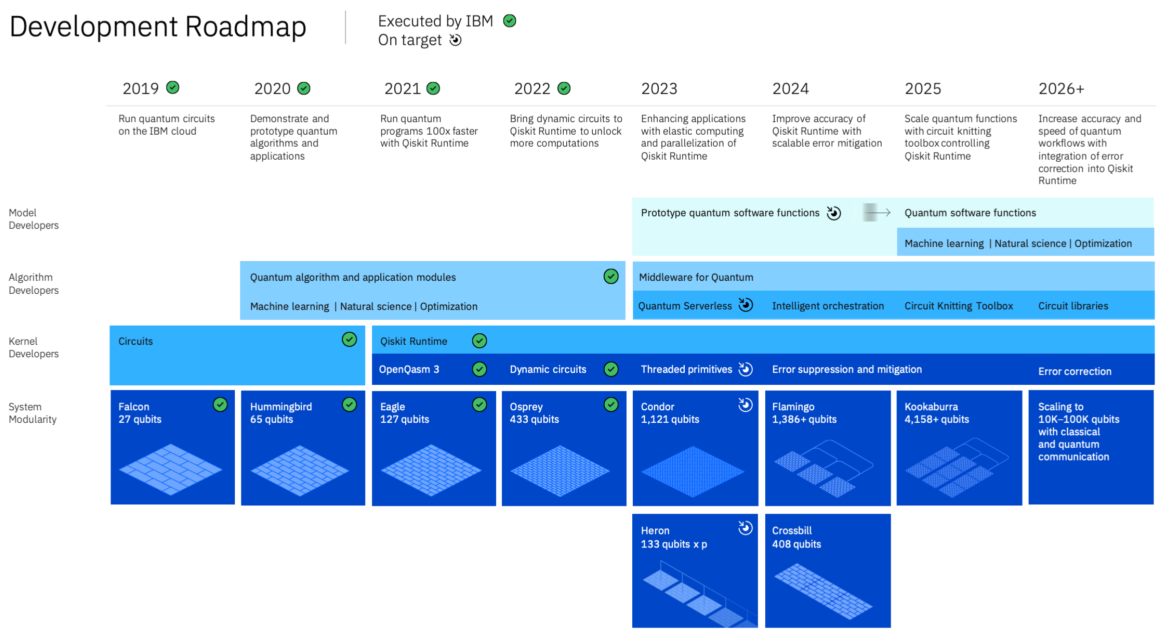
Based on an on-going collaboration (arXiv:2308.xxxxx)
with Masazumi Honda (YITP, Kyoto University & iTHEMS, RIKEN)

Digital Quantum Processor

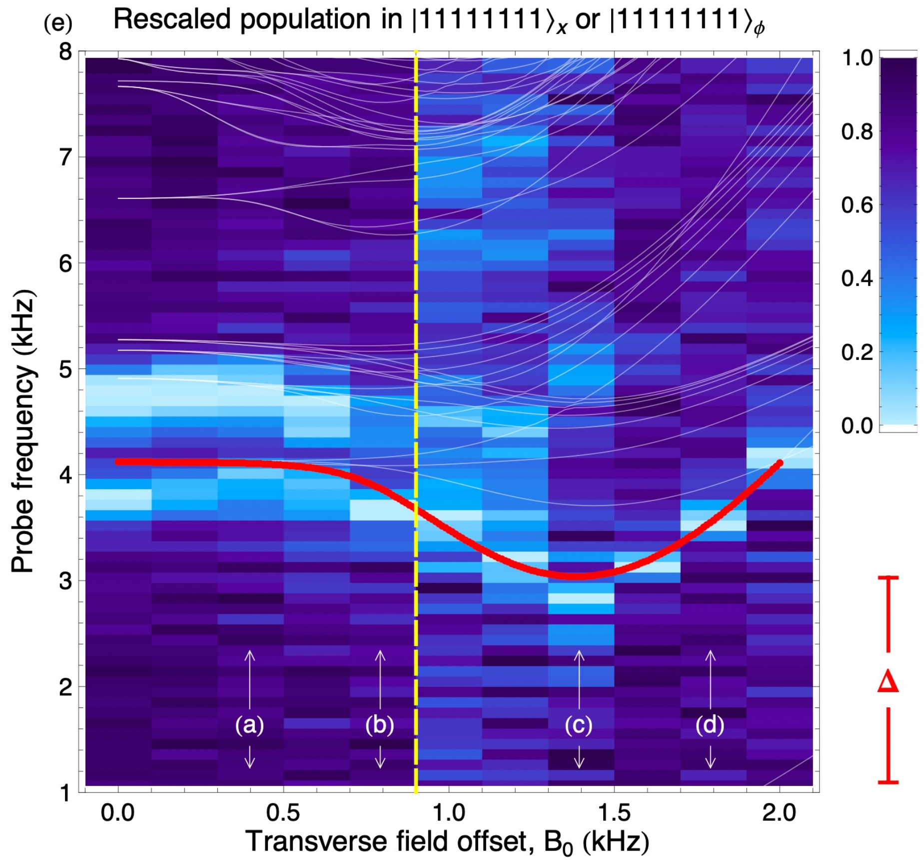
A natural embedding of Hamiltonian formulation on the Quantum Processor

[QC4HEP working group, Di Meglio, et al. 23]
See also [D. Grabowska]'s plenary talk



Application to **spectroscopy**
: to measure the excited state spectra

[Senko, Smith, Richerme, Lee, Campbell, Monroe 14]



Fermi Golden Rule

Transition rate from an initial state to a final state under the time-dependent (sinusoidal) perturbation $\Delta H = \Delta V \sin(\omega t)$

$$\dot{P}_{i \to f} = \frac{2\pi}{\hbar^2} \left| \langle f \, | \Delta V | \, i \rangle \right|^2 \delta(E_f - E_i - \omega)$$

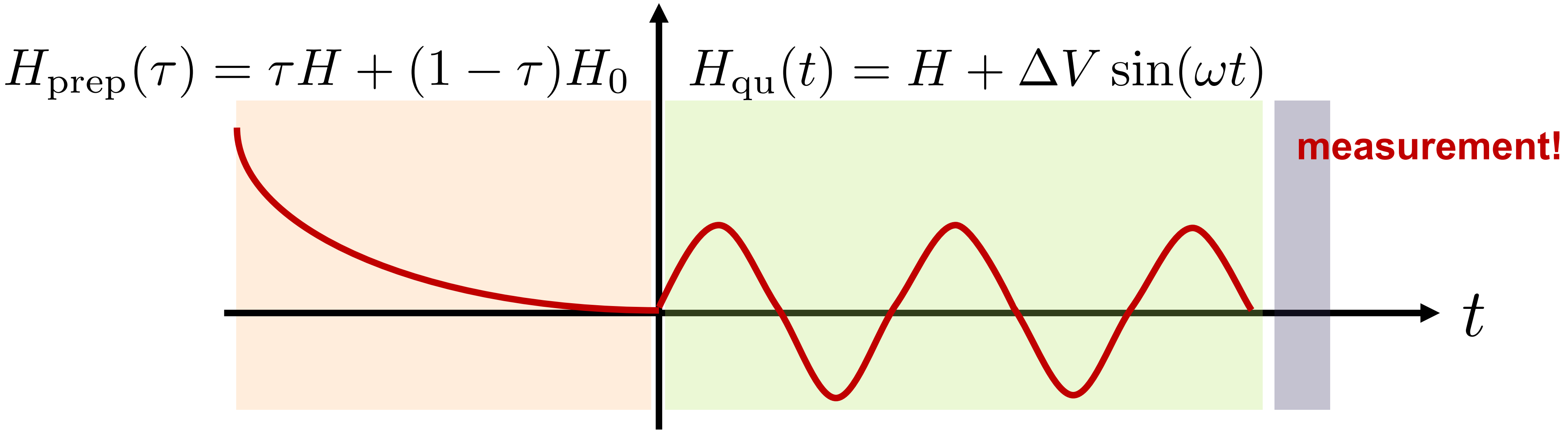
selection rule
e.g. parity, momentum (approximate)

frequency matching with spectral gap

[Dirac 1927], [Fermi 1950]
e.g. [Gottfried, Yan 03]

Spectroscopy with Digital Quantum Simulation

Outline



1. Ground state preparation by adiabatic procedure
2. Time-evolution with quenched Hamiltonian $H_{\mathrm{qu}} = H + \Delta H(t;x)$
3. Measurement of the vacuum persistency probability (or vac-to-vac probability)

Schwinger Model, i.e. 2d QED

[1] Lagrangian density [Schwinger 1951, 1962]

$$\mathcal{L} = \frac{1}{2g^2} F_{01}^2 + \frac{\theta}{2\pi} F_{01} + \overline{\psi} i \gamma^\mu (\partial_\mu + i A_\mu) \psi - m \overline{\psi} \psi$$

[2] Degree of freedom on lattice

- fermion with site-variables; staggered fermion $(\chi_{2n}\,,\chi_{2n+1})^{\mathbf{t}} \leftrightarrow \psi(x)$
- gauge field with link-variables $U_n \leftrightarrow e^{iaA_1(x)}\,, \quad L_n \leftrightarrow -\Pi(x)$

[3] Lattice formulation on an interval with Open Boundary Condition (OBC)

- Gauss constraint
- Jordan-Wigner transform
- dual boson description

$$\chi_n = \frac{X_n - iY_n}{2} \left(\prod_{j=0}^{n-1} -iZ_j \right)\,, \quad \chi_n^\dagger = \frac{X_n + iY_n}{2} \left(\prod_{j=0}^{n-1} iZ_j \right)$$

See also other parallel session talks by [L. Nagano], [C. Kane], [A. Matsumoto] and [J. W. Pedersen]

Schwinger Model, i.e. 2d QED

[4] Spin-system Hamiltonian (after solving Gauss constraint with OBC)

$$H_{\text{chiral}} = \frac{J}{2}\sum_{n=1}^{N-2}\sum_{k<n}(N-n-1)Z_k Z_n + \frac{1}{2}\sum_{n=0}^{N-2}\left\{w - (-1)^n \frac{m_{\text{lat}}}{2}\sin\theta\right\}(X_n X_{n+1} + Y_n Y_{n+1})$$

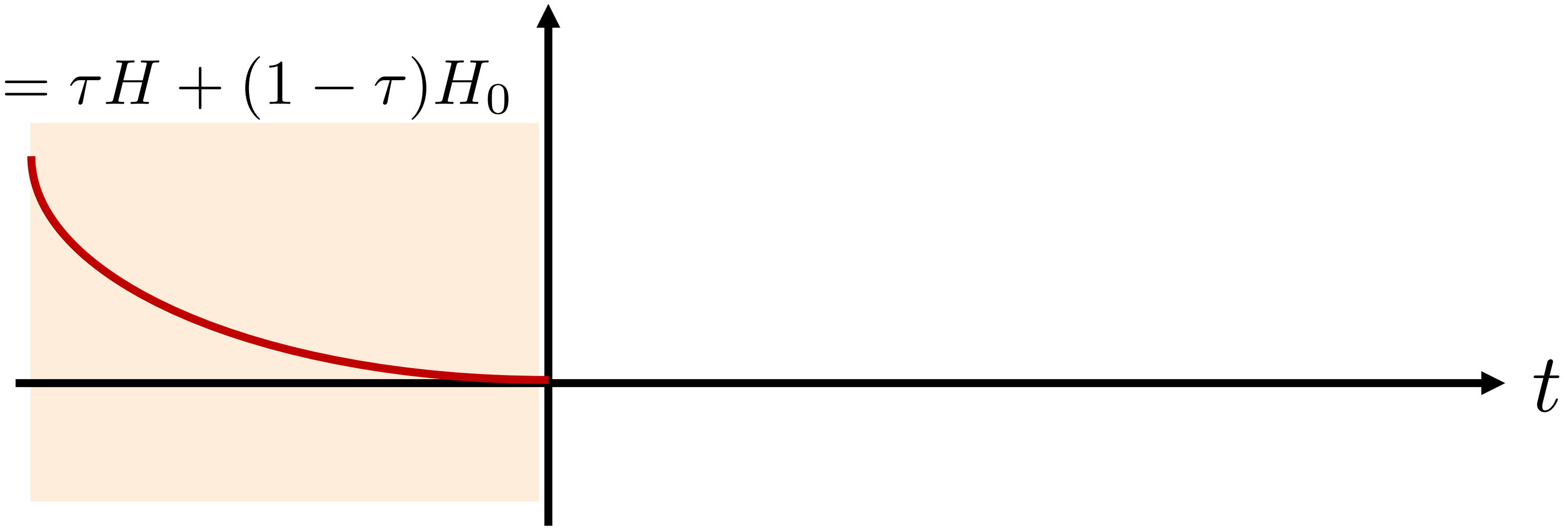
$$+\frac{m_{\text{lat}}\cos\theta}{2}\sum_{n=0}^{N-1}(-1)^n Z_n + \frac{J}{2}\sum_{n=0}^{N-2}(n+1 \bmod 2)\sum_{l=0}^{n} Z_l$$

e.g. [Chakraborty, Honda, Izubuchi, Kikuchi, Tomiya 20]

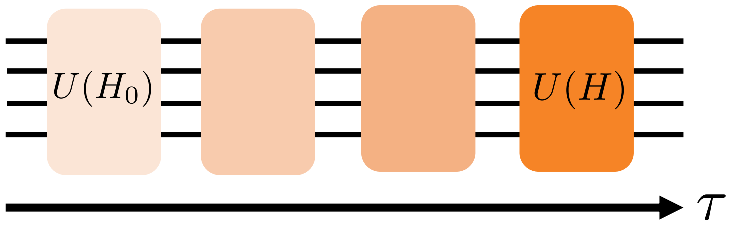
Spectroscopy with Digital Quantum Simulation

Adiabatic state preparation for the ground state

$$H_{\text{prep}}(\tau) = \tau H + (1-\tau) H_0$$



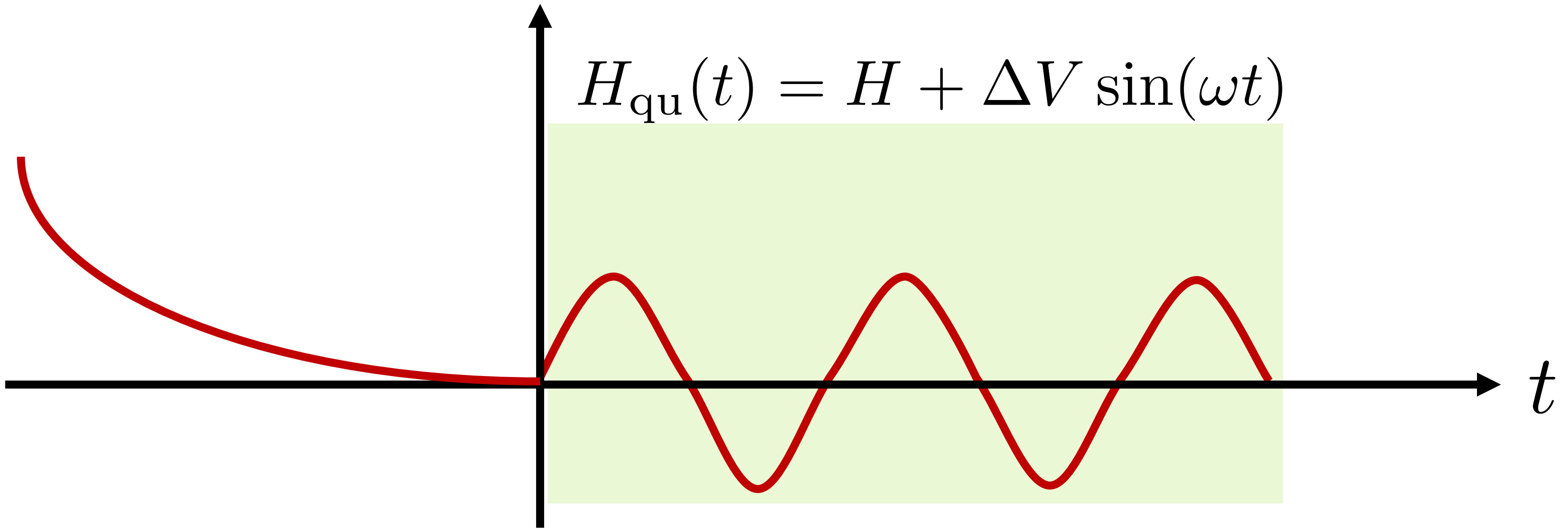
NB initial Hamiltonian with ground state of aligned spin



$$H_0 = \frac{M_0}{2} \sum_{n=0}^{N-1} (-1)^n Z_n + \frac{J}{2} \sum_{n=1}^{N-2} \sum_{k<n} (N-n-1) Z_k Z_n$$

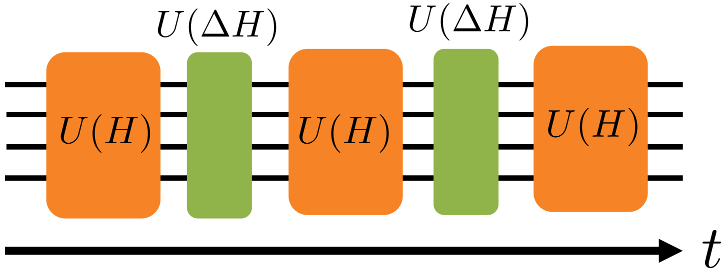
Spectroscopy with Digital Quantum Simulation

Quench with time-sinusoidal fluctuation



NB two different quench for Schwinger model on the lattice

- pseudo-scalar condensate $\overline{\psi}(x)\gamma_5\psi(x)$
- topological angle fluctuation $\widetilde{\theta}(t,x) = \theta + \delta\theta(t,x)$

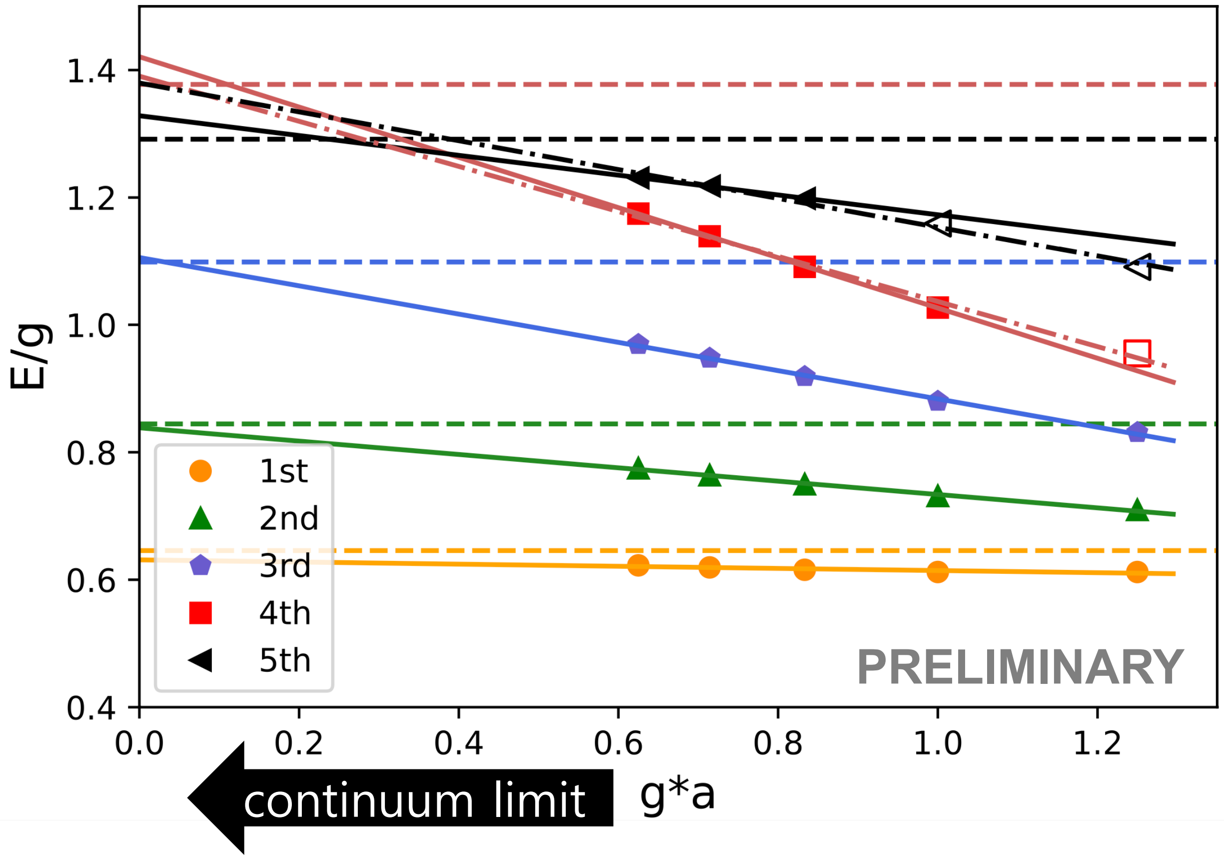


Towards the Continuum Limit

According to the duality with **Sine-Gordon model** (Schwinger meson),
Schwinger model with massless electron = free scalar theory

[Schwinger 1951, 1962]
[Coleman 1976]

Excited state level under fixed L=(N-1)a



5th excited state ~ 2-meson state (?)

2nd to 4th excited state ~
higher excitation mode

1st excited state ~ pseudo-scalar meson

e.g. [Banuls, Cichy, Cirac, Jansen 13]
[Dempsey, Klebanov, Pufu, Zan 22]

External Quench with(out) spatial modulation

[1] Pseudo-scalar condensate operator $\Delta V = \int dx\, f(x)\overline{\psi}(x)\gamma_5\psi(x)$

$$\Delta H = \frac{1}{2}\sum_{n=0}^{N-2}(-1)^{n+1}B_p\, f_n \sin(\omega t)\,(X_nX_{n+1}+Y_nY_{n+1})$$

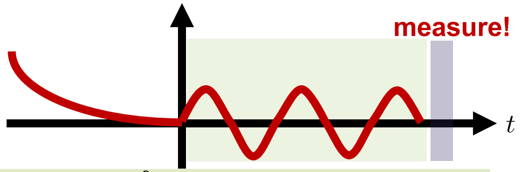
[2] Topological Angle Modulation $\widetilde{\theta}(t,x) = \theta + \delta\theta(t,x) = \theta + B_p\sin(\omega t)\cdot f(x)$

$$\Delta_0 H = \frac{m_{lat}}{4}\cos\theta\sum_{n=0}^{N-2}(-1)^n\, B_p\sin(\omega t) f_n\,(X_nX_{n+1}+Y_nY_{n+1})$$

$$+\frac{m_{lat}}{2}\sin\theta\sum_{n=0}^{N-1}(-1)^n\, B_p\sin(\omega t) f_n\, Z_n$$

$f_n = 1\,, \sin(\frac{\pi p n}{N})\,, \cos(\frac{\pi p n}{N})$

Pseudo-scalar condensate quench



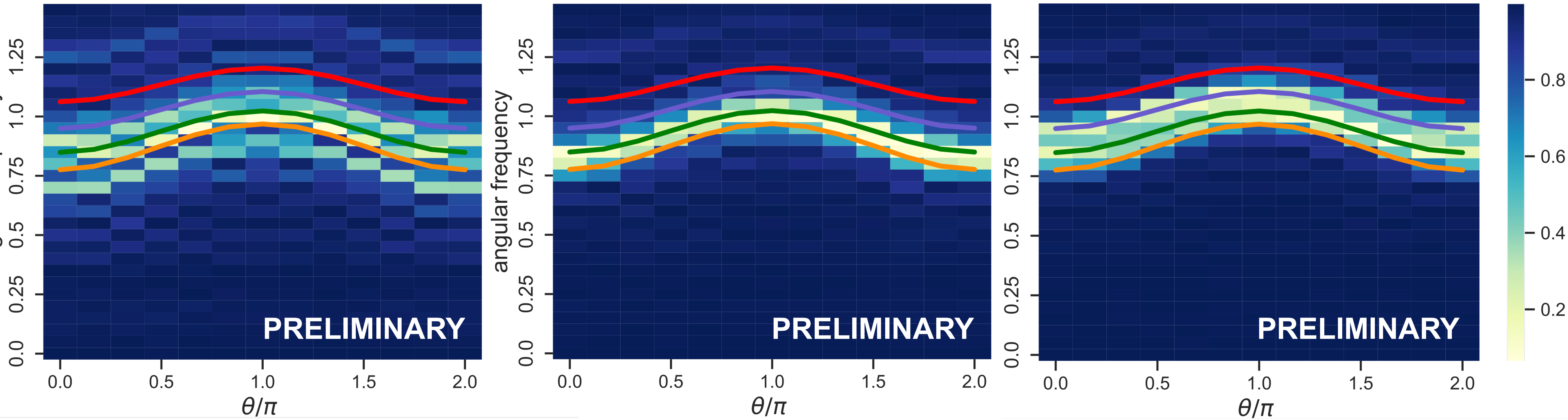
$$\Delta V = \int dx\, f(x)\overline{\psi}(x)\gamma_5\psi(x)$$

$$\dot{P}_{i\to f} = \frac{2\pi}{\hbar^2}\left|\langle f\,|\Delta V|\,i\rangle\right|^2 \delta(E_f - E_i - \omega)$$

$$\Delta H = \frac{1}{2}\sum_{n=0}^{N-2}(-1)^{n+1}B_p\, f_n \sin(\omega t)\,(X_nX_{n+1} + Y_nY_{n+1})$$

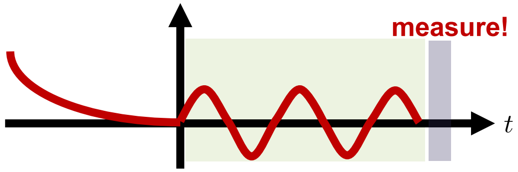
Vacuum-to-vacuum probability with 4 low excited states' energy spectra

$f_n = 1$ $\qquad$ $f_n = \cos(\frac{\pi n}{N})$ $\qquad$ $f_n = \cos(\frac{2\pi n}{N})$



$m = 0.1\,, B_p = 0.02\,, L = 10\,, N = 9\,, N_{shot} = 2000$

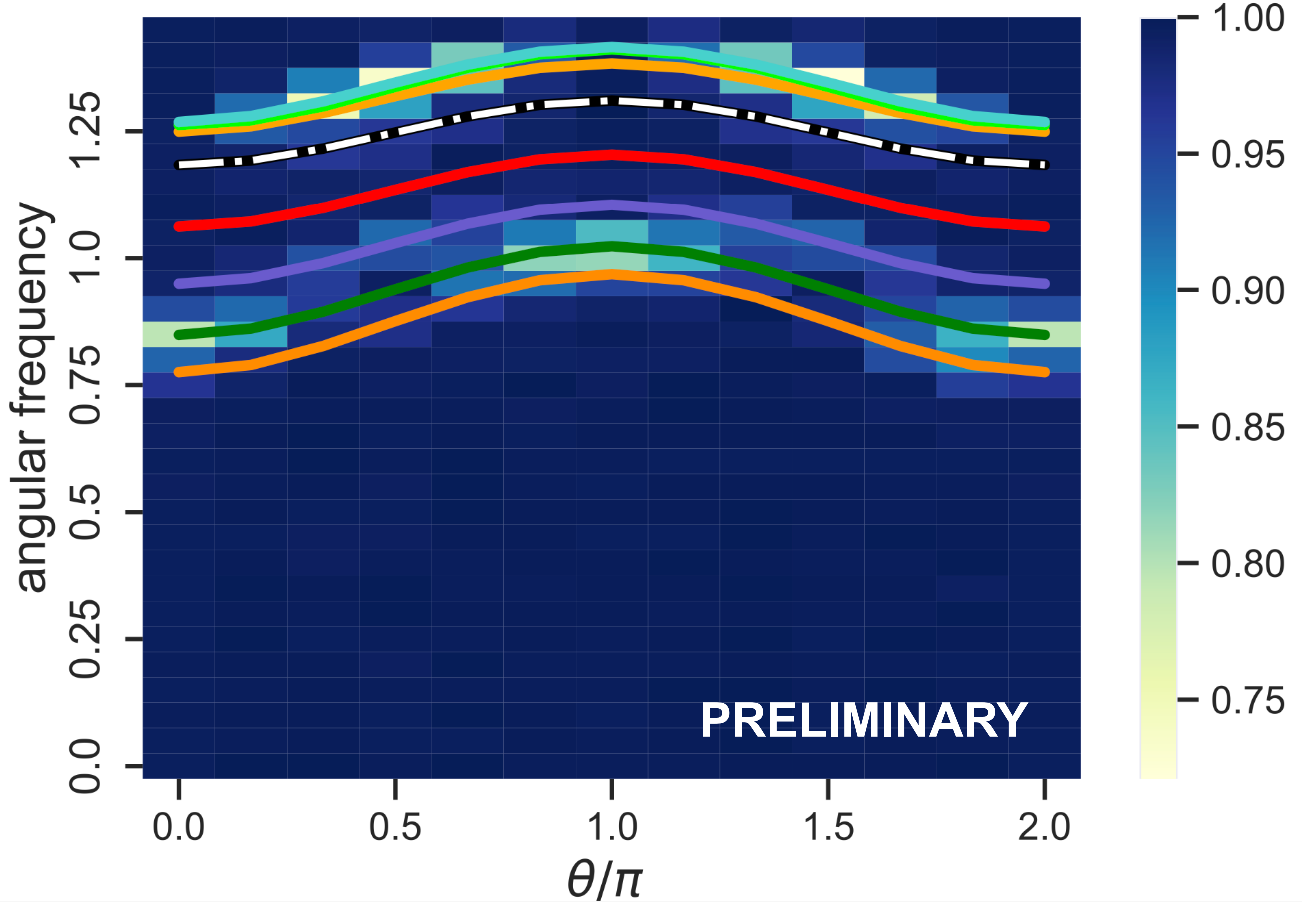
Topological angle quench



$$\widetilde{\theta}(t,x) = \theta + \delta\theta(t,x) = \theta + B_p \sin(\omega t) \cdot f(x)$$

$$f_n = \cos(\frac{\pi n}{N})$$

Vacuum-to-vacuum probability



$$\dot{P}_{i\to f} = \frac{2\pi}{\hbar^2} |\langle f\,|\Delta V|\,i\rangle|^2 \,\delta(E_f - E_i - \omega)$$

$$\Delta_0 H = \frac{m_{lat}}{4} \cos\theta \sum_{n=0}^{N-2} (-1)^n B_p \sin(\omega t) f_n \,(X_n X_{n+1} + Y_n Y_{n+1}) + \frac{m_{lat}}{2} \sin\theta \sum_{n=0}^{N-1} (-1)^n B_p \sin(\omega t) f_n \, Z_n$$

$$m = 0.1\,, B_p = 0.3\,, L = 10\,, N = 9\,, N_{shot} = 2000$$

Summary

[1] Hamiltonian formulation on a digital quantum processor simulates a state transition induced by external sinusoidal quench.

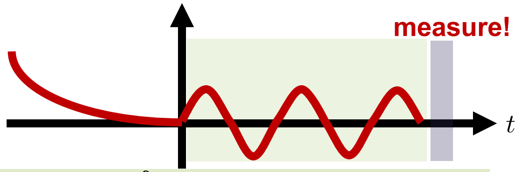
[2] Vacuum persistency probability signals the existence of excited state whose energy is identical to the frequency of quench.

[3] Selection rule on the quantum number matters.

[4] No-signal ≠ Nothing; new lesson from the various quench operators

Thank you

Pseudo-scalar condensate quench

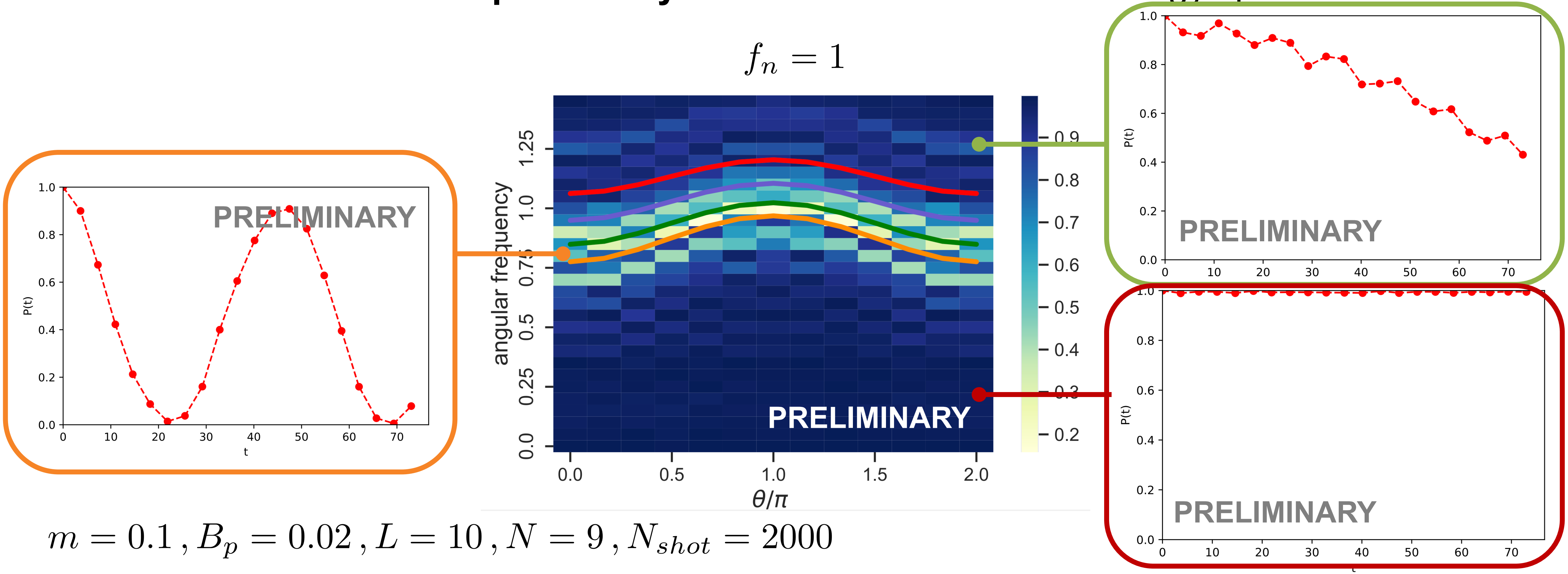


$$\Delta V = \int dx\, f(x)\overline{\psi}(x)\gamma_5\psi(x)$$

$$\dot{P}_{i\to f} = \frac{2\pi}{\hbar^2}\left|\langle f\,|\Delta V|\,i\rangle\right|^2 \delta(E_f - E_i - \omega) \qquad \Delta H = \frac{1}{2}\sum_{n=0}^{N-2}(-1)^{n+1}B_p\, f_n \sin(\omega t)\left(X_nX_{n+1} + Y_nY_{n+1}\right)$$

Vacuum-to-vacuum probability with 4 low excited states' energy spectra

$f_n = 1$



$$m = 0.1\,, B_p = 0.02\,, L = 10\,, N = 9\,, N_{shot} = 2000$$