The infinite volume based beta-function from the gradient flow with applications

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my LatHC collaborators at various stages of the project:

Szabolcs Borsanyi, Zoltan Fodor, Kieran Holland, Daniel Nogradi, Chik Him Wong

main purpose of the talk: to highlight results and questions about the new infinite volume based β-function on the gradient flow

- Lattice 2017: the origin of the method and the challenge of going to infinite volume

- Applications to the nf=10 BSM model including the newly claimed IRFP arXiv:2306.07236

- Short summary of recent focus: the scale dependent α_s coupling at the Z-pole in QCD

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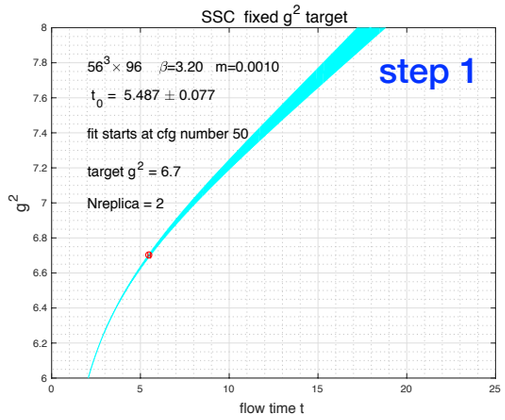
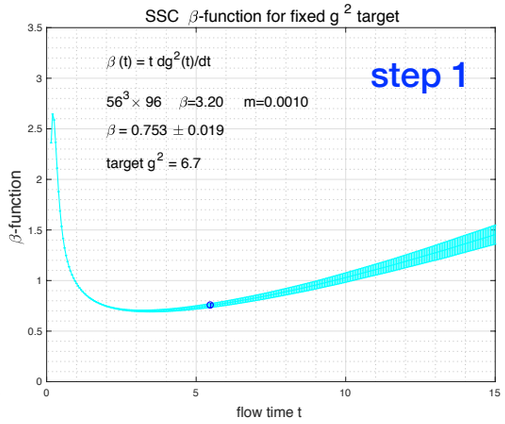
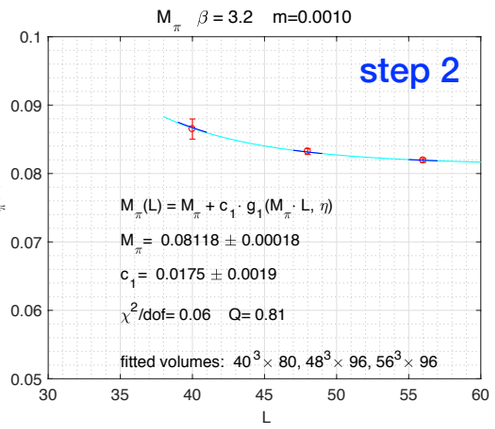
1711.04833 LatHC

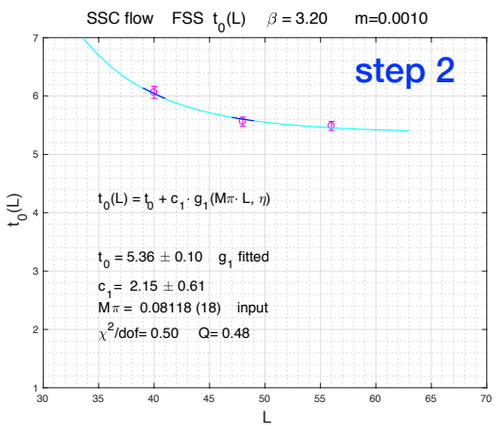
$$[-g^2(t+2\epsilon)+8g^2(t+\epsilon)-8g^2(t-\epsilon)+g^2(t-2\epsilon)]/(12\epsilon) = dg^2/dt + O(\epsilon^4)$$

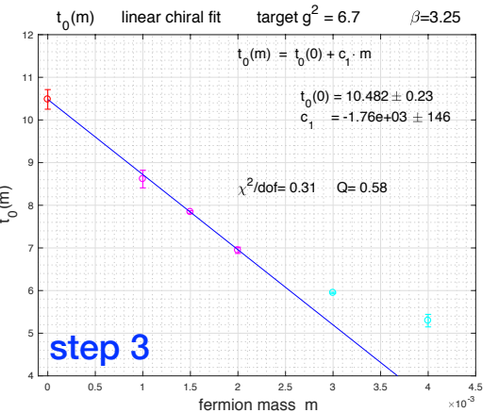
checked by 7-point stencil $\frac{1}{30} f^{(5)}(x)\cdot\epsilon^4$

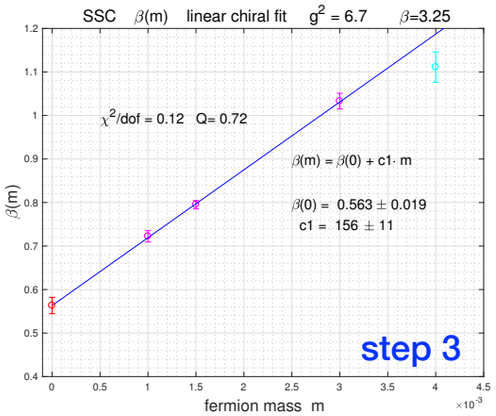












original sextet algorithm for $\beta(t(g^2)) = t\ dg^2/dt$:

sextet model β-function defined in infinite volume approached from p-regime

step 1 (a) select target renormalized coupling g^2 on the GF held fixed at several lattice spacings
(b) at each lattice spacing in the p-regime calculate $t\ dg^2/dt$ on the flow at several fermion masses, for several L at each m

step 2 at each lattice spacing for each m take L $\to \infty$ limit of M_π, $t_0(g^2)$, tdg^2/dt

$$t_0 = t_{0,\mathrm{ch}}\left(1 + k_1\frac{M_\pi^2}{(4\pi f)^2} + k_2\frac{M_\pi^4}{(4\pi f)^4}\log\left(\frac{M_\pi^2}{\mu^2}\right) + k_3\frac{M_\pi^4}{(4\pi f)^4}\right)$$ Golterman, Baer

step 3 at target g^2 take chiral limit $m \to 0$ at each lattice spacing for $t_0, tdg^2/dt$

step 4 repeat for 3 lattice spacings and take $a^2/t \to 0$ continuum limit of $\beta = tdg^2/dt$

Lattice 2017: original sextet algorithm of the infinite volume based β-function with massless fermions on the GF

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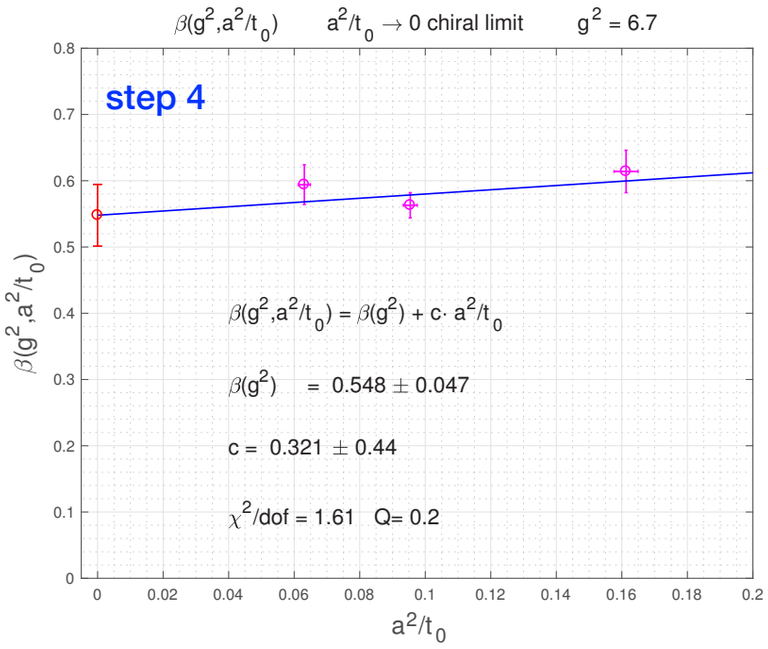
step 4 $a^2/t \to 0$ continuum limit of $\beta = t dg^2/dt$



The main idea behind the new infinite volume based $\beta_{m_f \to 0}$ function from the p-regime:

- If the limiting $\beta_{m_f \to 0}$ function from the p-regime matches the infinite volume based $\beta_{m_f=0}$ function, calculated directly with massless fermions, the model is probably not conformal with confidence
- There was concern about the reliability of the infinite volume extrapolation of the direct $\beta_{m_f=0}$ beta function and not implemented in the sextet model with insufficient data.
- Before 2019, experimenting with various m=0 direct implementations of $\beta_{m_f=0}$, tests worked better with mored data in the multi-flavor models after finding the $a^4/L^4 \to 0$ scheme at fixed lattice spacing a and fixed t, with hints from scaling arguments, but remained unpublished.
- Recent high precision data show the scaling behavior at strong coupling as well:

control of infinite volume limit is critical and more challenging than for the step β-function running with the physical volume

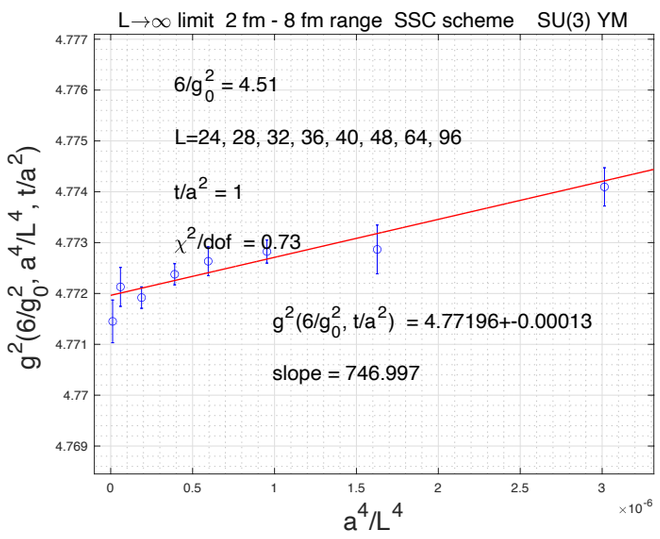
balance:

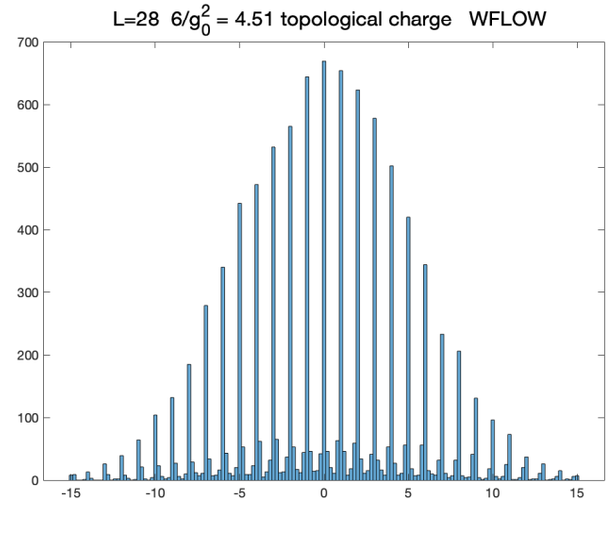
required large volumes can be matched to large volumes with mass deformations anyway

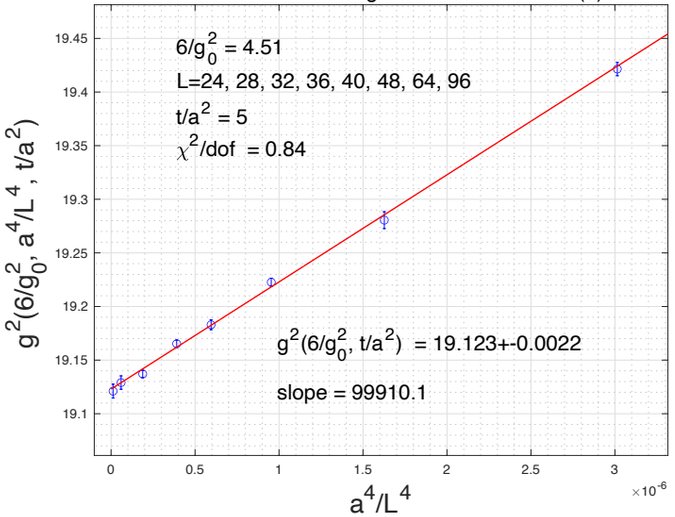
Lattice 2017: original sextet algorithm of the infinite volume based β-function with massless fermions on the GF

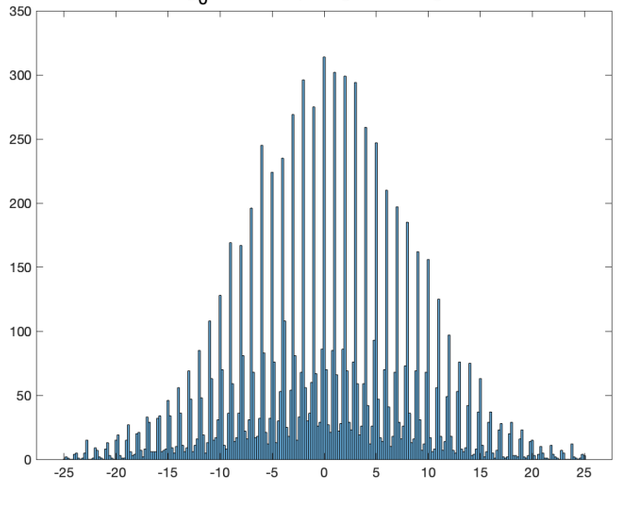
1711.04833 LatHC

high-precision tests of the volume dependence and its source









potential sources of a^4/L^4 scaling:

- Symanzik effective action at fixed a and t

  absence of a^2/L^2 term remains unexplained
  at small latt. spacing a but strong coupling, like here

- frozen topology

  not here, with well sampled topology

- form factor of the energy op on GF

  most likely origin, surprisingly indicates a larger extension

  of the form factor than the $C = \frac{\sqrt{8t}}{L}$ expectation

not under exponentially suppressed global control

care needed when infinite L taken at fixed a and fixed t

Lattice 2017: original sextet algorithm of the infinite volume based β-function with massless fermions on the GF

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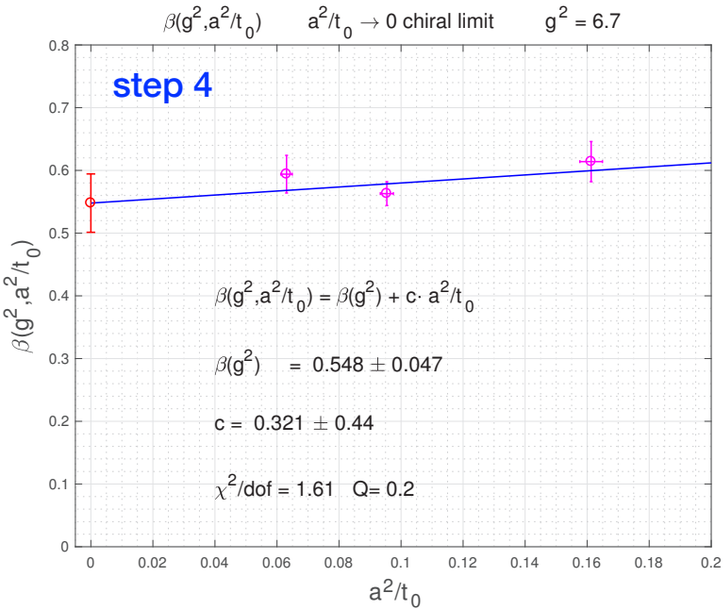
step 4 $a^2/t \to 0$ continuum limit of $\beta = tdg^2/dt$



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- Before 2019, experimenting with various m=0 direct implementations of $\beta_{m_f=0}$, tests worked better with mored data in the multi-flavor models after finding the $a^4/L^4 \to 0$ scheme at fixed lattice spacing and fixed t, with hints from scaling arguments, but remained unpublished. High precision data show that scaling behavior at strong coupling as well.
- Lattice 2019 surprise: first direct nf=2 calculation of $\beta_{m_f=0}$ goes public for the first time 1910.06408 switching $a^2/L^2 \to 0$ fitting of the talk to $a^4/L^4 \to 0$ in the publication.
- We chose to immediately publish our previously unpublished multi-flavor results of the $a^4/L^4 \to 0$ fitting procedure for $\beta_{m_f=0}$ as shown in 1912.07653.
- Back to the future? How to apply the $\beta_{m_f=0}$ and $\beta_{m_f \to 0}$ methods to the nf=10 model for testing if its chiral symmetry is broken in the $m \to 0$ limit? (Recent announcement claiming IRFP in the model). arXiv:2306.07236

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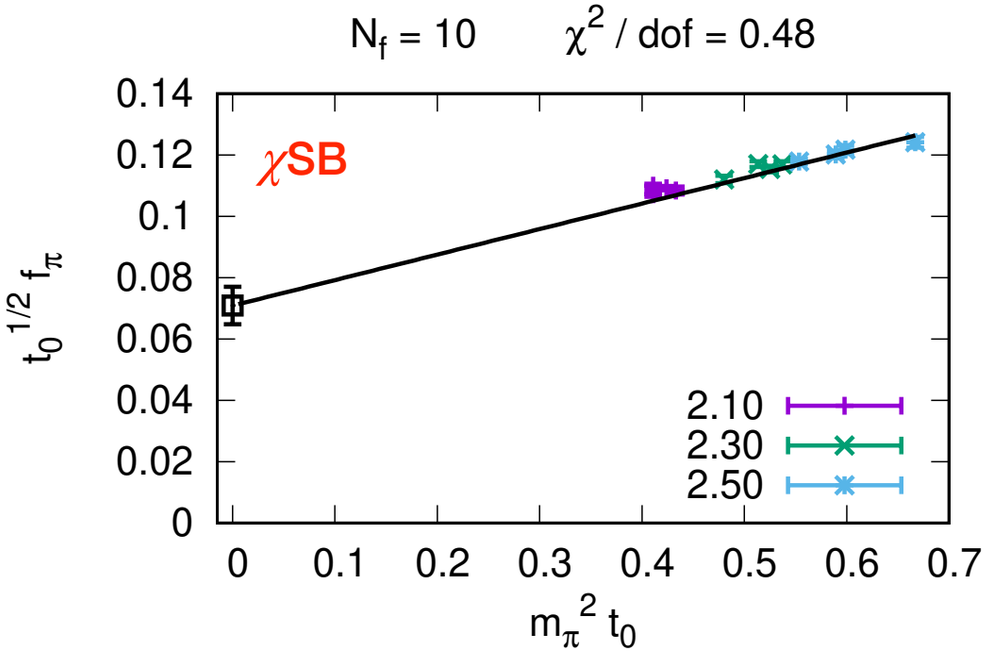
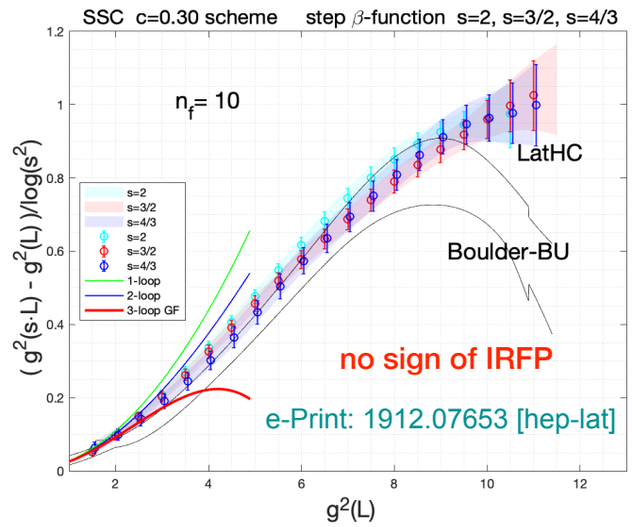
Back to the future?

Building on the ensembles of Dani Nogradi and his collaborators?

e-Print: 2107.05996 [hep-lat]





two findings are consistent but newly challenged

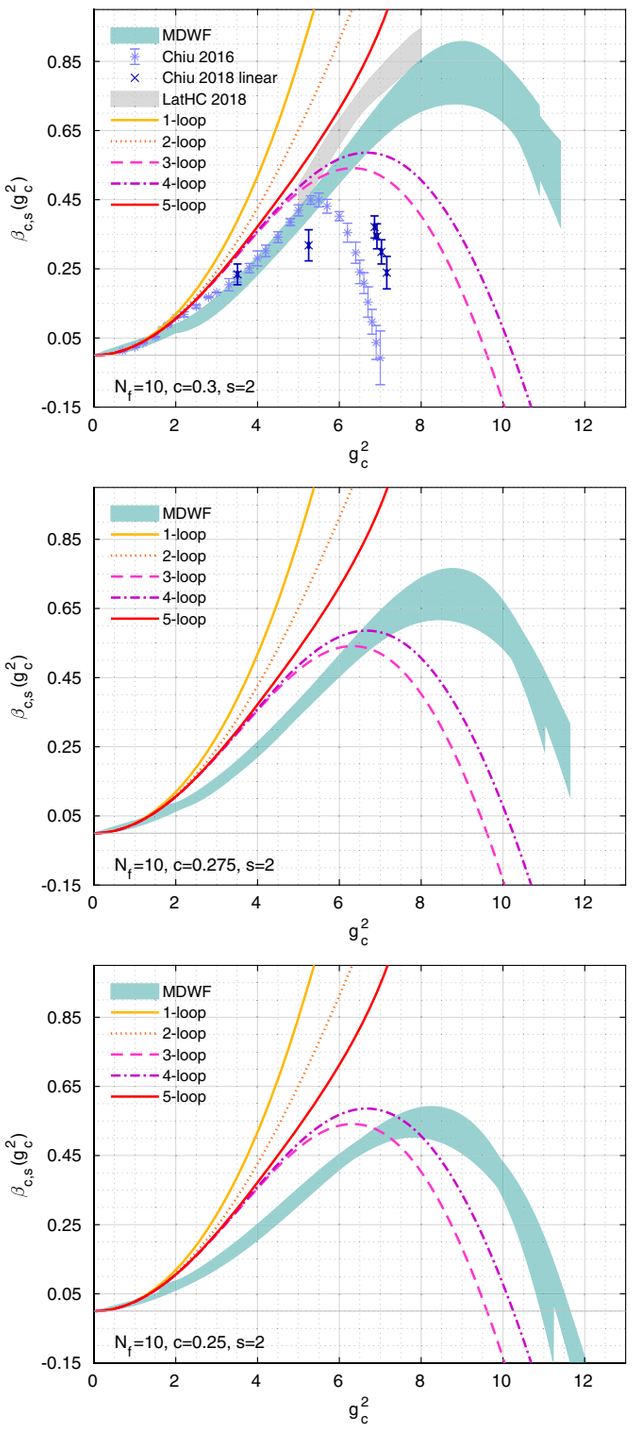
N_f	β	m	L/a	am_π	af_π	am_ϱ	t_0/a^2	$m_\pi L$	$f_\pi L$
10	2.10	0.0165	32	0.2423(7)	0.0416(2)	0.326(2)	7.00(7)	7.75(2)	1.330(5)
		0.0126	36	0.2037(7)	0.0342(1)	0.275(1)	10.2(1)	7.33(2)	1.230(5)
		0.0100	40	0.1801(9)	0.0297(2)	0.241(2)	13.4(3)	7.20(3)	1.186(6)
		0.0081	48	0.1535(6)	0.02567(9)	0.202(2)	17.4(2)	7.37(3)	1.232(4)
	2.30	0.0185	32	0.2386(7)	0.0390(1)	0.310(1)	9.0(1)	7.63(2)	1.247(5)
		0.0142	36	0.2005(9)	0.0320(1)	0.259(2)	13.4(2)	7.22(3)	1.152(5)
		0.0112	40	0.174(1)	0.0277(1)	0.227(1)	17.4(3)	6.95(4)	1.106(4)
		0.0091	48	0.1502(9)	0.0243(2)	0.194(2)	21.3(4)	7.21(4)	1.166(7)
	2.50	0.0233	28	0.2584(9)	0.0393(3)	0.328(2)	10.0(2)	7.23(3)	1.100(8)
		0.0178	36	0.2106(5)	0.0332(2)	0.267(2)	13.5(3)	7.58(2)	1.194(8)
		0.0141	40	0.1800(7)	0.0282(1)	0.227(2)	18.2(3)	7.20(3)	1.128(6)
		0.0114	48	0.1571(6)	0.0249(1)	0.202(1)	22.4(2)	7.54(3)	1.195(7)

- $f_\pi L \sim 2-3$ might be required at nf=10 for conventional p-regime χSB analysis
- the volume dependence of the GF time t_0 requires control
- original sextet analysis of $\beta(g^2)_{m_f \to 0}$ was based on χSB theory (Baer and Golterman)

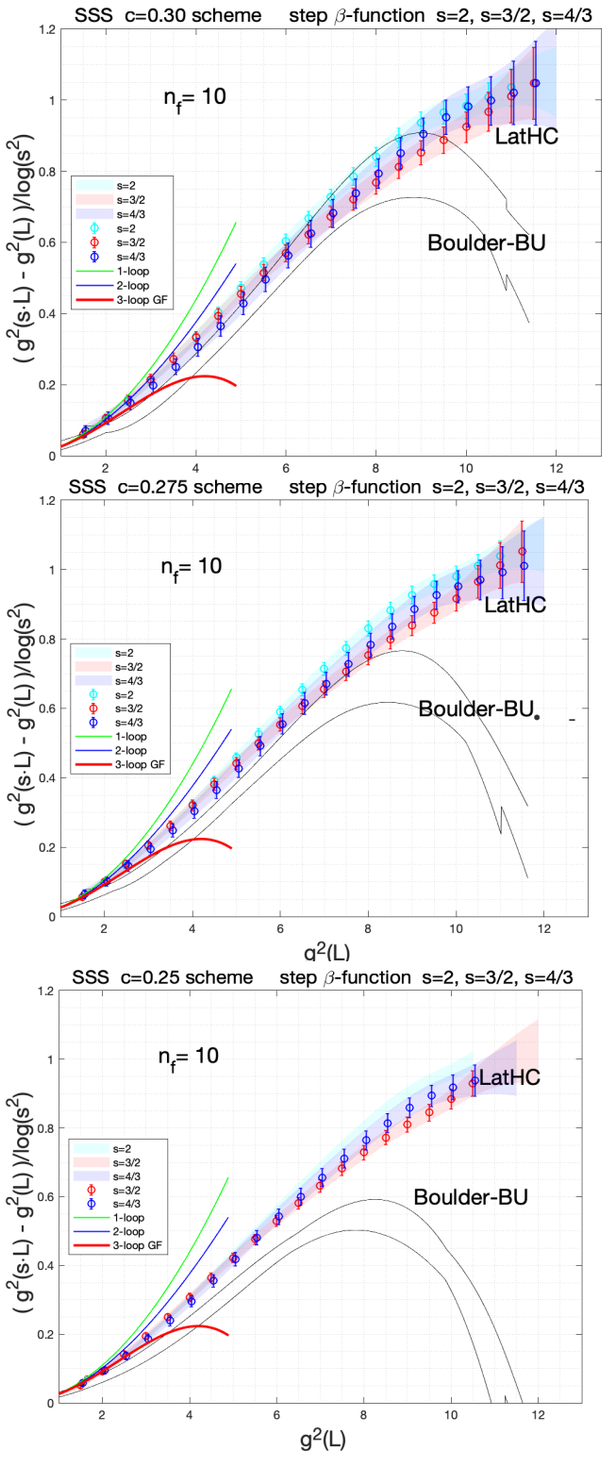
dilaton EFT is the alternate analysis perhaps needed to test matching $\beta(g^2)_{m_f \to 0} = \beta(g^2(t))_{m=0}$

- in our earlier $\beta(g^2(t))_{m=0}$ investigation we reached $g^2 \sim 12$ with $\beta(g^2(t))_{m=0} \sim 1$
- we could extend $\beta(g^2(t))_{m=0}$ into the $6/g_0^2 = 2.3$ range with $g^2 \sim 12-15$
- more ensembles would be needed

e-Print: 2004.00754 [hep-lat]

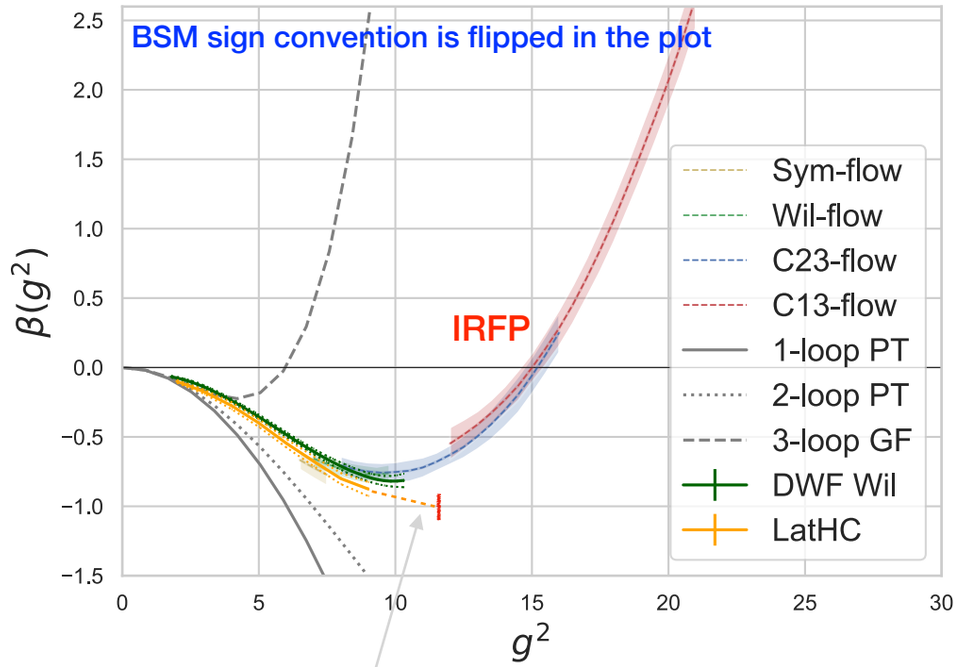


e-Print: 2203.15847 [hep-lat]



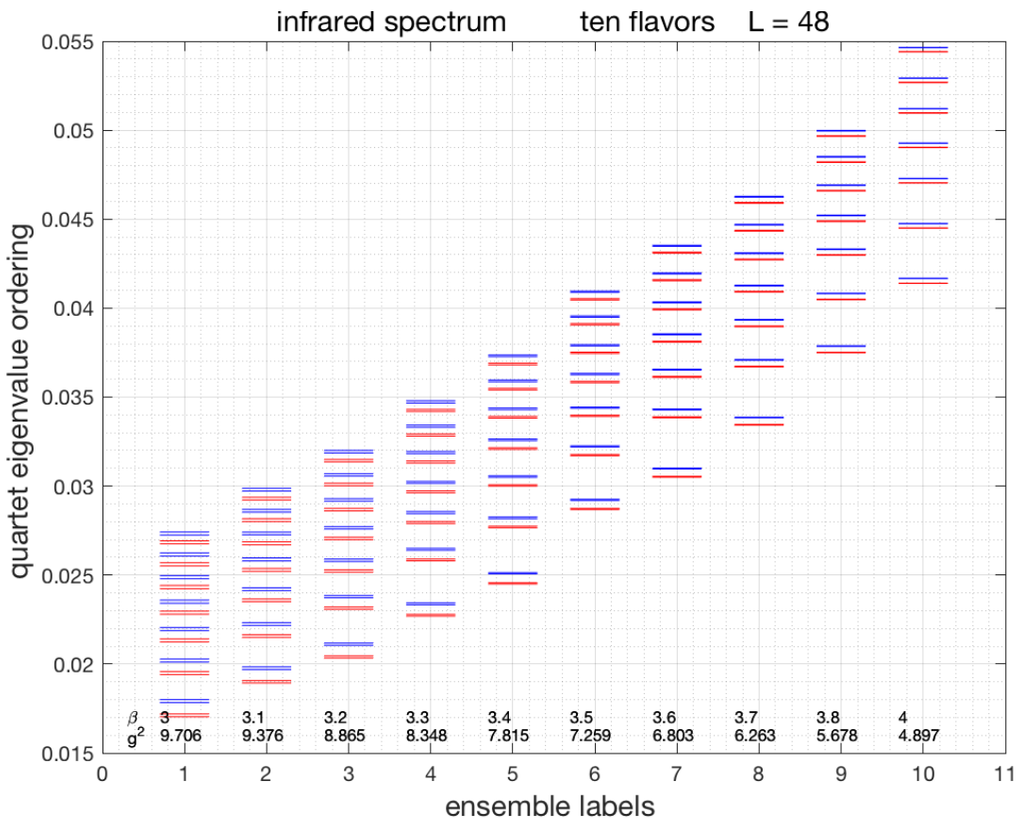
here comes the new challenge:

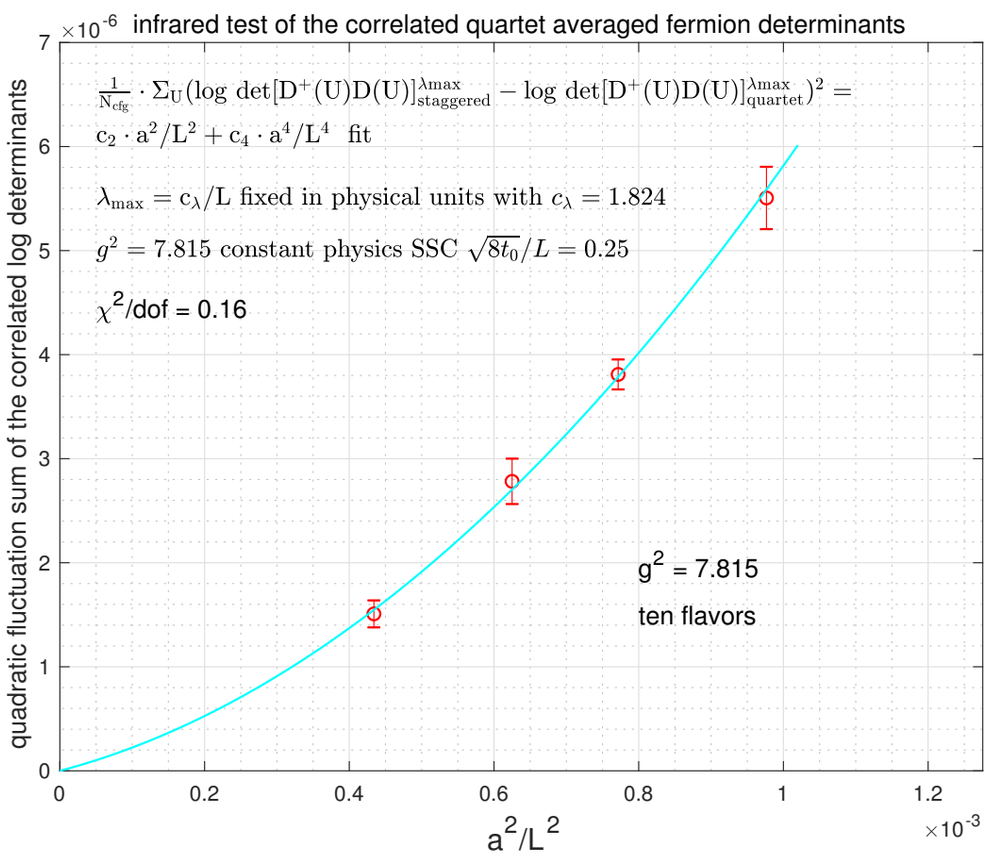
arXiv:2306.07236

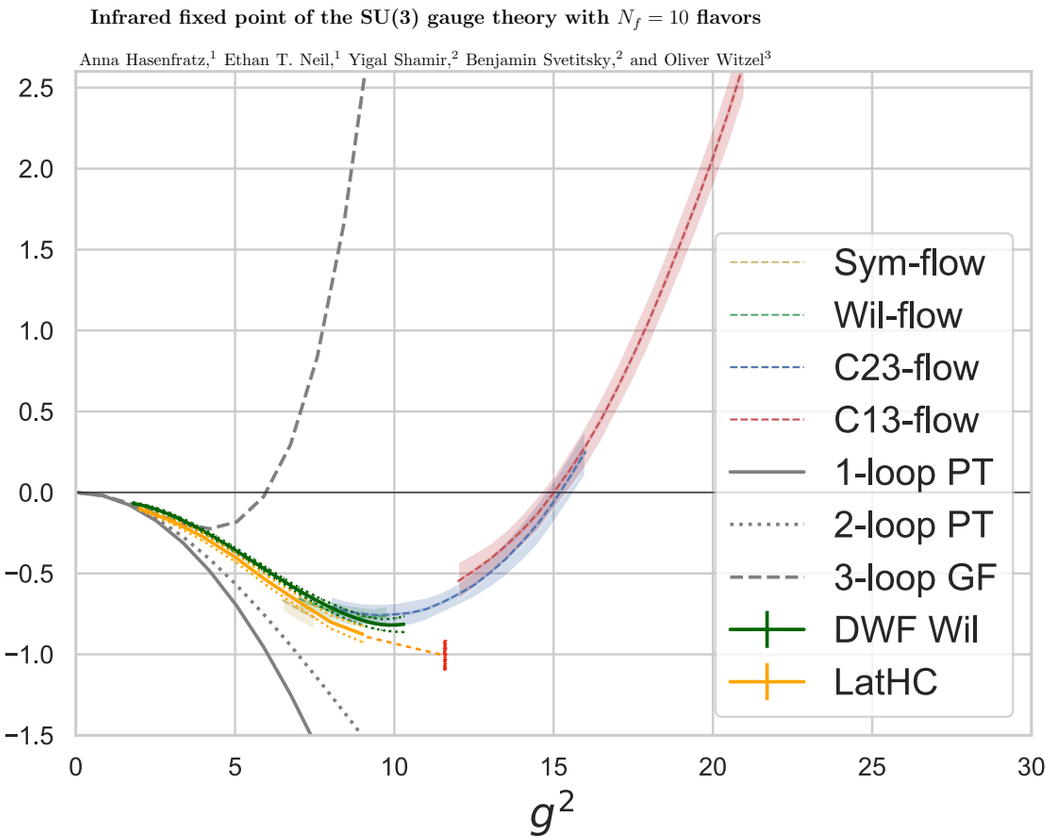


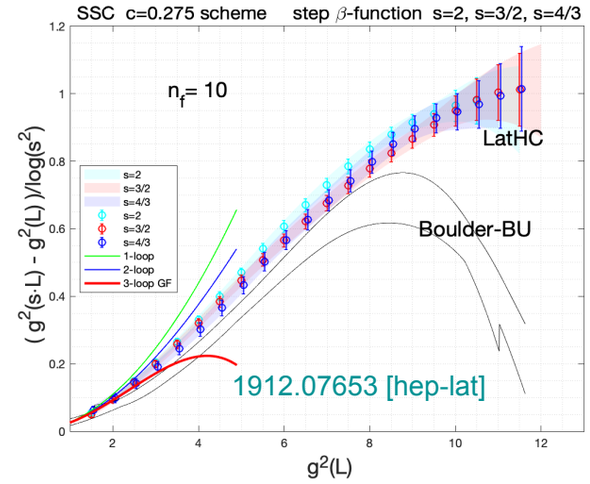
I added the dashed part from published LatHC

precision control on the staggered Dirac spectrum









comments for future discussions:

- $g^2 \sim 11.5$ reach of our earlier analysis is not shown
- if the IRFP is reached from the UVFP, this must be demonstrated without switching operator scheme and flow scheme
  nothing less should be acceptable in going from UV to IR
- only L=24 and L=28 for infinite volume extrapolation?
  why infinite volume method rather than running with the volume?
- what supports the continuum β-function beyond the IRFP?
  no second zero in β-function?
- tuning Wilson fermion with PCAC is not accuracy issue in the small lattice
  there is a residual mass ~ 1/L which effects how the infinite volume limit is taken
- why switch from staggered to Wilson fermion?
  earlier results with PV regulator were done with staggered fermions
- No control on the decoupling of the ghost sector at strong coupling
  Is this still the original nf=10 model at strong coupling?

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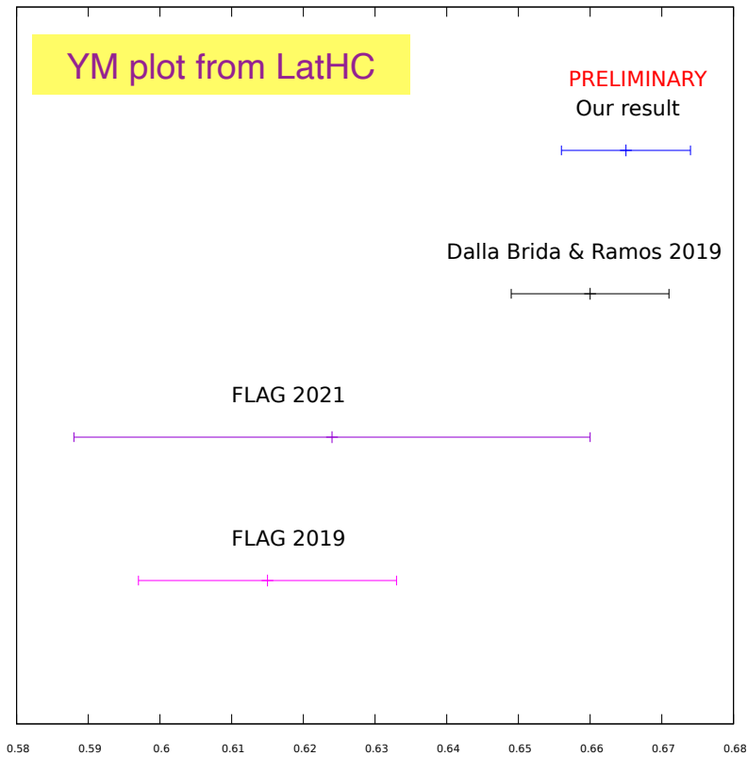
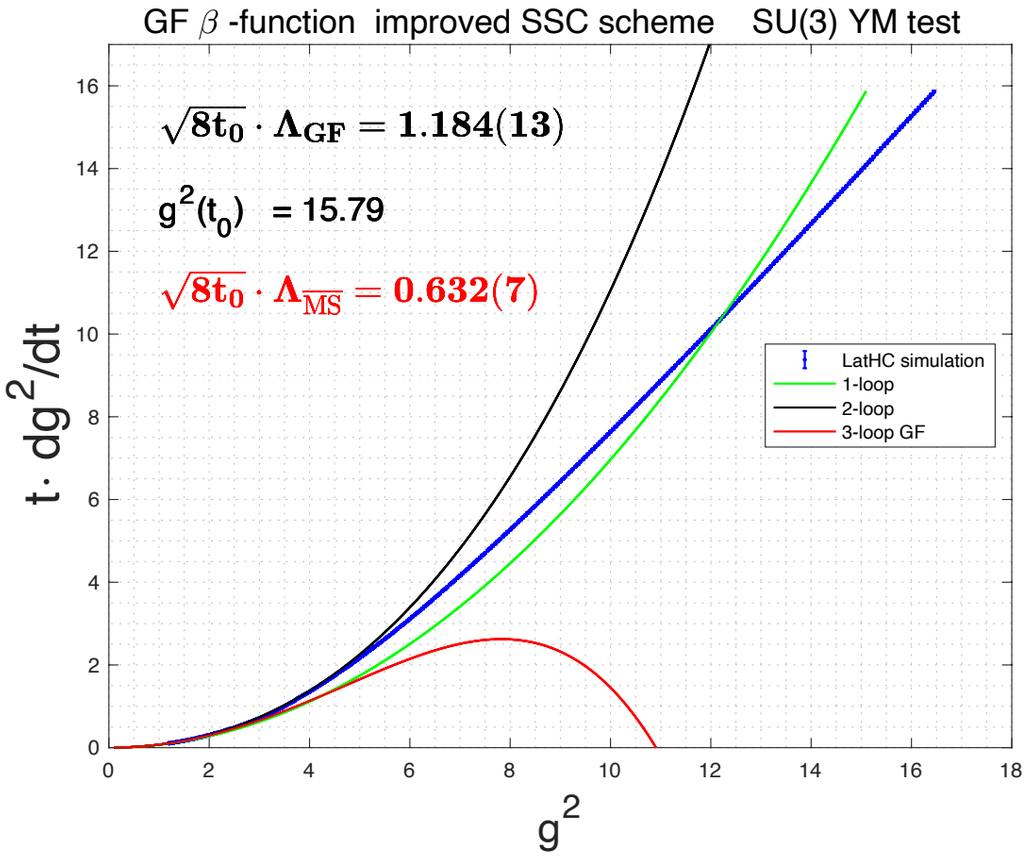
Wong et al. e-Print: 2301.06611 [hep-lat] Lattice 2022



Foreword:

- Our YM and multi-flavor analyses are motivated by the Harlander-Neumann 3-loop GF based beta-function over infinite Euclidean volume in the continuum limit — contact with PT in GF based renormalization needs new infinite volume β-function
- With all the BSM controversies, like ten flavors, can a new method help QCD?
- After careful precision calculations of the Alpha collaboration who needs another QCD strong coupling?
- But even the simplest SU(3) Yang-Mills model shows significant tension after repeated FLAG reviews
  - FLAG 2019 was without first high precision GF result of Dalla Brida and Ramos
  - combined FLAG 2021 error analysis hides the tension
  - results from new inf. vol. GF based β-function increase the tension with earlier lattice work



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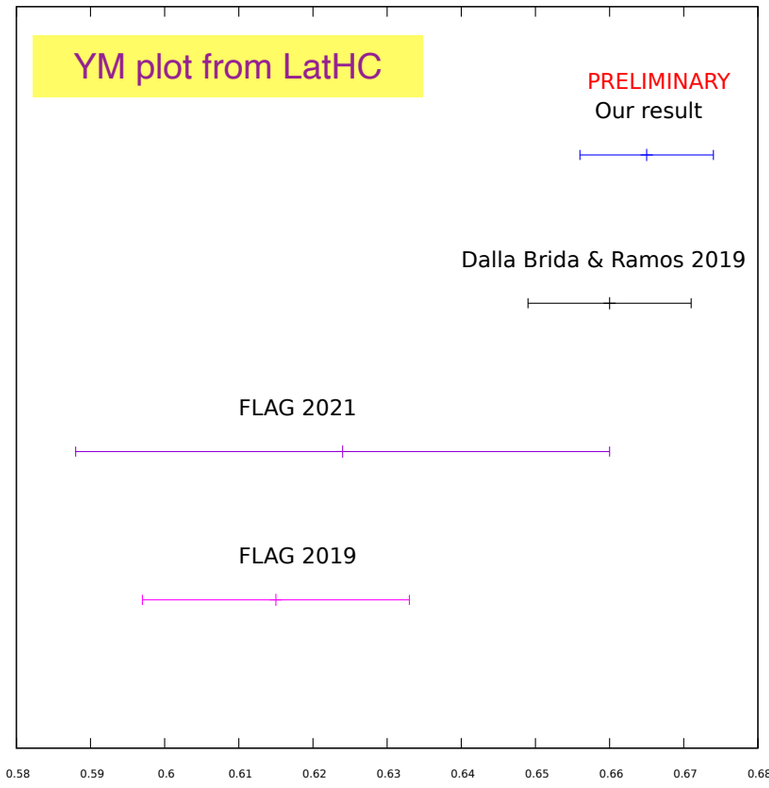

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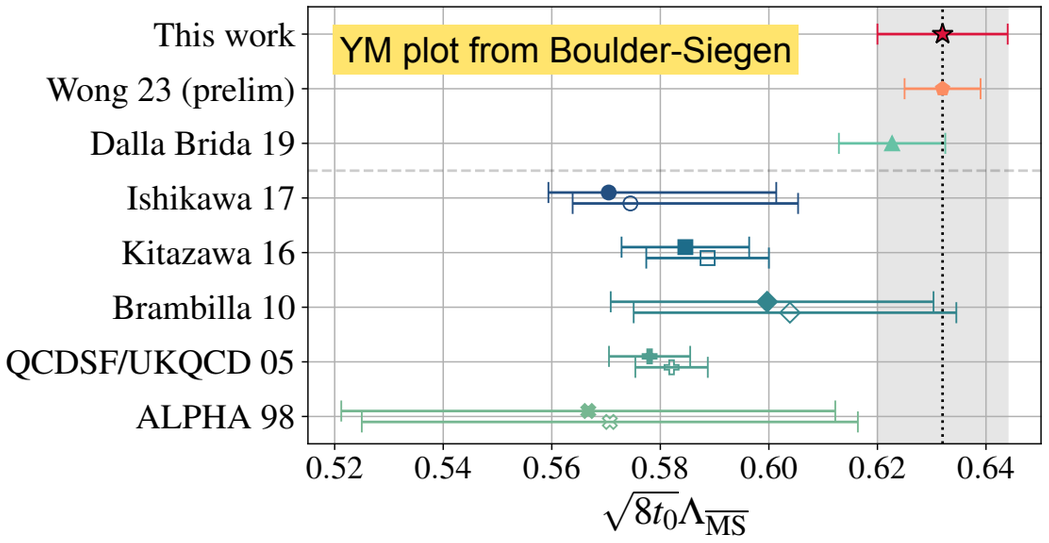
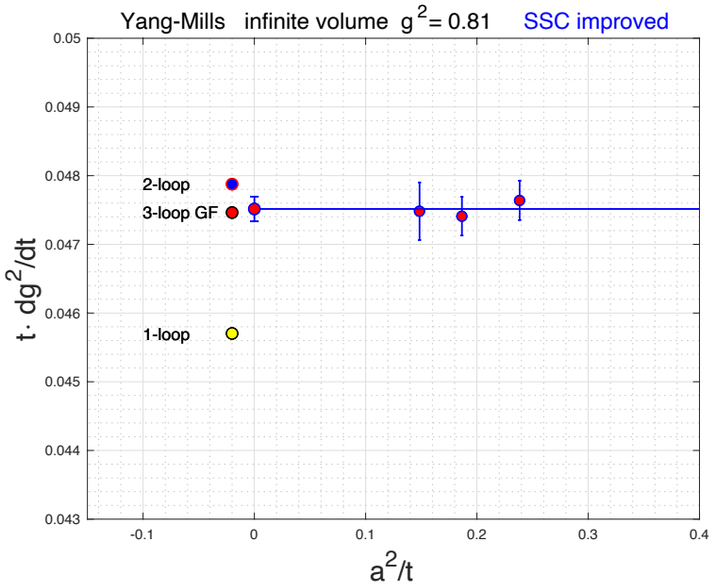
A. Hasenfratz et al. 2303.00704 [hep-lat]

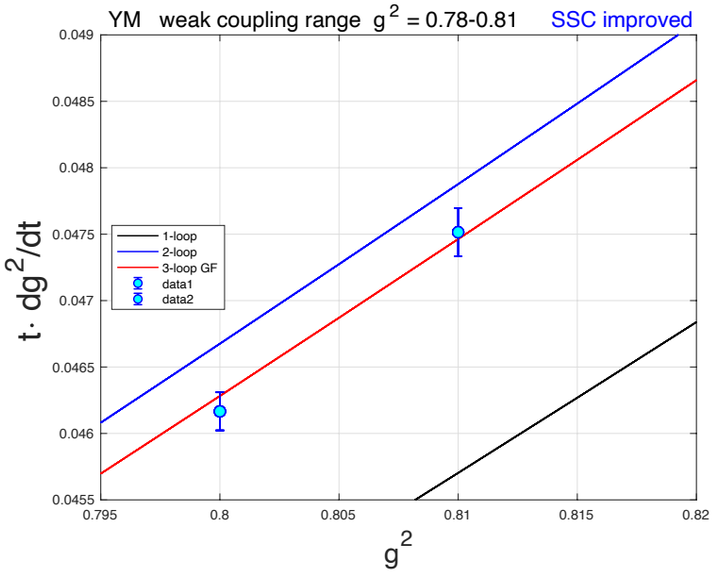


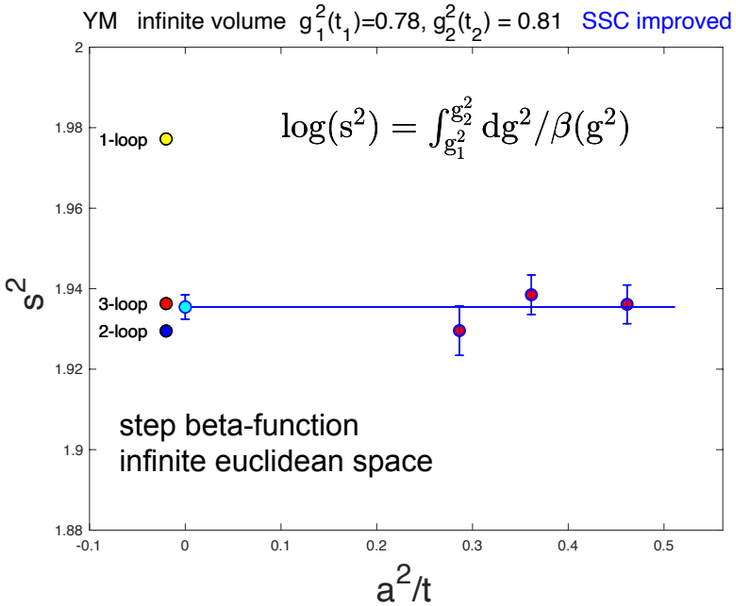
- Shortly after our publication, almost identical results of Boulder-Siegen YM project
- Move toward an ambitious new goal: to create similar competing result in QCD with three massless fermions?

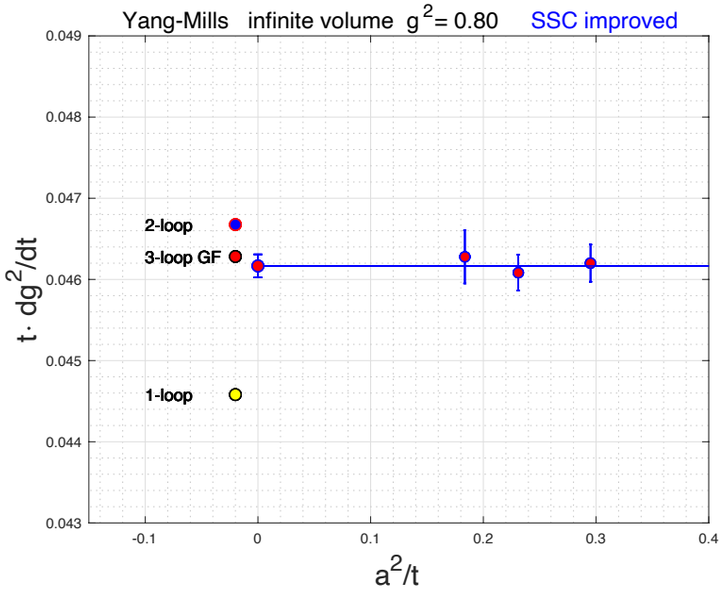
Yang-Mills project

tdg^2/dt and inf. vol. based step beta-function at weak coupling



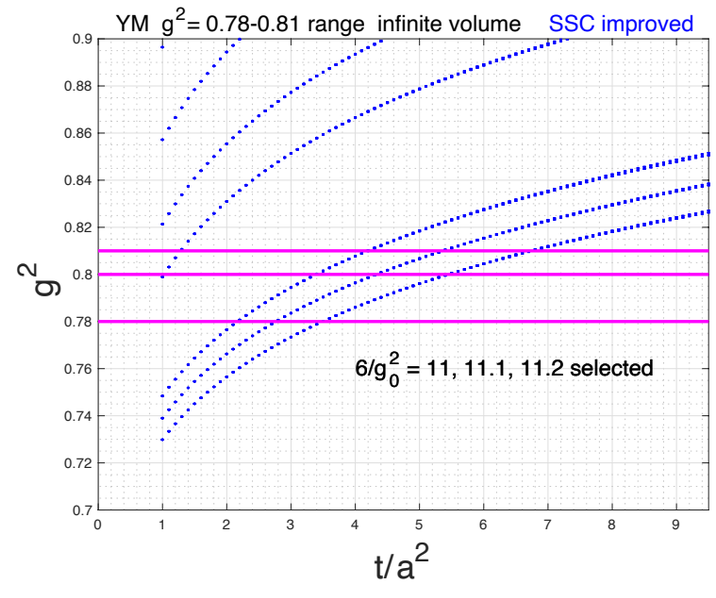




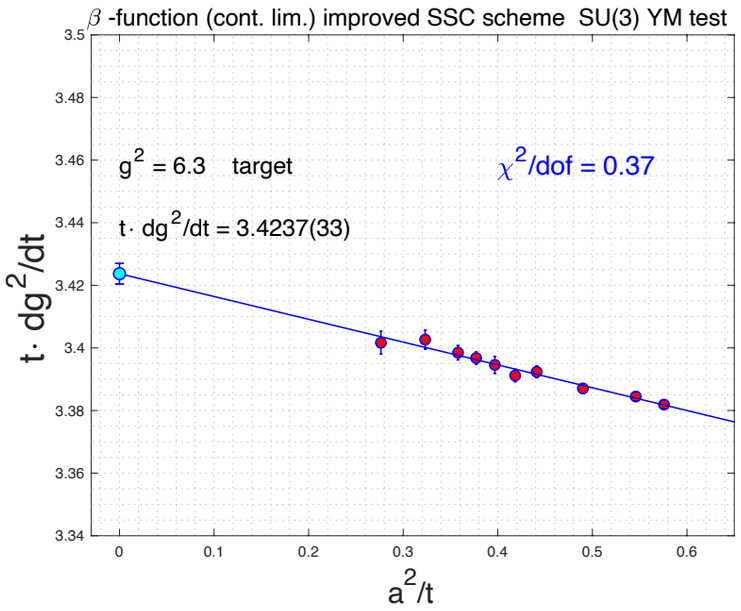


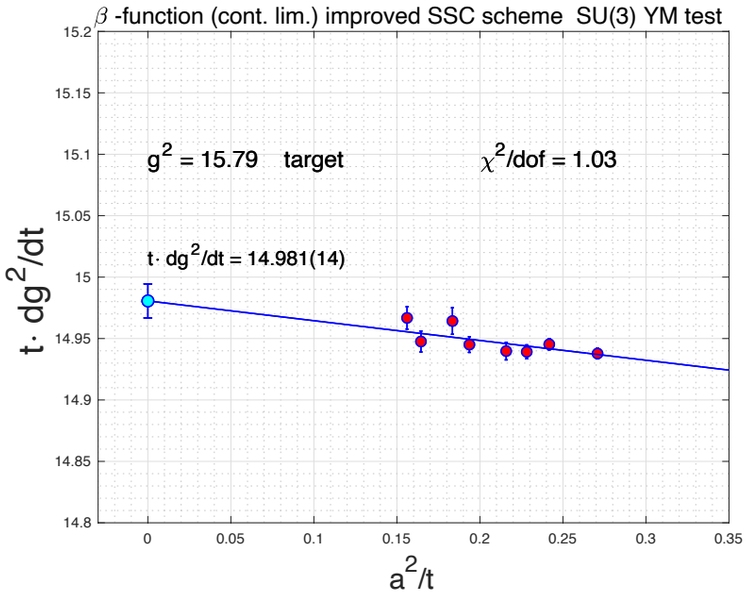
High precision contact with Harlander's 3-loop β-function in the continuum limit

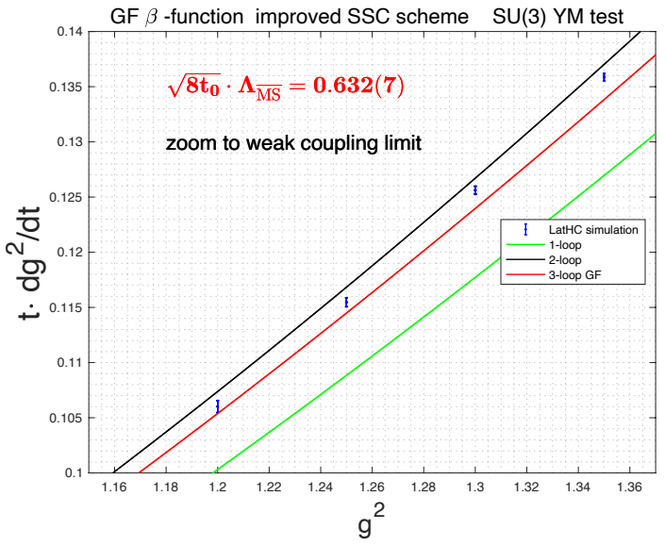
needed for precise scale setting of $\Lambda_{\overline{MS}}$

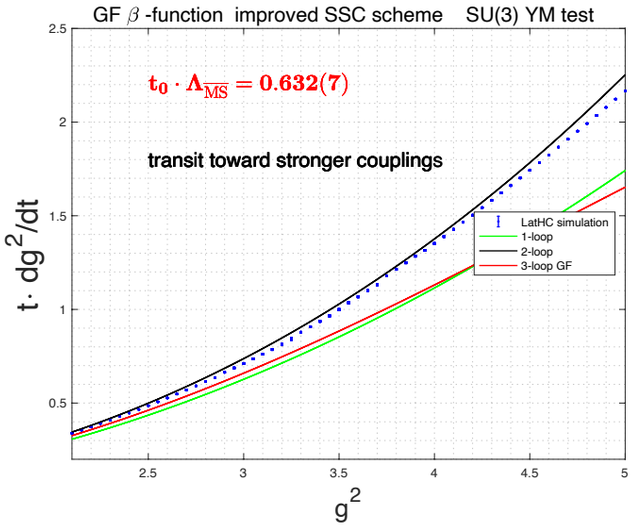


YM tdg^2/dt beta-function at strong coupling

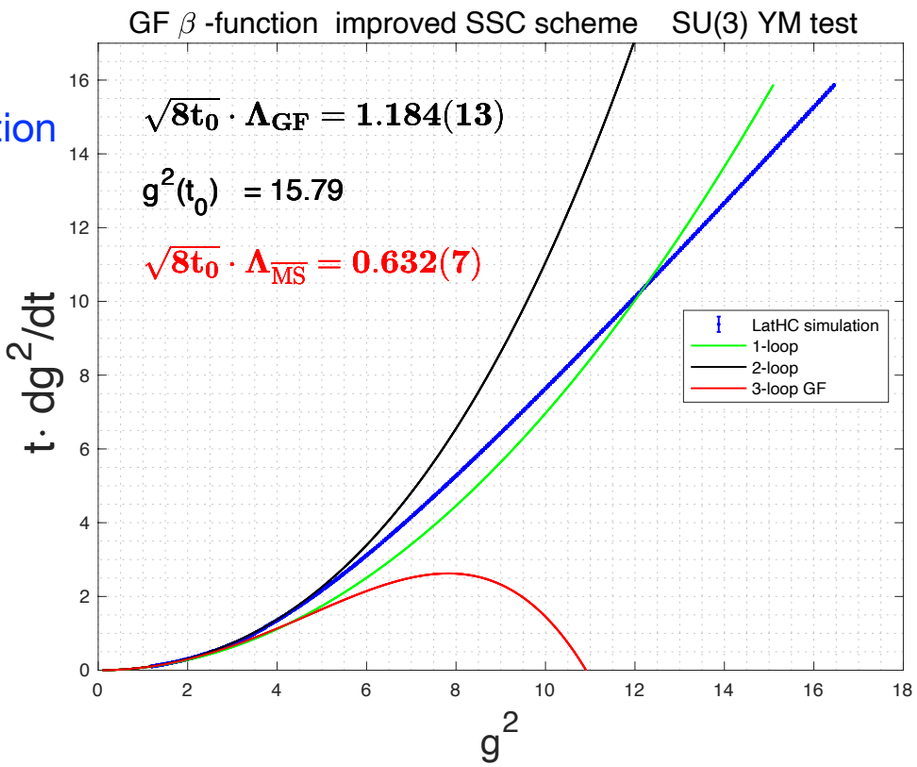


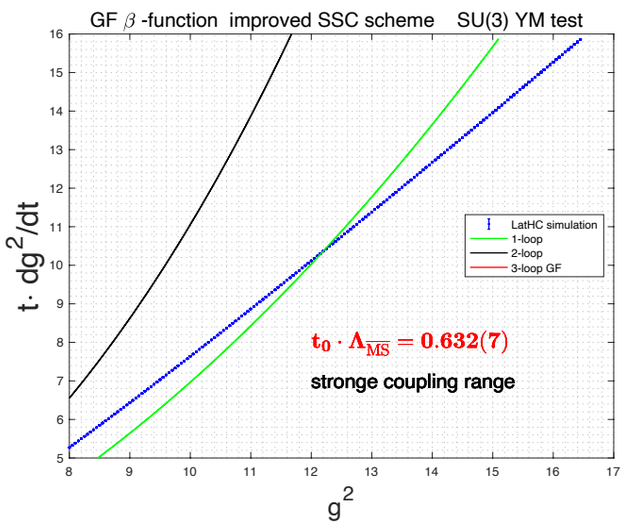






new analysis is being prepared for journal publication

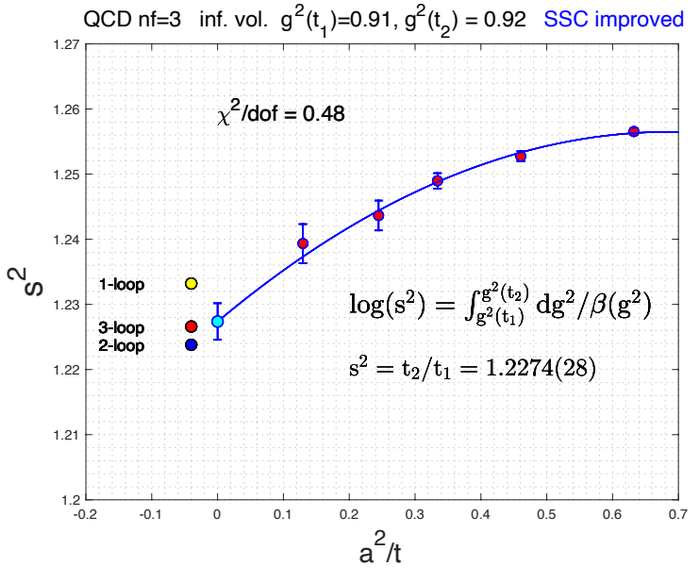


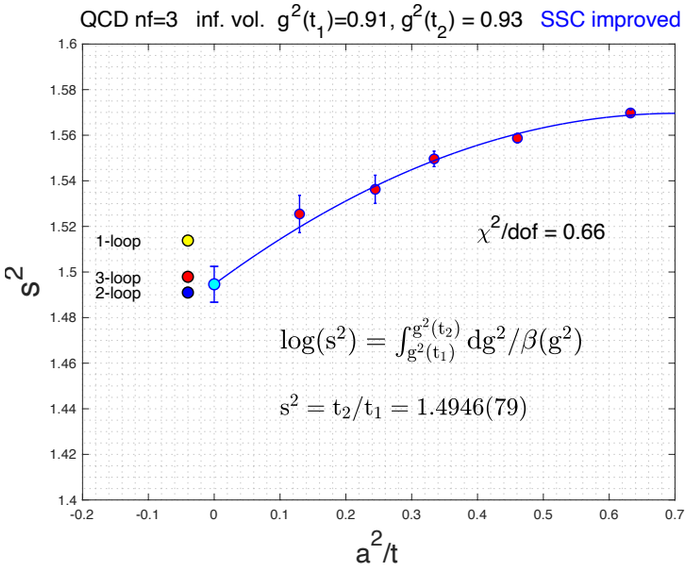


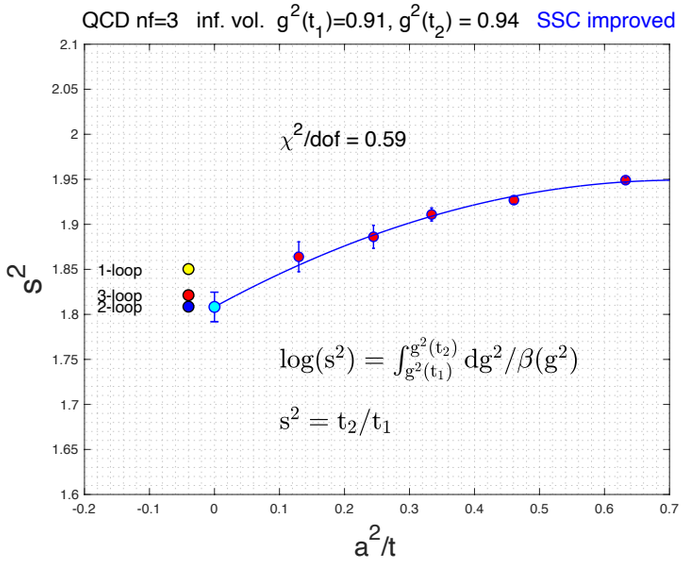
QCD with three massless flavors

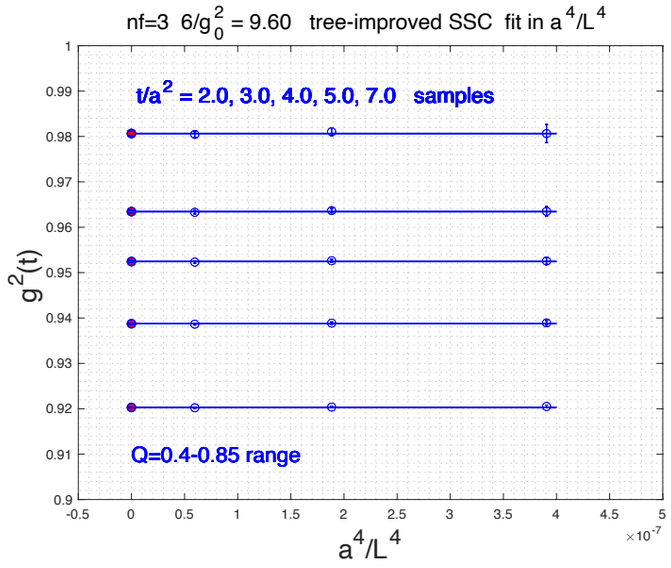
nf=3 infinite volume based step beta-function at weak coupling

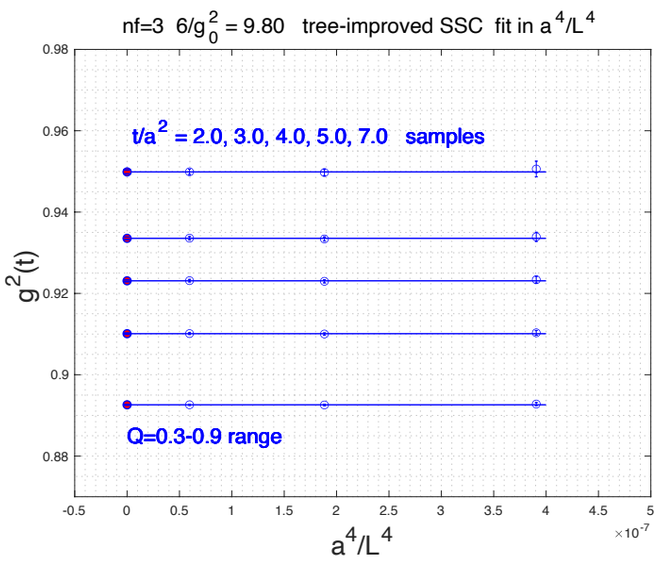
precision contact with Harlander's 3-loop β-function in the continuum limit
needed for precise scale setting of $\Lambda_{\overline{MS}}$

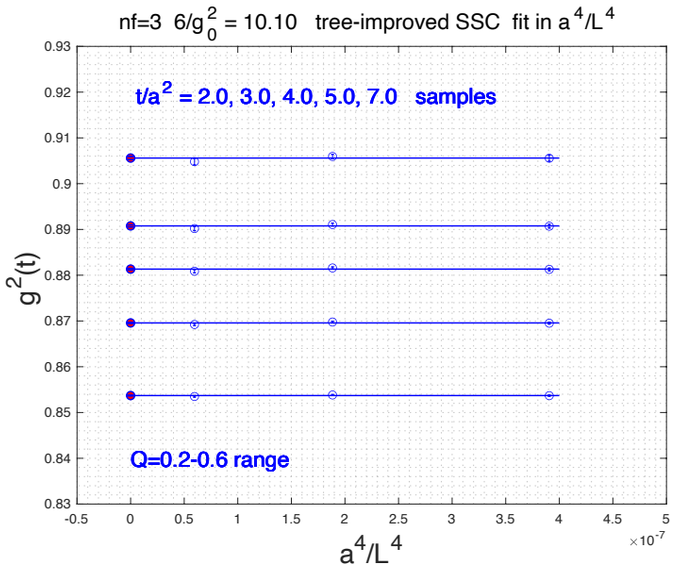












reaching the strong scale is in the works