The Muon g-2 Experiment at Fermilab

Alex Keshavarzi
University of Manchester

Lattice 2023 (Fermilab)
4th August 2023
The Muon g-2 Experiment at Fermilab

(NOT FOR DATA ANALYSIS…)

[Image of a Muon g-2 experiment setup]
Run 1 (2020) result: ~5% of full stats, 434 ppb stat ⊕ 157 ppb syst errors

Run 2/3 analysis complete, aiming to reduce experimental error by 2. Systematics on track for < 100 ppb.

Run 4/5/6, aiming for another factor of ~2 reduction in error: 70 ppb stat ⊕ 70 ppb syst errors

TDR target was 20 x BNL

Run-2/3 result announcement on August 10th.
The Run-1 Result

The g-2 Theory Initiative recommended SM value:
- 2020 compilation from published work only.
- HLbL includes data-driven theory and lattice.
- HVP entirely based on data-driven evaluation.
- Net uncertainty, driven by HVP is ~ 369 ppb.

The 2021 Run1 g-2 result:
- Confirmed the BNL result.
- Led to net increase in discrepancy with theory at 4.2σ.
- Statistical uncertainty: 434 ppb; Systematics: 159 ppb).
- World average uncertainty: 350 ppb.
Measurement principle

• Put a beam of polarized muons into a storage ring magnet.
• Both the muon spin and momentum precess.
• Because $g > 2$, the spin precesses faster than the momentum.
• Parity violation in muon decay means the highest energy positrons are emitted preferentially in the direction of the muon spin.

\[ a_\mu = \frac{eB}{mc} \]

\[ s_{\nu e} \quad p_{\nu e} \quad s_{\bar{\nu}_\mu} \quad p_{\bar{\nu}_\mu} \quad s_{\mu^+} \quad p_{\mu^+} \]

\[ \mu^+ \quad s_{e^+} \quad p_{e^+} \]

Count the number of positrons above a certain energy.
What we actually measure

The experiment actually measures two frequencies:

\[
\alpha_\mu = \frac{3\text{ppb}}{\mu_a} \frac{0.0003\text{ppb}}{\mu_p} \frac{m_\mu}{m_e} \frac{g_e}{2}
\]

What we measure

Magnetic Field Map, \(\omega'_p\)

Muon Distribution, \(M_\mu\)

Anomalous Precession Frequency, \(\omega_a\)
The experiment

24 Calorimeter stations located all around the ring

NMR probes and electronics located all around the ring

Kicker
The magnetic field map

Trolley measures the field in the ring every ~3 days

Fixed probes monitor the field in between trolley runs

Calibrated using the plunging probe and a spherical water and helium-3 probe

\[ a_\mu \propto \frac{\omega_a}{\langle \omega_p' \times M_\mu \rangle} \]
The muon beam distribution: $a_\mu \propto \frac{\omega_\alpha}{\langle \omega_p \times M_\mu \rangle}$

The trackers provide a non-destructive measurement of the beam position as a function of time.

This is used to convolute with the field to know the field the muons experienced at the point of decay.

It is also important due to detector acceptance effects.
The precession frequency $a_\mu \propto \left\langle \omega_p \times M_\mu \right\rangle$

Need to account for acceptance changes due to beam motions and slow effects on the exponential due to muon losses

Simple 5-parameter fit $\chi^2 / \text{ndf} = 8191 / 4149$

Fit with extra terms $\chi^2 / \text{ndf} = 4005 / 4134$
Real world complications

Beam Dynamics:
E-field, Pitch, Muon Losses, Phase-Acceptance

Transient Magnetic Fields:
Kicker Eddy Current, Quad Vibrations

Field Calibration “3 ingredients”

\[
\omega_a \left\langle \omega'_p \times M_\mu \right\rangle = \frac{f_{\text{clock}} \omega^m_a}{f_{\text{calib}} \left\langle \omega'_p \times M_\mu \right\rangle^m} \left( 1 + C_e + C_p + C_{ml} + C_{pa} \right) \frac{1}{1 + B_k + B_q}
\]

Clock Blinding

\[a_\mu \hat{B} - \left( a_\mu - \frac{1}{\gamma^2 - 1} \right) \vec{\beta} \times \vec{E} - a_\mu \left( \frac{\gamma}{\gamma + 1} \right) \left( \vec{\beta} \cdot \hat{B} \right) \vec{\beta} \]

E.g., the beam has a small vertical component which is focused using electrostatic quadrupoles, but this introduces extra terms:

We can minimise the first by choosing \( \gamma = 29.3 \) to give \( p_\mu = 3.1\text{GeV} \), the magic momentum

For a 1.45T field, this sets the radius of the ring to 7.11m

However we now have 2 corrections to make to \( a_\mu \) because:

- Not all muons are at the ‘magic’ momentum of 3.1GeV
- Vertical momentum component aligned with B field
- Both corrections depend on the quadrupole field strength, and are < 0.5ppm
Real world complications

e.g. corrections due to fast transient fields from the pulsed systems

Muons experience a field change which the fixed probes don’t see due to shielding

Effects measured in dedicated measurement campaigns
Systematics improvements since Run-1

**Coherent Betatron Oscillations**

Kickers upgraded during run-3 to provide a more optimal kick, reducing the CBO oscillation.

Introduction of the quad RF system in run 5 further reduced the amplitude of the oscillations.

**Damaged Quadruoiple Resistors**

Damaged quad resistors in Run-1 distorted beam distribution.

Led to a time dependent phase due to calorimeter acceptance.

Was fixed before Run-2.

**Quadrupole Field Transient**

Designed special NMR probe which is inserted into the storage reason to measure the transients at all positions.

**Temperature Stability**

Temperature stability of the hall and magnet was improved reducing variations and systematics.

And many more...
What are we heading towards?

Run-2/3

- Result announcement on August 10th 2023.
- Statistics ~ 200 ppb
- Systematics ~ 100 ppb
- e.g. field measurement systematic uncertainties:

Entire data set (Runs 1-6)

- Statistics <100 ppb
- Precession systematics <<70 ppb
- Field systematics <<70 ppb
- Not thought of yet ~50 ppb (a guess)

\[ \text{dm} = <140 \text{ ppb} \]

~133 ppb

*Warning: until we look at the data, we can’t be sure about final systematics, so this is just a good guess

Largest uncertainties were transients, now understood much better.

~133 ppb

We are here now
HVP: Dispersive Approach

Alex Keshavarzi
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Dispersive HVP: the challenge

\[ \Delta a_\mu = 279(76) \times 10^{-11} \rightarrow 2.39(0.65) \text{ ppm} \]

- \( a_\mu \) arises due to quantum corrections / higher order interactions / loop contributions

- All SM particles contribute \( \rightarrow \) Calculate and sum all sectors of the SM.

\[
a_\mu = a^{\text{SM}}_\mu + a^{\text{QED}}_\mu + a^{\text{EW}}_\mu + a^{\text{HVP}}_\mu + a^{\text{HLbL}}_\mu
\]

**QED**

1-loop + 2-loop + ... Perturbative (Known to five-loop)

\( a^{\text{SM}}_\mu \) portion \( \sim 99.99\% \)

\( \delta a^{\text{SM}}_\mu \) portion \( \sim 0.001\% \)

**EW**

\( \mu, \nu_\mu, W, Z, H \) Perturbative (Known to two-loop)

\( a^{\text{SM}}_\mu \) portion \( \sim 1\% \)

\( \delta a^{\text{SM}}_\mu \) portion \( \sim 0.2\% \)

**HVP**

Non-perturbative (Data-driven & lattice)

\( a^{\text{SM}}_\mu \) portion \( \sim 59\% \)

\( \delta a^{\text{SM}}_\mu \) portion \( \sim 84\% \)

**HLbL**

Non-perturbative (Data-driven & lattice)

\( a^{\text{SM}}_\mu \) portion \( \sim 1\% \)

\( \delta a^{\text{SM}}_\mu \) portion \( \sim 16\% \)
Dispersive HVP: the method

⇒ We want to calculate the leading order hadronic vacuum polarisation (HVP) contribution

1) Feynman rules for HVP insertion to photon propagator:
\[ \mu \sim q \quad q \quad \nu \sim \frac{-ig^{\mu\nu}}{(q^2 - i\varepsilon)}(i\varepsilon)i\Pi_{\alpha\beta}(q^2)(-i\varepsilon)\frac{-ig^{\beta\nu}}{(q^2 - i\varepsilon)} \]

2) Employ analyticity:
\[ \mu \sim q \quad q \quad \nu = \frac{i\varepsilon y_{\mu\nu}}{(q^2 - i\varepsilon)^2} \frac{q^4}{\pi} \int_{s_{th}}^{\infty} ds \frac{\text{Im} \Pi(s)}{s(s - q^2 - i\varepsilon)} \]

3) Insert to vertex correction, solve for \( a_{\mu}^{\text{had, LO VP}} \):
\[ a_{\mu}^{\text{had, LO VP}} = \frac{\alpha}{\pi^2} \int_{s_{th}}^{\infty} \frac{ds}{s} \text{Im} \Pi_{\text{had}}(s) K(s) \]

4) Utilise optical theorem:
\[ \text{Im} \gamma_{\text{had}} \Longleftrightarrow \gamma_{\text{had}} \quad \text{Im} \Pi_{\text{had}}(q^2) \]

5) Arrive at equation for \( a_{\mu}^{\text{had, LO VP}} \):
\[ a_{\mu}^{\text{had, LO VP}} = \frac{1}{4\pi^3} \int_{s_{th}}^{\infty} ds \sigma_{\text{had},\gamma}^0(s) K(s) \]
\[ \sigma_{\text{had},\gamma}^0 = \text{bare cross section, FSR included} \]

⇒ Similar dispersion integrals for NLO and NNLO HVP

Any and all permitted hadrons

Strongly weighted at low-energy (non-perturbative regime)
The measured data

Dedicated measurements of $e^+e^- \rightarrow$ hadrons.
• $\lesssim 2$ GeV = exclusive final states ($\pi^0\gamma$, $2\pi$, $3\pi$, $4\pi$, $5\pi$, $6\pi$, $7\pi$, $K\bar{K}$, $K\bar{K}\pi$, $K\bar{K}2\pi$, $2K\bar{K}$, $p\bar{p}$, $n\bar{n}$ ...).
• $\gtrsim 2$ GeV = inclusive hadronic R-ratio (all hadrons).

Two methods from cross section measurement:
• Direct energy scan - fixed CM energy measurement of production cross section.
• Radiative return – measure differential cross section with tagged ISR photon to reconstruct production cross section.

Radiative Return

**Babar ($E_{CM} = \Upsilon(4s)$)**
- Comprehensive (almost all) exclusive final states measured below 2 GeV.
- High statistics, from-threshold measurements of $\pi^+\pi^-$.  

**KLOE ($E_{CM} = \phi$)**
- 3 high-precision measurements of $\pi^+\pi^-$ on $\rho$-resonance, using different methods.
- Combination results in most precise measurement of $\pi^+\pi^-$. 

Direct scan

**SND and CMD-3 (Novosibirsk)**
- Both located at VEPP-2000 machine.
- Comprehensive (almost all) exclusive final states measured below 2 GeV.

**KEDR (Novosibirsk)**
- Inclusive measurement.

**Others**
- CLEO-c ($\pi^+\pi^-$).
- Belle-II (hopefully in the near future).

**BES-III ($E_{CM} = 2$-5 GeV)**
- High-precision measurement of $\pi^+\pi^-$ on $\rho$-resonance.
- Measurements of other modes, e.g. $\pi^+\pi^-\pi^0$, inclusive.

**Others**
- CLEO-c ($\pi^+\pi^-$).
- Belle-II (hopefully in the near future).

Plus, many older measurements from now inactive experiments…
Radiative Corrections: MC Generators

We need high-precision MC generators for radiative corrections at the experiment level:

**Direct scan:**
- For $2\pi$, radiative corrections account for ISR and FSR effects.
- For non-$2\pi$:
  - Radiative correction accounts for ISR effects only.
  - Efficiency is calculated via Monte Carlo + corrections for imperfect detector.

**Radiative return:**
- Precise knowledge of ISR-process through radiator function is paramount.

$$s \cdot \frac{d\sigma_{\pi\pi}}{ds_\pi} = \sigma_{\pi\pi}(s_\pi) \times H(s,s_\pi)$$

MC generators for exclusive channels (exact NLO + Higher Order terms in some approx)

<table>
<thead>
<tr>
<th>MC generator</th>
<th>Channel</th>
<th>Precision</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>MCGP.J (VEPP-2M, VEPP-2000)</td>
<td>$e^+e^- \rightarrow e^+e^-, \mu^+\mu^-, \pi^\pm\pi^\mp, \ldots$</td>
<td>0.2%</td>
<td>photon jets along all particles (collinear Structure function) with exact NLO matrix elements</td>
</tr>
<tr>
<td>BabaYaga@NLO (KLOE, BaBar, BESIII)</td>
<td>$e^+e^- \rightarrow e^+e^-, \mu^+\mu^-, \gamma\gamma$</td>
<td>0.1%</td>
<td>QED Parton Shower approach with exact NLO matrix elements</td>
</tr>
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MC generators for ISR (from approximate to exact NLO)

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<th>MC generator</th>
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<tbody>
<tr>
<td>EVA (KLOE)</td>
<td>$e^+e^- \rightarrow \pi^+\pi^-\gamma$</td>
<td>0(%)</td>
<td>Tagged photon ISR at LO + Structure Function FSR: point-like pions</td>
</tr>
<tr>
<td>AFKQED (BaBar)</td>
<td>$e^+e^- \rightarrow \pi^\pm\pi^-\gamma, \ldots$</td>
<td>depends on the event selection (can be as good as Phokhara)</td>
<td></td>
</tr>
<tr>
<td>PHOKHARA (KLOE, BaBar BESIII)</td>
<td>$e^+e^- \rightarrow \pi^\pm\pi^-\gamma, \mu^+\mu^-\gamma, 4\pi\gamma, \ldots$</td>
<td>0.5%</td>
<td>ISR and FSR(sQED+Form Factor) at NLO</td>
</tr>
</tbody>
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Radiative Corrections: MC Generators

We need high-precision MC generators for radiative corrections at the experiment level:

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Radiative return:
- Precise knowledge of ISR process through radiator function is paramount.

**Radiative corrections and MC generators for $e^+e^-\rightarrow$ hadrons, leptons should aim at 0.1% uncertainty. NNLO calculation needed!**

In desperate need of people-power!

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<td>0.5%</td>
<td>ISR and FSR(sQED+Form Factor) at NLO</td>
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</table>
Data tensions, e.g. KLOE vs BaBar

Difference between KLOE vs BaBar is still evident, but not at the level of the $g$-2 discrepancy!

Compared to $a_{\mu}^{\pi^+\pi^-} = 503.5 \pm 1.9 \rightarrow a_{\mu}^{\pi^+\pi^-}$ (BaBar data only) = $513.2 \pm 3.8$

Simple weighted average of all data $\rightarrow a_{\mu}^{\pi^+\pi^-}$ (weighted average) = $509.2 \pm 2.9$

(i.e. – no correlations in determination of mean value)

BaBar data dominate when no correlations are accounted for in the mean value.

- Data tensions also present in other channels.
- Accounted for with error inflation and additional uncertainties.

- Highlights the importance of incorporating available correlated uncertainties in fit.
Dispersive HVP: the real challenge

- Target: $\sim 0.2\%$ total error.
- Current dispersive uncertainty: $\sim 0.5\%$.
- Below $\sim 2$ GeV:
  - Radiative corrections.
  - Combine data for $> 50$ exclusive channels.
  - Use isospin / ChPT relations for missing channels (tiny, $< 0.05\%$).
  - Sum all channels for total cross section.
- Above $\sim 2$ GeV:
  - Combine inclusive data OR pQCD (away from flavour thresholds).
  - Add narrow resonances.
- Challenges:
  - How to combine data/errors/correlations from different experiments and measurements.
  - Accounting for tensions & sources of systematic error.

$$R(s) = \frac{\sigma_{\text{had}}^0(s)}{\frac{4\pi\alpha^2}{3s}}$$

Non-perturbative: experimental data (plus small isospin & ChPT estimations)

Non-perturbative & perturbative: experimental data OR pQCD (and Breit-Wigner for narrow resonances)

Perturbative: pQCD

## Analysis approaches: DHMZ & KNT

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Blinding</td>
<td>Included for upcoming update</td>
<td>None</td>
</tr>
<tr>
<td>VP Correction</td>
<td>Self-consistent VP routine + conservative uncertainty.</td>
<td>Self-consistent VP routine + some uncertainty (?).</td>
</tr>
<tr>
<td>FSR corrections</td>
<td>Scalar QED for two body + conservative uncertainty.</td>
<td>Scalar QED for two body + some uncertainty (?).</td>
</tr>
<tr>
<td>Re-binning</td>
<td>Re-bin data into “clusters”. Scans over cluster configurations for optimisation.</td>
<td>Quadratic splines of all data sets quadratically interpolated on fixed binning.</td>
</tr>
<tr>
<td>Additional constraints</td>
<td>None.</td>
<td>Analyticity constraints for $2\pi$ channel.</td>
</tr>
<tr>
<td>Fitting</td>
<td>$\chi^2$ minimisation with correlated uncertainties incorporated globally.</td>
<td>$\chi^2$ minimisation with correlated uncertainties incorporated <strong>locally</strong>.</td>
</tr>
<tr>
<td>Error inflation</td>
<td>Local $\chi^2$ error inflation.</td>
<td>Local $\chi^2$ error inflation.</td>
</tr>
<tr>
<td>Integration</td>
<td>Trapezoidal for continuum, quintic for resonances.</td>
<td>Quadratic interpolation.</td>
</tr>
</tbody>
</table>

\[
a_{\mu}^{\pi^+\pi^-}(\sqrt{s} < 2 \text{ GeV}) = 503.74 \pm 1.96
\]

\[
a_{\mu}^{\pi^+\pi^-}(\sqrt{s} < 2 \text{ GeV}) = 507.14 \pm 2.58
\]
Analyticity constraints
- Constraints to hadronic cross section applied from analyticity, unitarity, and crossing symmetry.
- These allow derivations of global fit functions based on fundamental properties of QCD.
- Can lead to reduction in uncertainties.
- Successfully applied for $2\pi, 3\pi, \pi^0\gamma$ channels.

Fred Jegerlehner’s combination
- Data-sets from the same experiment are combined in local regions of $\sqrt{s}$ using a global $\chi^2$ minimisation.
- Overlapping regions of combined data are then averaged.
- Resonances are parameterised using models (e.g. G-S, BW), with masses are fixed to PDG values.
- $\tau$ data are/aren’t included. Isospin corrections are made for e.g. $\rho - \gamma$ mixing.

Broken Hidden Local Symmetry (Benyanoun, Jegerlehner)
- Effective Lagrangian based on vector meson dominance and resonance ChPT.
- BHLS model parameters are extracted from experimental data.
- Can lead to drastically reduced uncertainties, but some data must be discarded.
Comparisons and the 2021 WP result

\[
a_{\mu}^{\text{had}, \ LOVP} = 693.84 \pm 1.19_{\text{stat}} \pm 1.96_{\text{sys}} \pm 0.22_{v\beta} \pm 0.71_{f\text{sr}} \]
\[
= 692.78 \pm 2.42_{\text{tot}}
\]

Precision better than 0.4% (uncertainties include all available correlations and \(\chi^2\) inflation)

Clear \(\pi^+\pi^-\) dominance

### Detailed comparisons by-channel and energy range between direct integration results:

<table>
<thead>
<tr>
<th>Channel</th>
<th>DHMZ19</th>
<th>KNT19</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\pi^+\pi^-)</td>
<td>507.85</td>
<td>504.23</td>
<td>3.62</td>
</tr>
<tr>
<td>(\pi^+\pi^-\pi^0)</td>
<td>46.21</td>
<td>46.63</td>
<td>-0.42</td>
</tr>
<tr>
<td>(\pi^+\pi^-\pi^+\pi^-)</td>
<td>13.68</td>
<td>13.99</td>
<td>-0.31</td>
</tr>
<tr>
<td>(\pi^1\pi^0\pi^0)</td>
<td>18.03</td>
<td>18.15</td>
<td>-0.12</td>
</tr>
<tr>
<td>(K^+K^-)</td>
<td>23.08</td>
<td>23.00</td>
<td>0.08</td>
</tr>
<tr>
<td>(K_S K_L)</td>
<td>12.82</td>
<td>13.04</td>
<td>-0.22</td>
</tr>
<tr>
<td>(\pi^0\gamma)</td>
<td>4.41</td>
<td>4.58</td>
<td>-0.17</td>
</tr>
<tr>
<td>Sum of the above</td>
<td>626.08</td>
<td>623.62</td>
<td>2.46</td>
</tr>
<tr>
<td>([1.8, 3.7] \text{ GeV (without c\bar{c})})</td>
<td>33.45(71)</td>
<td>34.45(56)</td>
<td>-1.00</td>
</tr>
<tr>
<td>(J/\psi, \psi(2S))</td>
<td>7.76(12)</td>
<td>7.84(19)</td>
<td>-0.08</td>
</tr>
<tr>
<td>([3.7, \infty] \text{ GeV})</td>
<td>17.15(31)</td>
<td>16.95(19)</td>
<td>0.20</td>
</tr>
<tr>
<td>Total (a_{\mu}^{HVP,LO})</td>
<td>694.0</td>
<td>692.8</td>
<td>2.4</td>
</tr>
</tbody>
</table>

+ evaluations using unitarity & analyticity constraints for \(\pi\pi\) and \(\pi\pi\pi\) channels

[CHS 2018, HHKS 2019]

Conservative merging to obtain a realistic assessment of the underlying uncertainties:

- Account for differences in results from the same experimental inputs.
- Include correlations between systematic errors

\[a_{\mu}^{HVP,LO} = 693.1 (4.0) \times 10^{-10}\]
New two-pion data from CMD-3

- New CMD-3 $2\pi$ measurement disagrees with all previous measurements at $2.5 \rightarrow 5\sigma$.
- This includes the CMD-2 measurements by the same group, using similar methods (cause unknown).
- The Muon g-2 Theory Initiative organised two scientific seminars and panel discussions, involving experts in these low-energy experiments [add link to indico].
- Discussions ongoing to scrutinise and hopefully identify possible reasons for the experimental discrepancies.
- Currently, no indication that CMD-3 measurement is incorrect (nor any previous measurements).
- Previous radiative corrections and Monte Carlo generators are being scrutinised, including higher-order and structure-dependent corrections.
- CMD-3 measurement still to be published.
- A lot more to be checked. No understanding of differences between data so far.

If confirmed, CMD-3 measurement will be consistent with lattice evaluations.

To be able to compare CMD-3 with KNT19 data combination:

- Data published as pion form factor, $|F_\pi|^2$.
- Must subtract vacuum polarisation effects using Fedor Ignatov’s VP correction update.
- Must include final-state-radiation effects.
- Put data on fine, common binning.

In the full $2\pi$ data combination range, the KNT19 analysis found:

$$a_\mu^{\pi^+\pi^-} (0.305 \rightarrow 1.937 \text{ GeV}) = (503.46 \pm 1.91) \times 10^{-10}.$$ 

Replacing KNT19 2pi data in the region $0.33 \rightarrow 1.20 \text{ GeV}$ with CMD-3 data:

$$a_\mu^{\pi^+\pi^-} (0.305 \rightarrow 1.937 \text{ GeV}) = (525.17 \pm 4.18) \times 10^{-10}.$$ 

Neglecting possible correlations between e.g. CMD-3 and CMD-2, this results in a difference of:

$$\Delta a_\mu^{\pi^+\pi^-} = (21.71 \pm 4.96) \times 10^{-10} \rightarrow 4.4\sigma,$$

This removes the experiment vs. SM Muon g-2 discrepancy.
**Impact of CMD-3**


<table>
<thead>
<tr>
<th>BMW 20</th>
</tr>
</thead>
<tbody>
<tr>
<td>LM 20</td>
</tr>
<tr>
<td>$\chi$QCD 23</td>
</tr>
<tr>
<td>ABGP 22</td>
</tr>
<tr>
<td>Mainz/CLS 22</td>
</tr>
<tr>
<td>ETMC 22</td>
</tr>
<tr>
<td>Fermilab/HPQCD/MILC 23</td>
</tr>
<tr>
<td>RBC/UKQCD 23</td>
</tr>
<tr>
<td>Data-based BBGKMP 23</td>
</tr>
<tr>
<td>Data-based (CMD3) BBGKMP 23</td>
</tr>
</tbody>
</table>

**DISCLAIMER:** these are **NOT** new updates or combinations including the CMD-3 data – simply demonstrations of the impact of the CMD-3 data alone.

**IMPORTANT:**
- TI White Paper result has been substituted by CMD-3 only for 0.33 → 1.0 GeV.
- The NLO HVP has not been updated.
- It is purely for demonstration purposes → should not be taken as final!

**Until differences are understood, and intense scrutiny of new/old results is complete, no conclusions can be drawn about the validity of SM estimates. A lot of work still to be done...**
Conclusions

• Muon g-2 Experiment has finished running.
• Reached statistics goal.
• On target to beat systematics goal with major experimental improvements since Run-1.
• Combined Run-2/3 result announcement on August 10th.

• Dispersive HVP technique and analysis under control.
• Even with different approaches, analysis groups are consistent.
• Future relies on new experimental data and improvements to e.g. MC generators.
• New CMD-3 result in major tension with all previous two-pion data.
  • Differences unknown – currently being scrutinised.
  • Results in no muon g-2 discrepancy.
  • But no conclusions to be drawn until differences have been understood.
• Major efforts of Muon g-2 Theory Initiative (for this and all other future work) ongoing.
\( \sigma_{\text{had},\gamma}^0 \) must be **bare** (undressed of VP effects) and **inclusive** of FSR effects. Must correct measured data not in this format:

\[
\Rightarrow \text{Reconsider the optical theorem: } \lim_{s \to 0} \frac{\Im \Pi_{\text{had}}(q^2)}{\Im \Pi_{\text{had}}(q^2)^2} \equiv \lim_{s \to 0} \frac{\sigma_{\text{had}}(q^2)}{\sigma_{\text{had}}(q^2)^2}.
\]

**VP corrections**

\[
\Rightarrow \text{Photon VP corresponds to higher order contributions to } \sigma_{\mu,\text{VP}}^\text{had}.
\]

\[
\Rightarrow \text{Must subtract VP: } \sigma(e^+e^- \to \gamma^* \to \text{had}) - \sigma(e^+e^- \to \gamma^* \to \text{had})
\]

\[
\Rightarrow \text{Fully updated, self-consistent VP routine: } \left[ \text{vp.knt.v3.0} \right], \text{ available for distribution}
\]

\[
\Rightarrow \text{Cross sections undressed with full photon propagator (must include imaginary part), } \sigma_{\text{had}}^0(s) = \sigma_{\text{had}}(s)[1 - \Pi(s)]^2.
\]

\[
\Rightarrow \text{If correcting data, apply corresponding radiative correction uncertainty}
\]

**FSR corrections**

\[
\Rightarrow \text{Photon FSR formally higher order corrections to } \sigma_{\mu,\text{VP}}^\text{had}.
\]

\[
\Rightarrow \text{Cannot be unambiguously separated, not accounted for in HO contributions}
\]

\[
\Rightarrow \text{Must be included as part of 1PI hadronic blobs}
\]

\[
\Rightarrow \text{Experiment may cut/miss photon FSR } \Rightarrow \text{Must be added back}
\]

\[
\]

\[
\Rightarrow \text{For higher multiplicity states,}
\]

\[
\Rightarrow \text{difficult to estimate correction } \Rightarrow \text{Apply conservative uncertainty}
\]

No showstoppers here. Estimates between groups consistent and **very** conservative uncertainties applied.
What about tau data?

From the 2020 Theory Initiative WP (Phys.Rept. 887 (2020) 1-166):

"at the required precision to match the \( e^+e^- \) data, the present understanding of the IB corrections to \( \tau \) data is unfortunately not yet at a level allowing their use for the HVP dispersion integrals."

Recent claims that including \( \rho - \gamma \) mixing can account for e.g. dispersive vs. lattice, Babar vs KLOE:

Commonly forgotten: mixing of \( \rho^0, \omega, \phi \) with the photon [\( \rho^0 + \gamma \) mixing] i.e. effect concerning relation

\[
\langle A(x) A(0) \rangle \leftrightarrow \langle j(x) j(0) \rangle
\]

\( e^+e^- \) measurement \( \Leftrightarrow \) LQCD calculation

- how to disentangle QED from QCD in \( e^+e^- \)-data?
- \( \rho^0 - \gamma \) absent in CC \( \tau \to \nu, \pi \pi \) data, but QED-QCD interference part incl. in \( e^+e^- \to \pi^+\pi^- \) data,
- for getting had blob in \( e^+e^- \) the \( \gamma - \rho^0 \) mixing has to be removed!
- for the \( l=1 \) part of \( a_{\mu}^{\text{had}}[\pi\pi] \) results in

\[
\delta a_{\mu}^{\text{had}}[\rho\gamma] \simeq (5.1 \pm 0.5) \times 10^{-10},
\]

Taking into account \( \rho - \gamma \) interference resolves \( \tau \) (charged channel) vs. \( e^+e^- \) (neutral channel) puzzle, F.J & R. Szafir [JS11], M. Benayoun et al. However, not accepted by WP as a possible effect, which is analogous to \( Z - \gamma \) interference established at LEP in the 90's.

\[
\rho - \gamma \text{ interference}
\]

(absent in charged channel)

often mimicked by large shifts

in \( M_\rho \) and \( \Gamma_\rho \)

\( \rho^0 \) is mixing with \( \gamma \);

propagators are obtained by inverting the symmetric \( 2 \times 2 \)

self-energy matrix

\[
\left( \begin{array}{cc}
q^2 + \Pi_{\gamma\gamma}(q^2) & \Pi_{\rho\gamma}(q^2) \\
\Pi_{\rho\gamma}(q^2) & q^2 - M_\rho^2 + \Pi_{\rho\rho}(q^2)
\end{array} \right)
\]

Irreducible self-energy contribution at one-loop
What about tau data?

From the 2020 Theory Initiative WP (Phys.Rept. 887 (2020) 1-166):

"at the required precision to match the \(e^+e^-\) data, the present understanding of the IB corrections to \(\tau\) data is unfortunately not yet at a level allowing their use for the HVP dispersion integrals."

Recent claims that including \(\rho - \gamma\) mixing can account for e.g. dispersive vs. lattice, Babar vs KLOE:

Commonly forgotten: mixing of \(\rho^0, \omega, \phi\) with the photon \([\rho^0 - \gamma\) mixing] i.e. effect concerning relation

1. In a model-independent description of strong physics (QCD), the \(\rho\) is not a physical final state that you should account for in interaction with the photon. All production mechanisms effects are encapsulated in the final state.

2. There is a power counting issue. The \(\rho - \gamma\) mixing diagram is part of the higher order HVP.

- how to disentangle \(\rho^0, \omega, \phi\) with the photon in \(e^+e^-\) data?

- \(\rho^0 - \gamma\) absent in CC \(\tau \rightarrow \nu_\tau \pi\pi\) data, but QED-QCD interference part incl. in \(e^+e^- \rightarrow \pi^+\pi^-\) data,

- for getting had blob in \(e^+e^-\) the \(\gamma - \rho^0\) mixing has to be removed!

- for the \(l=1\) part of \(\delta a_\mu^{\text{had}}[\pi\pi]\) results in

\[
\delta a_\mu^{\text{had}}[\rho\gamma] \approx (5.1 \pm 0.5) \times 10^{-10},
\]
Connection with $\Delta \alpha_{\text{had}}$

- $\Delta \alpha_{\text{had}}$ limits precision of EW precision fits and so the effectiveness of high-precision EW measurements.
- Can draw a direct parallel with evaluation of the Muon g-2 and probe the muon g-2 discrepancy.
- Is a test of low-energy hadronic theory, e.g. Lattice QCD vs dispersive $e^+e^-$ data.

Uncertainty from $e^+e^-$ data $\sim 0.5\%$

Experimentally measured hadronic cross section:

\[
d_{\text{had}}(s) = \frac{1}{4\pi^3} \int_{m_W}^{\infty} ds \sigma_{\text{had}}(s) K(s)
\]

Muon g-2:
- hadronic vacuum polarisation contribution
- ... sum with other SM contributions...

Running QED coupling:
- $\Delta \alpha_{\text{had}}^{(5)}(q^2) = \frac{q^2}{4\pi^2} \int_{m_W}^{\infty} ds \sigma_{\text{had}}(s) \frac{q^2}{(q^2 - s)}$
- ... evaluate at $q^2 = M_H^2$ and input into global EW fit...

The muon g-2 and $\Delta \alpha$ connection


- Shift KNT hadronic cross section in fully energy-dependent (point-like and binned) analysis to account for $\Delta a_\mu$.
- Input new values of $\Delta \alpha$ into Gfitter to predict EW observables.
- Analysis greatly constrained from more precise EW observables measurements and more comprehensive hadronic cross section.
  - Can $\Delta a_\mu$ be due to hypothetical mistakes in the hadronic $\sigma(s)$?
  - An upward shift of $\sigma(s)$ also induces an increase of $\Delta \alpha_{\text{had}}^{(5)}(M_Z)$.
  - Consider:

$$a_{\mu}^{\text{HLO}} \rightarrow a = \int_{4M_Z^2}^{s_u} ds f(s) \sigma(s), \quad f(s) = \frac{K(s)}{4\pi^3}, \quad s_u < M_Z^2,$$

$$\Delta \alpha_{\text{had}}^{(5)} \rightarrow b = \int_{4M_Z^2}^{s_u} ds g(s) \sigma(s), \quad g(s) = \frac{M_Z^2}{(M_Z^2 - s)(4\alpha\pi^2)}.$$

and the increase

$$\Delta \sigma(s) = \varepsilon \sigma(s)$$

$\varepsilon > 0$, in the range:

$$\sqrt{s} \in [\sqrt{s_0} - \delta/2, \sqrt{s_0} + \delta/2]$$

Note the very different energy-dependent weighting of the integrands...


Shifting $\Delta \sigma(s)$ to fix $\Delta a_\mu$ is possible, but:

- Excluded above $\sim 1$ GeV.
- Increases to cross section needed are orders of magnitude larger than experimental uncertainties.
New updates since KNT19

- $\pi^+\pi^-\pi^0$, BESIII (2019), arXiv:1912.11208
- $\pi^+\pi^-$, SND (2020), JHEP 01 (2021) 113
- etaomega $\rightarrow \pi^0\text{gamma}$, SND (2020), Eur.Phys.J.C 80 (2020) 11, 1008
- $\pi^+\pi^-\eta$, BaBar (2021), Phys.Rev.D 104 (2021) 11, 112003
- $\pi^+\pi^-2\pi^0\eta$, BaBar (2021), Phys. Rev. D 103, 092001
- etaoomega, BaBar (2021), Phys.Rev.D 104 (2021) 11, 112004
- $\pi^+\pi^-\pi^0\eta$, BaBar (2021), Phys.Rev.D 104 (2021) 11, 112004
- omegaetapi0, BaBar (2021), Phys. Rev. D 103, 092001
- $\pi^+\pi^-4\pi^0$, BaBar (2021), Phys.Rev.D 104 (2021) 11, 112004
- $\pi^+\pi^-4\pi^0\eta$, BaBar (2021), Phys.Rev.D 103 (2021) 9, 092001
- $\pi^+\pi^-3\pi^0\eta$, BaBar (2021), Phys.Rev.D 104 (2021) 11, 112004
- $2\pi^+2\pi^-3\pi^0$, BaBar (2021), Phys. Rev. D 103, 092001
- omega3pi0, BaBar (2021), Phys.Rev.D 104 (2021) 11, 112004
- $\pi^+\pi^-\pi^+\pi^-\eta$, BaBar (2021), Phys. Rev. D 103, 092001
- Inclusive R(s), BESIII (2021), Phys.Rev.Lett. 128 (2022) 6, 062004
- nnbar, SND (2022), arXiv:2206.13047
- K0sK3pi, CMD-3 (2022), arXiv:2207.04615

Plus, analysis updates to be presented at Edinburgh TI workshop...
Results for data-driven evaluations of $\Delta \alpha_{\text{had}}^{(5)}(M_Z^2)$ and $\alpha(M_Z^2)$

KNT19: $\Delta \alpha_{\text{had}}^{(5)}(M_Z^2) = 276.09(1.12) \times 10^{-4}$

$\Rightarrow a^{-1}(M_Z^2) = \left( 1 - \Delta \alpha_{\text{top}}(M_Z^2) - \Delta \alpha_{\text{had}}^{(5)}(M_Z^2) - \Delta \alpha_{\text{top}}(M_Z^2) \right) a^{-1}$

$\Delta \alpha_{\text{had}}^{(5)}(M_Z^2) = 276.09(2) \times 10^{-4}$

$\Delta \alpha_{\text{top}}(M_Z^2) = -0.7201(37) \times 10^{-4}$

Areas/plans for improvement from KNT:
- New data
  - New cross section measurements are currently being included in preparation for a new update, e.g. BESIII:
  - More cross section measurements due to be released.
  - Updated data analysis from KNT in the next year(s), including updated VP routine.
  - Future plans include a new evaluation of VP with significant improvements and a specific VP-dedicated publication.

Motivation for improvement: Future measurements
- FCC/FCC-ee (for example) would probe new physics at the precision of non-perturbative hadronic corrections to the running coupling for the first time.
  - Order(s) of magnitude improvement expected in e.g., $\sin^2 \theta_{\text{eff}}$ and $M_W$.

**World average:** $\sin^2 \theta_{\text{eff}} = 0.23151(14)$

**EW fit prediction:** $\sin^2 \theta_{\text{eff}} = 0.23152(4)_{\text{parametric}}(4)_{\text{th}}$

Parametric error $4\times10^{-3}$ on $\sin^2 \theta_{\text{eff}}$ is dominated by $\Delta a_{\text{had}}^{(5)} (M_Z^2)$ uncertainty.

- Without an improvement in the precision of $\Delta a_{\text{had}}^{(5)}$, the precision of the EW fit prediction will become more precise than the current best determination!
- Need an improvement $\sim 3$ in $\Delta a_{\text{had}}^{(5)}$ precision to make it compatible with such measurements (e.g. $\sin^2 \theta_{\text{eff}}$ precision $\lesssim 1\times10^{-5}$).
Prospects and motivation for improvement

Motivation for improvement: tensions with lattice QCD

- Tension with data-driven results washed out at the Z pole.
- Up to 3.5σ tension with data-driven results between 1 and 7 GeV² (comparable to g-2 discrepancy...).

Other prospects for improvement:
- New low-energy data for $\sigma^{0\hbar}(s)$ (CMD-3, SND, KEDR, BESIII, Belle-2, ...).
- Direct determination of $\Delta a^{(5)}_{\text{had}}(M_2^2)$ measuring the muon asymmetry $A_{\mu}(s)$ in the vicinity of the Z-pole (see Patrick Janot’s talk in this workshop).
- Euclidean split method (Adler function). Needs spacelike offset $\Delta a^{(5)}_{\text{had}}(-M_2^2)$ with $-M_2^2 \sim 2$ GeV and pQCD (see Fred Jegerlehner’s talk in this workshop).
- Direct measurement of $\Delta a^{(5)}_{\text{had}}(q^2)$ from MUonE muon-electron scattering experiment.
- More lattice QCD evaluations...

Simon Kuberski, Mainz Lattice, SchwingerFest 2022