Testing formalism for $\gamma^{\star} \to 3\pi\,, K^{0} \to 3\pi\,, \ldots$

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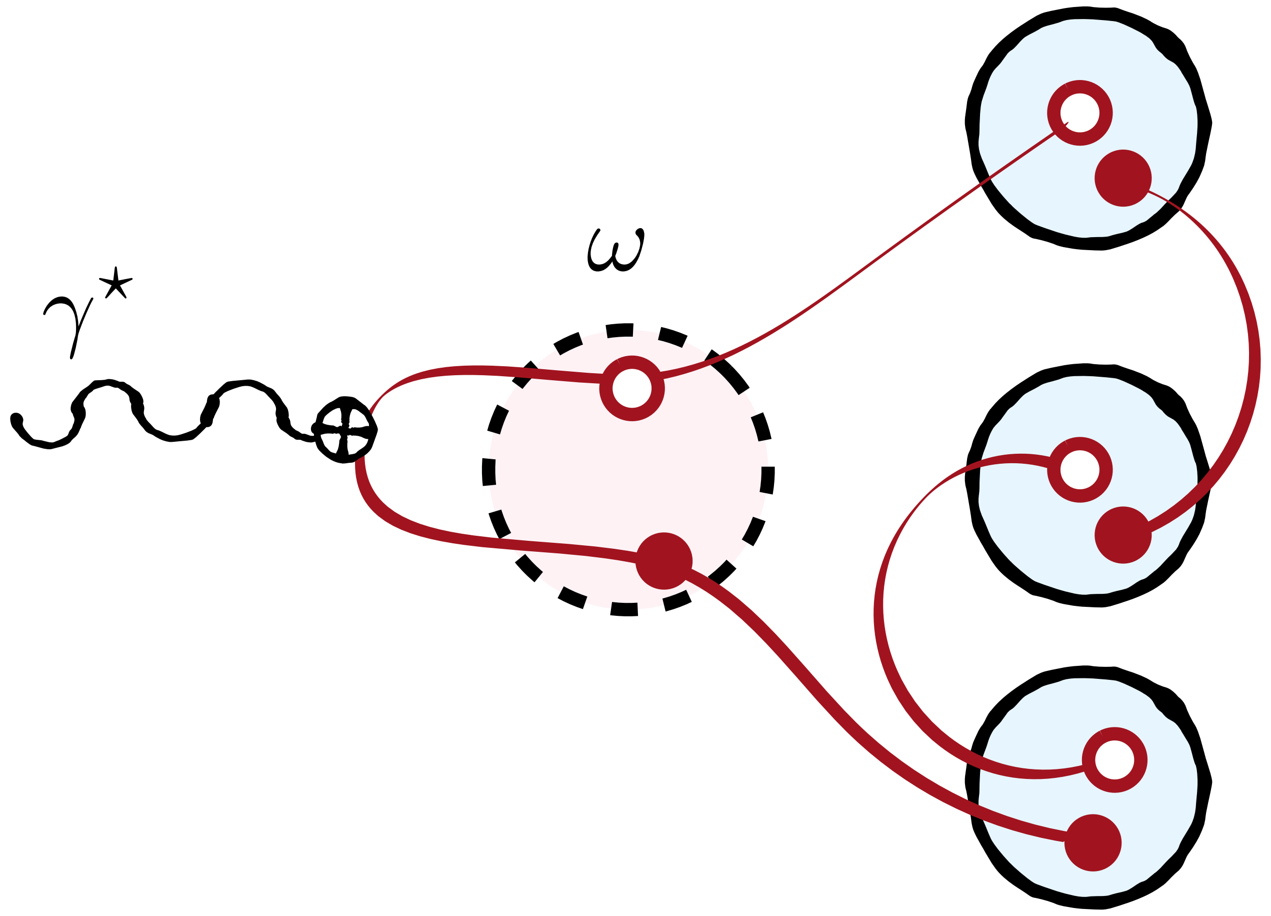
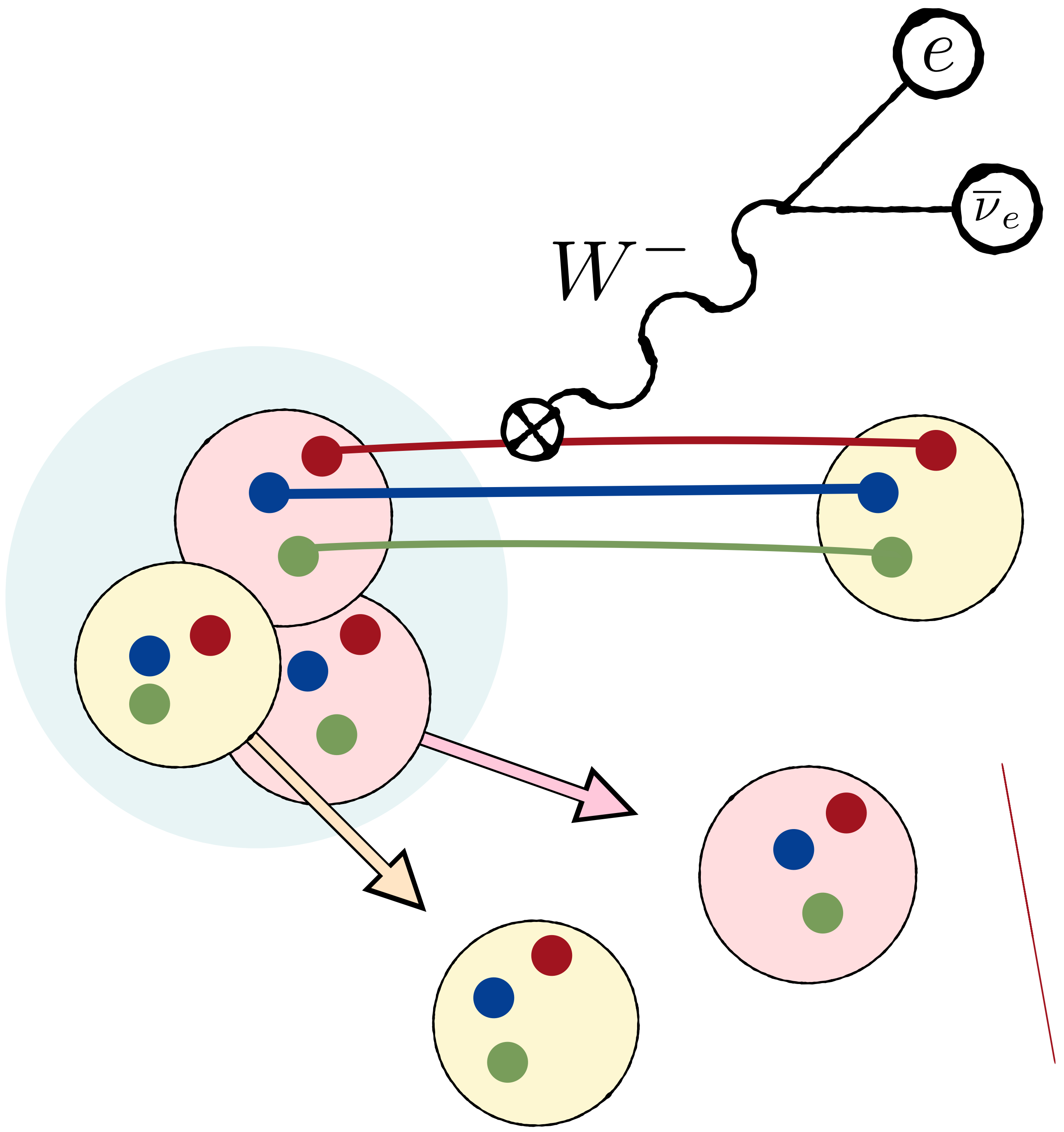
big picture

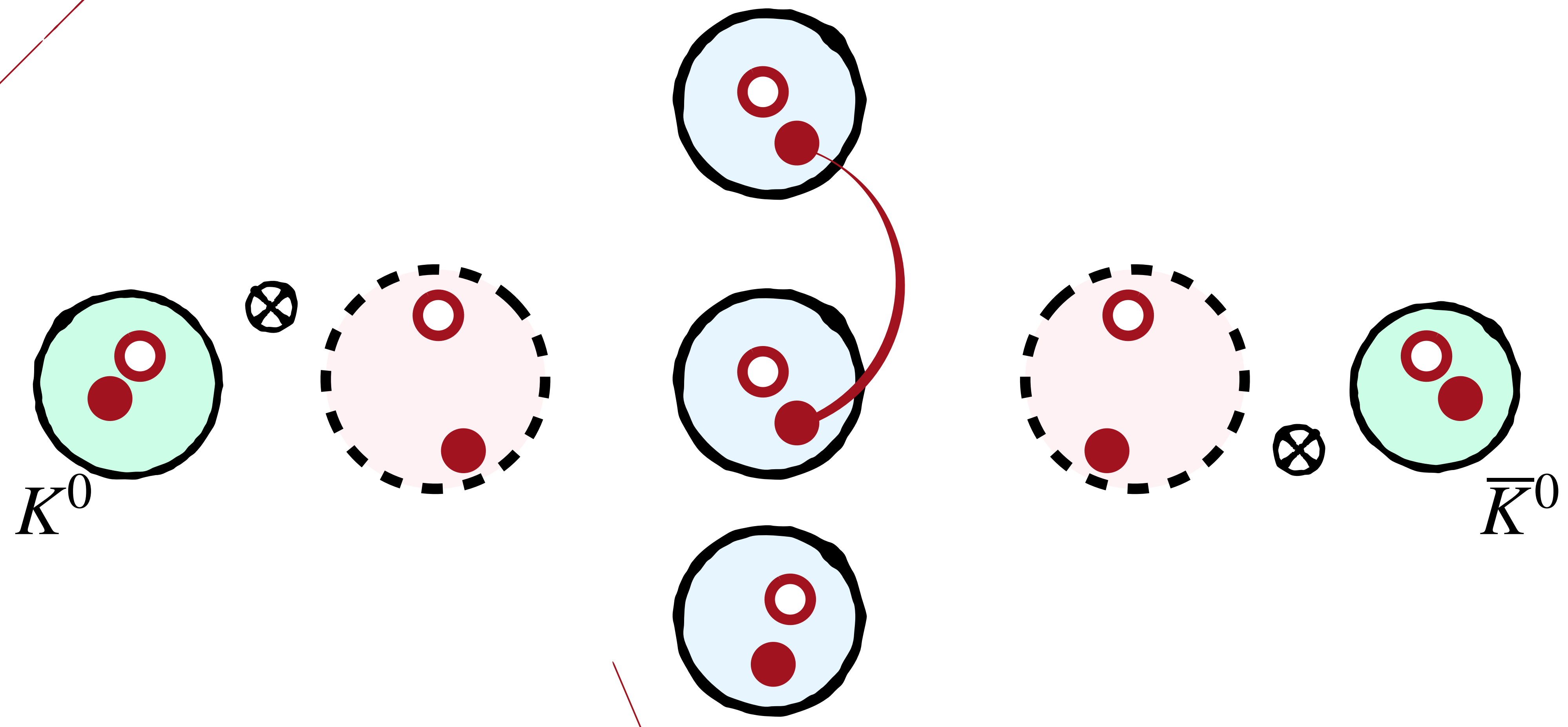
- hadron spectroscopy



- nuclear structure



- precision electroweak



workflow for three-body spectroscopy

$$\det\left[\mathcal{K}_3 + F_3^{-1}\right] = 0$$

Hansen & Sharpe ('14, '15)

☑ formalism derivation

workflow for three-body spectroscopy

Dawid, Islam, RB (2023)

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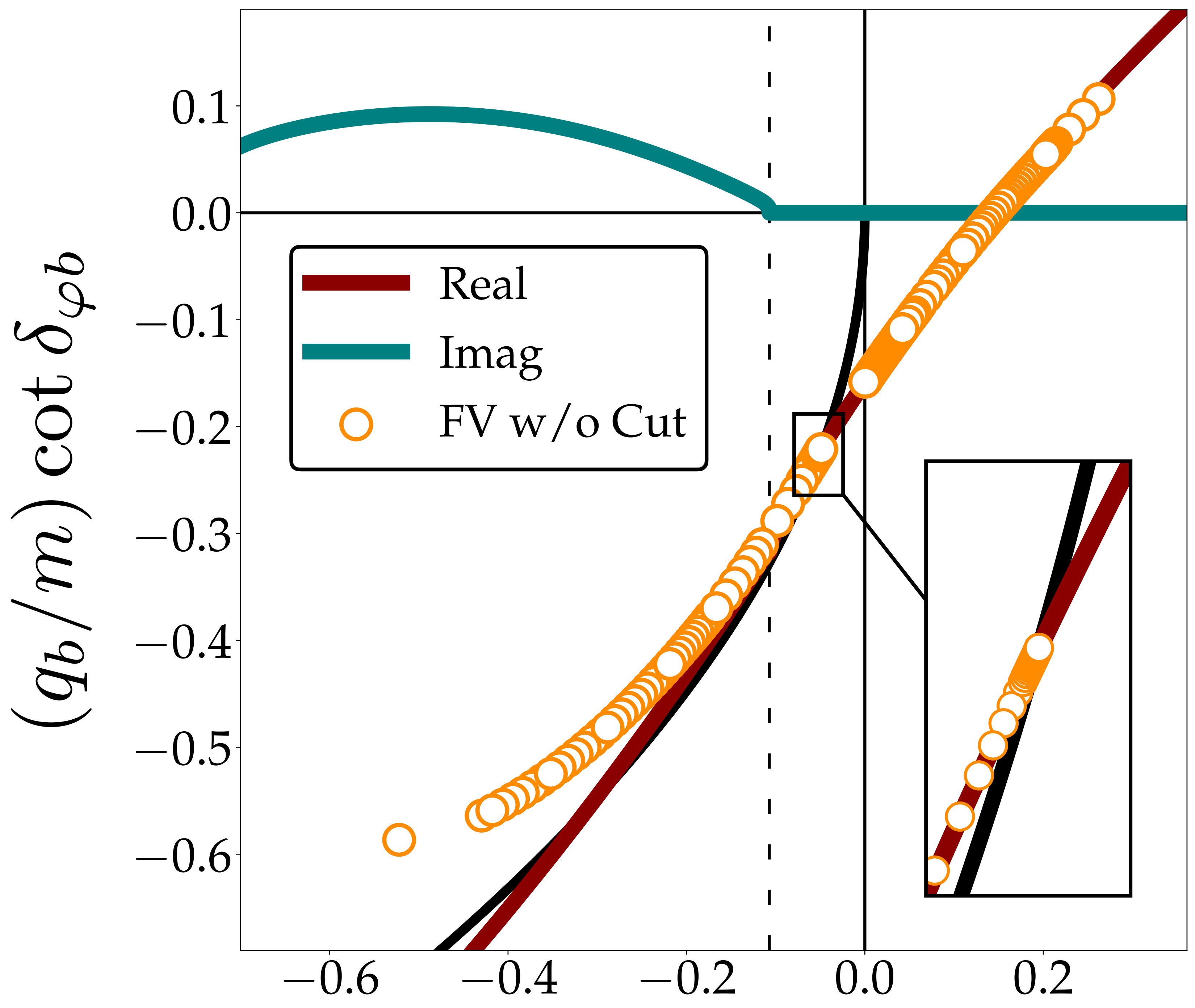
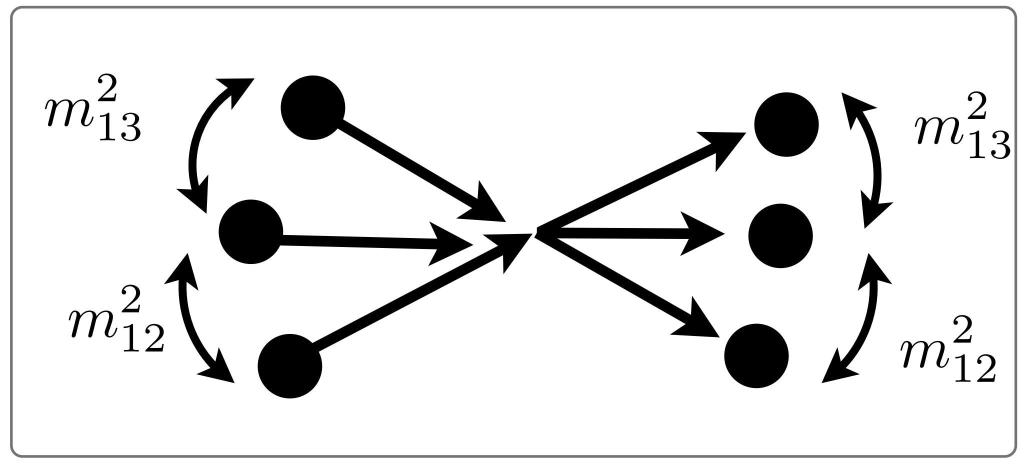


☑ checks of the formalism

workflow
for three-body spectroscopy



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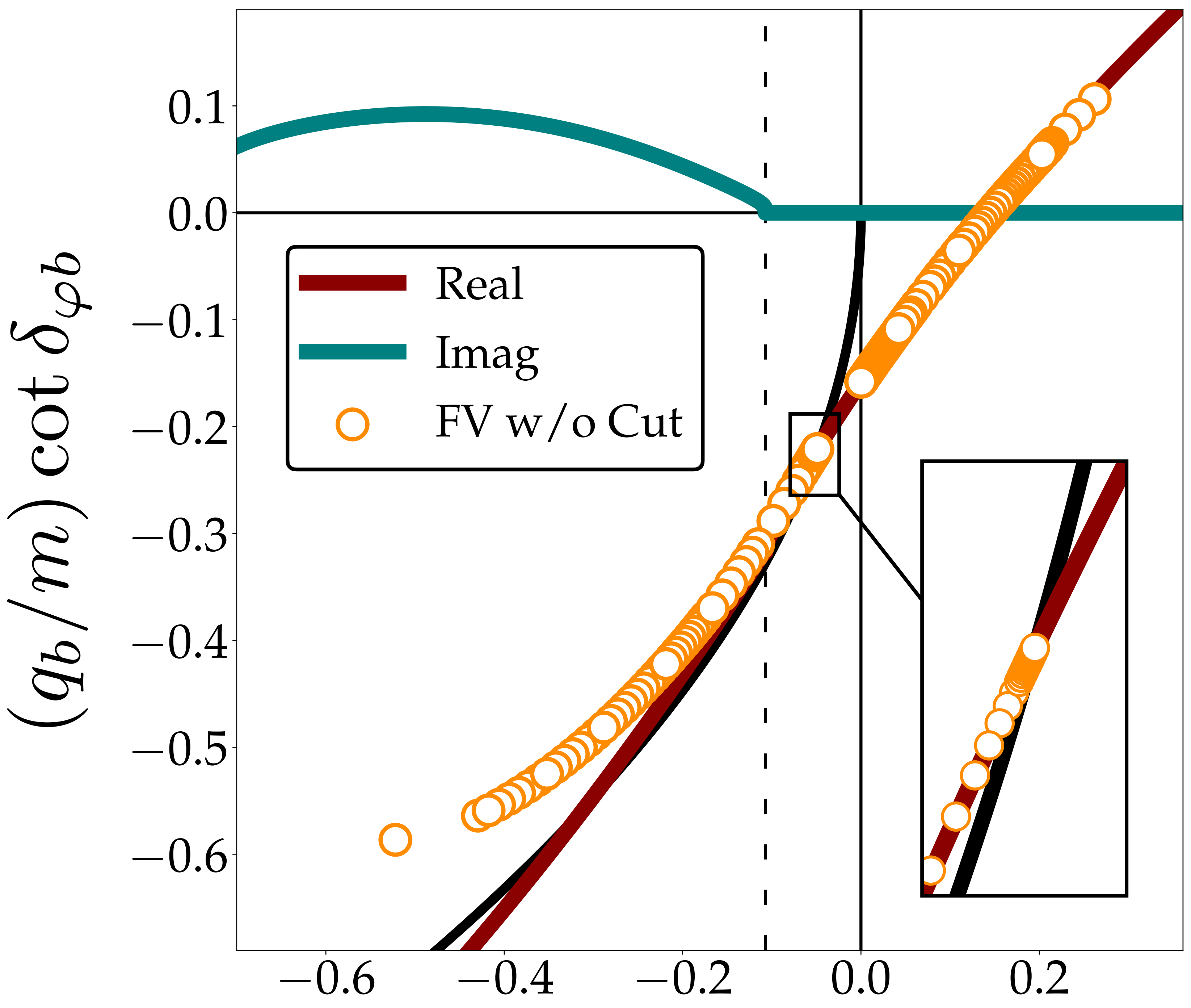
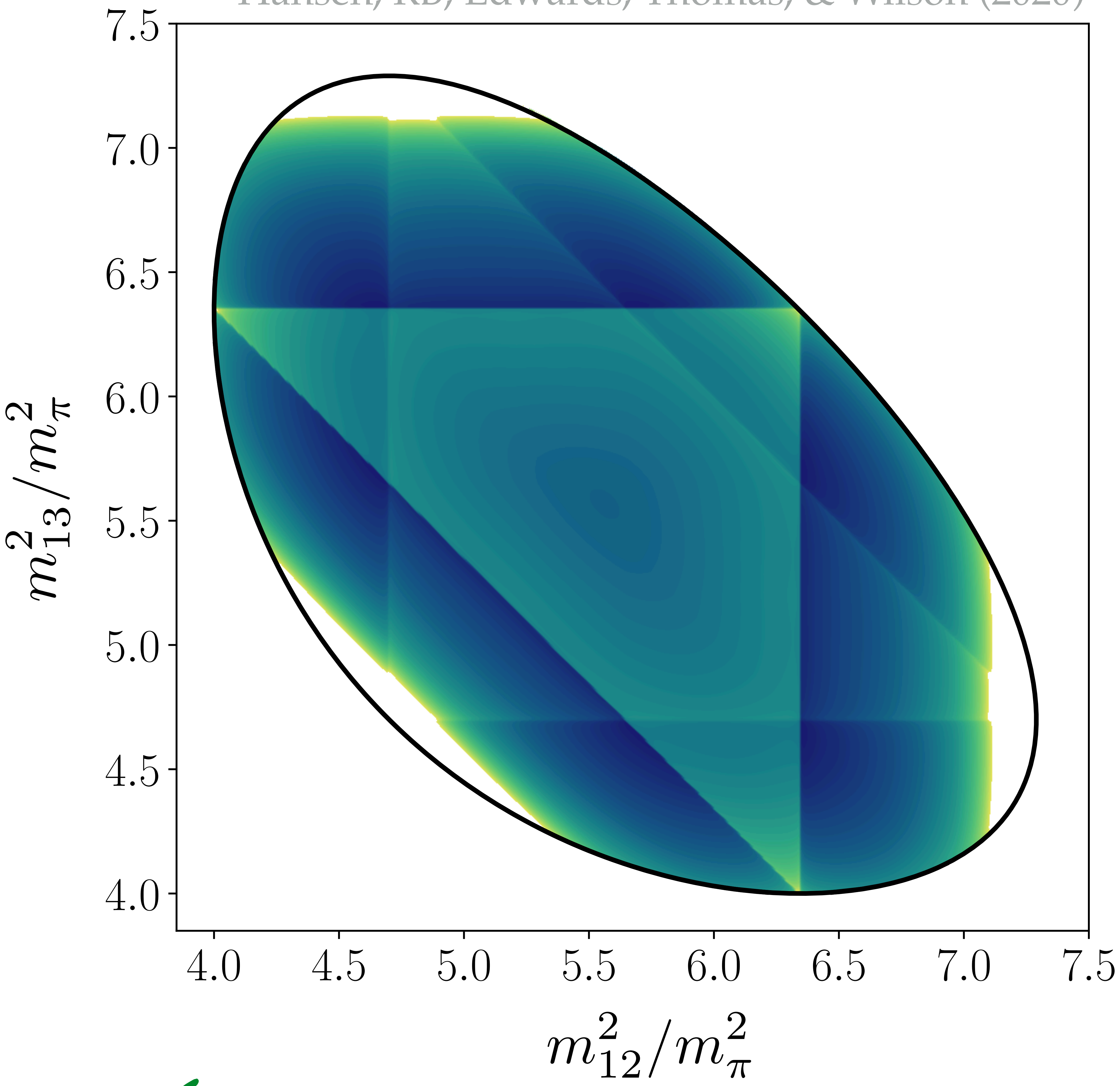


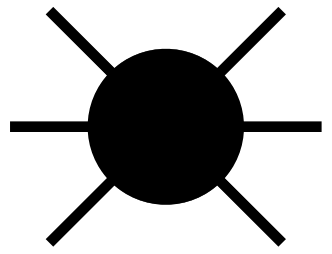
☑ checks of the formalism

Hansen, RB, Edwards, Thomas, & Wilson (2020)



☑ actual lattice QCD calculation

formalism for $\gamma^\star \to 3\pi$, …

$i\mathcal{M}_3 =$  $= i\mathcal{D} + i\mathcal{L}\dfrac{1}{\mathcal{K}_3^{-1} + F_3^{\infty}}\mathcal{L}$

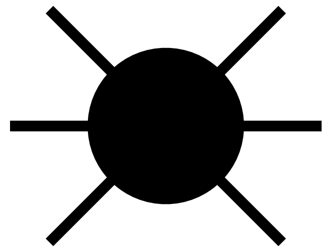
satisfies an integral equations

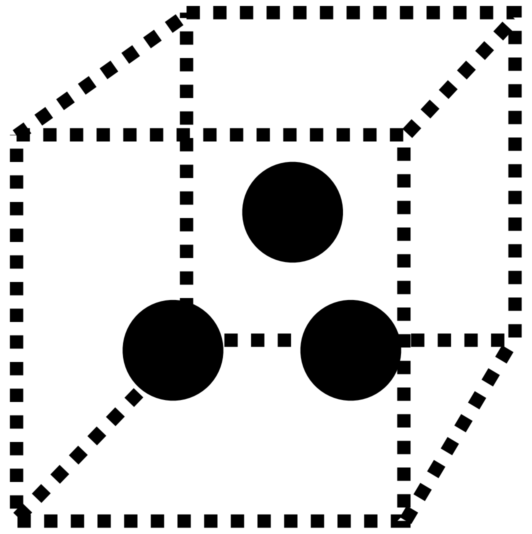
$$i\mathcal{D} = i\mathcal{M}_2 iG i\mathcal{M}_2 + \int i\mathcal{M}_2 iG\, i\mathcal{D}$$

Hansen & Sharpe ('14, '15)

Romero-López, Hansen, & Sharpe ('21)

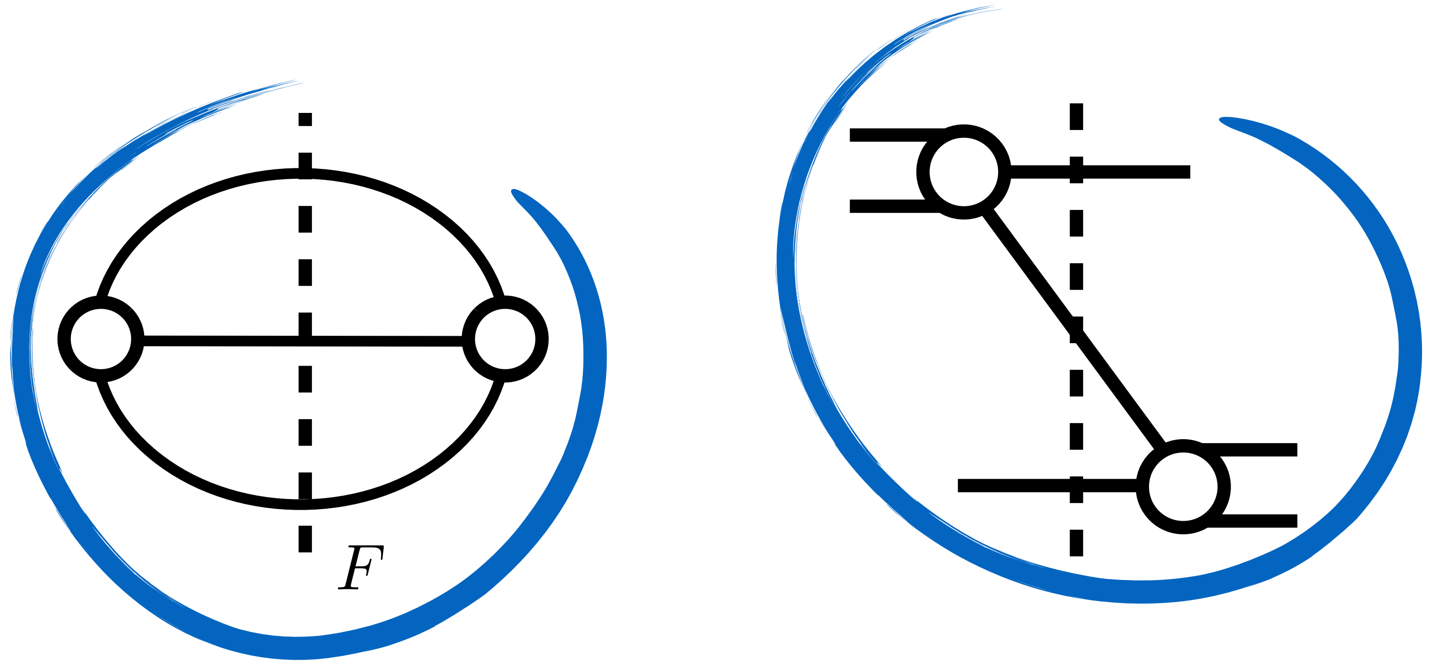
formalism for $\gamma^\star \to 3\pi$, …

$i\mathcal{M}_3 =$  $= i\mathcal{D} + i\mathcal{L}\frac{1}{\mathcal{K}_3^{-1} + F_3^{\infty}}\mathcal{L}$



$$\det\left[\mathcal{K}_3 + F_3^{-1}\right] = 0$$

$$[F_3]_{kp} = \frac{1}{L^3}\left[\frac{F}{3} - F\frac{1}{1/(2\omega\mathcal{K}_2) + F + G}F\right]_{kp}$$

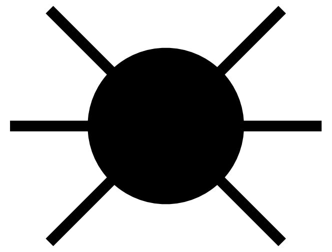


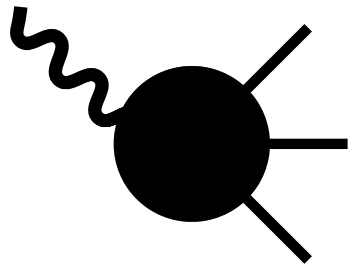
infinite volume | finite volume

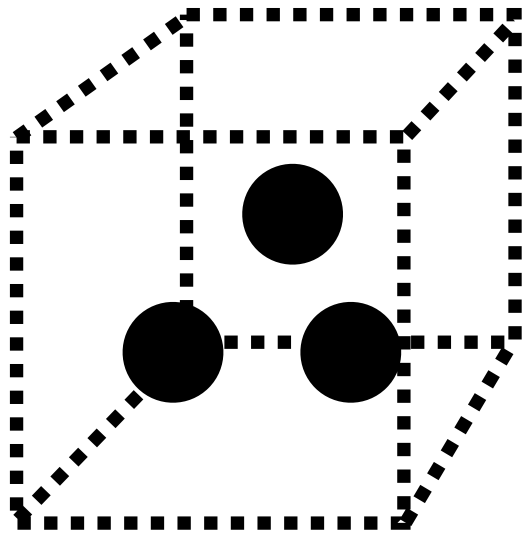
Hansen & Sharpe ('14, '15)

Romero-López, Hansen, & Sharpe ('21)

formalism for $\gamma^\star \to 3\pi, \ldots$

$$i\mathcal{M}_3 = $$  $$= i\mathcal{D} + i\mathcal{L}\frac{1}{\mathcal{K}_3^{-1} + F_3^{\infty}}\mathcal{L}$$

$$i\mathcal{T}_3 = $$  $$= iA\left(\frac{1}{3} - \mathcal{M}_2\tilde{\rho} + \int i\mathcal{D}i\tilde{\rho}\right) + \mathcal{O}(\mathcal{K}_3)$$



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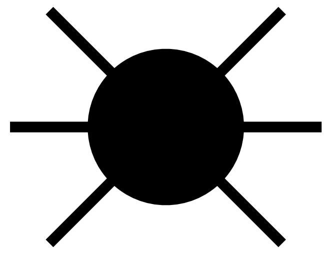
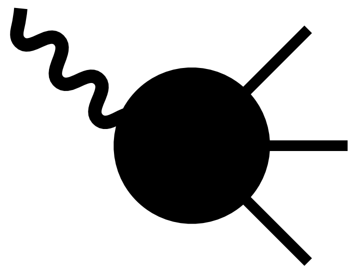
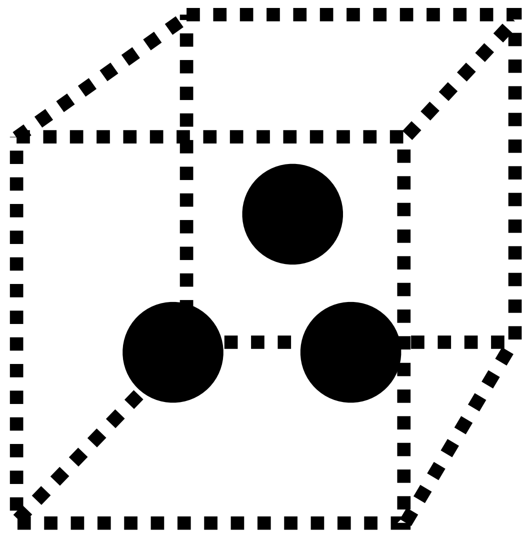
$$i\mathcal{M}_3 =$$  $$= i\mathcal{D} + i\mathcal{L}\frac{1}{\mathcal{K}_3^{-1} + F_3^{\infty}}\mathcal{L}$$

$$i\mathcal{T}_3 =$$  $$= iA\left(\frac{1}{3} - \mathcal{M}_2\tilde{\rho} + \int i\mathcal{D}i\tilde{\rho}\right) + \mathcal{O}(\mathcal{K}_3)$$



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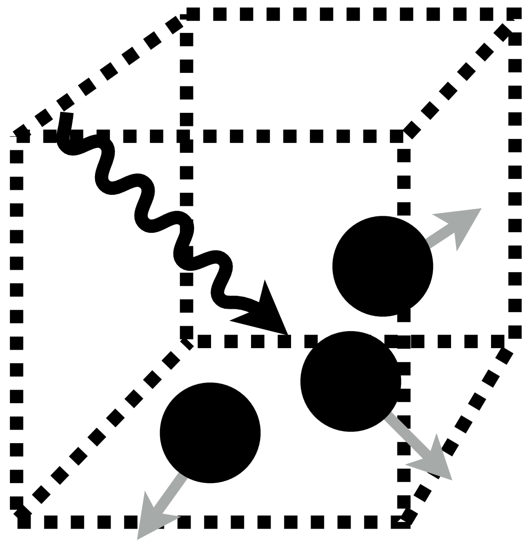


$$A^2 = L^3\langle P_f; L|J(0)|0\rangle^2 \frac{\partial[F_3^{\text{iso}}]^{-1}}{\partial E}\bigg|_{E=E_{nf}} + \mathcal{O}(\mathcal{K}_3)$$

infinite volume | finite volume

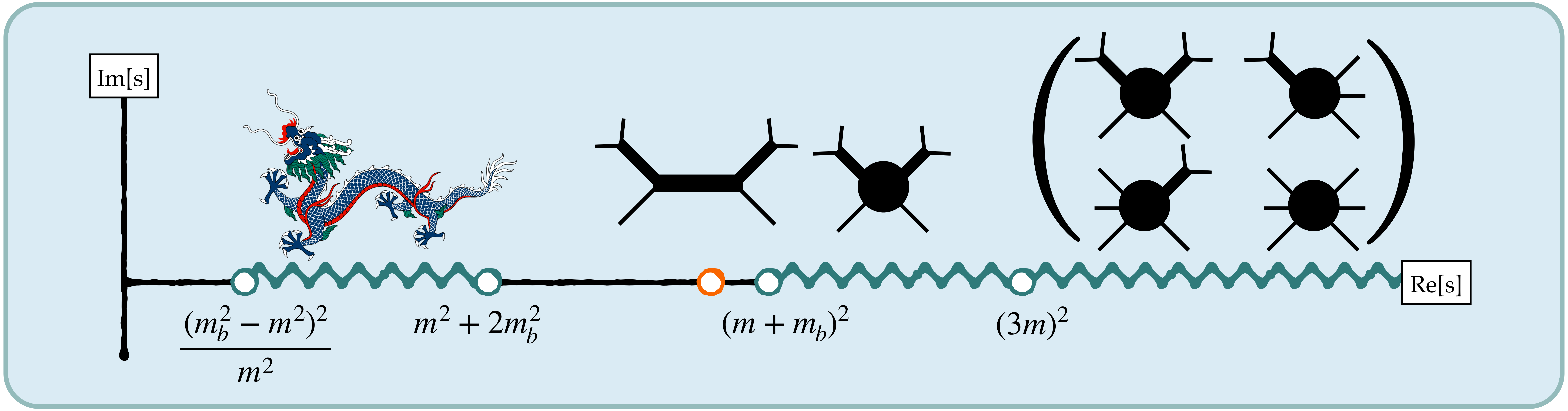
Hansen & Sharpe ('14, '15)

Romero-López, Hansen, & Sharpe ('21)

Consistency checks using toy models

Consider a toy theory with a two-body bound state

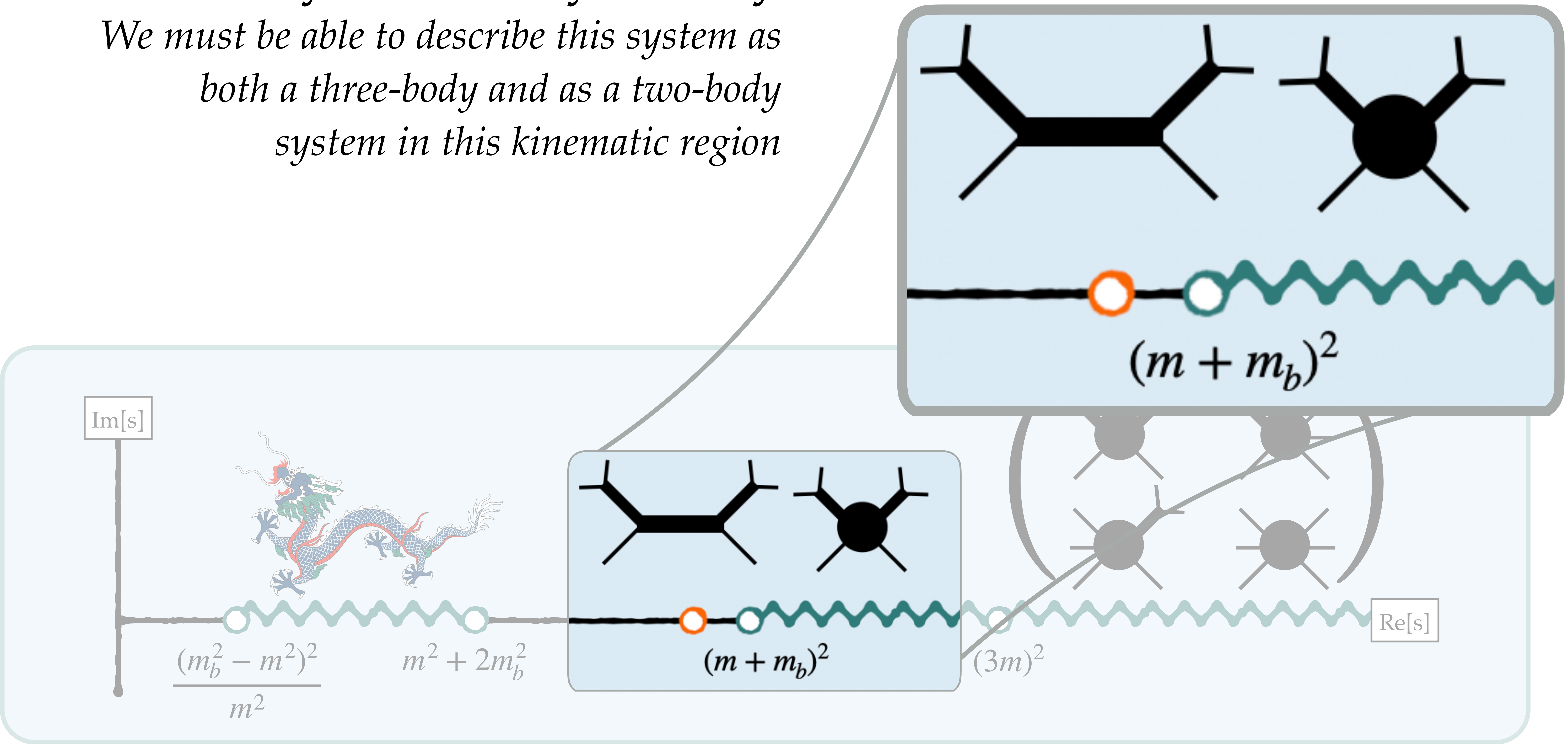
- Simple case $q \cot \delta = -\frac{1}{a}$
- if $a > 0$, there is a bound state with binding energy $m_b = 2\sqrt{m^2 - \frac{1}{a^2}}$
- $\mathcal{M}_2(s) =$ [diagram] $\sim$ [diagram] $\sim \frac{-g^2}{s - m_b^2}$
- will fix $am = 1.5$ and $\mathcal{K}_3 = 0$



Consistency checks using toy models

three-body / two-body duality

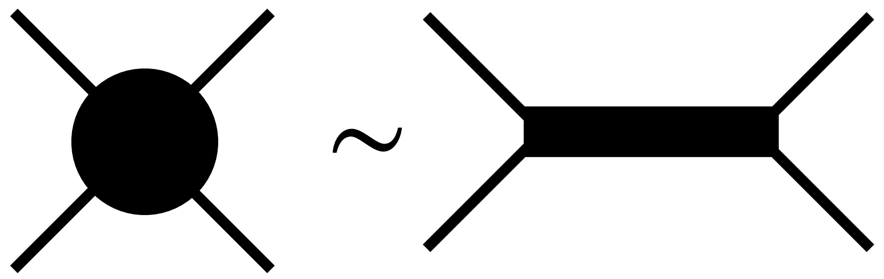
We must be able to describe this system as both a three-body and as a two-body system in this kinematic region



infinite-volume formalism for $\gamma^\star \to b\varphi$, …

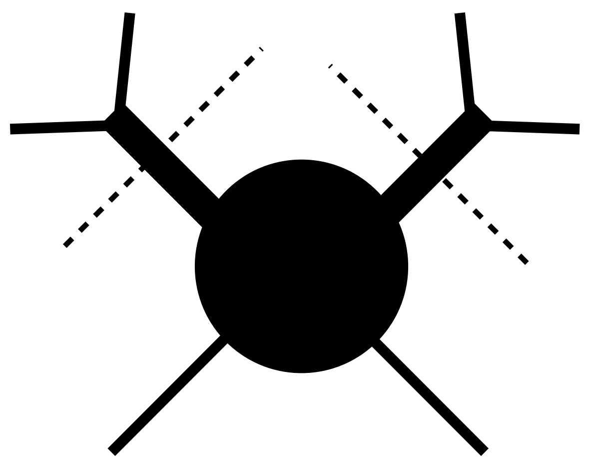
Some scattering theory [LSZ on steroids 💪🏽]

☑ Two-body bound state:

$$\mathscr{M}_2(s) =$$  $$\sim \frac{-g^2}{s - m_b^2}$$

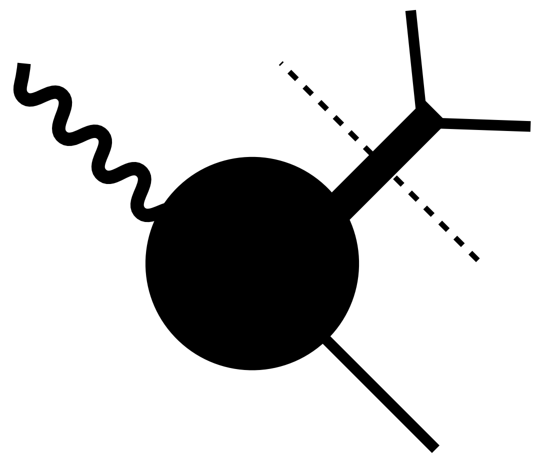
☑ Bound state / spectator scattering amplitude

$$\mathcal{M}_{\varphi b}(s) = \frac{1}{\mathcal{K}_{\varphi b}^{-1}(s) - i\rho_{\varphi b}} = \lim_{\sigma_k,\sigma_p \to m_b^2} \mathcal{D}(s,k,p)\frac{(\sigma_k - m_b^2)(\sigma_p - m_b^2)}{g^2}$$



☑ Bound state / spectator transition amplitude

$$\mathcal{T}_{\varphi b}(s) = \mathcal{A}_{\varphi b}(s)\mathcal{M}_{\varphi b}(s) = \lim_{\sigma_k \to m_b^2} \mathcal{T}_3(s,k,)\frac{(\sigma_k - m_b^2)}{-g}$$



finite-volume formalism for $\gamma^\star \to b\varphi, \ldots$

For energies below the three-particle threshold:

☑ both two and three-body QC should describe the spectrum:

["Lüscher" formalism]

$$[F_3^{\text{iso}}]^{-1} = 0 \Longleftrightarrow \left(F_{\varphi b}^{-1} + \mathcal{K}_{\varphi b}\right) = 0$$

[up to exponentially suppressed effects]

finite-volume formalism for $\gamma^\star \to b\varphi, \ldots$

For energies below the three-particle threshold:

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$$[F_3^{\text{iso}}]^{-1} = 0 \iff \left(F_{\varphi b}^{-1} + \mathcal{K}_{\varphi b}\right) = 0$$

☑ the finite-volume matrix elements should be described by the three- and two-body formalism:

$$\langle P_f; L|J(0)|0\rangle^2 = \frac{A^2}{L^3}\left[\frac{\partial [F_3^{\text{iso}}]^{-1}}{\partial E}\right]^{-1} = \frac{|\mathcal{T}_{\varphi b}|^2}{L^3}\left[\frac{\partial (F_{\varphi b}^{-1} + \mathcal{K}_{\varphi b})}{\partial E}\right]^{-1}$$

[RB, Hansen, Walker-Loud (2014)]

finite-volume formalism for $\gamma^\star \to b\varphi$, …

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$$\Longrightarrow \quad \frac{|\mathcal{T}_{\varphi b}|^2}{A^2} = \frac{\partial (F_{\varphi b}^{-1} + \mathcal{K}_{\varphi b})}{\partial E} \Big/ \frac{\partial [F_3^{\rm iso}]^{-1}}{\partial E}$$

Solving integral equations

- Introduce $\mathcal{D} = \mathcal{M}_2 d \mathcal{M}_2$
- Assuming $\mathscr{K}_{\text{df}} = 0$ and partial wave-projecting onto $\ell = 0$ sector,

$$d(p', s, p) = -G(p', s, p) - \int_0^{q_{\max}} \frac{dq\, q^2}{(2\pi)^2 \omega_q} G(p', s, q) \mathcal{M}_2(q, s)\, d(q, s, p)$$

- Deform contour to miss singularities and discretize momenta
- Write as a matrix or vector over discrete momenta:

$$\boldsymbol{d} = -\boldsymbol{G} - \boldsymbol{K} \cdot \boldsymbol{d}$$

Dawid

Washington

Islam

ODU

Jackura, RB, Dawid, Islam, & McCarty (2020)

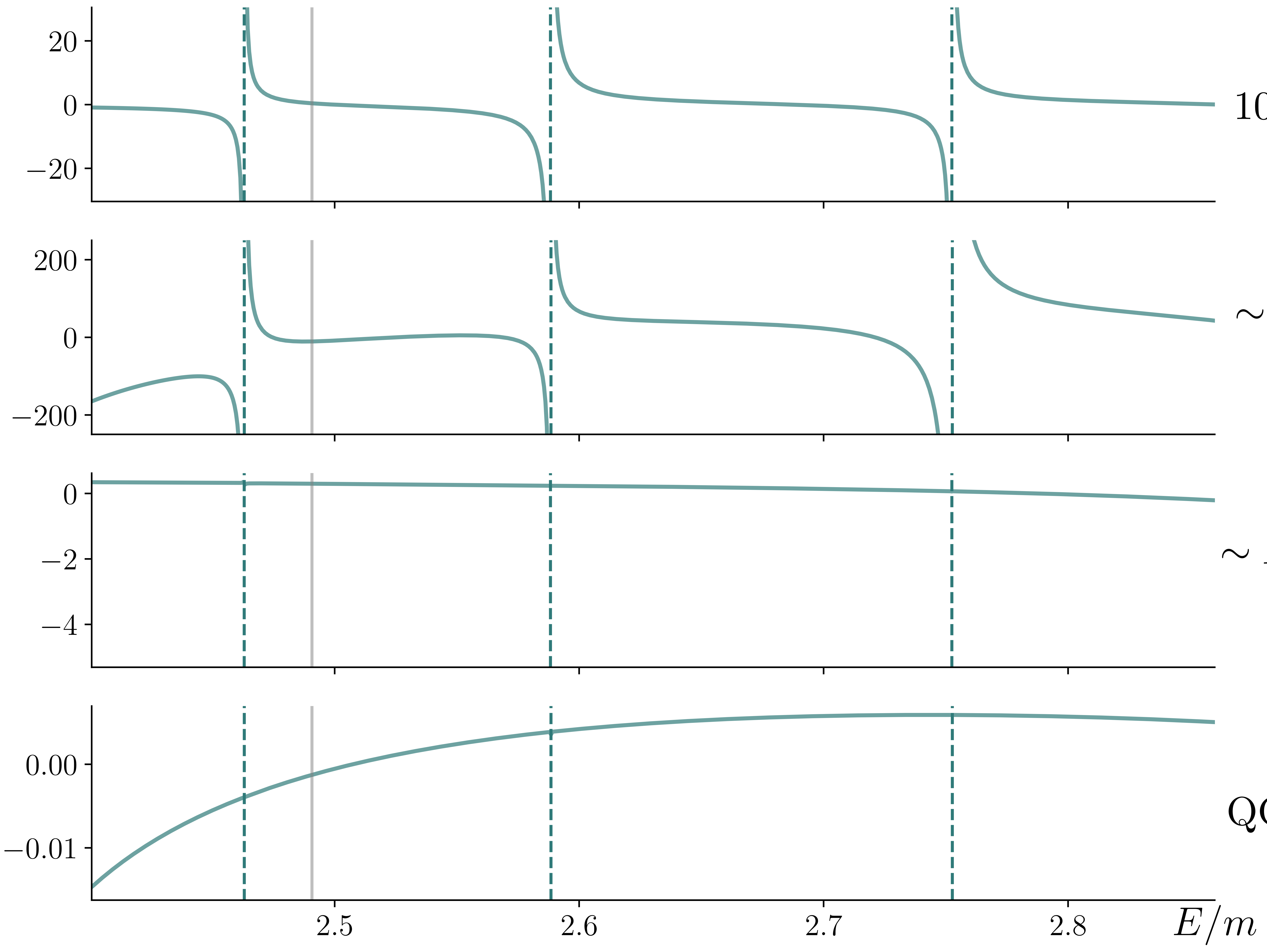
Dawid, Islam, & RB (2023)

check out their talks!

pole hunting preliminary

$am = 1.5$
$mL = 14$

a = scattering length



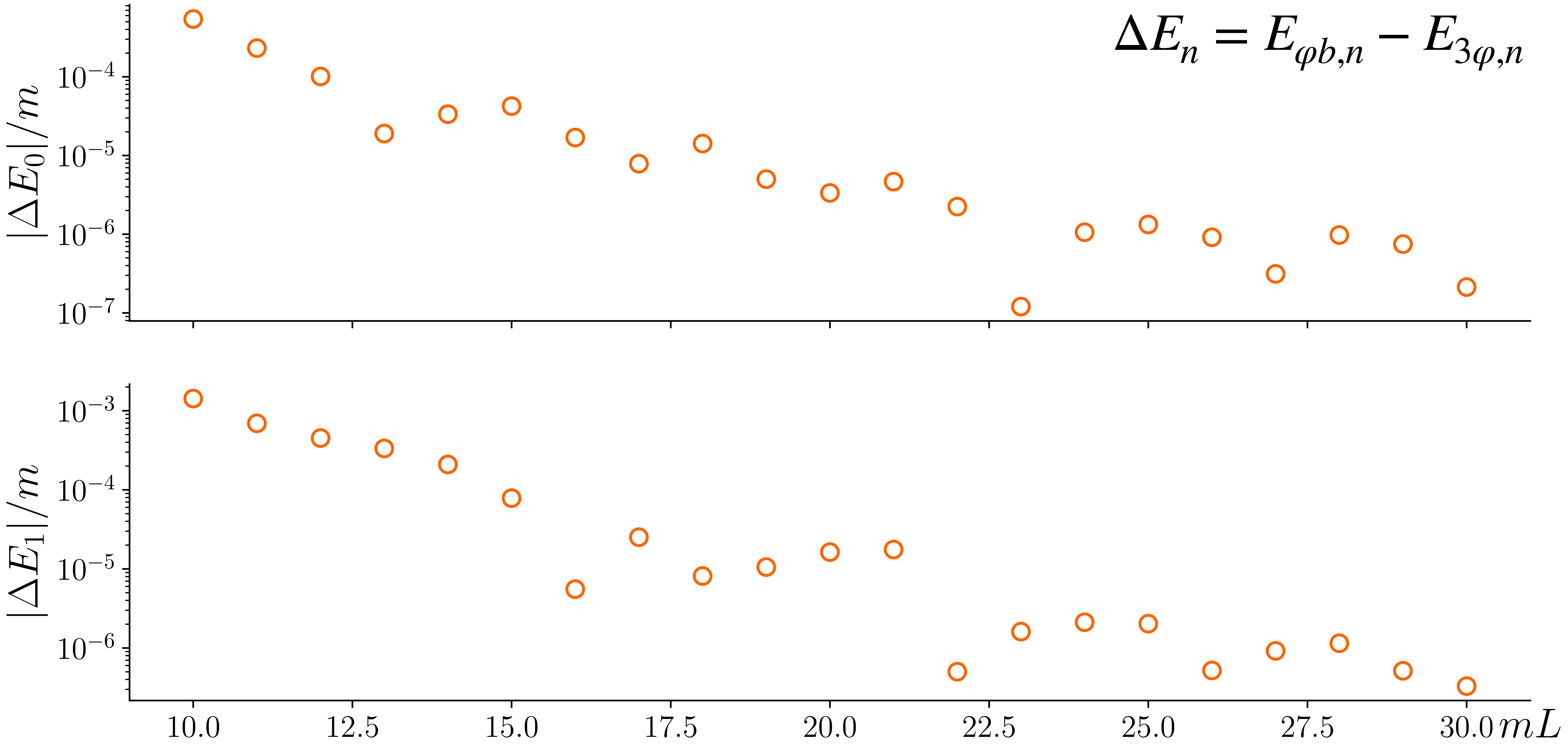
$$10^4 \times F_3^{\text{iso}} = 10^4 \times \sum_{k,p} [F_3]_{kp}$$

$$\sim QC_{\phi b}^{-1} = \left(F_{\varphi b}^{-1} + \mathcal{K}_{\varphi b}\right)^{-1}$$

$$\sim F_3^{\text{iso}} - \sum_i \frac{\mathcal{R}_{3\varphi,i}}{E - E_{3\varphi,i}}$$

$$\text{QC}_{\phi\text{b}}^{-1} - \sum_i \frac{\mathcal{R}_{i,\phi b}}{E - E_{\varphi b,i}}$$

exponential effects preliminary



finite-volume spectrum preliminary

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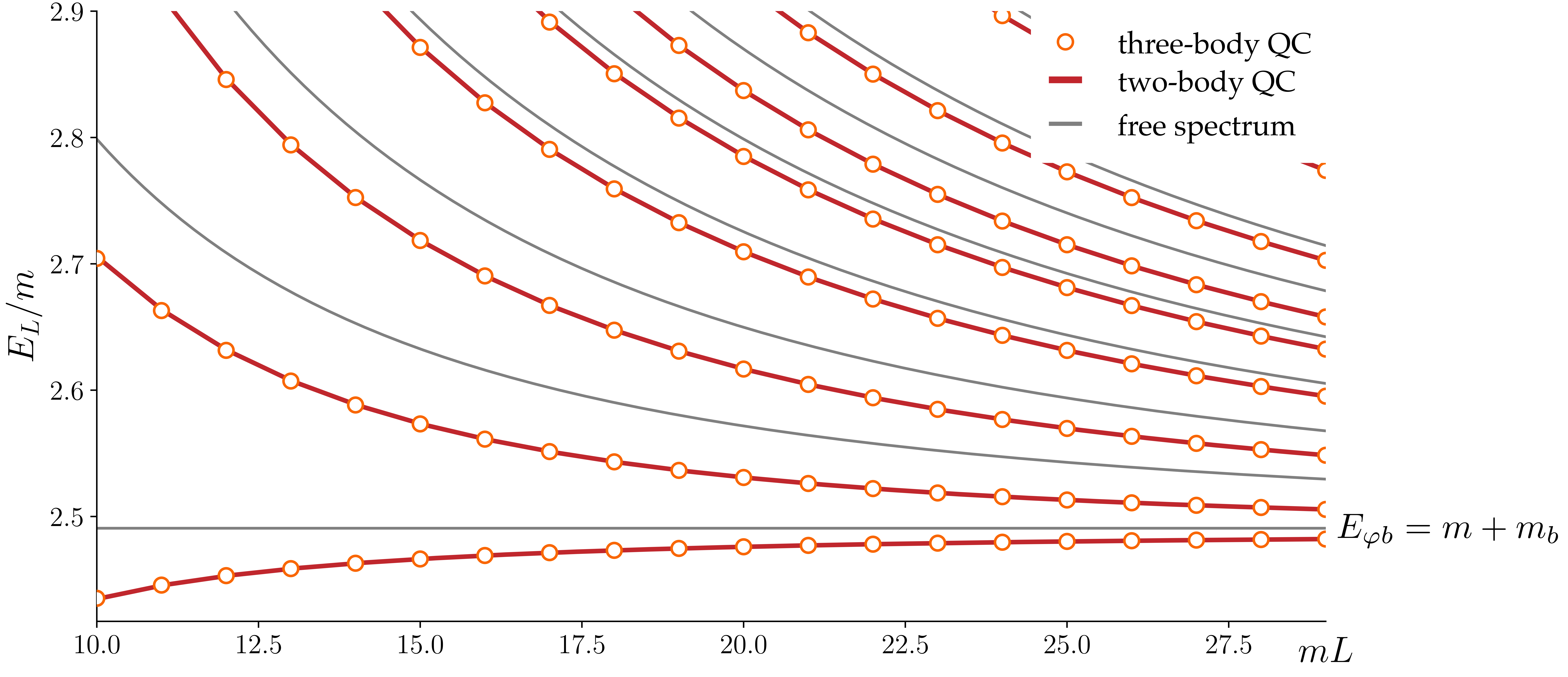

hadronic amplitude preliminary

$am = 1.5$

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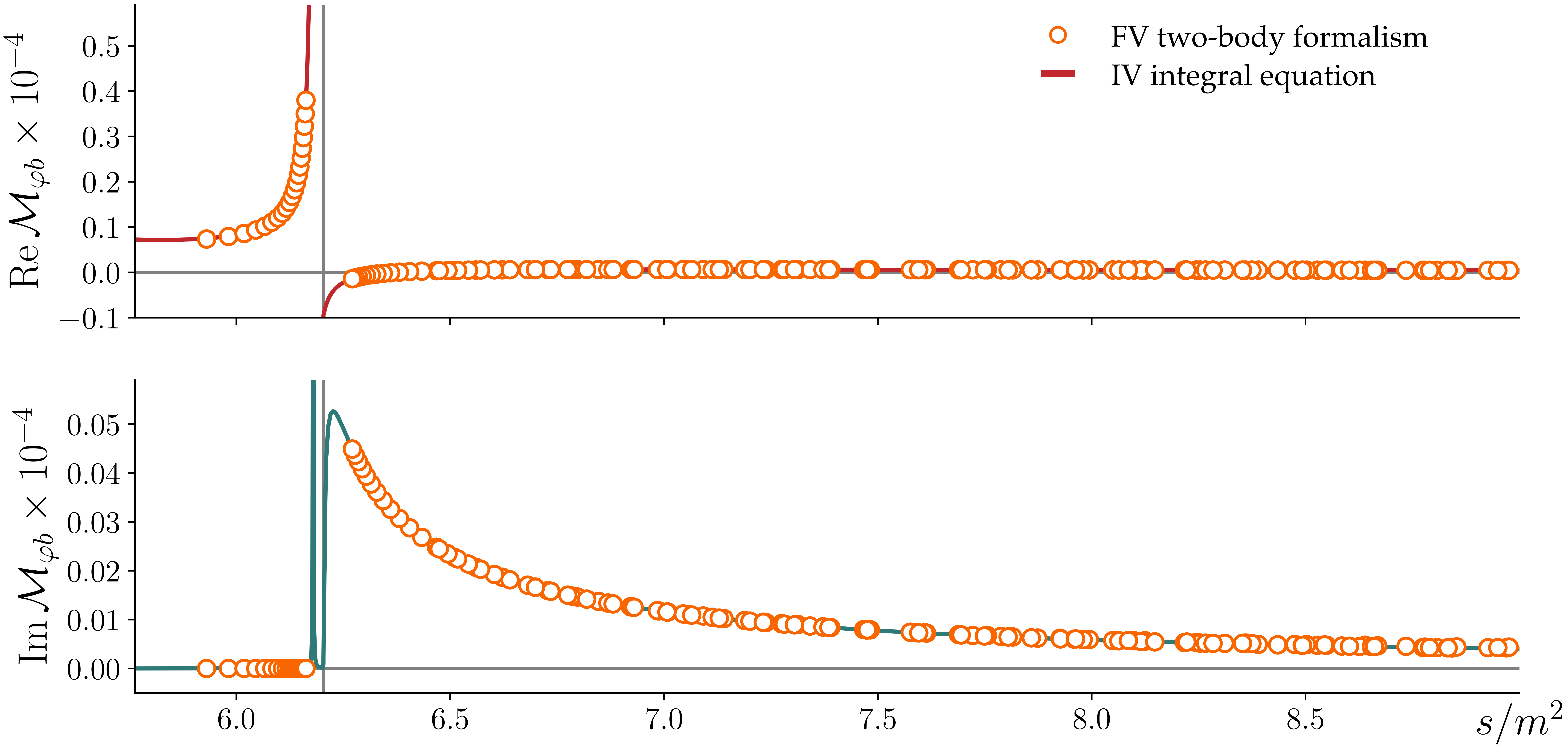
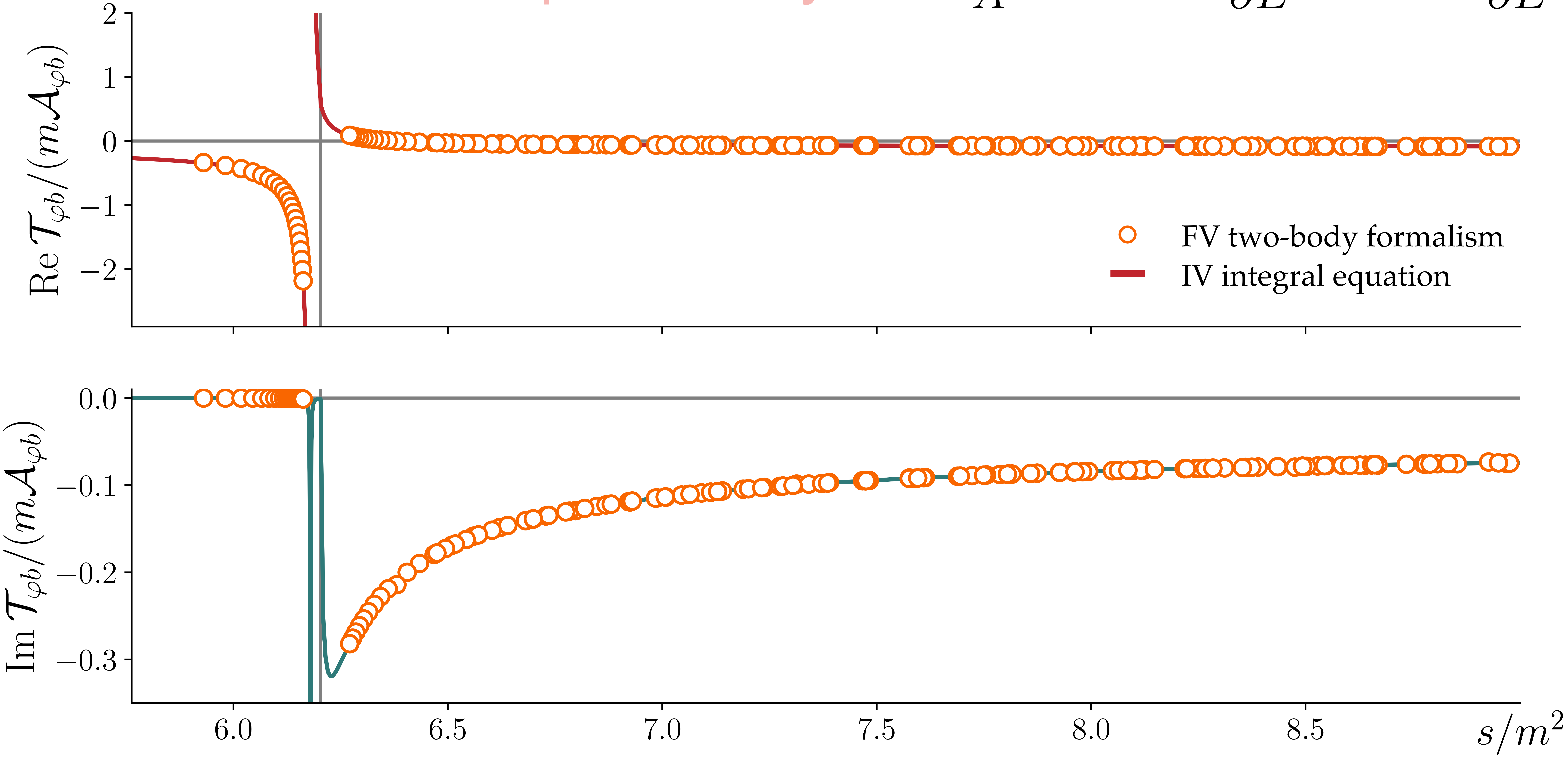


transition amplitude preliminary

$$\frac{|\mathcal{T}_{\varphi b}|^2}{A^2} = \frac{\partial(F_{\varphi b}^{-1} + \mathcal{K}_{\varphi b})}{\partial E} \Big/ \frac{\partial [F_3^{\text{iso}}]^{-1}}{\partial E}$$

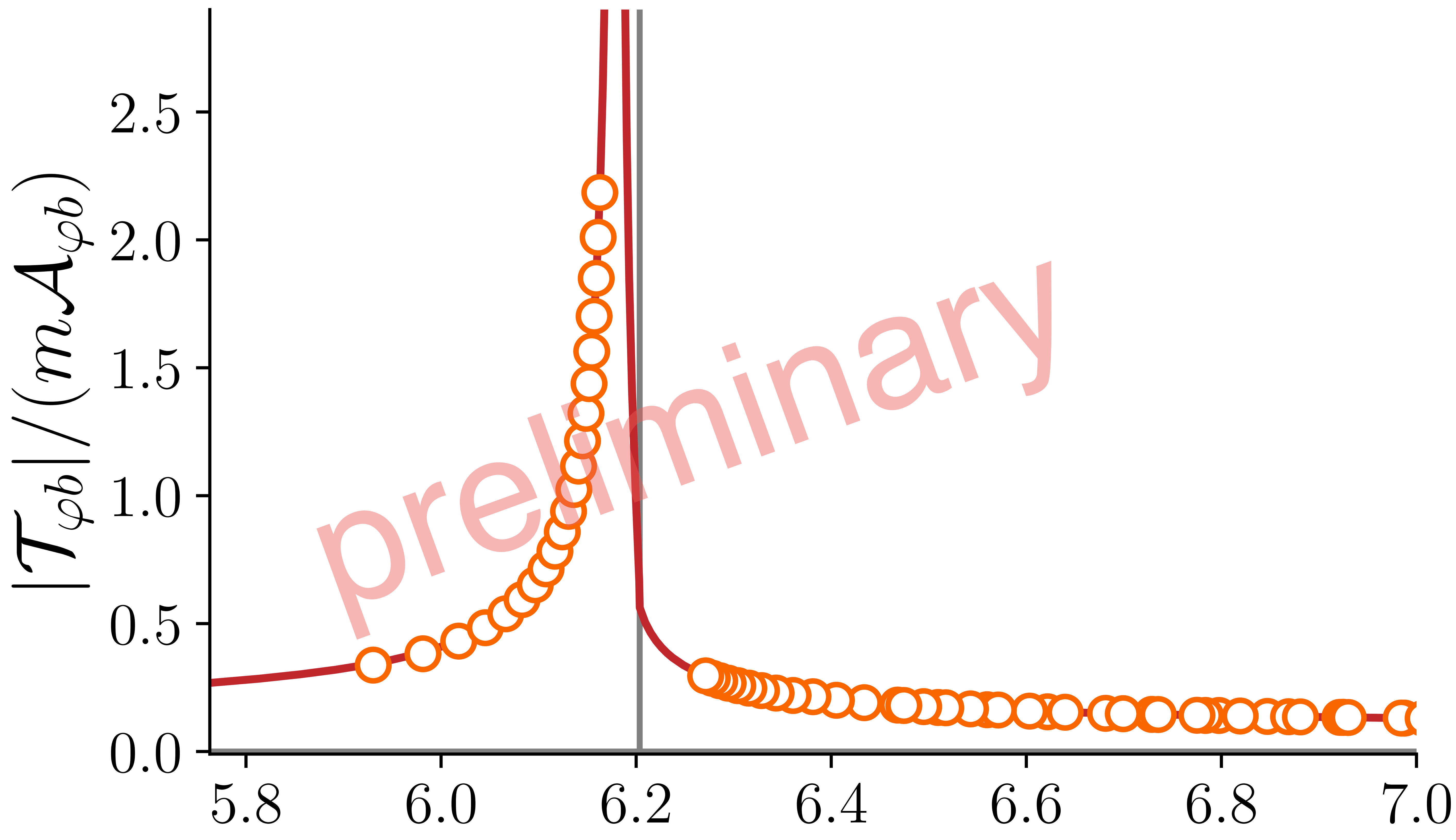


conclusion

$$\mathcal{T} \leftrightarrow \langle P_f; L|J(0)|0\rangle_L$$

Hansen, Romero-López & Sharpe ('21)

☑ formalism derivation



☑ checks of the formalism

Jackura Pefkou Romero-López

Formalism presented by Hansen, Romero-López & Sharpe (2021)

studying for $\gamma^\star \to 3\pi$, $K^0 \to 3\pi$, … passes non-trivial unitarity

and analyticity self-consistent checks.