Dynamical Dark Energy from Lattice Quantum Gravity

Jack Laiho
Mingwei Dai, Walter Freeman, Gwen Hartshaw,
Marc Schiffer, Judah Unmuth-Yockey

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Einstein Hilbert Action

We want to see if we can make sense of the (Euclidean) path integral over the Einstein-Hilbert action using non-perturbative methods.

The hope is to make contact with Weinberg's asymptotic safety scenario, where the theory would be renormalizable nonperturbatively.

Continuum Euclidean path-integral:

$$Z = \int \mathscr{D}g \; e^{-S[g]}, \tag{1}$$

$$S[g_{\mu\nu}] = -\frac{k}{2}\int d^d x \sqrt{\det g}(R - 2\Lambda), \tag{2}$$

where $k = 1/(8\pi G_N)$.

Discrete form of path integral

The formulation of Euclidean Dynamical Triangulations (EDT) is an attempt to discretize this in a form suitable for numerical simulations. The path integral is then

$$Z_E = \sum_T \frac{1}{C_T} \left[\prod_{j=1}^{N_2} \mathscr{O}(t_j)^\beta \right] e^{-S_{ER}}, \tag{3}$$

$$S_{ER} = k \sum 2 V_2 \delta - \lambda \sum V_4, \tag{4}$$

where $\delta = 2\pi - \mathscr{O}(t_j) \arccos(1/4)$ is the deficit angle around a triangular face, V_i is the volume of an i-simplex, and $\lambda = k\Lambda$. The term in (3) in brackets is a local measure term $\prod_x \sqrt{\det g}^\beta$, where we take β as an additional free parameter in the simulations. C_T is a symmetry factor that divides out the number of equivalent ways of labeling the vertices in the triangulation. Must fine-tune β to recover physical geometries.

Main problems to overcome

- Must show recovery of semiclassical physics in 4 dimensions, reproducing general relativity in the appropriate limit.
- Must show existence of continuum limit at continuous phase transition. This would realize asymptotic safety.
- Must make contact with experiment.

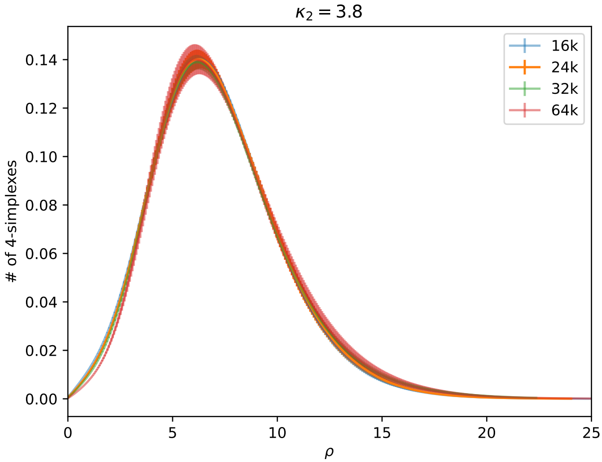
Simulations

Methods for doing these simulations were introduced in the 90's. We wrote new code from scratch.

New rejection free algorithm introduced by Walter Freeman. Big win with a couple orders of magnitude speed-up for fine lattices. Paper in progress. Talk by Marc Shiffer (theoretical developments, previous session).

Three volume distribution

New results at finer lattice spacing. Rescaled curves consistent with Hausdorff dimension 4.



Euclidean de Sitter space

Einstein equations for de Sitter space reduce to:

$$\frac{\dot{a}^2}{a^2} + \frac{1}{a^2} = \frac{\Lambda}{3}, \tag{5}$$

with solution

$$a(t) = \sqrt{\frac{3}{\Lambda}} \cosh\left(\sqrt{\frac{\Lambda}{3}}\, t\right). \tag{6}$$

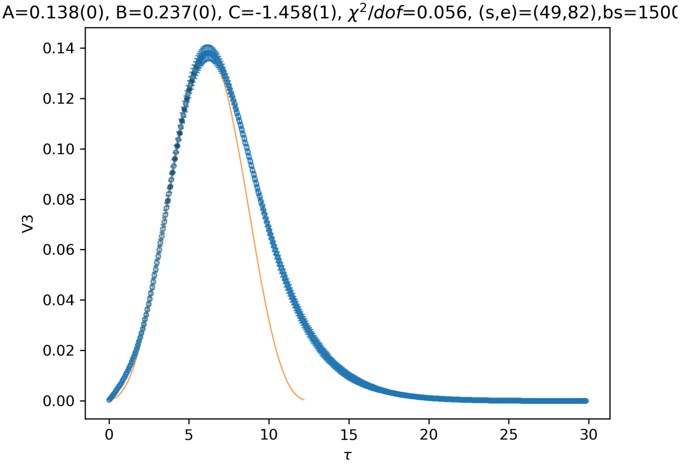
In Euclidean space, this becomes

$$a(\tau) = \sqrt{\frac{3}{\Lambda}} \cos\left(\sqrt{\frac{\Lambda}{3}}\, \tau\right). \tag{7}$$

We adopt the fit function:

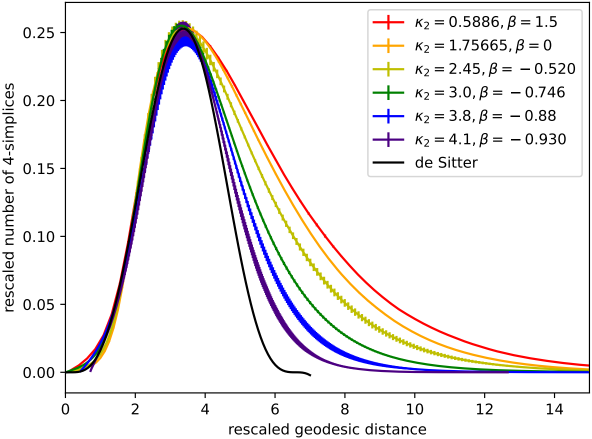
$$N_4^{\mathrm{shell}} = a(\tau)^3 = A \cos^3(B\tau + C), \tag{8}$$

de Sitter fit



$$N_4^{\text{shell}} = A \sin^3(B\tau + C), \tag{9}$$

Approaching the continuum limit

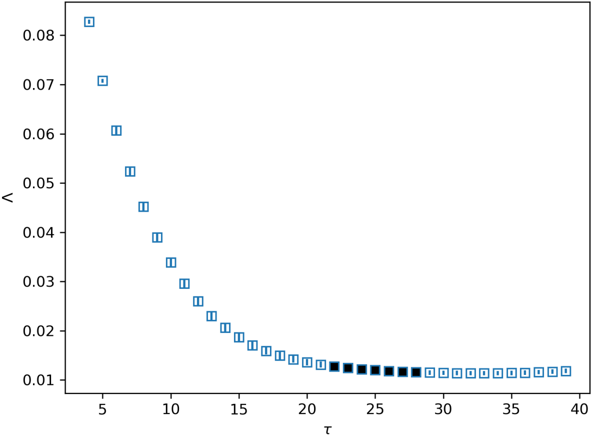


As the lattice spacing becomes finer, the lattice results approach the classical curve.

Dynamics of the vacuum

Euclidean Einstein equation in de Sitter space:

$$-\frac{\dot{a}^2}{a^2} + \frac{1}{a^2} = \frac{\Lambda}{3}, \tag{10}$$



Running Vacuum Model

The cosmological constant evolves as a function of renormalization scale as a power law

$$\Lambda(\mu) = \Lambda_0 + 3\nu|\mu^2|, \tag{11}$$

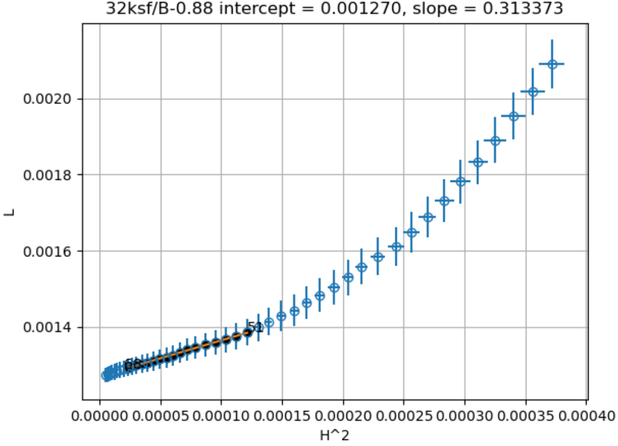
$$\nu(\mu_f^2) = \frac{\nu(\mu_i^2)}{1 + b\log(\frac{\mu_i^2}{\mu_f^2})}, \tag{12}$$

where we associate the renormalization scale μ with the Hubble scale H. $\nu(\mu^2)$ is a dimensionless running coupling.

This is the running vacuum model of Sola and collaborators. See, e.g. arXiv:2301.05205.

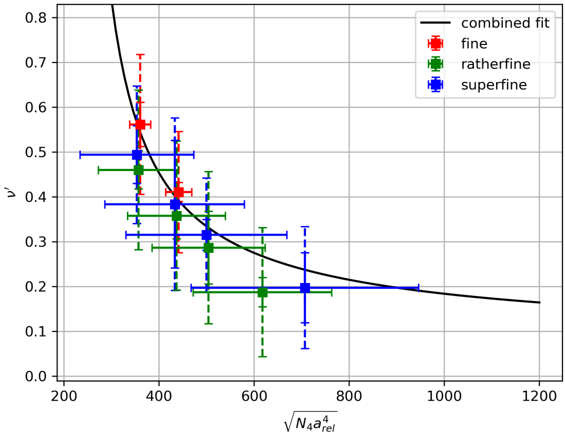
Running Vacuum Model from lattice data

$$\Lambda(H) = \Lambda_0 + 3\nu H^2 \tag{13}$$



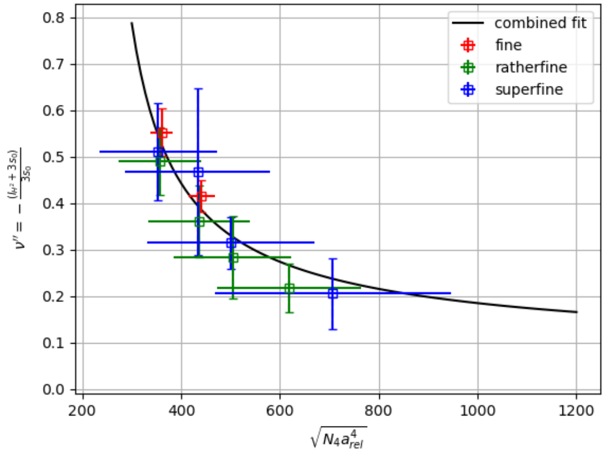
Plots and analysis for running vacuum courtesy of Mingwei Dai. (I neglect the term $3\tilde{\nu}\dot{H}$ since we find it small and consistent with zero.)

Running of ν versus $\sqrt{N_4}$



Coupling is consistent across lattice spacings. Shows scaling! A positive value of ν is consistent with a decaying vacuum. ν run logarithmically to current universe (three orders of magnitude smaller) is roughly consistent with existing bounds.

Alternative determination of ν



Parameter ν can be overconstrained within the model. Can also be determined by a linear combination of intercept in earlier plot and second derivative $\ddot{a}/a$. This is required by covariant energy conservation (a consequence of diffeomorphism invariance). Consistent!

Application to cosmology

The vacuum model

$$\Lambda(H) = \Lambda_0 + 3\nu H^2 \tag{14}$$

cannot be consistent across cosmological history when matter or radiation dominate the universe unless energy is transferred from vacuum to some type of matter or radiation. Vacuum must couple to a matter sector. Likely must be "dark" to not break precision cosmology.

Not many candidates within pure gravity itself. Gravitons would be the most likely decay product. They also happen to be the most viable for phenomenology.

Outlook for cosmology

Dynamical dark energy emerges from lattice gravity simulations. Behaves like “running vacuum model", with free parameters determined from first principles. Appears to be a prediction of quantum gravity not put in by hand.

Leads to observable effects, and given emerging tensions in cosmology, would be a high priority to make predictions in advance of the vast amounts of new observational data that will appear in the coming decade.

If vacuum decays to gravitons (dark radiation!) might be compatible with observations. Decay to Standard Model particles would probably break precision cosmology.

Most likely have to take into account "threshold” effects in the running of Λ. Can be investigated by unquenching the simulations. A code with dynamical fermions exists and is being tested. Stay tuned!