Collins-Soper kernel from lattice QCD at close-to-physical pion mass

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in collaboration with
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arXiv:2307.12359

2023 LATTICE

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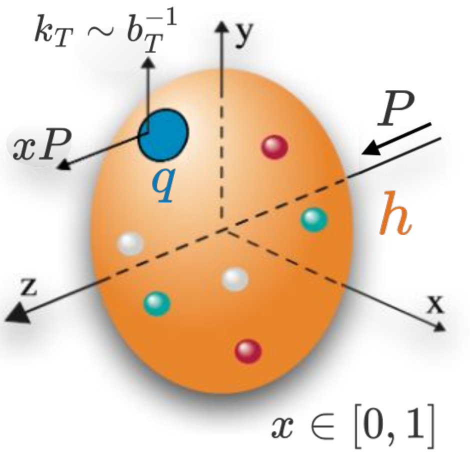
The Collins-Soper (CS) kernel

- Related to TMDs (transverse-momentum-dependent distributions): a generalization of hadronic structure functions, e.g. PDFs:

PDFs $f_{q/h}(x, \mu)$ $\longrightarrow$ TMD PDFs $f_{q/h}(x, b_T, \mu, \zeta)$



Based on Fig. 1.1 in TMD Handbook, 2304.03302

- Describes RG evolution of TMDs along ζ :

$$f_{q/h}(x, b_T, \mu, \zeta) = f_{q/h}(x, b_T, \mu, \zeta_0) \exp\left[\frac{1}{2}\gamma_q(b_T, \mu) \ln \frac{\zeta}{\zeta_0}\right]$$

- $\Rightarrow$ Computed as a ratio of TMDs at different ζ:

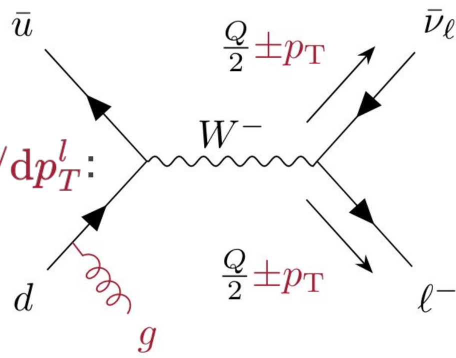
- Independent of hadronic state ($\Rightarrow$ choose pion)
- Non-perturbative at large b_T (for any μ)

$$\gamma_q(b_T, \mu) = \frac{2}{\ln(\zeta_1/\zeta_2)} \ln \frac{f_{q/h}(x, b_T, \mu, \zeta_1)}{f_{q/h}(x, b_T, \mu, \zeta_2)}$$

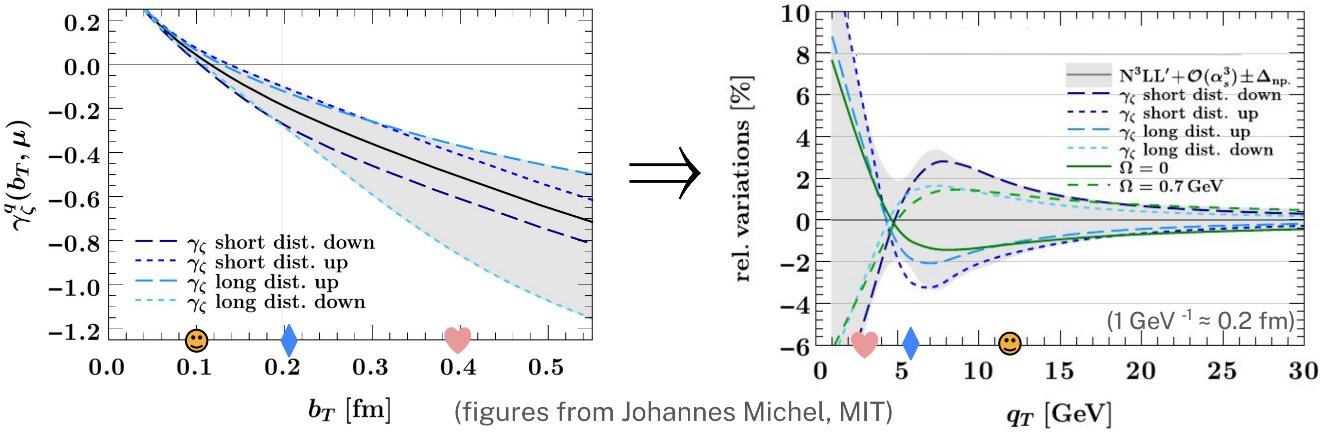
- Encoded by light-like matrix elements

Proportional to hadron momentum P

Example: CS kernel in W mass measurement



- Extract M_W from $p\bar{p} \to W^- \to l^- \nu_l$ via lepton's p_T^l + template fits of $d\sigma/dp_T^l$:
- p_T^l depends on q_T: transverse momentum of the $\bar{u}d$ pair.
- Variations in CS kernel model $\Rightarrow$ % variations in $d\sigma/dq_T$:



(figures from Johannes Michel, MIT)

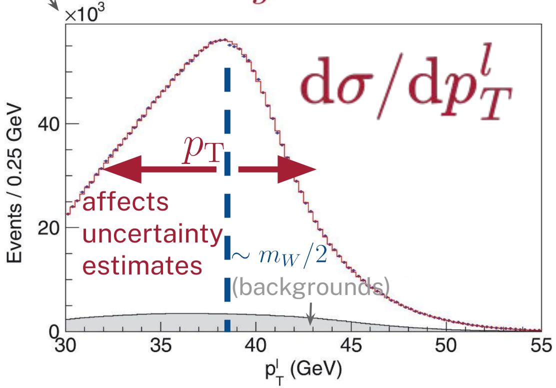
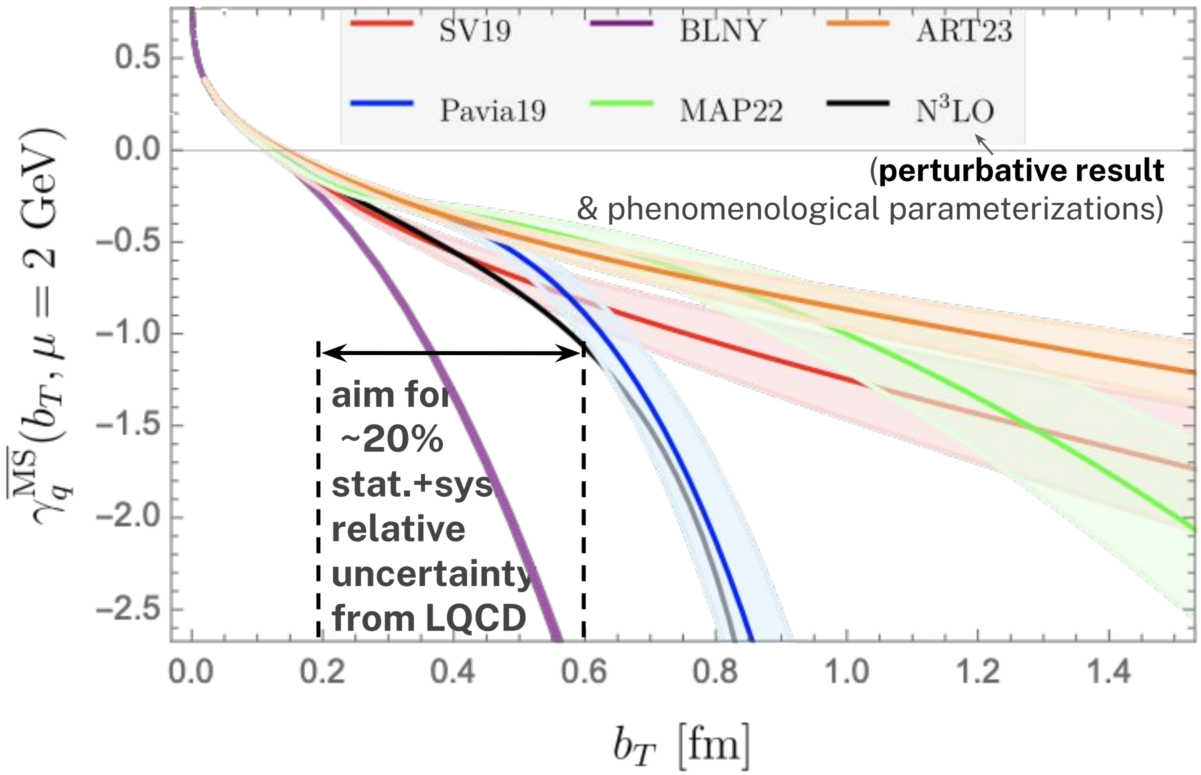


Fig. 4(B) from High-precision measurement of the W boson mass with the CDF II detector, CDF Collaboration *et al.*, *Science* **376**, 170–176 (2022) *(annotated for presentation)*

- Non-perturbative CS kernel affects M_W measurement through the template shape for $d\sigma/dp_T^l$ (but not enough to explain the discrepancy).

Non-perturbative CS kernel

- Consistent for $b_T \lesssim 0.2\,\mathrm{fm}\,(\approx 1\,\mathrm{GeV}^{-1})$
- Non-perturbative modeling significant for $b_T \gtrsim 0.2\,\mathrm{fm}$
- LQCD goal: sufficient precision for direct comparison

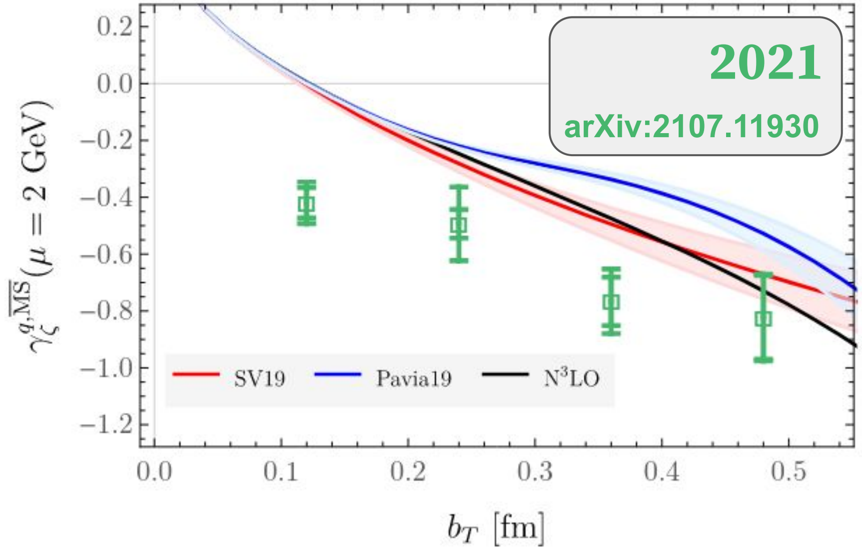


BLNY: F. Landry et. al, PRD 67 (2003), [hep-ph/0212159]
SV19: I. Scimemi and A. Vladimirov, JHEP 06, 137 [1912.06532]
Pavia19: A. Bacchetta et. al, JHEP 07, 117, [1912.07550]
MAP22: A. Bacchetta et. al, JHEP 10, 127, [2206.07598]
ART23: V. Moos et. al, [2305.07473]

Status of our group's calculations of the CS kernel

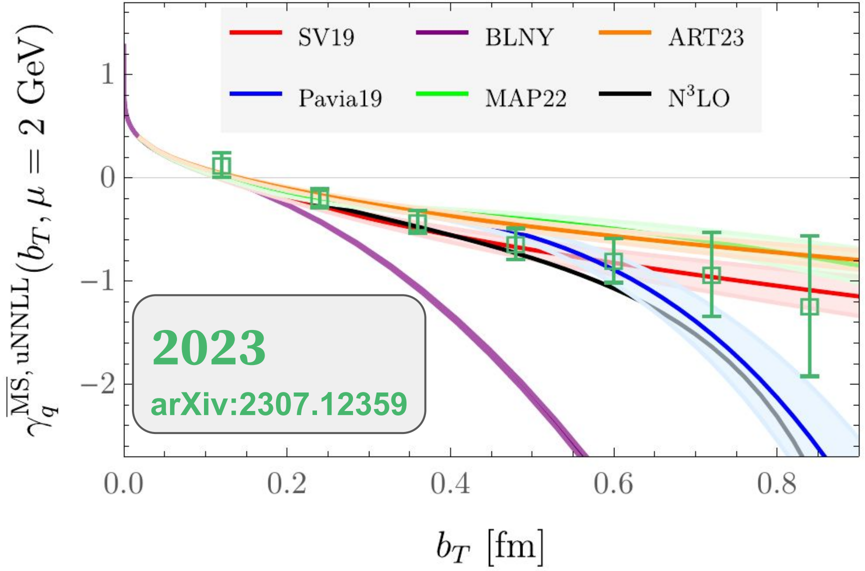
Proof of concept

- $M_\pi \approx 540\,\text{MeV}$
- $0.12\,\text{fm} \leq b_T \leq 0.48\,\text{fm}$
- **NLO** matching.



Improved systematics

- $M_\pi \approx 150\,\text{MeV}$
- $0.12\,\text{fm} \leq b_T \leq 0.86\,\text{fm}$ Increased range
- **NNLL** matching.



CS kernel from LQCD: outline

X. Ji et. al., Phys. Lett. B 811 [1911.03840]

2. Fourier transform

1. Quasi-TMDs in position space

$$\gamma_q(b_T,\mu) = \lim_{a\to 0} \frac{1}{\ln(P_1^z/P_2^z)} \ln \left[\frac{\int db^z e^{ib^z x P_1^z} P_1^z N_\Gamma(P_1^z) \sum_{\Gamma'} Z_{\Gamma\Gamma'}(\mu) \lim_{\ell\to\infty} W^{(0)}_{\Gamma'}(b^z, b_T, \ell, P_1^z)}{\int db^z e^{ib^z x P_2^z} P_2^z N_\Gamma(P_2^z) \sum_{\Gamma'} Z_{\Gamma\Gamma'}(\mu) \lim_{\ell\to\infty} W^{(0)}_{\Gamma'}(b^z, b_T, \ell, P_2^z)} \right] + \delta\gamma_q(\mu, P_1^z, P_2^z) + \mathrm{p.\,c.}$$

4. Power corrections

3. EFT matching

X. Ji et. al, PRD91 (2015);
Ebert et. al, PRD99 (2019), JHEP09 (2019) 037;

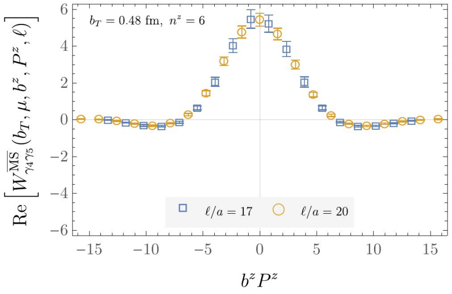
Calculations done on single 2+1+1 ensemble

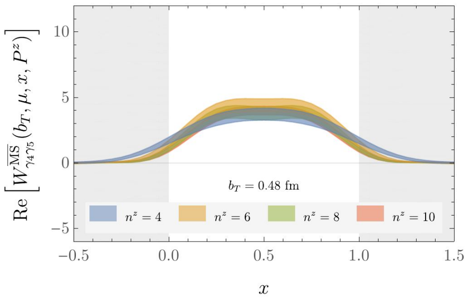
- 48^3 x 64, a = 0.12 fm
- Wilson-flowed
- Clover-on-HISQ

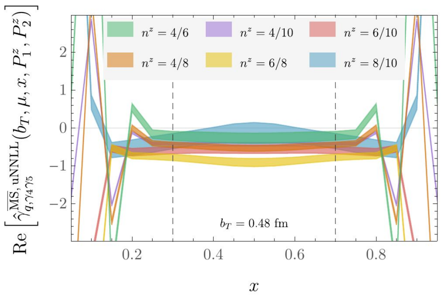
A. Bazavov et al. (MILC), PRD 87 (2013), [1212.4768]

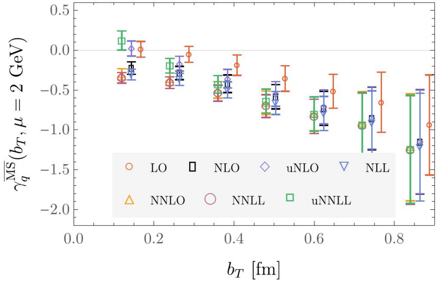
CS kernel from LQCD: outline

position-space MEs → **momentum-space MEs** → **ratios of MEs** → **...repeated for each bT**









$$\sum_{\Gamma'} Z_{\Gamma\Gamma'}(\mu) \lim_{\ell\to\infty} W^{\Gamma'}_{\mathcal{O}}(b^z, b_\mathrm{T}, \ell, P_1^z)$$

$$\int \mathrm{d}b^z e^{ib^z x P_1^z} P_1^z N_\Gamma(P_1^z) \sum_{\Gamma'} Z_{\Gamma\Gamma'}(\mu) \lim_{\ell\to\infty} W^{\Gamma'}_{\mathcal{O}}(b^z, b_\mathrm{T}, \ell, P_1^z)$$

$$\gamma_q(b_T, \mu) = \lim_{a\to 0} \frac{1}{\ln(P_1^z/P_2^z)} \ln \left[\frac{\int \mathrm{d}b^z e^{ib^z x P_1^z} P_1^z N_\Gamma(P_1^z) \sum_{\Gamma'} Z_{\Gamma\Gamma'}(\mu) \lim_{\ell\to\infty} W^{\Gamma'}_{\mathcal{O}}(b^z, b_\mathrm{T}, \ell, P_1^z)}{\int \mathrm{d}b^z e^{ib^z x P_2^z} P_2^z N_\Gamma(P_1^z) \sum_{\Gamma'} Z_{\Gamma\Gamma'}(\mu) \lim_{\ell\to\infty} W^{\Gamma'}_{\mathcal{O}}(b^z, b_\mathrm{T}, \ell, P_2^z)}\right] + \delta\gamma_q(\mu, P_1^z, P_2^z) + \boxed{\mathrm{p.\,c.}}$$

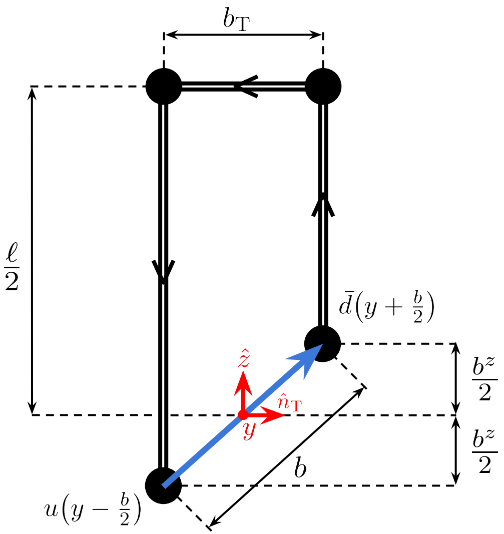
Position-space quasi-TMDs

$$\gamma_q(b_T,\mu) = \lim_{a\to 0} \frac{1}{\ln(P_1^z/P_2^z)} \ln\left[\frac{\int db^z e^{ib^z x P_1^z} P_1^z N_\Gamma(P_1^z) \sum_{\Gamma'} Z_{\Gamma\Gamma'}(\mu) \lim_{\ell\to\infty} W_{\Gamma'}^{(0)}(b^z, b_T, \ell, P_1^z)}{\int db^z e^{ib^z x P_2^z} P_2^z N_\Gamma(P_2^z) \sum_{\Gamma'} Z_{\Gamma\Gamma'}(\mu) \lim_{\ell\to\infty} W_{\Gamma'}^{(0)}(b^z, b_T, \ell, P_2^z)}\right] + \delta\gamma_q(\mu, P_1^z, P_2^z) \boxed{+\,\mathrm{p.c.}}$$

- Compute **quasi-TMD wavefunctions (WFs)**

$$\phi_\Gamma(b_T, b^z, P^z, \ell) = \langle 0 | \mathcal{O}_\Gamma(b_T, b^z, 0, \ell) | \pi(P^z) \rangle$$

- Operators $\mathcal{O}_\Gamma(b_T, b^z, y, \ell)$ with staple-shaped Wilson lines:

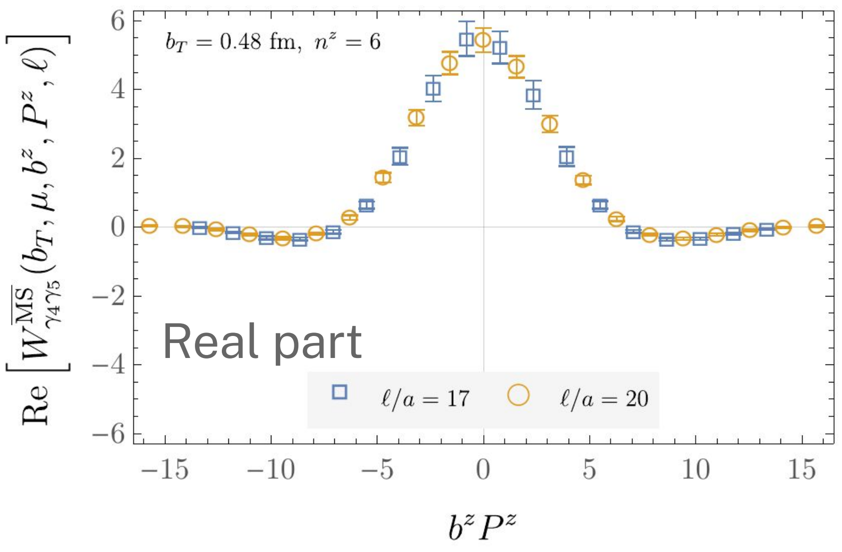


- For each $P^z,\ b_T,\ b^z,\ \ell$ – expensive!

- Matrix elements have divergences $\sim \ell + b_T$

- Subtract divergences in **quasi-TMD WF ratios**

$$W_\Gamma^{(0)}(b_T, b^z, P^z, \ell) = \frac{\tilde{\phi}_\Gamma(b_T, b^z, P^z, \ell)}{\tilde{\phi}_{\gamma^4\gamma^5}(b_T, 0, 0, \ell)}$$



Position-space quasi-TMDs

$$\gamma_q(b_T,\mu) = \lim_{a\to 0} \frac{1}{\ln(P_1^z/P_2^z)} \ln\left[\frac{\boxed{\int db^z e^{ib^z x P_1^z} P_1^z N_\Gamma(P_1^z)} \boxed{\sum_{\Gamma'} Z_{\Gamma\Gamma'}(\mu) \lim_{\ell\to\infty} W^{(0)}_{\Gamma'}(b^z, b_T, \ell, P_1^z)}}{\boxed{\int db^z e^{ib^z x P_2^z} P_2^z N_\Gamma(P_2^z)} \boxed{\sum_{\Gamma'} Z_{\Gamma\Gamma'}(\mu) \lim_{\ell\to\infty} W^{(0)}_{\Gamma'}(b^z, b_T, \ell, P_2^z)}}\right] \boxed{+\delta\gamma_q(\mu, P_1^z, P_2^z)} \boxed{+\,\mathrm{p.c.}}$$

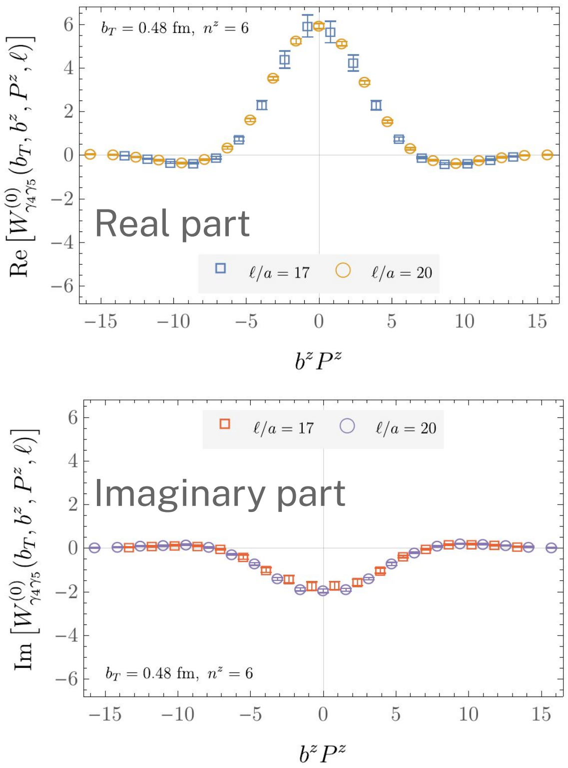
- Mixing effects included via **RIxMOM** scheme (in backup)

$$W_\Gamma^{\overline{\mathrm{MS}}}(b_T,\mu,b^z,P^z,\ell) = \sum_{\Gamma'} Z_{\Gamma\Gamma'}^{\overline{\mathrm{MS}}}(\mu)\, W_\Gamma^{(0)}(b_T,b^z,P^z,\ell)$$

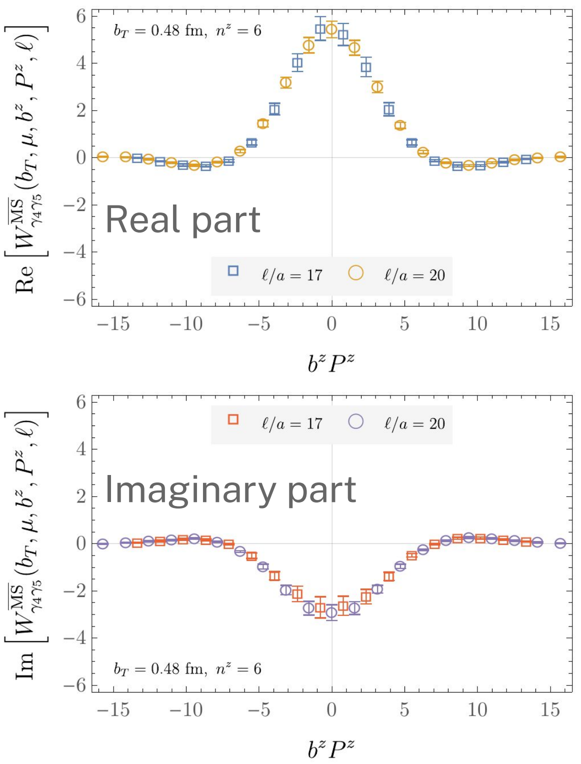
$$\Gamma \in \{\gamma_4\gamma_5, \gamma_3\gamma_5\}$$

- Shown for bT = 0.48 fm, Pz = 1.29 GeV.
- Consistent between different staple lengths ℓ.
- Decay to zero within computed bz ranges.

without mixing



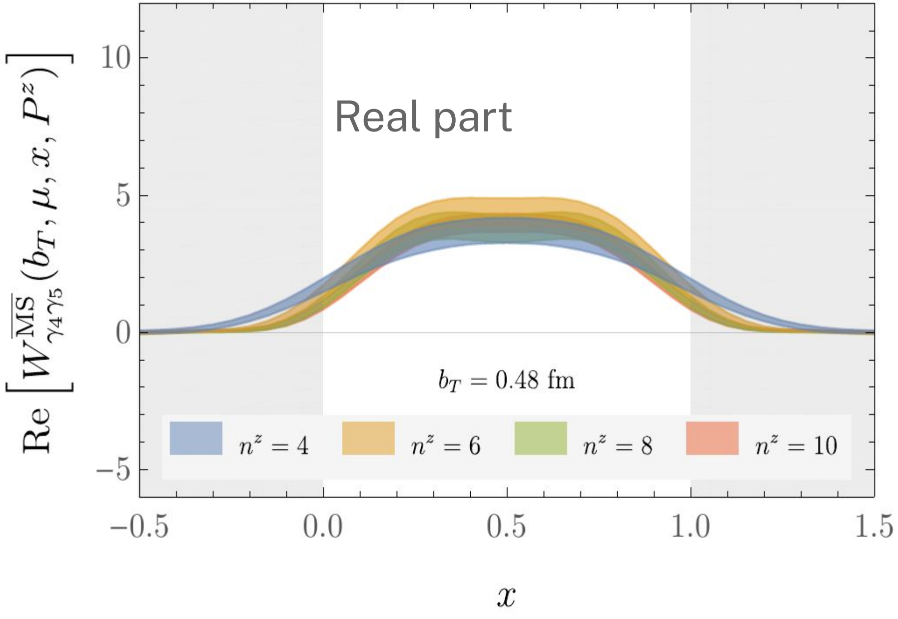
with mixing (via **RIxMOM**)

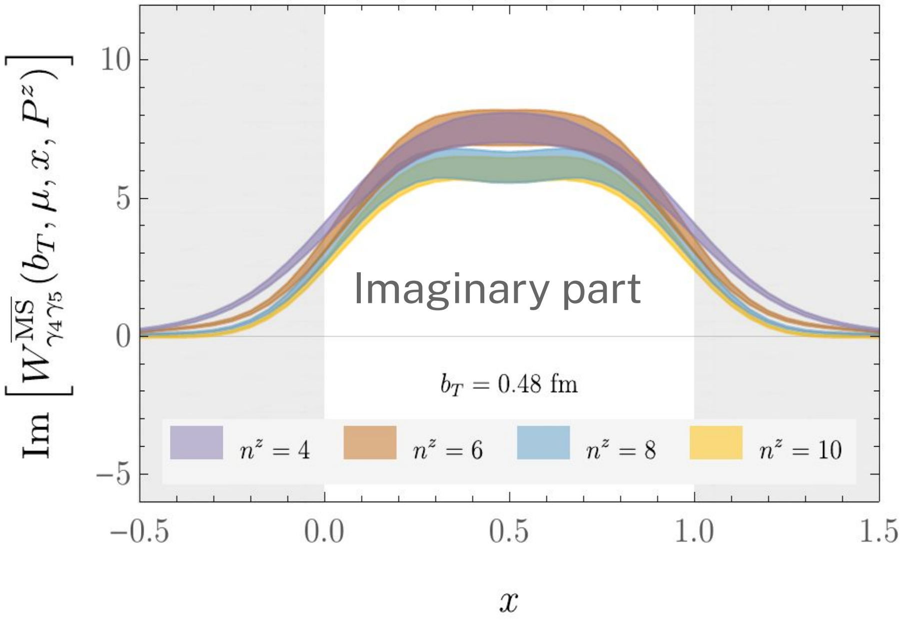


Momentum-space quasi-TMDs

$$\gamma_q(b_T,\mu) = \lim_{a\to 0} \frac{1}{\ln(P_1^z/P_2^z)} \ln\left[\frac{\int \mathrm{d}b^z e^{ib^z x P_1^z} P_1^z N_\Gamma(P_1^z) \sum_{\Gamma'} Z_{\Gamma\Gamma'}(\mu) \lim_{\ell\to\infty} W_{\Gamma'}^{(0)}(b^z, b_\mathrm{T}, \ell, P_1^z)}{\int \mathrm{d}b^z e^{ib^z x P_2^z} P_2^z N_\Gamma(P_2^z) \sum_{\Gamma'} Z_{\Gamma\Gamma'}(\mu) \lim_{\ell\to\infty} W_{\Gamma'}^{(0)}(b^z, b_\mathrm{T}, \ell, P_2^z)}\right] + \delta\gamma_q(\mu, P_1^z, P_2^z) \boxed{+\,\mathrm{p.\,c.}}$$

- Have support outside $x \in [0,1]$, as expected.
- Converge to physical range $x \in [0,1]$ with increasing $P^z = \frac{2\pi}{L} n^z$.





CS kernel estimate

$$\gamma_q(b_T,\mu) = \lim_{a\to 0} \frac{1}{\ln(P_1^z/P_2^z)} \ln\left[\frac{\int \mathrm{d}b^z e^{ib^z x P_1^z} P_1^z N_\Gamma(P_1^z) \sum_{\Gamma'} Z_{\Gamma\Gamma'}(\mu) \lim_{\ell\to\infty} W_{\Gamma'}^{(0)}(b^z, b_\mathrm{T}, \ell, P_1^z)}{\int \mathrm{d}b^z e^{ib^z x P_2^z} P_2^z N_\Gamma(P_2^z) \sum_{\Gamma'} Z_{\Gamma\Gamma'}(\mu) \lim_{\ell\to\infty} W_{\Gamma'}^{(0)}(b^z, b_\mathrm{T}, \ell, P_2^z)}\right] + \delta\gamma_q(\mu, P_1^z, P_2^z) \boxed{+\,\mathrm{p.\,c.}}$$

$$\hat{\gamma}_\Gamma^{\overline{\mathrm{MS}}}(b_T, x, P_1^z, P_2^z, \mu) = \frac{1}{\ln(P_1^z/P_2^z)} \ln\left[\frac{W_\Gamma^{\overline{\mathrm{MS}}}(b_T, x, P_1^z, \ell)}{W_\Gamma^{\overline{\mathrm{MS}}}(b_T, x, P_2^z, \ell)}\right] + \delta\gamma_q^{\overline{\mathrm{MS}}}(x, P_1^z, P_2^z, \mu)$$

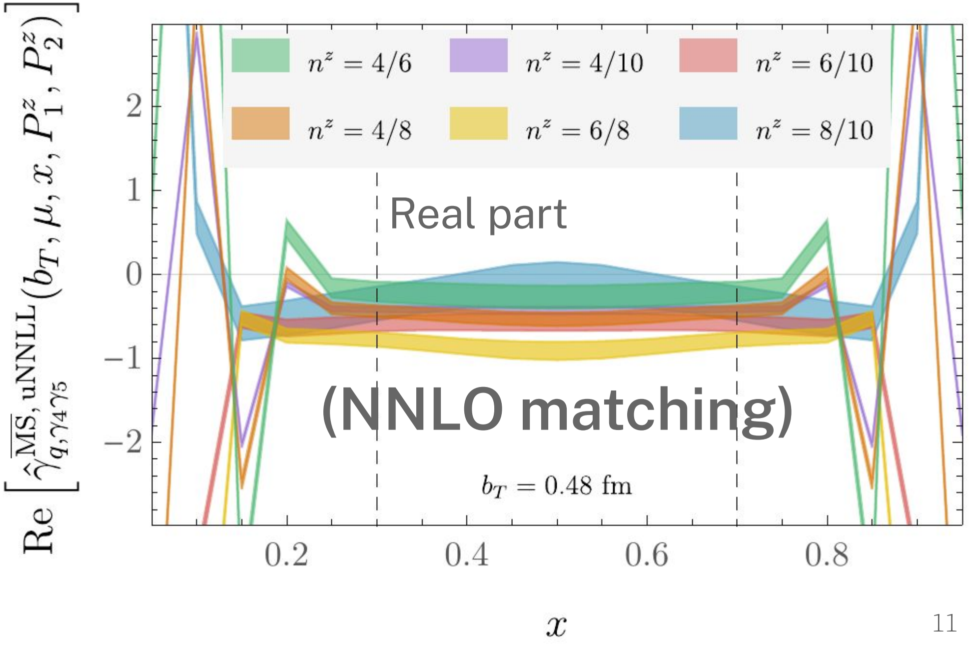
- Separate for each momentum pair, bT, Dirac structure, and matching accuracy.

- Differ by power corrections:

$$\boxed{\mathcal{O}\left(\frac{1}{(xP^z b_T)^2}, \frac{m_\pi^2}{(xP^z)^2}\right) + (x \leftrightarrow 1-x)}$$

P^z-dependent ⇒ cannot disentangle from O(a) effects at finite a.

- Fit each estimator separately to a constant in $x \in [0.3, 0.7]$, then average fits at fixed bT and matching accuracy.

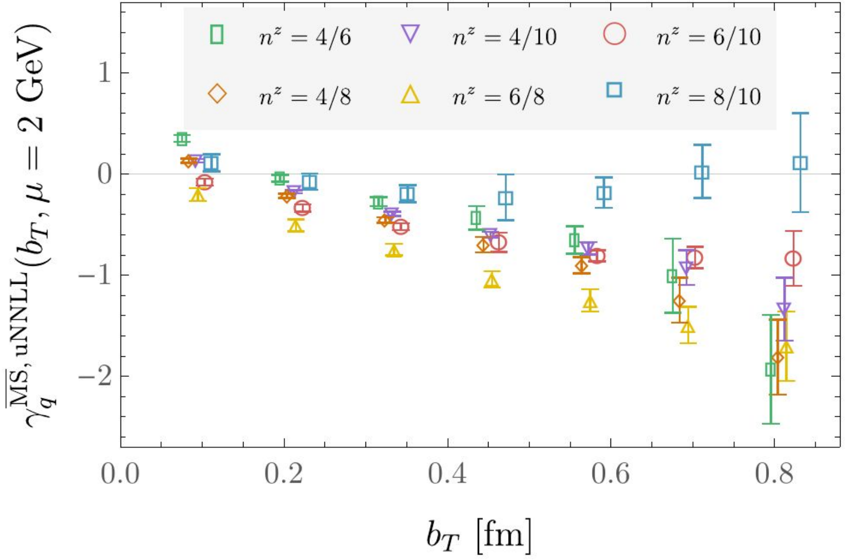


X. Ji et. al., Phys. Lett. B 811 [1911.03840]
X. Ji and Y. Liu, PRD 105, [2106.05310]
Z.-F. Deng et. al, JHEP 09, [2207.07280]

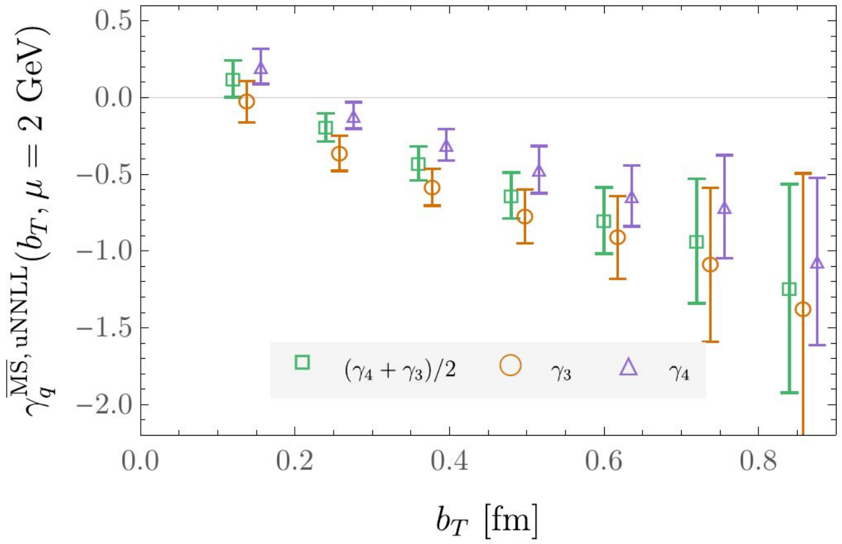
CS kernel estimate

$$\gamma_q(b_T,\mu) = \lim_{a\to 0}\frac{1}{\ln(P_1^z/P_2^z)}\ln\left[\frac{\int db^z e^{ib^z x P_1^z} P_1^z N_\Gamma(P_1^z) \sum_{\Gamma'} Z_{\Gamma\Gamma'}(\mu)\lim_{\ell\to\infty} W_{\Gamma'}^{(0)}(b^z,b_\mathrm{T},\ell,P_1^z)}{\int db^z e^{ib^z x P_2^z} P_2^z N_\Gamma(P_2^z) \sum_{\Gamma'} Z_{\Gamma\Gamma'}(\mu)\lim_{\ell\to\infty} W_{\Gamma'}^{(0)}(b^z,b_\mathrm{T},\ell,P_2^z)}\right] + \delta\gamma_q(\mu,P_1^z,P_2^z) \boxed{+\,\mathrm{p.\,c.}}$$

Before averaging over momentum pairs:



Before averaging over Dirac structures:



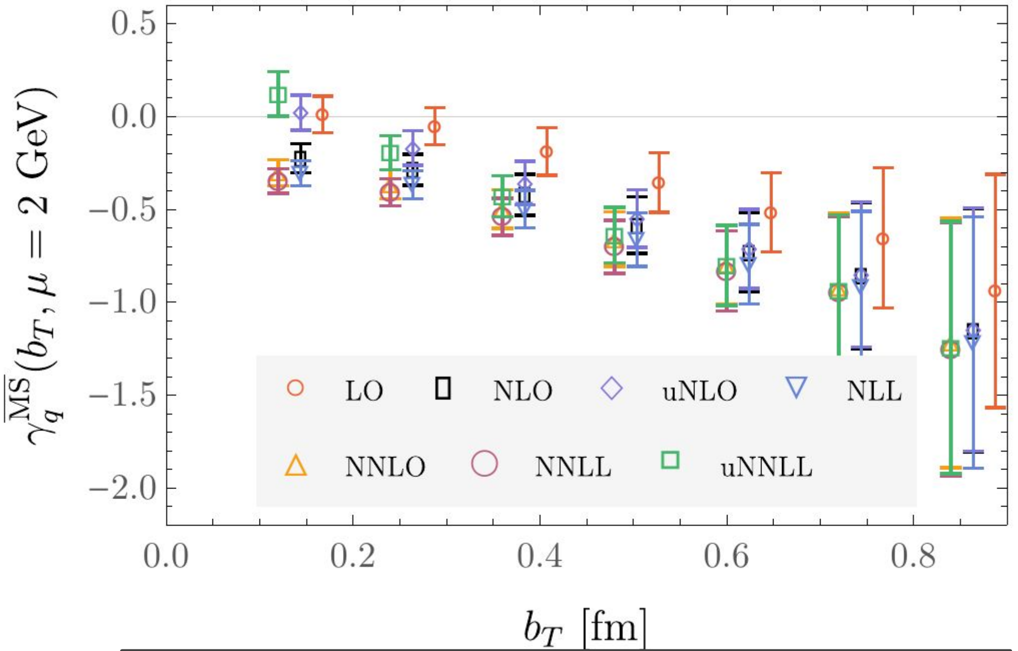
Matching corrections

$$\gamma_q(b_T,\mu) = \lim_{a\to 0} \frac{1}{\ln(P_1^z/P_2^z)} \ln \left[\frac{\int \mathrm{d}b^z e^{ib^z x P_1^z} P_1^z N_\Gamma(P_1^z) \sum_{\Gamma'} Z_{\Gamma\Gamma'}(\mu) \lim_{\ell\to\infty} W_{\Gamma'}^{(0)}(b^z, b_\mathrm{T}, \ell, P_1^z)}{\int \mathrm{d}b^z e^{ib^z x P_2^z} P_2^z N_\Gamma(P_2^z) \sum_{\Gamma'} Z_{\Gamma\Gamma'}(\mu) \lim_{\ell\to\infty} W_{\Gamma'}^{(0)}(b^z, b_\mathrm{T}, \ell, P_2^z)} \right] + \delta\gamma_q(\mu, P_1^z, P_2^z) \boxed{+\,\mathrm{p.\,c.}}$$

- New results at **NNLO** and **NNLL**.
  O. del Río and A. Vladimirov, [2304.14440], and X. Ji et. al, [2305.04416].
- $b_T \gtrsim 0.36\,\mathrm{fm}$: consistent between matching corrections above LO.
- $b_T \lesssim 0.36\,\mathrm{fm}$: deviations related to power corrections: $\boxed{\mathcal{O}\left(\frac{1}{(P^z b_T)^2}\right)}$
- Power corrections reduced by **uNLO**:

$$\delta\gamma_q^{\mathbf{NLO}}(\mu, P_1^z, P_2^z) + \mathrm{p.\,c.}$$
$$= \delta\gamma_q^{\mathbf{uNLO}}(b_T, \mu, P_1^z, P_2^z) + \mathrm{p.\,c.}'$$

incorporates some of the bT-dependent power corrections in $\mathrm{p.\,c.}$ (more in backup)



Final determination:

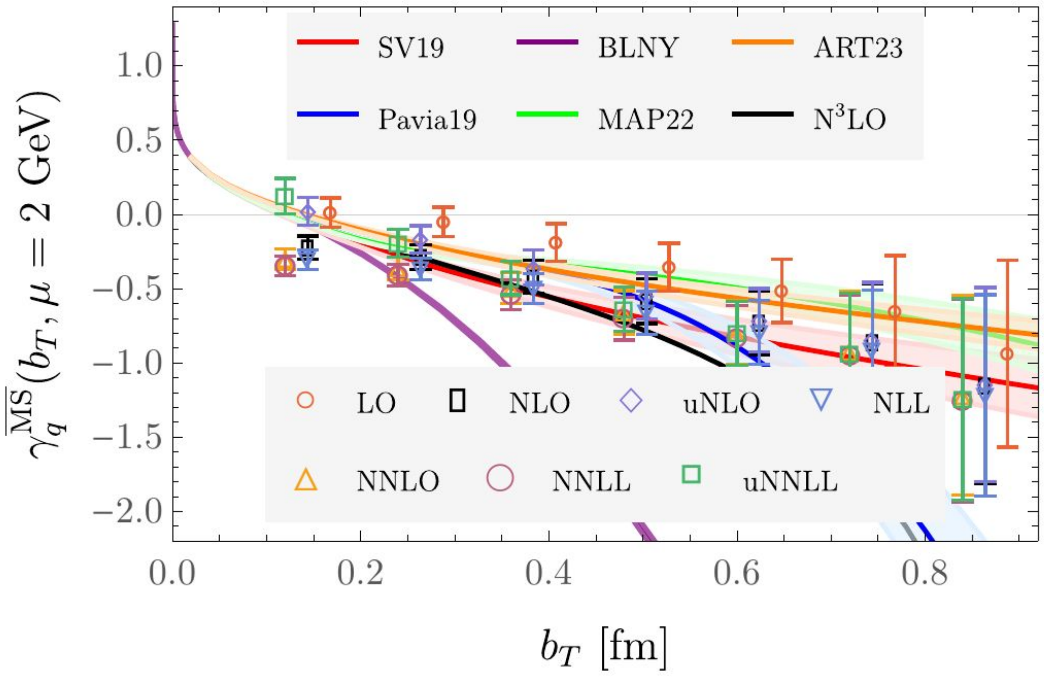
uNNLL = **uNLO** + resumption

Artur Avkhadiev, MIT

Conclusion and outlook

$$\gamma_q(b_T,\mu) = \lim_{a\to 0} \frac{1}{\ln(P_1^z/P_2^z)} \ln\left[\frac{\int db^z e^{ib^z x P_1^z} P_1^z N_\Gamma(P_1^z) \sum_{\Gamma'} Z_{\Gamma\Gamma'}(\mu) \lim_{\ell\to\infty} W_{\Gamma'}^{(0)}(b^z, b_T, \ell, P_1^z)}{\int db^z e^{ib^z x P_2^z} P_2^z N_\Gamma(P_2^z) \sum_{\Gamma'} Z_{\Gamma\Gamma'}(\mu) \lim_{\ell\to\infty} W_{\Gamma'}^{(0)}(b^z, b_T, \ell, P_2^z)}\right] + \delta\gamma_q(\mu, P_1^z, P_2^z) + \mathrm{p.c.}$$

- First calculation at ~physical pion mass and **NNLO** + **NNLL** matching.
- Can begin to discriminate between phenomenological parameterizations.
- Perturbative convergence for bT > .36 fm.
- Power corrections for bT < .36 fm accounted by **uNLO**, **uNNLL**.
- **Significant progress from the 2021 calculation.**
- Next steps: better quantify power corrections by disentangling O(a) effects at multiple lattice spacings.

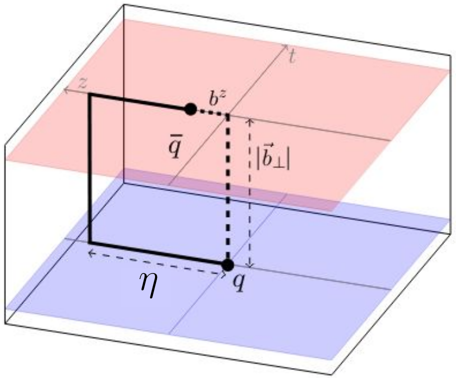


Backup slides

CS Kernel from Lattice QCD

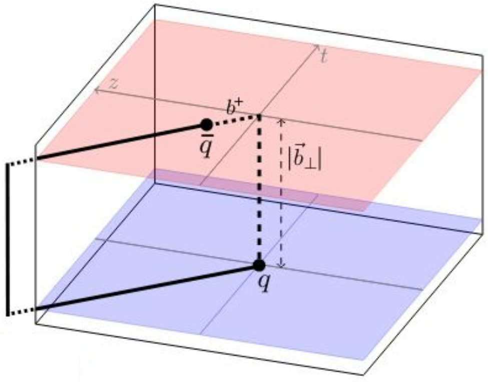
- CS kernel defined through ratios of **light-like MEs** of staple-shaped operators.

- Corresponding **space-like MEs** computed in LQCD, then matched onto the **light-like MEs** via Large-Momentum Effective Theory (LaMET).



Lorentz boost and $\eta \to \infty$

$$\boxed{\frac{1}{P} \ll b_T \ll \eta}$$



Ji, Sun, Xiong and Yuan, PRD91 (2015);
Ji, Jin, Yuan, Zhang and Zhao, PRD99 (2019);
Ebert, Stewart, Zhao, PRD99 (2019), JHEP09 (2019) 037;
Ji, Liu and Liu, NPB 955 (2020), PLB 811 (2020);
Vladimirov and Schäfer, PRD 101 (2020);
Ebert, Schindler, Stewart and Zhao, JHEP04 (2022) 178.

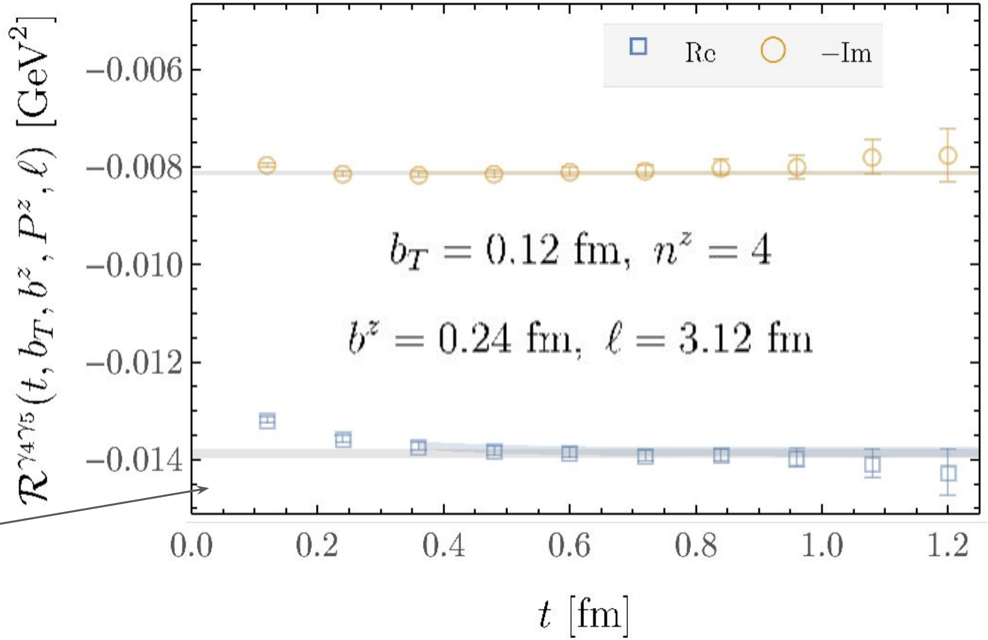
Fig. by Ebert, Stewart, Zhao, JHEP 1909 (2019) (notation changed).

Unsubtracted quasi-TMD WFs: examples

- Extracted from correlation functions

$$\sum_{\mathbf{y}} e^{i\mathbf{P}\cdot\mathbf{y}} \left\langle \mathcal{O}_\Gamma(b_T, b^z, y, \ell)\chi^\dagger_{\mathbf{P}}(0)\right\rangle \xrightarrow{t\gg 0} \frac{Z^S_\pi(\mathbf{P})}{2E_\pi(\mathbf{P})}\tilde{\phi}_\Gamma(b_T, b^z, \mathbf{P}, \ell)e^{-E_\pi(\mathbf{P})t}$$

- Momentum-smeared interpolators $\chi^\dagger_{\mathbf{P}}$
- $E_\pi(\mathbf{P})$ and $Z^S_\pi(\mathbf{P})$ fit and cancelled in ratios $\mathcal{R}^\Gamma(t, b_T, b^z, P^z, \ell)$:
- Plateau gives $\tilde{\phi}_\Gamma(b_T, b^z, \mathbf{P}, \ell)$.
- Repeated for each P^z, b_T, b^z, ℓ.



- A range of time windows chosen systematically
- Covariance matrix from bootstrap + linear shrinkage
- Correlated determinations between staple geometries
- AIC-preferred fits (1 + 2 state)
- Further selection cuts + combine in weighted average

Mixing effects quantified with RIxMOM

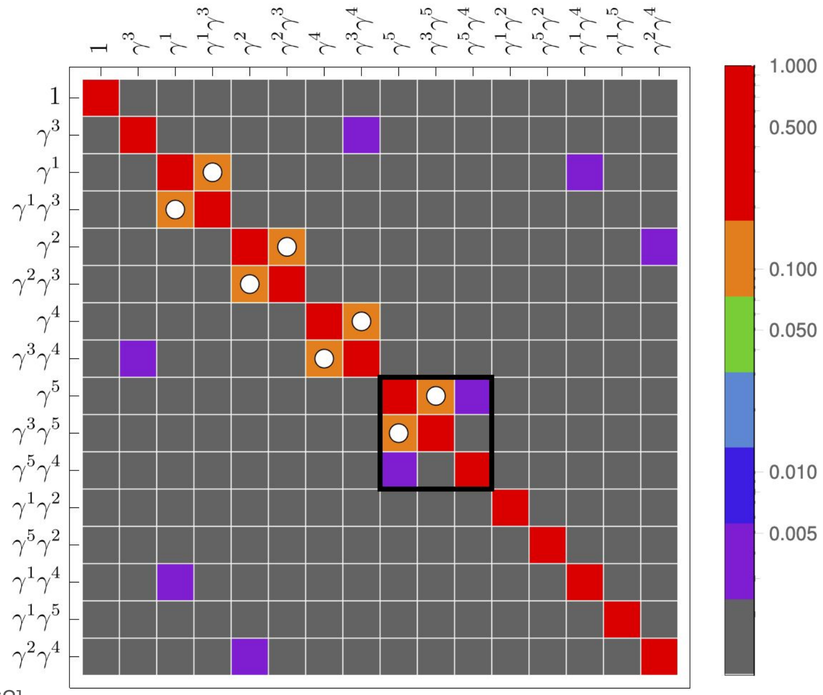
- Calculation of mixing effects in RIxMOM independent of staple geometry.

$$W_\Gamma^{\overline{\mathrm{MS}}}(b_T,\mu,b^z,P^z,\ell)=\sum_{\Gamma'} Z_{\Gamma\Gamma'}^{\overline{\mathrm{MS}}}(\mu)\,W_{\Gamma}^{(0)}(b_T,b^z,P^z,\ell)$$

- Full 16x16 mixing matrix computed

$$\mathcal{M}_{\Gamma\Gamma'}^{\mathrm{RI/xMOM}}(p_{\mathrm{R}},\,\xi_{\mathrm{R}},a) \equiv \frac{\mathrm{Abs}[Z_{\Gamma\Gamma'}^{\mathrm{RI/xMOM}}(p_{\mathrm{R}},\xi_{\mathrm{R}},a)]}{\frac{1}{16}\sum_\Gamma \mathrm{Abs}[Z_{\Gamma\Gamma}^{\mathrm{RI/xMOM}}(p_{\mathrm{R}},\xi_{\mathrm{R}},a)]}$$

- Dominant mixings consistent with lattice perturbation theory at 1-loop.*



$$p_{\mathrm{R}}^{\mu}=\frac{2\pi}{L}\times(0,0,10,0),\ \xi=0.24\ \mathrm{fm}$$

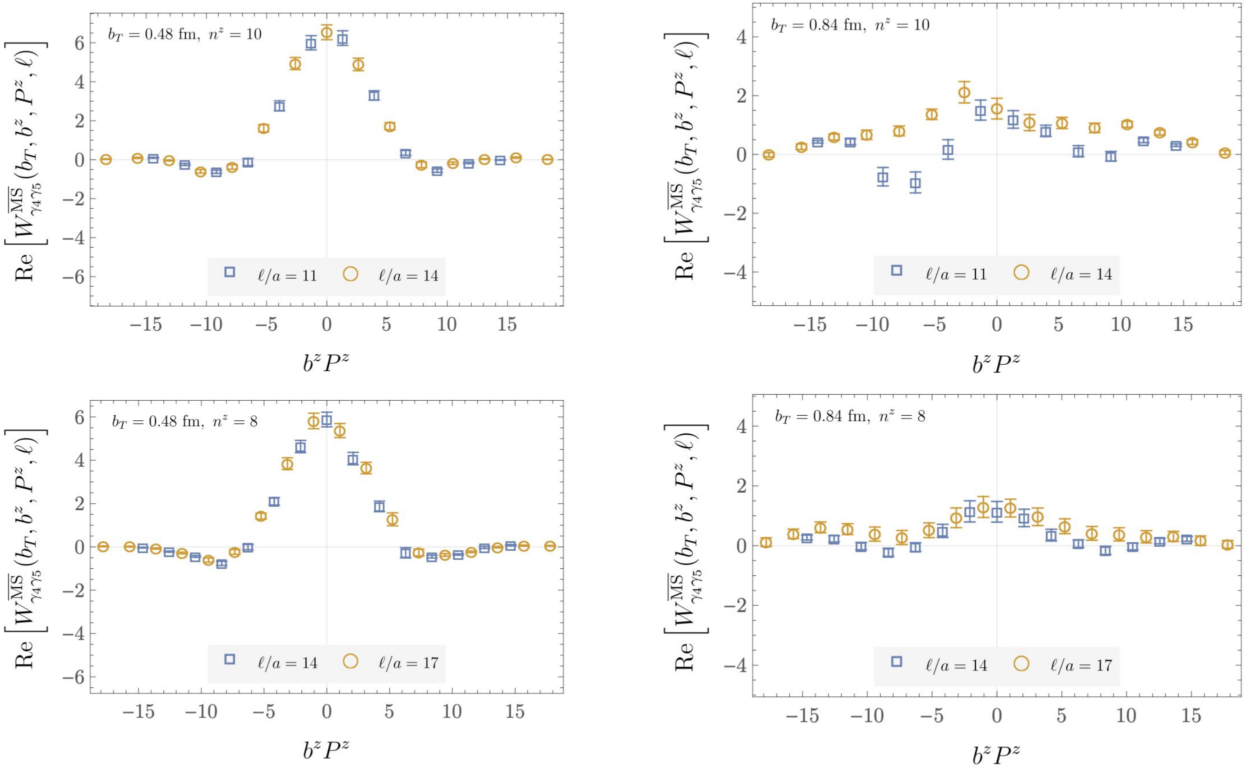
X. Ji, et. al, PRL 120 (2018), [1706.08962]
J. Green et. al, PRL 121 (2018), [1707.07152]
J. Green et. al, PRD 101 (2020), [2002.09408]
*M. Constantinou et al., PRD 99 (2019), [1901.03862]
Y. Ji et. al., PRD 104 (2021), [2104.13345]
C. Alexandrou et al., [2305.11824]

TMD WFs in position space

$$\gamma_q(\mu, b_{\mathrm{T}}) = \boxed{\lim_{a\to 0}} \frac{1}{\ln(P_1^z/P_2^z)} \ln \left[\frac{\int \mathrm{d}b^z e^{ib^z x P_1^z} P_1^z \sum_{\Gamma'} Z_{\Gamma\Gamma'}(\mu, a) \lim_{\ell\to\infty} W_{\mathcal{O}}^{\Gamma'}(b^z, b_{\mathrm{T}}, \ell, P_1^z, a)}{\int \mathrm{d}b^z e^{ib^z x P_2^z} P_2^z \sum_{\Gamma'} Z_{\Gamma\Gamma'}(\mu, a) \lim_{\ell\to\infty} W_{\mathcal{O}}^{\Gamma'}(b^z, b_{\mathrm{T}}, \ell, P_2^z, a)} \right] + \delta\gamma_q(\mu, b_{\mathrm{T}}, P_1^z, P_2^z) + \boxed{\mathcal{O}\left(\frac{1}{(xP^z b_{\mathrm{T}})^2}, \frac{M^2}{(xP^z)^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{(xP^z)^2}\right)}$$

Statistical noise makes computation challenging for large $P^z, \ell,$ and b_T

- $b_T = 0.48\,\mathrm{fm},\ 0.84\,\mathrm{fm}$ left to right
- $P^z = 2.15\,\mathrm{GeV},\ 1.72\,\mathrm{GeV}$ top to bottom
- Our group's previous calculation had $b_T^{\max} = 0.48\,\mathrm{fm}$, $P^z_{\max} = 1.51\,\mathrm{GeV}$



TMD WFs in momentum space

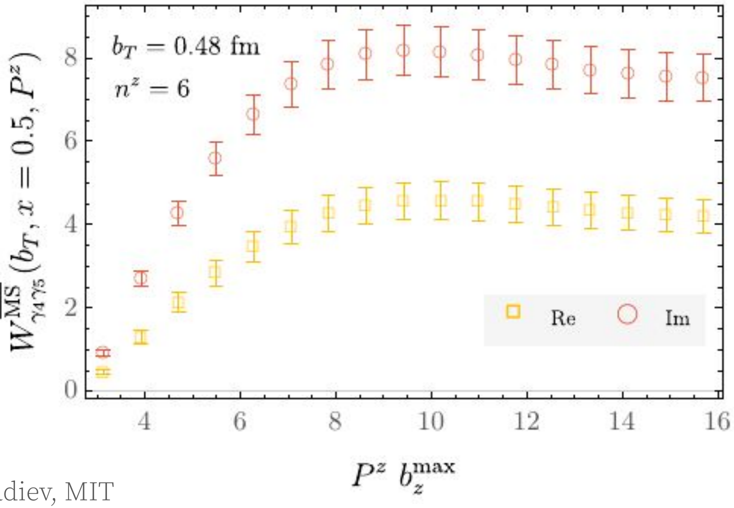
$$\gamma_q(\mu, b_T) = \boxed{\lim_{a\to 0}} \frac{1}{\ln(P_1^z/P_2^z)} \ln\left[\frac{\int db^z e^{ib^z x P_1^z} P_1^z \sum_{\Gamma'} Z_{\Gamma\Gamma'}(\mu,a) \lim_{\ell\to\infty} W_{\mathcal{O}}^{\Gamma'}(b^z, b_T, \ell, P_1^z, a)}{\int db^z e^{ib^z x P_2^z} P_2^z \sum_{\Gamma'} Z_{\Gamma\Gamma'}(\mu,a) \lim_{\ell\to\infty} W_{\mathcal{O}}^{\Gamma'}(b^z, b_T, \ell, P_2^z, a)}\right] + \delta\gamma_q(\mu, b_T, P_1^z, P_2^z) + \boxed{\mathcal{O}\left(\frac{1}{(xP^z b_T)^2}, \frac{M^2}{(xP^z)^2}, \frac{\Lambda_{QCD}^2}{(xP^z)^2}\right)}$$

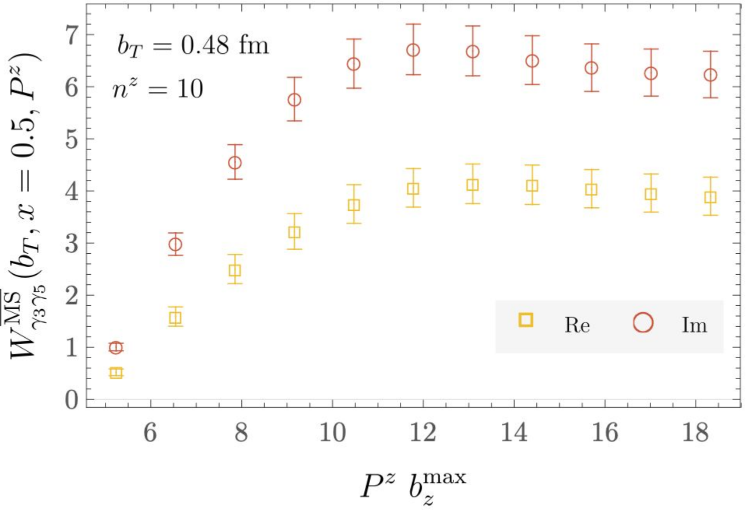
bz range sufficient to use a Discrete Fourier Transform

$$\bar{W}_\Gamma^{\overline{MS}}(b_T, \mu, x, P^z) = \frac{P^z}{2\pi} N_\Gamma(P) \sum_{|b_z| \le b_z^{max}} e^{i\left(x - \frac{1}{2}\right)P^z b^z} \bar{W}_\Gamma^{\overline{MS}}(b_T, \mu, b^z, P^z)$$

Normalization factor to compare between Dirac structures

The DFT is stable to decreasing the range in b_T^{max}:

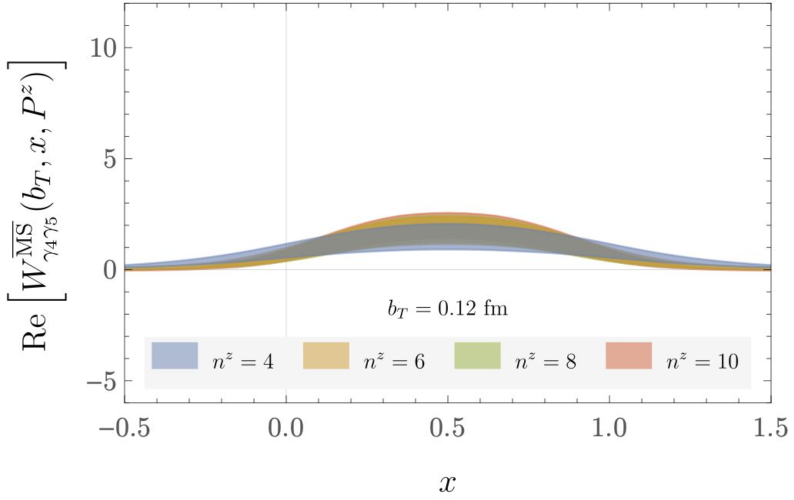


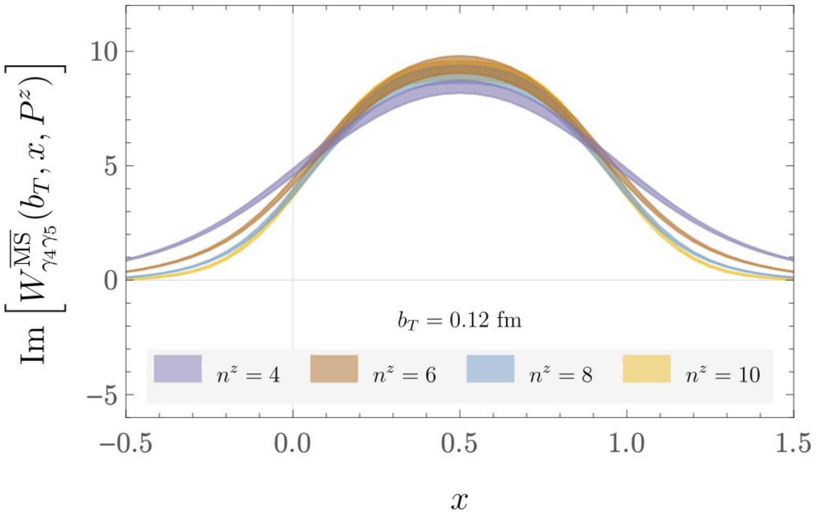


TMD WFs in momentum space

$$\gamma_q(\mu, b_T) = \boxed{\lim_{a\to 0}} \frac{1}{\ln(P_1^z/P_2^z)} \ln \left[\frac{\boxed{\int db^z e^{ib^z x P_1^z} P_1^z} \boxed{\sum_{\Gamma'} Z_{\Gamma\Gamma'}(\mu, a) \lim_{\ell\to\infty} W_{\mathcal{O}}^{\Gamma'}(b^z, b_T, \ell, P_1^z, a)}}{\boxed{\int db^z e^{ib^z x P_2^z} P_2^z} \boxed{\sum_{\Gamma'} Z_{\Gamma\Gamma'}(\mu, a) \lim_{\ell\to\infty} W_{\mathcal{O}}^{\Gamma'}(b^z, b_T, \ell, P_2^z, a)}} \right] \boxed{+\delta\gamma_q(\mu, b_T, P_1^z, P_2^z)} + \boxed{\mathcal{O}\left(\frac{1}{(xP^z b_T)^2}, \frac{M^2}{(xP^z)^2}, \frac{\Lambda_{QCD}^2}{(xP^z)^2} \right)}$$

See convergence to the physical range $x \in [0, 1]$ with increasing $P^z = \frac{2\pi}{L} n^z$

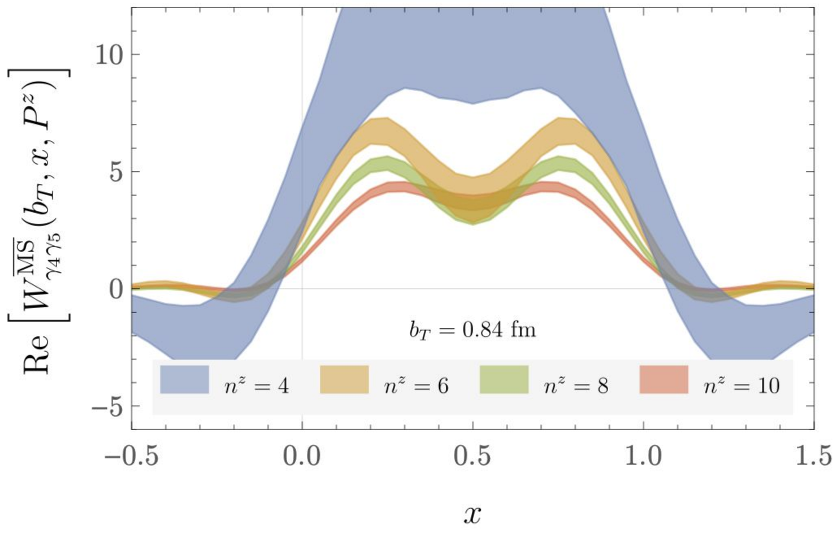


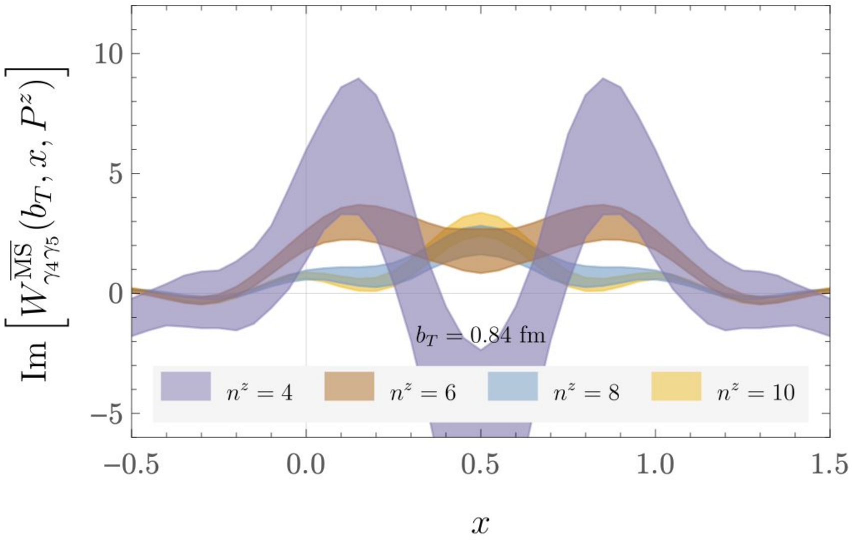


TMD WFs in momentum space

$$\gamma_q(\mu, b_T) = \boxed{\lim_{a\to 0}} \frac{1}{\ln(P_1^z/P_2^z)} \ln \left[\frac{\boxed{\int db^z e^{ib^z x P_1^z} P_1^z} \boxed{\sum_{\Gamma'} Z_{\Gamma\Gamma'}(\mu, a) \lim_{\ell\to\infty} W_{\mathcal{O}}^{\Gamma'}(b^z, b_T, \ell, P_1^z, a)}}{\boxed{\int db^z e^{ib^z x P_2^z} P_2^z} \boxed{\sum_{\Gamma'} Z_{\Gamma\Gamma'}(\mu, a) \lim_{\ell\to\infty} W_{\mathcal{O}}^{\Gamma'}(b^z, b_T, \ell, P_2^z, a)}} \right]$$
$$\boxed{+\delta\gamma_q(\mu, b_T, P_1^z, P_2^z)} + \boxed{\mathcal{O}\left(\frac{1}{(xP^z b_T)^2}, \frac{M^2}{(xP^z)^2}, \frac{\Lambda_{QCD}^2}{(xP^z)^2} \right)}$$

See convergence to the physical range $x \in [0,1]$ with increasing $P^z = \frac{2\pi}{L} n^z$





NLO, NNLO, and resummations

The correction is given by coefficients

$$\delta\gamma_q(x, P_1^z, P_2^z, \mu) \equiv \frac{1}{\ln(P_1^z/P_2^z)}\left(\ln\frac{C_\phi(xP_2^z,\mu)}{C_\phi(xP_1^z,\mu)} + (x \leftrightarrow \bar{x})\right)$$

$C_\phi(p^z,\mu)$ appear in the TMD WF matching formula and are computed perturbatively as

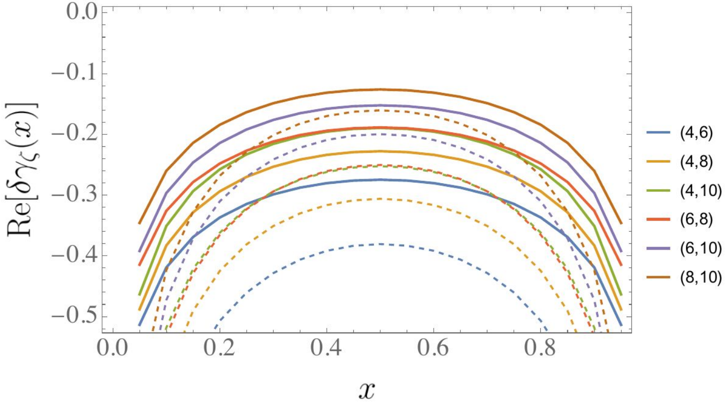
$$C_\phi(p^z,\mu) = 1 + \sum_{n=1}\left(\frac{\alpha_s(\mu)}{4\pi}\right)^n C_\phi^{(n)}(p^z,\mu)$$

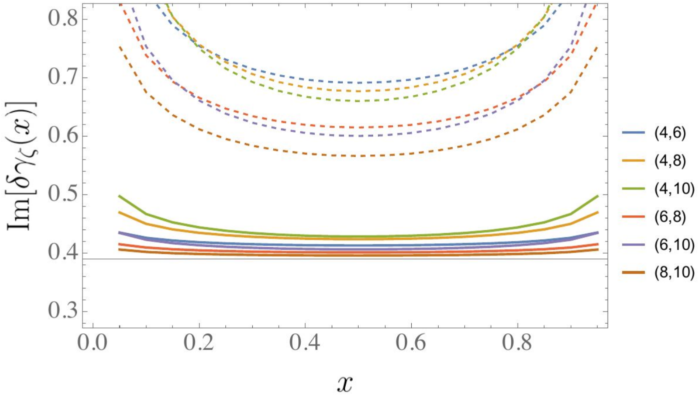
at LO, NLO and recently at NNLO, and resummed as

O. del Río and A. Vladimirov, [2304.14440]
X. Ji et. al, [2305.04416]

Resummation kernel

$$C_\phi(p^z,\mu) = C_\phi(p^z, 2p^z) \times \exp[K_\phi(p^z, 2p^z)]$$





NLO (solid) and NNLO (dashed);
No convergence in the imaginary part

NLL and NNLL

X. Ji et. al., Phys. Lett. B 811 [1911.03840]
Ebert et. al, JHEP 04 (2022), [2201.08401]

Resummation kernel is $K_\phi(2p^z,\mu) = 2K_\Gamma(2p^z,\mu) - K_{\gamma_\mu}(2p^z,\mu) - i\pi\eta(2p^z,\mu)$

$$K_{\gamma_\mu}(\mu_0,\mu) = \int_{\alpha_s(\mu_0)}^{\alpha_s(\mu)} \frac{\mathrm{d}\alpha_s}{\beta(\alpha_s)}\gamma_\mu(\alpha_s),$$

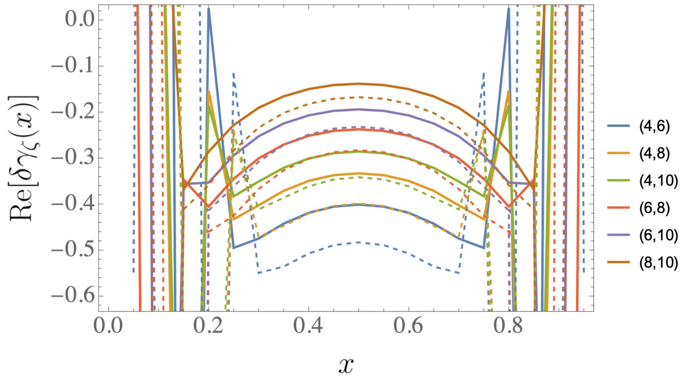
$$K_\Gamma(\mu_0,\mu) = \int_{\alpha_s(\mu_0)}^{\alpha_s(\mu)} \frac{\mathrm{d}\alpha_s}{\beta(\alpha_s)}\Gamma_{\mathrm{cusp}}(\alpha_s)\int_{\alpha_s(\mu_0)}^{\alpha_s} \frac{\mathrm{d}\alpha_s'}{\beta(\alpha_s')},$$

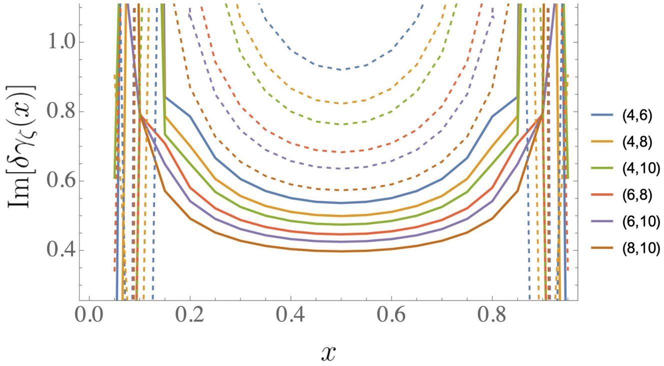
$$\eta_\Gamma(\mu_0,\mu) = \int_{\alpha_s(\mu_0)}^{\alpha_s(\mu)} \frac{\mathrm{d}\alpha_s}{\beta(\alpha_s)}\Gamma_{\mathrm{cusp}}(\alpha_s)$$

where $\Gamma_{\mathrm{cusp}}(\alpha_s(\mu)) = \dfrac{\mathrm{d}\gamma_\mu(p^z,\mu)}{\mathrm{d}\ln p^z}$ and $\gamma_\mu(p^z,\mu) \equiv \dfrac{d\ln C_\phi(p^z,\mu)}{d\ln\mu}$

are computed perturbatively at following loop orders for each resummation accuracy:

	K_Γ	K_{γ_C}	K_{γ_μ}	η	C_ϕ
NLL	2	1	1	1	0
NNLL	3	2	2	2	1





NLL (solid) and NNLL (dashed)
No convergence in the imaginary part

bT-dependent matching

Matching coefficients C include are a $P^z b_T \gg 1$ limit of

$$\boxed{\mathbf{C}_\phi(p^z, b_T, \mu) = C_\phi(p^z, \mu) + \delta C_\phi(p^z, b_T)}$$

uNLO

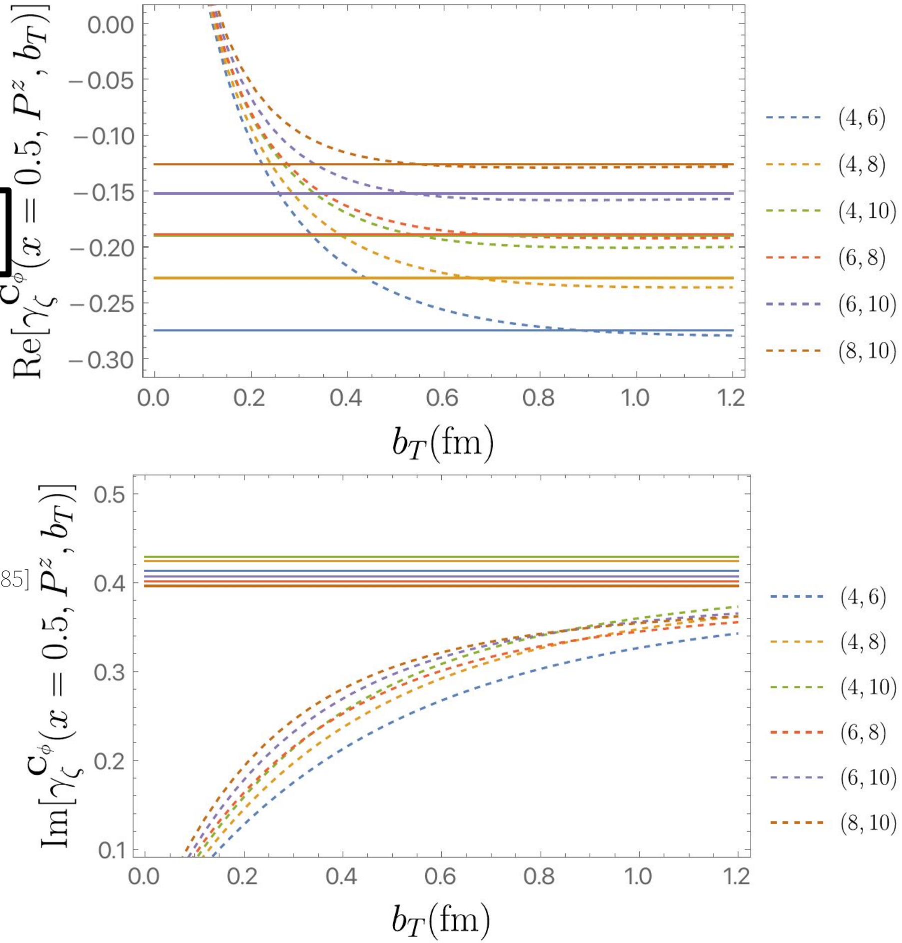
$$p^z \in xP^z, \bar{x}P^z$$

- $\delta C_\phi(p^z, b_T)$ contains bT-dependent terms on $x \in (-\infty, \infty)$ suppressed in $P^z b_T$
- Has been computed at NLO.

M. A. Ebert et. al,, JHEP 09, 037, [1901.03685]
Z.-F. Deng et. al, JHEP 09, [2207.07280]

- Corresponding unexpanded (in bT) matching correction reveals power corrections in 1/Pz bT.
- Imaginary part more sensitive to power corrections => <u>not taken as a systematic uncertainty directly.</u>

M.-H. Chu et al. (LPC), PRD 106, 034509, [2204.00200]
M.-H. Chu et al. (LPC), [2302.09961]
M.-H. Chu et al. (LPC), [2306.06488]

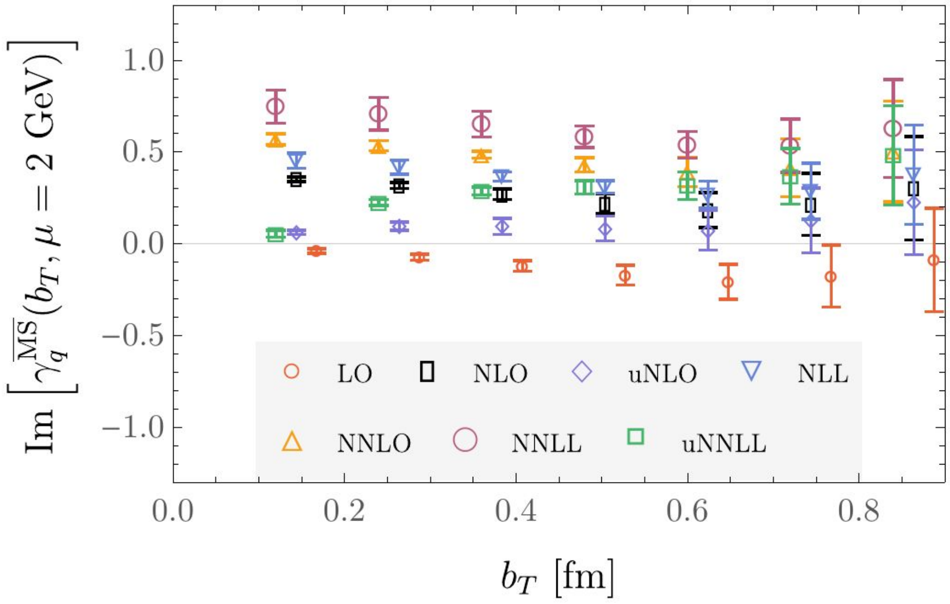


Dashed: **uNLO**, solid: NLO.

The imaginary part in the CS kernel estimate

- The CS kernel is real-valued.
- The CS kernel *estimate* has a non-zero imaginary part, primarily from matching.
- This is explained by poor perturbative convergence and power corrections in bT <u>=> not treated as a systematic directly</u>

  M.-H. Chu et al. (LPC), PRD 106, 034509, [2204.00200]
  M.-H. Chu et al. (LPC), [2302.09961]
  M.-H. Chu et al. (LPC), [2306.06488]
- Estimates of power corrections expected to improve with multiple lattice spacings, by disentangling O(a) effects
- For this calculation, uNNLL dominated by uNLO at small bT – unexpanded matching accounts for power corrections.



$$\boldsymbol{\delta\gamma_q}(x,b_T,P_1^z,P_2^z,\mu) = -\frac{1}{\ln(P_1^z/P_2^z)}\left(\ln\frac{\boldsymbol{C}_\phi(xP_1^z,b_T,2xP_1^z)}{\boldsymbol{C}_\phi(xP_2^z,b_T,2xP_2^z)} - (K_\phi(2xP_1^z,\mu)-K_\phi(2xP_2^z,\mu)) + (x\leftrightarrow\bar{x})\right)$$

Using auxiliary fields for non-perturbative renormalization

Get a renormalized staple-shaped operator

$$\mathcal{O}^{\text{ren.}}_{\ell,\Gamma} = Z^{\text{ren.}}_{\mathcal{O}_\ell \Gamma\Gamma'} \mathcal{O}^{\text{bare}}_{\ell,\Gamma}$$

By solving for Z_O in a renormalization scheme where it is given by matrix elements computed non-perturbatively, such as

$$\Lambda^{\text{bare}}_{\ell,\Gamma}(p,b) = \langle q(p)|\mathcal{O}^{\text{bare}}_{\ell,\Gamma}(b)|q(p)\rangle_{\text{gf,amp.}}$$

renormalized as

$$\Lambda^{\text{RI}'-\text{MOM}}_{\ell,\Gamma}(p,b) = [Z_q'(p)]^{-1} Z^{\text{RI}'-\text{MOM}}_{\mathcal{O}_\ell(b),\Gamma\Gamma'}(p)\Lambda^{\text{bare}}_{\ell,\Gamma}(p,b)$$

Set to its tree-level value at p = p_R, together with some renormalization condition for Z_q. This is RI'-MOM, *with a different Z_O for each staple configuration.*

[1]Green, Jansen, and Steffens, PRL 121 (2018) and PRD 101(2020).

With the auxiliary-field approach, renormalization of extended staples is simplified to that of point-like objects:

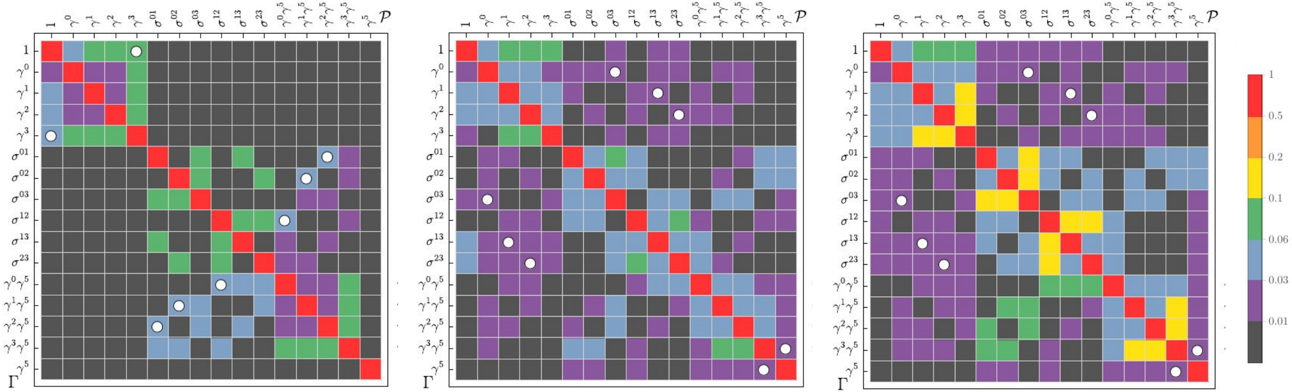
$$\begin{aligned}
&\bar{q}(b)\,\Gamma W_{-z} W_{\text{T}} W_{+z}\, q(0) \\
&\quad = \langle \bar{q}(b)\ \underbrace{\Gamma\zeta_{-z}(b)\bar{\zeta}_{-z}(\eta+b_{\text{T}})}_{W_{-z}}\,\underbrace{\zeta_{\text{T}}(\eta+b_{\text{T}})\bar{\zeta}_{\text{T}}(\eta)}_{W_{\text{T}}}\,\underbrace{\zeta_{+z}(\eta)\bar{\zeta}_{+z}(0)}_{W_{+z}}\ q(0)\rangle_\zeta \\
&\quad = \langle \underbrace{\bar{q}(b)\zeta_{-z}(b)}_{\phi_{-z}(b)}\,\Gamma\,\underbrace{\bar{\zeta}_{-z}(\eta+b_{\text{T}})\zeta_{\text{T}}(\eta+b_{\text{T}})}_{C_{-z,\text{T}}(\eta+b_{\text{T}})}\,\underbrace{\bar{\zeta}_{\text{T}}(\eta)\zeta_{+z}(\eta)}_{C_{\text{T},+z}(\eta)}\,\underbrace{\bar{\zeta}_{+z}(0)q(0)}_{\phi_{+z}(0)}\rangle_\zeta
\end{aligned}$$

where Wilson lines are given by zeta propagators in the extended theory, and Z_0 is broken down as

$$\begin{aligned}
\mathcal{O}^{\text{ren.}}_{\ell,\Gamma} &= e^{-\delta m(l+b_{\text{T}})}(Z^\dagger_{\phi_{-z}}\Gamma Z_{\phi_{+z}}) \\
&\quad\times \langle \phi_{-z}(Z_{C_{-z,\text{T}}}C_{-z,\text{T}})(Z_{C_{\text{T},+z}}C_{\text{T},+z})\phi_{+z}\rangle_\zeta
\end{aligned}$$

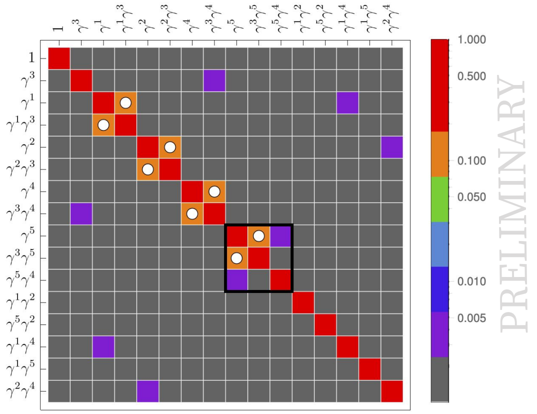
with one renormalization condition for each Z, independent of staple configurations. This is RI-xMOM[1].

New renormalization scheme leads to reduced mixing



Figures from Shanahan, Wagman, and Zhao, PRD 101 (2020)

Showing mixing patterns for RI'-MOM from left to right for: straight-line, symmetric, and asymmetric staples.



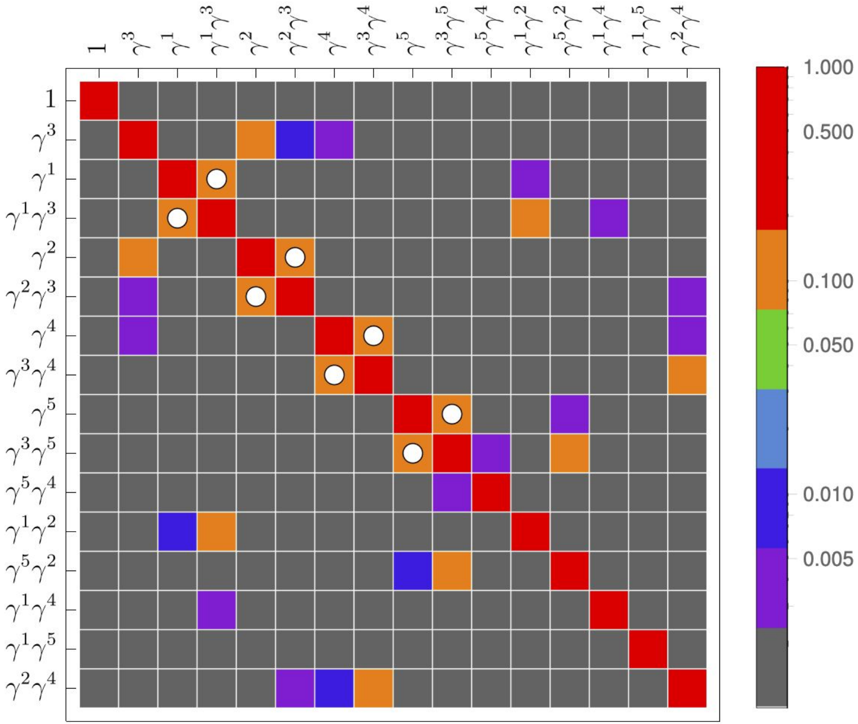
$$p_R^\mu = \frac{2\pi}{L} \times (0, 0, 10, 0),\ \xi = 0.24\ \text{fm}$$

For short, straight-line configurations, mixing patterns in RI'-MOM agree with lattice perturbation theory at one-loop[1] (white circles), but deviations become large for staple-shaped Wilson lines; in comparison, mixing effects in RI-xMOM are well-controlled (for collinear momenta and Wilson lines)

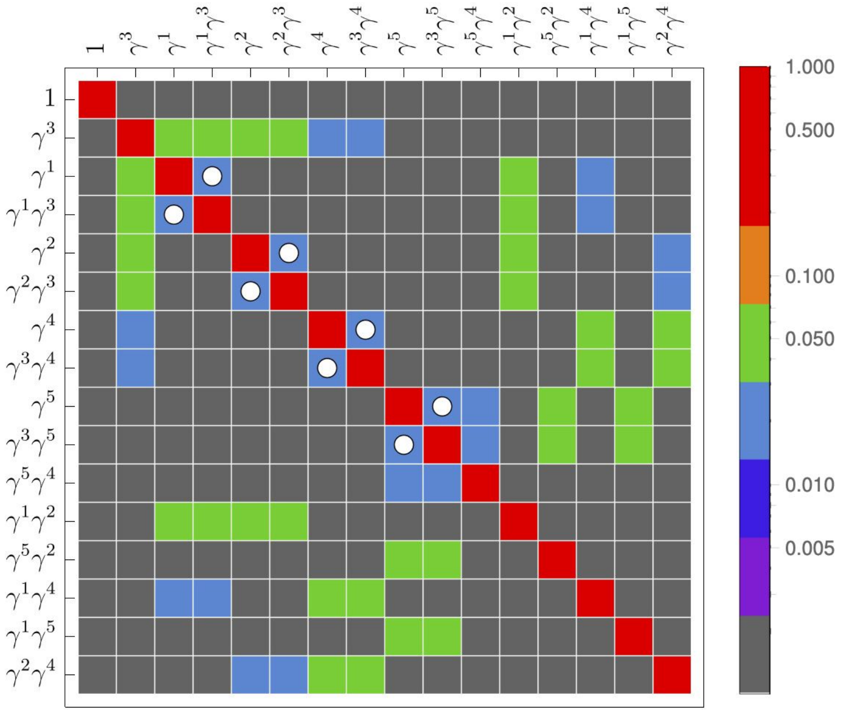
[1]Constantinou, Panagopoulos, and Spanoudes, PRD 99 (2019) and PRD 96 (2017).

Preliminary figure from this work (different ensemble and renormalization scale)

Scheme dependence of mixing patterns

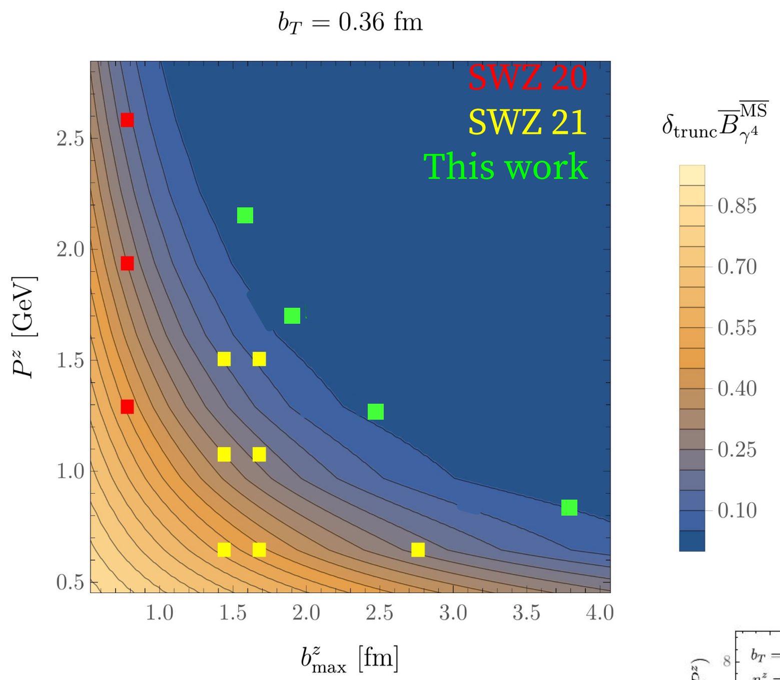


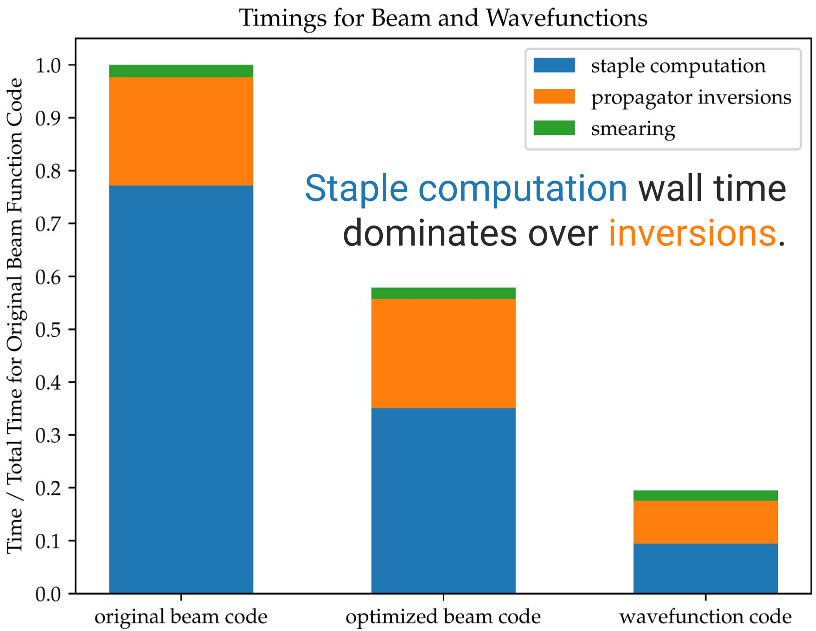
$$p_{\mathrm{R}}^{\mu} = \frac{2\pi}{L} \times (0, 10, 0, 0),\ \xi = 0.24\ \mathrm{fm}$$



$$p_{\mathrm{R}}^{\mu} = \frac{2\pi}{L} \times (6, 6, 6, 6),\ \xi = 0.24\ \mathrm{fm}$$

Code improvements





Staple computation wall time dominates over inversions.