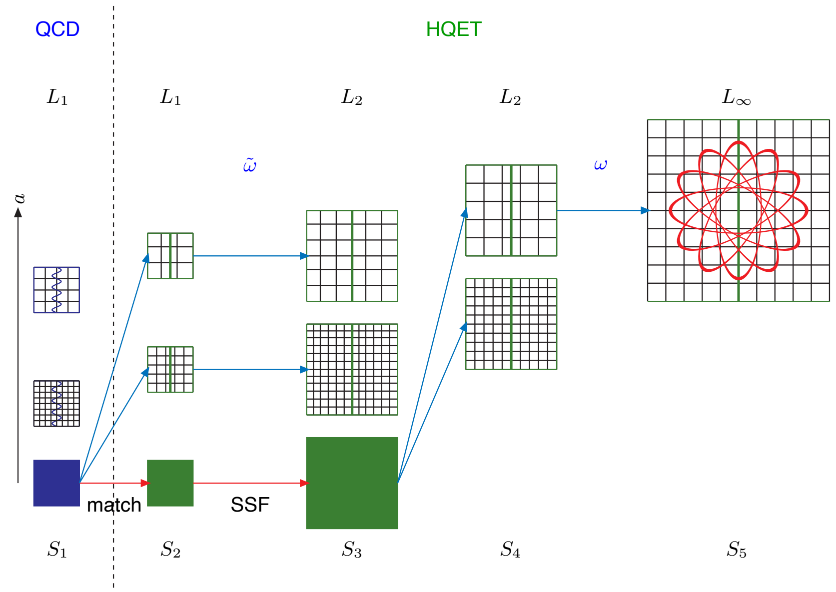
A strategy for B-physics observables in the continuum limit



Alessandro Conigli; Julien Frison; Patrick Fritzsch; Antoine Gérardin; Jochen Heitger; Simon Kuberski; Gregorio Herdoiza; Hubert Simma; Carlos Pena; Rainer Sommer

Speaker: RS (John von Neumann Institute for Computing, DESY & Humboldt University, Berlin)

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- Now and for some time to come

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  Therefore lattice computations are impossible without EFTs

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  - renormalisable <=> continuum limit
  - but: $1/m_b$ corrections are needed for precision and they are challenging

Combination of static ($m_h = \infty$) and relativistic $m_h < m_b$

[Guazzini, S., Tantalo, 0710.2229
Heitger, S., hep-lat/0310035
Guagnelli et al.,hep-lat/0206023
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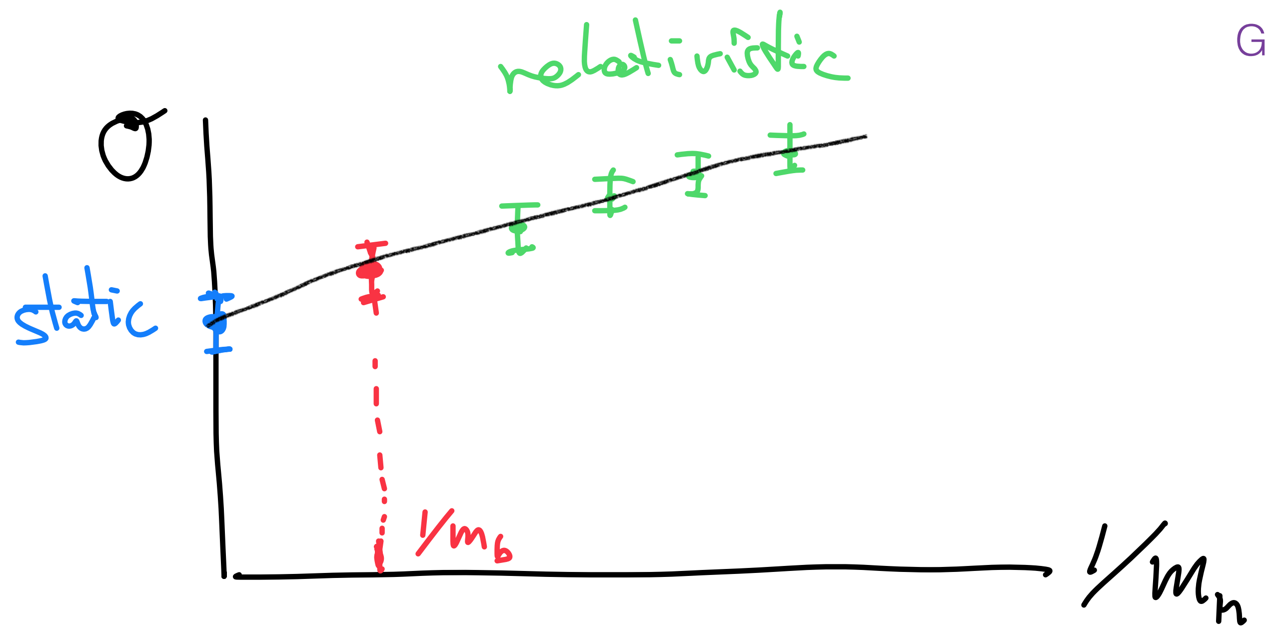
- simple idea



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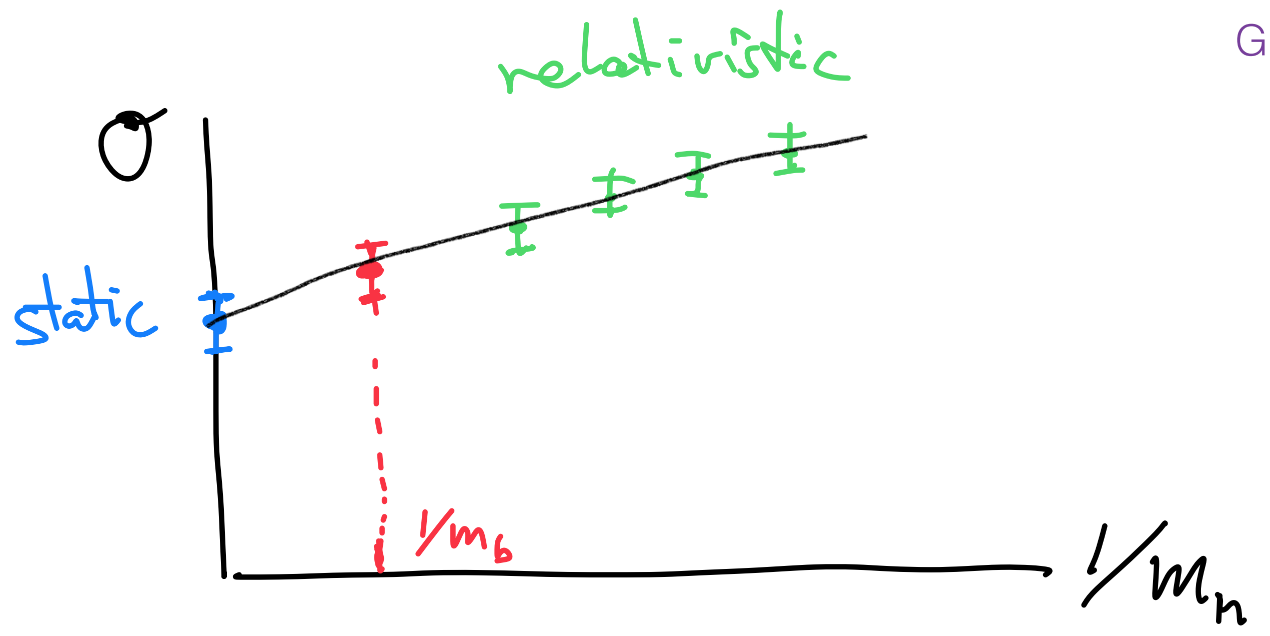

- this can't be controlled in practice because

  even static approximation requires non-trivial renormalisation+matching (perturbation theory not well behaved)

  —> NP treatment needs again $am_b \ll 1$

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- Quick reminder of renormalisation+matching
  - static action: $\mathscr{L}^{\rm stat} = \bar\psi_h \, D_0 \, \psi_h$, —> $E^{\rm stat} \sim g_0^2 \dfrac{1}{a}$

    —> add $\delta m \sim g_0^2 \dfrac{1}{a}$ and $m_b^{\rm finite}$: $E = E^{\rm stat} + \delta m + m_b^{\rm finite}$

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    renormalisation matching

  - heavy-light currents:

$$V_k^{\rm stat} = C_{V_k}(m_b) \; Z^{\rm stat}(g_0) \; \bar\psi_h \gamma_k \psi_l$$

$$V_0^{\rm stat} = C_{V_0}(m_b) \; Z^{\rm stat}(g_0) \; \bar\psi_h \gamma_0 \psi_l$$

matching renorm.

$C_J(m_b)$ log-divergent as $m_b \to \infty$

Combination of static ($m_h = \infty$) and relativistic $m_h < m_b$

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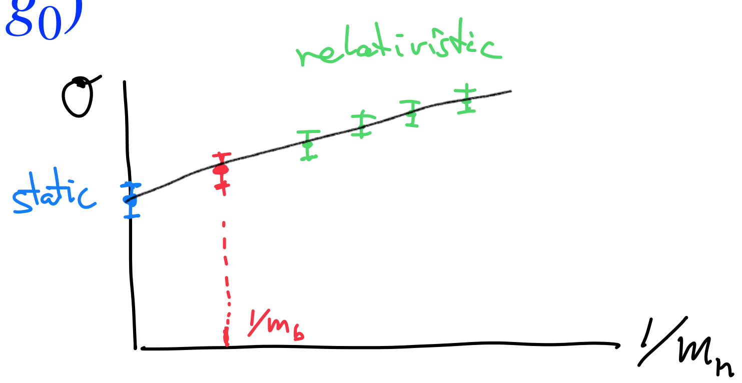
- Example 1: q^2 - dependence of semi-leptonic form-factors

$$\frac{1}{\sqrt{2p_k^\pi}}\ {}_{\rm rel}\langle \pi(p_\pi)\,|\,V_k(0)\,|\,B(\vec{p}=0)\rangle_{\rm NR} = h_\perp(E_\pi) = h_\perp^{\rm stat}(E_\pi) + \mathrm{O}(1/m_b)$$

$$h_\perp^{\rm stat}(E_\pi) = C(m_b)\, Z^{\rm stat}(g_0)\, h_\perp^{\rm stat,bare}(E_\pi, g_0)$$

therefore

$$\frac{h_\perp(E_\pi)}{h_\perp(E_\pi^{\rm ref})} = \frac{h_\perp^{\rm stat,bare}(E_\pi)}{h_\perp^{\rm stat,bare}(E_\pi^{\rm ref})} + \mathrm{O}(1/m_b)$$



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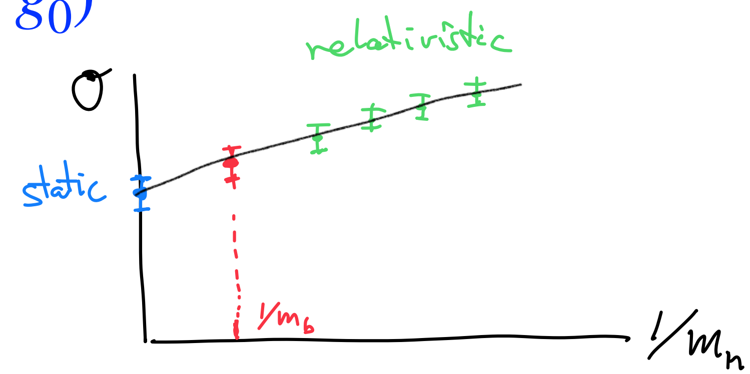

- static limit computable without renormalisation and matching

$\alpha_s(m_b)$ modifications only in $\mathrm{O}(1/m_b)$ terms

captured by effective fit descriptions of corrections

$$c_1/m_b + c_2/m_b^2 + \ldots \;\approx\; c_1'[\alpha_s(m_b)]^{\hat\gamma_1}/m_b + c_2'[\alpha_s(m_b)]^{\hat\gamma_2}/m_b^2 + \ldots$$

Combination of static ($m_h = \infty$) and relativistic $m_h < m_b$

- Example 2: determination of the quark mass

  - heavy-light mass <—> quark mass

$$\frac{m_{\rm PS}(L_1)}{m_h^{\rm RGI}} = \pi_m$$ computable with relativistic b-quarks in finite vol. $L = L_1 \approx 0.5$ fm

using NP relation $m_i^{\rm RGI} = Z(g_0)\, m_i^{\rm PCAC}$
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  - need $m_{\rm PS}(L_1)$ [Heitger, S. hep-lat/0110016]

$$m_{\rm PS}(L_1) = m_{\rm PS} - [m_{\rm PS} - m_{\rm PS}(L_2)] - [m_{\rm PS}(L_2) - m_{\rm PS}(L_1)]$$

dimensionless by reference scale $L_{\rm ref}$

$$L_{\rm ref} m_{\rm PS}(L_1) = L_{\rm ref} m_{\rm PS} - L_{\rm ref}[m_{\rm PS} - m_{\rm PS}(L_2)] - L_{\rm ref}[m_{\rm PS}(L_2) - m_{\rm PS}(L_1)]$$
$$= L_{\rm ref} m_{\rm PS} - \rho_m - \sigma_m$$

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dimensionless variables for PS masses (proxy for quark mass)

$$y_B = L_{\rm ref} m_{\rm B}\,, \quad y_2 = L_{\rm ref} m_{\rm PS}(L_2)\,, \quad y_1 = L_{\rm ref} m_{\rm PS}(L_1)$$

dimensionless variables for box-lengths

$$u_i = \bar{g}^2(L_i)\,, \quad u_{i+1} = \sigma(u_i)\,, \quad L_2 = 2L_1$$

then

$$y_2 = y_B - \rho_m(u_2, y_B)\,, \qquad y_1 = y_2 - \sigma_m(u_1, y_2)$$

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$$\sigma_m(u_1, y) = \lim_{a/L \to 0} \Sigma_m(u_1, y, a/L)$$
$$\bar{g}^2(L) = u_1, \quad \text{LCP1}$$

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$$\rho_m(u_2, y) = \lim_{a/L \to 0} P_m(u_2, y, a/L) \qquad \sigma_m(u_1, y) = \lim_{a/L \to 0} \Sigma_m(u_1, y, a/L)$$

$$\bar{g}^2(L) = u_2, \quad \text{LCP2} \qquad \bar{g}^2(L) = u_1, \quad \text{LCP1}$$

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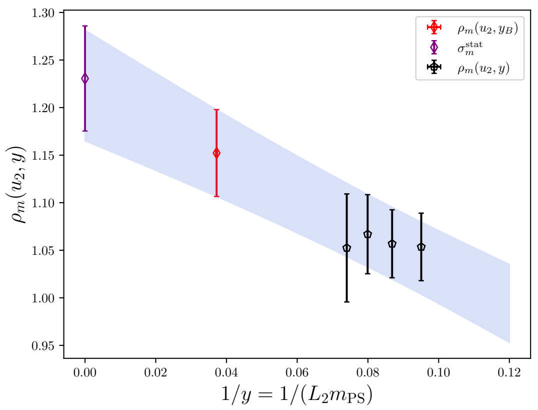
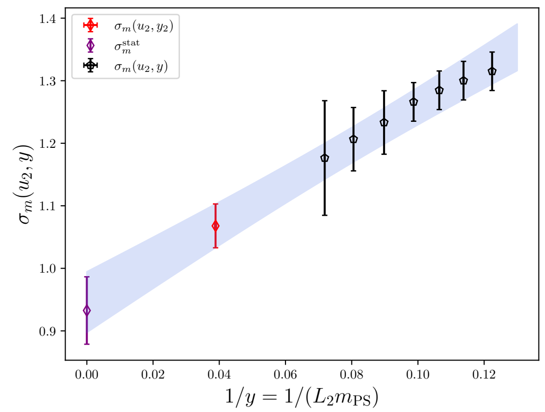




Combination of static ($m_h = \infty$) and relativistic $m_h < m_b$

- Example 3: determination of vector meson decay constant

  - matrix element of vector current

$$\hat{f}_V = \sqrt{2}\,\langle 0 \,|\, V_k(0) \,|\, V(\vec{p} = 0, k)\rangle_{\rm NR} = \hat{f}_V^{\rm stat} + \mathrm{O}(1/m_b)$$

multiplicative renormalisation + matching —> take log —> additive, looks as mass

$$\Phi_V = \log\left(L_{\rm ref}^{3/2}\,\hat{f}_V\right)$$

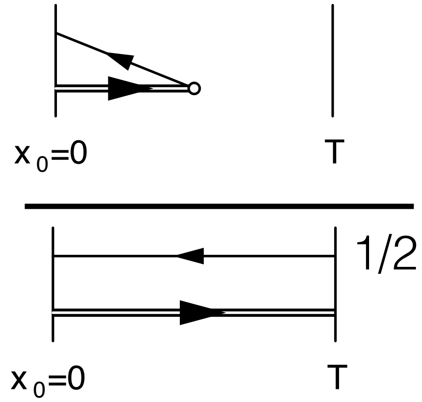
$$= \Phi_V(L_1) + [\Phi_V(L_2) - \Phi_V(L_1)] + [\Phi_V - \Phi_V(L_2)]$$

$$= \Phi_V(L_1) + \sigma_V(u_1, y_2) + \rho_V(u_2, y_B)$$

static: divergences cancel

Definition of $\Phi_V(L)$ (at finite L)

$$\Phi_V(L) = \log\left(L_{\rm ref}^{3/2}\,\langle \Omega(L) \,|\, V_k(0) \,|\, V(L,k)\rangle\right) =$$



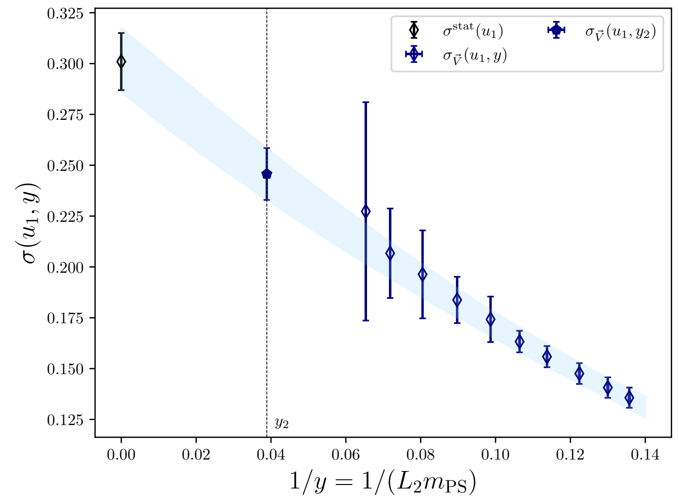
SF boundary states: vacuum QN, Vector HL QN's

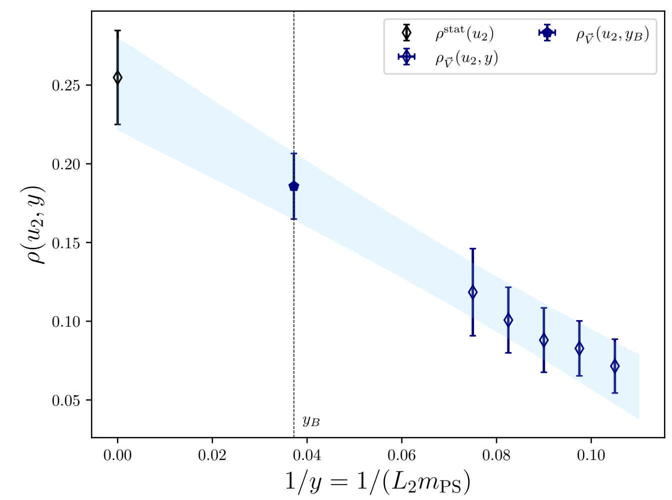
Combination of static $(m_h = \infty)$ and relativistic $m_h < m_b$

- Example 3: determination of vector meson decay constant

$$\Phi_V = \log\left(L_{\rm ref}^{3/2}\,\hat{f}_V\right) = \Phi_V(L_1) + \sigma_m(u_1, y_2) + \rho_m(u_2, y_B)$$

relativistic





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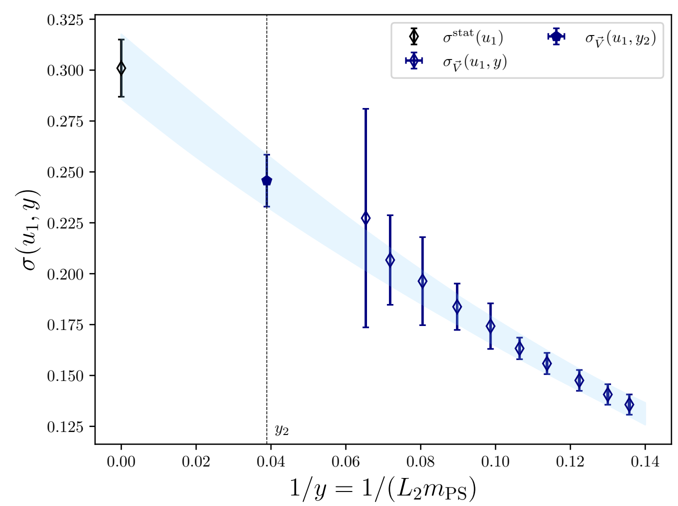

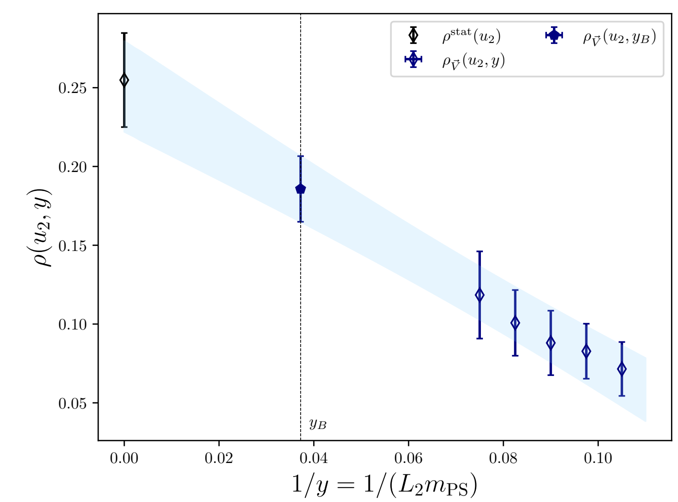

- And back to semi-leptonic decays:

$$h_\perp(E_\pi) = \hat{f}_V \,\frac{h_\perp(E_\pi)}{\hat{f}_V} = \hat{f}_V \left[\frac{h_\perp^{\rm stat}(E_\pi)}{\hat{f}_V^{\rm stat}} + {\rm O}(1/m_b)\right]$$

==> also normalisation of the form factor is determined => $V_{\rm ub}$

Summary

- for low energy physics, b-quarks are close to static

  corrections are computable by HQET but progress has been slow in practice

- a good alternative is a combination of static + relativistic with $m_h < m_b$

  provided that functions are formed which need

  neither renormalisation nor matching in the static approximation <— THE ONLY REQUIREMENT

- this applies directly in large volume, e.g. to form-factor ratios

$$\frac{h_\perp(E_\pi)}{h_\perp(E_\pi^{\rm ref})} = \frac{h_\perp^{\rm stat,bare}(E_\pi)}{h_\perp^{\rm stat,bare}(E_\pi^{\rm ref})} + \mathrm{O}(1/m_b)$$

($h_\parallel$ works basically the same way)

- it becomes more powerful by the use of finite volume and step scaling
  - $L = L_1 \approx 0.5$ fm relativistic at b-mass
  - $L_1 \to L_2 = 2L_1, \quad L_2 \to L_\infty \gg 1/m_\pi$ by combination

- note that most quantities are universal, in the continuum,

  and can be computed with any action, by any collaboration. E.g. one can do "just" the large volume $\frac{h_\perp(E_\pi)}{\hat{f}_V}, \frac{h_\parallel(E_\pi)}{h_\parallel(0)}, \ldots$

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Thank you for your attention.
Listen to Alessandro Conigli for a first proof of concept!