Strongly coupled gauge theories towards physics beyond the standard models

An overview of recent lattice efforts to the extension of the Standard Model based on strongly coupled gauge theories

Jong-Wan Lee

(Institute for Basic Science)

Lattice 2023 @ Fermilab, Chicago
August 4, 2023

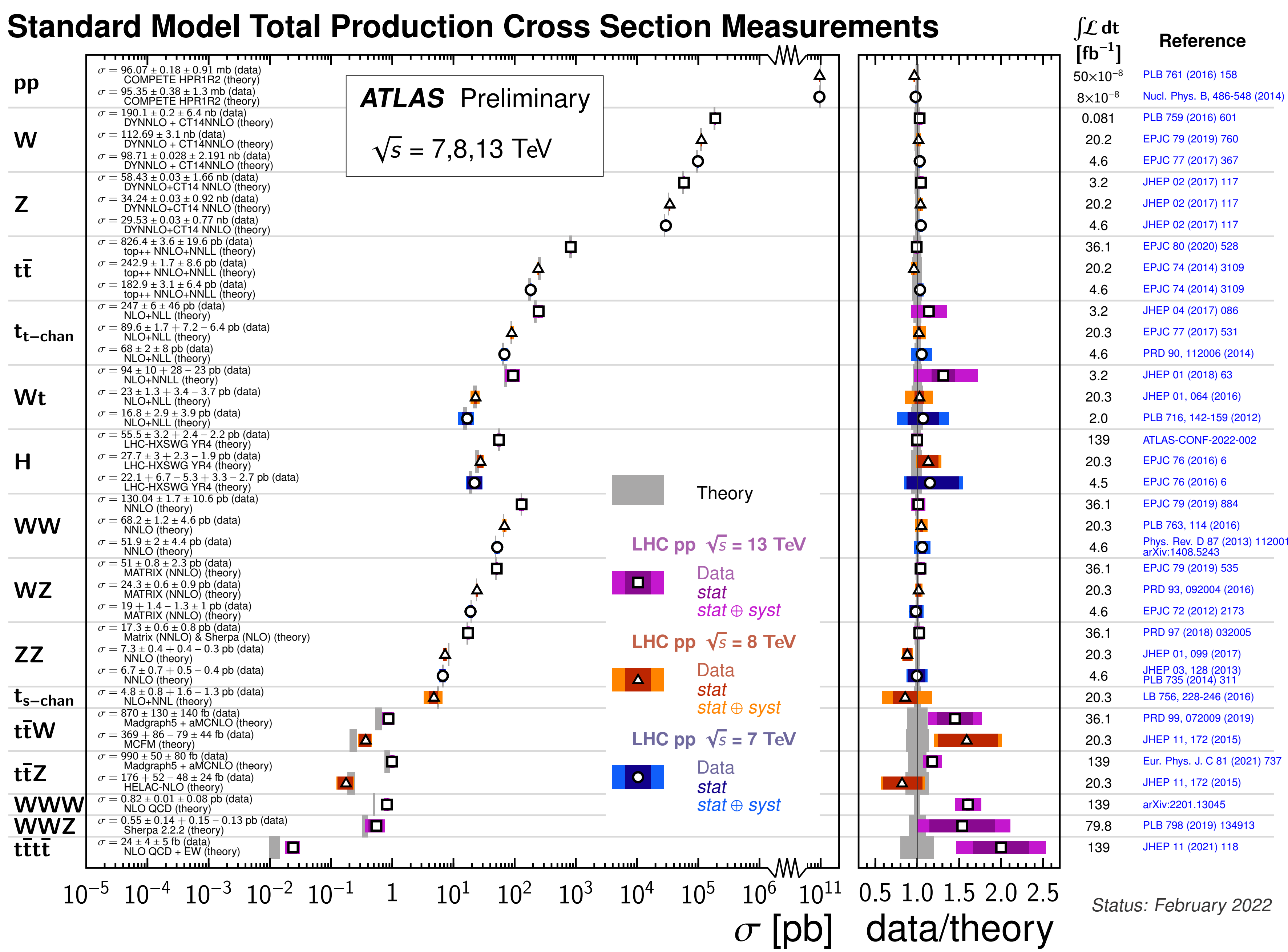
- ✓ *I would like to thank the organizers for giving me an opportunity to review recent lattice studies of strongly coupled gauge theories other than QCD in the context of physics beyond the standard model (BSM).*

- ✓ *I also thank everyone who sent me emails with a nice summary, discussion, and useful materials of their wonderful work. I really enjoyed their talks in this conference and learned a lot!*

- ✓ *And, I must apologize to those who did a great job, but not covered in my review. Not because their work isn't great, but it is due to the lack of my understanding and the limited time.*

Status of Standard Model (SM)

- The Standard Model (SM) well describes physics below TeV scale, as strongly supported by collider experiments.



ATLAS wiki

Standard Model (SM) and new physics

- The standard model (SM) well describes physics below TeV scale, as strongly supported by collider experiments. However, we know that SM is *incomplete* and should be *extended.*

Astronomical observations and experimental results

Obvious

- Dark matter
- matter/antimatter asymmetry
- Neutrino mass

Theoretical problems

Less obvious

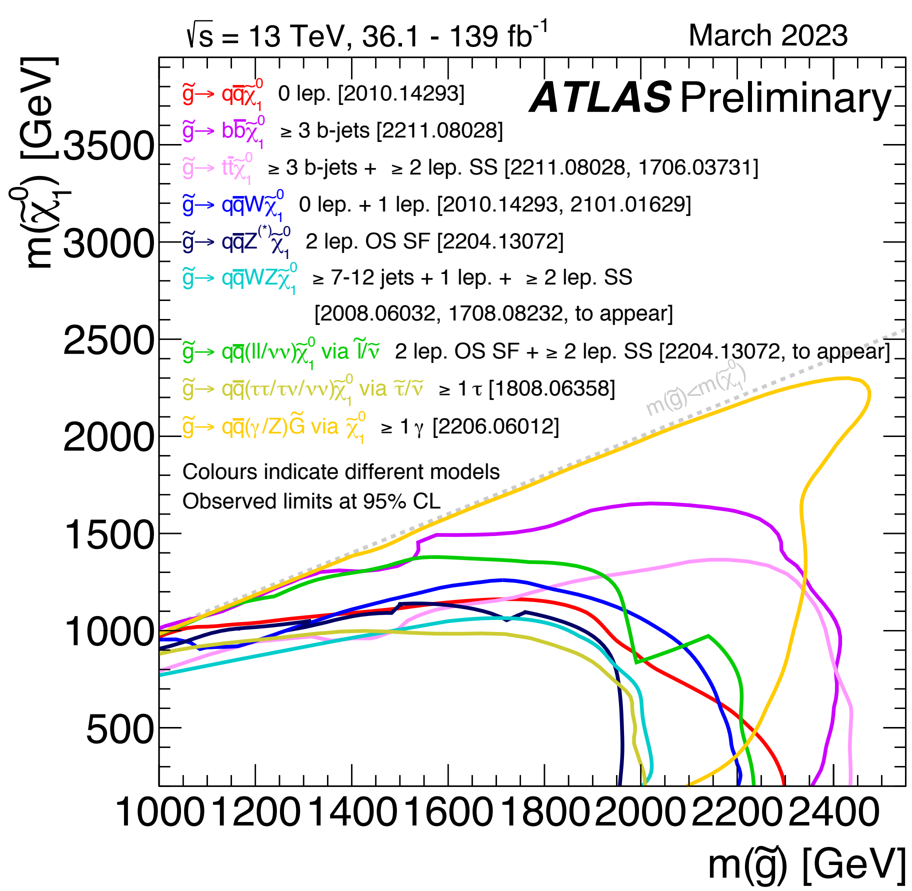
- Naturalness/hierarchy problem
- Strong CP problem
- Fermion mass hierarchy

We want to find a more fundamental description which underlies these theoretical and experimental issues!

Why strongly coupled gauge theories matters for BSM?

- Why not?
- For the past few decades supersymmetric (SUSY) BSM models have received much attention: not only do they solve the hierarchy problem, but also provide an excellent candidate for the cold dark matter, in particular, WIMP dark matter. However, *so far no evidence of SUSY particles has been found.*

SUSY search at LHC



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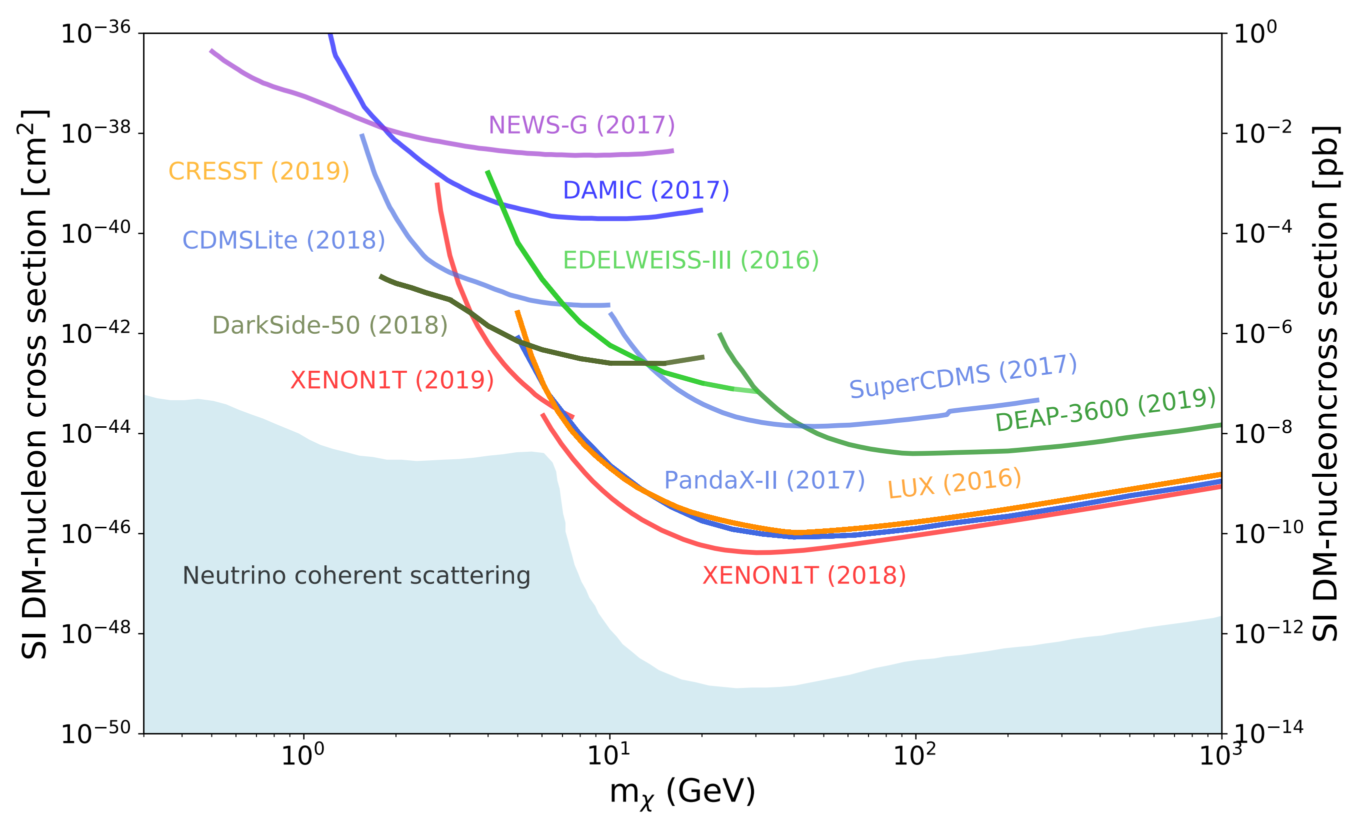
WIMP search in direct detection



Particle Data Group (2022)

Why strongly coupled gauge theories matters for BSM?

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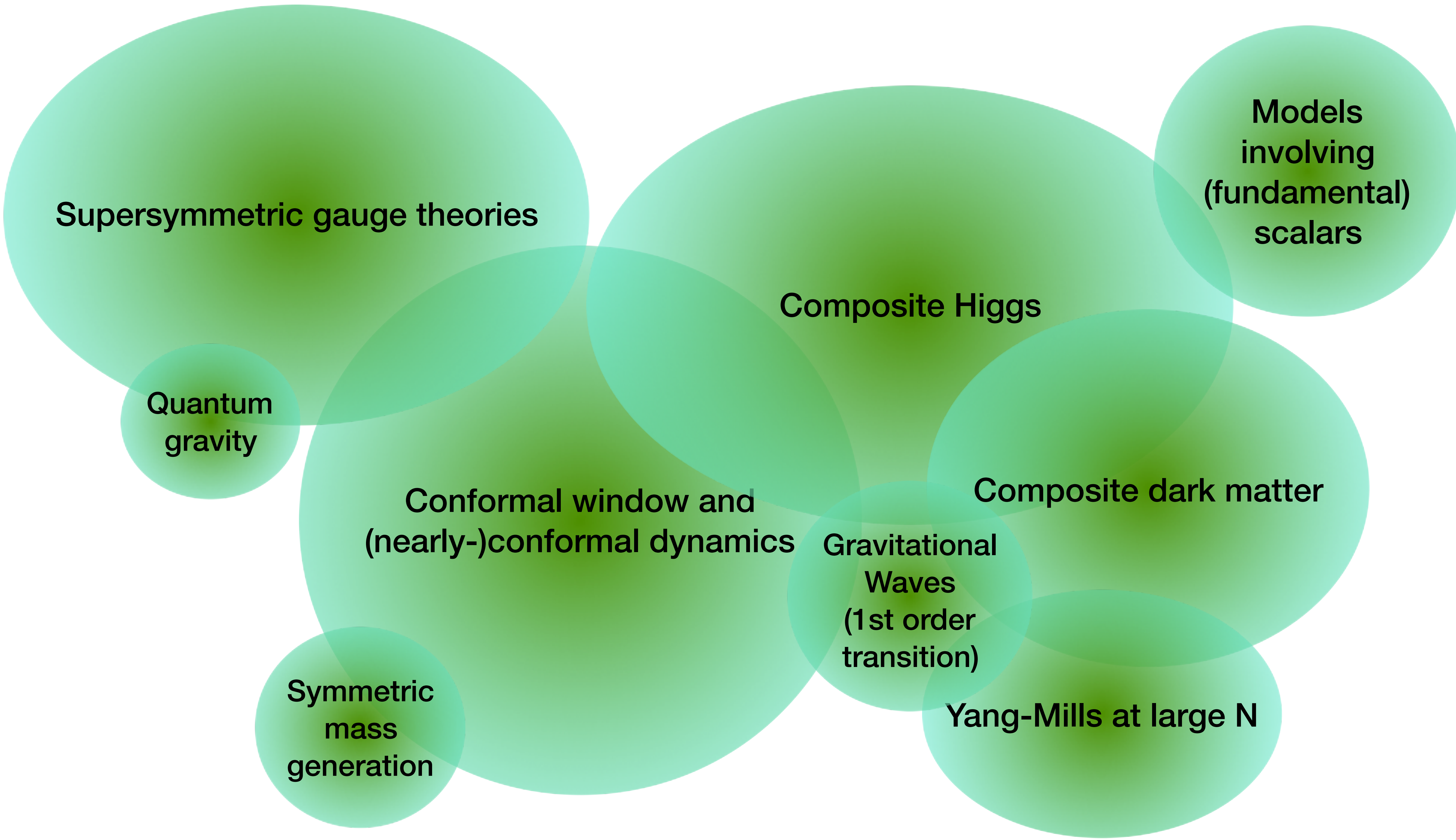
- Among many other alternatives, BSM models based on strongly coupled gauge theories as their UV descriptions are appealing.

✓ We have pretty good understandings of QCD from theoretical and experimental studies for the last half a century.

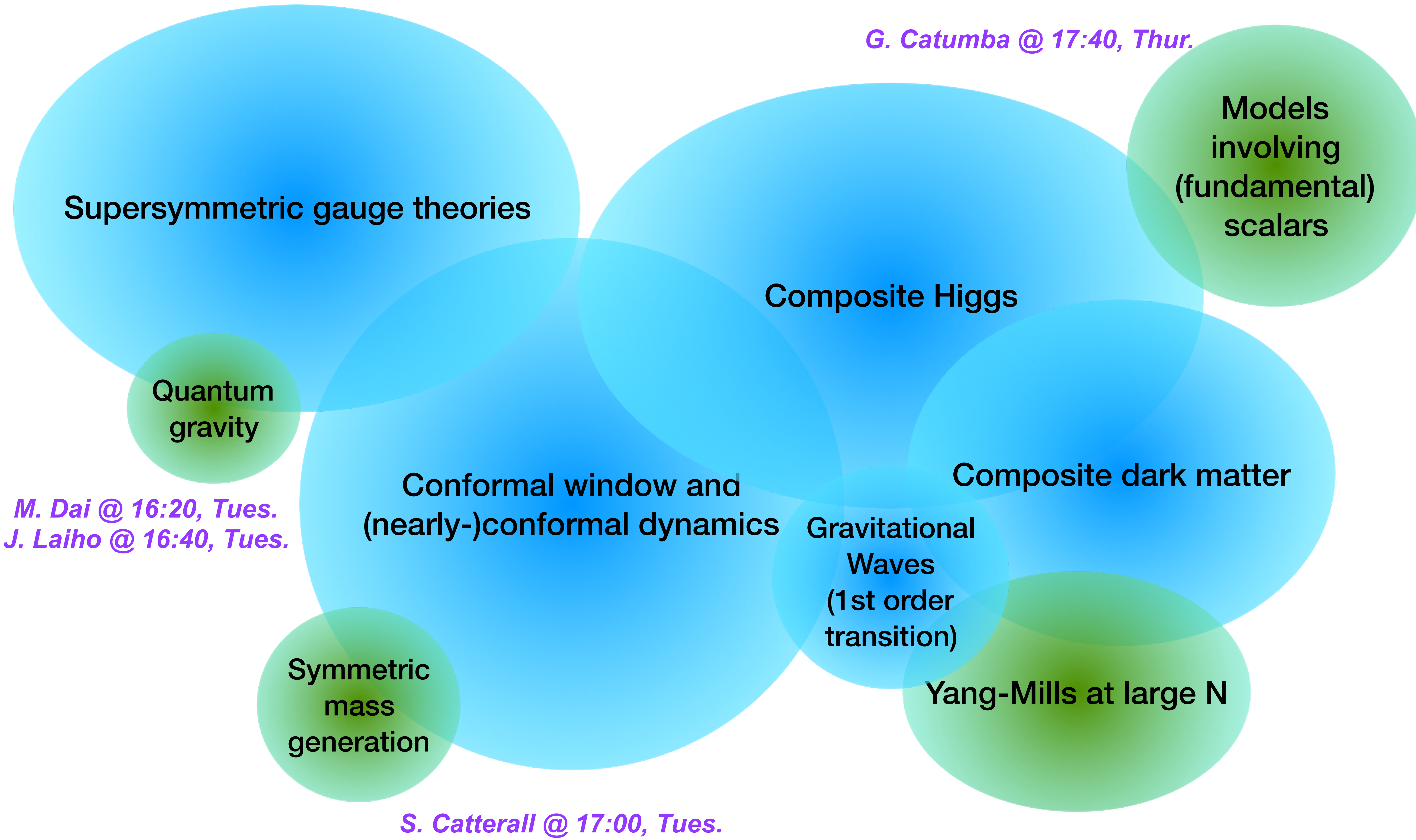
✓ We could find novel features of strongly coupled gauge theories other than QCD, not yet explored, but may have potential impact on the BSM physics.

Many interesting and important questions can only be answered by first-principle ***lattice calculations****!*

BSM models on the lattice for the last three years (2021~23)



BSM models on the lattice for the last three years (2021~23)



Some distinct features of lattice BSM compared to lattice QCD

- The local gauge group doesn't have to be $SU(3)$, but could be a generic non-abelian group, *hypercolor*. Many pheno. models still enjoy some key features of QCD: asymptotic freedom, confinement, spontaneous breaking of global symmetry.

- Fermions do not have to be in the fundamental representation or even in a single representation.

complex	*pseudoreal*	*real*
$SU(N_f) \times SU(N_f) \to SU(N_f)$	$SU(2N_f) \to Sp(2N_f)$	$SU(2N_f) \to SO(2N_f)$

- pNGBs (pions, kaons in QCD) do not have to be (very) light.

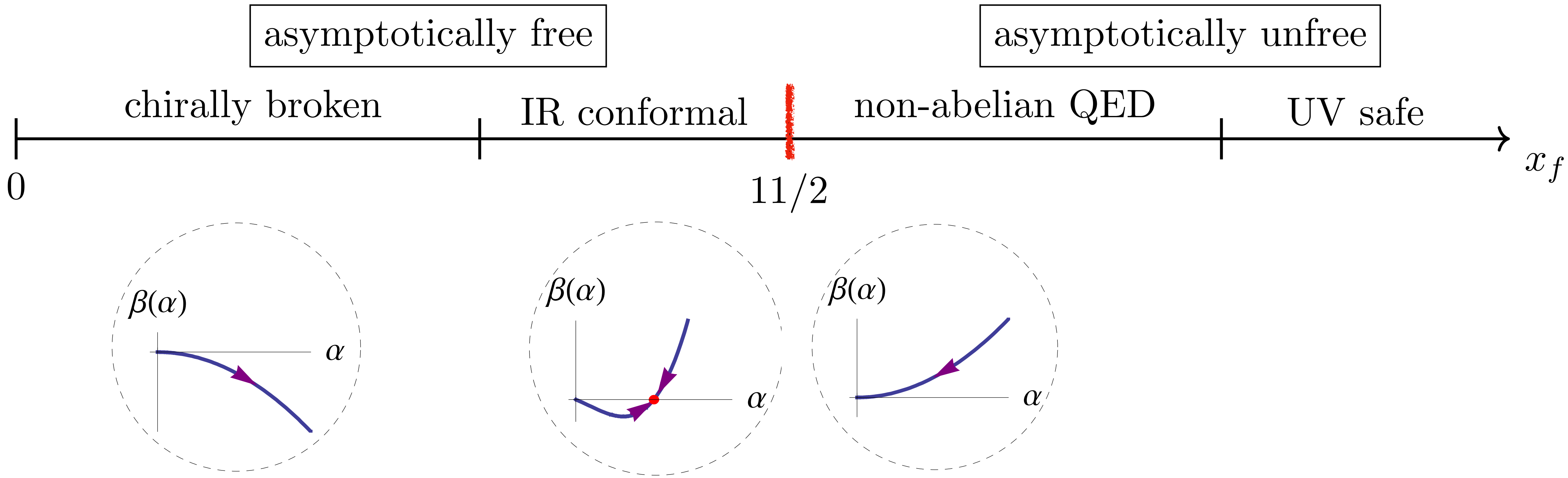
- In addition to pNGBs, other parametrically light states, especially flavor-singlet mesons, can show up, which can be used for many phenomenological models for BSM.

- Novel hypercolor-singlet composite states, which may play a crucial role in phenomenological model buildings, can also appear in the low-energy spectrum.

- Finite temperature thermal transition could be first-order rather than smooth crossover in real-world QCD.

Conformal window

- Consider QCD in the Veneziano limit and define a continuous variable, $x_f = N_f/N$



$\beta(\alpha^\star) = 0$

Conformal window

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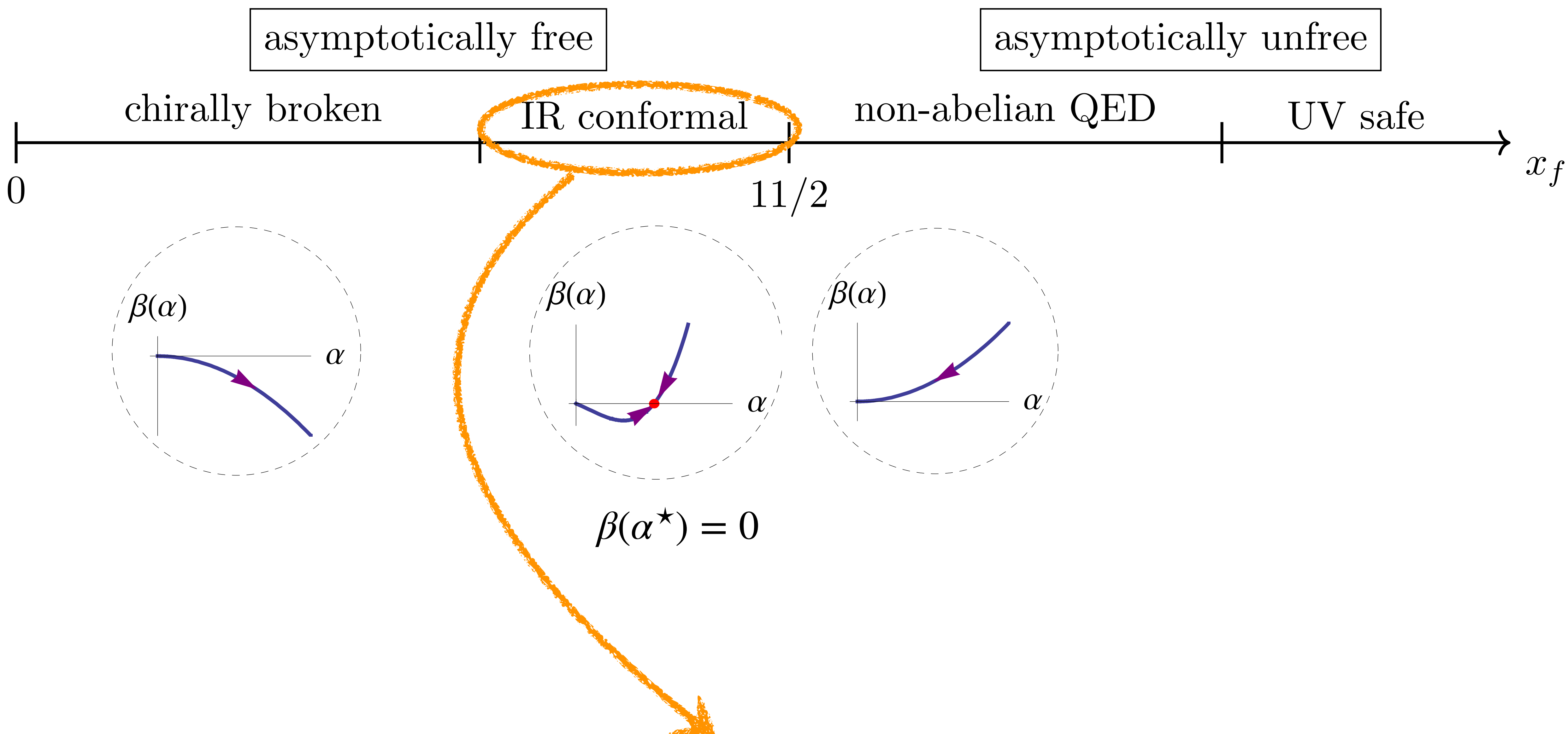

Conformal window (CW)

$x_f^{\rm cr} < x_f < 11/2$ in general, $N_f^{\rm cr} < N_f < N_f^{\rm AF}$

Conformal window

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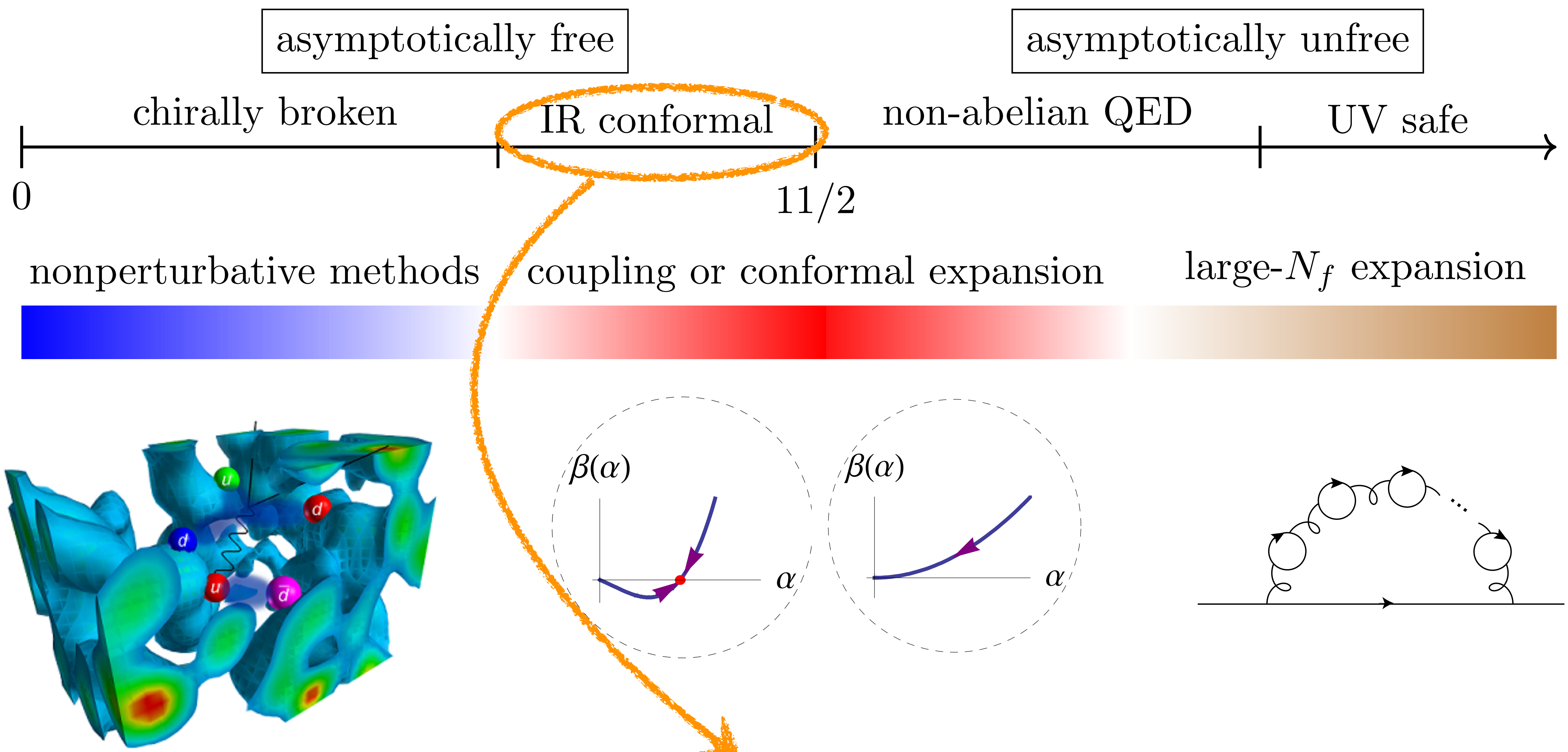


Derek Leinweber, CSSM, University of Adelaide

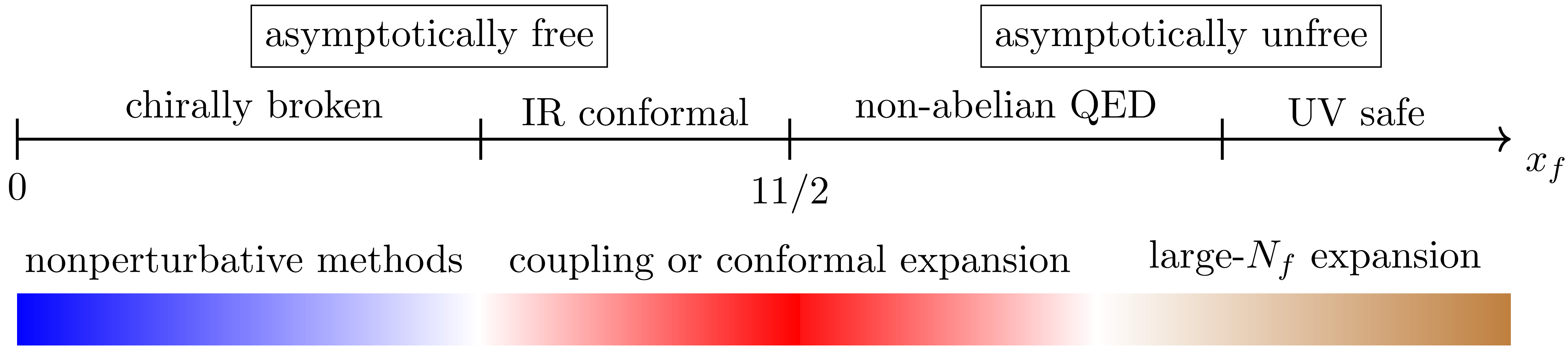
Conformal window (CW)

Lattice method

$x_f^{\mathrm{cr}} < x_f < 11/2$ in general, $N_f^{\mathrm{cr}} < N_f < N_f^{\mathrm{AF}}$

Conformal window and near-conformal dynamics

- Consider QCD in the Veneziano limit and define a continuous variable, $x_f = N_f/N$



- At the lower edge of of the conformal window the theory is expected to be strongly coupled and the nature of the chiral phase transition is largely unknown.

- Notoriously difficult problem in both perturbative and nonperturbative approaches, but worth to tackle this problem as the theory may exhibit novel features and thus may have huge impact on phenomenological BSM model buildings and beyond.

Light scalar (dilaton), large anomalous dimension of composite operators, novel phase (confinement, but not chiral symmetry breaking), large scale separation (walking), etc

Sill of the conformal window - perturbative approach

- Coupling expansion suffers from the scheme dependence for $\ell \geq 3$.

$$\beta(\alpha) = -2\alpha \sum_{\ell=1}^{\infty} b_\ell \left(\frac{\alpha}{4\pi}\right)^\ell \qquad\qquad \alpha = g^2/4\pi$$

- For the past few years much progress has been made in an alternative scheme-independent expansion, so called Bank-Zaks (BZ) or conformal expansion, of certain physical observables in terms of $\Delta_{N_f} = N_f^{\rm AF} - N_f^{\rm IR}$ by R. Shrock & T. Ryttov.

$$\gamma_{\bar\psi\psi,\,{\rm IR}}(\Delta_{N_f}) = \sum_{j=1}^{\infty} c_j (\Delta_{N_f})^j$$

PRD 94 (2016) 105014; PRD 95 (2017) 085012; PRD 95 (2017) 105004; PRD 96 (2017) 105015

Ryttov, PRL 117 (2016) 071601

Ryttov & Shrock, arXiv:2307.12426

Requirement: $(j+1)$-loop beta function and j-loop mass anomalous dim.

Talk by R. Shrock @ 15:40, Mon.

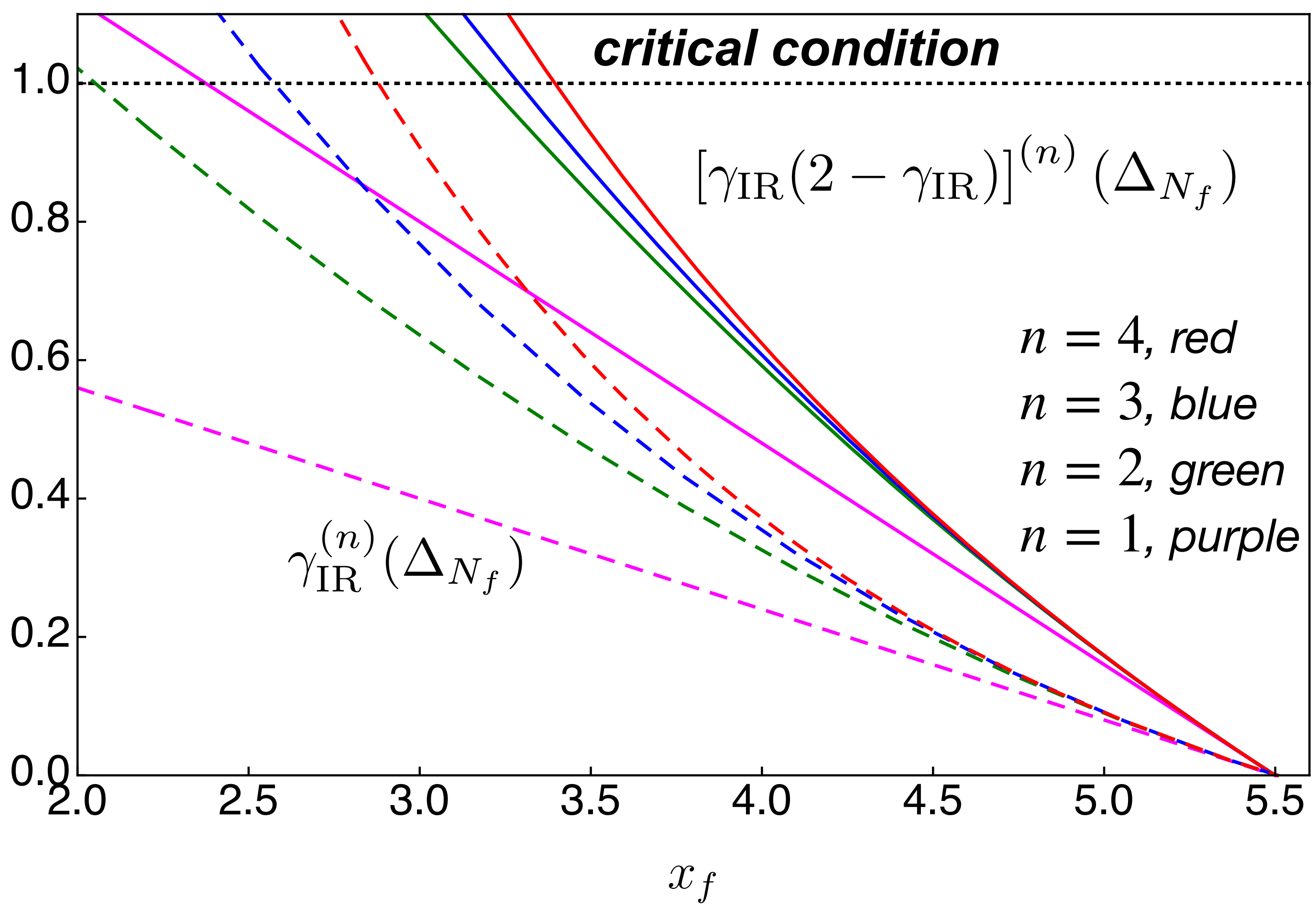
- Combining with the conjectured critical conditions to the anomalous dimension of fermion bilinear, $\gamma_{\bar\psi\psi} = 1$ or equivalently $\gamma_{\bar\psi\psi}(2 - \gamma_{\bar\psi\psi}) = 1$, the conformal window has been estimated in a scheme independent way.

B. S. Kim, D. K. Hong & JWL, PRD 101 (2020), 056008

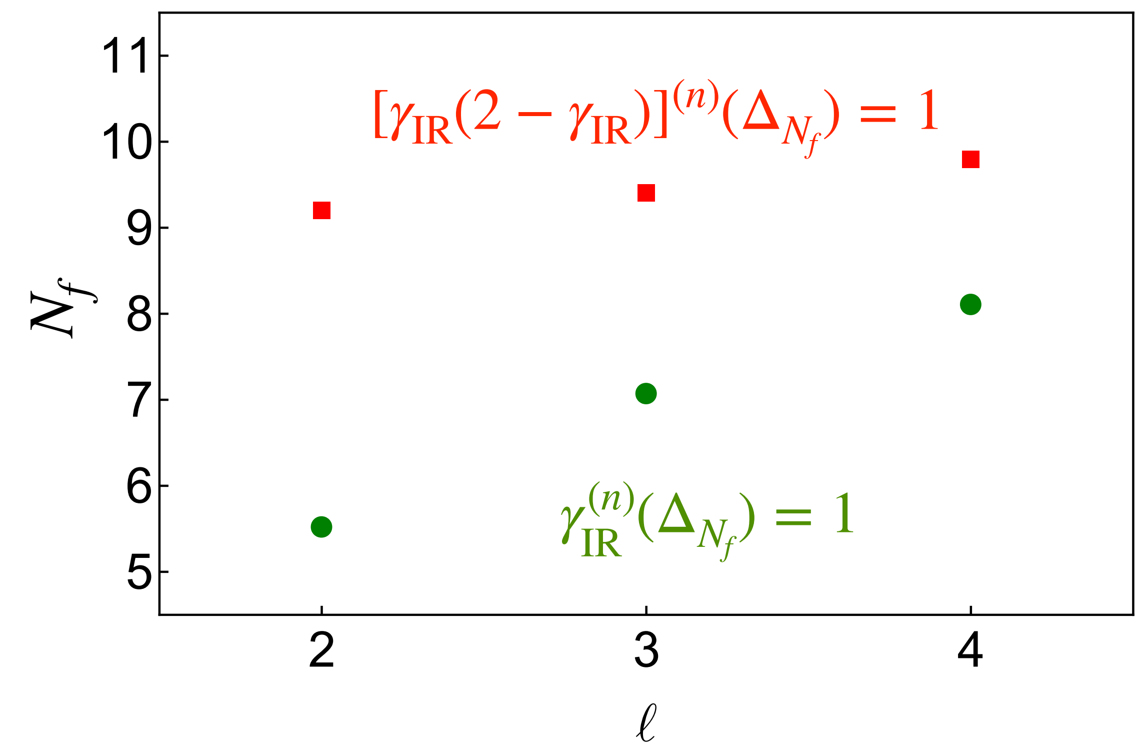
At finite order in Δ_{N_f}, $\gamma_{\bar\psi\psi}(2 - \gamma_{\bar\psi\psi}) = 1$ turns out to show a better convergence.

Sill of the conformal window - perturbative approach

QCD in the Veneziano limit



SU(3) + N_f fund. fermions



$$[N_f^{\rm cr},\, N_f^{\rm AF}] = \left[9.79\,{}^{+0.94}_{-0.82}\,{}^{+0.31}_{-0.36},\, 16.5\right]$$

JWL, PRD 103 (2021), 076006

- Another scheme independent estimation of the conformal window has also been proposed by computing f_π/m_V and f_V/m_V using (p)NRQCD at NNLO in CW and matching them to lattice results for $N_f = [2,10]$ QCD, which finds $N_f^{\rm cr} \sim 12$ or 13.

Talk by D. Nogradi @ 14:10, Thur.

Sill of the conformal window - nonperturbative approach

- GF as continuous space RG with $\mu \propto 1/\sqrt{8t}$ for local gauge-invariant observables

0) It is not necessary, but it is convenient to take the zero quark mass.

1) Infinite volume limit at fixed t/a^2 for each bare lattice coupling β

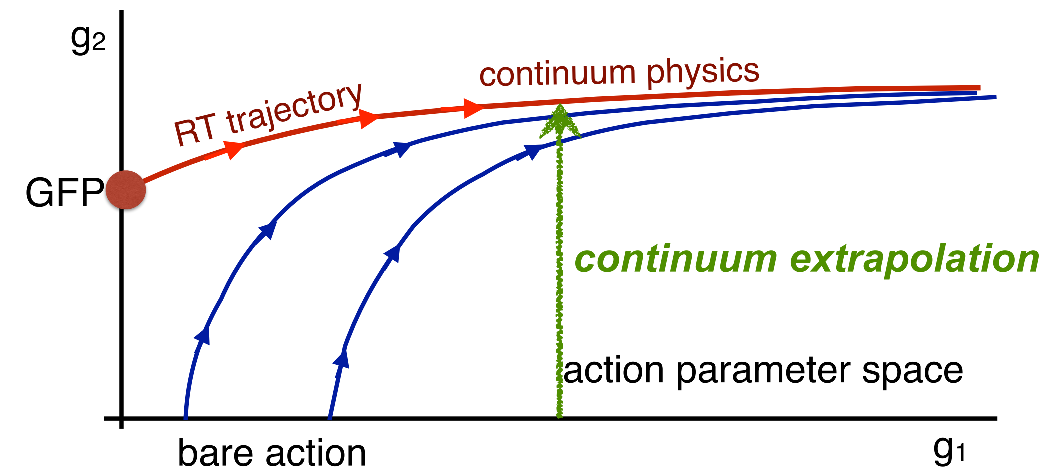
2) Continuum limit at fixed $g^2_{\rm GF}$, which brings β to infinity.

Z. Fodor et al, EPJ Web Conf. 175 (2018), 08027

A. Hasenfratz & O. Witzel, PRD 101 (2019) 3, 034514

- Beta function

$$g^2_{\rm GF}(t) = \mathcal{N} t^2 \langle E(t) \rangle \quad \Rightarrow \quad \beta_{\rm GF}(a; g^2_{\rm GF}) = -\, t \frac{d g^2_{\rm GF}(a;t)}{dt}$$



- Mass anomalous dimension

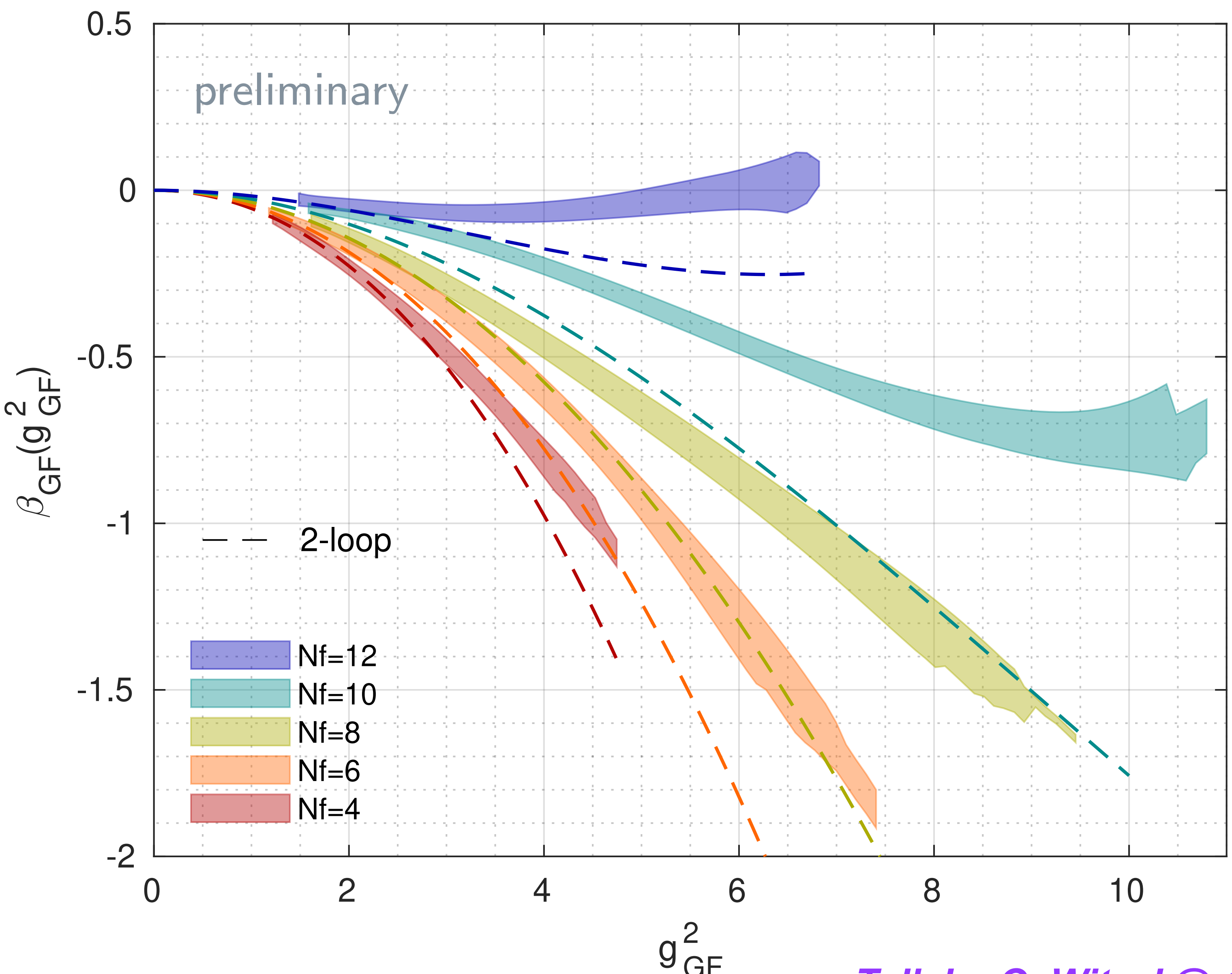
$$G_{\mathcal{O}}(x_4; t) = \langle \mathcal{O}(\overrightarrow{p} = 0, x_4; t) \mathcal{O}(\overrightarrow{p} = 0, 0; 0) \rangle \quad \text{with} \quad \mathcal{O}(x) = \overline{\psi}(x) \Gamma \psi(x)$$

$$R_{\mathcal{O}}(x_4; t) = \frac{G_{\mathcal{O}}(x_4, t)}{G_V(x_4; t)} \quad \Rightarrow \quad \gamma_{\mathcal{O}}(a; g^2_{\rm GF}) = t \frac{d \log R_{\mathcal{O}}(a; t)}{dt}$$

for sufficiently large x_4

SU(3) + many fund. fermions

Continuous RG beta function



- Confirm the previous step-scaling results published in a series of papers by A. Hasenfratz, C. Rebbit & O. Witzel.

PLB 798 (2019), 134937; PRD 100 (2019), 114508; PRD 101 (2020), 114508; PRD 106 (2022), 114509; PRD 107 (2023), 114508

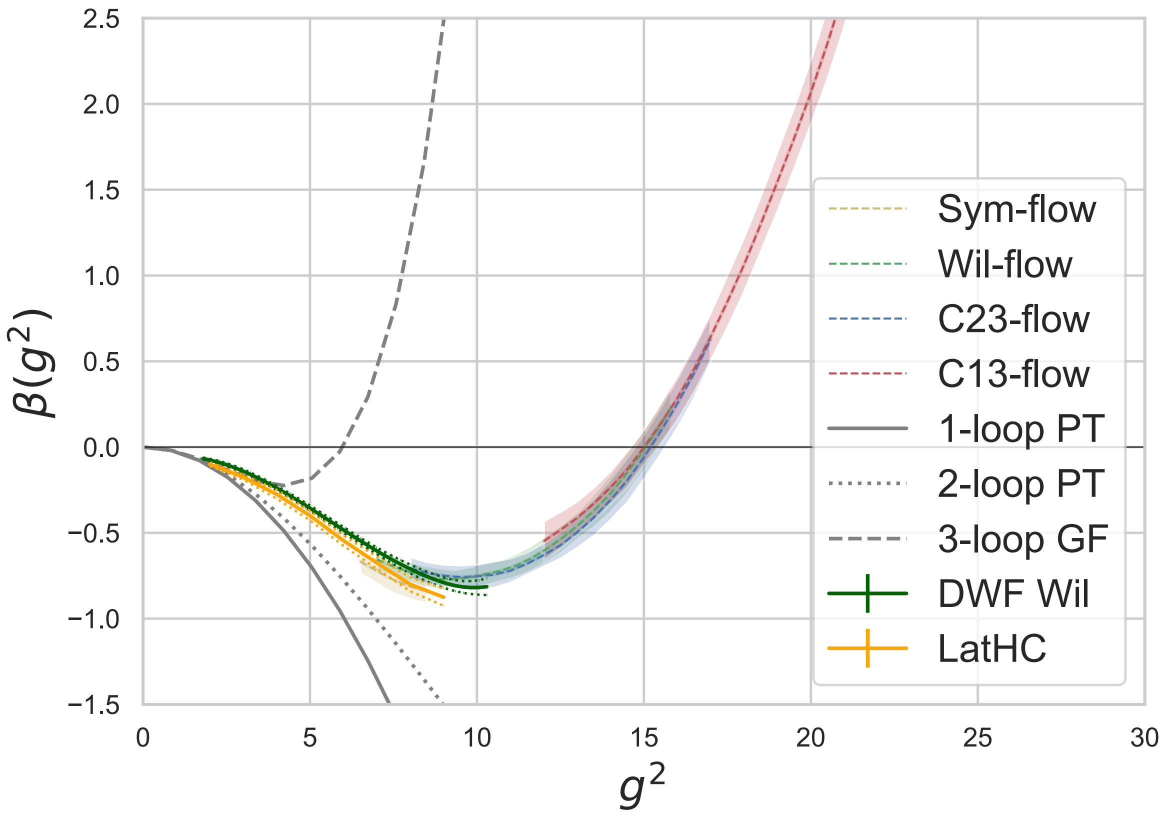
- Extension to the stronger GF coupling is hindered by large UV effects or even 1st order *bulk* phase transition.

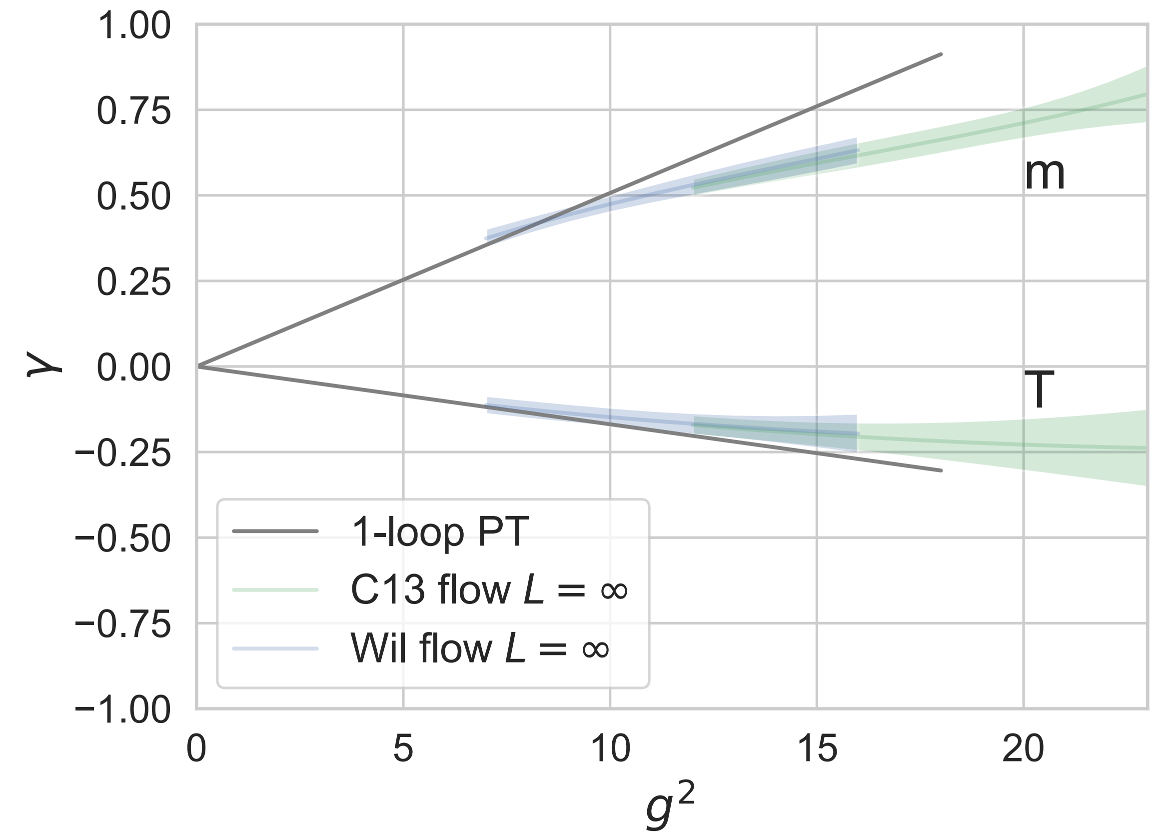
Talk by O. Witzel @ 14:30, Mon.

SU(3) + 10 fund. fermions

- Idea: Introduce heavy Pauli-Villars bosons, $am_{\rm PV} \sim \mathcal{O}(1)$, to reduce the cutoff effects by compensating the screening effects from many flavors of fermion.

A. Hasenfratz, Y. Shamir & B. Svetitsky, PRD 104 (2019) 7, 074509





A. Hasenfratz et al, arXiv:2306.07236

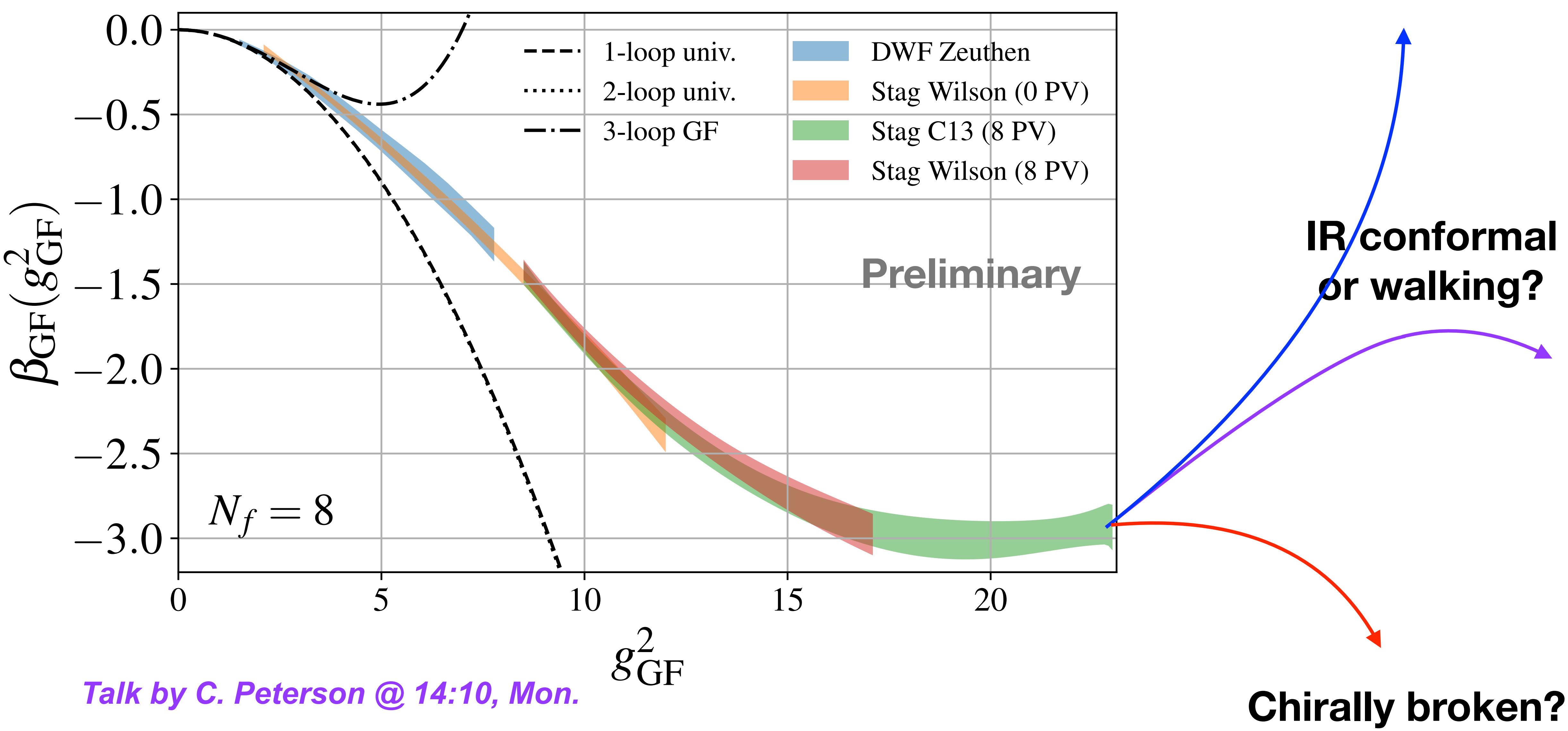
Talk by A. Hasenfratz @ 13:30, Mon.

- Find an IR fixed point at $g^2_{\rm IR} \sim 15$ and the mass anomalous dimension $\gamma^\star_{\bar\psi\psi,{\rm IR}} \simeq 0.6$

cf) Scheme-independent perturbative result: $\gamma^\star_{\bar\psi\psi,IR}(\Delta^4_{N_f}) = 0.615$

Ryttov & Shrock, PRD 94 (2016) 105014

SU(3) + 8 fund. fermions



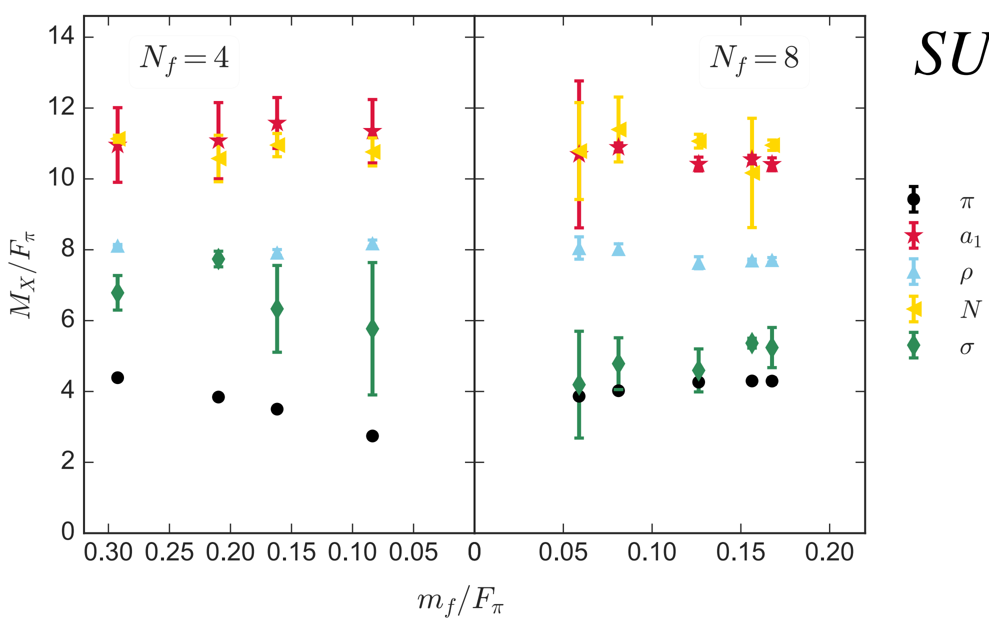
Talk by C. Peterson @ 14:10, Mon.

- The GF coupling has been extended to ~ 22. No sign of IR fixed point, yet (?).
- Also, it has been found that BKT scaling is preferred. *Poster by C. Peterson @ 19:00, Tues.*

Evidence of walking scenario?

Dilaton effective field theory (dilaton EFT)

- A revival of EFT for dilaton has been triggered by the discovery of light dilaton, in addition to pNGBs, in lattice calculations of *would-be* near-conformal gauge theories.

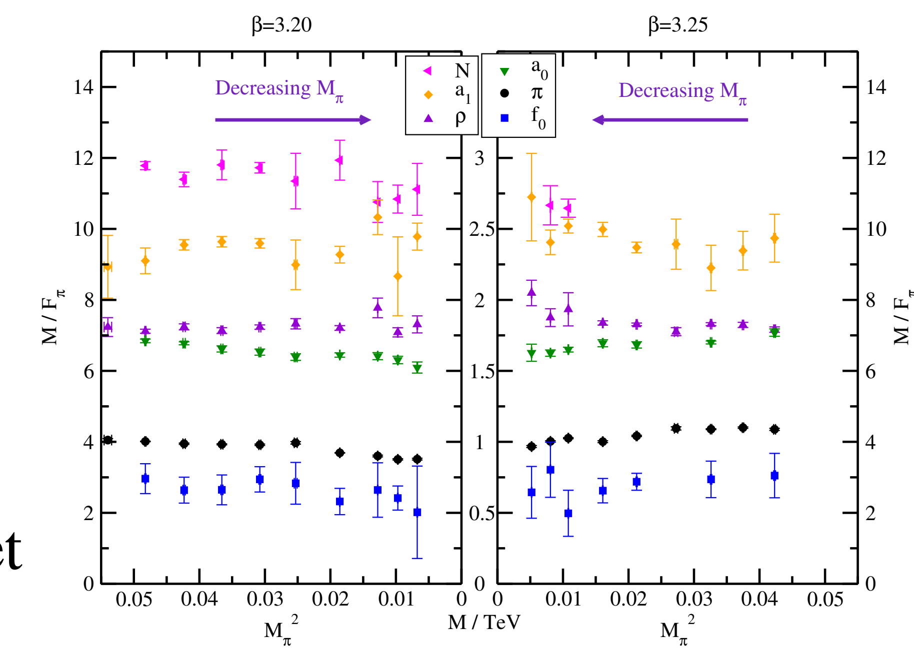


$SU(3)$ + N_f Fund.

Lattice Strong Dynamics (LSD), PRD 99 (2019), 014509

Also, LatKMI, PRD 96 (2017) 014508

$SU(3)$ + 2 Sextet



Z. Fodor, K. Holland, J. Kuti, S. Mondal, D. Nogradi & C. Wong, PoS LATTICE2015 219

- The parametrically light scalar might be identified as a *dilaton*, associated with the spontaneous breaking of scale symmetry.

Scale transformation: $x^\mu \longrightarrow e^\alpha x^\mu, \quad \chi(x) \longrightarrow e^\alpha \chi(e^\alpha x), \quad \mathscr{L}(x) \longrightarrow e^{4\alpha}\mathscr{L}(e^\alpha x)$

If $\langle\chi\rangle = f_d$*, dilaton can be realized in a non-linear way:* $\chi(x) \equiv f_d e^{\sigma(x)/f_d}$ **S. Coleman (1971)**

Dilaton effective field theory (dilaton EFT)

- The dilation EFT at LO is given by

J. Incoldby, M. Piai & T. Appelquist, JHEP 1707 (2017) 035; JHEP 1803 (2018) 039; PRD 101 (2020) 075025

M. Golterman & Y. Shamir, PRD 94 (2016) 054502; PRD 98 (2018) 056025; PRD 102 (2020) 034515 (with E. Neil); PRD 102 (2020) 114507

$$\mathcal{L}_{\mathrm{LO}} = \frac{1}{2}\partial_\mu\chi\partial^\mu\chi + \mathcal{L}_K + \mathcal{L}_M - V_\Delta(\chi)$$

scaling dim. of $\bar{\psi}\psi$ → y

$$\mathcal{L}_K = \frac{f_\pi^2}{4}\left(\frac{\chi}{f_d}\right)^2 \mathrm{Tr}\left[\partial_\mu\Sigma(\partial^\mu\Sigma)^\dagger\right] \qquad \mathcal{L}_M = \frac{m_\pi^2 f_\pi^2}{4}\left(\frac{\chi}{f_d}\right)^y \mathrm{Tr}\left[\Sigma + \Sigma^\dagger\right]$$

pNGBs kinetic term — *pNGBs mass term*

$$V_\Delta(\chi) \equiv \frac{m_d^2\chi^4}{4(4-\Delta)f_d^2}\left[1 - \frac{4}{\Delta}\left(\frac{f_d}{\chi}\right)^{4-\Delta}\right]$$

$\Delta = 2$: Higgs potential of SM

$$V_1 \equiv \frac{m_d^2}{2f_d^2}\left(\frac{\chi^2}{2} - \frac{f_d^2}{2}\right)^2$$

$\Delta = 4$: Coleman-Weinberg type potential

$$V_2 \equiv \frac{m_d^2}{16f_d^2}\chi^4\left(4\ln\frac{\chi}{f_d} - 1\right)$$

- Lattice measurements at finite fermion mass am include $M_\pi^2, M_d^2, F_\pi^2, F_S^2$ and $a_0^{\mathrm{I}=2}$. One extracts the low-energy constants from the fits to the lattice data.

$$y,\ \Delta,\ a^2f_\pi^2,\ f_\pi^2/f_d^2,\ m_d^2/f_d^2,\ a\mathrm{B}_\pi$$

SU(3) + 8 Fund. fermions: dilaton EFT

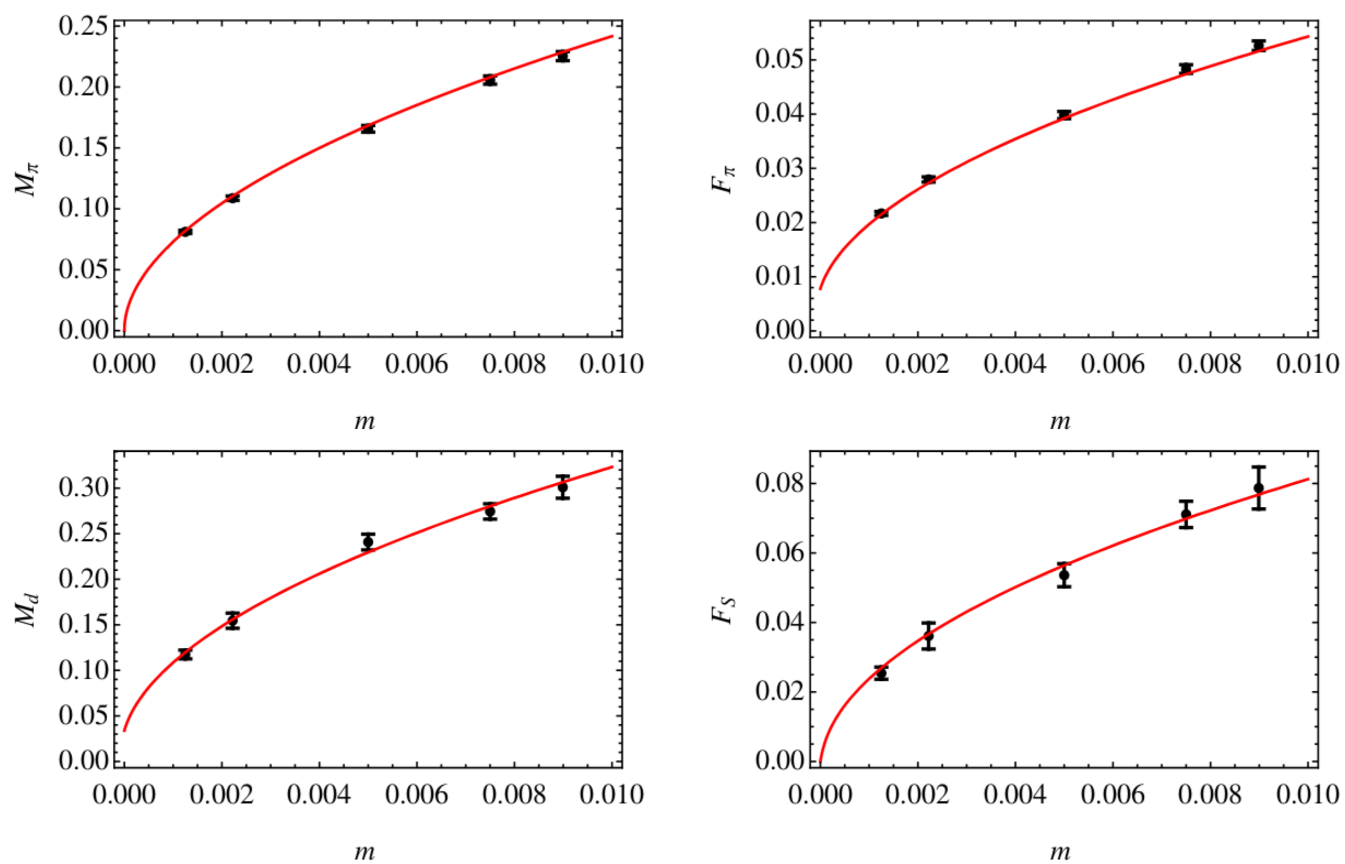
Talk by J. Ingolby @ 13:50, Thur.

- LSD collaboration updated the lattice results of meson spectrum in 2023. Combining the scattering data in 2021, they performed the global fit using dilaton EFT.

LSD, PRD 99, 014509; PRD 105 (2022) 034505; arXiv:2306.06095

  - *infinite volume extrapolation*
  - *improved measurements for flavor-single scalar meson*
  - *measured a new observable, scalar decay constant* F_S

LSD, arXiv:2305.03665



Parameter	LO	NLO
y	2.091(32)	2.069(32)
B_π	2.45(13)	2.46(13)
Δ	3.06(41)	2.88(49)
f_π^2	$6.1(3.2) \times 10^{-5}$	$5.8(3.4) \times 10^{-5}$
f_π^2/f_d^2	0.1023(35)	0.1089(41)
m_d^2/f_d^2	1.94(65)	2.24(80)
l_a	—	0.78(27)
χ^2/dof	21.3/19	10.3/18
AIC	33.3	24.3

- They also fit the data to the mass-deformed CFT scalings, which shows a less quality. This result seems to support the *walking behavior* in the 8-flavor $SU(3)$ theory.

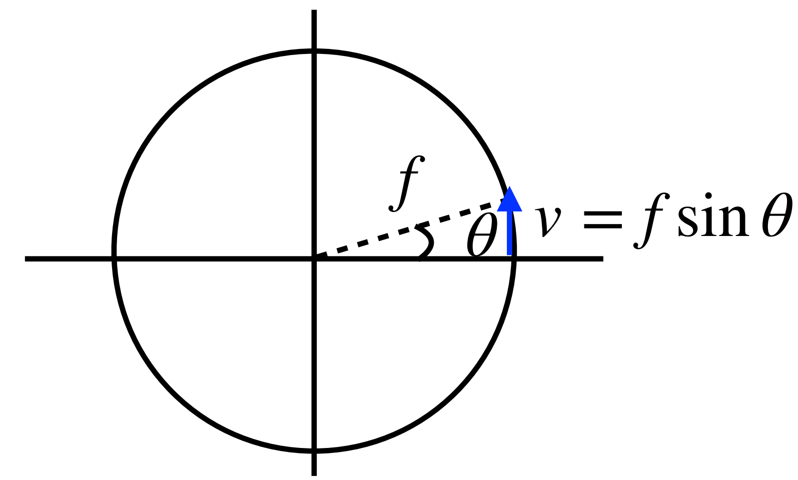
Composite Higgs and (top-)partial compositeness

- **Composite Higgs (CH):** an alternative description of $SU(2)_L \times U(1)_Y$ electroweak symmetry breaking by vacuum misalignment

Georgi & Kaplan; Kaplan, Georgi & Dimopolous (1984);
Dugan, Kaplan & Geoorgi (1985)

Key requirement: electroweak symmetry not broken by new strong interaction

Global symmetry G —— *spontaneously broken* at scale, f ——→ subgroup $SU(2)_L \times U(1)_Y \subset H$



Higgs = pseudo Nambu-Goldstone boson

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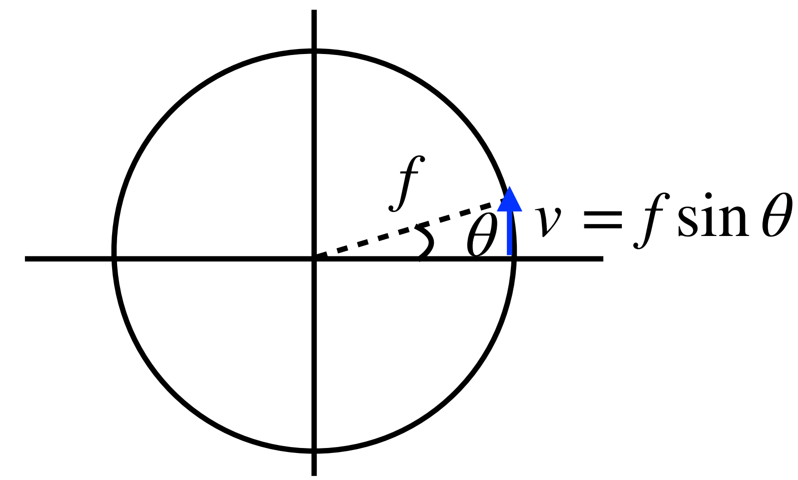


Higgs = pseudo Nambu-Goldstone boson

- **Partial compositeness:** mixing between SM quarks and hybrid (Chimera) baryons $\mathcal{O}_{L,R}$, formed by fermions in two different representations, can explain quark mass hierarchy

  Kaplan (1991)

  Key requirement: large anomalous dim. of the chimera baryon, e.g. top-partner

$$m_f \sim \lambda_L \lambda_R v \quad \text{where} \quad \lambda_{L,R} \sim \left(\frac{\Lambda}{\Lambda_{UV}}\right)^{dim\,\mathcal{O}_{L,R} - \frac{5}{2}}$$

Light fermions (~elementary), $dim\,\mathcal{O}_{L,R} > \frac{5}{2}$, top quark (~composite) $dim\,\mathcal{O}_{L,R} \sim \frac{5}{2}$

or $\gamma_{\mathcal{O}} \sim 2$

4D UV models for comp. Higgs + partial compositeness

Coset	HC	ψ	χ	$-q_\chi/q_\psi$	Baryon	Name
$\frac{SU(5)}{SO(5)} \times \frac{SU(6)}{SO(6)}$	SO(7)	$5 \times \mathbf{F}$	$6 \times \mathbf{Sp}$	5/6	$\psi\chi\chi$	M1
	SO(9)	$5 \times \mathbf{F}$	$6 \times \mathbf{Sp}$	5/12	$\psi\chi\chi$	M2
	SO(7)	$5 \times \mathbf{Sp}$	$6 \times \mathrm{F}$	5/6	$\psi\psi\chi$	M3
	SO(9)	$5 \times \mathbf{Sp}$	$6 \times \mathrm{F}$	5/3	$\psi\psi\chi$	M4
$\frac{SU(5)}{SO(5)} \times \frac{SU(6)}{Sp(6)}$	Sp(4)	$5 \times \mathbf{A}_2$	$6 \times \mathbf{F}$	5/3	$\psi\chi\chi$	M5
$\frac{SU(5)}{SO(5)} \times \frac{SU(3)^2}{SU(3)}$	SU(4)	$5 \times \mathbf{A}_2$	$3 \times (\mathbf{F}, \overline{\mathbf{F}})$	5/3	$\psi\chi\chi$	M6
	SO(10)	$5 \times \mathbf{F}$	$3 \times (\mathbf{Sp}, \overline{\mathbf{Sp}})$	5/12	$\psi\chi\chi$	M7
$\frac{SU(4)}{Sp(4)} \times \frac{SU(6)}{SO(6)}$	Sp(4)	$4 \times \mathbf{F}$	$6 \times \mathbf{A}_2$	1/3	$\psi\psi\chi$	M8
	SO(11)	$4 \times \mathbf{Sp}$	$6 \times \mathbf{F}$	8/3	$\psi\psi\chi$	M9
$\frac{SU(4)^2}{SU(4)} \times \frac{SU(6)}{SO(6)}$	SO(10)	$4 \times (\mathbf{Sp}, \overline{\mathbf{Sp}})$	$6 \times \mathbf{F}$	8/3	$\psi\psi\chi$	M10
	SU(4)	$4 \times (\mathbf{F}, \overline{\mathbf{F}})$	$6 \times \mathbf{A}_2$	2/3	$\psi\psi\chi$	M11
$\frac{SU(4)^2}{SU(4)} \times \frac{SU(3)^2}{SU(3)}$	SU(5)	$4 \times (\mathbf{F}, \overline{\mathbf{F}})$	$3 \times (\mathbf{A}_2, \overline{\mathbf{A}_2})$	4/9	$\psi\psi\chi$	M12

G. Cacciapaglia, G. Ferretti, T. Flacke & H. Serodio, arXiv:1902.06890

F: fundamental rep.

A_2: 2-index antisymmetric rep.

- 4D UV minimal models are classified - nonabelian gauge theories coupled to fermions in two different reps.

G. Ferretti & T. Karataev, arXiv:1312:5330

→ SU(4) + (2 A_2 + 2 F) Dirac

→ Sp(4) + (3 A_2 + 2 F) Dirac

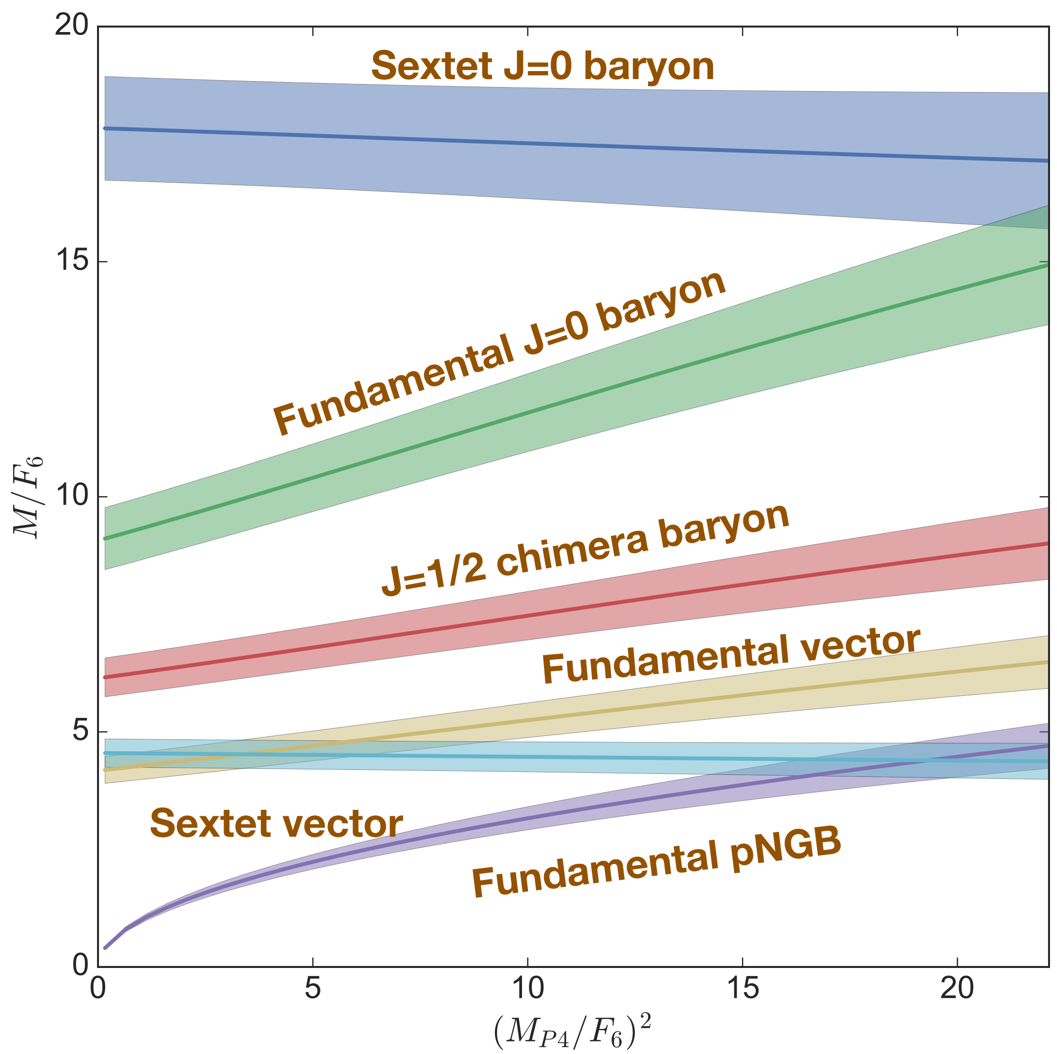
no modification

→ SU(4) + (4 A_2 + 4 F) Dirac

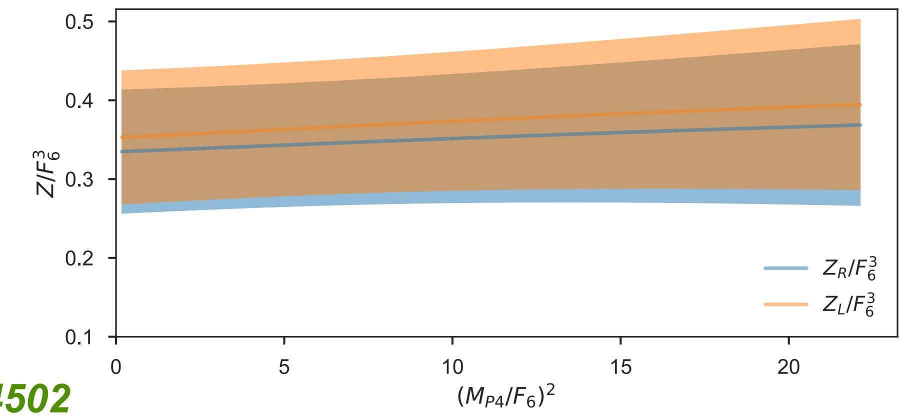
✤ In SU(4) models, the number of flavors are modified to be amenable on the lattice without much difficulties.

SU(4) + 2 Fund. + 2 AS fermions

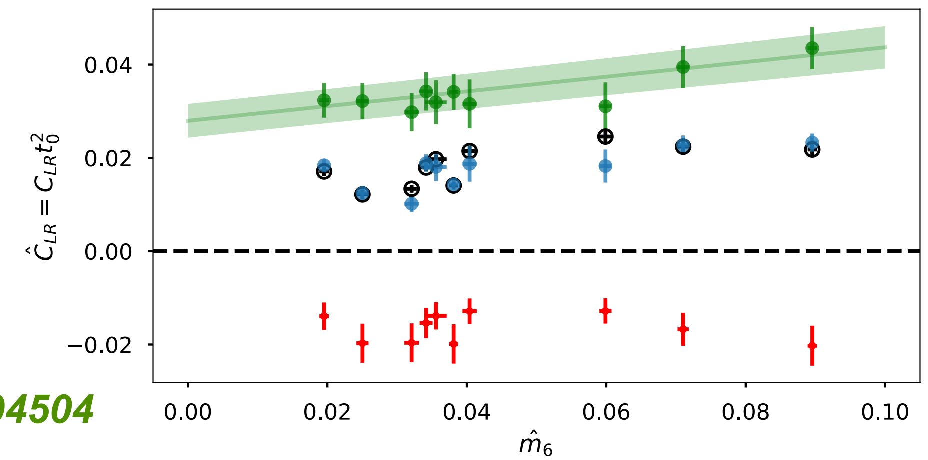
- Extensively studied on the lattice by TACoS collaboration for the past years: the meson and baryon spectra, the low-energy constants entering to the Higgs potential and the S parameter, and the baryon matrix elements entering to the top Yukawa coupling.



PRD 97 (2018), 074505;
PRD 97 (2018), 114505



PRD 99 (2019), 094502



PRD 99 (2019), 094504

- First lattice simulation of a gauge theory with multiple representation, but this prototype lattice model is QCD-like and no pheno. interesting features have been found.

⚠ : # of flavors are different to Ferretti model

SU(4) + 2 Fund. + 2 AS fermions

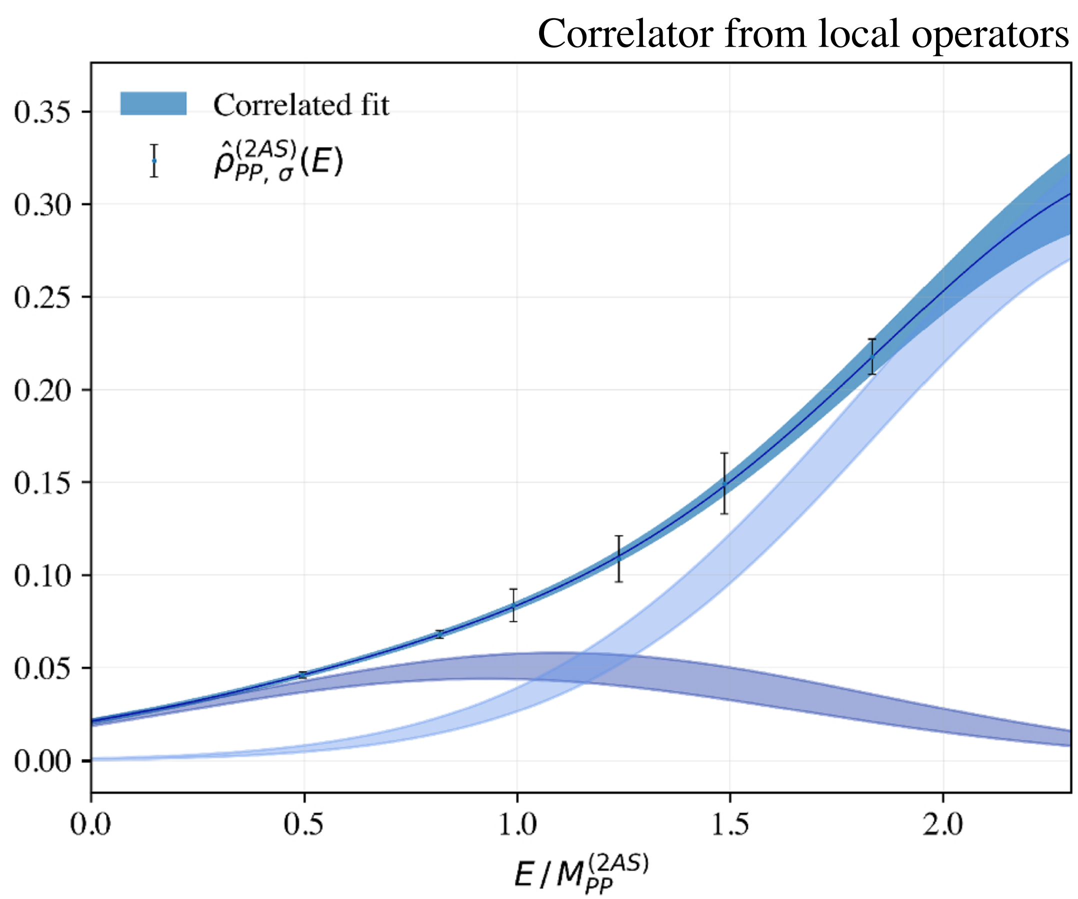
- Possible excited states in the fundamental and antisymmetric sector are constrained by symmetry.

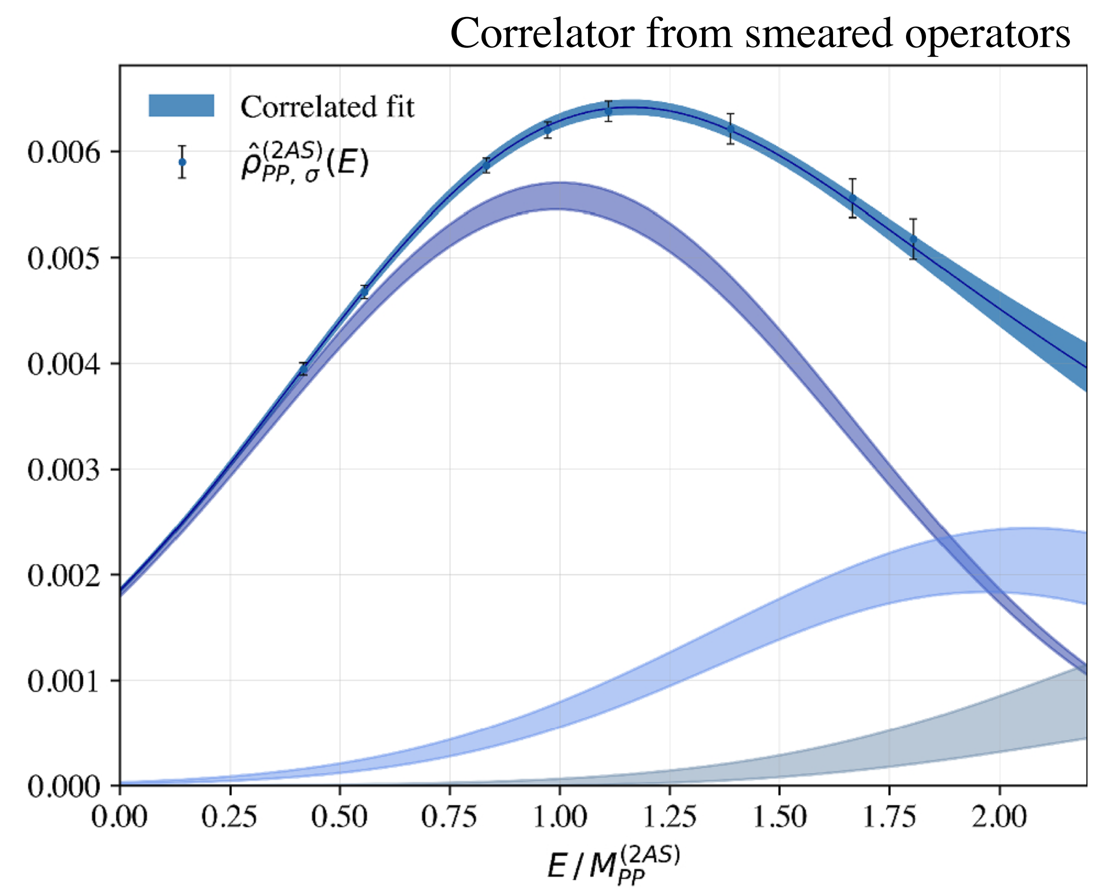
$\langle 0|\pi|\pi\rangle \;, \quad \langle 0|\pi|\pi\pi\pi\rangle \;, \quad \langle 0|\pi|\pi\Pi\Pi\rangle \;, \ldots$

$\langle 0|\Pi|\Pi\rangle \;, \quad \langle 0|\Pi|\Pi\,\Pi\rangle \,, \quad \langle 0|\Pi|\Pi\pi\pi\rangle \;, \quad \langle 0|\Pi|\Pi\pi\pi\pi\pi\rangle \;, \ldots$

- Explored the excite states by using the spectral density method

M. Hansen, A. Lupo & N. Tantalo, PRD 99 (2019) 094508



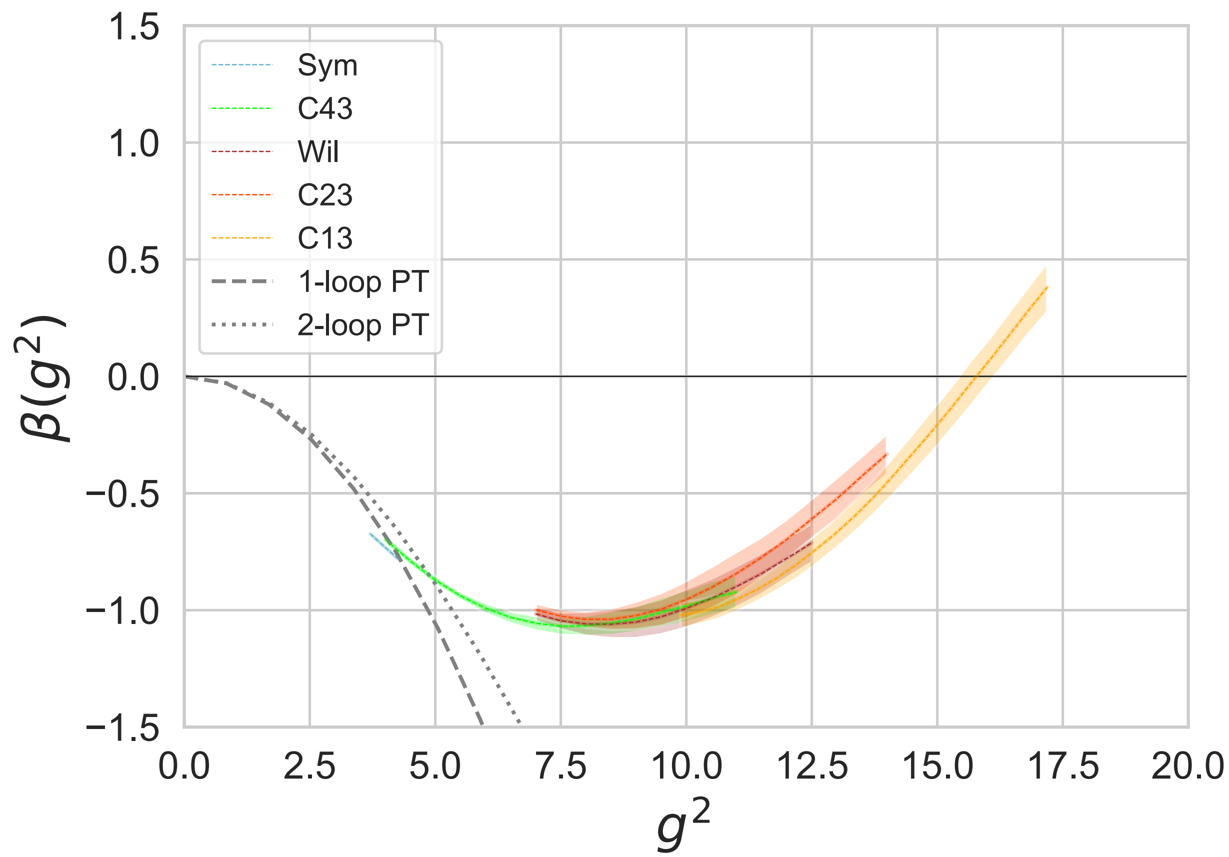


L. Del Debbio, A. Lupo, M. Panero, N. Tantalo (2023), arXiv:2211.09581

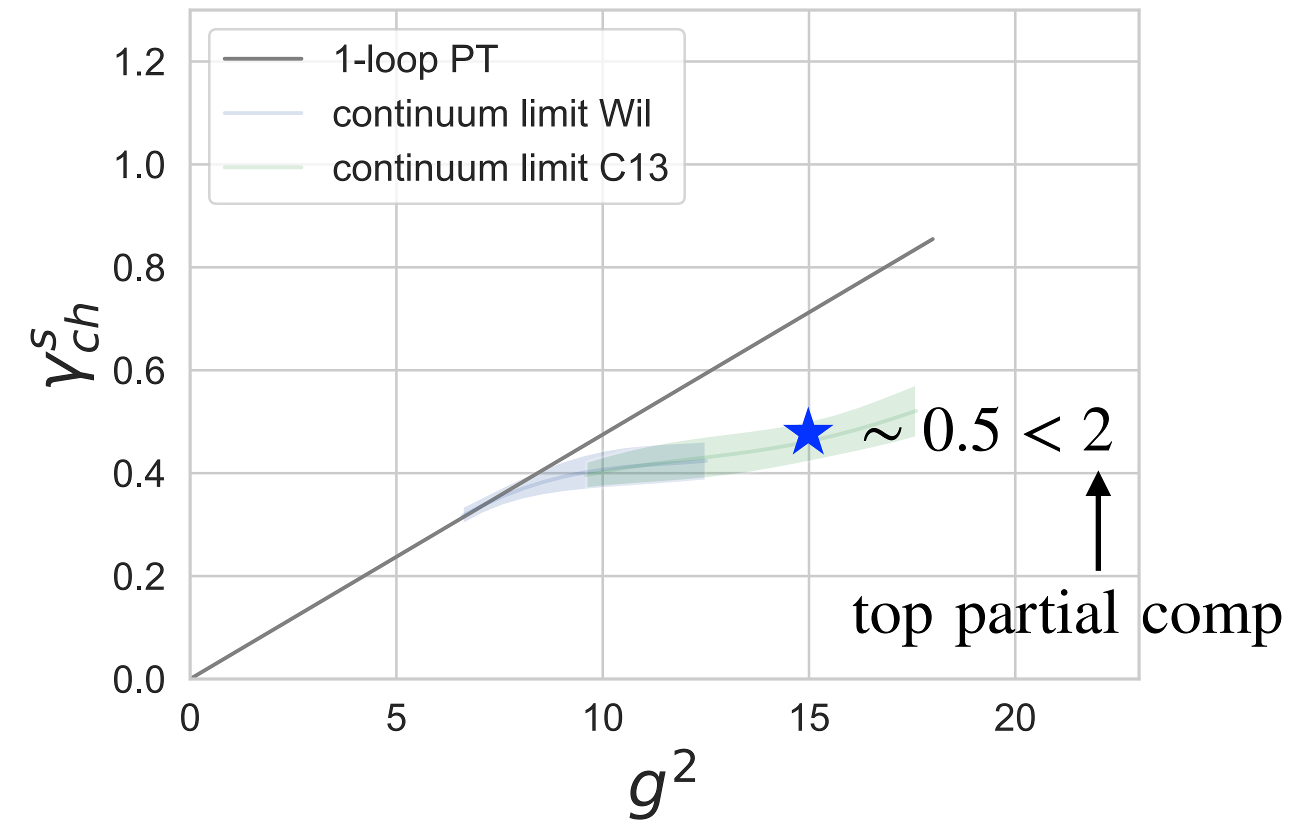
SU(4) + 4 Fund. + 4 AS fermions

- The other prototype SU(4) model with 4 fundamental and 4 antisymmetric Dirac fermions has been studied on the lattice using the continuous RG method.

Continuous RG beta function



anomalous dim. of chimera baryon



A. Hasenfratz et al, PRD 107 (2023), 114504

Talk by Y. Shamir @ 13:50, Mon.

- The IR fixed point indicates that the model is inside the conformal window.
- The mass anomalous dim. $\gamma^{\star(6)}_{\bar{\psi}\psi,\mathrm{IR}} \simeq 1.0$, which is consistent with the pert. result.

Ryttov & Shrock, arXiv:2307.12426

- But, the anomalous dim. of chimera baryon is too small to be used for top partial comp.

⚠ : # of flavors are different to Ferretti model

Theory space of Sp(4) gauge theory

Plot: B. S. Kim, D. K. Hong & JWL, PRD 101 (2020), 056008

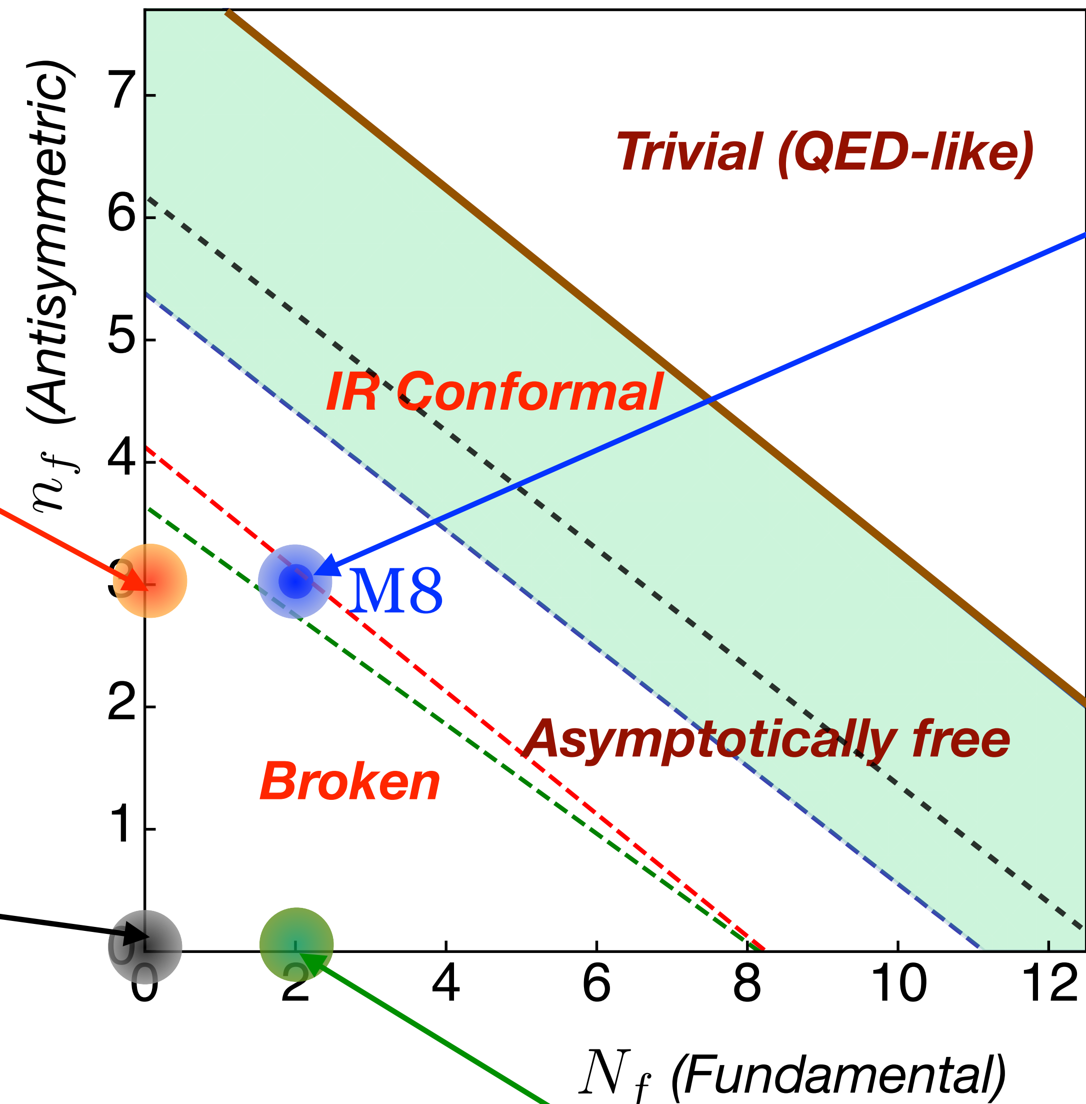
E. Bennett et al (2022), work in progress

- ☑ Meson spectrum in n_f=3 dynamical simulations

Talk by N. Forzano @ 17:00, Thur.

E. Bennett et al, JHEP 03 (2018) 185; PRD 101 (2020), 074516

- ☑ Glueballs & quenched meson spectrum



E. Bennett et al, PRD 106 (2022), 014501

- ☑ Exploratory studies of model M8: mesons & chimera baryons

E. Bennett et al, JHEP 12 (2019) 053

- ☑ Meson spectrum in N_f=2 dynamical simulations

SIMP dark matter

Theory space of Sp(2N) gauge theory

Plot: B. S. Kim, D. K. Hong & JWL, PRD 101 (2020), 056008

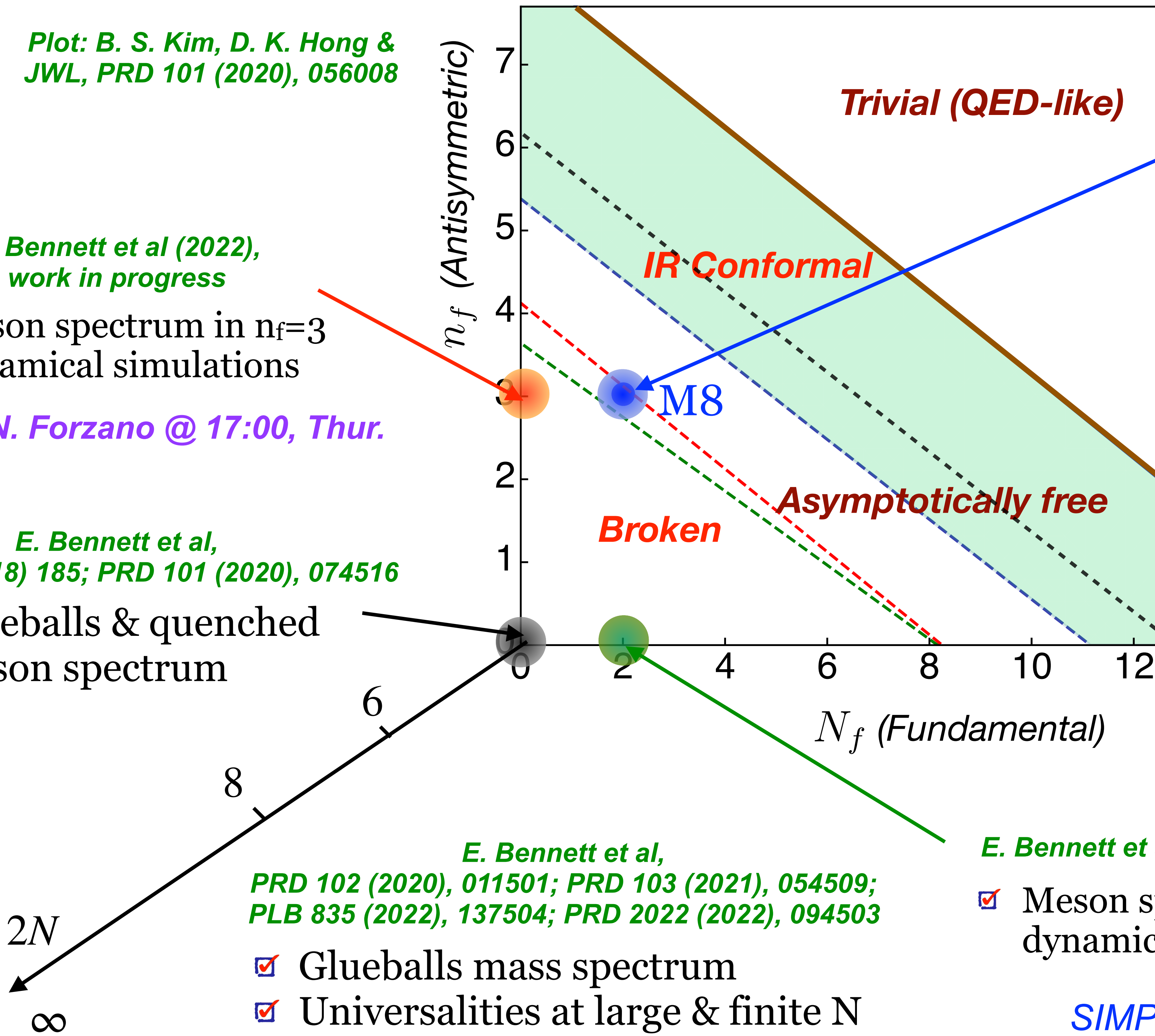
E. Bennett et al (2022), work in progress

- ☑ Meson spectrum in $n_f=3$ dynamical simulations

Talk by N. Forzano @ 17:00, Thur.

E. Bennett et al, JHEP 03 (2018) 185; PRD 101 (2020), 074516

- ☑ Glueballs & quenched meson spectrum



E. Bennett et al, PRD 106 (2022), 014501

- ☑ Exploratory studies of model M8: mesons & chimera baryons

E. Bennett et al, PRD 102 (2020), 011501; PRD 103 (2021), 054509; PLB 835 (2022), 137504; PRD 2022 (2022), 094503

- ☑ Glueballs mass spectrum
- ☑ Universalities at large & finite N
- ☑ Toplogical susceptibilities

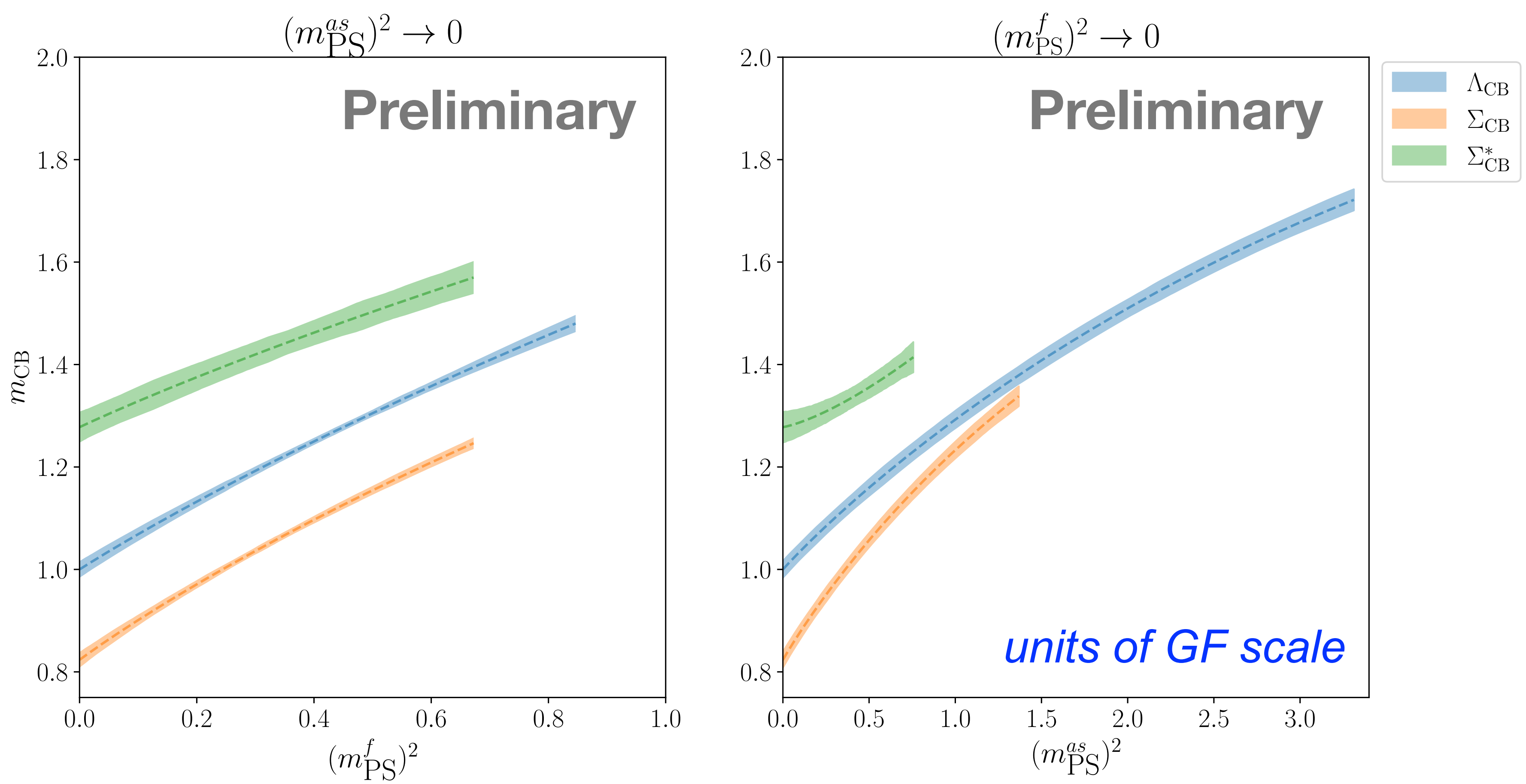
E. Bennett et al, JHEP 12 (2019) 053

- ☑ Meson spectrum in $N_f=2$ dynamical simulations

SIMP dark matter

Chimera baryon masses in quenched Sp(4)

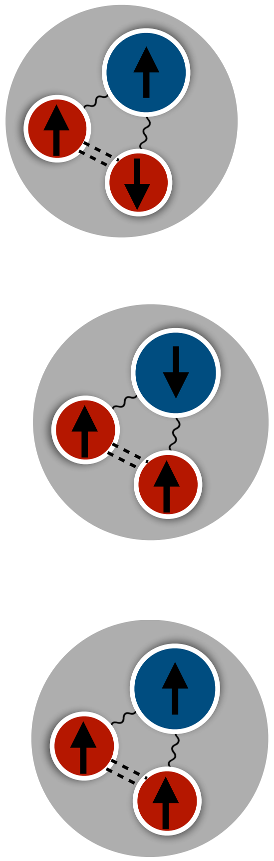
- Generate configurations at several values of the gauge coupling for pure Sp(4).
- Construct spin-1/2 & spin-3/2 chimera baryon operators in analogy to Λ & Σ baryons in QCD, and carry out $\mathcal{O}(100)$ measurements at wide range of masses am_0^f and am_0^{as}.
- Perform the massless and continuum extrapolations using an ansatz inspired by the heavy baryon chiral perturbation theory.



$\Lambda_{\rm CB}\left(\frac{1}{2}, 10\right)$

$\Sigma_{\rm CB}\left(\frac{1}{2}, 10\right)$

$\Sigma^*_{\rm CB}\left(\frac{3}{2}, 10\right)$



- $\Sigma_{\rm CB}$ is the lightest baryon state!

⚠ : quenched $Sp(4)$

Talk by H. Hsiao @ 16:40, Thur.

Composite dark matter

- Dark matter arises from new strong dynamics in the dark sector (isolated) or from SM extension with new strong extension, e.g. composite Higgs

- Dark matters are composite particles like hadrons in SM

  Mesons, Baryons, Glueballs

- Dynamical scale (Λ_D) associated with the confinement *Gravitational wave?*

- *Stability* can be guaranteed by accidental symmetries at low energy, e.g. proton

- Dark matters are neutral to SM interactions (*invisibility*), but constituents may or may not interact with SM particles.

- Self interactions are naturally accommodated *Solution to small scale problem*

Composite dark matter

Baryon-like

- Asymmetric DM (ADM)
  Kaplan, Luty, Zurek (2009)
- Technibaryons
  LSD collaboration (2013), LatKMI (2014), Fofor et al (2015)
- Stealth DM
  LSD collaboration (2013)
- Squeezeout DM
  Asadi, Kramer, Kuflik, Ridgway, Slatyer, Smirnov (2021)

U(1) baryon number

Glueball-like

- SUNonia
  Soni & Zhang (2016)

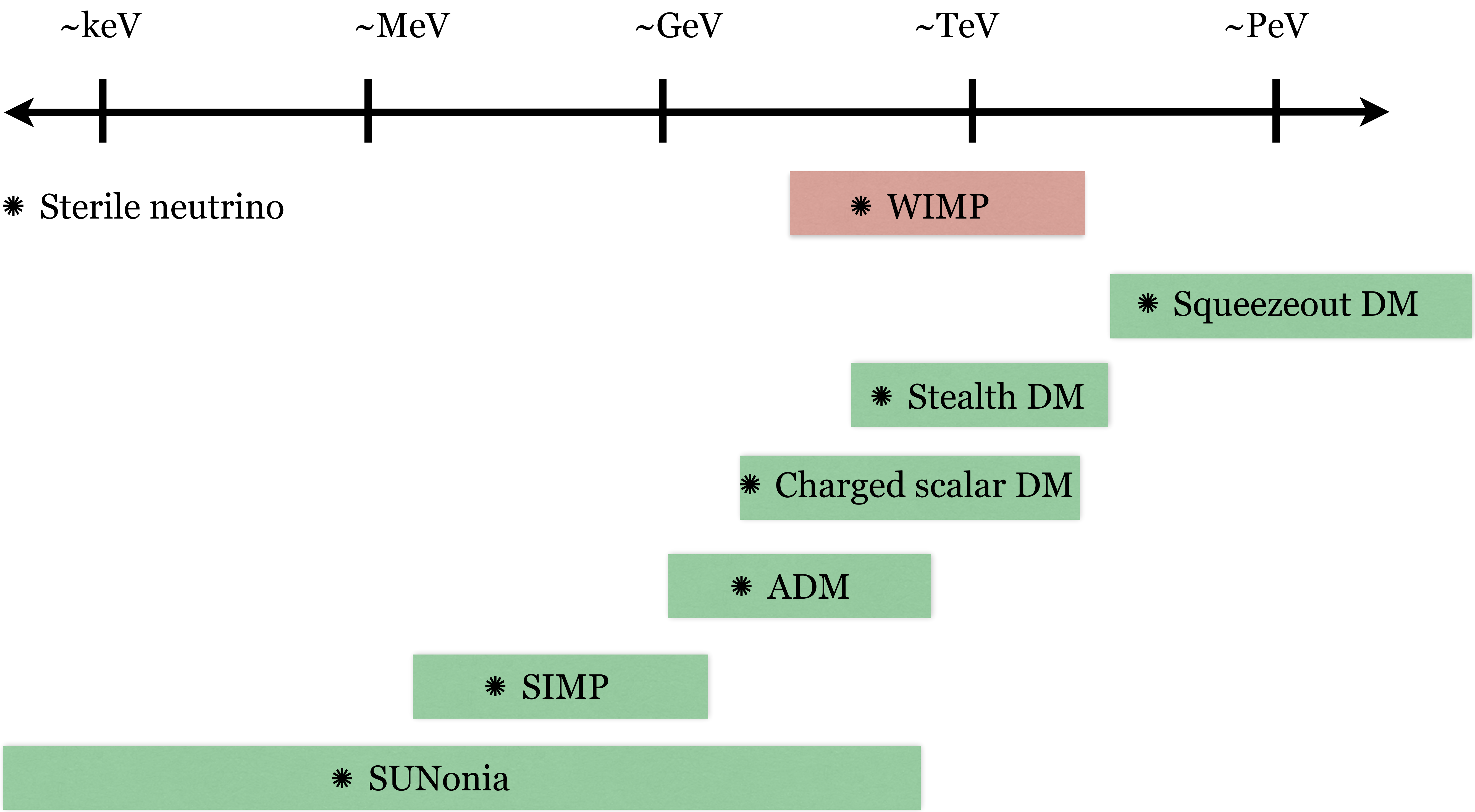
Gauge singlet of dim. 4

Meson-like

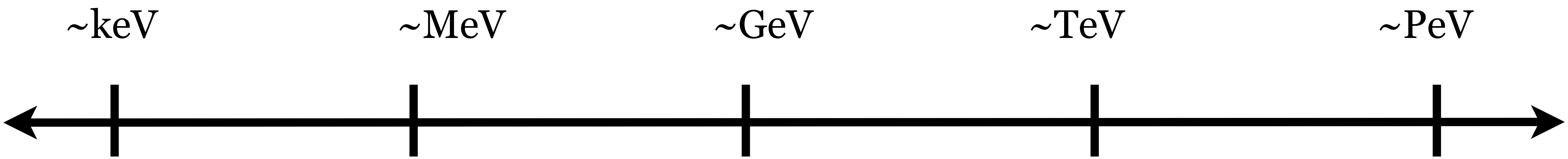
- pNGB DM
  Bai & Hill (2010), Buckley & Neil (2013), Hietanen, Lewis, Pica, Sannino (2014), …
- Charged scalar dark matter
  Frigerio, Pomarol, Riva, Urbano (2012), Cacciapaglia, Ma, Zhang, Wu (2017), Ballestreros , Carmona, Chala (2017), Balkin, Ruhdorfer, Salvioni, Weiler (2017), …
- Strongly interacting massive particle (SIMP)
  Hochberg, Kuflik, Murayama, Volansky, Wacker (2014)
- Quirky composite DM
  Kribs, Roy, Terning, Zuerk (2010)
- Composite Inelastic DM
  Alves, Behbahani, Shuster, Wacker (2010)

Species number (e.g. G parity)

Composite dark matter



Composite dark matter

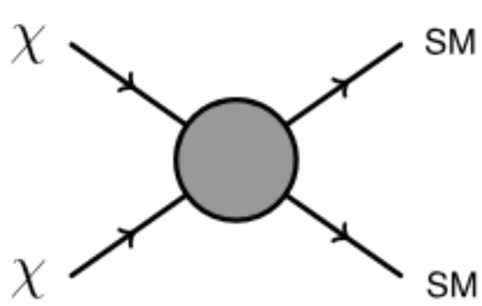


- Sterile neutrino
- WIMP
- Squeezeout DM
- Stealth DM
- Charged scalar DM
- ADM
- SIMP
- SUNonia

Strongly interacting massive particles (SIMPs)

Y. Hochberg, E. Kuflik, T. Volansky, J. Wacker, PRL 113 (2014) 171301

- Thermal freezeout is dominated by number changing process in the dark sector.

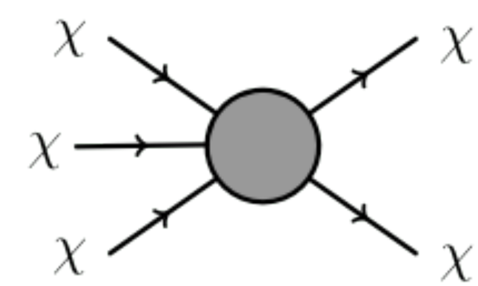


WIMP Freeze Out

$$\langle \sigma_{\rm ann} v \rangle_{2\to 2} = \frac{\alpha^2}{m_{\rm DM}^2}$$

$$m_{\rm DM} \sim \alpha_{\rm ann} \left(T_{\rm eq} M_{\rm Pl}\right)^{1/2}$$

$$\sim \ {\rm TeV}$$



Strongly Interacting Dark Matter

$$\langle \sigma v^2 \rangle_{3\to 2} \equiv \frac{\alpha_{\rm eff}^3}{m_{\rm DM}^5}$$

$$m_{\rm DM} \sim \alpha_{\rm eff} \left(T_{\rm eq}^2 M_{\rm Pl}\right)^{1/3}$$

$$\sim 100 \ {\rm MeV}$$

- The 5-point self interaction can arise by the Wess-Zumino-Witten anomaly term.

$$\mathcal{L}_{\rm WZW} = \frac{2N_c}{15\pi^2 f_\pi^5} \epsilon^{\mu\nu\rho\sigma} {\rm Tr}\left[\pi \partial_\mu \pi \partial_\nu \pi \partial_\rho \pi \partial_\sigma \pi\right]$$

cf) In QCD, $K^+ K^- \rightarrow \pi^+ \pi^0 \pi^-$

- DM sector should be remained in kinetic equilibrium with SM sector.

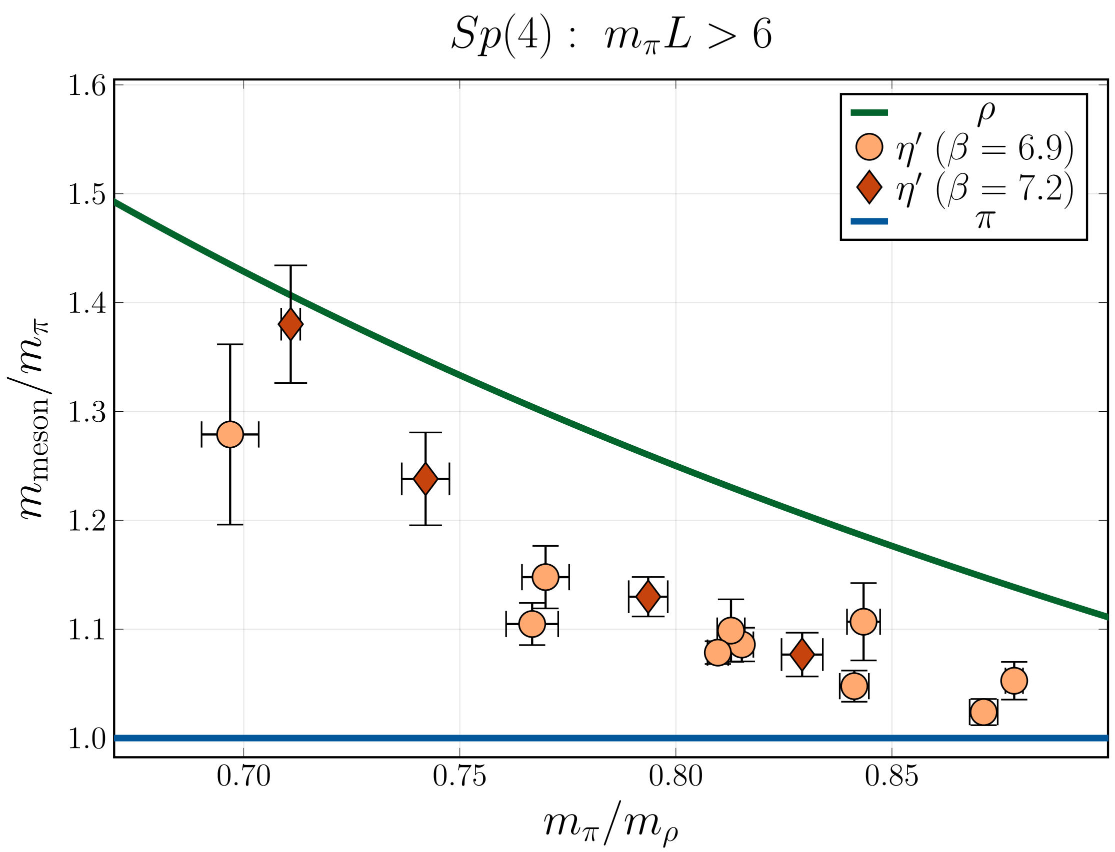
$$\frac{\Gamma_{\rm kin}}{\Gamma_{\rm ann}} = \frac{n_{\rm SM} \langle \sigma v \rangle_{\rm kin}}{n_{\rm DM} \langle \sigma v \rangle_{\rm ann}} \simeq 5 \times 10^6 \quad (m_{\rm DM} = 40 \ {\rm MeV})$$

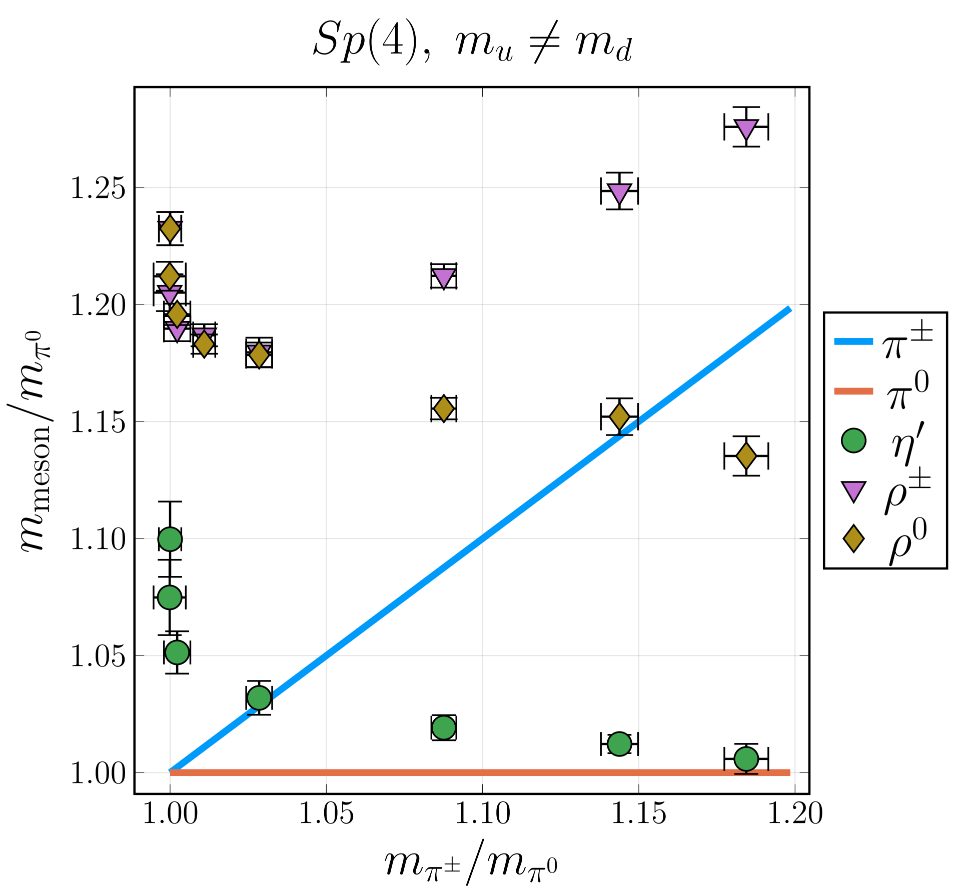
SIMP condition: $\left.\frac{\Gamma_{\rm kin}}{\Gamma_{3\to 2}}\right|_{T=T_F} \gtrsim 1, \quad \left.\frac{\Gamma_{\rm ann}}{\Gamma_{3\to 2}}\right|_{T=T_F} \lesssim 1$

Sp(4) + 2 Fund. fermions

- Flavor non-singlet mesons have been studied on the lattice for $0.65 \lesssim m_\pi/m_V \lesssim 0.87$

  E. Bennett et al, JHEP 12 (2019) 053
- Flavor-singlet pseudoscalar may play an important role in the dark matter pheno., e.g. destabilized by dark photon, involved in the dark pion scattering, etc.
- Involves disconnected diagrams - noice source, vacuum subtractions, …





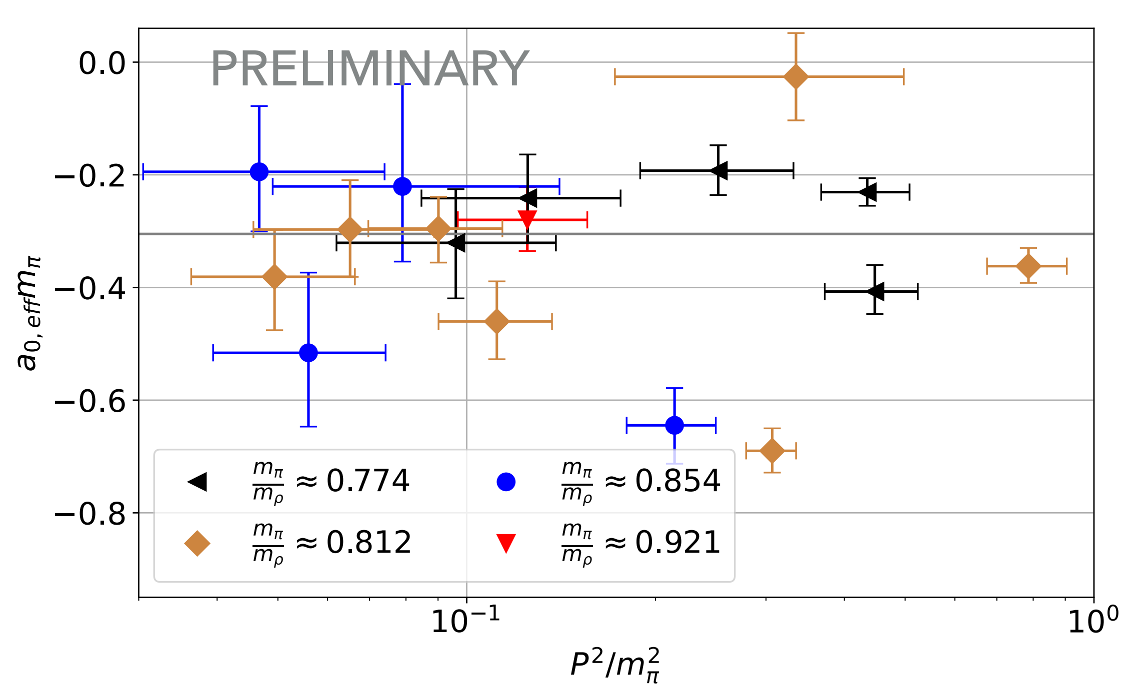
E. Bennett et al, arXiv:2304.07191

- The results are similar to 2-flavour QCD.

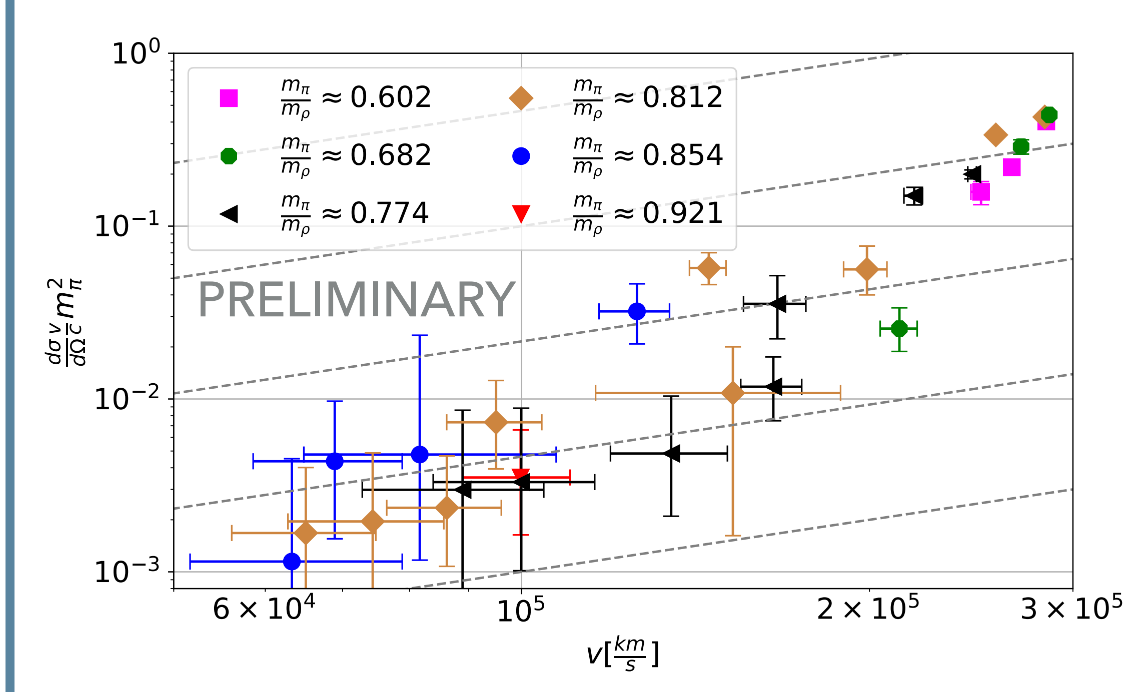
Sp(4) + 2 Fund. fermions

- Calculate the phase shift of dark pion s-wave scattering using Luscher's method, and extract the s-wave scattering length & explore the velocity dependent cross-section.

Talk by Y. Dengler @ 16:00, Thur.



Effective scattering length



Cross-section vs velocity

- The density profiles of galaxy clusters are constrained by astronomical observations by $\sigma/m_{\mathrm{DM}} < 0.19\mathrm{cm}^2/\mathrm{g}$, which indicates $m_{\mathrm{DM}} > 75\mathrm{MeV}$.

Stealth dark matter

LSD collaboration, PRD 89 (2014) 094508

- $SU(N)$ coupled to N_f fund. flavors: if N is even, baryons are composite scalars.
- Stability: $U(1)$ dark baryon number
- Typical setup: $N > 2$ & confinement scale ~ dark fermion mass
- Dark fermions are charged under electroweak symmetry in vector reps., interactions to SM particles can be highly suppressed - *stealth DM*

☑ Scalar dark baryon: magnetic dipole interaction (dim. 5) is absent

$$\frac{\bar{\psi}\sigma^{\mu\nu}\psi F_{\mu\nu}}{\Lambda_{\rm dark}}$$

☑ Proper charge assignment (e.g. custodial $SU(2)$ symmetry): no charge radius operators (dim. 6)

$$\frac{(\bar{\psi}\psi)v_\mu\partial_\nu F^{\mu\nu}}{(\Lambda_{\rm dark})^2}$$

☑ The leading interaction with photon is electromagnetic polarizability (dim. 7)

$$\frac{(\bar{\psi}\psi)F_{\mu\nu}F^{\mu\nu}}{(\Lambda_{\rm dark})^3}$$

SU(4) + 4 Fund. fermions

- Assuming the Higgs exchange between dark and ordinary baryons, the matrix elements have been calculated using the lattice spectroscopy & Feynman-Hellmann theorem.

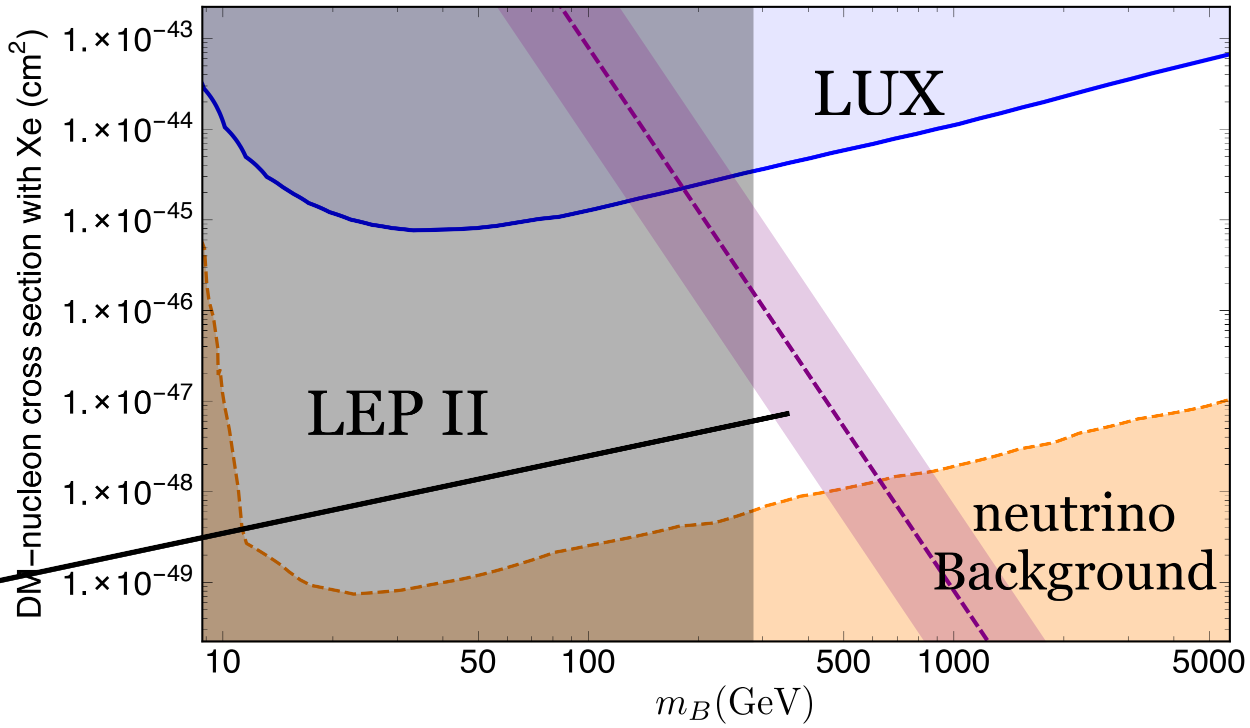
T. Appelquist et al, PRD 92 (2015), 075030

- Electric polarizability has been computed via lattice background method.

T. Appelquist et al, PRL 115 (2015), 171803

- Taking account of experimental and constraints, it has been found that

$$200 GeV \lesssim m_{\rm DM} \lesssim 700\,\mathrm{GeV}$$



- Quantitative study of the dark baryon scattering will play a crucial role in explaining the dark matter self-interaction, but the calculations are very challenging because of the signal-to-noise problem.

Laplacian Heaviside, Irreducible representations, ... **Talk by K. Cushman @ 14:30, Thur.**

- $SU(4)$ + 1 Fund. fermion: light dark matter, 1st order phase transition, ...

Talk by V. Ayyar @ 14:50, Thur.

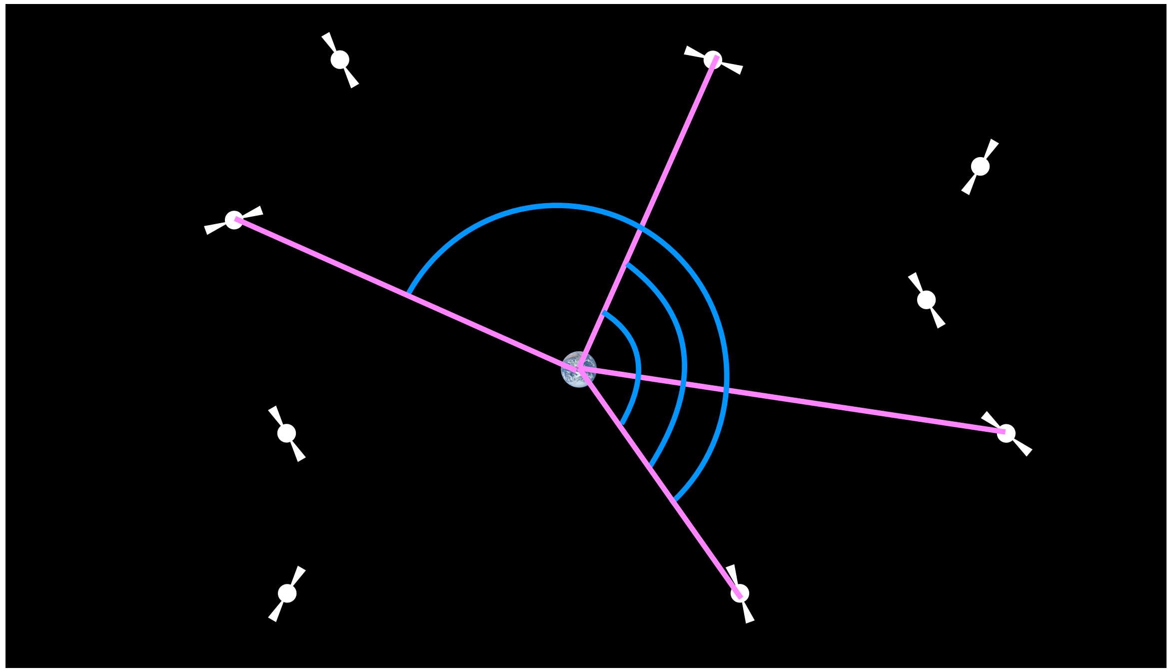
Gravitational waves (GWs)

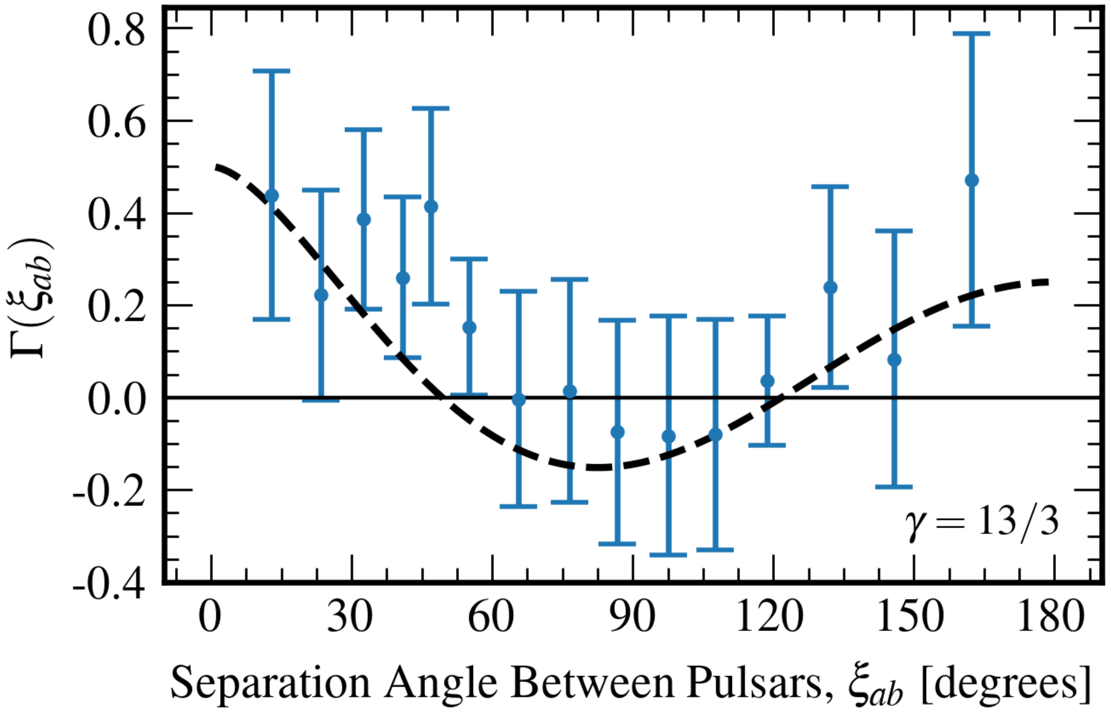
- In Jun. 28, 2023, NANOGrav (North American Nanohertz Observatory for Gravitational Waves) collaboration released the 15-year pulsar timing data set.

- Pulsar Timing Array (PTA) - A galactic-scale nHz GW detector using highly stable millisecond pulsars, rapidly rotating and highly magnetized neutron stars which act as highly accurate clocks.

Romani (1989); Foster & Backer (1990)

15-year results

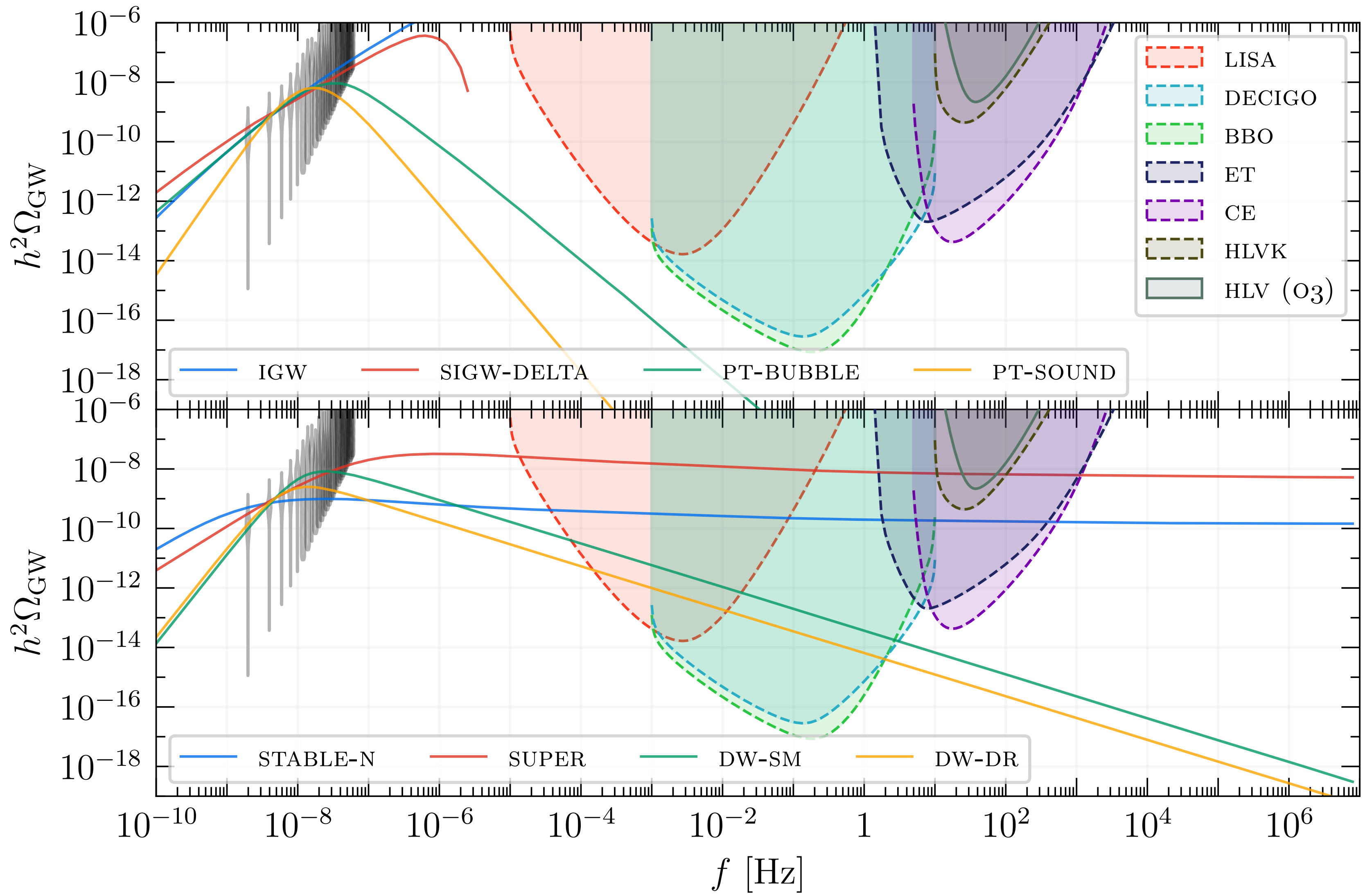




NANOGrav collaboration
Astrophys.J.Lett 951 (2023), L8

Gravitational waves (GWs)

- From the 15-year PTA data set, NANOGrav found positive evidence of a low-frequency stochastic gravitational wave (GW) background!



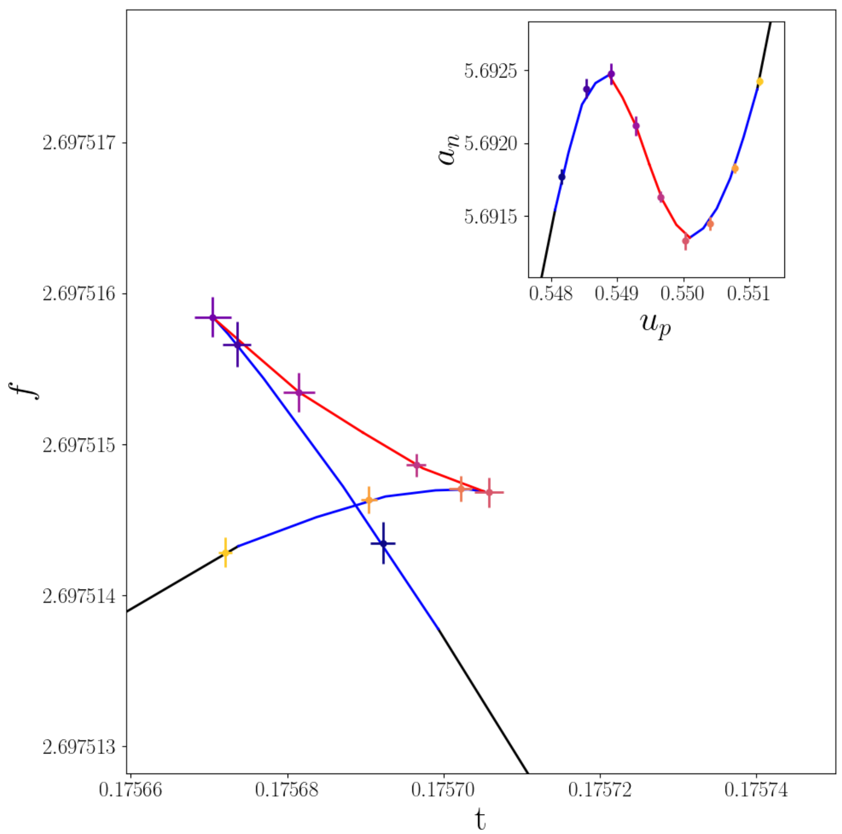
NANOGrav collaboration
Astrophys.J.Lett.951 (2023) L11

- Could it be a foot-print of a *1st order phase transition* in early universe? If so, what is the source for the transition? A noble strong dynamics? Maybe.

SU(3) YM - 1st order phase transition

Langfeld, Lucini & Rago, PRL 109, 111601 (2012)

- Density of state method using Logarithmic Linear Relaxation algorithm (LLR) has been applied to characterize the 1st order phase transition of $SU(3)$ Yang-Mills.



$$\rho(E) \equiv \int D\phi \, \delta(S(\phi) - E)$$

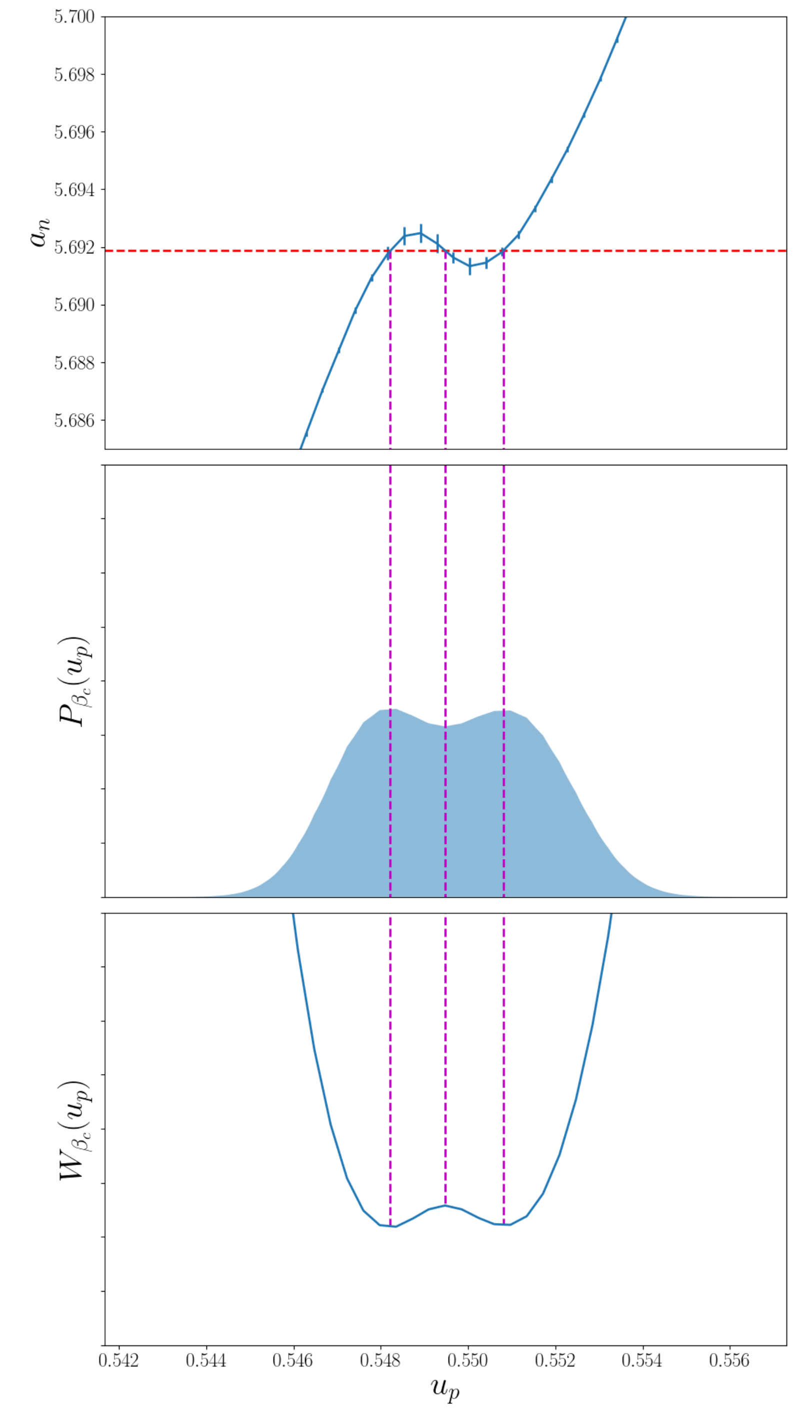
$$\log \tilde{\rho}(E) \equiv a_n(E - E_n) + c_n$$

$$F(t) \equiv E - ts$$

$$s = \log \rho(E)$$

$$1/t(E) \equiv \partial s/\partial E = a_n$$

Lucini, Mason, Rinaldi & Vadacchino (2023), arXiv: 2305.07463



- First lattice calculation exposing the multi-valued nature of the free energy in the critical region.
- The strength of the 1st order phase transition might be too weak to produce GWs. Other gauge groups?

Sp(4): talk by D. Mason @ 16:20, Thur.

Supersymmetric gauge theories

- Supersymmetry - a space-time symmetry extended by $4\mathcal{N}$ spinor operators

  Super-Poincare algebra: $\{Q^I_\alpha, Q^J_{\dot\alpha}\} = i2\delta^{IJ}\sigma^\mu_{\alpha\dot\alpha}p^\mu$, where $I, J = 1, 2, \cdots, \mathcal{N}$.

  Lattice discretization breaks the supersymmetry - may require severe fine-tuning.

- $\mathcal{N} = 1$ SYM - a minimal supersymmetric extension of $SU(N)$ Yang-Mills

  $SU(N)$ gauge theory + a massless adjoint Majorana fermion

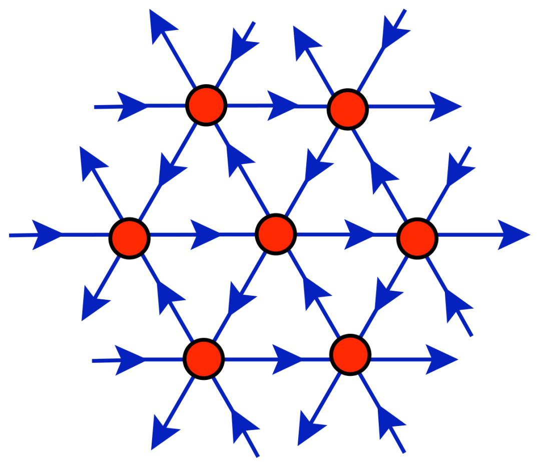
  Recent work focuses on the extension to supersymmetric QCD

  Talk by H. Herodotou @ 17:20, Tues.

- $\mathcal{N} = 4$ SYM - a maximal supersymmetric extension of $SU(N)$ Yang-Mills

  $SU(N)$ gauge theory + 4 fermions + 10 scalars (conformal)

  Much progress has been made to minimize the tuning of the parameters by reformulating the theory (topological twisting or orbifold dimensional reconstruction) and repackaging the fields contents to preserve close subalgebra $\{Q, Q\} = 0$.



A comprehensive review by D. Schaich, arXiv:2208.03580

$\mathcal{N} = 4$ supersymmetric Yang-Mills

- $\mathcal{N} = 4$ SYM exhibits a line of conformal fixed points, and conjectured to be described by a holographic dual of type IIb string theory on five dimensional anti-deSitter space. (Ads/CFT correspondence).

$$V(r) = -\frac{4\pi^2}{\Gamma^4\left(\frac{1}{4}\right)}\frac{\sqrt{\lambda}}{r}\left[1 + \frac{\kappa}{\sqrt{\lambda}} + O\left(\frac{1}{\lambda}\right)\right]$$

S.-x. Chu, D. Hou, H.-c. Hen, JHEP 08 (2009) 004

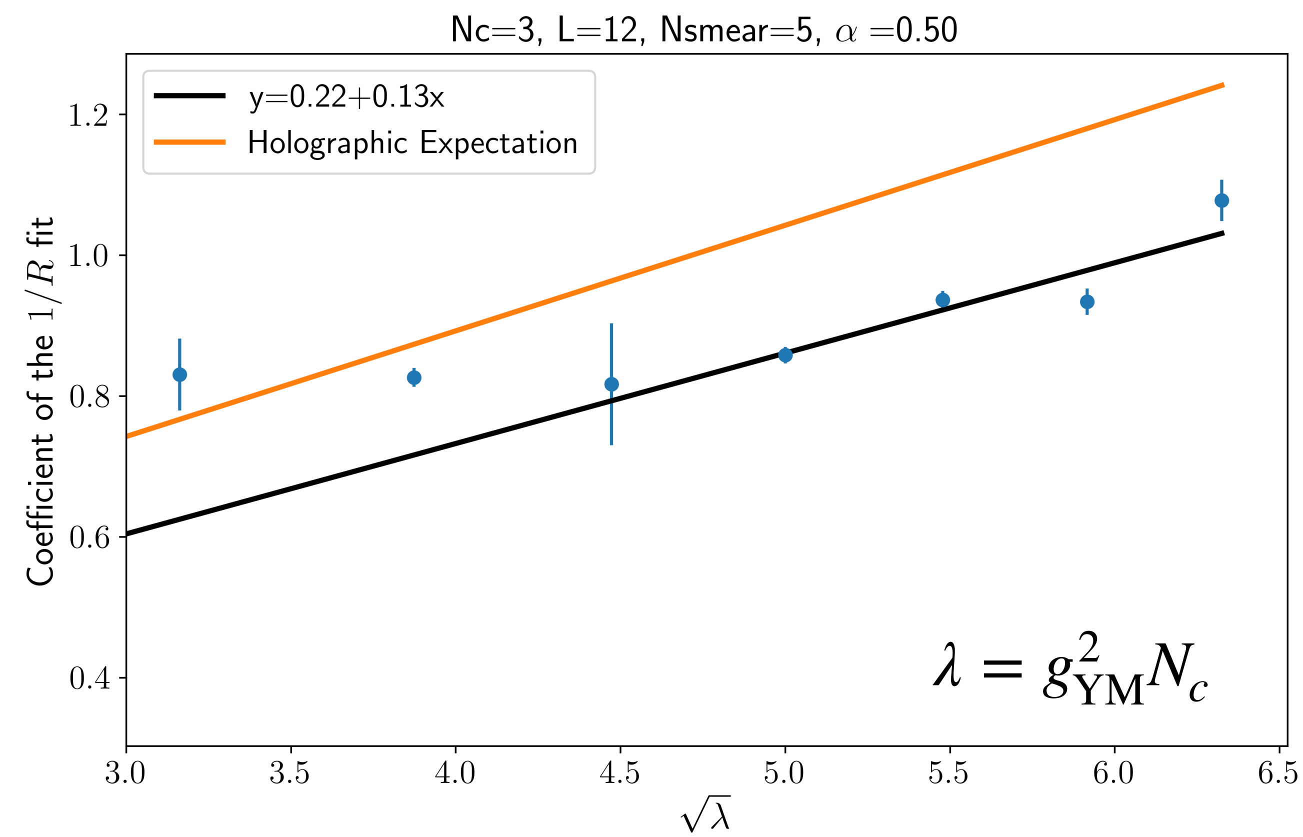
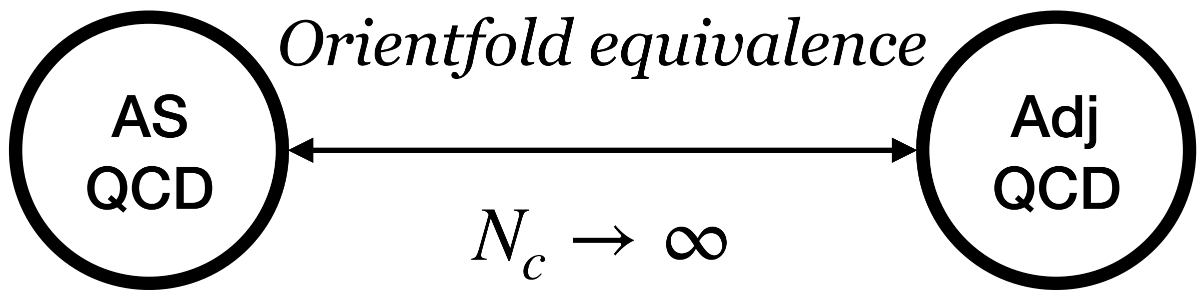


Figure 5. Coefficient of the $1/r$ vs $\sqrt{\lambda}$ for 12^4 lattices at $\mu = 0.05$

S. Catterall, J. Giedt, G. Toga (2023), arXiv:2303.16025

SU(3) + 1 AS fermion



Armoni, Shifman, Veneziano (2003)

- At large N_c, the bosonic sector of a gauge theory with a antisymmetic Dirac fermions is non-perturbatively equivalent to that of $\mathcal{N} = 1$ SYM.

- To start with, M. Morte et al consider one flavor QCD assuming $N_c = 3$ is large.

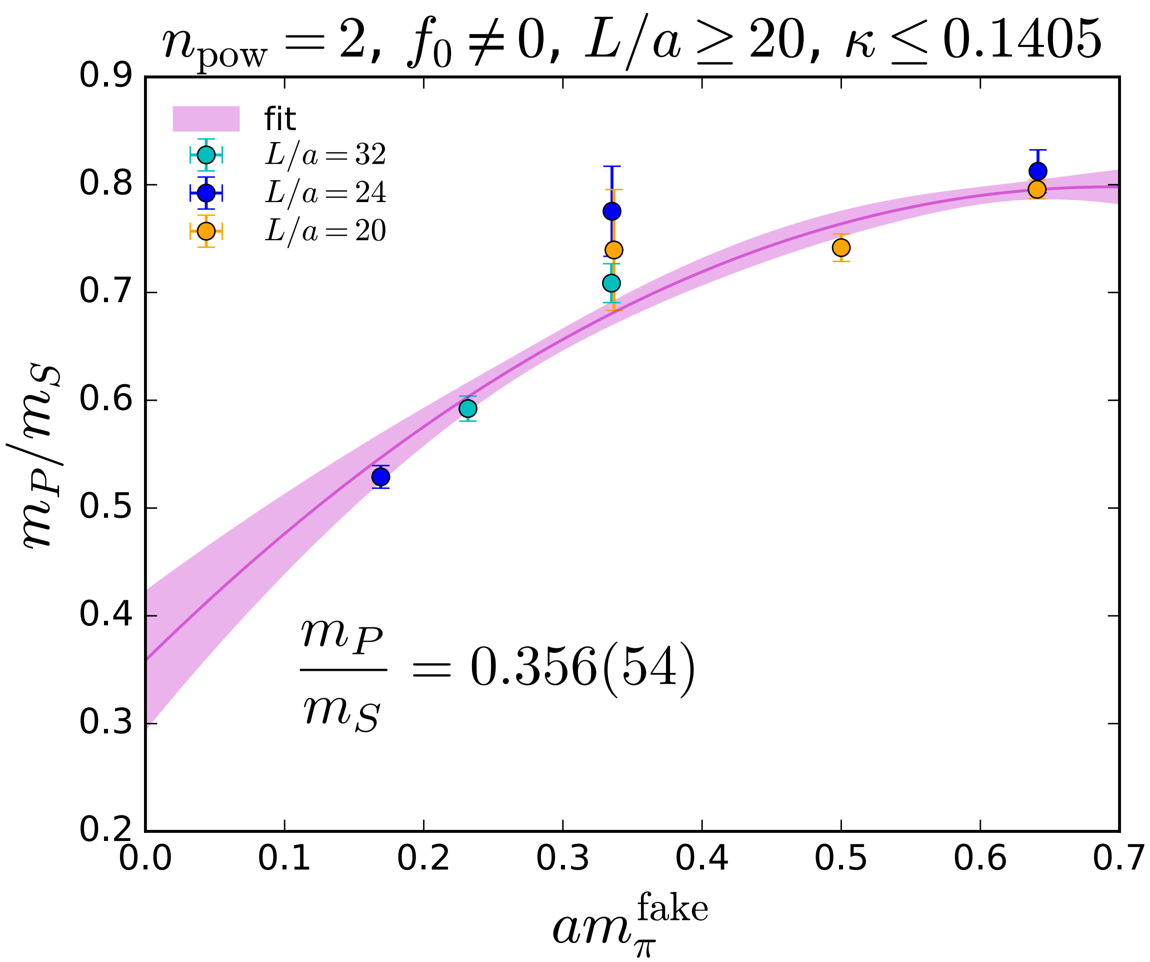
In the case of $SU(3)$, Fund. rep. $\equiv$ Two-index antisymm. (AS) rep

- Simulation setup

M. Morte, B. Jager, F. Sannino, J. Tsang, F. Ziegler (2023), arXiv:2302.10514

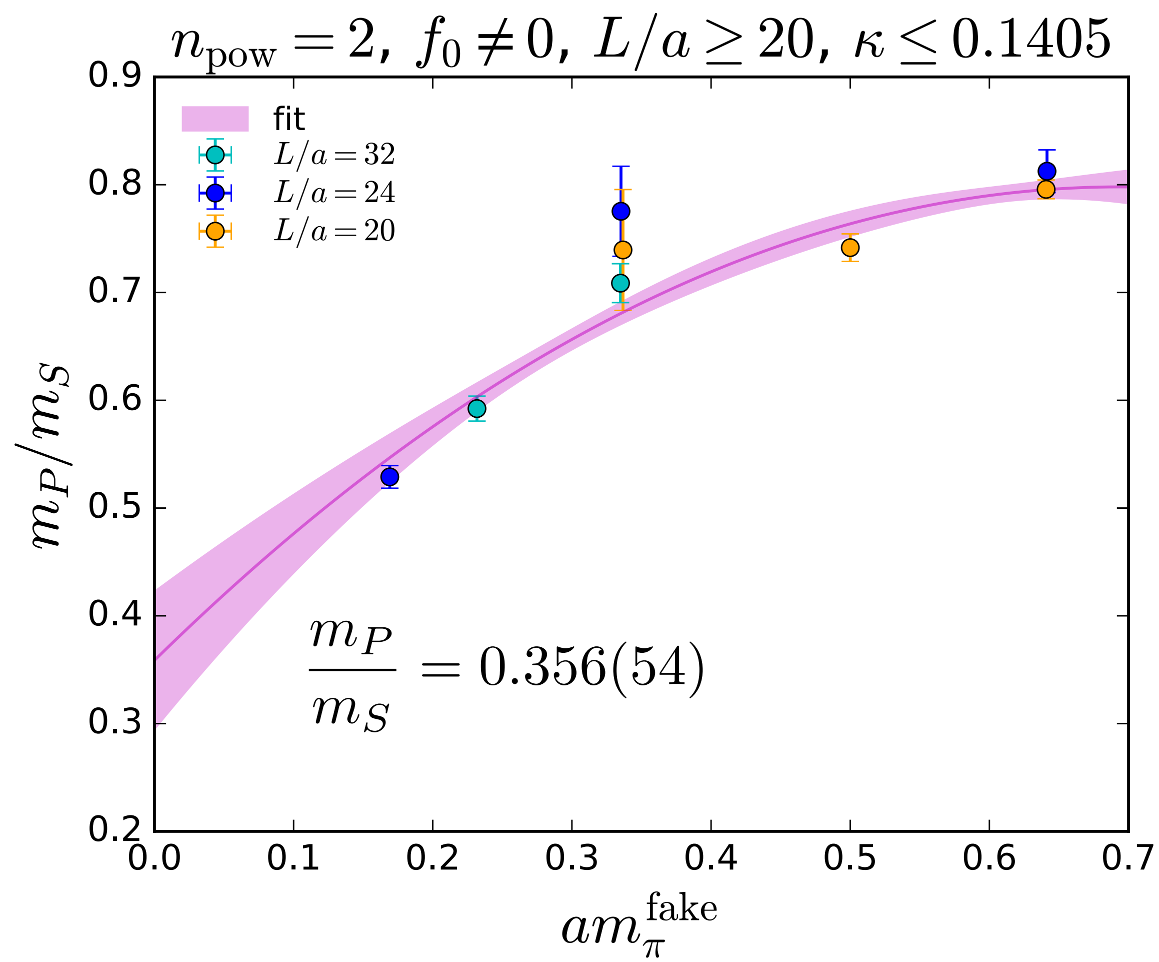
  - Wilson fermions with tree-level improvement in both the gauge and fermionic actions with $c_{SW} = 1$.
  - Rational hybrid Monte Carlo (RHMC) + reweighting for negative fermion determinant

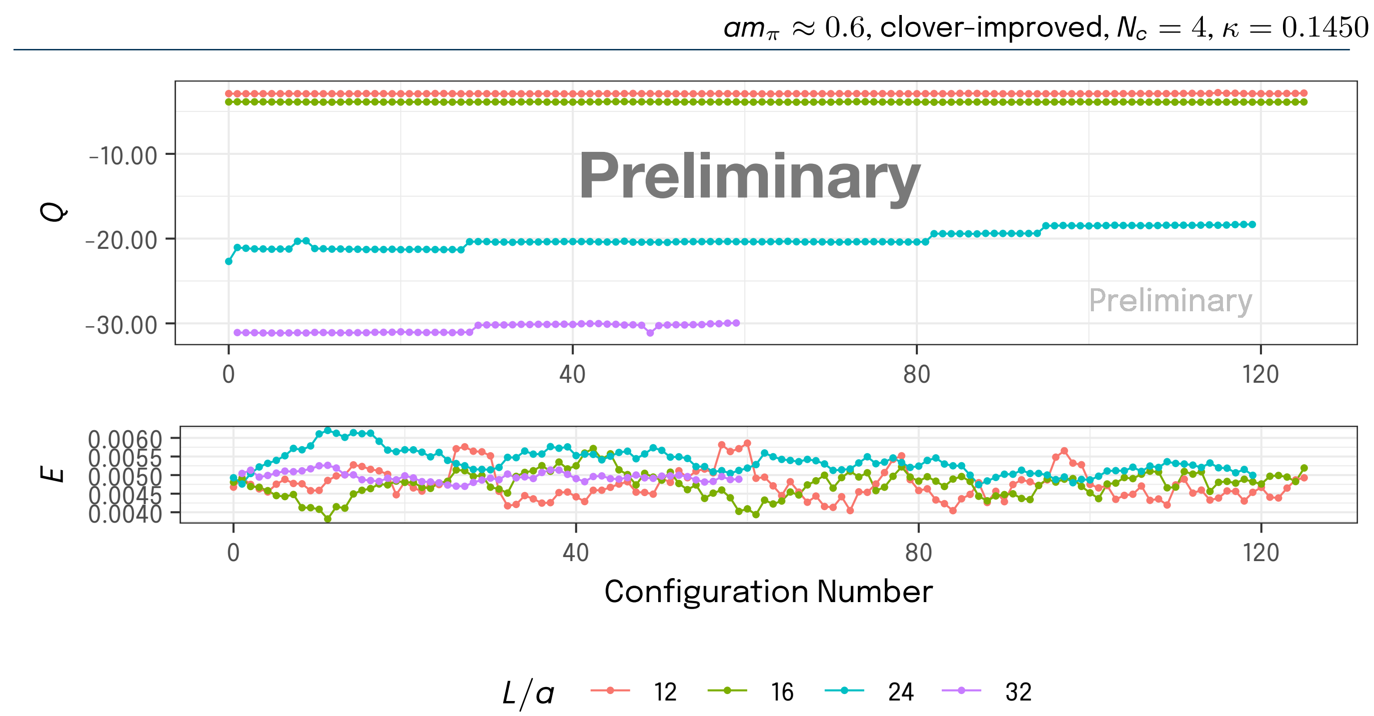
SU(N) + 1 AS fermion



- In the massless limit, EFT calculation yields $\dfrac{m_P}{m_S} \lesssim 0.185 + \mathcal{O}(\dfrac{1}{N_c^2})$ ***Saninno & Shifman(2004)***

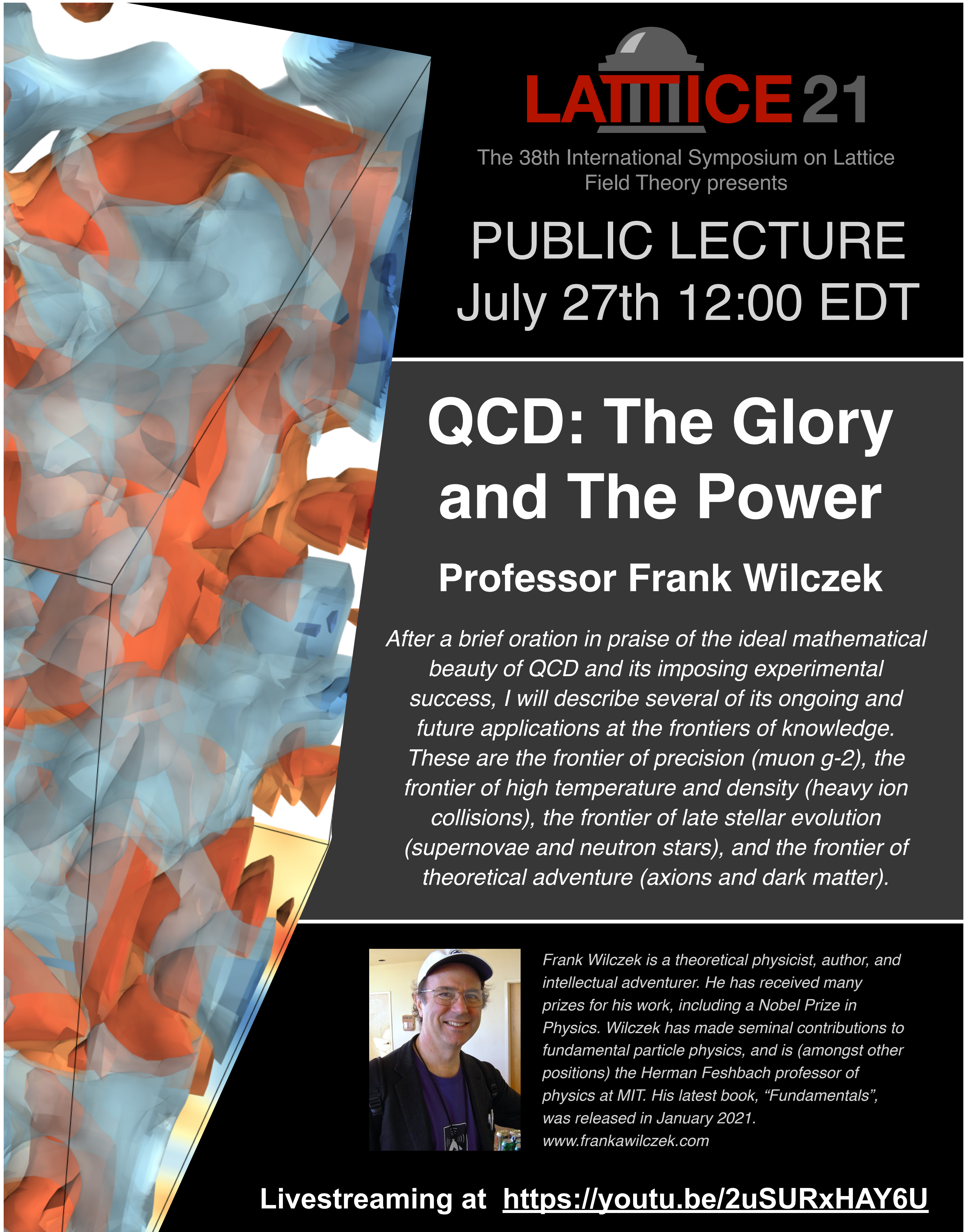
SU(N) + 1 AS fermion





- In the massless limit, EFT calculation yields $\frac{m_P}{m_S} \lesssim 0.185 + \mathcal{O}(\frac{1}{N_c^2})$ *Saninno & Shifman(2004)*
- To verify the orlentfold equivalence and obtain a better insight on $\mathcal{N} = 1$ SYM, they have extended their studies to larger values of $N_c = 4, 5, 6$.
- Challenges: opological freezing problem gets severe as $N_c \to \infty$ and $am_\pi \to 0$.

Talk by S. Martins @ 15:10, Thur.



In his response to the question from Z. Davoudi

Another exploratory aspect that's of special interest to physicists is to look at variants of QCD, with four quarks or a different color group or some that might show up in future models of fundamental interactions, but also would enable us to frame the properties of QCD in a broader context and see what is contingent in a sense and what really is crucial. And there are also attempts to make anthropic arguments, quantitative, how much could you vary the parameters and still get something to friendly biology, things like that.

Thank you for your attention!

Any questions?

Acknowledgement

M. D. Morte	$SU(N_c)$ gauge theories couple to $N_f = 1$ antisymmetric Dirac fermion
Y. Shamir	$SU(4)$ gauge theories couple to $N_f = 4$ fundamental and $n_f = 4$ antisymmetric Dirac fermions
S. Catterall	$\mathcal{N} = 4$ supersymmetric Yang-Mills; Symmetric mass generation
D. Nogradi	Conformal window of $SU(3)$ gauge theory + N_f fundamental flavors
B. Lucini	Density of states method and first order phase transition
A. Lupo	Spectral density calculations for $SU(4)$ gauge theories couple to $N_f = 2$ fundamental and $n_f = 2$ antisymmetric Dirac fermions
R. Shrock	Scheme-independent BZ expansion and conformal window
A. Hasenfratz **C. Peterson**	Continuous RG method and conformal window
D. Schaich	EFT analysis of $SU(3) + N_f = 8$ flavors results; Stealth dark matter
C. -J. D. Lin **G. Catumba**	2 Higgs Doublet Model (2HDM) on the lattice