Flavour-breaking effects in the Hyperon charges

James Zanotti
The University of Adelaide

QCDSF Collaboration

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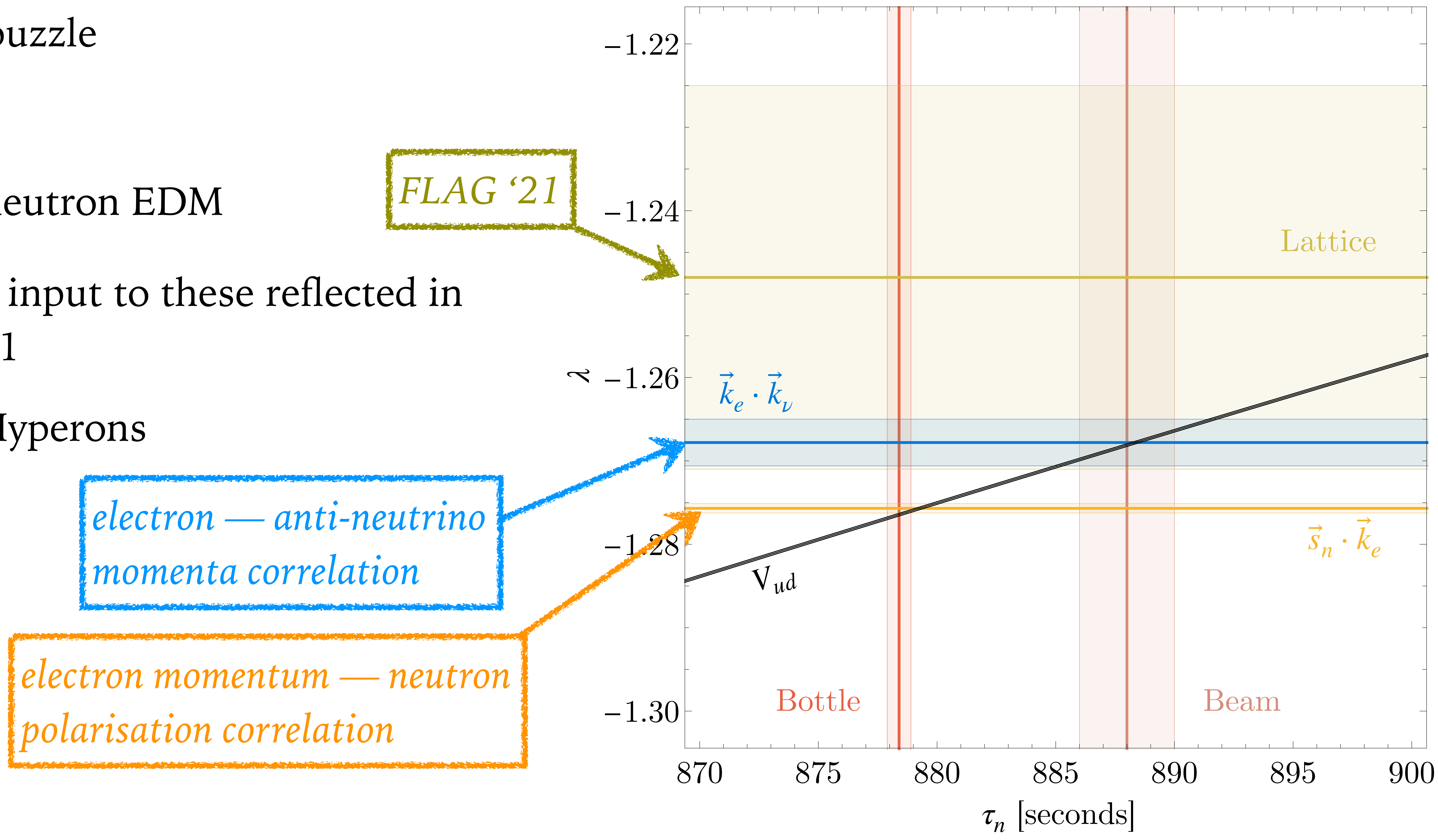
CSSM/QCDSF/UKQCD Collaborations

- M. Batelaan (Adelaide, PhD 2023 -> W&M)
- K. U. Can (Adelaide)
- A.Chambers (Adelaide, PhD 2018)
- A. Hannaford-Gunn (Adelaide, PhD 2023)
- R. Horsley (Edinburgh)
- T. Howson (Adelaide, PhD 2023)
- Y. Nakamura (RIKEN)
- H. Perlt (Leipzig)
- D. Pleiter (KTH)
- P. Rakow (Liverpool)
- G. Schierholz (DESY)
- **R. Smail (Adelaide, PhD)**
- K. Somfleth (Adelaide, PhD 2020)
- H. Stüben (Hamburg)
- R. Young (Adelaide)

Motivation

[arXiv:2304.02866]

- Nucleon isovector charges ($g_A^{u-d}, g_T^{u-d}, g_S^{u-d}$) can have an impact on searches for New Physics
  - Neutron lifetime puzzle
  - Neutron β-decay
  - CP-violation and neutron EDM
- Importance of lattice input to these reflected in appearing in FLAG 21
- Not much work on Hyperons



Feynman-Hellmann Theorem

Suppose we want: $\langle H | \mathcal{O} | H \rangle$

Modify action with external field:

real parameter

$$S \to S + \lambda \int d^4x\, \mathcal{O}(x)$$

local operator, e.g. $\bar{q}(x)\gamma_3 q(x)$

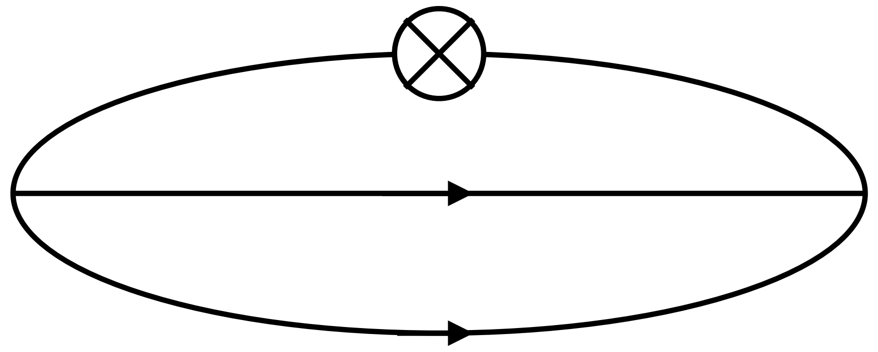
Measure hadron energy while changing λ

$$G(\lambda; \vec{p}; t) = \int dx\, e^{-i\vec{p}\cdot\vec{x}} \langle \chi'(x)\chi(0)\rangle \overset{\text{large t}}{\propto} e^{-E_H(\lambda,\vec{p})t}$$

Calculation of matrix elements ≡ hadron spectroscopy $\left.\frac{\partial E_H(\lambda,\vec{p})}{\partial \lambda}\right|_{\lambda=0} = \frac{1}{2E_H(\vec{p})}\langle H(\vec{p})|\mathcal{O}(0)|H(\vec{p})\rangle$

Feynman-Hellmann Theorem

- Can modify fermion action in 2 places:
  - quark propagators
  - fermion determinant



Connected

g_A, $\Delta\Sigma$ [PRD90 (2014)]

NPR [PLB740 (2015)]

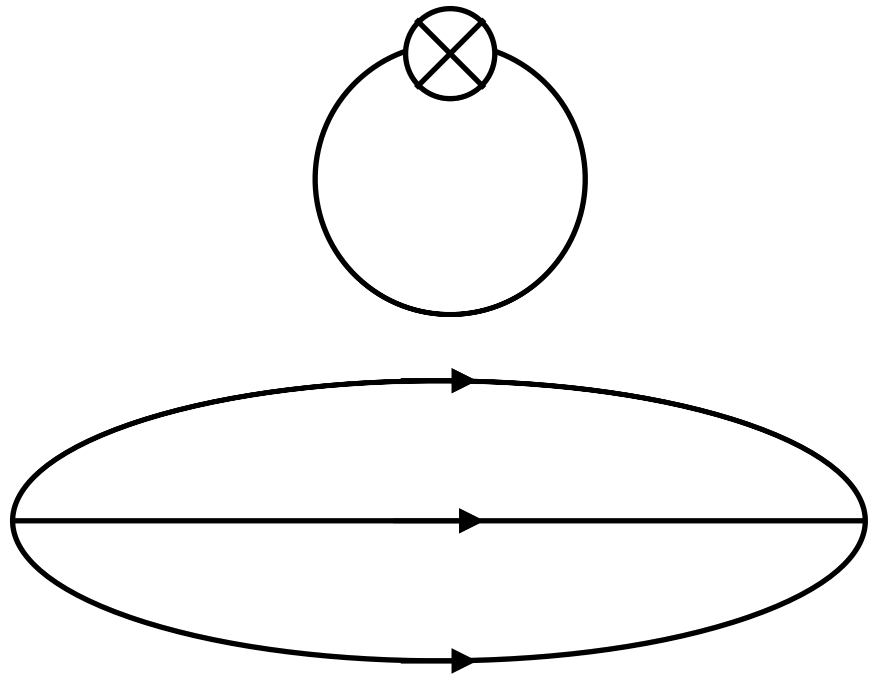
G_E,G_M [PRD96 (2017)]

$F_{1,2}(\omega, Q^2)$ [PRL118 (2017), PRD102 (2020), PRD107 (2023)]

GPDs [PRD104 (2022)]

$\Sigma \to n$ [2305.05491]

g_A, g_T, g_S [2304.02866]



Disconnected

(Requires new gauge configurations)

$\langle x \rangle_g$ [PLB714 (2012)]

NPR [PLB740 (2015)]

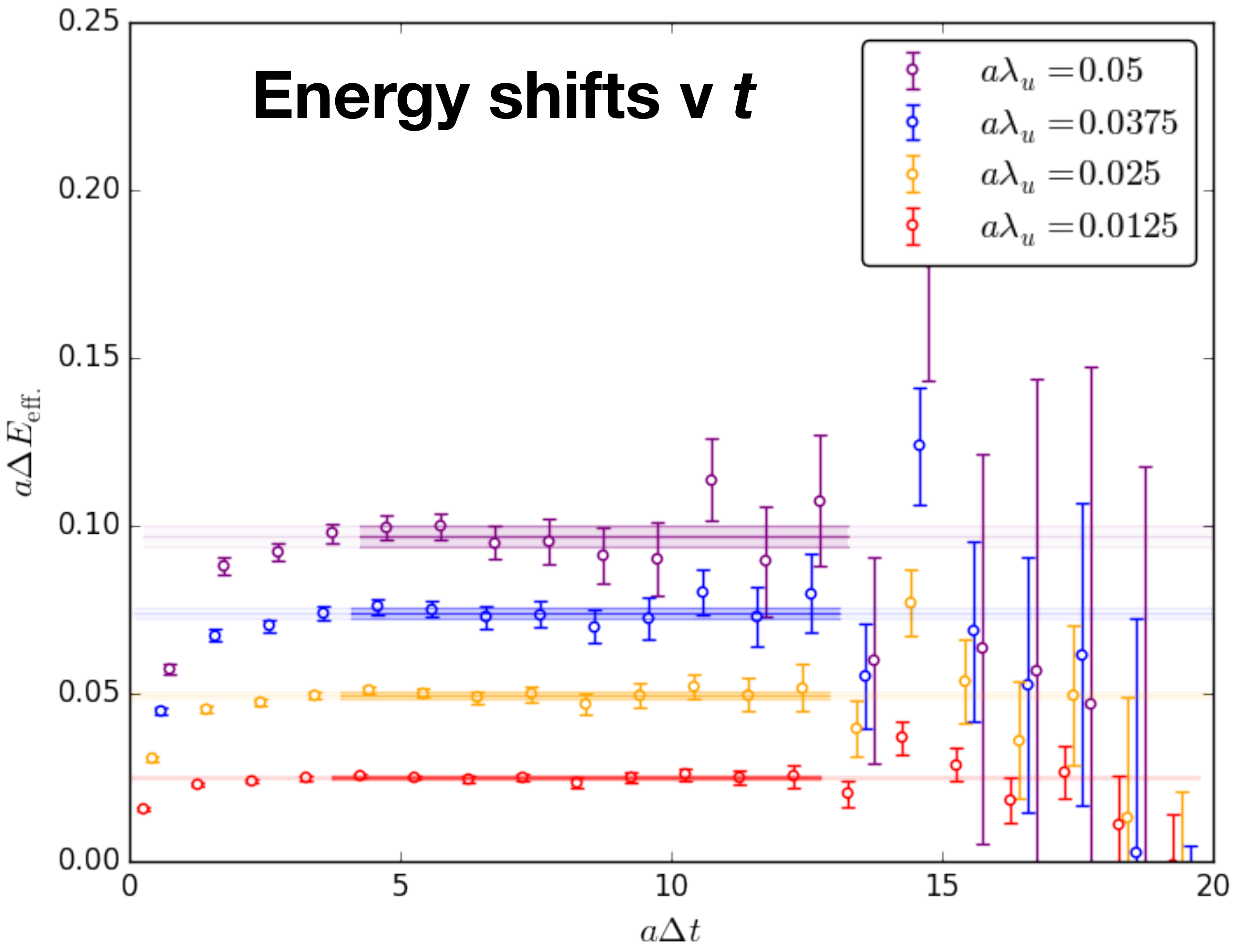
Δs [PRD92 (2015)]

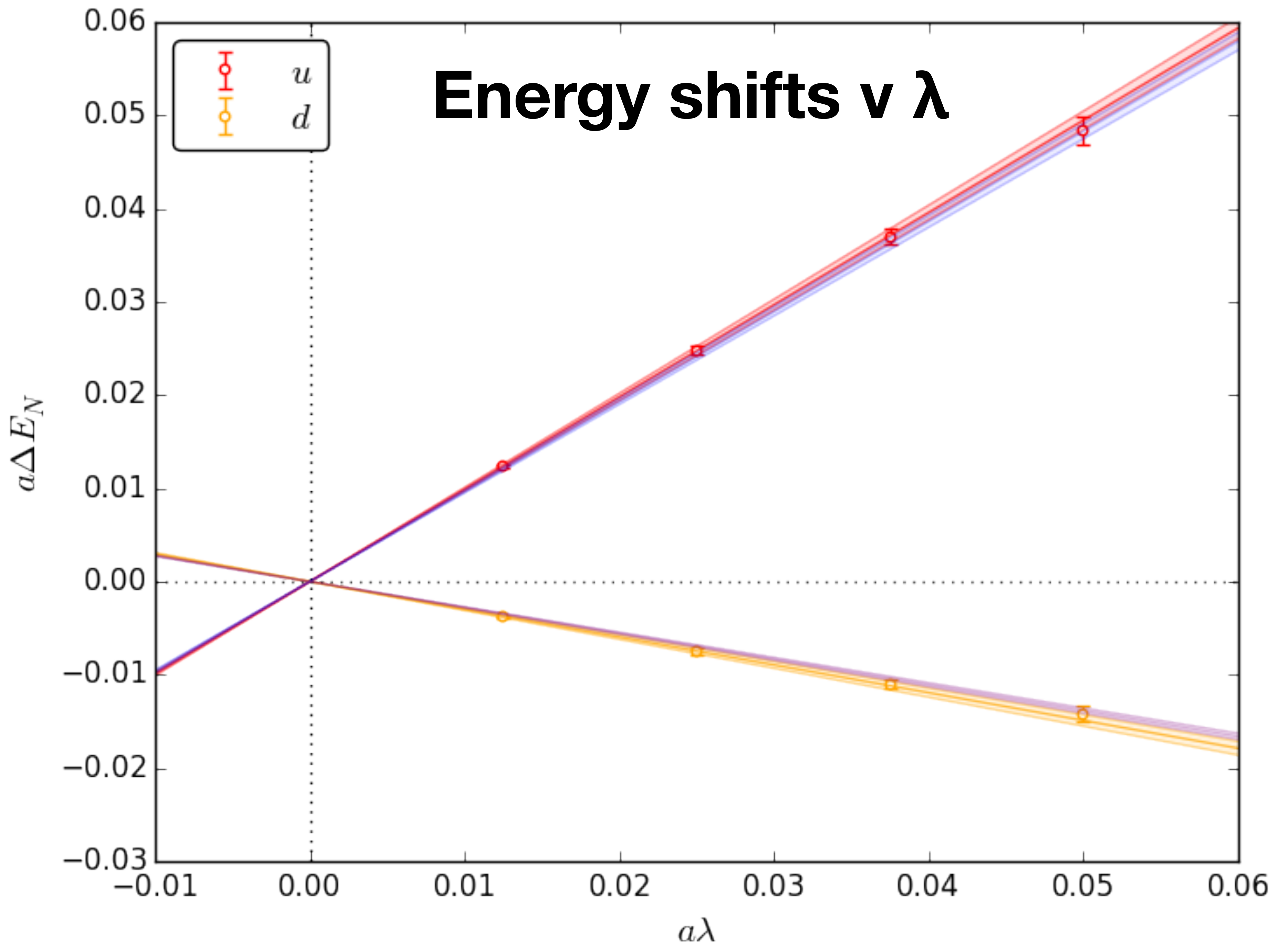
Demonstration: Axial charges

(Connected only, [PRD90 (2014)])

- Want $\langle N_s(\vec{p})|\bar{q}(0)\gamma_\mu\gamma_5 q(0)|N_s(\vec{p})\rangle = 2is_\mu \Delta q \qquad q \in (u,d)$

- Employ $\mathcal{L} \to \mathcal{L} + \lambda\bar{q}(-i\gamma_3\gamma_5)q \implies \left.\frac{\partial E_N(\lambda)}{\partial\lambda}\right|_{\lambda=0}^{\Gamma\pm} = \pm\Delta q_{\mathrm{conn.}}$

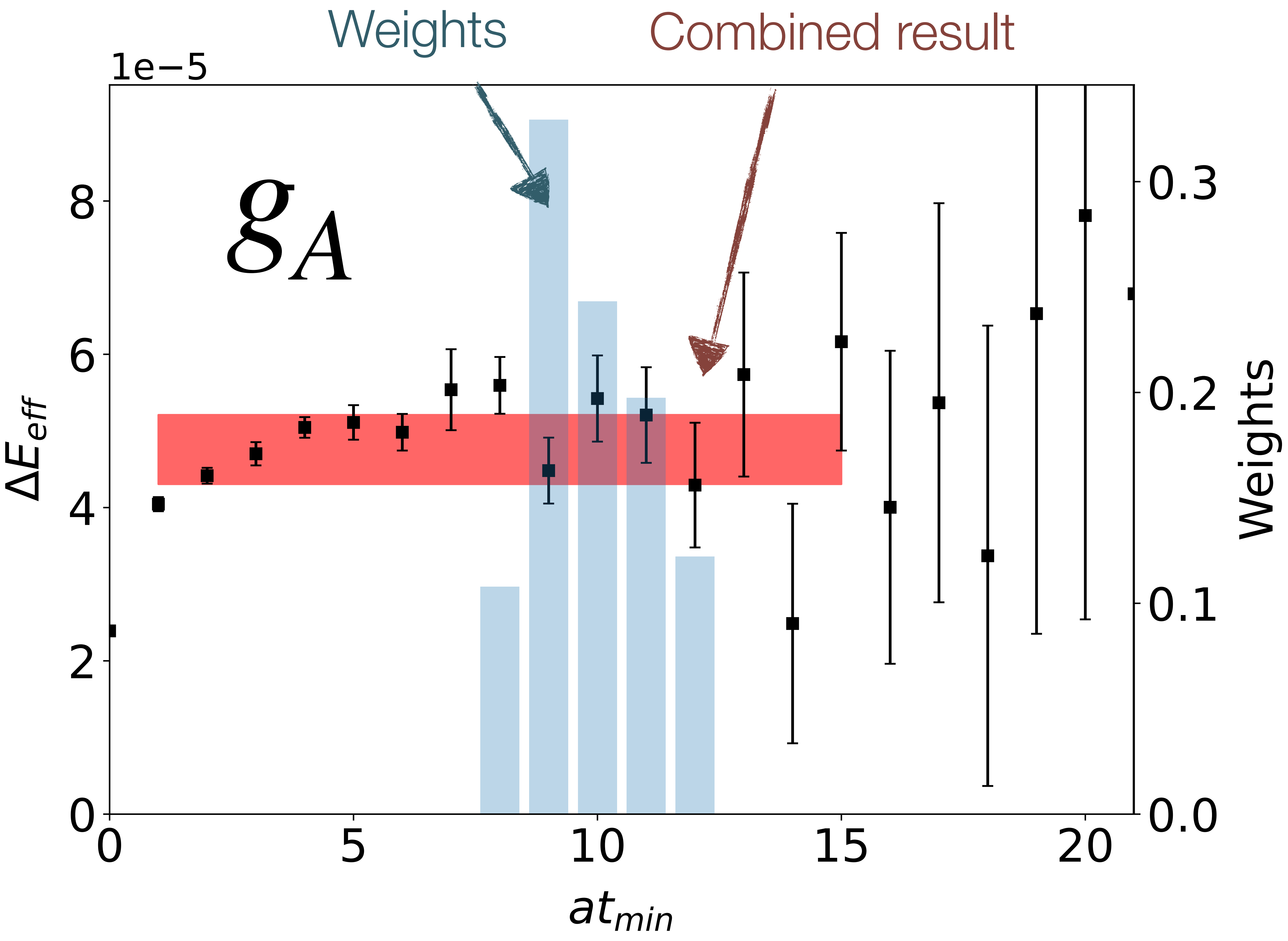


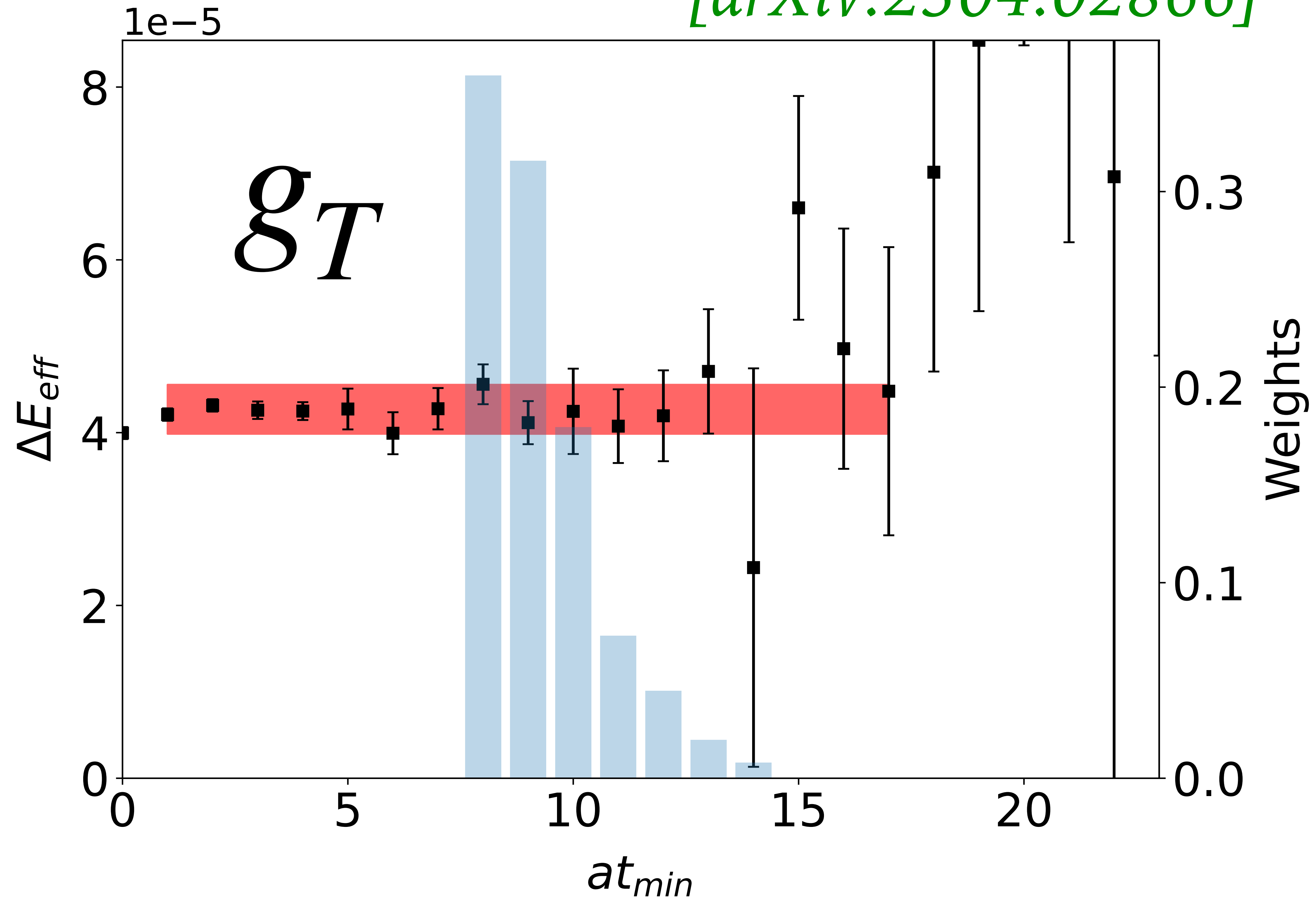


$m_\pi \approx 470$ MeV 350 configurations $32^3 \times 64$

Energy shifts: weighted average

[arXiv:2304.02866]





Minimum time used in fit~0.5-0.55fm

$t = 8, 9, 10, 11, 12$ for $a = 0.052,\ 0.058, 0.068, 0.074, 0.082$ fm

$m_\pi \approx 265\,\mathrm{MeV},\ \mathrm{a} = 0.068\,\mathrm{fm},\ \mathrm{V} = 48^3 \times 96,\ \lambda = 5 \times 10^{-4}$

see also: Beane *et al*. NPLQCD/QCDSF, PRD(2021), Rinaldi *et al*., PRD(2019)

(Non-normalised) weights:

$$\tilde{w}_f = \frac{p_f}{\sigma_f^2}$$

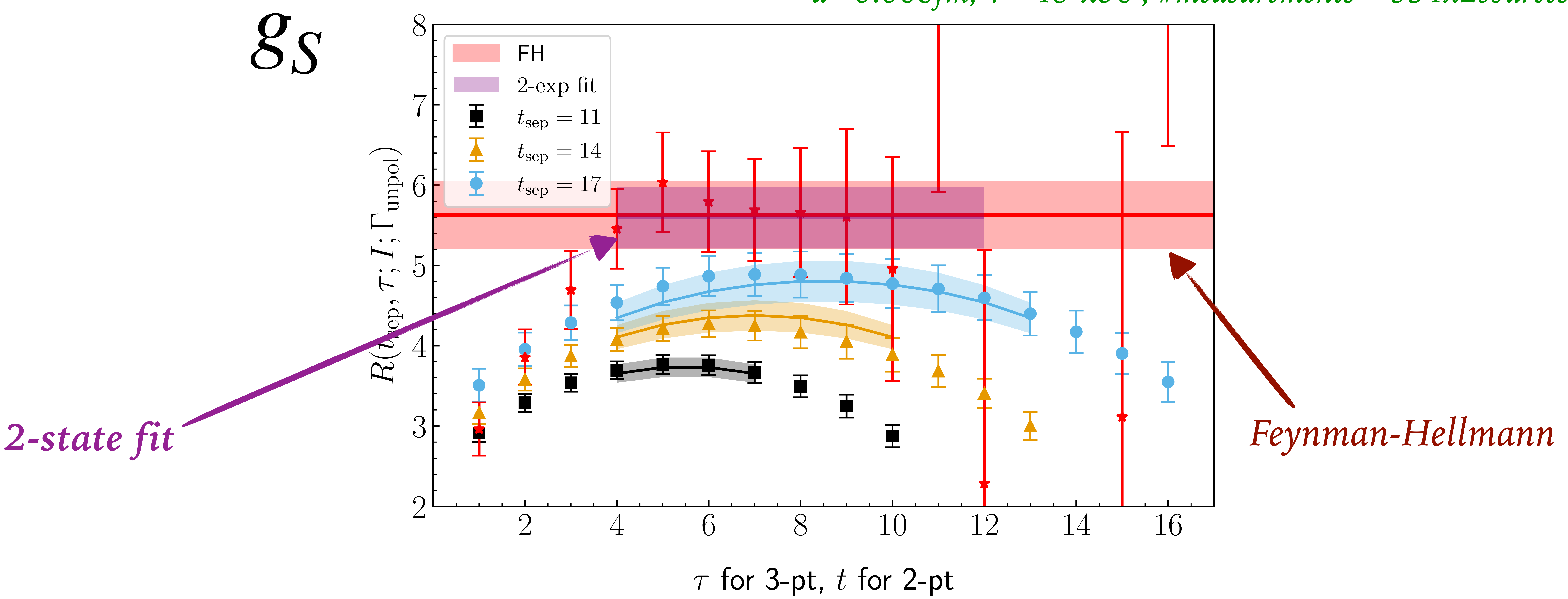
fit p-value

result uncertainty

Comparison to 3-point functions

$m_\pi \approx 265\,\text{MeV}$

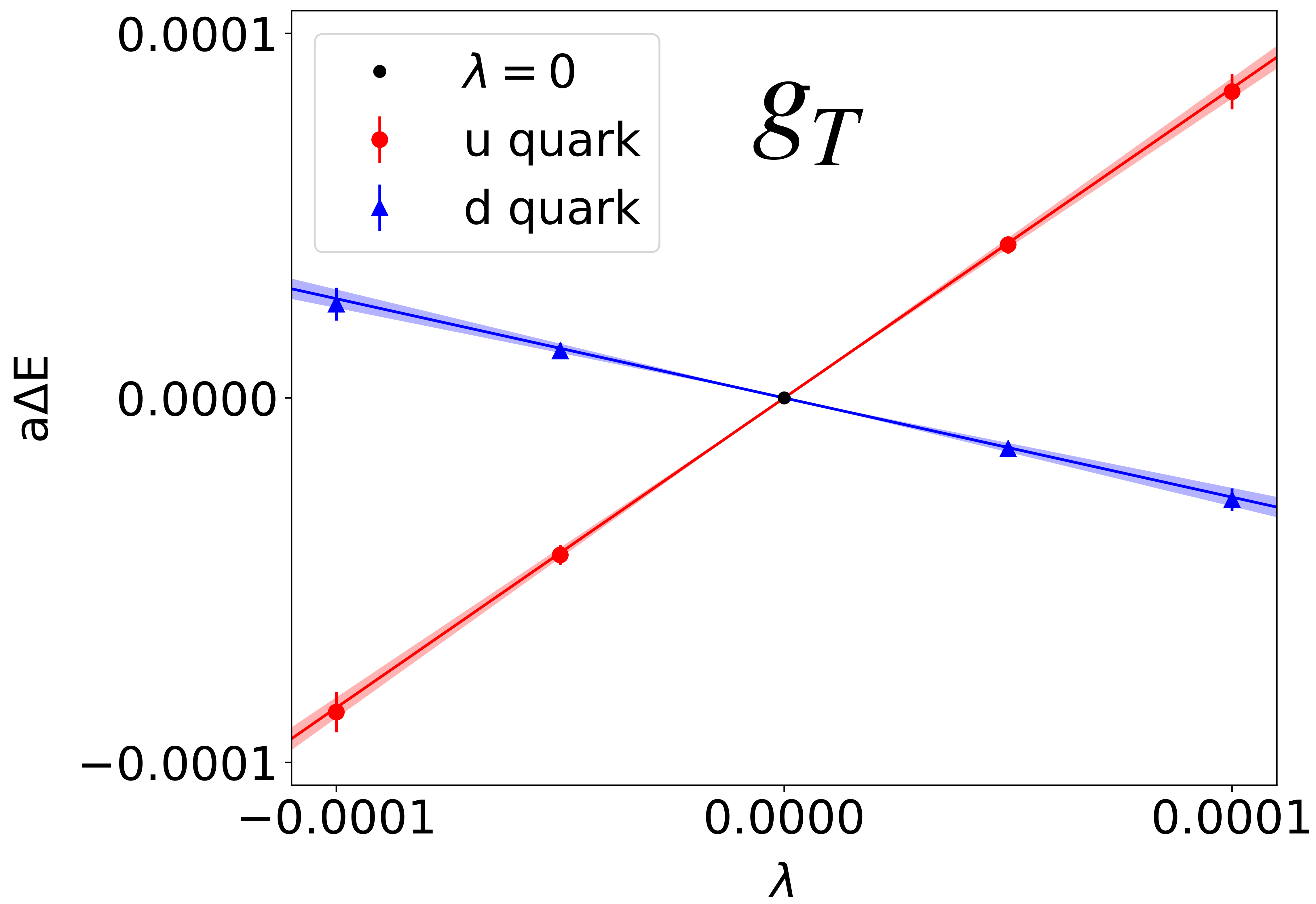
a=0.068fm, V=48³x96 , #measurements= 534x2sources

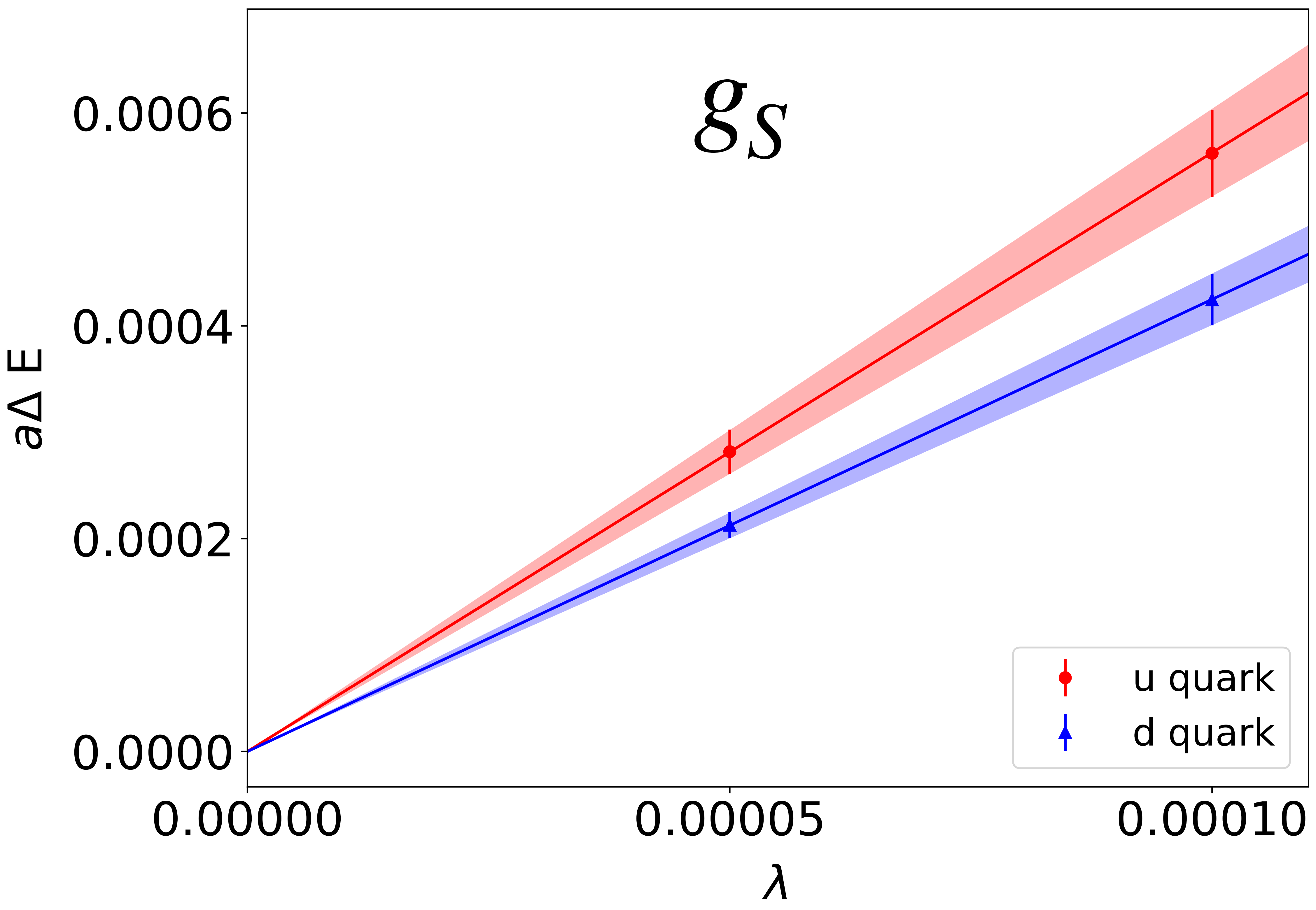


Excellent agreement between Feynman-Hellmann and standard 3-point function methods

Lambda dependence

$m_\pi \approx 265\,\mathrm{MeV},\ \mathrm{a} = 0.068\,\mathrm{fm},\ \mathrm{V} = 48^3 \times 96$





Spin-dependent:

$$\left.\frac{\partial E^{\uparrow}(\lambda)}{\partial\lambda}\right|_{\lambda=0} = +\,g_T^q \qquad \left.\frac{\partial E^{\downarrow}(\lambda)}{\partial\lambda}\right|_{\lambda=0} = -\,g_T^q$$

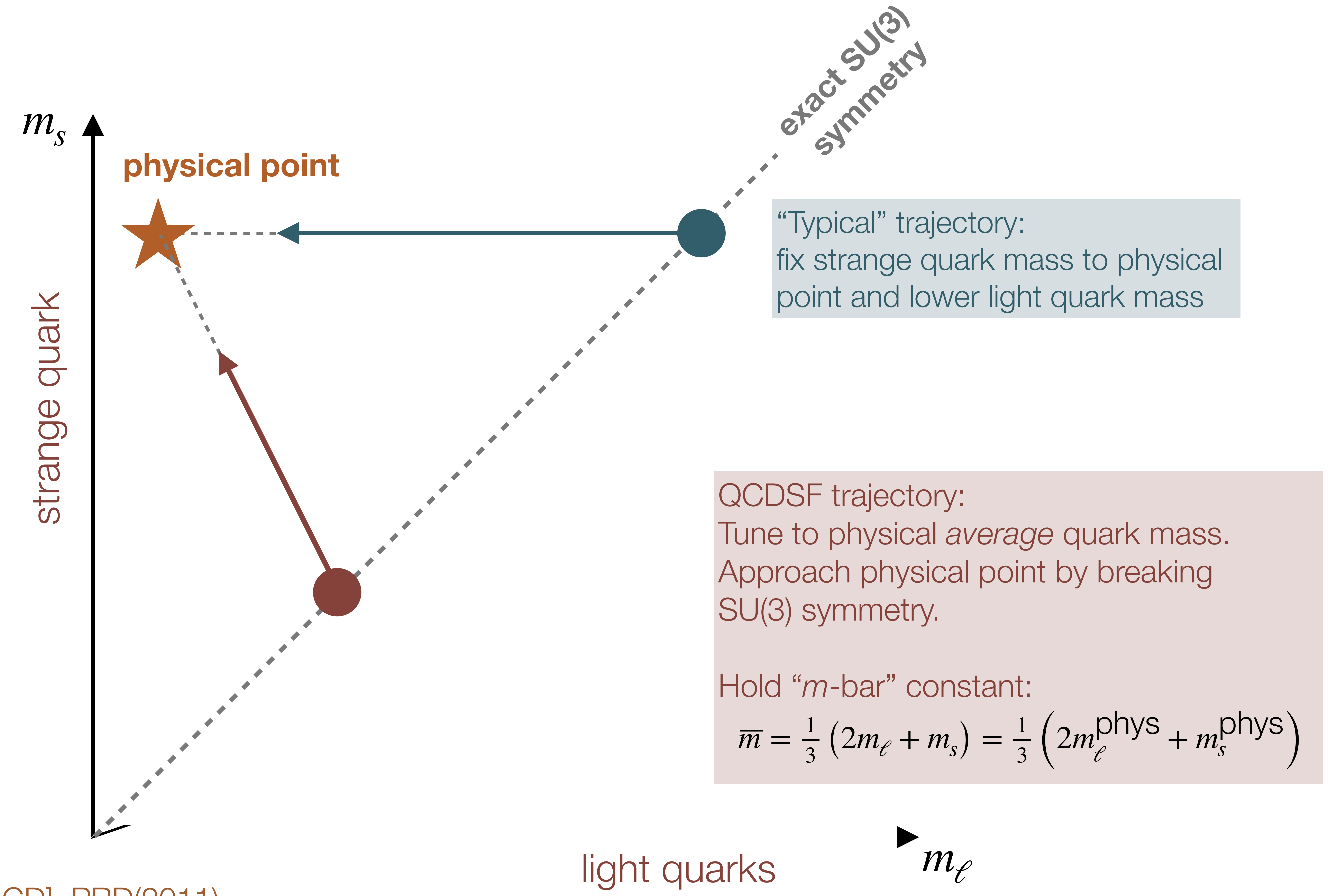
$$E(\lambda) = E(0) \pm \lambda g_T^q + \mathcal{O}(\lambda^2)$$

Spin-independent:

$$\left.\frac{\partial E(\lambda)}{\partial\lambda}\right|_{\lambda=0} = +\,g_S^q$$

$$E(\lambda) = E(0) + \lambda g_S^q + \mathcal{O}(\lambda^2)$$

Quark mass trajectory



"Typical" trajectory:
fix strange quark mass to physical point and lower light quark mass

QCDSF trajectory:
Tune to physical *average* quark mass.
Approach physical point by breaking SU(3) symmetry.

Hold "*m*-bar" constant:

$$\overline{m} = \frac{1}{3}\left(2m_\ell + m_s\right) = \frac{1}{3}\left(2m_\ell^{\text{phys}} + m_s^{\text{phys}}\right)$$

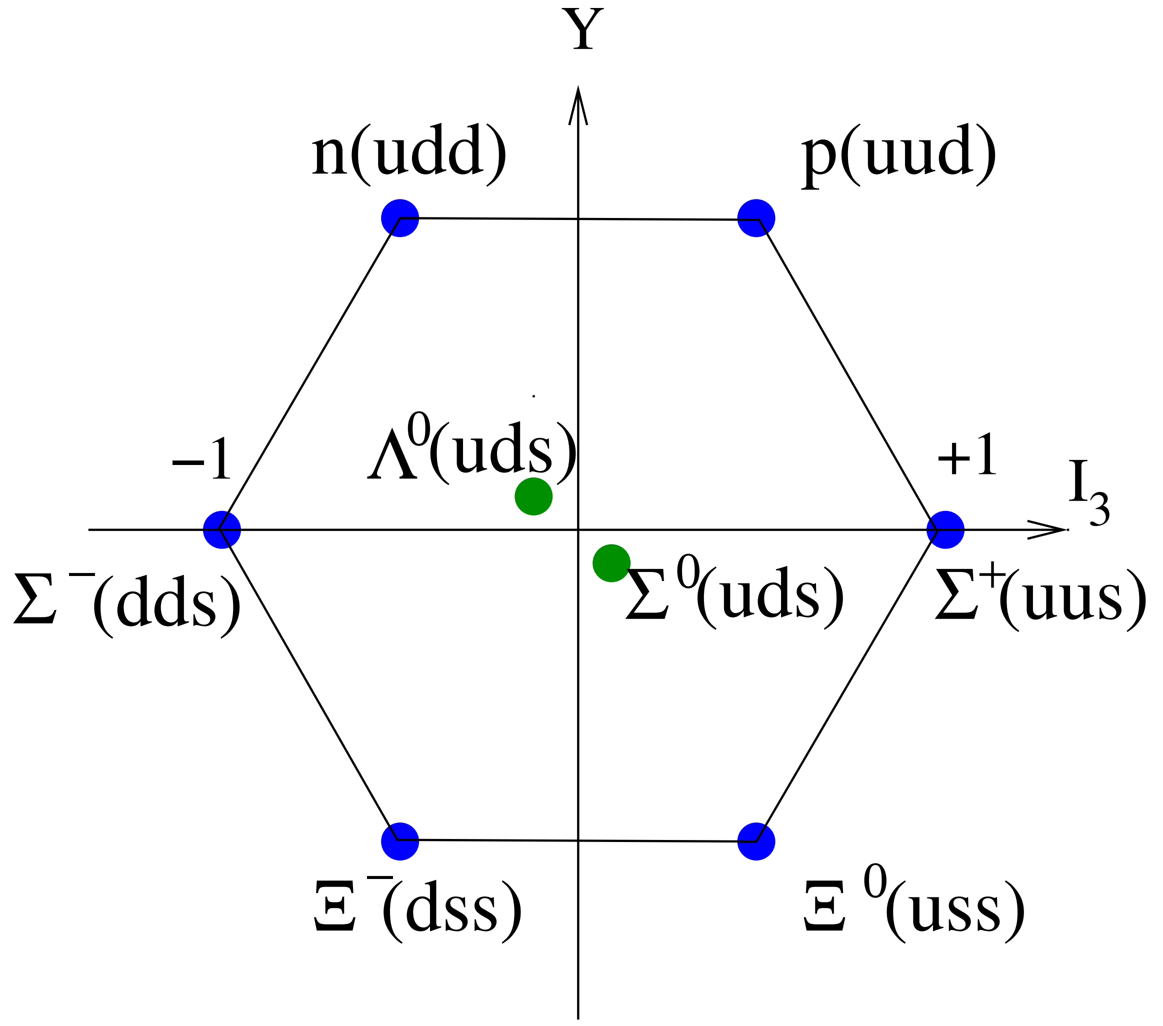
Bietenholz *et al*. [QCDSF-UKQCD], PRD(2011)

Flavour-breaking expansion

Bickerton, Horsley *et al.* [CSSM/QCDSF/UKQCD], PRD(2019)

Consider general flavour matrix elements of octet baryons:

$$\langle B' | J^F | B \rangle = A_{B'FB}$$



In exact SU(3) limit, just 2 independent constants:

- *F*- and *D*-type couplings

At linear order in SU(3) breaking: 5 slope parameters (3 D's & 2 F's)

- # of parameters (polynomials/operators) reduced by restricting to $\bar{m}$ = constant line

$$\begin{aligned}
F_1 \equiv \frac{1}{\sqrt{3}}(A_{\bar{N}\eta N} - A_{\bar{\Xi}\eta\Xi}) &= 2f - \frac{2}{\sqrt{3}} s_2 \delta m_l, \\
F_2 \equiv (A_{\bar{N}\pi N} + A_{\bar{\Xi}\pi\Xi}) &= 2f + 4 s_1 \delta m_l, \\
F_3 \equiv A_{\bar{\Sigma}\pi\Sigma} &= 2f + (-2 s_1 + \sqrt{3} s_2) \delta m_l, \\
F_4 \equiv \frac{1}{\sqrt{2}}(A_{\bar{\Sigma}K\Xi} - A_{\bar{N}K\Sigma}) &= 2f - 2 s_1 \delta m_l, \\
F_5 \equiv \frac{1}{\sqrt{3}}(A_{\bar{\Lambda}K\Xi} - A_{\bar{N}K\Lambda}) &= 2f + \frac{2}{\sqrt{3}}(\sqrt{3} s_1 - s_2) \delta m_l.
\end{aligned}$$

All matrix elements identical in the SU(3) symmetric limit

Index	Baryon (B)	Meson (F)	Current (J^F)
1	n	K^0	$\bar{d}\gamma s$
2	p	K^+	$\bar{u}\gamma s$
3	Σ^-	π^-	$\bar{d}\gamma u$
4	Σ^0	π^0	$\frac{1}{\sqrt{2}}(\bar{u}\gamma u - \bar{d}\gamma d)$
5	Λ^0	η	$\frac{1}{\sqrt{6}}(\bar{u}\gamma u + \bar{d}\gamma d - 2\bar{s}\gamma s)$
6	Σ^+	π^+	$\bar{u}\gamma d$
7	Ξ^-	K^-	$\bar{s}\gamma u$
8	Ξ^0	$\bar{K}^0$	$\bar{s}\gamma d$
0		η'	$\frac{1}{\sqrt{6}}(\bar{u}\gamma u + \bar{d}\gamma d + \bar{s}\gamma s)$

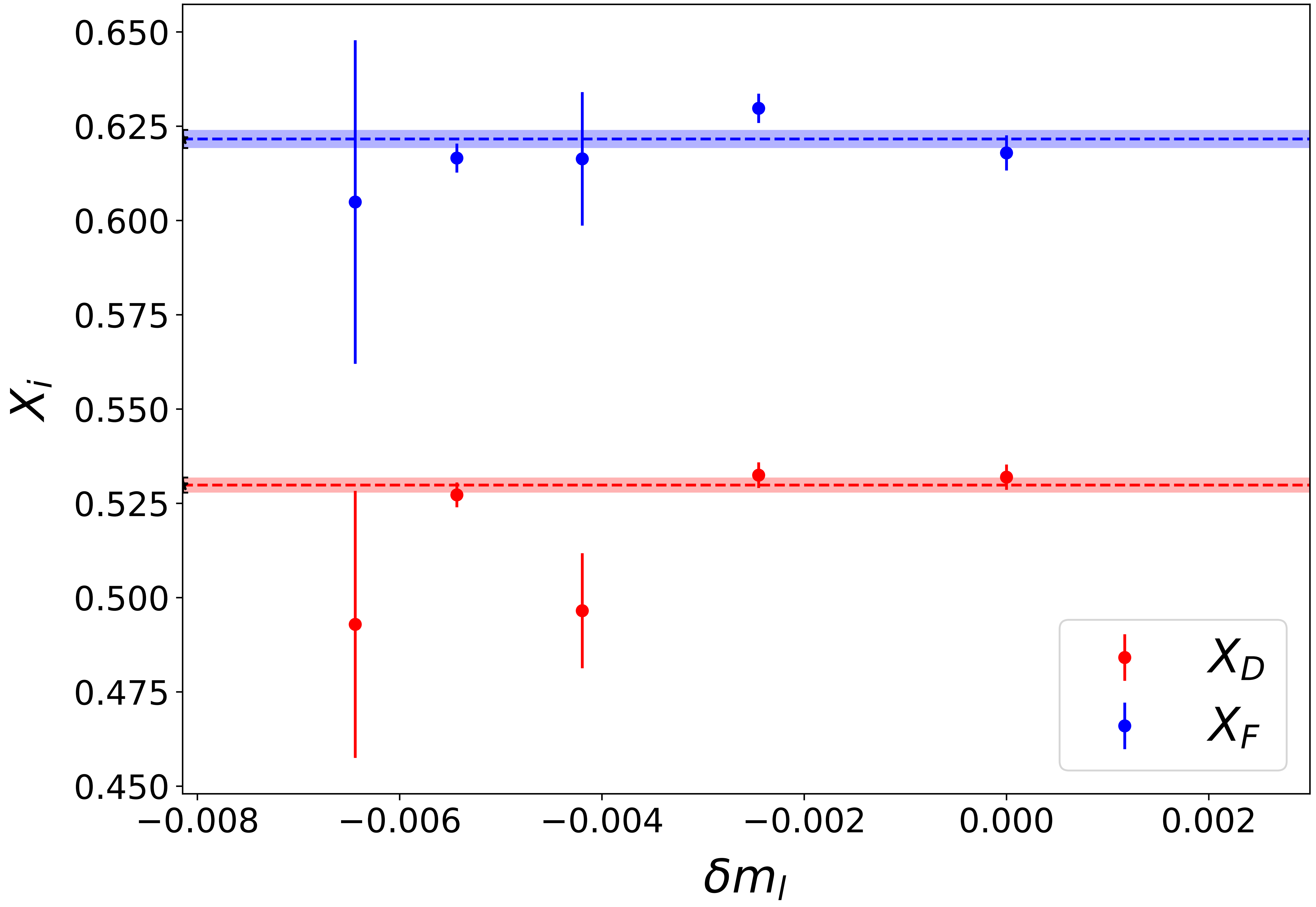
Fan plots

a=0.068fm

Can form a "singlet" combination

$$X_F = \frac{1}{6}(3F_1 + F_2 + 2F_3) = 2f + \mathcal{O}(\delta m_\ell^2)$$

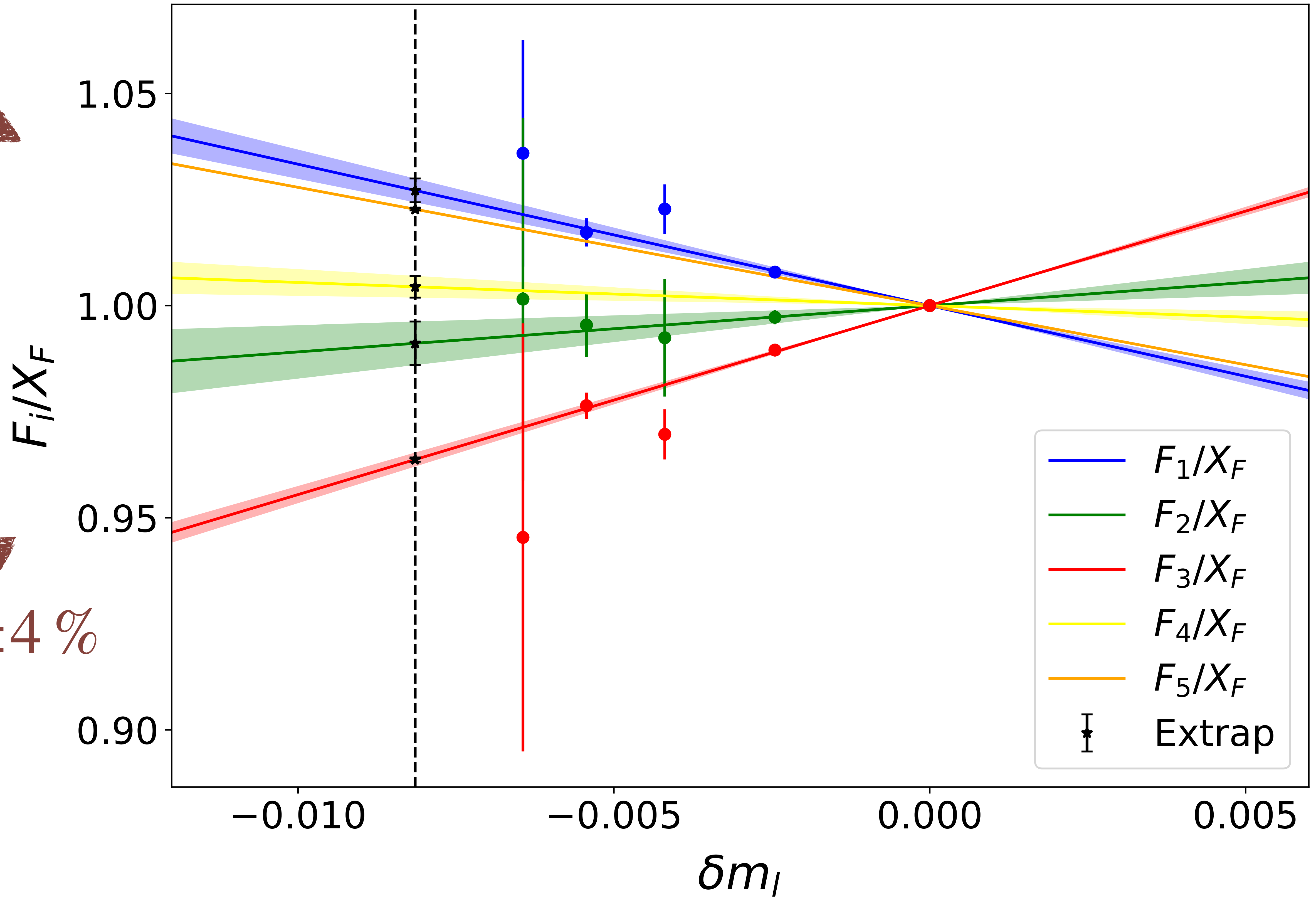
General result: Singlet quantities only vary at 2nd-order in SU(3) breaking.



$$\begin{aligned}
F_1 &\equiv \frac{1}{\sqrt{3}}(A_{\bar{N}\eta N} - A_{\bar{\Xi}\eta\Xi}) &= 2f - \frac{2}{\sqrt{3}}s_2\delta m_l,\\
F_2 &\equiv (A_{\bar{N}\pi N} + A_{\bar{\Xi}\pi\Xi}) &= 2f + 4s_1\delta m_l,\\
F_3 &\equiv A_{\bar{\Sigma}\pi\Sigma} &= 2f + (-2s_1 + \sqrt{3}s_2)\delta m_l,\\
F_4 &\equiv \frac{1}{\sqrt{2}}(A_{\bar{\Sigma}K\Xi} - A_{\bar{N}K\Sigma}) &= 2f - 2s_1\delta m_l,\\
F_5 &\equiv \frac{1}{\sqrt{3}}(A_{\bar{\Lambda}K\Xi} - A_{\bar{N}K\Lambda}) &= 2f + \frac{2}{\sqrt{3}}(\sqrt{3}s_1 - s_2)\delta m_l.
\end{aligned}$$

F fan

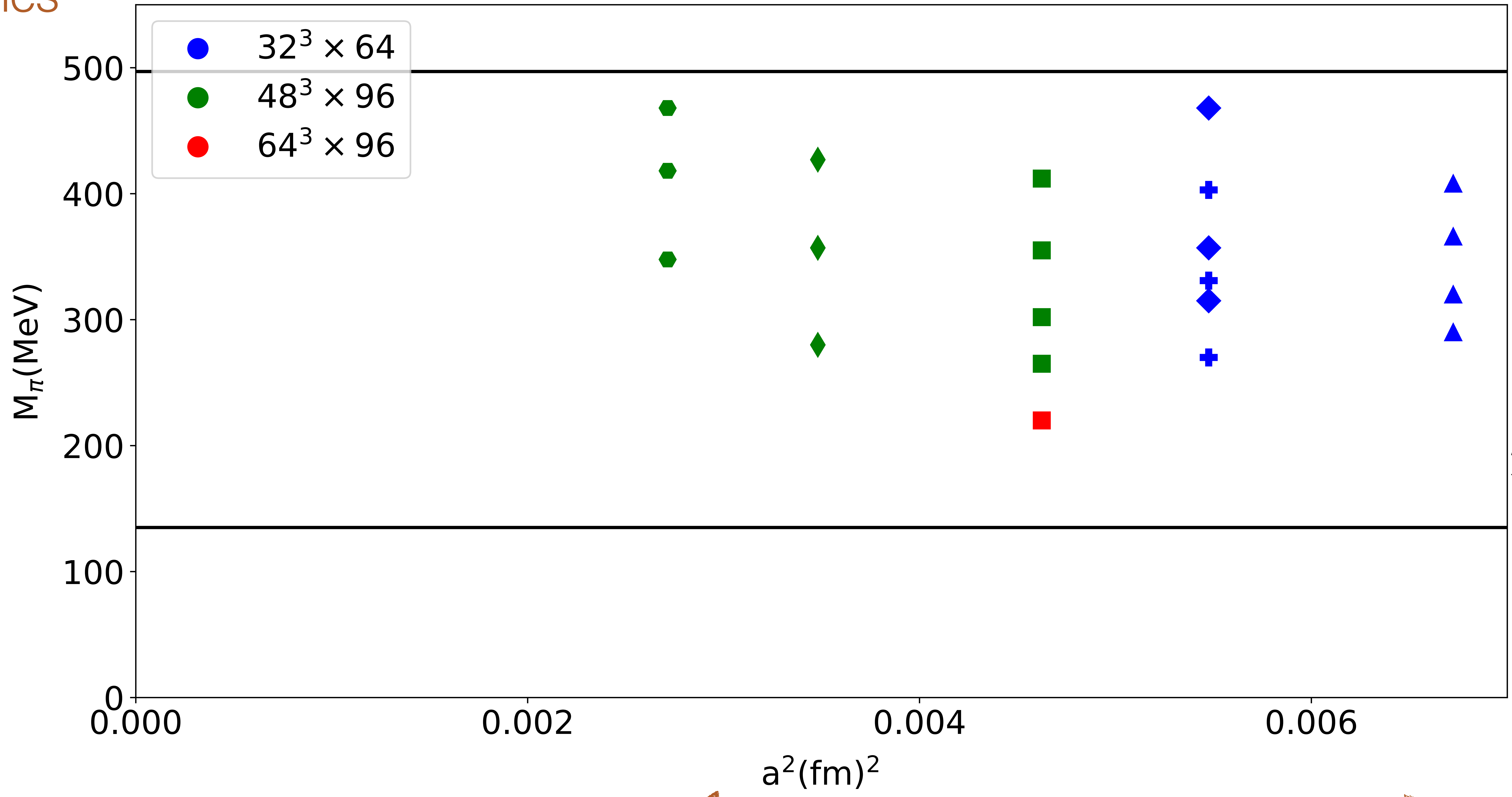
$\pm 4\,\%$



Simulation details

2+1 flavour, NP-improved Wilson fermions

3 volumes



pion masses

$220 \leftrightarrow 500\,\mathrm{MeV}$

5 lattice spacings $0.052 \leftrightarrow 0.082\,\mathrm{fm}$

Global fits

Want result

- in continuum and infinite volume limits
- at physical quark masses

Global fit

$$\delta m_l \to \delta m_l = \frac{m_\pi^2 - X_\pi^2}{X_\pi^2}$$

- Include $O(a)$ or $O(a^2)$ terms in X (singlet) and slope parameters

$$X_{D,F} = X^*_{D,F}(1 + c_1 \frac{1}{3}[f_L(m_\pi) + 2f_L(m_\pi)]) + c_2 a + c_3 \delta m_l^2$$

e.g. $\tilde{D}_1 = 1 - 2(\tilde{r}_1 + \tilde{b}_1 a)\delta m_l + \tilde{d}_1 \delta m_l^2$

- Free parameter to encode leading finite-volume correction on singlet:

$$f_L(m) = \left(\frac{m}{X_\pi}\right)^2 \frac{e^{-mL}}{\sqrt{mL}}$$

[functional form from chiral EFT, see Beane & Savage PRD(2004)]

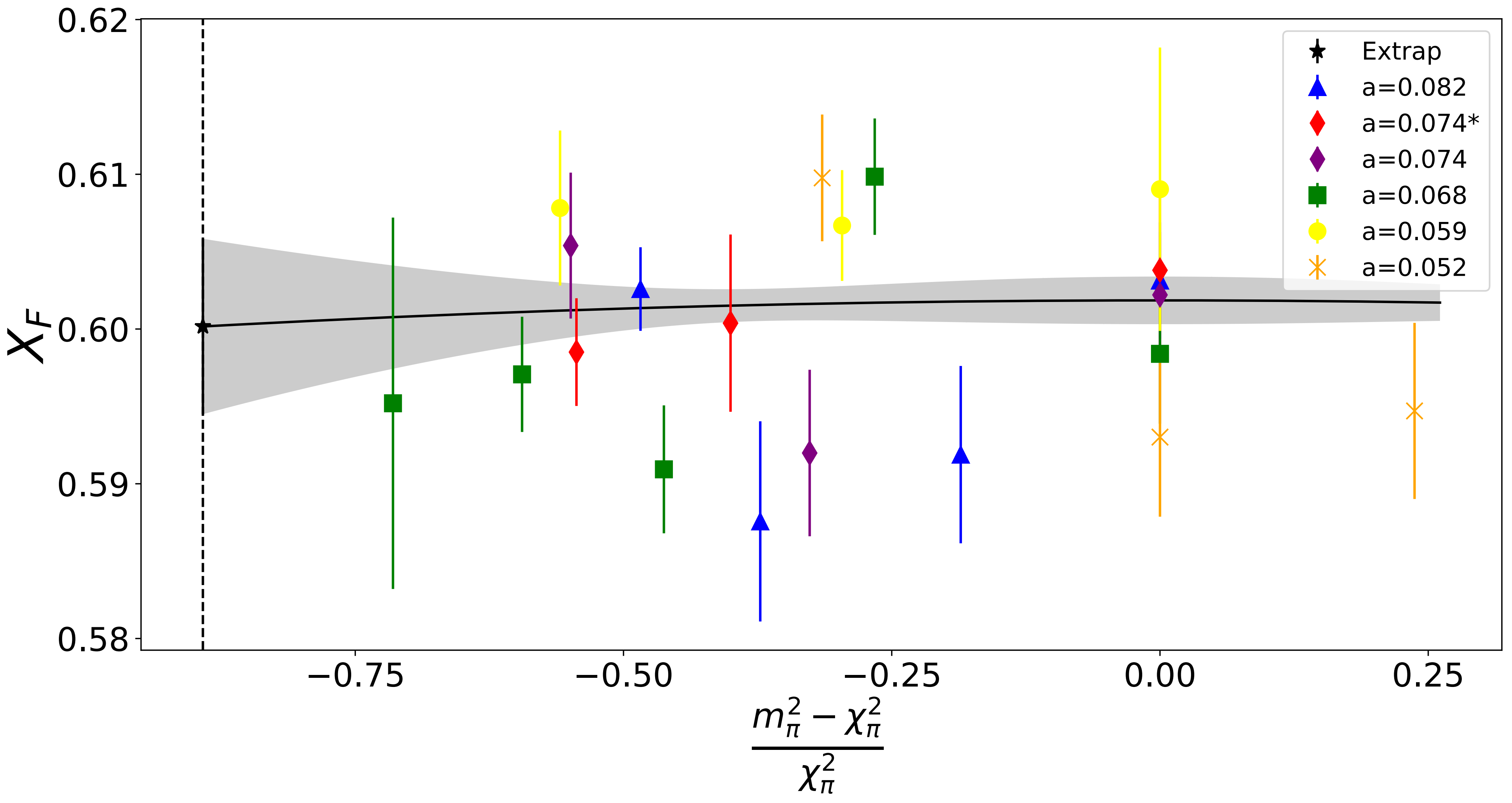
- Work to $O(\delta m_l^2)$ in flavour expansion

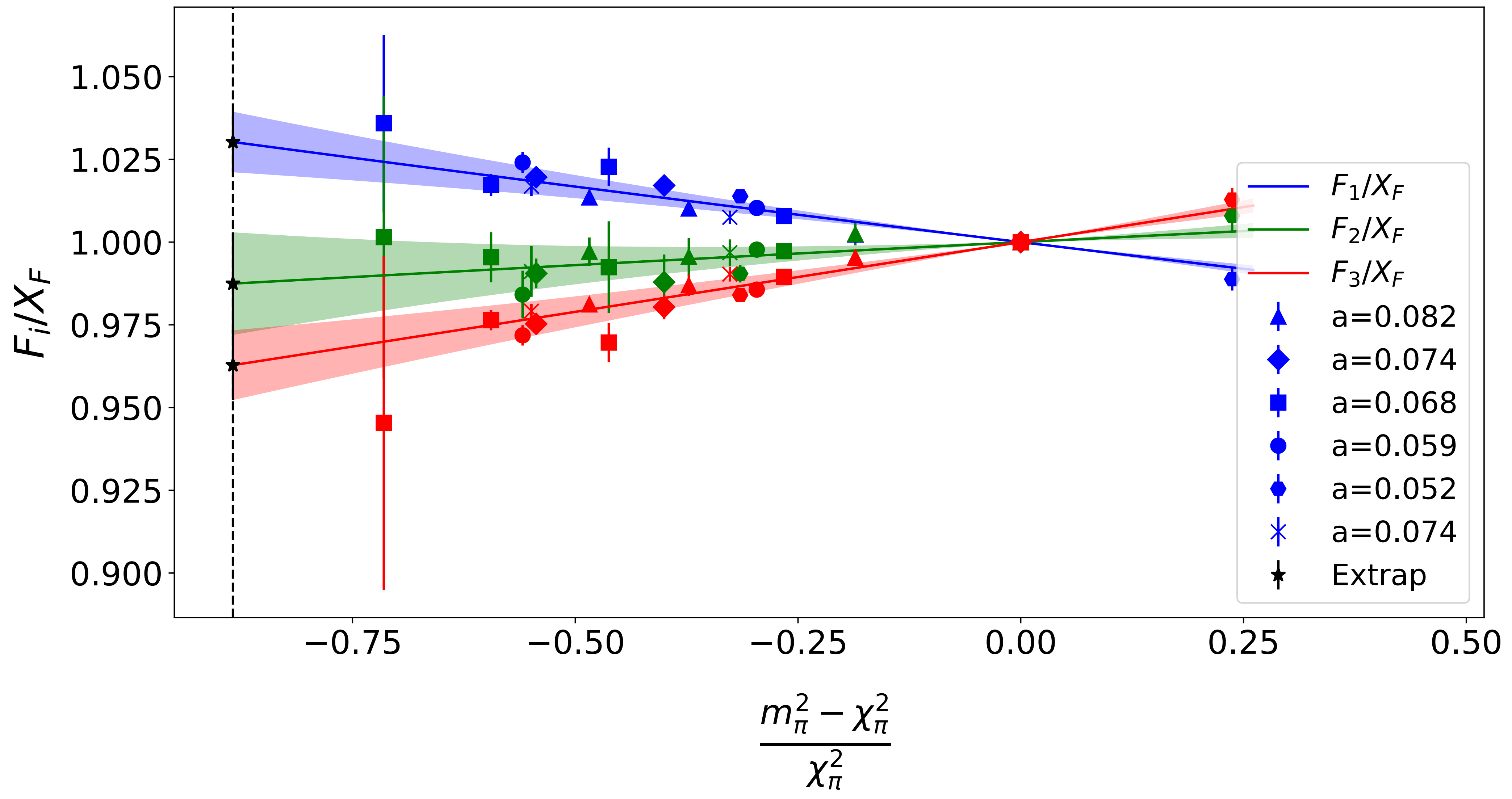
Global fits

Singlet X_F

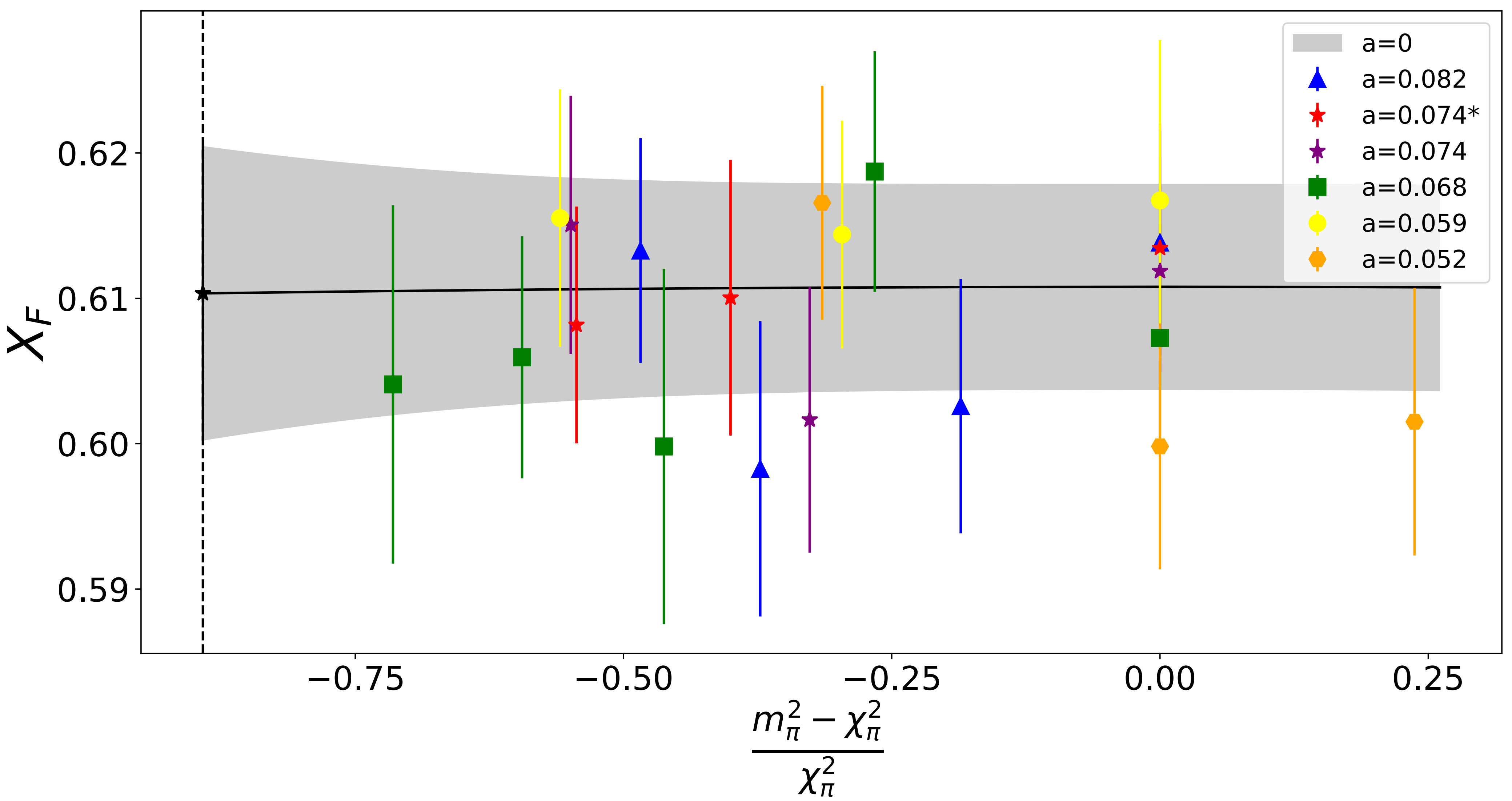
F slope parameters

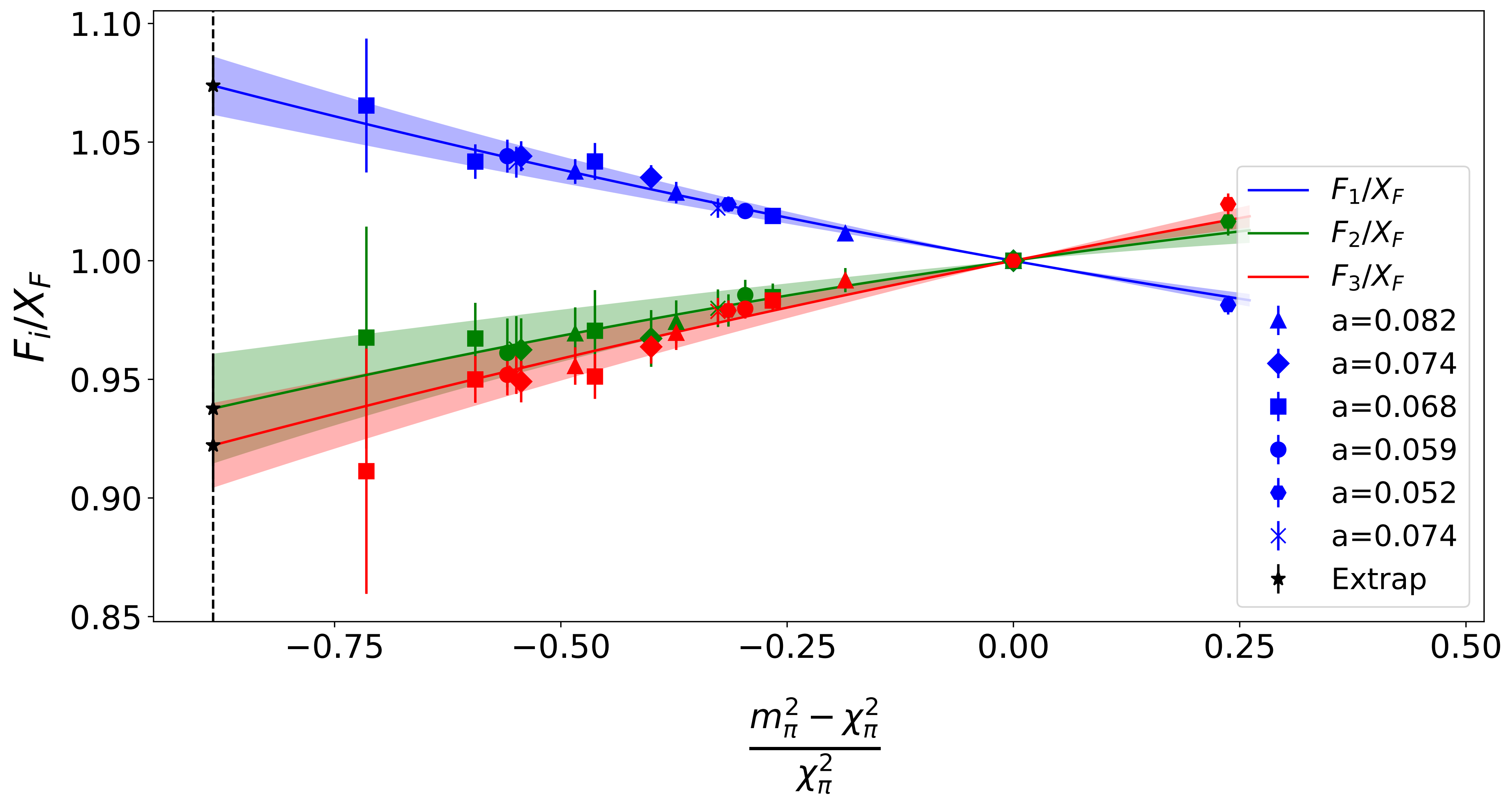
Quark mass only





With O(a) and FV

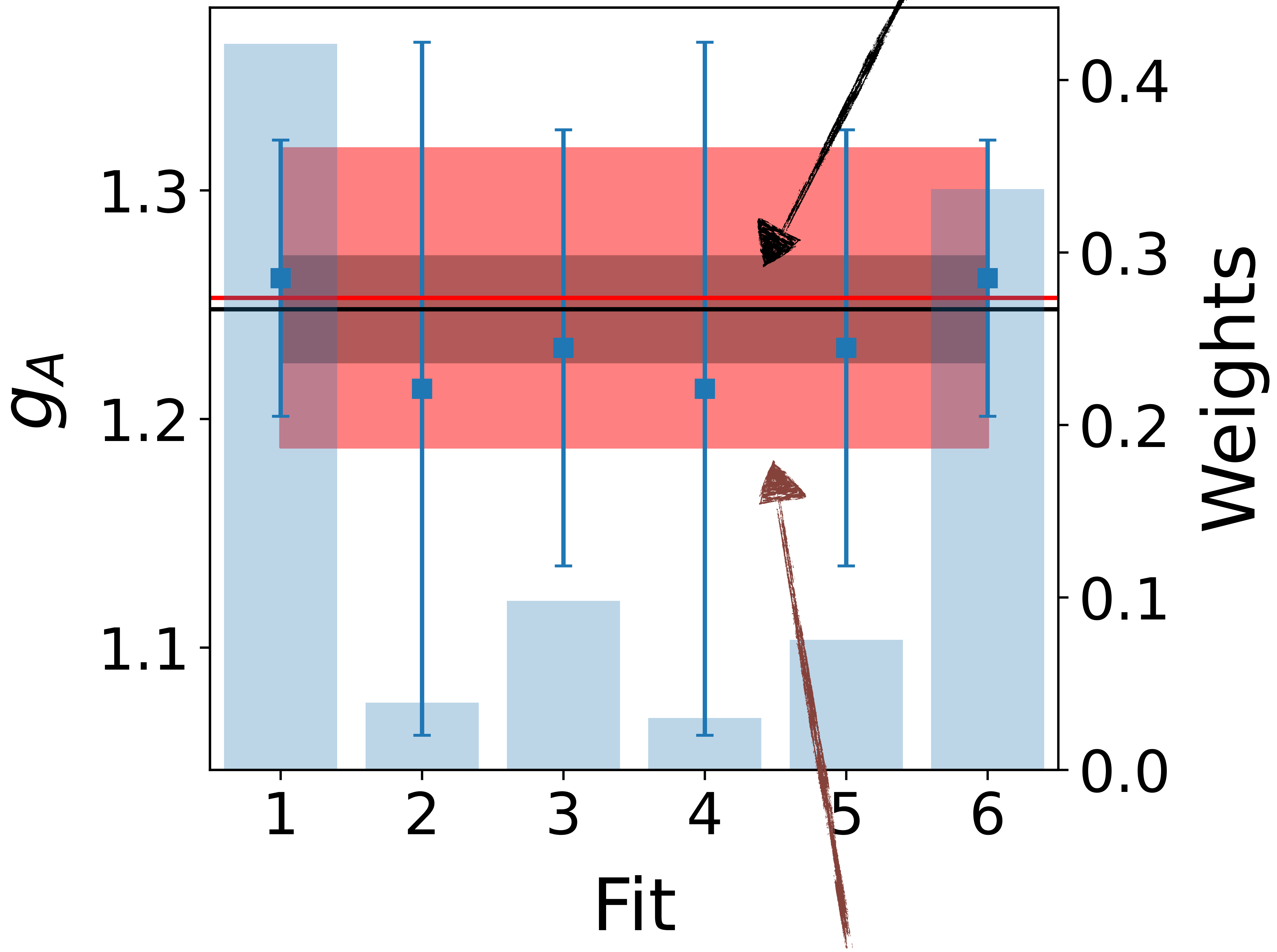




Results - g_A (isovector)

weighted average among models (as above)

FLAG result, ~2.2%

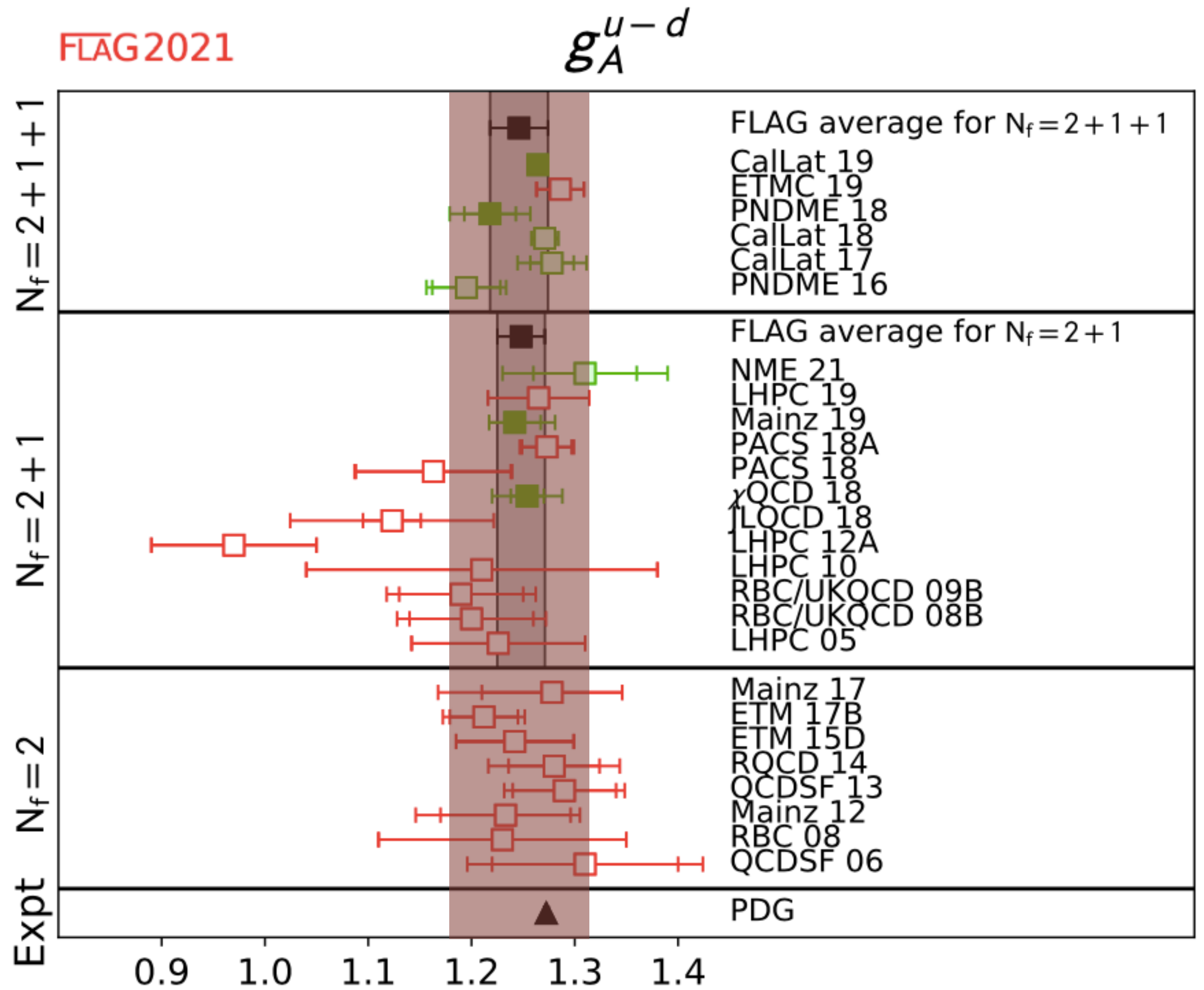


Our result, (stat+sys)~5.5%

$$g_A^{u-d} = 1.253(63)_{\text{stat}}(41)_a(03)_{\text{FV}}$$

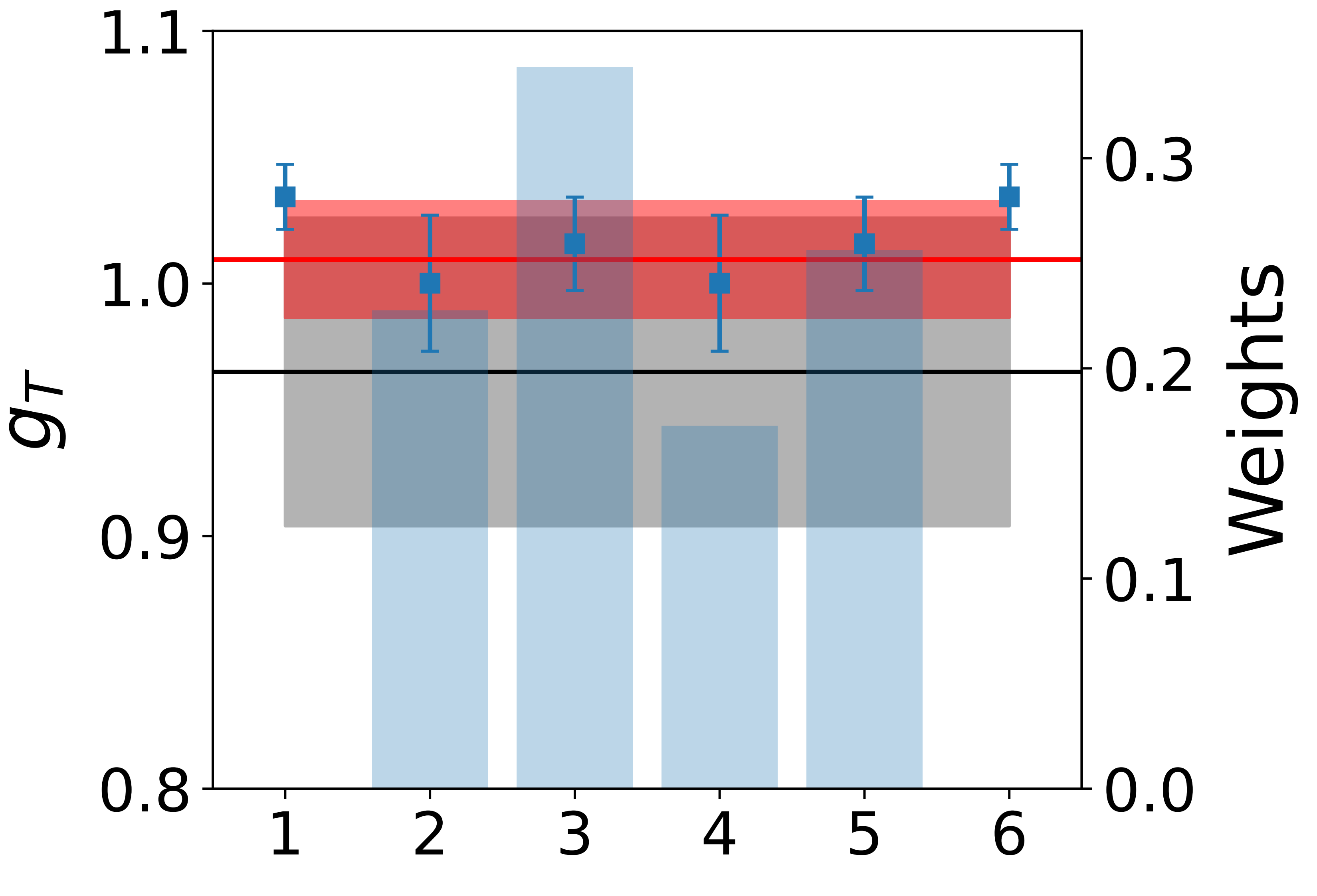
Different model parameterisations

1. δm_l^2
2. $a,\ \delta m_l^2$
3. $a^2,\ \delta m_l^2$
4. $a,\ \delta m_l^2,\ m_\pi L$
5. $a^2,\ \delta m_l^2,\ m_\pi L$
6. $\delta m_l^2,\ m_\pi L$



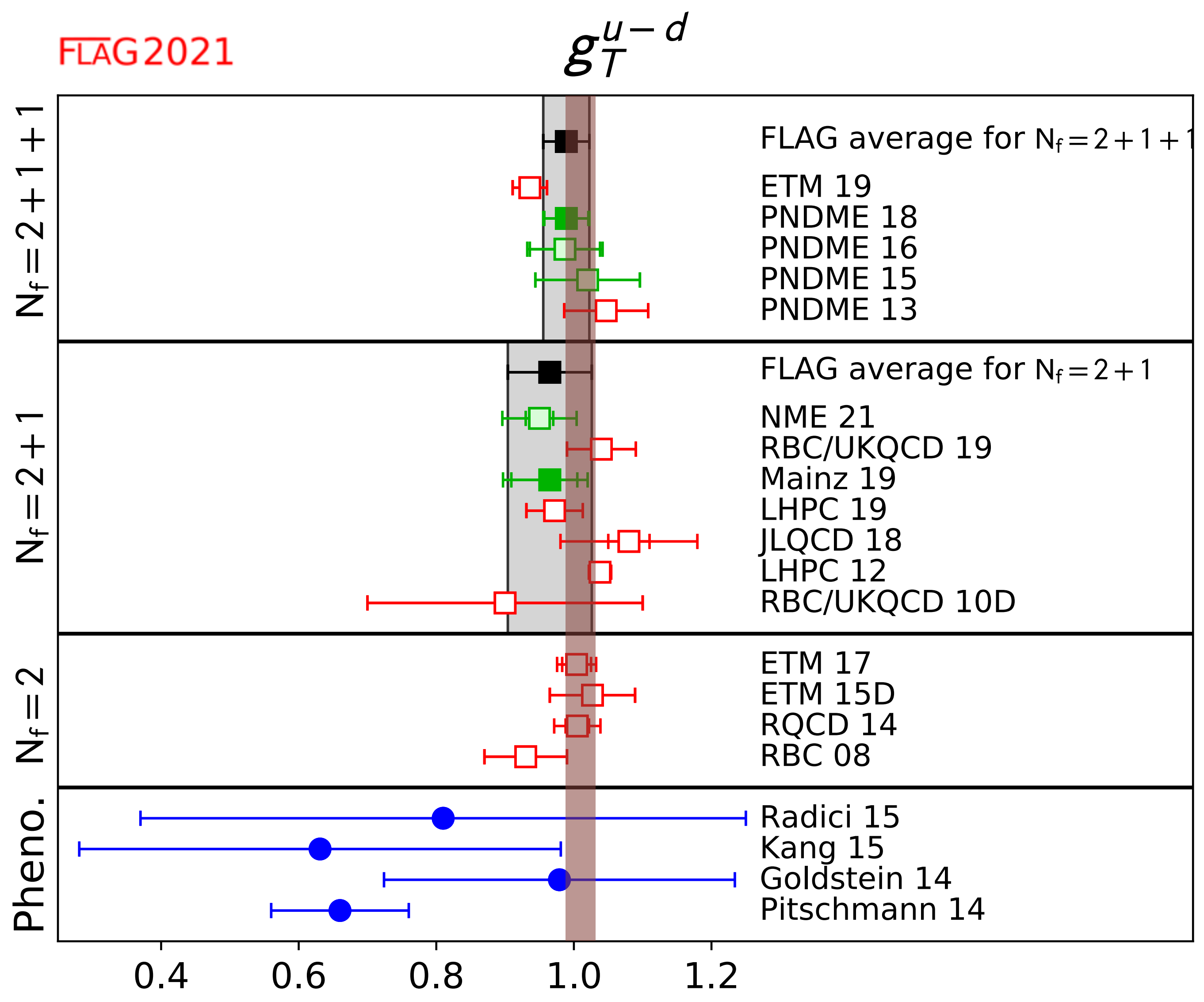
Results - isovector charges $N_f = 2 + 1$

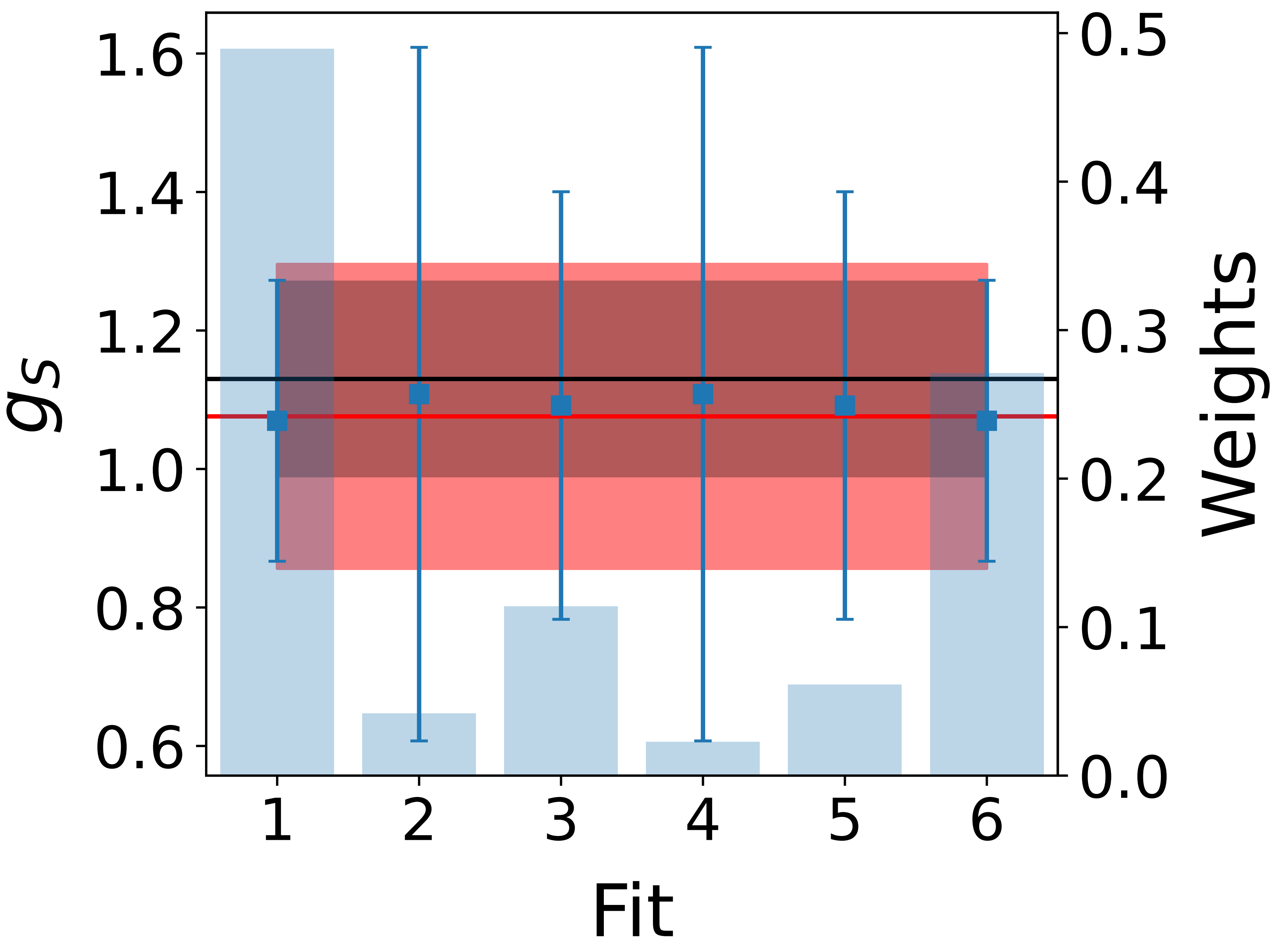
$\overline{\mathrm{MS}}, \mu = 2\,\mathrm{GeV}$



$$g_T^{u-d} = 1.010(21)_{\mathrm{stat}}(12)_a(01)_{\mathrm{FV}}$$

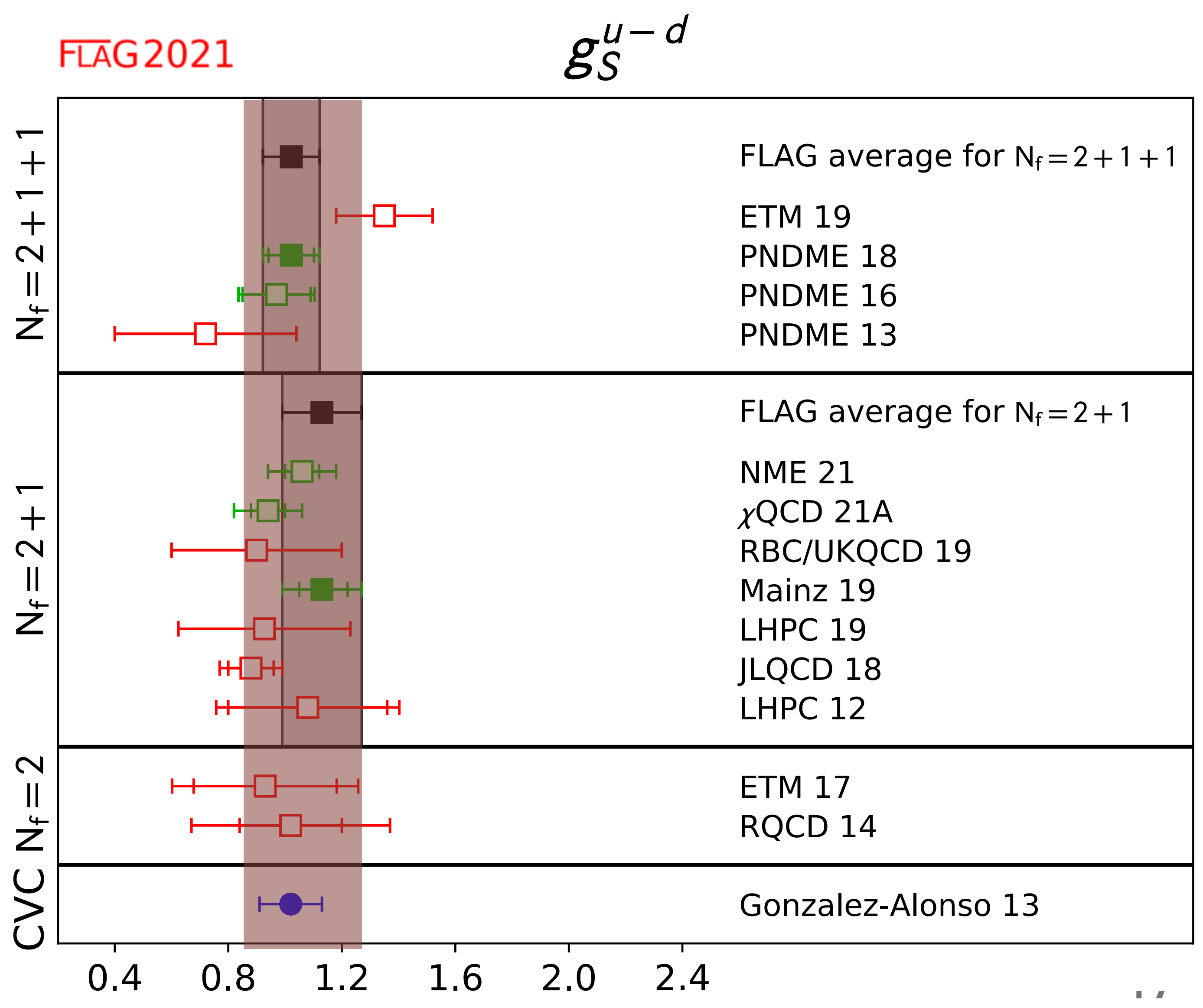
FLAG 2+1: ~6%
FLAG 2+1+1: ~3%
Our result: ~2%





$$g_S^{u-d} = 1.08(21)_{\mathrm{stat}}(03)_a(01)_{\mathrm{FV}}$$

FLAG 2+1: ~12%
Our result: ~19%



Results - Hyperon charges

Not in FLAG, but recent results by RQCD [2305.04717]

(see also previous talk)

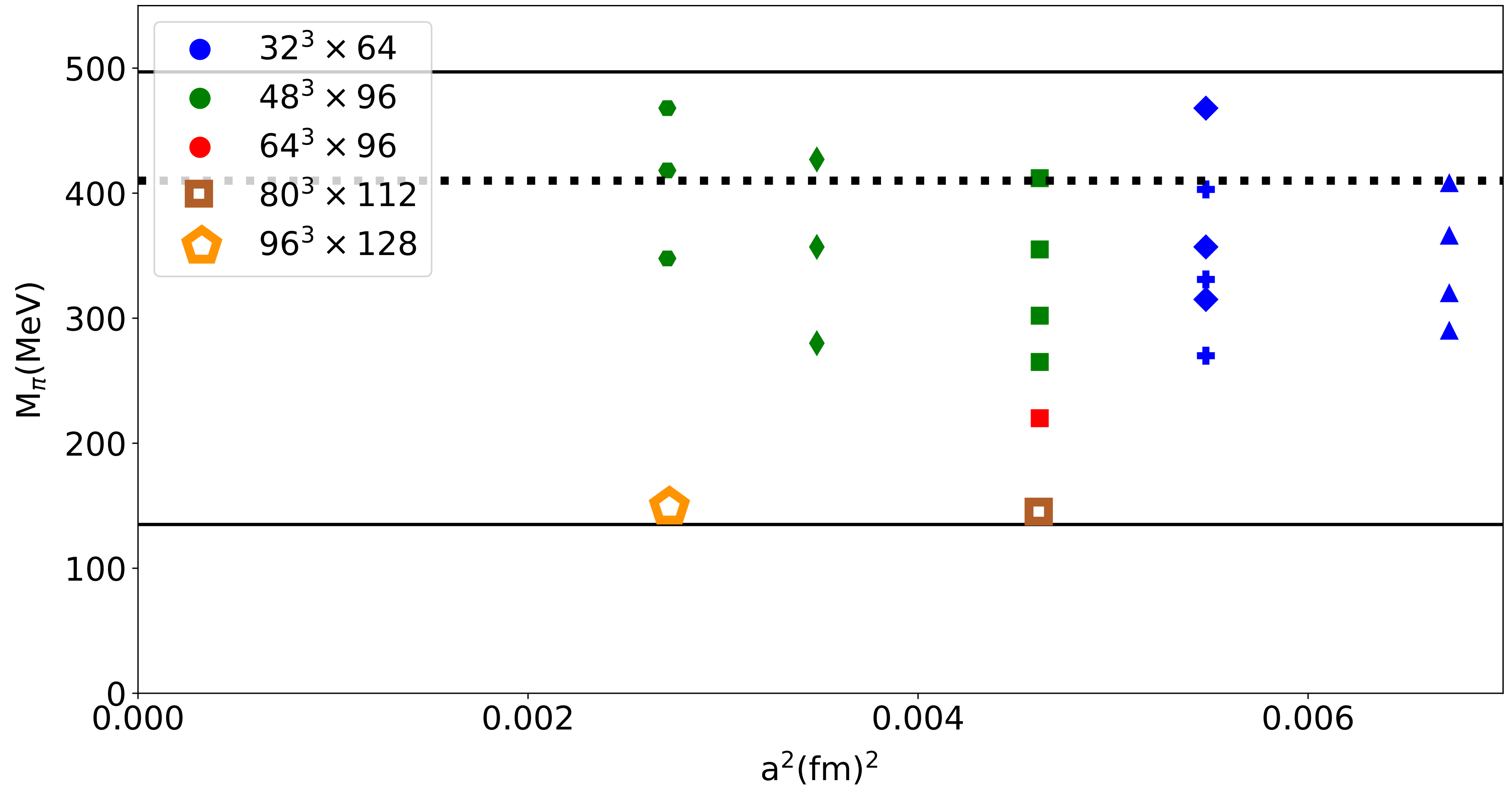
This work	*RQCD*
$g_T^\Sigma = 0.805(15)$	$g_T^\Sigma = 0.798(26)$
$g_T^\Xi = -0.1952(75)$	$g_T^\Xi = -0.1872(72)$
$g_A^\Sigma = 0.876(28)$	$g_A^\Sigma = 0.875(49)$
$g_A^\Xi = -0.206(21)$	$g_A^\Xi = -0.267(18)$
$g_S^\Sigma = 2.80(25)$	$g_S^\Sigma = 3.98(33)$
$g_S^\Xi = 1.59(12)$	$g_S^\Xi = 2.57(16)$

some tension

Summary and outlook

- Feynman Hellman theorem
  - provides a viable alternative to 3-pt function methods for computing hadronic matrix elements
- Flavour-breaking expansion along the $\bar{m}$ = constant line
  - allows for a controlled extrapolation from the SU(3)-symmetric point
- Future improvements
  - ensembles with near-physical quark masses and $4 \lesssim m_\pi L$
  - strong isospin breaking effects [c.f. QCDSF PLB(2012)]
  - gamma-W box (dispersion integral over moments of $F_3^{\gamma W}$)

$$\Box^b_{\gamma W}(E_e) = \frac{3\alpha}{2\pi}\int_0^\infty \frac{dQ^2}{Q^2}\frac{M_W^2}{M_W^2+Q^2}\left[M_{3,-}(1,Q^2)+\frac{8E_eM}{9Q^2}M_{3,+}(2,Q^2)\right]+\mathcal{O}(E_e^2)$$



For progress on moments of $F_3^{\gamma Z}$ via the Compton Amplitude, see K.U. Can, Fri, 9:40 (WH1W)