Isospin-$\frac{1}{2}, \frac{3}{2}$ $D\pi$ scattering and the D_0^* resonance from lattice QCD

Lattice 2023

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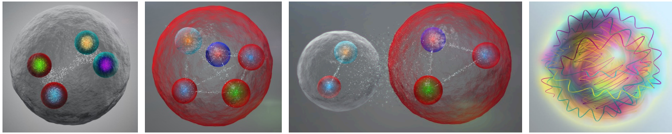
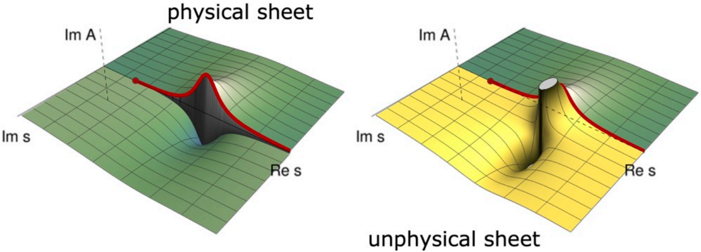
Hadron spectroscopy on the lattice

- Each year people discover new particles, what are they?



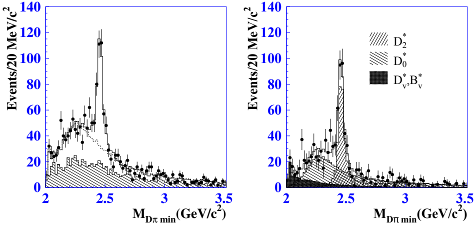
- Most particles are hadronic resonances – scattering experiments



- Turn off the weak and electromagnetic interactions
- Since the invention of lattice field theory[1], the calculation of hadron spectroscopy in the non-perturbative regime has been pursued to understand the structure of particles from the first principle

[1] Wilson, PRD 10 (1974) 2445

- The D_0^* was found in 2004 by Belle collboration[2]



- The mass of $D_0^*(2300)$ is almost identical to $D_{s0}^*(2317)$, which is **not** consistent with the traditional quark model predictions[3]. This can be explained by the strong coupling to DK[4]
- UChPT: $D_0^*(2100)$ should be the lightest charmed scalar meson[5]
- The possible two-pole structure
- Towards the understanding of $\psi_0(4360) \to D^* \bar{D}_1$ (0^{--})[6]

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[2]Satpathy *et al.*, PRB 159 (2003) 553.
[3]Du *et al.*, PRD 98 (2018) 094018.
[4]Chen *et al.*, Rep. Prog. Phys. 80 (2017) 076201
[5]Albaladejo *et al.*, PLB 767 (2017) 465.
[6]Ji *et al.*, PRL 129 (2022) 102002.

Previous works on $D\pi$ scattering

- Spectroscopy:
  - Liu *et al.*, JHEP 07 (2012) 126: charmonium excited and exotic spectroscopy
  - Moir *et al.*, JHEP 05 (2013) 021: D excited spectroscopy
  - Cheung *et al.*, JHEP 12 (2016) 089: D_s excited spectroscopy
  - ... ...
- Scattering:
  - Mohler *et al.*, PRD 87 (2013) 034501: $D\pi$ $I = \frac{1}{2}$ scattering
  - Moir *et al.*, JHEP 10 (2016) 011: Coupled-Channel $D\pi$, $D\eta$ and $D_s\bar{K}$ scattering
  - Gayer *et al.*, JHEP 07 (2021) 123: $D\pi$ $I = \frac{1}{2}$ scattering
  - Cheung *et al.*, JHEP 02 (2021) 100: DK $I = 0$, $D\bar{K}$ $I = 0, 1$ scattering
  - ... ...

We try to conduct a study on isospin-$\frac{1}{2}, \frac{3}{2}$ $D\pi$ scattering on a set of new ensembles

Configurations generated by the CLQCD collaboration (中国格点合作组)

By 24:00, Jul. 31, 2023

configuration	volume	a/fm	β	m_π/MeV	m_{η_s}/MeV	$m_\pi L$	N_{cfgs}
C24P34	$24^3 \times 64$	0.1053	6.20	340	748	4.38	301
C24P29	$24^3 \times 72$	0.1053	6.20	292	658	3.75	879
C32P29	$32^3 \times 64$	0.1053	6.20	292	658	5.01	984
C32P23	$32^3 \times 64$	0.1053	6.20	228	643	3.91	451
C48P23	$48^3 \times 96$	0.1053	6.20	225	643	5.79	278
C48P14	$48^3 \times 96$	0.1053	6.20	135	706	3.56	203
F32P30	$32^3 \times 96$	0.0775	6.41	303	681	3.81	568
F48P30	$48^3 \times 96$	0.0775	6.41	303	679	5.72	278
F32P21	$32^3 \times 64$	0.0775	6.41	210	665	2.67	459
F48P21	$48^3 \times 96$	0.0775	6.41	207	667	3.91	270
H48P32	$48^3 \times 144$	0.0519	6.72	321	709	4.06	274
H64P32	$64^3 \times 128$	0.0519	6.72	321	709	5.41	preparing

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- Create them from the vacuum! (in a world where $m_\pi \approx 300$ MeV)
- Severe partial wave mixing – need many many operators
- Project the operators into specific quantum numbers – irrep. and group[7]

$$O_{|p|,\Gamma,r,n} = \sum_{\tilde{R}\in G} T^{\Gamma}_{r,r}(\tilde{R})\tilde{R}D(p_1)\pi(p_2)\tilde{R}^{-1}$$

$$I = \frac{1}{2}, Dic_4(A_1) \begin{cases} \mathcal{O}_{D_0^*} & = D_0^{*+}(e_z), \\ \mathcal{O}_{D^*} & = D_z^{*+}(e_z), \\ \mathcal{O}_{D(0)\pi(1),|\vec{p}^2_{\text{rel}}|=|\frac{1}{4}|} & = \sum_\alpha D(\vec{p}_\alpha)\pi(\vec{P}_{\text{tot}} - \vec{p}_\alpha), \alpha \in [0], \\ \mathcal{O}_{D(1)\pi(0),|\vec{p}^2_{\text{rel}}|=|\frac{1}{4}|} & = \sum_\alpha D(\vec{p}_\alpha)\pi(\vec{P}_{\text{tot}} - \vec{p}_\alpha), \alpha \in [e_z], \\ \mathcal{O}_{D(1)\pi(2),|\vec{p}^2_{\text{rel}}|=|\frac{5}{4}|} & = \sum_\alpha D(\vec{p}_\alpha)\pi(\vec{P}_{\text{tot}} - \vec{p}_\alpha), \alpha \in [e_{-x}, e_x, e_{-y}, e_y], \\ \mathcal{O}_{D(2)\pi(1),|\vec{p}^2_{\text{rel}}|=|\frac{5}{4}|} & = \sum_\alpha D(\vec{p}_\alpha)\pi(\vec{P}_{\text{tot}} - \vec{p}_\alpha), \alpha \in [e_{xz}, e_{-x,z}, e_{yz}, e_{-y,z}], \\ \dots \end{cases}$$

- No single-body operator for $\frac{3}{2}$
- These operators are constructed to map out the scattering phase shift

[7] Prelovsek *et al.*, JHEP 2017 (2017) 1.

- Create $D\pi$ from a spacetime point, and annihilate them later

$$\langle \mathcal{O}^{[I=\frac{1}{2},I_z=\frac{1}{2}]}_{D^{(*)}\pi,\Gamma,p}(t')\mathcal{O}^{[I=\frac{1}{2},I_z=\frac{1}{2}]\dagger}_{D^{(*)}\pi,\Gamma,p}(t)\rangle = \sum_{\beta\alpha ji}(6\mathbb{E}+9\mathbb{F}-3\mathbb{G})^{[\beta,P-\beta;-\alpha,-(P-\alpha)]}_{[\gamma_j,\gamma_5;\gamma_i,\gamma_5]}$$

$$\mathbb{F} = \langle \bar{u}\Box e^{-ip_\delta\cdot x}\Omega\Box c(t')\cdot \bar{d}\Box e^{-ip_\gamma\cdot x}\Xi\Box u(t')\cdot \bar{c}\Box e^{-ip_\beta\cdot x}\Lambda\Box u(t)\cdot \bar{u}\Box e^{-ip_\alpha\cdot x}\Gamma\Box d(t)\rangle$$

- The Wick contractions contain the following diagrams

(a) $\mathbb{E}$ (b) $\mathbb{F}$ (c) $\mathbb{J}$ (d) $\mathbb{G}$

We apply the distillation method[8] to make the calculation possible

$$\Box(t) = V(t)\,V^\dagger(t) \longrightarrow \Box_{xy}(t) = \sum_{k=1}^{N} v_x^{(k)}(t)v_y^{(k)\dagger}(t)$$

[8] Peardon *et al.*, PRD 80 (2009) 054506.

Correlation functions

- Create $D\pi$ from a spacetime point, and annihilate them later

$$\langle \mathcal{O}^{[I=\frac{1}{2},I_z=\frac{1}{2}]}_{D^{(*)}\pi,\Gamma,p}(t')\mathcal{O}^{[I=\frac{1}{2},I_z=\frac{1}{2}]\dagger}_{D^{(*)}\pi,\Gamma,p}(t)\rangle = \sum_{\beta\alpha j i}(6\mathbb{E}+9\mathbb{F}-3\mathbb{G})^{[\beta,P-\beta;-\alpha,-(P-\alpha)]}_{[\gamma_j,\gamma_5;\gamma_i,\gamma_5]}$$

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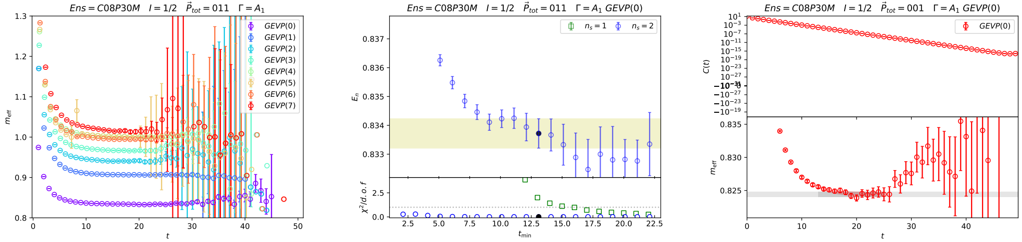
Spectrum analysis

- Inserting a complete basis, we know

$$\langle \mathcal{O}^{[J=L=S=0]}_{D\pi,A_1^+,p}(t')\mathcal{O}^{[J=L=S=0]\dagger}_{D\pi,A_1^+,p}(t)\rangle = \sum_n |\langle n|\mathcal{O}^{[J=L=S=0]}_{D\pi,A_1^+,p}|0\rangle|^2\, \mathrm{e}^{-E_n t}$$

- For more than one operator, we use the GEVP method to diagonalize them
- The effective mass $m_{\rm eff}(t)$ would go asymptotic to a plateau of energy levels



- The effect of non-local operators is to be examined

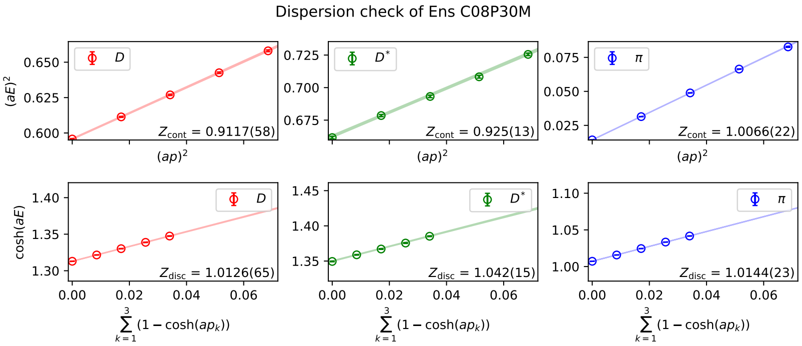
Dispersion check

- The dispersion relation

$$E(\vec{p}) = \sqrt{m_H^2 + Z_{\text{cont}}\vec{p}^2}(1 + \mathcal{O}(ap)) \quad (1)$$

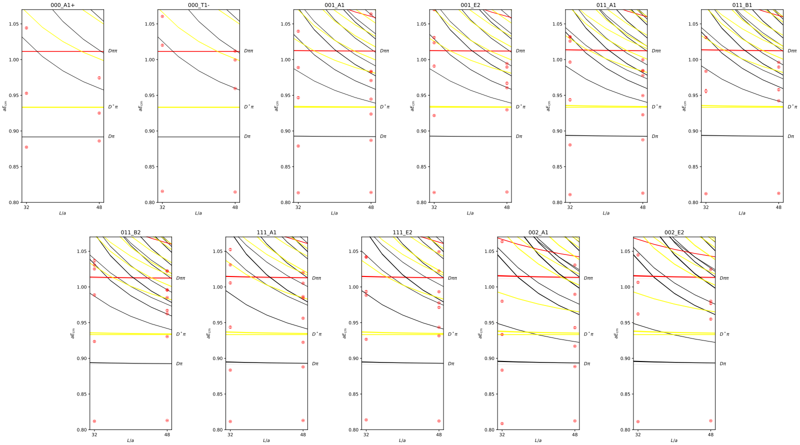
should be replaced by the discretized version

$$\cosh(aE(\vec{p})) = \cosh(am_H) + Z_{\text{disc}} \sum_{k=1}^{3} (1 - \cos(ap_k)) \quad (2)$$



The spectra

- The extracted finite volume scattering spectra



- The emergence of D^*
- Strong attraction in S-wave and small δ_1

Scattering analysis (preliminary)

- The Lüscher's equation[10]

$$\det\left[e^{2i\delta} - U(\Gamma)\right] = 0$$

  relates the spectrum to the scattering phase shifts in infinite volume
- Underconstrained problem
- Parametrize the phase shifts by the effective range expansion

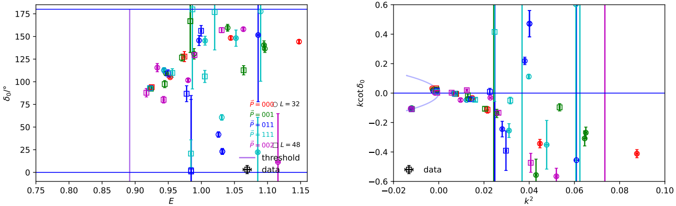
$$k^{2l+1}\cot\delta_l = \frac{1}{a_l} + \frac{1}{2}r_l k^2 + P_2 k^4 + \mathcal{O}\left(k^6\right),$$

- Coupling to $D^*\pi$ is to be considered

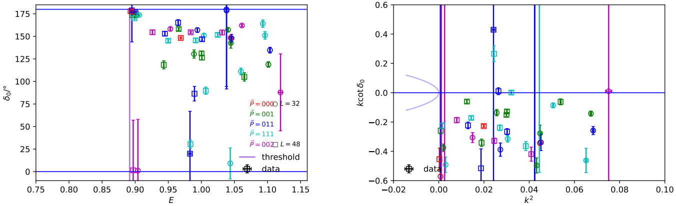
[10] Lüscher, NPB 354 (1991) 531.

Scattering analysis (preliminary)

- Ignore all $l > 0$ partial wave
- $I = \frac{1}{2}$

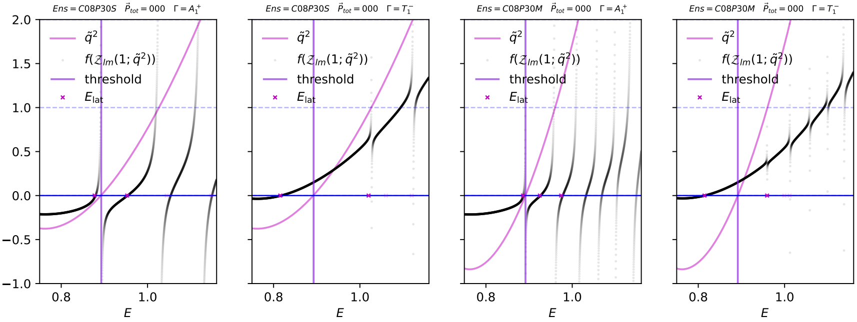


- $I = \frac{3}{2}$



Scattering analysis (preliminary)

- Using only data from $\vec{P} = 0$
- The Lüscher's equations

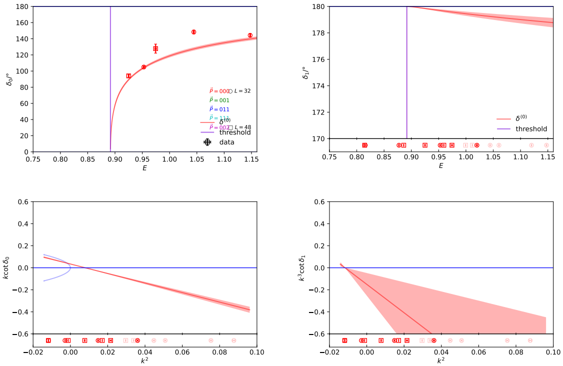


- The scattering length and the effective range

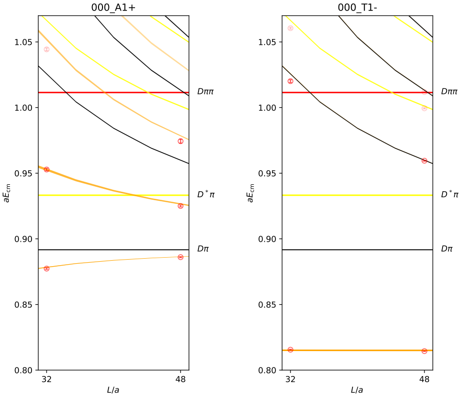
$$\begin{cases} a_0 = 2.26(19)\text{fm} \\ r_0 = -0.670(47)\text{fm} \end{cases} \quad \begin{cases} a_1 = -0.52(15)\text{fm} \\ r_1 = -2.0(1.3)\text{fm} \end{cases} \tag{3}$$

Scattering analysis (preliminary)

The $I = \frac{1}{2}$ phase shifts



The predicted spectrum



- For now, D_0^* looks like a virtual state on our lattice
- Consistent with the JLab results in the sense of pion mass dependence

Conclusions

- Obtained many finite-volume energy levels in the $D\pi$ system
- Found the D_0^* virtual state on our configuration
- To-dos:
  - Analysis for non-inertial frames
  - Interpretation of the pole(s)
  - Chiral extrapolation
  - Continuum extrapolation

Thank you!