Flavour-breaking effects in the Hyperon charges

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QCDSF Collaboration

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Motivation

- Nucleon isovector charges \((g_A^{u-d}, g_T^{u-d}, g_S^{u-d})\) can have an impact on searches for New Physics
  - Neutron lifetime puzzle
  - Neutron \(\beta\)-decay
  - CP-violation and neutron EDM
  - Importance of lattice input to these reflected in appearing in FLAG 21
  - Not much work on Hyperons

![Graph](image-url)
Feynman-Hellmann Theorem

Suppose we want: \[ \langle H | \mathcal{O} | H \rangle \]

Modify action with external field:

\[ S \rightarrow S + \lambda \int d^4 x \mathcal{O}(x) \]

Measure hadron energy while changing \( \lambda \)

\[ G(\lambda; \vec{p}; t) = \int dx \, e^{-i\vec{p} \cdot \vec{x}} \langle \chi'(x) \chi(0) \rangle \]

Calculation of matrix elements \( \equiv \) hadron spectroscopy

\[ \frac{\partial E_H(\lambda, \vec{p})}{\partial \lambda} \bigg|_{\lambda=0} = \frac{1}{2E_H(\vec{p})} \langle H(\vec{p}) | \mathcal{O}(0) | H(\vec{p}) \rangle \]
Feynman-Hellmann Theorem

> Can modify fermion action in 2 places:

- quark propagators

\[ \langle x \rangle_g \text{ [PLB714 (2012)]} \]

\[ \Delta s \text{ [PRD92 (2015)]} \]

\[ \Sigma \rightarrow n \text{ [2305.05491]} \]

\[ g_A, g_S \text{ [2304.02866]} \]

- fermion determinant

\[ g_A, \Delta \Sigma \text{ [PRD90 (2014)]} \]

\[ NPR \text{ [PLB740 (2015)]} \]

\[ G_E, G_M \text{ [PRD96 (2017)]} \]

\[ F_{1,2}(\omega, Q^2) \text{ [PRL118 (2017), PRD102 (2020), PRD107 (2023)]} \]

\[ GPDs \text{ [PRD104 (2022)]} \]

\[ \Sigma ightarrow n \text{ [2305.05491]} \]

\[ g_A, g_T, g_S \text{ [2304.02866]} \]
Demonstration: Axial charges

- Want
  \[ \langle N_s(p) | \bar{q}(0) \gamma_\mu \gamma_5 q(0) | N_s(p) \rangle = 2i s_\mu \Delta q \quad q \in (u, d) \]

- Employ
  \[ \mathcal{L} \rightarrow \mathcal{L} + \lambda \bar{q}(-i \gamma_3 \gamma_5) q \implies \frac{\partial E_N(\lambda)}{\partial \lambda} \bigg|_{\lambda=0}^{\Gamma_{\pm}} = \pm \Delta q_{\text{conn.}} \]

Energy shifts v t

Energy shifts v \lambda

\[ m_\pi \approx 470 \text{ MeV} \quad 350 \text{ configurations} \quad 32^3 \times 64 \]
Energy shifts: weighted average

Weights

Combined result

Minimum time used in fit~0.5-0.55fm

$g_A \sim w_f = p_f \sigma_f^2$

fit $p$-value

result uncertainty

see also: Beane et al. NPLQCD/QCDSF, PRD(2021), Rinaldi et al., PRD(2019)

$m_\pi \approx 265 \text{ MeV}, a = 0.068 \text{ fm}, V = 48^3 \times 96, \lambda = 5 \times 10^{-4}$

$\bar{a} = 0.052, 0.058, 0.068, 0.074, 0.082 \text{ fm}$ 

[arXiv:2304.02866]
Comparison to 3-point functions

Excellent agreement between Feynman-Hellmann and standard 3-point function methods
**Lambda dependence**

\[ E(\lambda) = E(0) \pm \lambda g^q_T + \mathcal{O}(\lambda^2) \]

\[ E(\lambda) = E(0) + \lambda g^q_S + \mathcal{O}(\lambda^2) \]

Spin-dependent:

\[ \left. \frac{\partial E^\uparrow(\lambda)}{\partial \lambda} \right|_{\lambda=0} = + g^q_T \quad \left. \frac{\partial E^\downarrow(\lambda)}{\partial \lambda} \right|_{\lambda=0} = - g^q_T \]

Spin-independent:

\[ \left. \frac{\partial E(\lambda)}{\partial \lambda} \right|_{\lambda=0} = + g^q_S \]

\( m_\pi \approx 265 \text{ MeV}, \ a = 0.068 \text{ fm}, \ V = 48^3 \times 96 \)
Quark mass trajectory

“Typical” trajectory:
fix strange quark mass to physical point and lower light quark mass

QCDSF trajectory:
Tune to physical average quark mass. Approach physical point by breaking SU(3) symmetry.

Hold “\(m\)-bar” constant:
\[
\bar{m} = \frac{1}{3} \left(2m_\ell + m_s\right) = \frac{1}{3} \left(2m_\ell^{\text{phys}} + m_s^{\text{phys}}\right)
\]
Flavour-breaking expansion

Consider general flavour matrix elements of octet baryons:

\[ \langle B' | J^F | B \rangle = A_{B'FB} \]

In exact SU(3) limit, just 2 independent constants:

- \( F \)- and \( D \)-type couplings

At linear order in SU(3) breaking: 5 slope parameters (3 D’s & 2 F’s)

- \# of parameters (polynomials/operators) reduced by restricting to \( \bar{m} = \text{constant line} \)

\[
\begin{align*}
F_1 &\equiv \frac{1}{\sqrt{3}} (A_{N_N} - A_{\Xi_\Xi}) = 2f - \frac{2}{\sqrt{3}} s_2 \delta m_l, \\
F_2 &\equiv (A_{N_N} + A_{\Xi_\Xi}) = 2f + 4s_1 \delta m_l, \\
F_3 &\equiv A_{\Sigma_\Sigma} = 2f + (-2s_1 + \sqrt{3}s_2) \delta m_l, \\
F_4 &\equiv \frac{1}{\sqrt{2}} (A_{\Lambda_K} - A_{\bar{N}_K}) = 2f - 2s_1 \delta m_l, \\
F_5 &\equiv \frac{1}{\sqrt{3}} (A_{\Lambda_K} + A_{\bar{N}_K}) = 2f + \frac{2}{\sqrt{3}} (\sqrt{3}s_1 - s_2) \delta m_l.
\end{align*}
\]

<table>
<thead>
<tr>
<th>Index</th>
<th>Baryon (B)</th>
<th>Meson (F)</th>
<th>Current ((J^F))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(n)</td>
<td>(K^0)</td>
<td>(d\bar{\gamma}s)</td>
</tr>
<tr>
<td>2</td>
<td>(p)</td>
<td>(K^+)</td>
<td>(\bar{u}\bar{\gamma}s)</td>
</tr>
<tr>
<td>3</td>
<td>(\Sigma^-)</td>
<td>(\pi^-)</td>
<td>(\bar{d}\bar{\gamma}u)</td>
</tr>
<tr>
<td>4</td>
<td>(\Sigma^0)</td>
<td>(\pi^0)</td>
<td>(\frac{1}{\sqrt{6}} (\bar{u}\bar{\gamma}u + \bar{d}\bar{\gamma}d - 2\bar{s}\bar{\gamma}s))</td>
</tr>
<tr>
<td>5</td>
<td>(\Lambda^0)</td>
<td>(\eta)</td>
<td>(\frac{1}{\sqrt{6}} (\bar{u}\bar{\gamma}u + \bar{d}\bar{\gamma}d - 2\bar{s}\bar{\gamma}s))</td>
</tr>
<tr>
<td>6</td>
<td>(\Sigma^+)</td>
<td>(\pi^+)</td>
<td>(\bar{u}\bar{\gamma}d)</td>
</tr>
<tr>
<td>7</td>
<td>(\Xi^-)</td>
<td>(K^-)</td>
<td>(\bar{s}\bar{\gamma}u)</td>
</tr>
<tr>
<td>8</td>
<td>(\Xi^0)</td>
<td>(\bar{K}^0)</td>
<td>(\bar{s}\bar{\gamma}d)</td>
</tr>
<tr>
<td>0</td>
<td>(\eta')</td>
<td>(\eta')</td>
<td>(\frac{1}{\sqrt{6}} (\bar{u}\bar{\gamma}u + \bar{d}\bar{\gamma}d + \bar{s}\bar{\gamma}s))</td>
</tr>
</tbody>
</table>

All matrix elements identical in the SU(3) symmetric limit
Can form a “singlet” combination

\[ X_F = \frac{1}{6} (3F_1 + F_2 + 2F_3) = 2f + O(\delta m_f^2) \]

General result: Singlet quantities only vary at 2nd-order in SU(3) breaking.

\[
\begin{align*}
F_1 &\equiv \frac{1}{\sqrt{3}} (A_{\bar{N}N} - A_{\bar{\Xi}\Xi}) = 2f - \frac{2}{\sqrt{3}} s_2 \delta m_l, \\
F_2 &\equiv (A_{\bar{N}\pi N} + A_{\bar{\Xi}\pi \Xi}) = 2f + 4s_1 \delta m_l, \\
F_3 &\equiv A_{\Sigma \pi \Sigma} = 2f + (-2s_1 + \sqrt{3}s_2) \delta m_l, \\
F_4 &\equiv \frac{1}{\sqrt{2}} (A_{\bar{N}K\Xi} - A_{N\Lambda \Xi}) = 2f - 2s_1 \delta m_l, \\
F_5 &\equiv \frac{1}{\sqrt{3}} (A_{\bar{N}K\Xi} - A_{N\Lambda \Xi}) = 2f + \frac{2}{\sqrt{3}} (\sqrt{3}s_1 - s_2) \delta m_l.
\end{align*}
\]
Simulation details

2+1 flavour, NP-improved Wilson fermions

3 volumes

500

400

300

200

100

M_{\pi}(MeV)

0.000

0.002

0.004

0.006

a^2(fm)^2

5 lattice spacings

0.052 ↔ 0.082 fm

pion masses

220 ↔ 500 MeV

FIGURE 1. Simulation details. The table lists the lattice ensembles used in this work. The data points correspond to different values of the quark mass parameter, indicating ensembles with different values of the physical bare quark mass. The graph shows the relationship between the physical bare quark mass and a^2(fm)^2 for each of the ensembles. The table and graph together provide a clear overview of the simulation details and help in understanding the relationship between the physical bare quark mass and the lattice spacings.
Global fits

Want result

➤ in continuum and infinite volume limits
➤ at physical quark masses

Global fit

➤ Include $O(a)$ or $O(a^2)$ terms in $X$ (singlet) and slope parameters

\[ X_{D,F} = X_{D,F}^* (1 + c_1 \frac{1}{3} [f_L(m_\pi) + 2f_L(m_\pi)]) + c_2 a + c_3 \delta m_i^2 \]

➤ Free parameter to encode leading finite-volume correction on singlet:

\[ f_L(m) = \left( \frac{m}{X_\pi} \right)^2 \frac{e^{-mL}}{\sqrt{mL}} \]

➤ Work to $O(\delta m_i^2)$ in flavour expansion

\[ \delta m_i \rightarrow \delta m_i = \frac{m_\pi^2 - X_\pi^2}{X_\pi^2} \]

[functional form from chiral EFT, see Beane & Savage PRD(2004)]
Global fits

Singlet $X_F$

$F$ slope parameters

With O(a) and FV Quark mass only

Global fits
Results - $g_A$ (isovector)

weighted average among models (as above)

FLAG result, ~2.2%

Our result, (stat+sys) ~5.5%

$g_A^{u-d} = 1.253(63)_{\text{stat}}(41)_{\text{a}(03)}_{\text{FV}}$

Different model parameterisations

1. $\delta m_i^2$
2. $a, \delta m_i^2$
3. $a^2, \delta m_i^2$
4. $a, \delta m_i^2, m_\pi L$
5. $a^2, \delta m_i^2, m_\pi L$
6. $\delta m_i^2, m_\pi L$
Results - isovector charges $N_f = 2 + 1$

$g_{T}^{u-d} = 1.010(21)_{\text{stat}}(12)_{\text{a}(01)}_{\text{FV}}$

**FLAG 2+1**: $\sim 6\%$

**FLAG 2+1+1**: $\sim 3\%$

**Our result**: $\sim 2\%$

$g_{S}^{u-d} = 1.08(21)_{\text{stat}}(03)_{\text{a}(01)}_{\text{FV}}$

**FLAG 2+1**: $\sim 12\%$

**Our result**: $\sim 19\%$

**MS, $\mu = 2\,\text{GeV}$**
Results - Hyperon charges

Not in FLAG, but recent results by RQCD [2305.04717]

(see also previous talk)

This work

\[ g_\Sigma^T = 0.805(15) \]
\[ g_\Xi^T = -0.1952(75) \]
\[ g_\Sigma^A = 0.876(28) \]
\[ g_\Xi^A = -0.206(21) \]
\[ g_\Sigma^S = 2.80(25) \]
\[ g_\Xi^S = 1.59(12) \]

RQCD

\[ g_\Sigma^T = 0.798(26) \]
\[ g_\Xi^T = -0.1872(72) \]
\[ g_\Sigma^A = 0.875(49) \]
\[ g_\Xi^A = -0.267(18) \]
\[ g_\Sigma^S = 3.98(33) \]
\[ g_\Xi^S = 2.57(16) \]

some tension
Summary and outlook

- Feynman Hellman theorem
  - provides a viable alternative to 3-pt function methods for computing hadronic matrix elements
- Flavour-breaking expansion along the $\bar{m} = \text{constant}$ line
  - allows for a controlled extrapolation from the SU(3)-symmetric point
- Future improvements
  - ensembles with near-physical quark masses and $4 \lesssim m_\pi L$
  - strong isospin breaking effects [c.f. QCDSF PLB(2012)]
  - gamma-W box (dispersion integral over moments of $F_{3}^{\gamma W}$)

\[
\Box_{\gamma W}^{b}(E_e) = \frac{3\alpha}{2\pi} \int_{0}^{\infty} \frac{dQ^2}{Q^2} \frac{M_W^2}{M_W^2 + Q^2} \left[ M_{3,-}(1, Q^2) + \frac{8E_eM}{9Q^2} M_{3,+}(2, Q^2) \right] + \mathcal{O}(E_e^2)
\]

For progress on moments of $F_{3}^{\gamma Z}$ via the Compton Amplitude, see K.U. Can, Fri, 9:40 (WH1W)