

Simulating neutrino interactions and their impact on oscillation experiments



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The precision era of ν oscillations?

Latest results

- Indication of CP violation!
- Currently largely limited by statistics ... but not for long!



Current systematic uncertainties

Source (T2K)	$N(\nu_e)$
$\sigma_{\nu N}$ and FSI	3.8%
Total Syst.	5.2%

NEUTRINO 2022

XXX International Conference on Neutrino Physics and Astrophysics

Source (NOVA)	$N(\nu_e)$
$\sigma_{\nu N}$ and FSI	7.7%
Total Syst.	9.2%

Phys. Rev. D **98**, 032012

- Tables show **largest** and **total** syst. uncertainty on samples most sensitive to CP-violation

- Current results have $\sim 100 \nu_e$ events, expect **1000-2000** for DUNE/HK

The precision era of ν oscillations?

Latest results

- Indication of CP violation!
- Current results largely limited by statistics for long!



Neutrino interaction uncertainties must be reduced for DUNE/Hyper-K to succeed



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- Current results have $\sim 100 \nu_e$ events, expect **1000-2000** for DUNE/HK

Focus of these lectures

- There is not time to cover all the interesting physics associated with neutrino interactions
- You'll get a slightly biased choice of topics!
- We'll stay mostly qualitative, I'll try to give a conceptual overview of the topics most relevant to ongoing experiments
- Lots of places to learn more:
 - **INSS 2023 talks from Noemi and Deborah** on neutrino interaction theory and cross-section measurements
- [Other INSS lecture slides](#)
- [From eV to EeV \(Formaggio and Zeller\)](#)
- [NuSTEC White Paper](#)
- [Xsecs for Oscillations \(Katori and Martini\)](#)
- [e-scat vs nu-scat \(SuSAv2 group\)](#)
- [Semi-inclusive interactions \(Donnelly talk at ECT* 2018\)](#)
- [K. McFarland's Lectures](#)
- [S. Boyd's Lectures](#)
- [T. Golan's thesis](#)
- [G. Megias' thesis](#)
- GiBUU based summaries ([1](#),[2](#))

(Which I liberally borrowed material from when making these slides!)

Overview

- Neutrino Interactions: A History
 - Weak interactions from Fermi to SM
- Neutrino-nucleon interactions
 - QE, RES and DIS
- Neutrino-nucleus interactions
 - Nuclear effects
 - The rise and fall of $M_A = 1.3 \text{ GeV}$
- Neutrino event generators
 - Theory inputs
 - Filling in the gaps
- Neutrino-nucleus interaction measurements
 - Inclusive successes and exclusive failures
- Why do we care?
 - Neutrino interactions for neutrino oscillations
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- Don't Panic! The future of neutrino interaction simulations

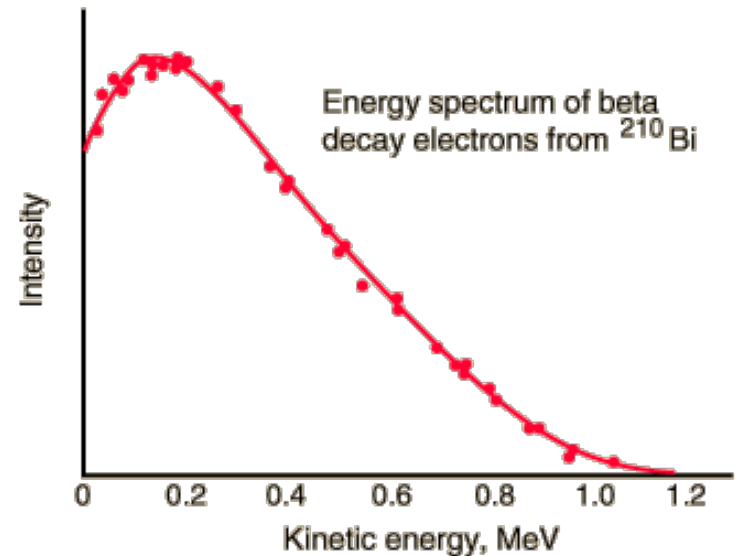
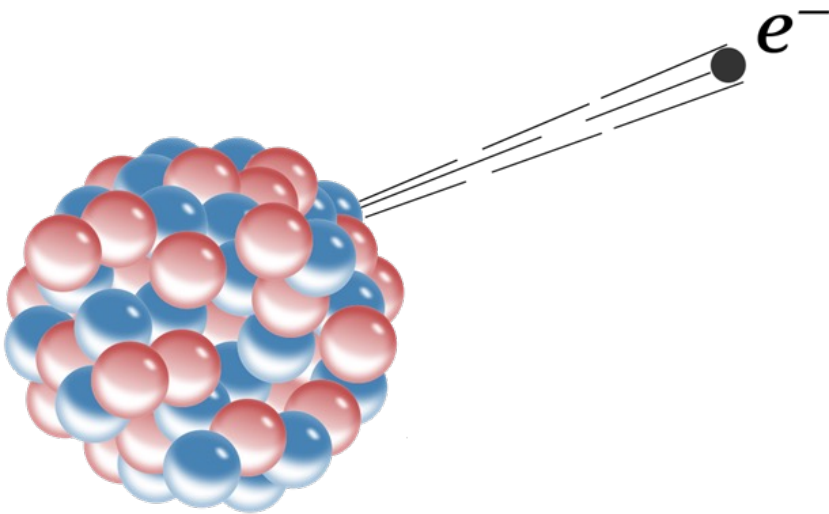
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Neutrinos: a desperate remedy

- It's the summer of 1930 and β decay doesn't appear to conserve energy
- α and γ are emitted with discrete spectra: the energy difference between the initial and final state nucleus
- β decays give a continuous spectrum ...

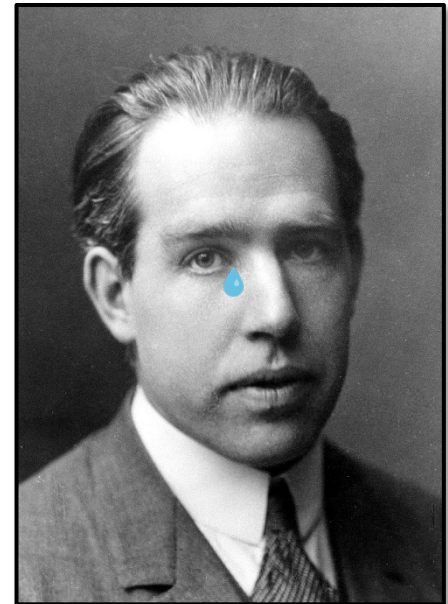
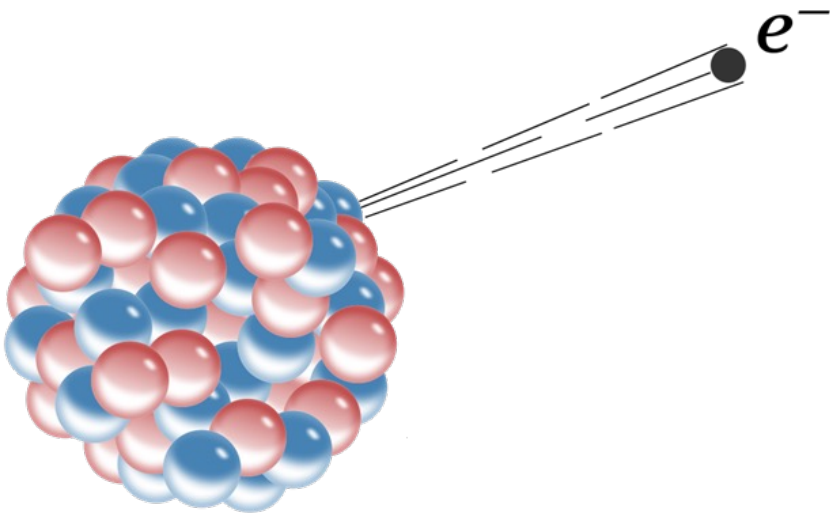


Neutrinos: a desperate remedy

- It's the summer of 1930 and β decay doesn't appear to conserve energy

"At the present stage of atomic theory, however, we may say that we have no argument, either empirical or theoretical, for upholding the energy principle in the case of β -ray disintegrations". *Niels Bohr*

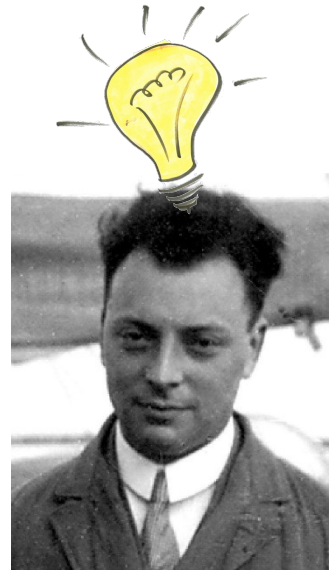
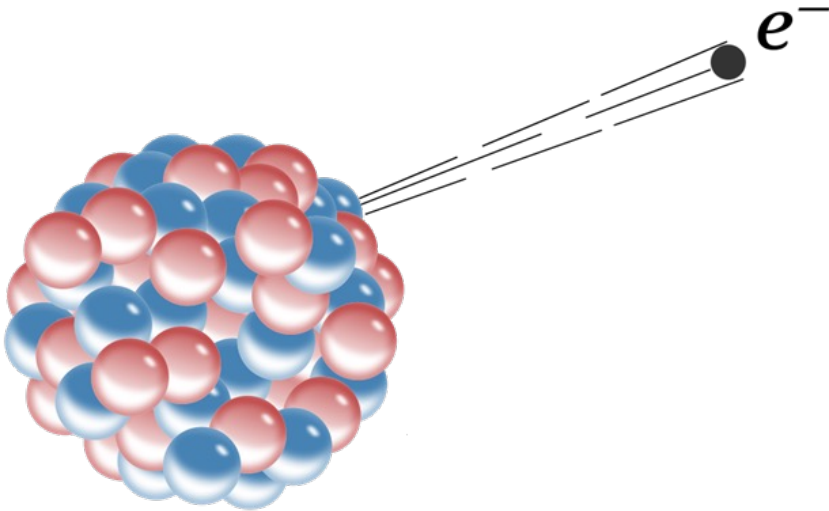
- Bohr was close to giving up the conservation of energy!



Neutrinos: a desperate remedy

- Pauli writes a letter to colleagues attending a conference in Tübingen proposing a “solution”:

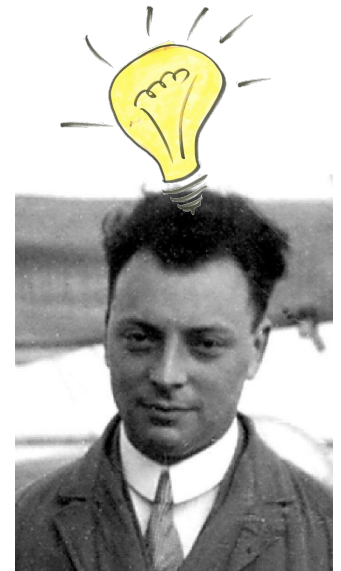
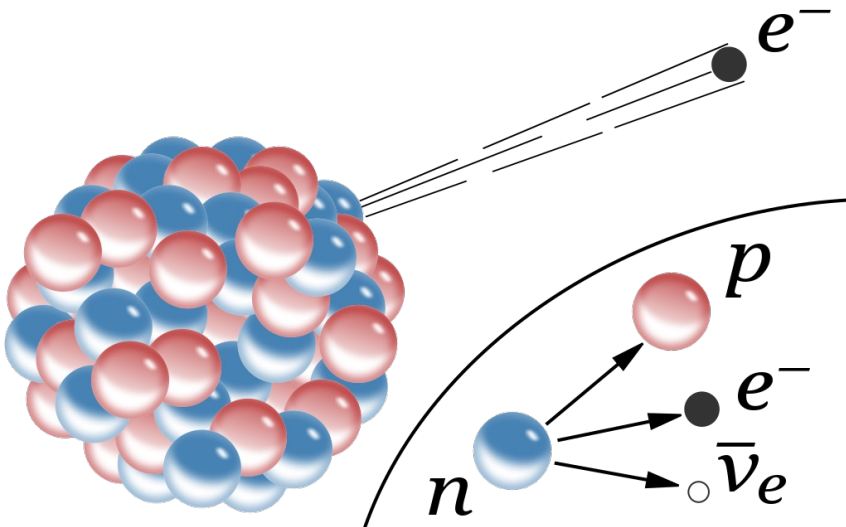
“Dear Radioactive Ladies and Gentlemen, ... I have hit upon a desperate remedy to save the law of conservation of energy. Namely, the possibility that there could exist ... [light] electrically neutral particles. The β spectrum would then become understandable by the assumption that [these are] emitted in addition to the electron” *Wolfgang Pauli*



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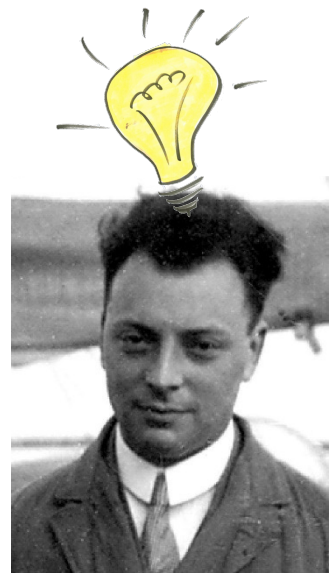
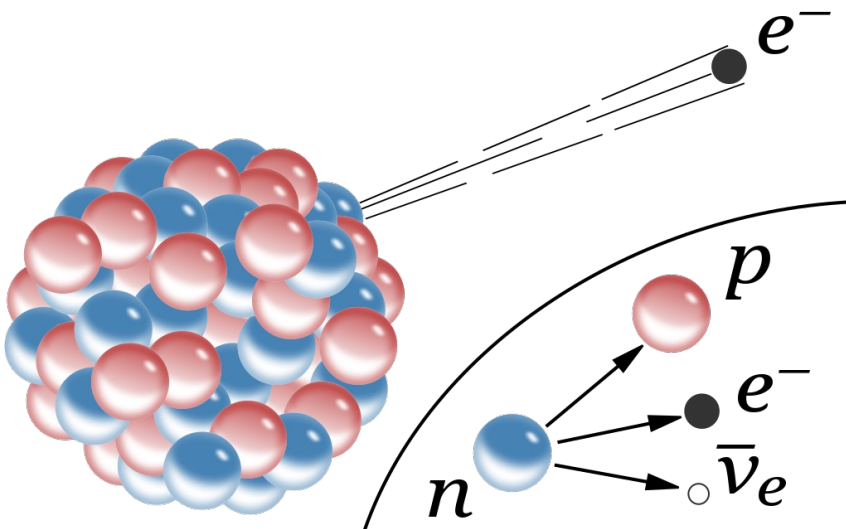
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Neutrinos: a desperate remedy

- He's not proud of it ...

"I have done something very bad today by proposing a particle that cannot be detected; it is something no theorist should ever do."



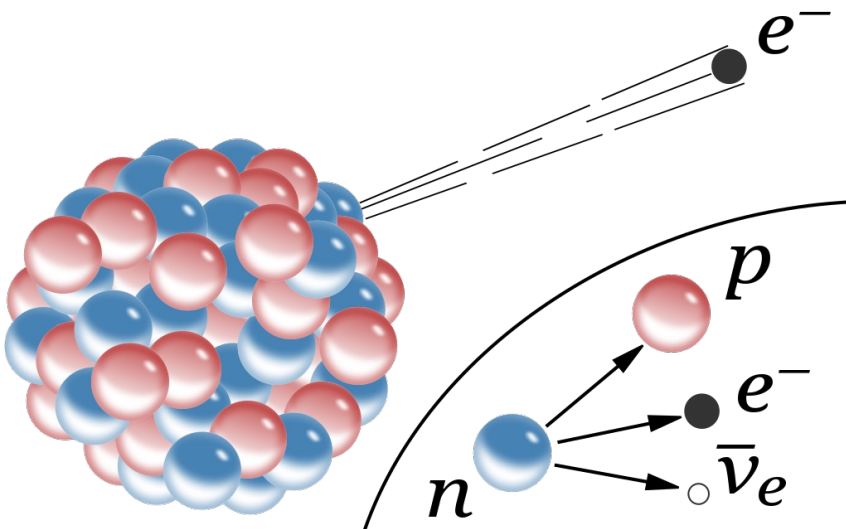
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"Unfortunately, I cannot personally appear in Tübingen since I am indispensable here in Zürich because of a ball."



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- He wagers a case of champagne that no one would ever detect his elusive postulated particle



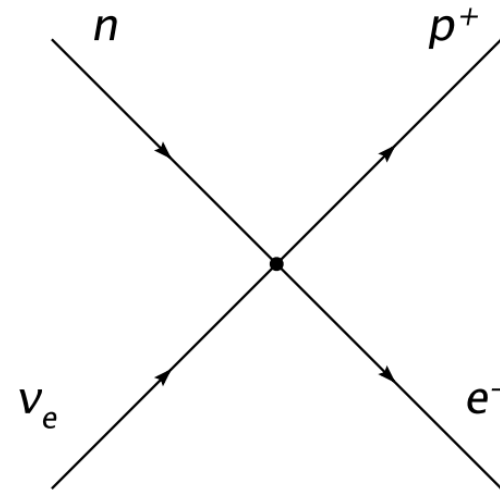
The Weak Interaction (1932)

- Enrico Fermi takes Pauli seriously, naming this new particle “neutrino” (little neutral one) and constructing a theory of β decay

Z. Physik, 88, 161 (1934)



- He constructs a new fundamental interaction analogous to EM but with a different strength
- His theory also predicts neutrino interaction cross sections
- The strength of the interaction (G_F) can be tuned using β decay data



$$M = \frac{G_F}{\sqrt{2}} [\bar{u}_P \gamma^\mu u_N] [\bar{u}_e \gamma^\nu u_\nu]$$

The Weak Interaction (1932)

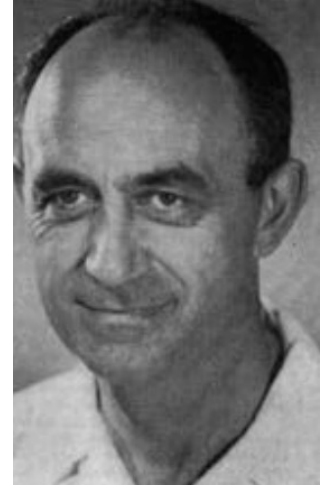
- The theory can tell us the cross section for a neutrino from β decay to interact:

$$\sigma_{\bar{\nu}p}(E_{\bar{\nu}} \sim 1 \text{ MeV}) \sim 5 \times 10^{-44} \text{ cm}^2$$

- Compared to EM interactions of similar energies:

$$\sigma_{\gamma p} \sim 10^{-25} \text{ cm}^2$$

- Suggests a $\sim \text{MeV}$ neutrino's mean free path in steel to be around 10 light years ...
- Fermi submitted his paper to *Nature*. The response:



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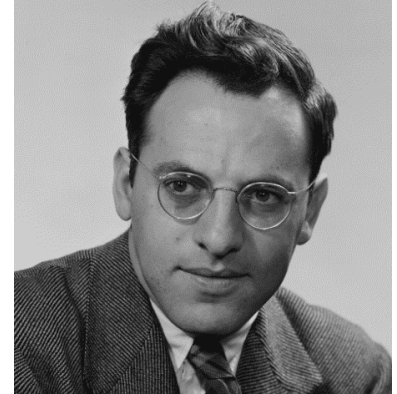
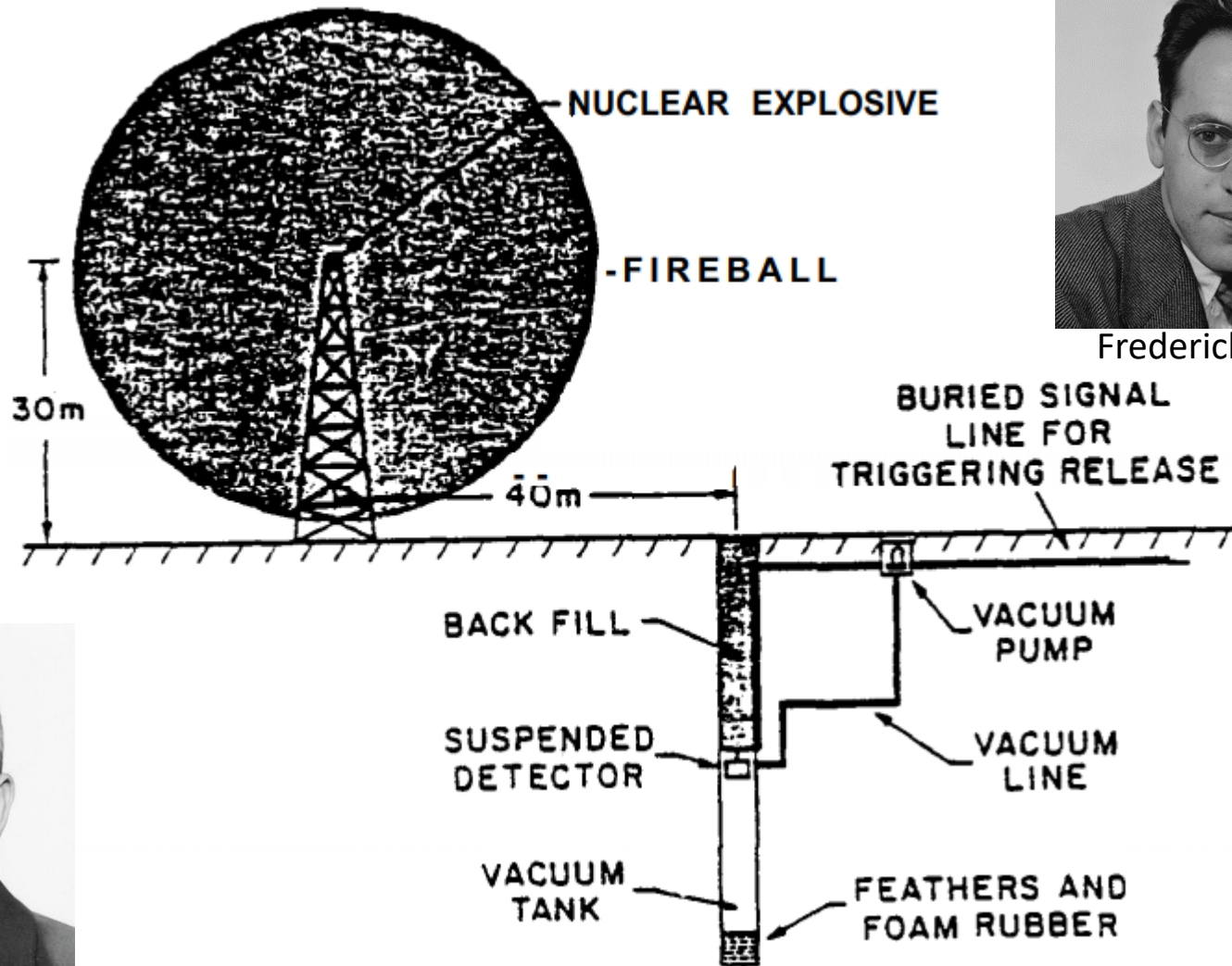
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“it contains speculations too remote from reality to be of interest to the reader”



Where can I get a lot of ν s ...

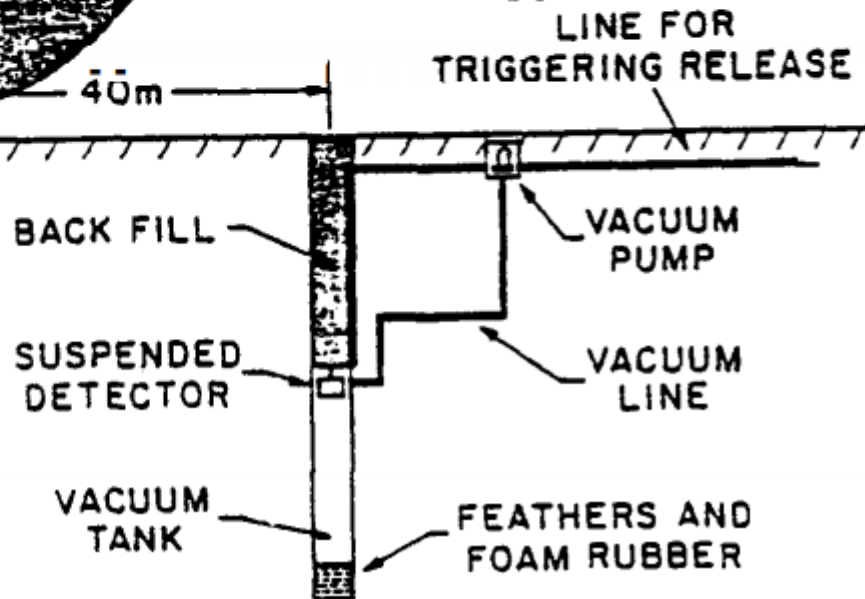
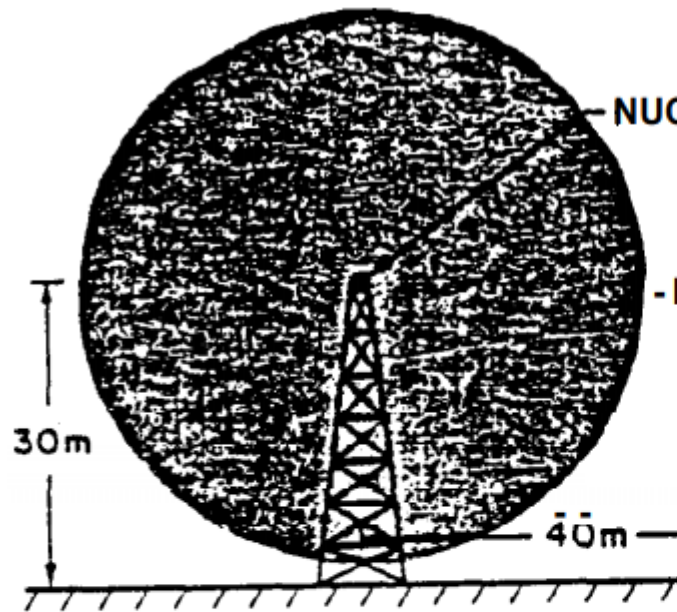


Frederick Reines



Clyde Cowan

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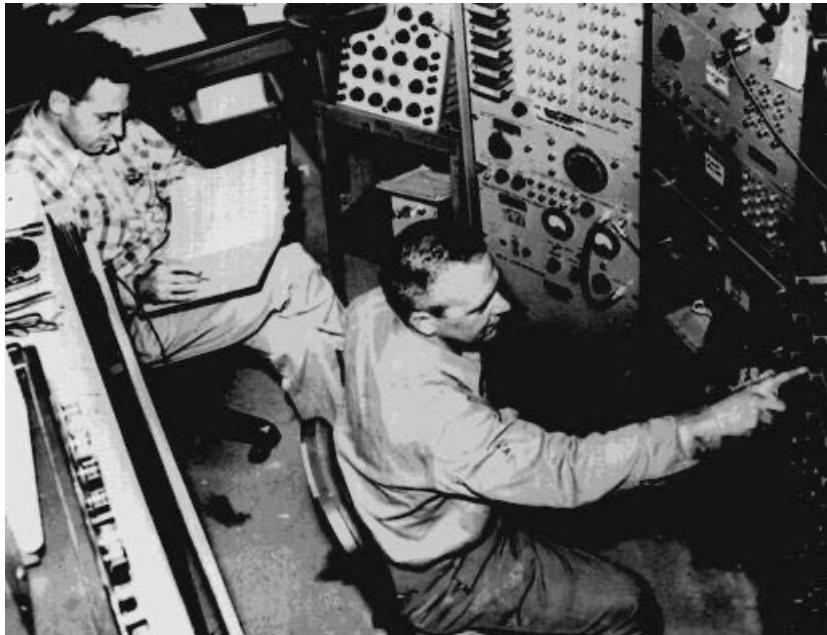


“The idea that such a sensitive detector could be operated in the close proximity (within a hundred meters) of the most violent explosion produced by man was somewhat bizarre”

Frederick Reines, Nobel Lecture

Discovery of the neutrino

- Reins and Cowan detect neutrinos from a nuclear reactor (Savannah River) in 1956 by observing inverse beta decay ($\bar{\nu}_e p \rightarrow e^+ n$)



- Letter to Pauli:

"We are happy to inform you that we have definitely detected neutrinos..."

- 1995 Nobel Prize

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- And a case of champagne !



Discovery of the neutrino

Frederick REINES and Clyde COWAN

Box 1663, LOS ALAMOS, New Mexico

Thanks for message. Everything comes to
him who knows how to wait.

Pauli

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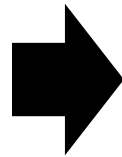
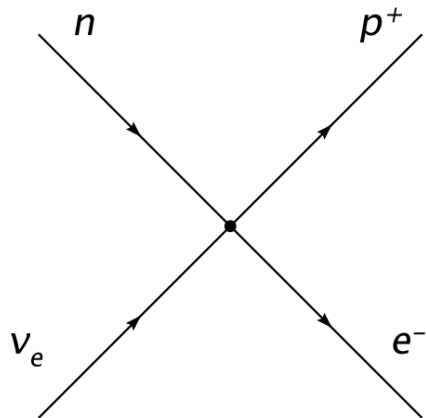
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also night letter

Borrowed from [K. McFarland's INSS 2015 lectures](#)

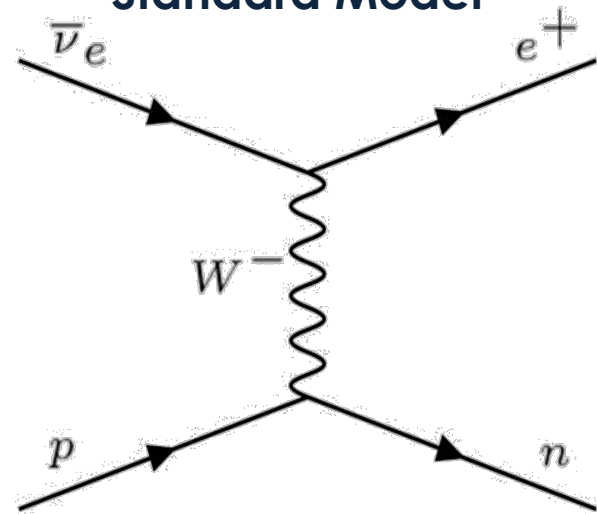
Back to the Weak Interaction

- Fermi's interaction works remarkably well, but the cross-section rises linearly with energy forever!
- The modern standard model theory of weak interactions is a little different:

Fermi



Standard Model

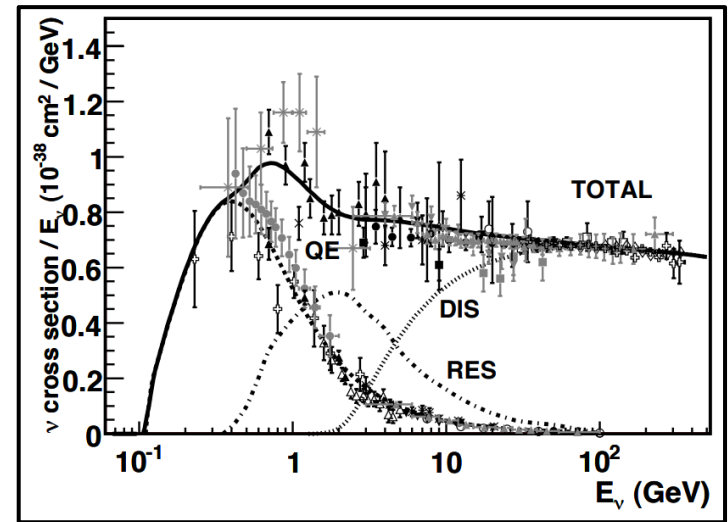


$$M \sim \frac{G_F}{\sqrt{2}} [\bar{u}_e \gamma_\mu u_\nu] [\bar{u}_n V^\mu u_p]$$

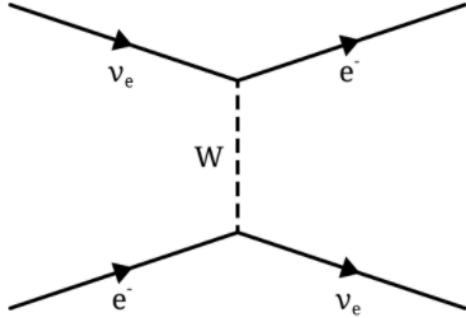
$$M \sim \frac{g_w^2}{8} \frac{1}{M_W^2 - q^2} [\bar{u}_e \gamma_\mu (1 - \gamma_5) u_\nu] [\bar{u}_n (V^\mu - A^\mu) u_p]$$

Overview

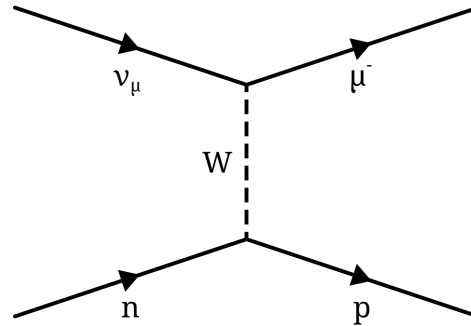
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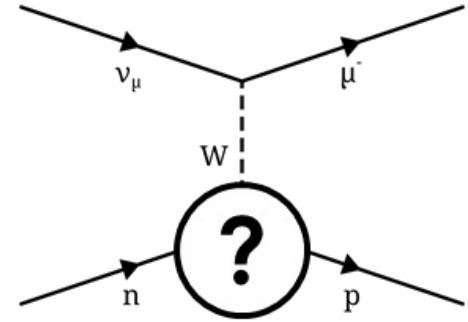
Beyond point-like scattering



Point-like: Masters
homework problem



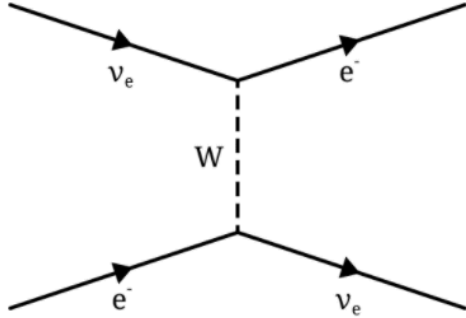
Nucleon: mostly harmless



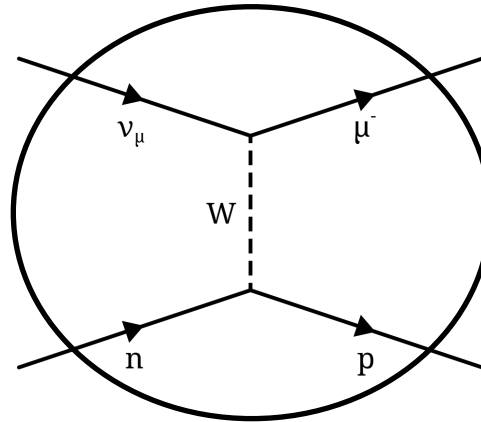
Nucleus: very hard

- Cross-Sections for **point-like neutrino scattering** with electrons or quarks are relatively **easy to calculate**
- In most experiments, **neutrinos interact with nucleons or a nucleus**

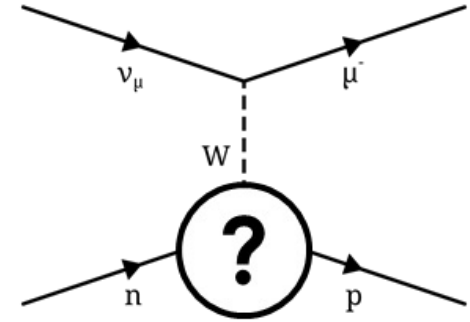
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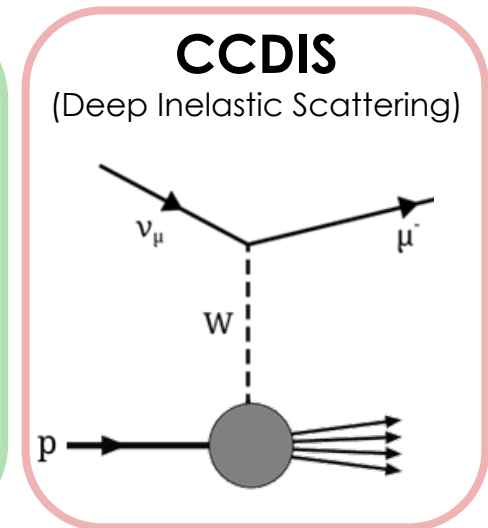
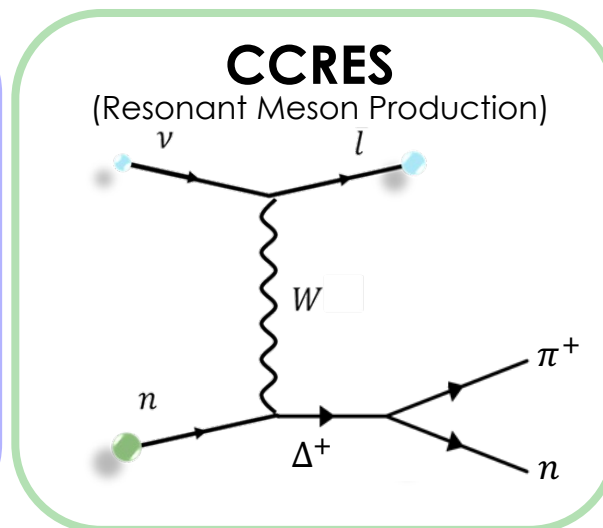
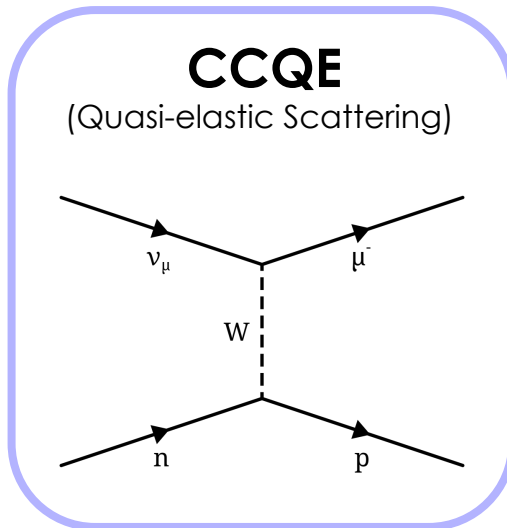
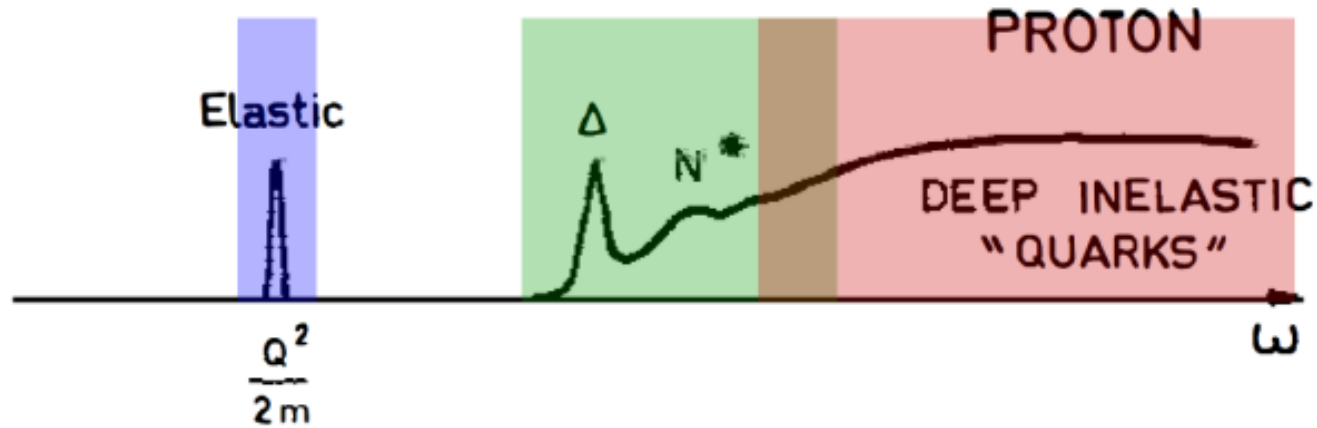
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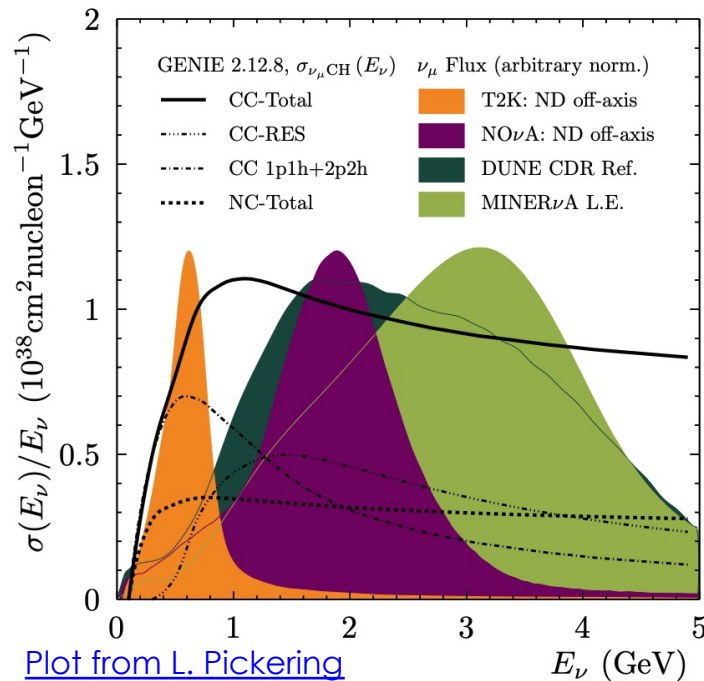
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- Next slides describe our baseline models for simulating neutrino-nucleon interactions

Neutrino nucleon scattering



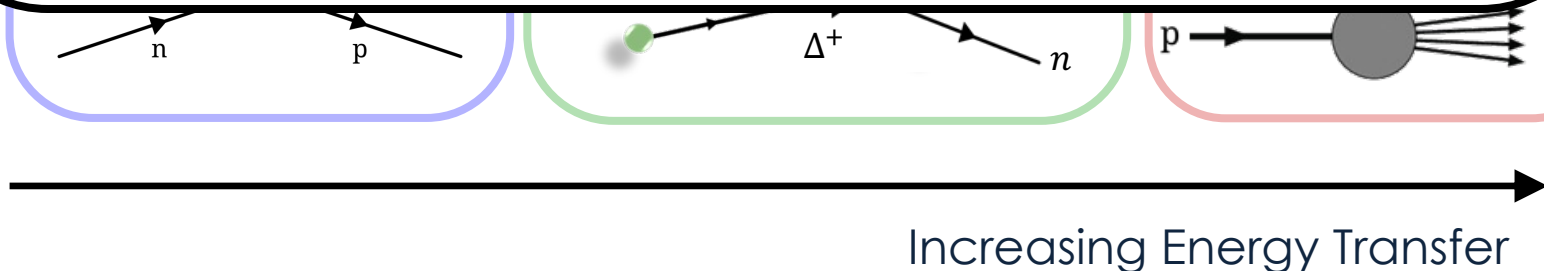
Increasing Energy Transfer

Neutrino nucleon scattering

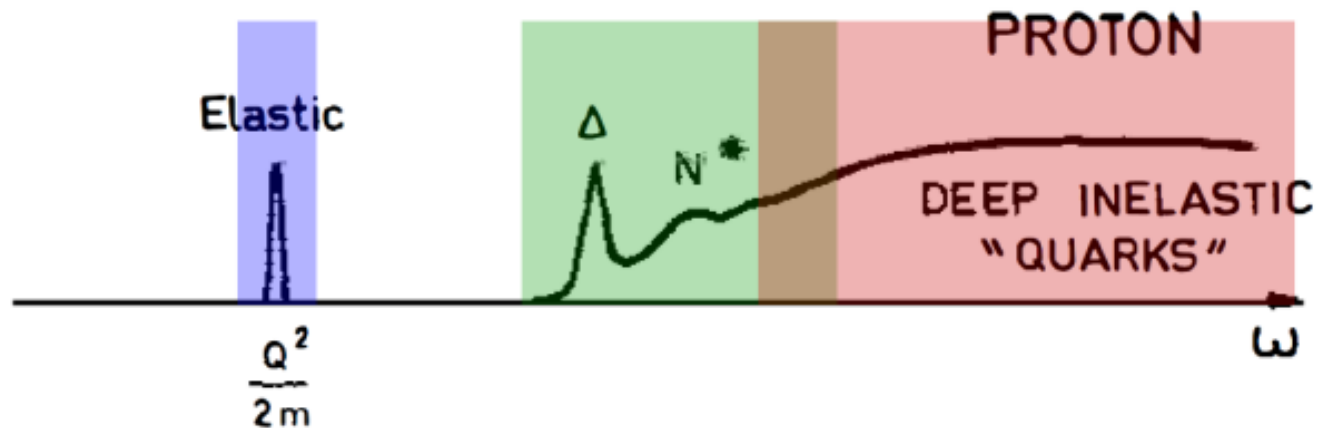


Whilst interaction channels live in **relatively narrow regions of energy transfer**, they are **spread out in neutrino energy**

Few-GeV experiments must care about all the channels!

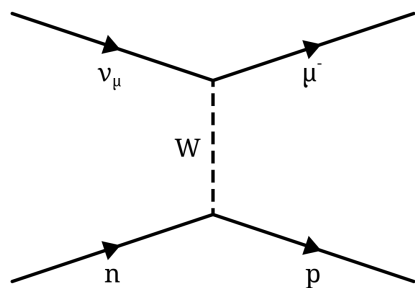


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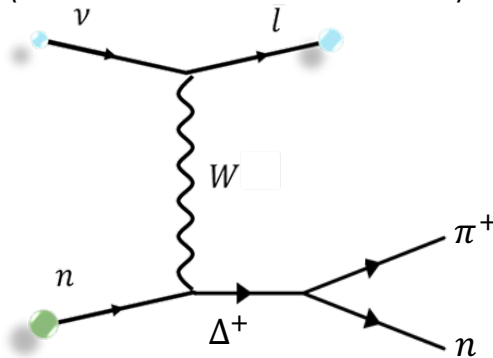
CCQE

(Quasi-elastic Scattering)



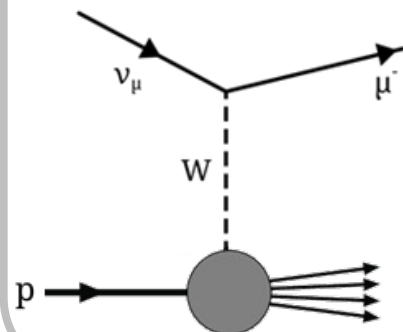
CCRES

(Resonant Meson Production)



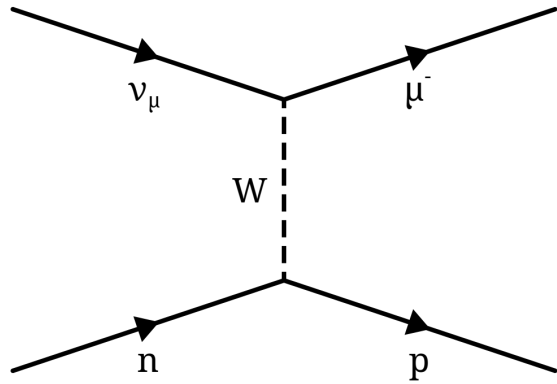
CCDIS

(Deep Inelastic Scattering)



Increasing Energy Transfer

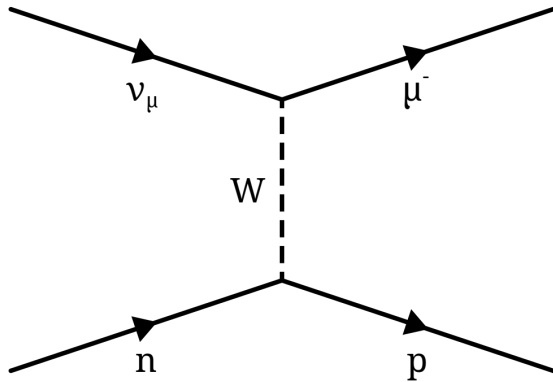
Quasi-elastic Scattering



- Let's work with the “easiest”
neutrino-nucleon interaction: CCQE
(= *charged current quasi elastic*)

$$M \sim \frac{g_w^2}{8} \frac{1}{M_W^2} [\bar{u}_\mu \gamma_\mu (1 - \gamma_5) u_\nu] [\bar{u}_p (\dots) u_n]$$

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There are four parts of this matrix element

(1) The *coupling factor*, which determines the interaction strength

$\frac{g_w^2}{8}$

OK...

(2) The *propagator*, accounting for the w exchange

$\frac{1}{M_W^2}$

It also has some other parts at higher energies

ALSO WHAT?

(3) The *leptonic current*, which helps violate Parity and ... , uh comes from

... WELL, UMM...

... Its pretty important in QFT

I SEE.

$[\bar{u}_\mu \gamma_\mu (1 - \gamma_5) u_\nu]$

(4) The *hadronic current*, it [mumble mumble] extended objects [mumble mumble]

THAT'S NOT A SENTENCE. YOU JUST SAID 'RADIO—

- And those are the four parts of the matrix element!

Adapted from XKCD

The hadronic current

$$J_H^\beta = \bar{u}_p \left[f_{1V} \gamma^\beta + i \frac{\xi f_{2V}}{2M} \sigma^{\beta\delta} q_\delta + \frac{f_{3V}}{M} q^\beta + f_A \gamma^\beta \gamma_5 + \frac{f_p}{M} q^\beta \gamma_5 + \frac{f_{3A}}{M} (P_p^\beta + P_n^\beta) \gamma_5 \right] u_n$$

$$M = (M_p + M_n) / 2 \quad q = p_\nu - p_\mu = P_p - P_n \quad \xi = \mu_p - \mu_n \quad \sigma^{\mu\nu} = \frac{i}{2} [\gamma^\mu, \gamma^\nu]$$

ξ is the difference between proton and neutron anomalous magnetic moments

- The f factors are the “form factors”
- Parameterise the nucleon as an extended object.
- The Fourier transform of form factors represent a physical distribution
- Dipole \rightarrow exponential

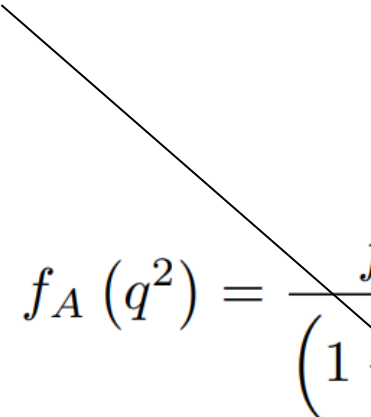
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- f_{3V}, f_{3A} are “second class currents”, typically set to 0
- f_{1V}, f_{2V} (vector form factors) can be extracted from electron scattering experiments. f_p can be related to f_A (“Partially Conserved Axial Current Hypothesis”)
- f_A , we guess the form of! Usually we take a dipole with one free parameter: the infamous nucleon axial mass (M_A)

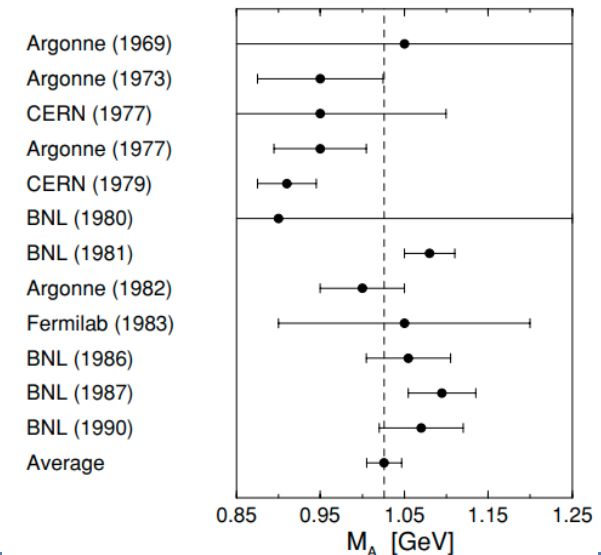
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- f_A , we guess the form of! Usually we take a dipole with one free parameter: the infamous nucleon axial mass (M_A)
- We usually constrain the axial form factor with old bubble chamber neutrino-nucleon (or light nucleus) data from the 1960s-80s.
- Result: $M_A \approx 1 \text{ GeV}$



The hadronic current

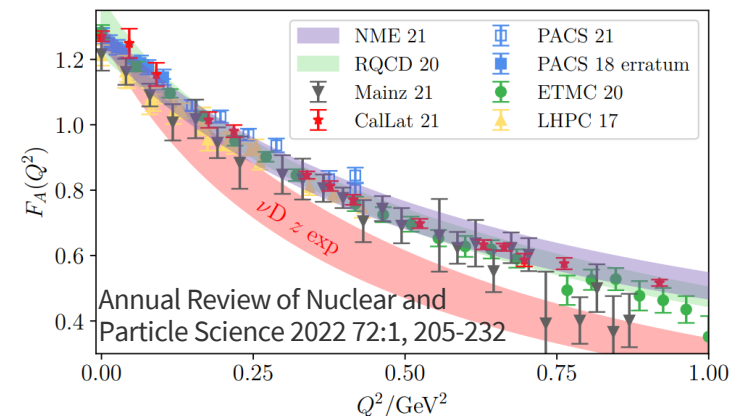
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$$M = (M_p + M_n) / 2 \quad q = p_\nu - p_\mu = P_p - P_n \quad \xi = \mu_p - \mu_n \quad \sigma^{\mu\nu} = \frac{i}{2} [\gamma^\mu, \gamma^\nu]$$

- f_{3V}, f_{3A} are “second class currents”, typically set to 0
- f_{1V}, f_{2V} (vector form factors) can be extracted from electron scattering experiments. f_p can be related to f_A (“Partially Conserved Axial Current Hypothesis”)
- f_A , we guess the form of! Usually we take a dipole with one free parameter: the infamous nucleon axial mass (M_A)

Aside: recent lattice QCD updates

- Recent work has allowed LQCD to calculate the axial form factor
- These suggest a dipole doesn't work
- See Noemi's slides for more details!



The hadronic current

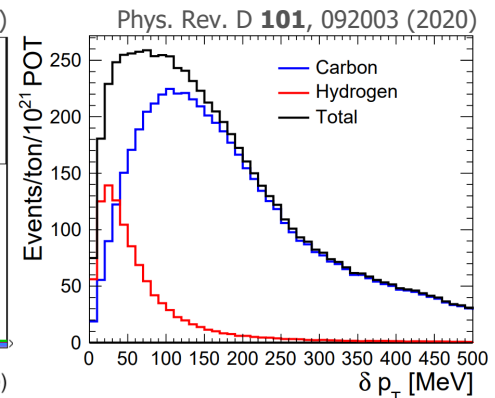
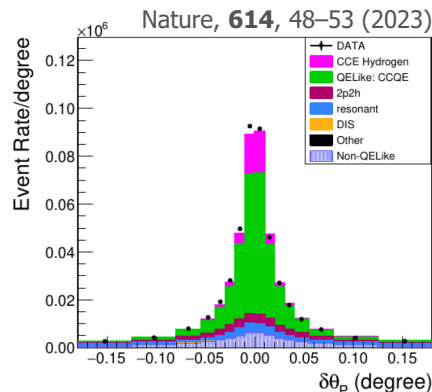
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Aside: recent new measurements

- Recent work permits new measurements on free nucleons
- Proof-of-principle from MINERvA!
- T2K-Upgrade aims to follow up



Llewellyn-Smith CCQE



- Putting this all together gets us to the cross section

$$\frac{d\sigma}{d|q^2|} \left(\begin{array}{c} \nu n \rightarrow \ell^- p \\ \bar{\nu} p \rightarrow \ell^+ n \end{array} \right) = \frac{M^2 G^2 \cos^2 \theta_c}{8\pi E_\nu^2} \left[A(q^2) \mp B(q^2) \frac{(s-u)}{M^2} + \frac{C(q^2)(s-u)^2}{M^4} \right]$$

$$(s-u = 4ME_\nu + q^2 - m^2) .$$

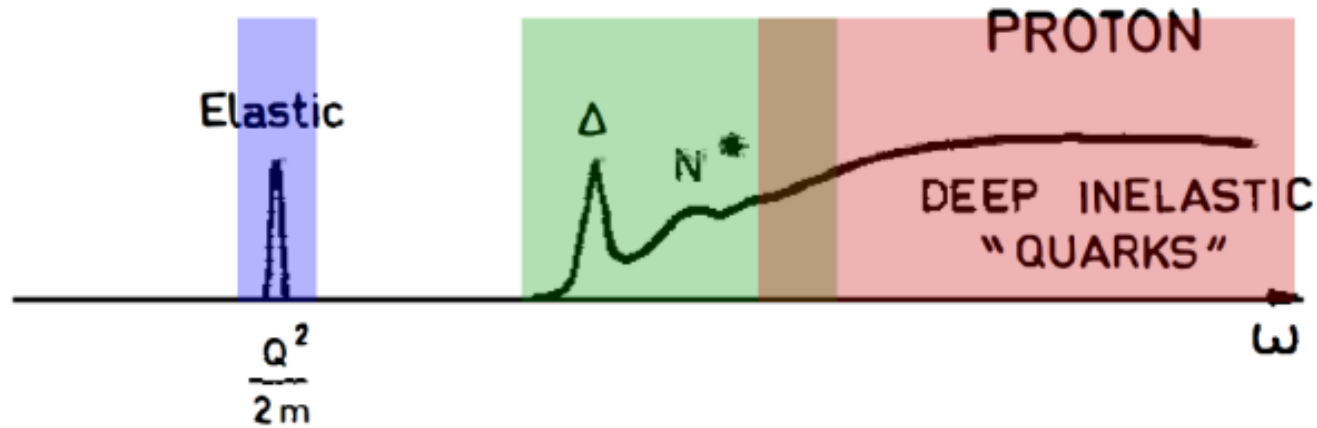
Neutrino reactions at accelerator energies, Llewellyn Smith, 1972

$$A \simeq \frac{t}{M^2} \left(|f_{1V}|^2 - |f_A|^2 \right) + \frac{t^2}{4M^2} \left(|f_{1V}|^2 + \xi^2 |f_{2V}|^2 + |f_A|^2 + 4\xi \text{Re}(f_{1V} f_{2V}^*) \right) \\ + \frac{t^3 \xi^2}{16M^6} |f_{2V}|^2$$

$$B \simeq \frac{1}{M^2} \left(\text{Re}(f_{1V} f_A^*) + \xi \text{Re}(f_{2V} f_A^*) \right) t \quad C = \frac{1}{4} \left(|f_{1V}|^2 + |f_A|^2 - \frac{\xi^2 |f_{2V}|^2}{4M^2} t \right)$$

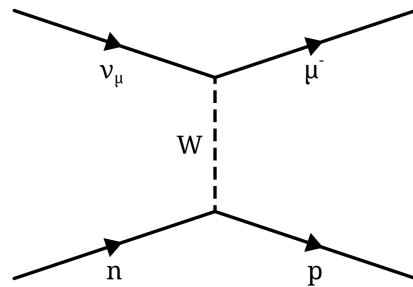
- It's a long expression, but only one unknown (in a dipole model): M_A

Neutrino nucleon scattering



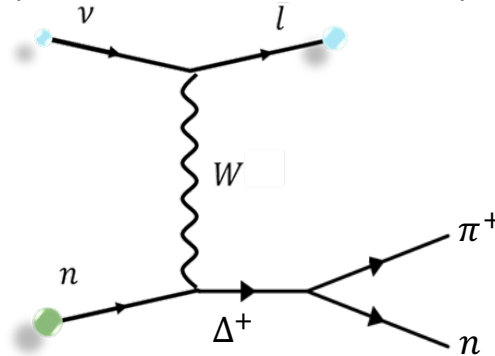
CCQE

(Quasi-elastic Scattering)



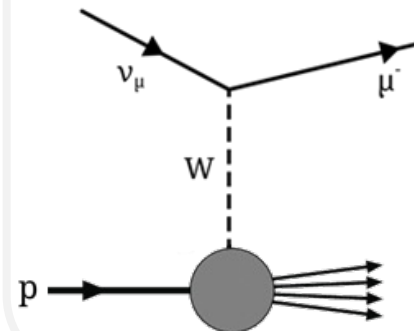
CCRES

(Resonant Meson Production)



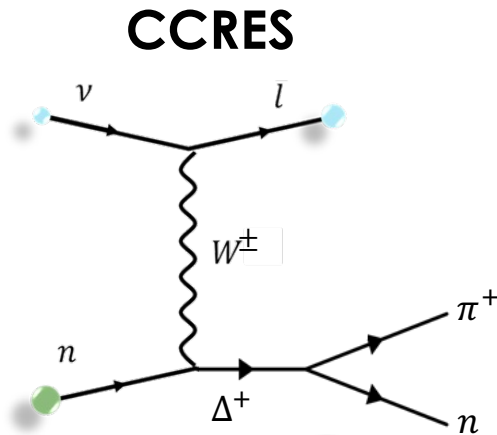
CCDIS

(Deep Inelastic Scattering)



Increasing Energy Transfer

Resonant Pion Production



CC Single Pion Production (SPP) final states

$$\begin{aligned}
 \nu_\mu p &\rightarrow \mu^- p \pi^+, & \bar{\nu}_\mu p &\rightarrow \mu^+ p \pi^- \\
 \nu_\mu n &\rightarrow \mu^- p \pi^0, & \bar{\nu}_\mu p &\rightarrow \mu^+ n \pi^0 \\
 \nu_\mu n &\rightarrow \mu^- n \pi^+, & \bar{\nu}_\mu n &\rightarrow \mu^+ n \pi^-
 \end{aligned}$$

D. Rein and L. Sehgal, Ann. Phys.
133, 79 (1981)

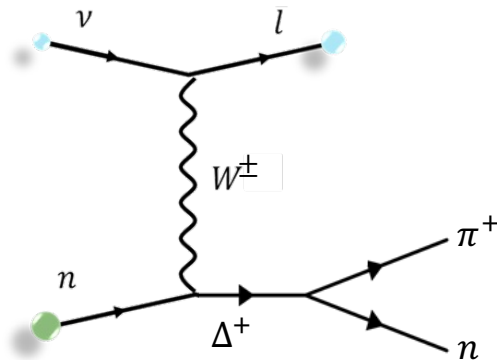
- Neutrinos can excite a nucleon into a resonance state, which decays to give a nucleon + meson final state
- The dominant resonance is $\Delta(1232)$ but others can contribute, as can non-resonant pion production
- And the contributions from each should have interference terms ...
- Resonance models are complicated!
- Whilst CCQE scattering on the nucleon can be described fully with 1 variable the multi-particle final state for SPP requires 4:

$$\frac{d\sigma}{dW dQ^2 d\Omega_\pi}$$

Contains polar and azimuthal angle

Resonant Pion Production

CCRES



Current Matrix Elements from a Relativistic Quark Model*

R. P. Feynman, M. Kislinger, and F. Ravndal

Lauritsen Laboratory of Physics, California Institute of Technology, Pasadena, California 91109

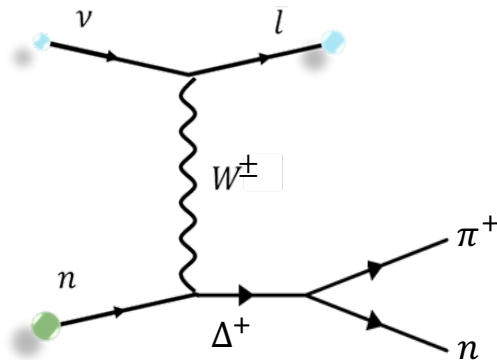
(Received 17 December 1970)

- The model's used in today's neutrino experiments are based on an approximate model from the 1970s

ficing theoretical adequacy for simplicity. We shall choose a relativistic theory which is naive and obviously wrong in its simplicity, but which is definite and in which we can calculate as many things as possible – not expecting the results to agree exactly with experiment, but to see how closely our “shadow of the truth” equation gives a partial reflection of reality. In our attempt to maintain simplicity, we shall evidently have to violate known principles of a complete relativistic field theory (for example, unitarity). We shall attempt to modify our calculated results in a general way to allow, in a vague way, for these errors.

Resonant Pion Production

CCRES



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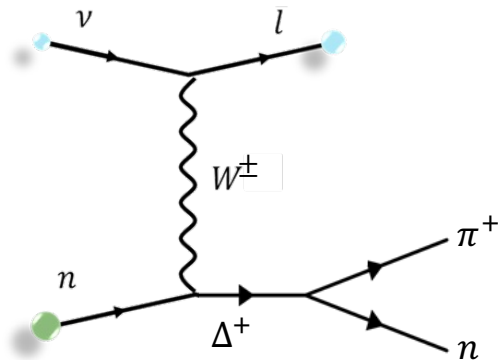
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gence of the axial-vector current matrix elements. Starting only from these two constants, the slope of the Regge trajectories, and the masses of the particles, 75 matrix elements are calculated, of which more than $\frac{3}{4}$ agree with the experimental values within 40%. The prob-

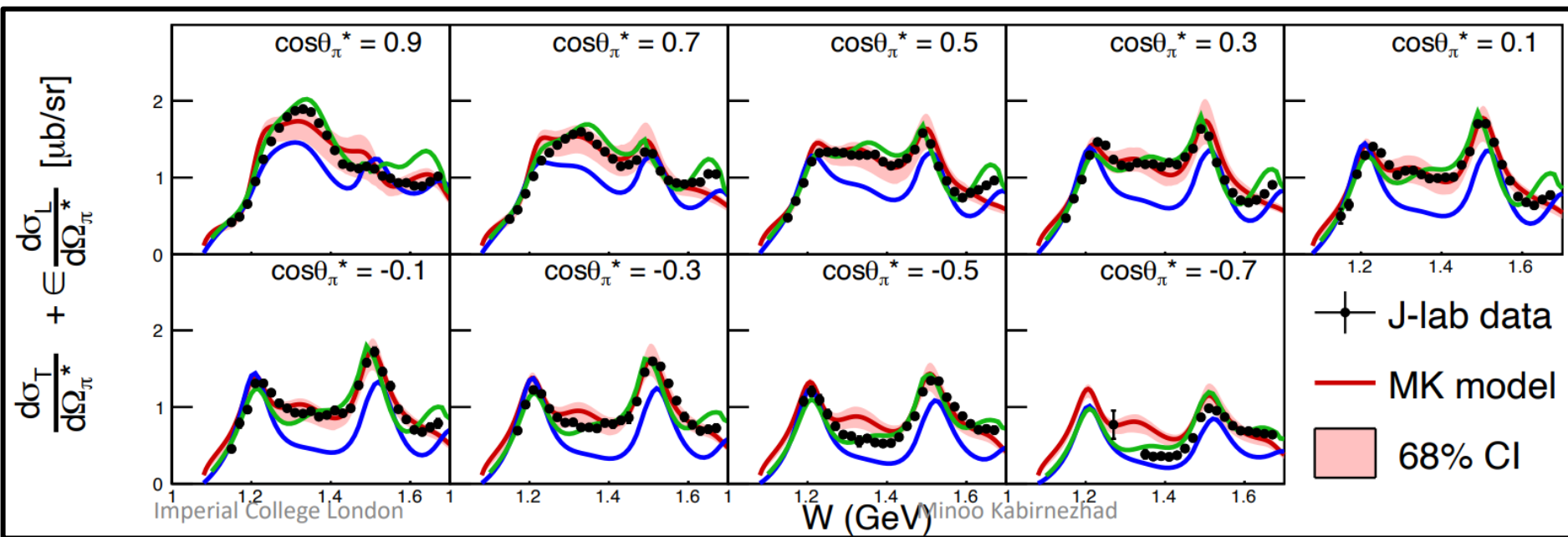
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Resonant Pion Production

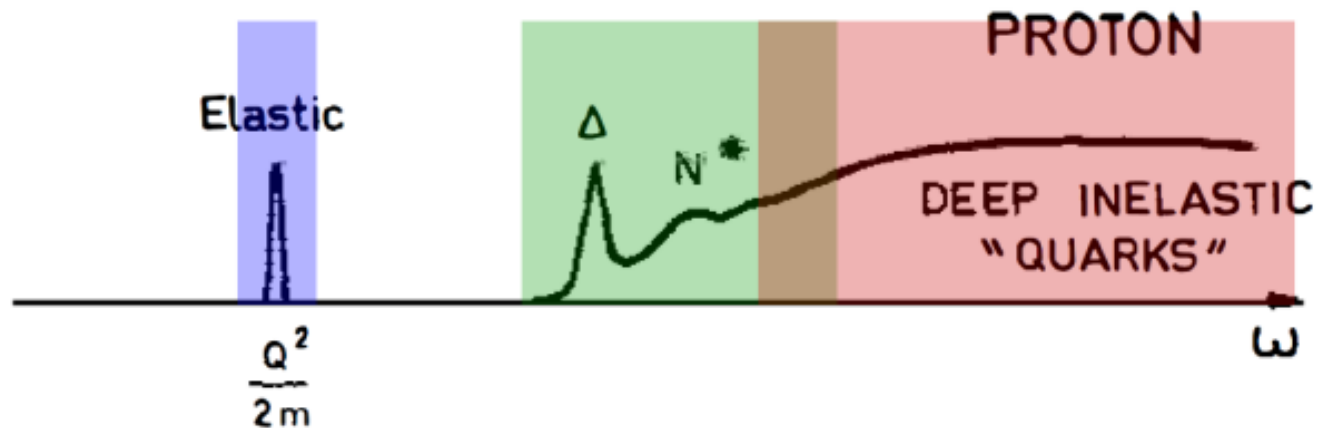
CCRES



- **New theory calculations** tuned to precision electron scattering data are on the horizon
 - E.g.: [MK model at NuINT 2022](#)
- The axial component remains a challenge

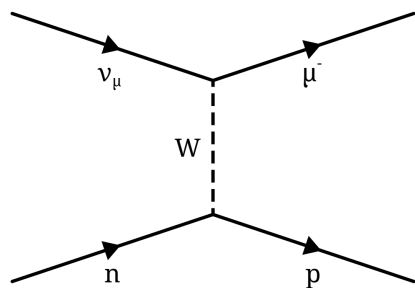


Neutrino nucleon scattering



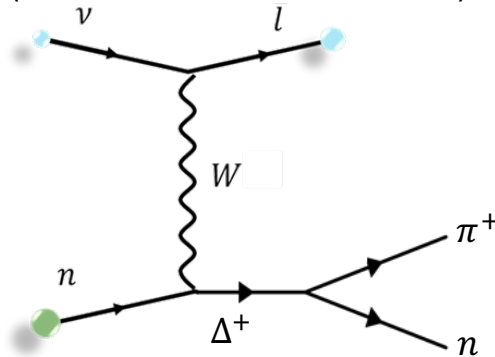
CCQE

(Quasi-elastic Scattering)



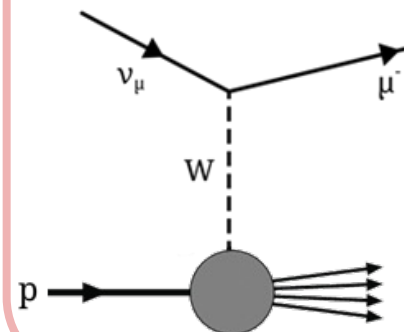
CCRES

(Resonant Meson Production)



CCDIS

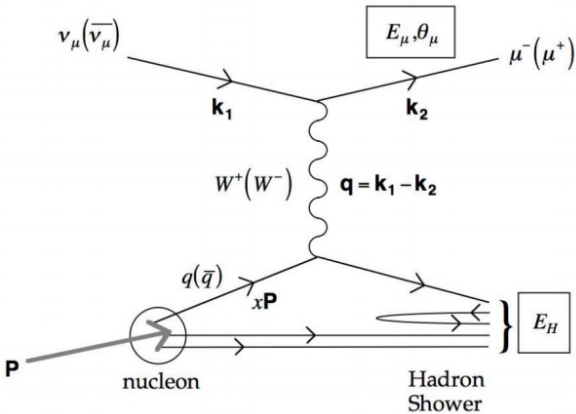
(Deep Inelastic Scattering)



Increasing Energy Transfer

Deep inelastic scattering

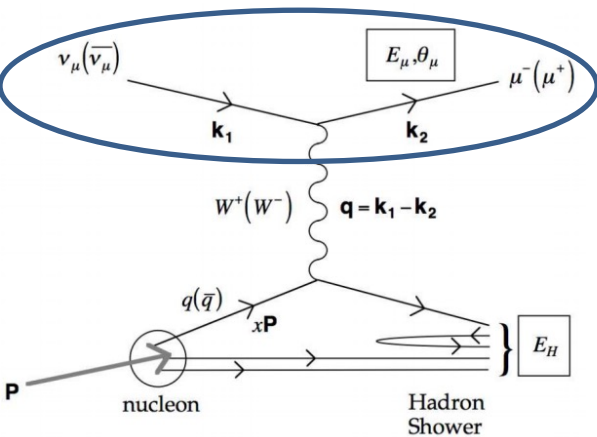
CCDIS



- Given enough energy, neutrinos can resolve the quarks within a nucleon. This is deep inelastic scattering.
- At high energies, the *inclusive* (i.e. integrating over possible hadronic final states) cross-section is fairly well understood (perturbative QCD):

Deep inelastic scattering

CCDIS



- Given enough energy, neutrinos can resolve the quarks within a nucleon. This is deep inelastic scattering.
- At high energies, the *inclusive* (i.e. integrating over possible hadronic final states) cross-section is fairly well understood (perturbative QCD):

$$\frac{d^2 \sigma^{\nu, \bar{\nu}}}{dx dy} = \frac{G_F^2 M E_\nu}{\pi (1 + Q^2/M_{W,Z}^2)^2} \left[\frac{y^2}{2} 2xF_1(x, Q^2) + \left(1 - y - \frac{Mxy}{2E}\right) F_2(x, Q^2) \pm y \left(1 - \frac{y}{2}\right) xF_3(x, Q^2) \right]$$

Bjorken x and y

$$x = \frac{Q^2}{2M\nu} = \frac{Q^2}{2ME_\nu y}$$

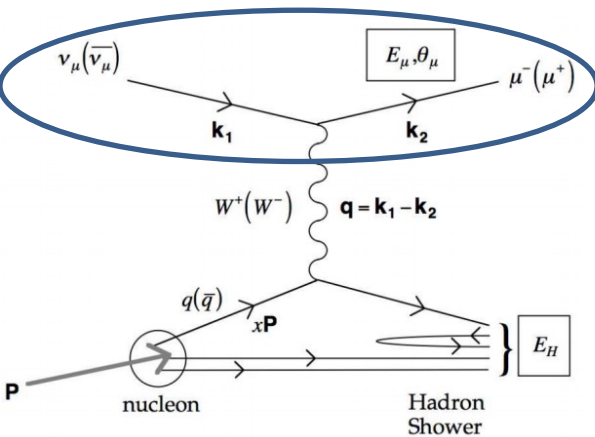
$$y = E_{had}/E_\nu$$

$$Q^2 = -m_\mu^2 + 2E_\nu(E_\mu - p_\mu \cos \theta_\mu)$$

- The $F_i(x, Q^2)$ are nuclear structure functions, which are dimensionless and encompass the quark structure of nucleons
- The first two can be measured with e-scattering, the last one is from the weak VA interference term: **only accessible with neutrinos!**

Deep inelastic scattering

CCDIS



- At low energies (or actually low Q^2) QCD becomes non-perturbative.
- Bodek-Yang: extrapolate down to low Q^2 assuming some parametrised scaling. Fix the details with e-scattering, apply to ν -scattering
- But this is an empirical treatment that comes with uncertainties

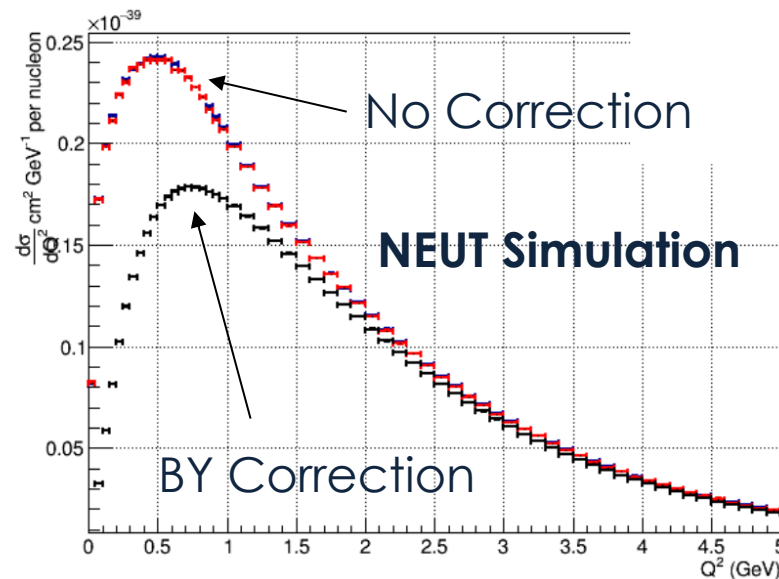
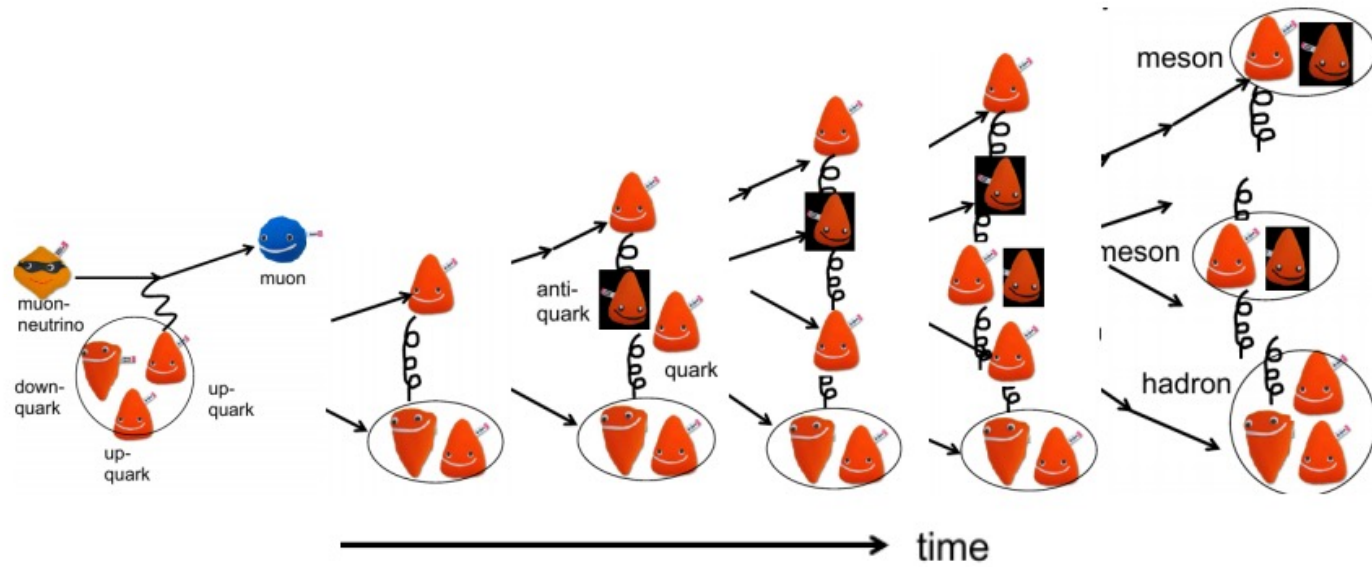


Diagram illustrating a neutrino interaction with a nucleon:

- An incoming neutrino with momentum ν_μ and energy E_μ, θ_μ interacts via a $W^+(W^-)$ boson with a nucleon.
- The interaction produces a muon ($\mu^-(\mu^+)$) and a hadron shower.
- The momentum transfer is $q = k_1 - k_2$.
- The hadron shower is represented by a series of lines and a box labeled E_H .

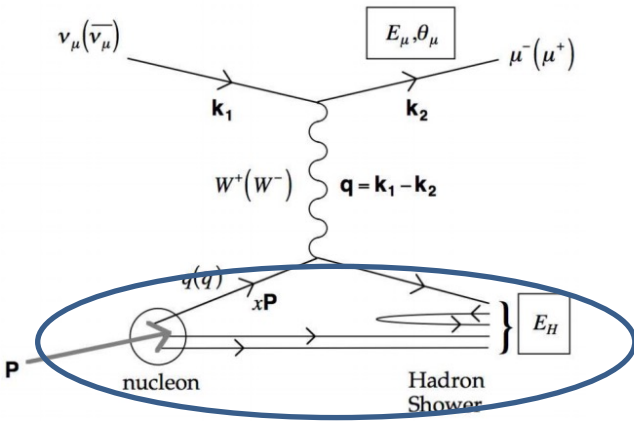
- The hadronic side of DIS interactions requires more empirical treatments
- Often the PYTHIA generator is used, but this is really built for much higher energies than used in most neutrino experiments



Stephen Dolan

Deep inelastic scattering

CCDIS

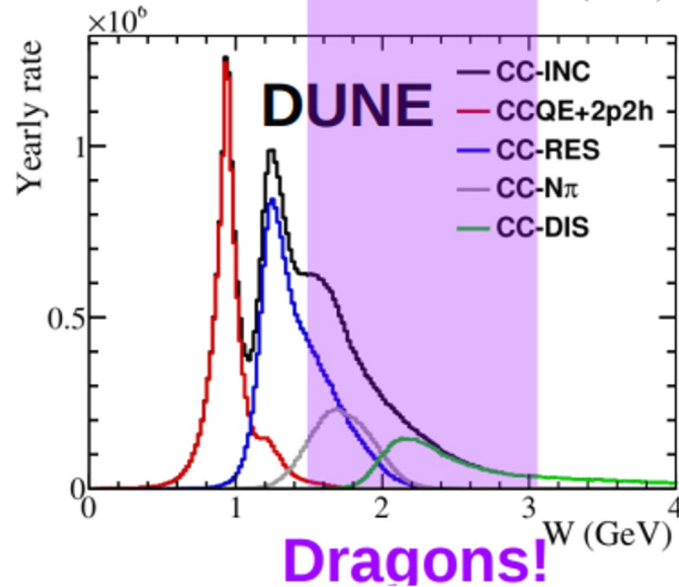
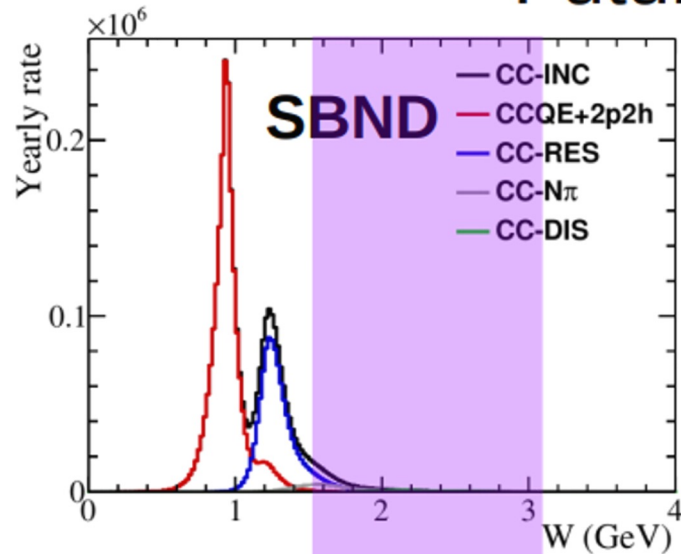


- The hadronic side of DIS interactions requires more empirical treatments
- Often the PYTHIA generator is used, but this is really built for much higher energies than used in most neutrino experiments

“I would not trust PYTHIA for anything with less than 6 pions”

S. Prestel (a PYTHIA author)

Future (^{40}Ar)



Dragons!

- Low- W , CC0 π and Δ region well covered by SBN data
- Timely data to guide ongoing theory efforts
- DUNE (ND here) has a significant high- W component
- Not well covered by theory
- No relevant data on ^{40}Ar

23

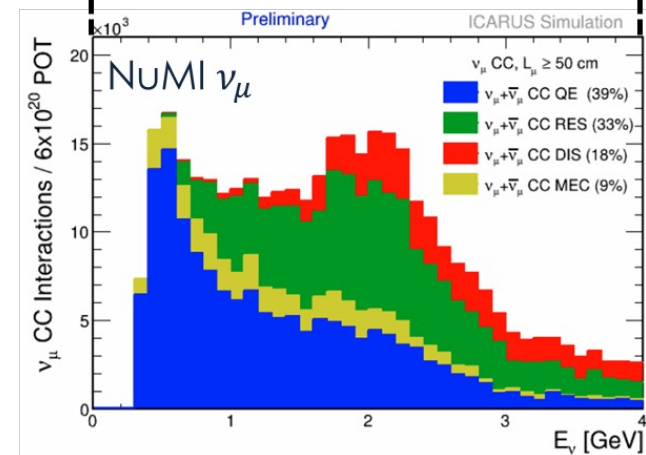
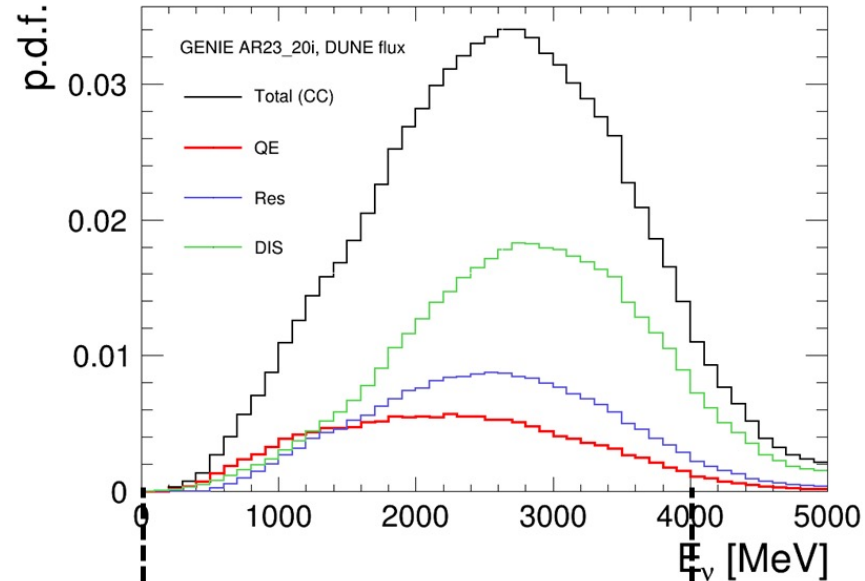
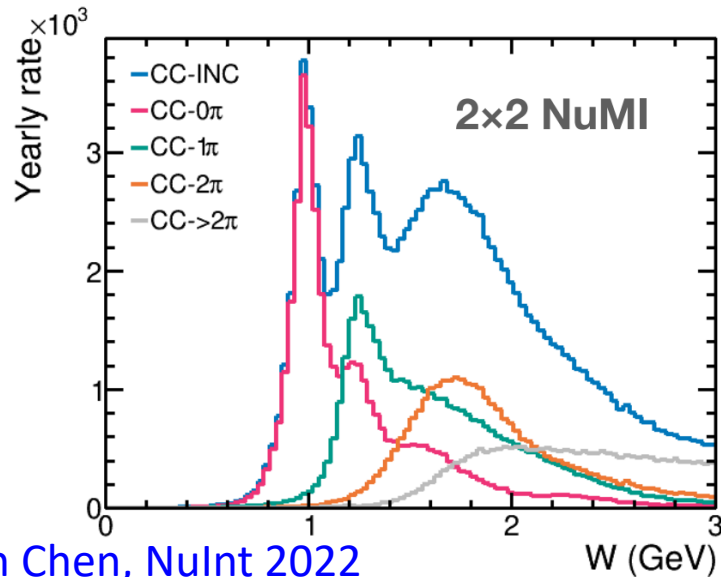
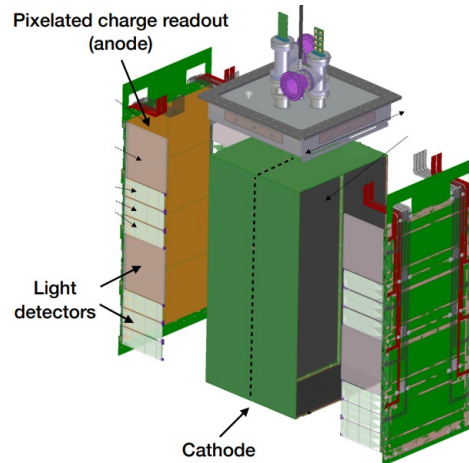
W = hadronic invariant mass

C. Wilkinson, NuPhys 2018

Escaping the dragons: 2x2 and ICARUS

A prototype **DUNE “2x2” detector** is now operating in the “NuMI” beam

ICARUS sees the “NuMI” beam with energies that overlap better with DUNE's



Yifan Chen, NuInt 2022

Summary so far

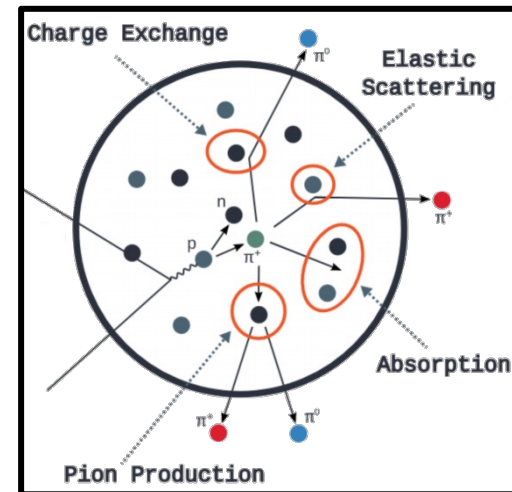
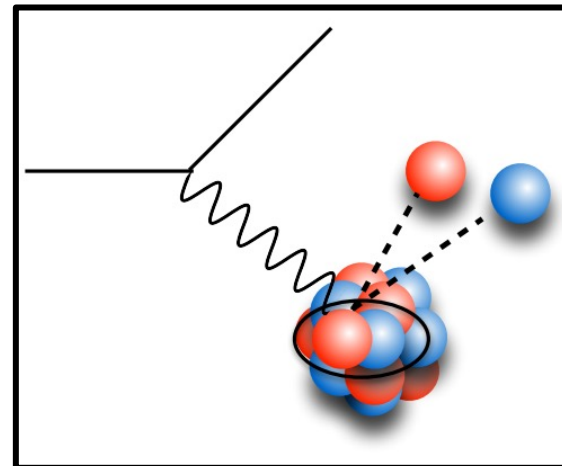
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 - Point-like scattering is “easy” to calculate
 - Interactions with nucleons is more challenging **due to their finite extent**

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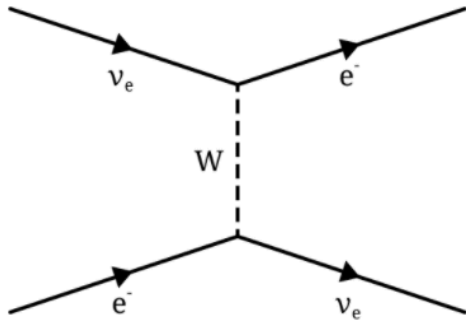
- Weak Interactions with neutrinos
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- Neutrino-nucleon interactions
 - QE: almost calculable with some form factors
 - RES: **much more difficult**, lots of diagrams to consider
 - DIS: easy for *inclusive* high Q^2 , hard at low Q^2 , **hadronic side a total guess**

Overview

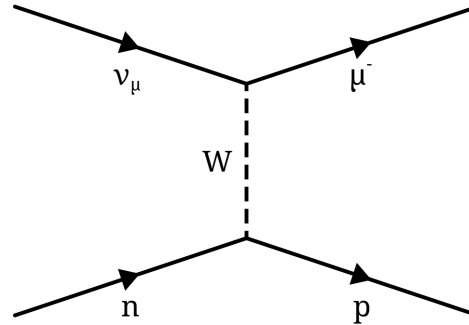
- Neutrino Interactions: A History
 - Weak interactions from Fermi to SM
- Neutrino-nucleon interactions
 - QE, RES and DIS
- Neutrino-nucleus interactions
 - Nuclear effects
 - The rise and fall of $M_A = 1.3 \text{ GeV}$
- Neutrino event generators
 - Theory inputs
 - Filling in the gaps
- Neutrino-nucleus interaction measurements
 - Inclusive successes and exclusive failures
- Why do we care?
 - Neutrino interactions for neutrino oscillations
 - Neutrino energy reconstruction
- Don't Panic! The future of neutrino interaction simulations



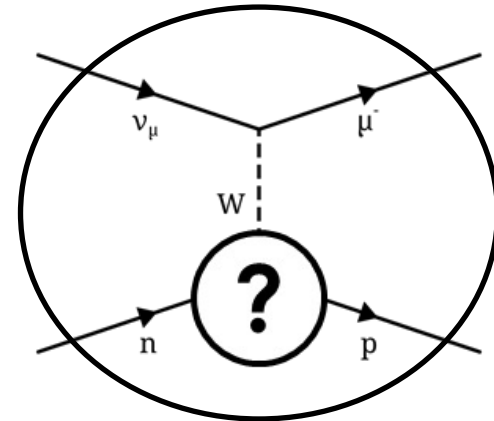
Beyond nucleon scattering



Point-like: Masters
homework problem



Nucleon: mostly harmless



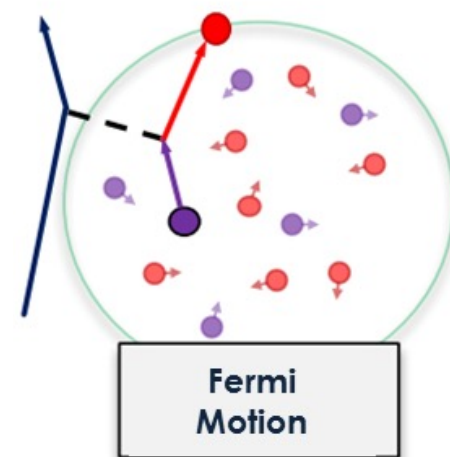
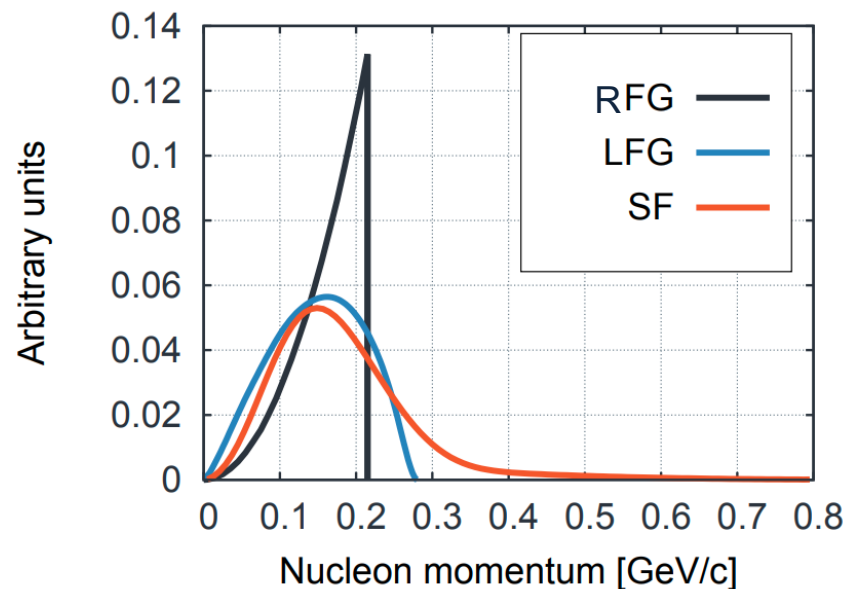
Nucleus: very hard

- For few-GeV neutrino interactions with nuclei, the impact of nuclear physics processes cause leading-order alterations from the nucleon scattering case
- I leave most details to Noemi's talks

Nuclear effects in a nutshell

- **Fermi Motion**

- Nucleons are moving targets
- Their momenta are not so different than typical E_ν for our experiments ...



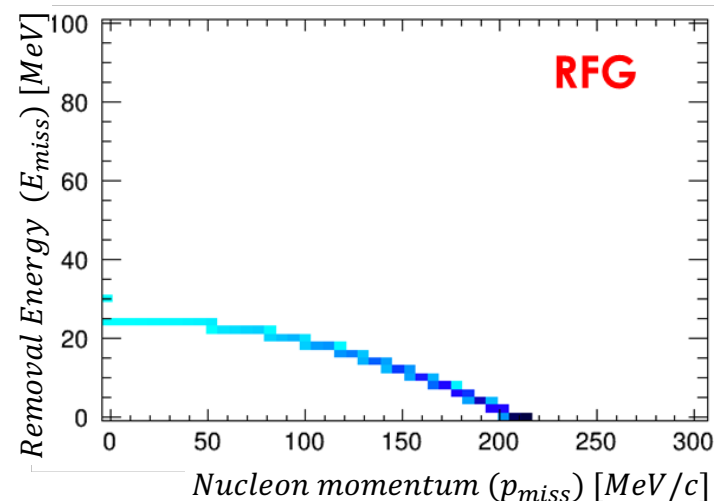
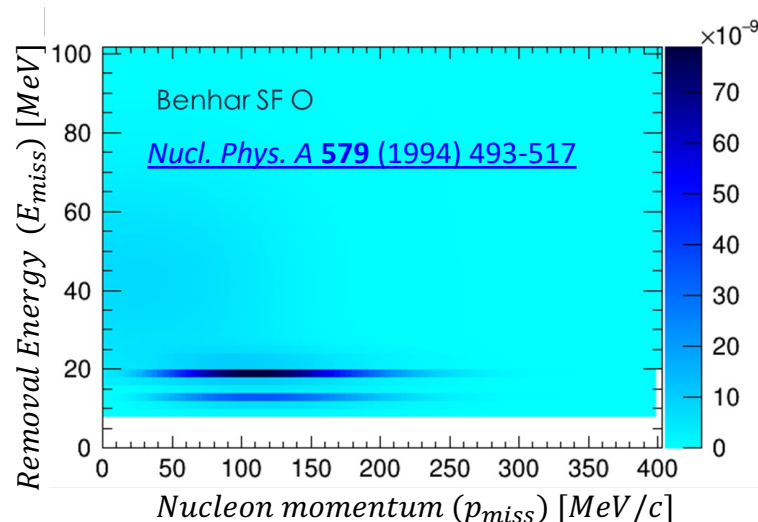
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- Some amount of energy is needed to free them
- Most models predict that removal energy and Fermi motion should be correlated



Nuclear effects in a nutshell

- **Fermi Motion**

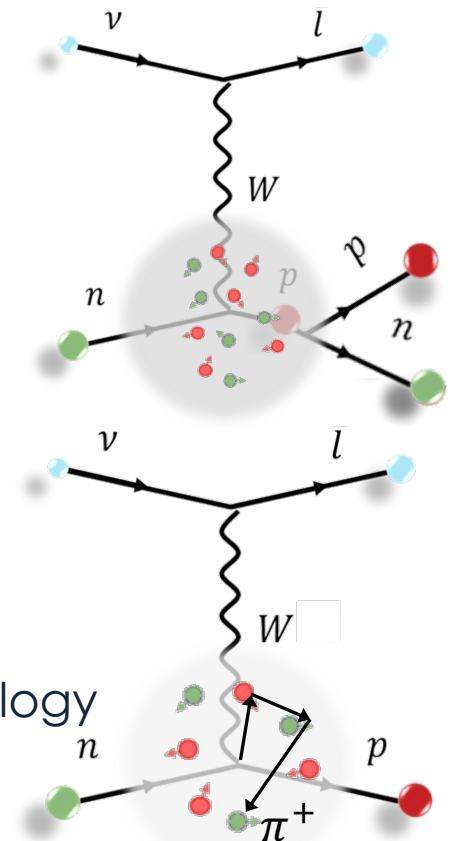
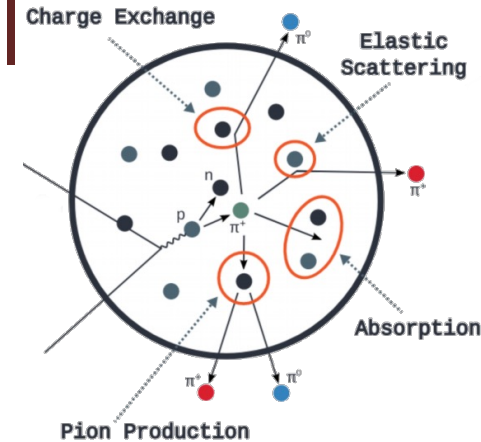
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- **Final State Interactions (FSI)**

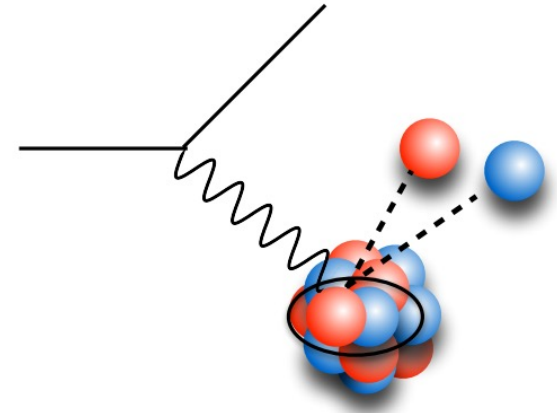
- Hadrons don't exit the nucleus cleanly
- They can re-interact inside the nucleus
- Distorts kinematics and changes the final state topology
- Full calculation also changes *inclusive* cross section



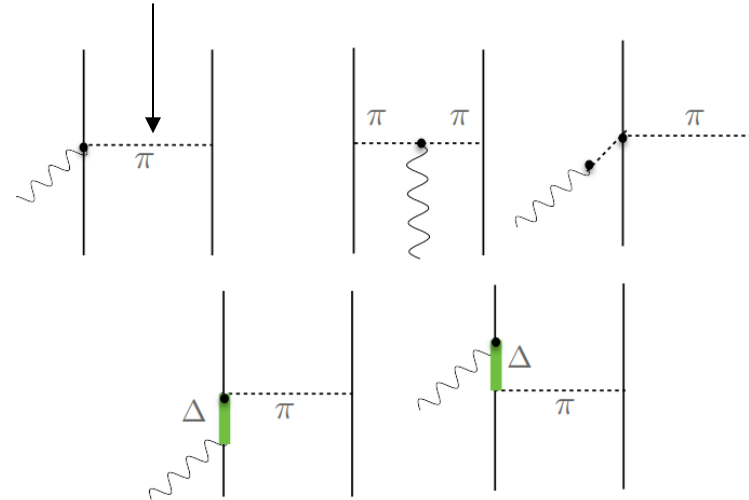
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- **Multi-nucleon Interactions**

- Nucleons are interacting with each other inside the nucleus
- Some interactions are with nucleons bound together somehow
- Multi-nucleon “2p2h” final states



“Meson-exchange currents” (MEC)



[N. Rocco INSS 2019](#)

Nuclear effects in a nutshell

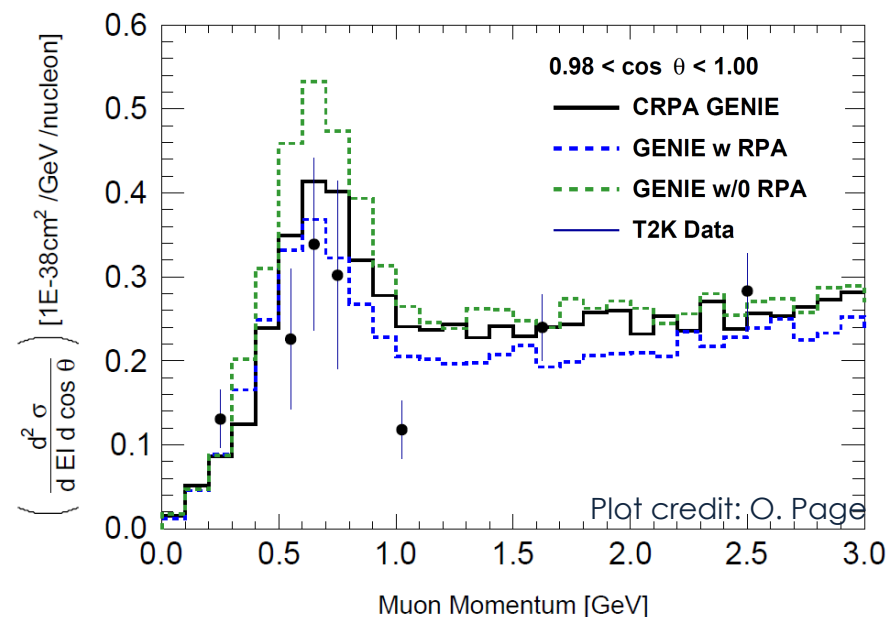
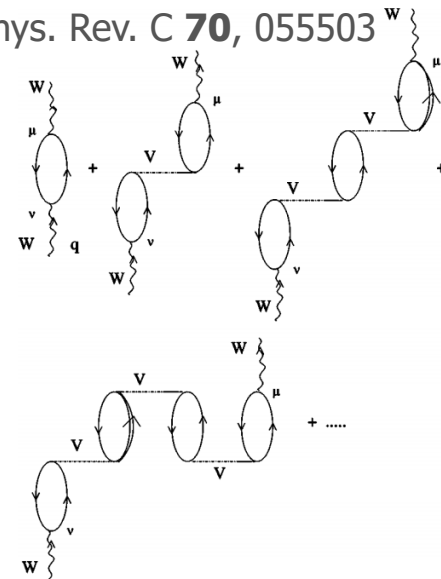
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- Difficult physics, usually parameterised and treated via “RPA” (random phase approximation)

Phys. Rev. C **70**, 055503



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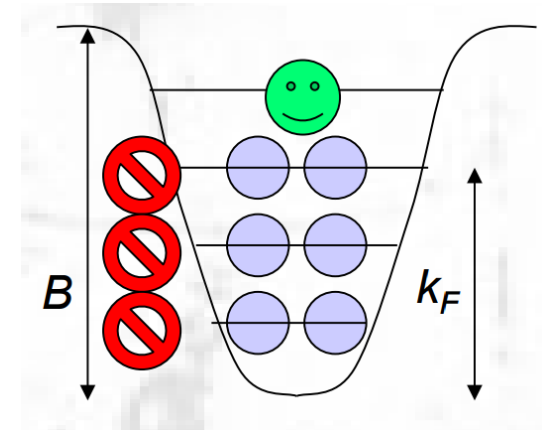
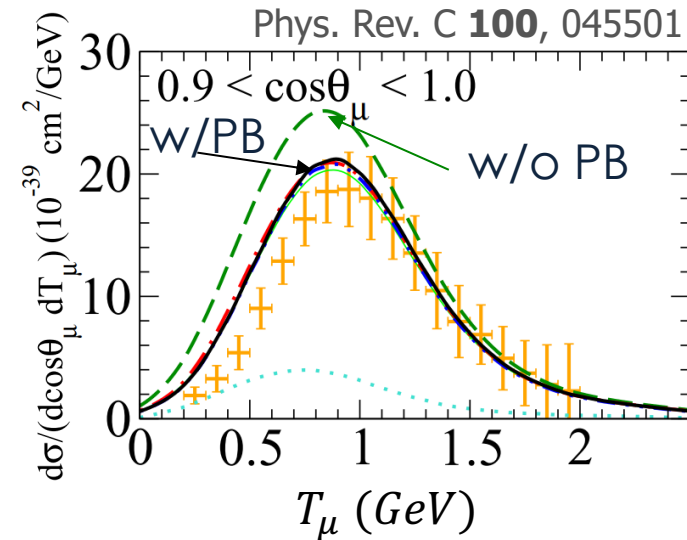
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- **Pauli Blocking**

- Nucleons cannot be excited into nuclear states that are already filled
- Reduction of cross section at low energy transfer



Borrowed from [K. McFarland's INSS 2014 lectures](#)

Consequences of nuclear effects

- **Altered cross section**
 - Nuclear effects significantly alter the cross section with respect to the nucleon case

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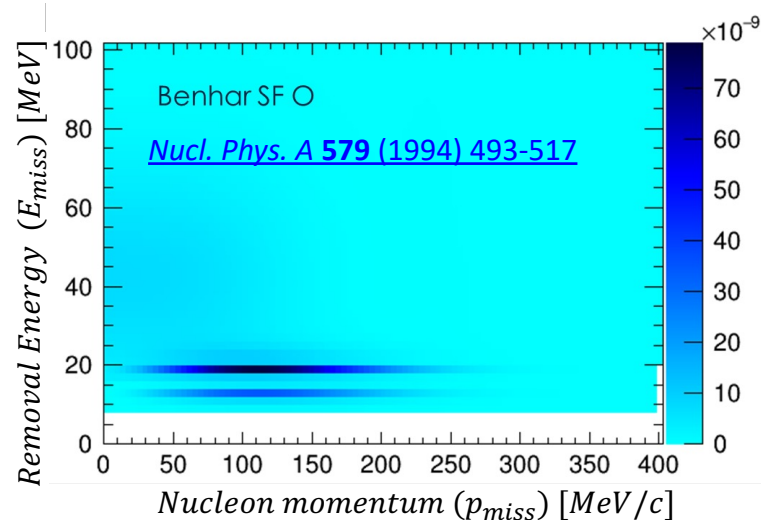
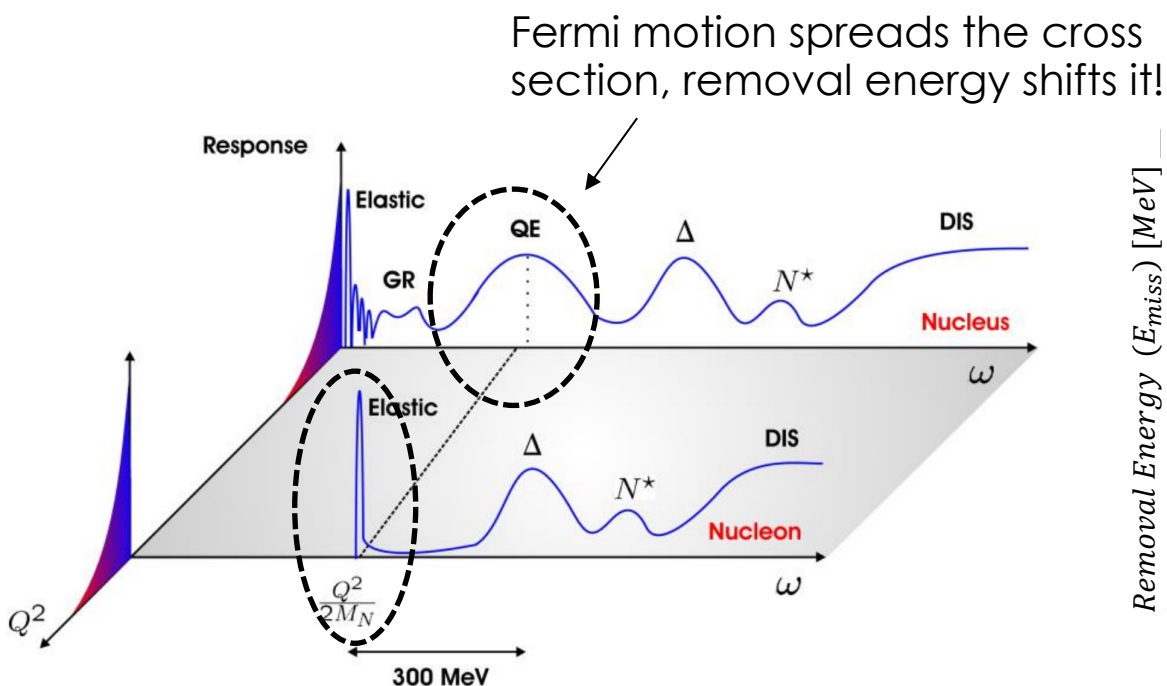
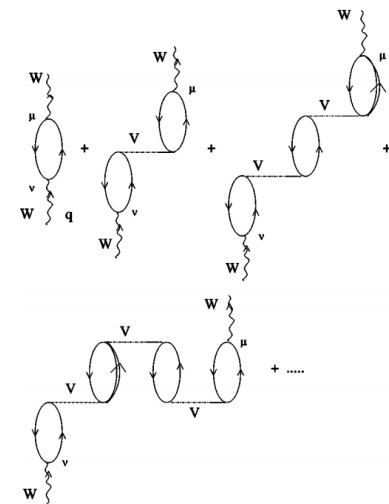


Fig. from N. Jachowicz

Consequences of nuclear effects

- **Altered cross section**

- Nuclear effects significantly alter the cross section with respect to the nucleon case



RPA and Pauli blocking further suppresses the cross section at low energy transfers

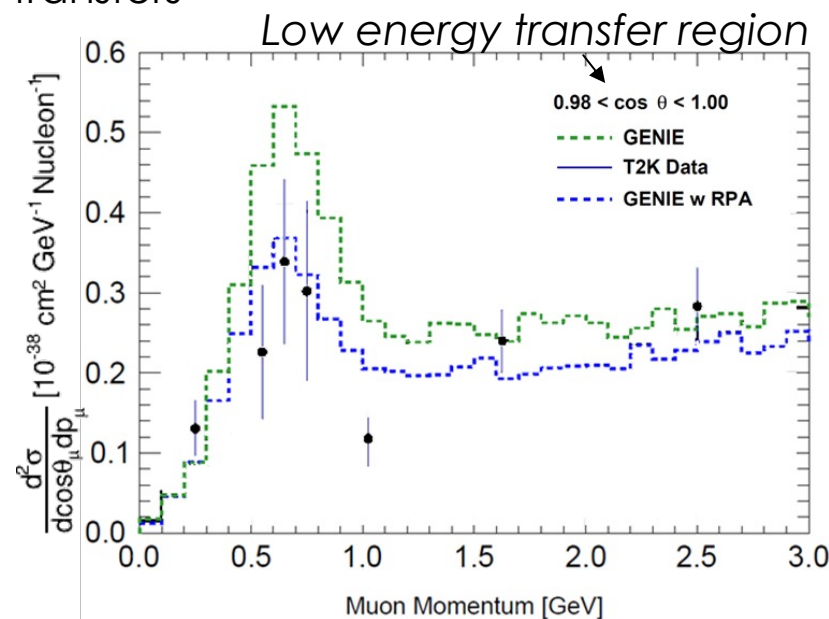
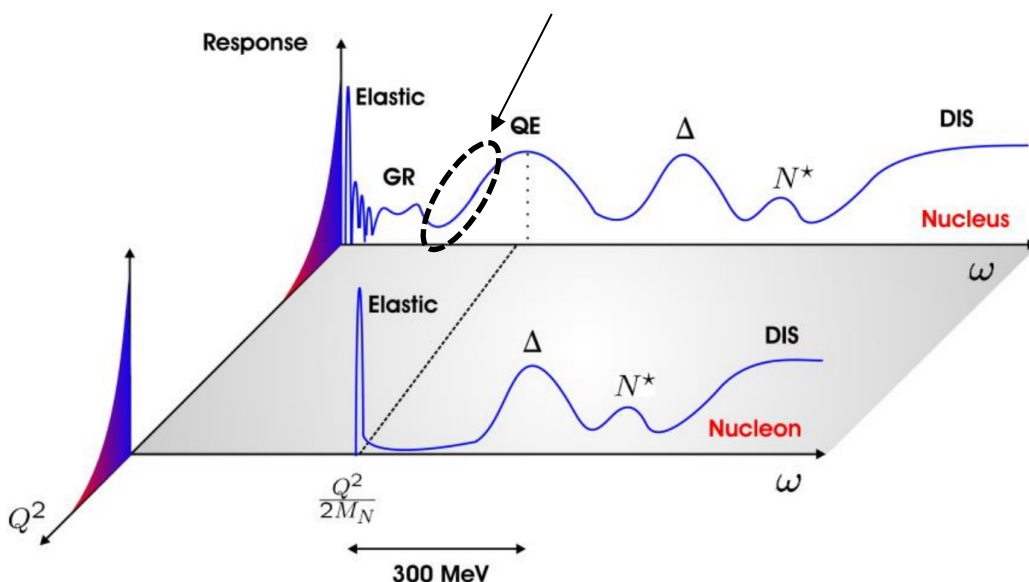


Fig. from N. Jachowicz

Consequences of nuclear effects

- **Altered cross section**

- Nuclear effects significantly alter the cross section with respect to the nucleon case

2p2h adds a contribution where there previously wasn't any "the dip region"

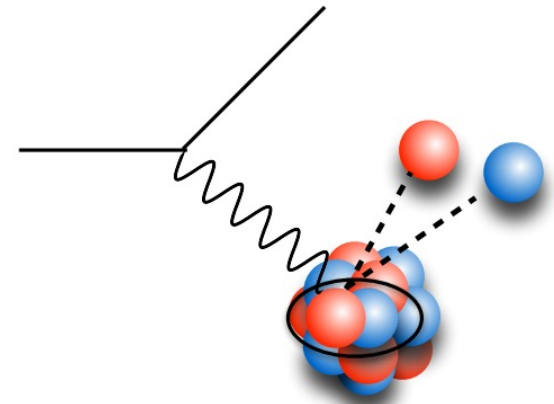
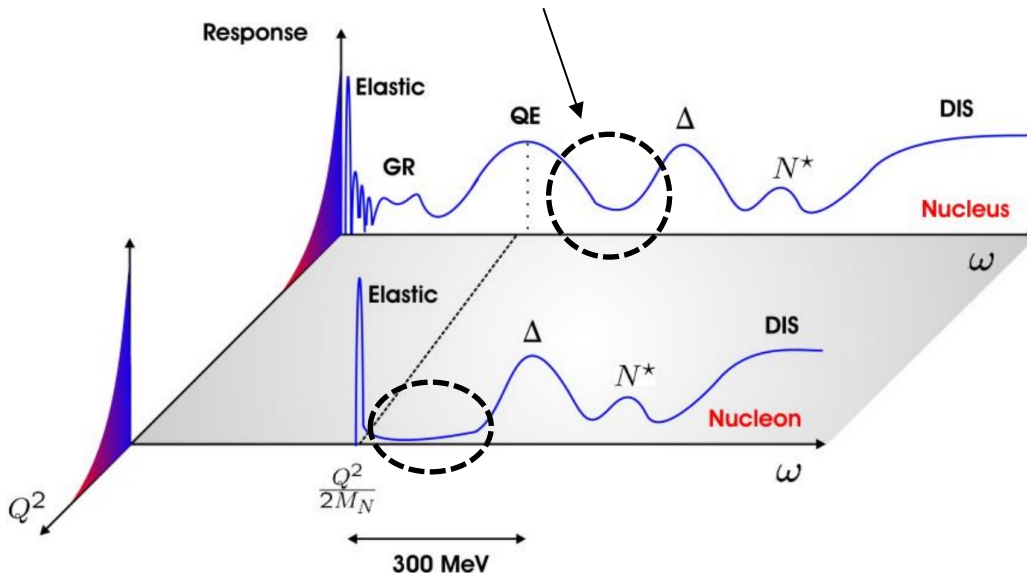


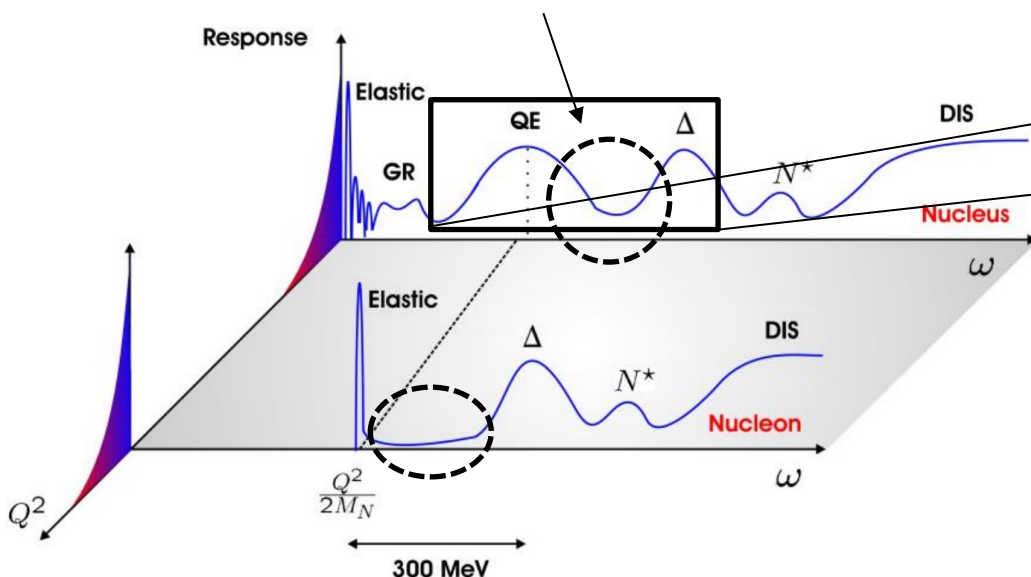
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Electron scattering data

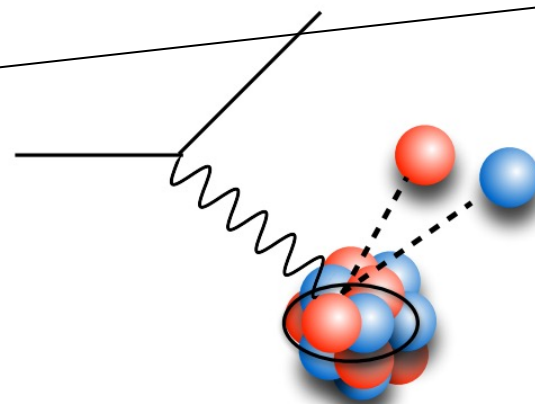
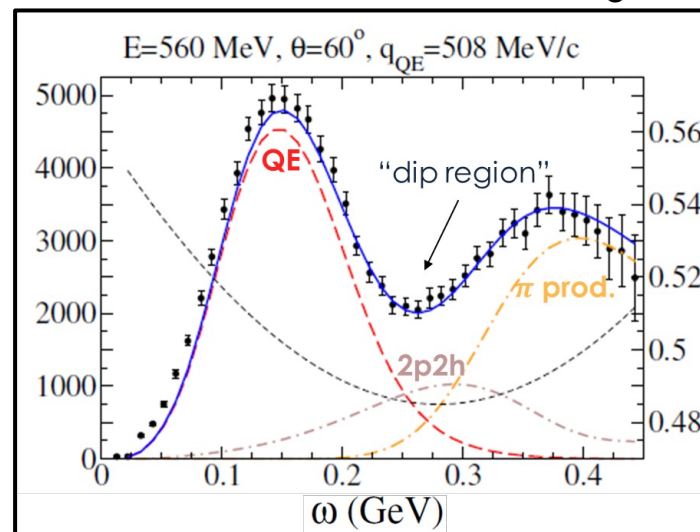


Fig. from N. Jachowicz

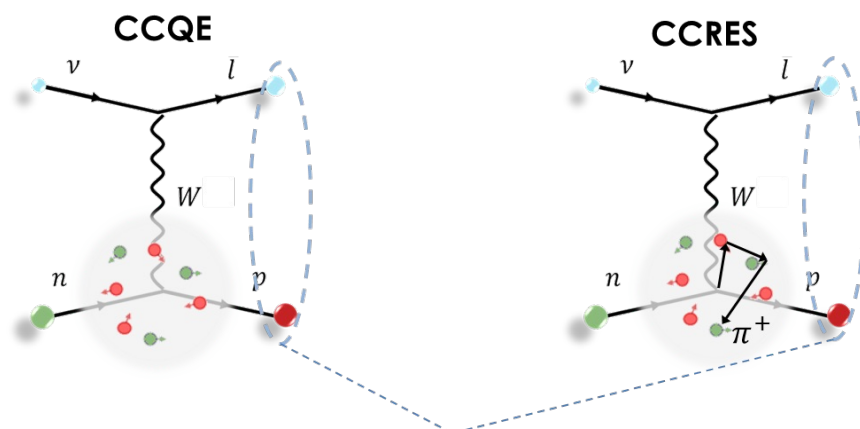
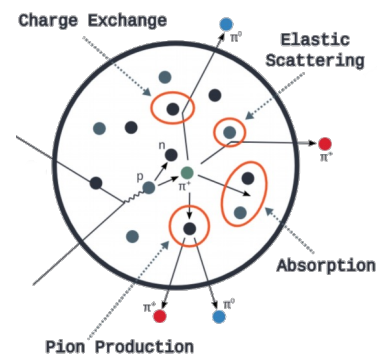
Consequences of nuclear effects

- **Altered cross section**

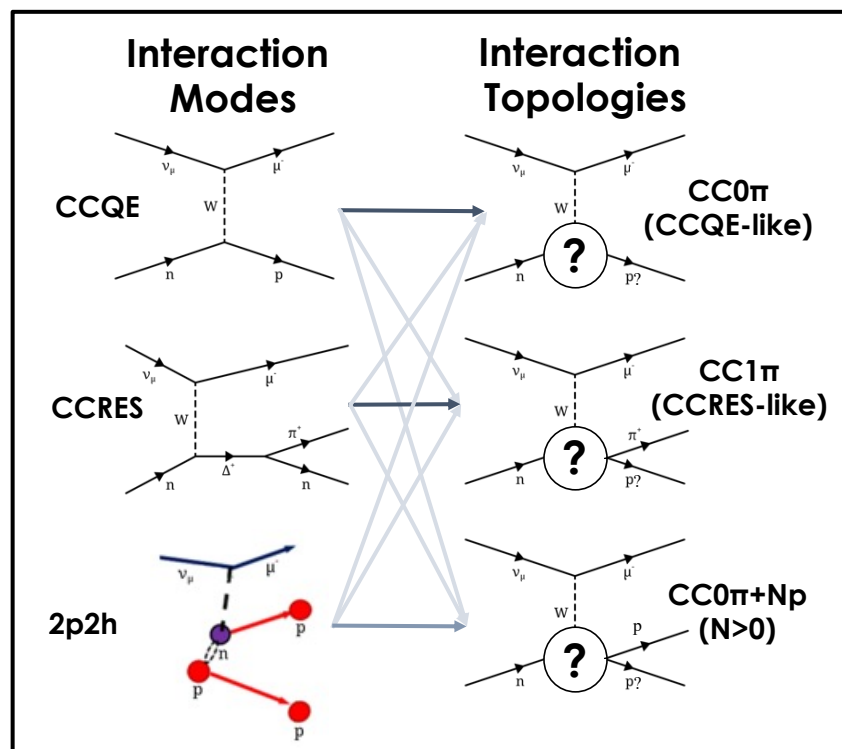
- Nuclear effects significantly alter the cross section with respect to the nucleon case

- **Altered hadronic final state**

- Final state interactions hide/distort the interaction channel

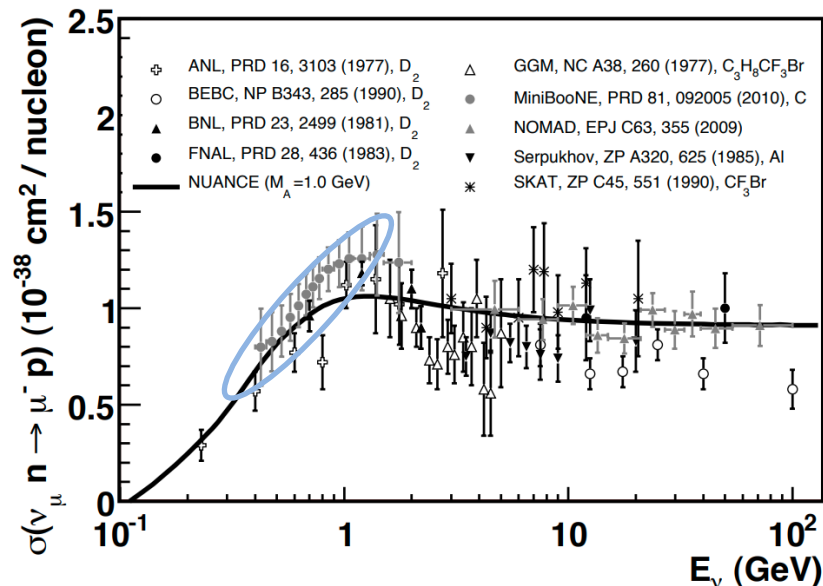


Final state interactions (FSI) can cause different interaction modes to have the same final state

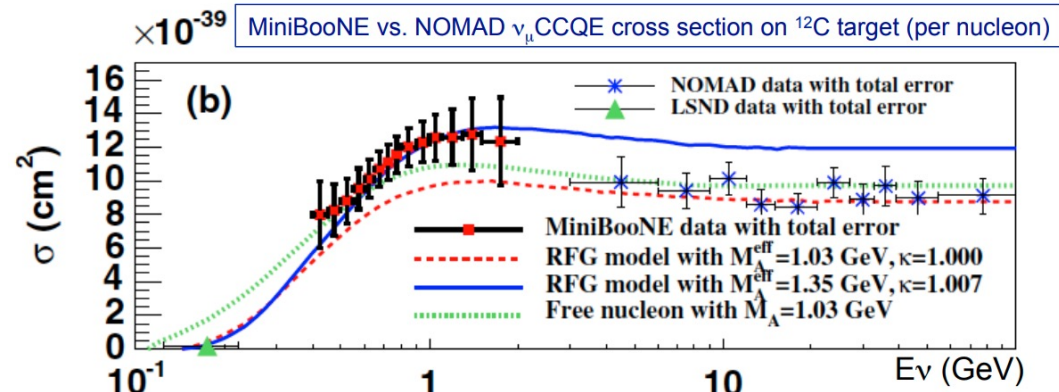


Example: nucleon axial mass “puzzle”

- Some heavier nuclear target experiments also try to measure M_A
- The MiniBooNE experiment (carbon-based target) prefers a much higher M_A to the bubble chambers

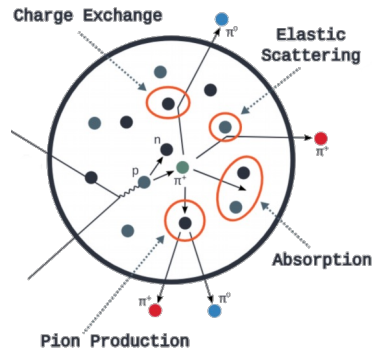
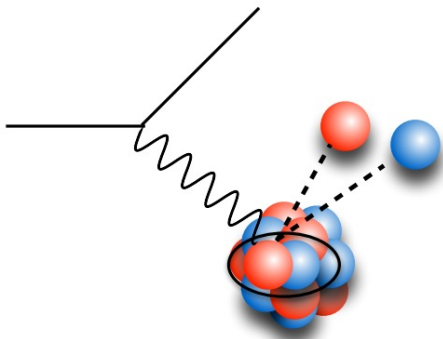


J. A. Formaggio and G. P. Zeller Rev. Mod. Phys. **84**, 1307



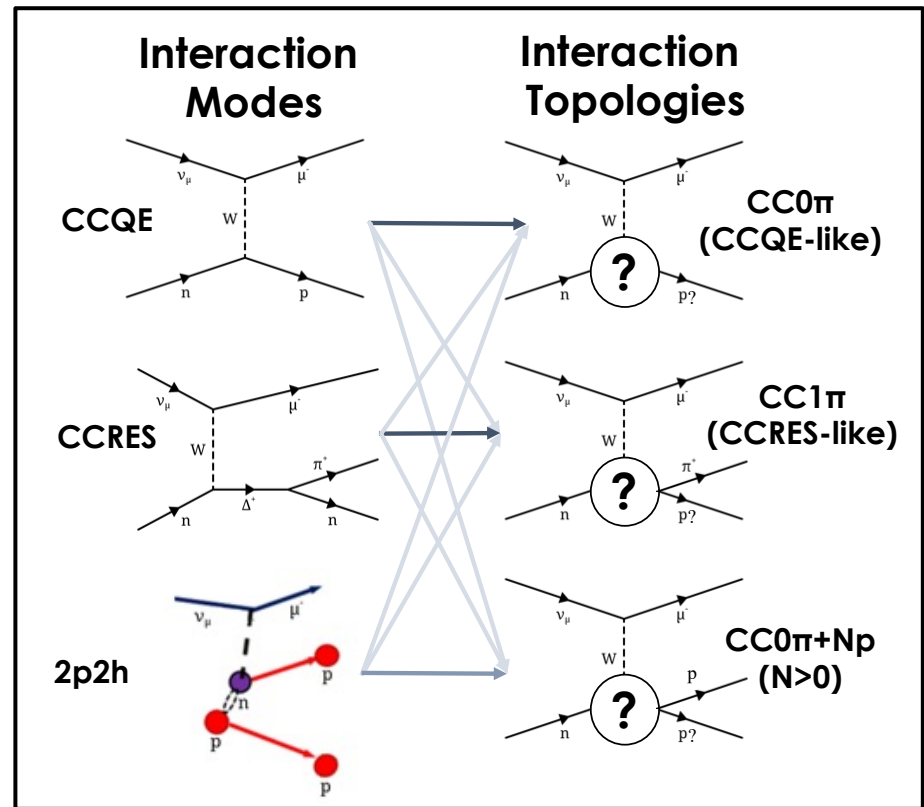
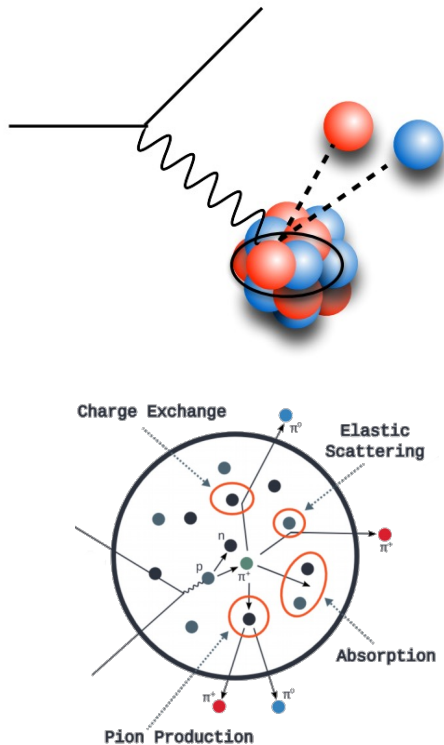
The nucleon axial mass “puzzle”

- What MiniBooNE really measured wasn't CCQE, they just looked for interactions with no mesons in the final state



The nucleon axial mass “puzzle”

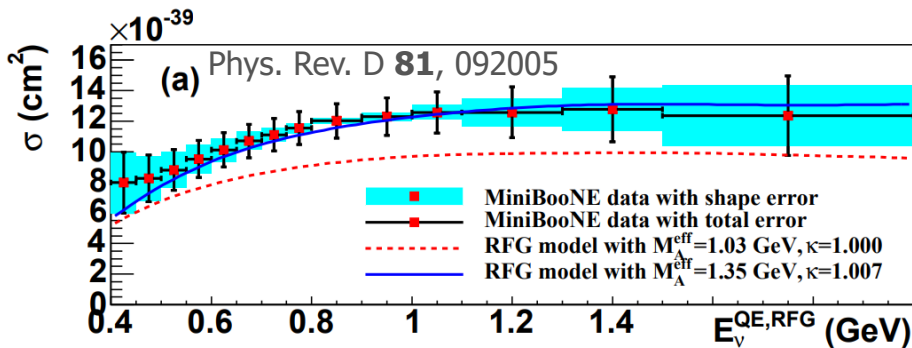
- What MiniBooNE really measured wasn't CCQE, they just looked for interactions with no mesons in the final state
- This should include contributions from 2p2h (and FSI with pion absorption)!



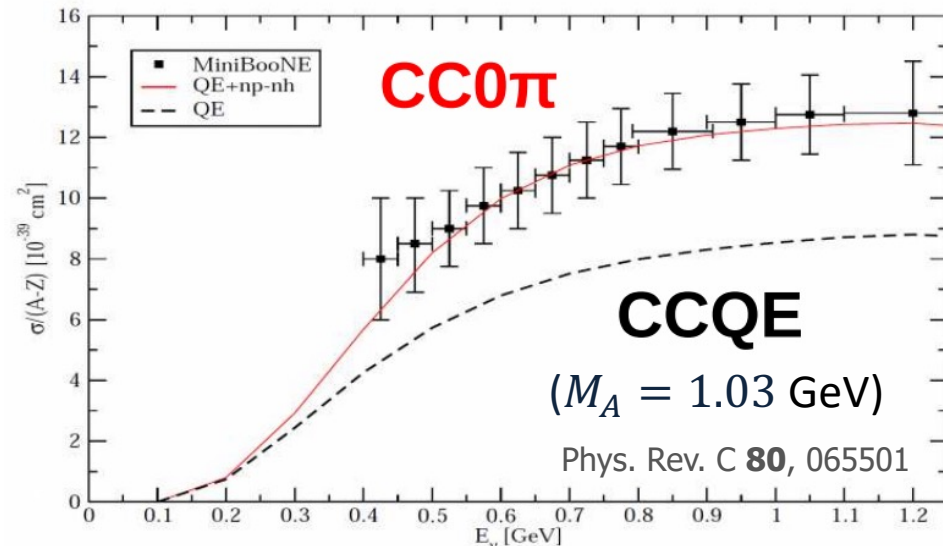
The nucleon axial mass “puzzle”

- What MiniBooNE really measured wasn't CCQE, they just looked for interactions with no mesons in the final state
- This should include contributions from 2p2h (and FSI with pion absorption)!

If we only consider CCQE, $M_A \sim 1.3$ GeV



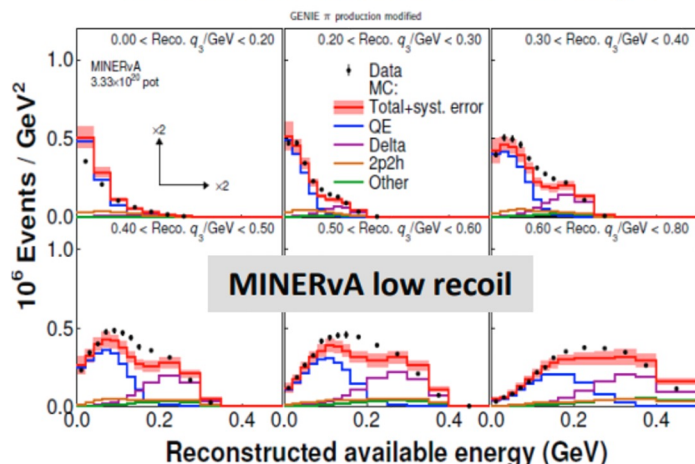
But with 2p2h, M_A is compatible with 1.0 GeV suggested by bubble chambers



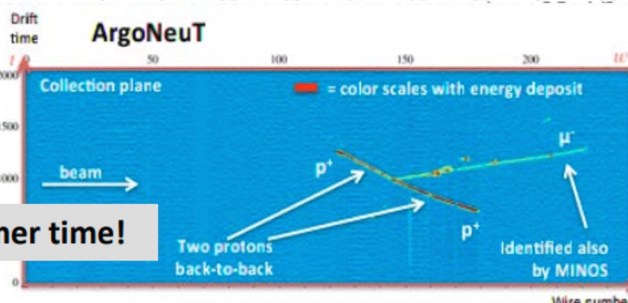
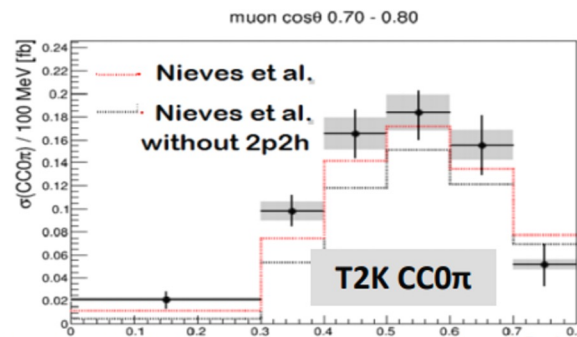
The nucleon axial mass “puzzle”

Dramatic Conclusion

- For the first time, we have multiple observables pointing to a two body current contribution to CCQE



It's Hammer time!



21 November 2015

NuInt15 QE Summary: KSM, JN, RW

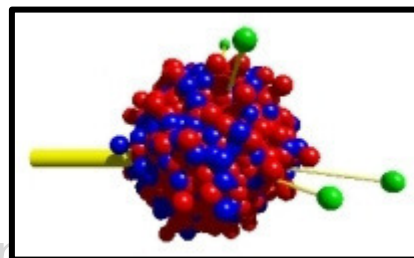
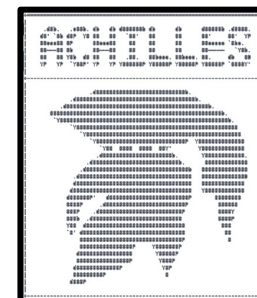
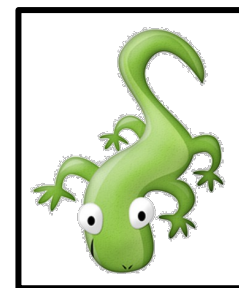
NuInt 2015 Summary
by Kevin McFarland

- It's time to say goodbye to $M_A^{\text{effective}}$



Overview

- Neutrino Interactions: A History
 - Weak interactions from Fermi to SM
- Neutrino-nucleon interactions
 - QE, RES and DIS
- Neutrino-nucleus interactions
 - Nuclear effects
 - The rise and fall of $M_A = 1.3 \text{ GeV}$
- Neutrino event generators
 - Theory inputs
 - Filling in the gaps
- Neutrino-nucleus interaction measurement
 - Inclusive successes and exclusive failures
- Why do we care?
 - Neutrino interactions for neutrino oscillations
 - Neutrino energy reconstruction
- Don't Panic! The future of neutrino interaction simulations



Meet the generators

Experiments model all this using **neutrino interaction event generators**



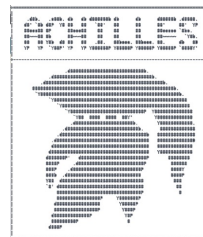
NEUT



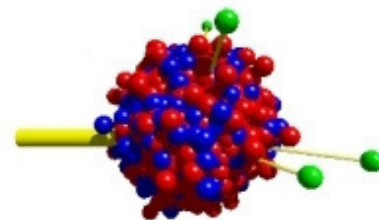
GENIE



NuWro



AChILLES



GiBUU

- Used by T2K, SK, Hyper-K
- Updated according to experiments needs

- Very widely used
- Large dev team separate from experiments

- Wide range of models available
- Driven by theory
- Few developers

Meet the generators

Experiments model all this using **neutrino interaction event generators**



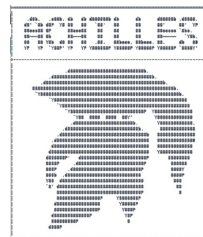
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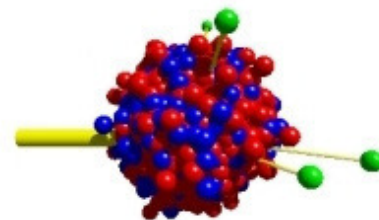
GENIE



NuWro



AChILLES



GiBUU

- New generator from theorists
- Only QE for the moment, but with novel FSI
- e/nu-scattering equivalence built in from the ground-up

- Full theory in its own right
- Predicts nu/e/hadron scattering in the same framework
- Very different philosophy to other generators
- Few developers

Meet the generators

Experiments model all this using **neutrino interaction event generators**



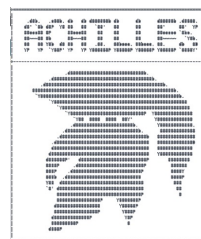
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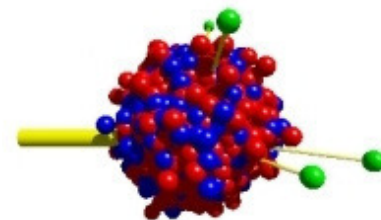
GENIE



NuWro



AChILLES



GiBUU

- To be used in experimental analyses, generators must be able to produce fully exclusive neutrino interactions. I.e.:
 - The full list of final state particles
 - The 4-momentum of each one
 - For all interaction channels

Meet the generators

Experiments model all this using **neutrino interaction event generators**



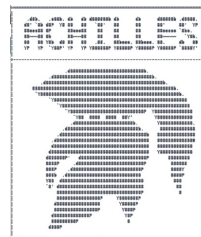
NEUT



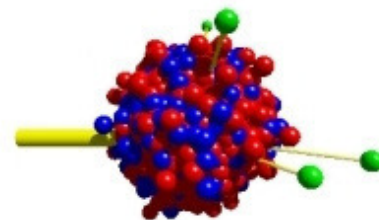
GENIE



NuWro



AChILLES



GiBUU

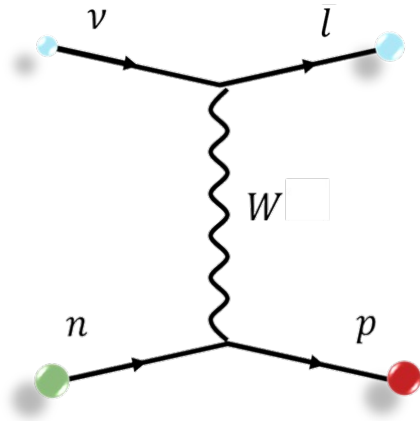
- To be used in experimental analyses, generators must be able to produce fully exclusive neutrino interactions. I.e.:
 - The full list of final state particles
 - The 4-momentum of each one
 - For all interaction channels
- The generators take theory inputs where possible, but ultimately *ad-hoc* approximations to “fill in the gaps” are needed

Theory inputs

- Generators take theory inputs where possible
- But theory inputs are limited in what they can predict
- Typical inputs include:
 - **Nucleon-level calculations**
 - **Inclusive calculations** (only predicts outgoing lepton kinematics)
 - **Factorized calculations**
 - **Exclusive calculations**

Neutrino-nucleon calculations

The most basic inputs are only neutrino-nucleon calculations: no nuclear effects



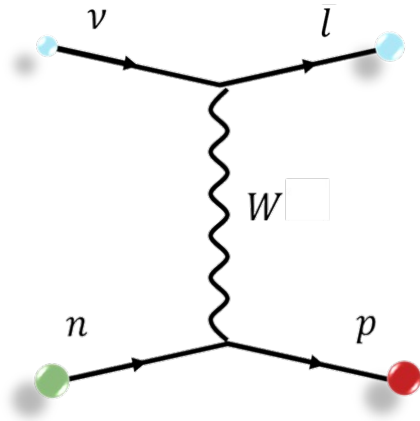
$$\frac{d\sigma}{d|q^2|} \left(\begin{matrix} \nu n \rightarrow l^- p \\ \bar{\nu} p \rightarrow l^+ n \end{matrix} \right) = \frac{M^2 G^2 \cos^2 \theta_c}{8\pi E_\nu^2} \left[A(q^2) \mp B(q^2) \frac{(s-u)}{M^2} + \frac{C(q^2)(s-u)^2}{M^4} \right]$$

$$(s-u = 4ME_\nu + q^2 - m^2) .$$

Generators are forced to “dress” the interaction with nuclear effects themselves

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$$(s-u = 4ME_\nu + q^2 - m^2) .$$

Generators are forced to “dress” the interaction with nuclear effects themselves

This is often still the level of input we work with

E.g.:

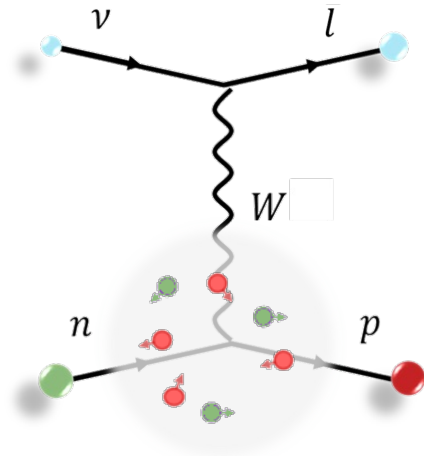
CCQE GENIEv2 or NEUT's “Smith-Moniz” Fermi gas model
All RES interactions in GENIE or NEUT

Inclusive calculations

Inclusive calculations come “pre-integrated” over hadron kinematics

All of the nuclear dynamics lives in here

$$\frac{d^2\sigma_{\nu l}}{d\Omega(\hat{k}')dE'_l} = \frac{|\vec{k}'|}{|\vec{k}|} \frac{G^2}{4\pi^2} L_{\mu\sigma} W^{\mu\sigma}$$



E.g. Inclusive quasielastic charged-current neutrino-nucleus reactions, J. Nieves et al, 2004

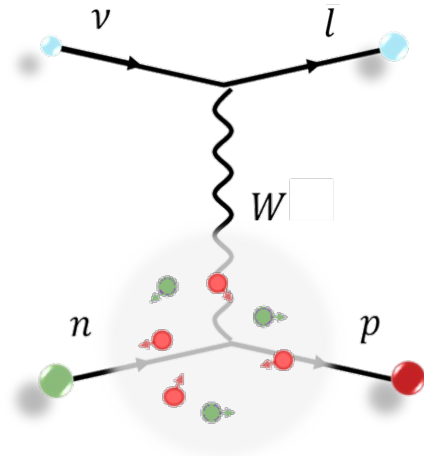
Nuclear effects are “baked in” to the model used for the integration

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Nuclear effects are “baked in” to the model used for the integration

Integrating over the nucleon kinematics makes this calculation more tractable:

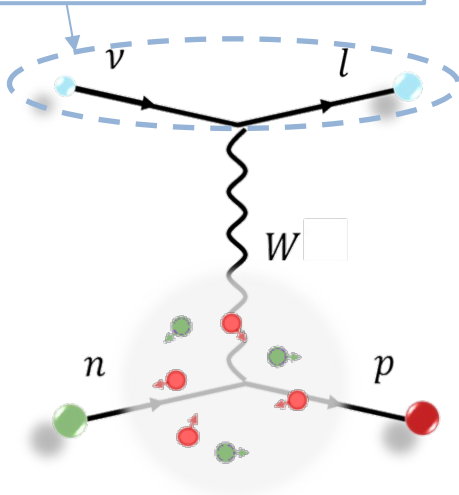
- Needs 6 “structure functions” built from 5 hadron tensor elements dependent on two variables

$$\begin{aligned} \frac{d^2\sigma_{\nu l}}{d\Omega(\hat{k}')dE'_l} = \frac{|\vec{k}'|E'_l M_i G^2}{\pi^2} & \left\{ 2W_1 \sin^2 \frac{\theta'}{2} + W_2 \cos^2 \frac{\theta'}{2} \right. \\ & - W_3 \frac{E_\nu + E'_l}{M_i} \sin^2 \frac{\theta'}{2} + \frac{m_l^2}{E'_l(E'_l + |\vec{k}'|)} \left[W_1 \cos \theta' \right. \\ & - \frac{W_2}{2} \cos \theta' + \frac{W_3}{2} \left(\frac{E'_l + |\vec{k}'|}{M_i} - \frac{E_\nu + E'_l}{M_i} \cos \theta' \right) \\ & + \frac{W_4}{2} \left(\frac{m_l^2}{M_i^2} \cos \theta' + \frac{2E'_l(E'_l + |\vec{k}'|)}{M_i^2} \sin^2 \theta' \right) \\ & \left. \left. - W_5 \frac{E'_l + |\vec{k}'|}{2M_i} \right] \right\} \end{aligned} \quad (10)$$

Inclusive calculations

Inclusive calculations come “pre-integrated” over hadron kinematics

Only predicts lepton kinematics!



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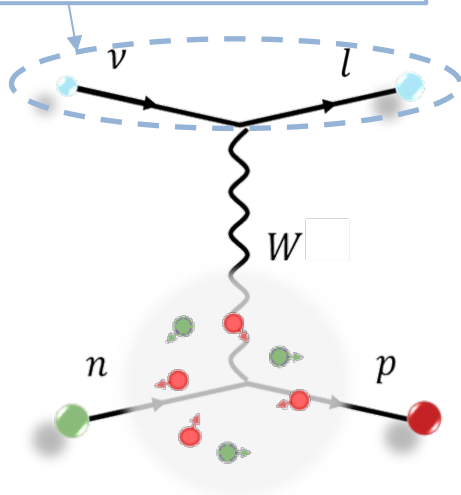
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Inclusive calculations

Inclusive calculations come “pre-integrated” over hadron kinematics

Only predicts lepton kinematics!



This is what we have for most 2p2h and some CCQE models

E.g.:

SuSA or Valencia 2p2h
SuSAv2 or CRPA in GENIE v3

All of the nuclear dynamics lives in here

$$\frac{d^2\sigma_{\nu l}}{d\Omega(\hat{k}')dE'_l} = \frac{|\vec{k}'|}{|\vec{k}|} \frac{G^2}{4\pi^2} L_{\mu\sigma} W^{\mu\sigma}$$

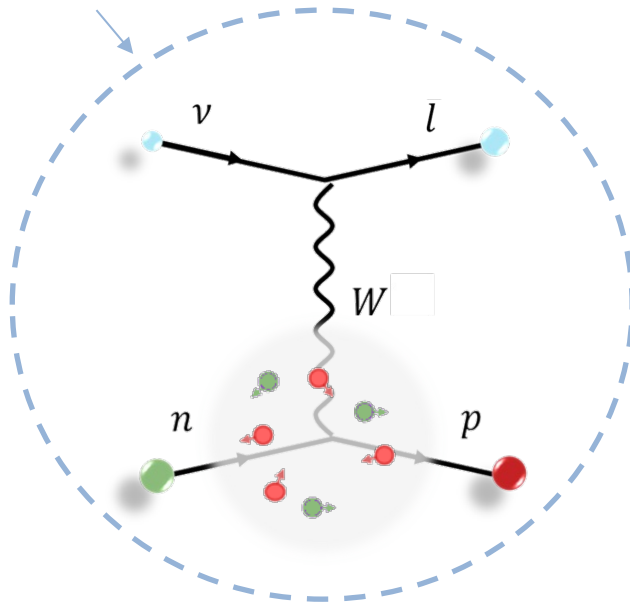
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Exclusive calculations

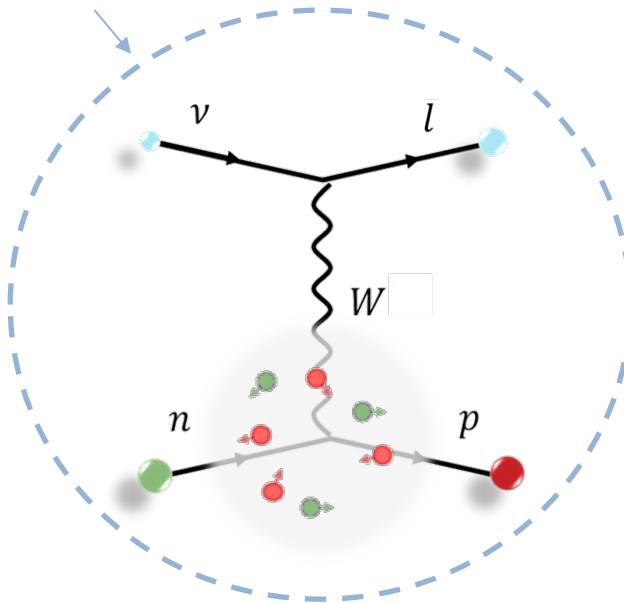
Exclusive model: can describe all final state particle kinematics



Exclusive calculations

All of the nuclear dynamics still lives in here

Exclusive model: can describe all final state particle kinematics



E.g. Semi-inclusive charged-current neutrino-nucleus reactions, O. Moreno et al, 2014

But now there's 10 tensor elements ...

$$\eta_{\mu\nu}^s W_s^{\mu\nu} \sim \hat{V}_{CC} W_{semi}^{CC} + \hat{V}_{CL} W_{semi}^{CL} + \hat{V}_{LL} W_{semi}^{LL} + \hat{V}_T W_{semi}^T + \hat{V}_{TT} W_{semi}^{TT} + \hat{V}_{TC} W_{semi}^{TC} + \hat{V}_{TL} W_{semi}^{TL}$$

$$\eta_{\mu\nu}^a W_a^{\mu\nu} \sim \hat{V}_{T'} W_{semi}^{T'} + \hat{V}_{TC'} W_{semi}^{TC'} + \hat{V}_{TL'} W_{semi}^{TL'}$$

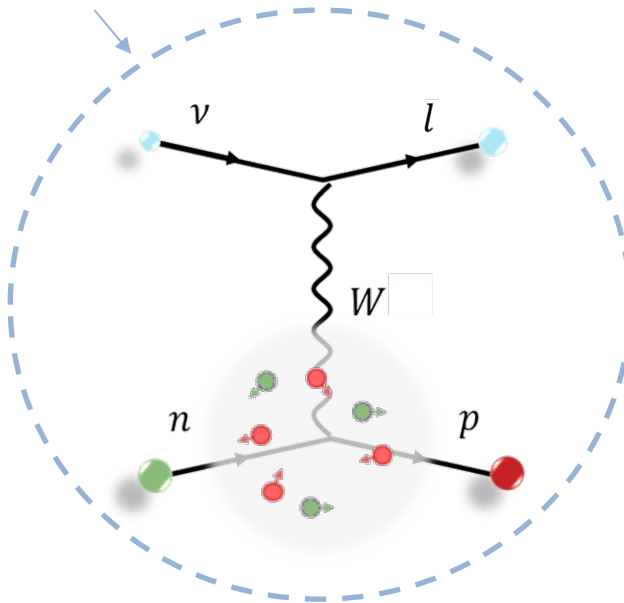
... and these become challenging to calculate

- Some models can do this, e.g. Relativistic Mean Field (RMF)

Exclusive calculations

All of the nuclear dynamics still lives in here

Exclusive model: can describe all final state particle kinematics



$$\frac{d^5\sigma_{\nu\ell}}{d\Omega(\hat{k}')d\Omega(p_N)dE_{\ell'}} \sim L_{\mu\sigma} W^{\mu\sigma}$$

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But now there's 10 tensor elements ...

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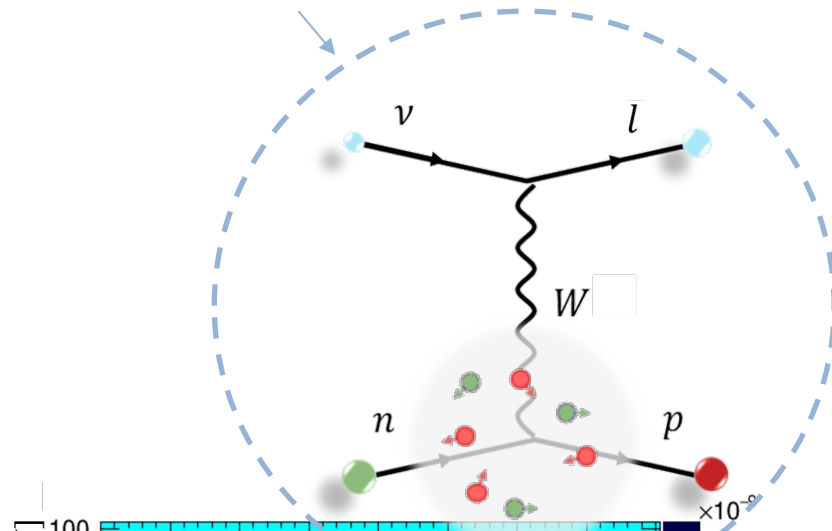
... and these become challenging to calculate

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No generator model does this

Factorized Calculations

Exclusive model: can describe all final state particle kinematics



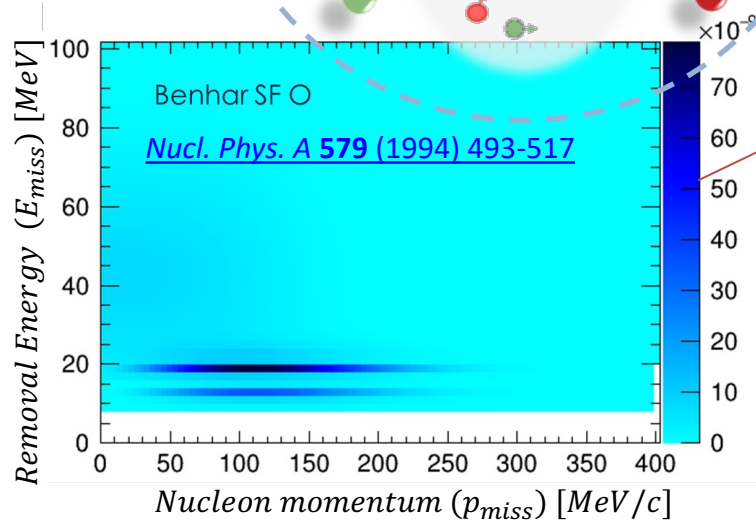
Plane Wave Impulse Approximation (PWIA)

If we assume an interaction with a **single**, non-relativistic nucleon and that there's **no FSI or RPA correlations** we can write the cross section like this:

$$\frac{d^5\sigma_{\nu\ell}}{d\Omega(\hat{k}')d\Omega(p_N)dE_{\ell'}} \sim S(E_m, \mathbf{p}_m) L_{\mu\nu} W^{\mu\nu} \delta(\omega + M - E_m - E_{p'})$$

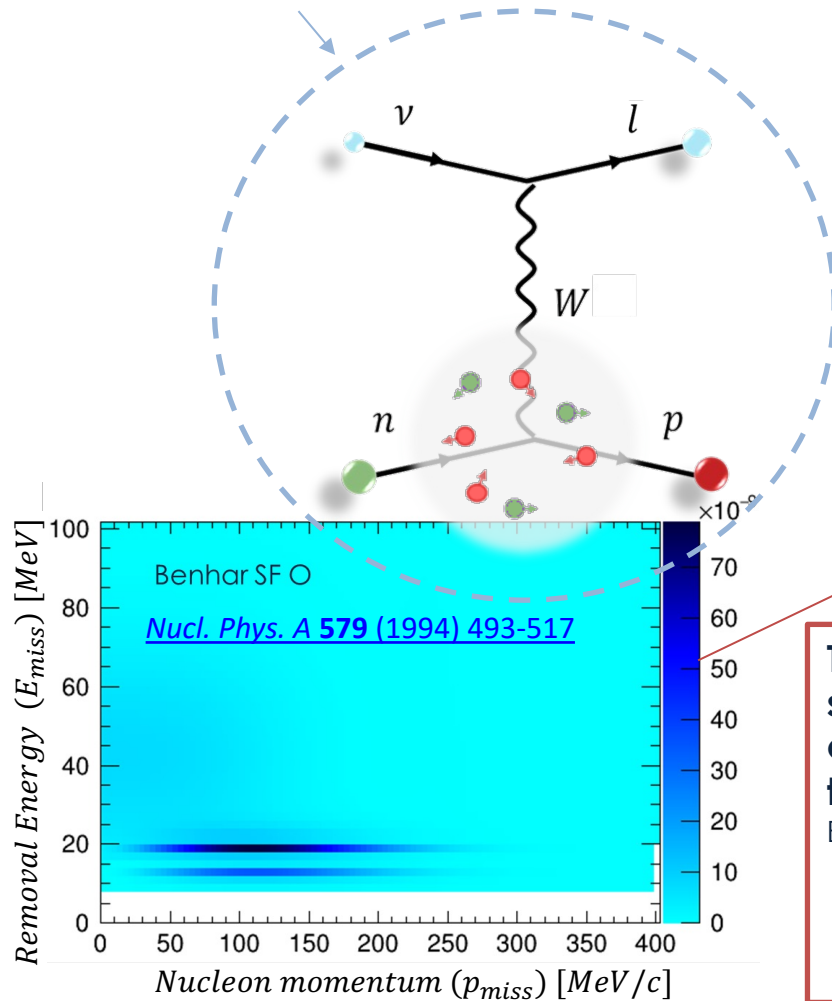
“Spectral Function”

Single nucleon tensor contraction
(no nuclear effects)



Factorized Calculations

Exclusive model: can describe all final state particle kinematics



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"Spectral Function"

Single nucleon tensor contraction
(no nuclear effects)

This is what some newer generators use for CCQE – some predictive power for nucleon kinematics. An FSI cascade is added on top *ad hoc* but this only affects the nucleon.

E.g.:

- SF in NEUT
- SF in NuWro
- Starting point in Achilles
- Fermi-gas QE in GENIE v3

What can I calculate?

- Summarising what the theory inputs give us:

Theory input	What kinematics can I calculate?
Nucleon-level calculation	Lepton and nucleon before FSI
Inclusive calculation	Lepton only
Factorized calculation	Lepton and nucleon before FSI
Exclusive calculation	Lepton and nucleon

* Possible to include in an ad-hoc way, but doesn't reliably allow for a calculation for alteration of outgoing nucleon kinematics

See e.g.: Phys. Rev. D **91**, 033005

What can I calculate?

- Summarising what the theory inputs give us:

Theory input	What kinematics can I calculate?	How accurate is the calculation?
Nucleon-level calculation	Lepton and nucleon before FSI	Do not trust!
Inclusive calculation	Lepton only	As accurate as the underlying theory
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What can I calculate?

- Summarising what the theory inputs give us:

Theory input	What kinematics can I calculate?	How accurate is the calculation?	FSI/RPA?	Example use in generators
Nucleon-level calculation	Lepton and nucleon before FSI	Do not trust!	Not included	Most nonQE/2p2h + older QE calcs.
Inclusive calculation	Lepton only	As accurate as the underlying theory	Can be included	Most 2p2h, SuSAv2 / CRPA QE in GENIEv3
Factorized calculation	Lepton and nucleon before FSI	Approximations can limit predications	Not without approximations*	SFQE in NEUT, NuWro, AChILLES. Default QE in GENIEv3
Exclusive calculation	Lepton and nucleon	As accurate as the underlying theory	Can be included	Not yet available

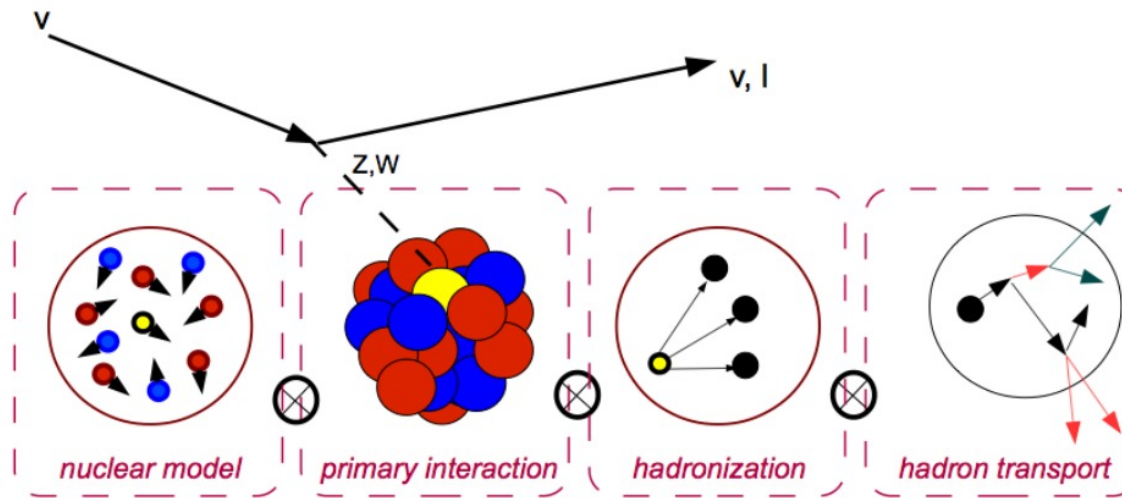
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Filling in the gaps

- Generators take theory inputs where possible, but we found these are often limited:
 - Only capable of predicting a subset of observables
 - Only valid within some range of kinematic phase space
 - Only valid for certain processes
- Need to “fill in the gaps” to get to a useable event simulation

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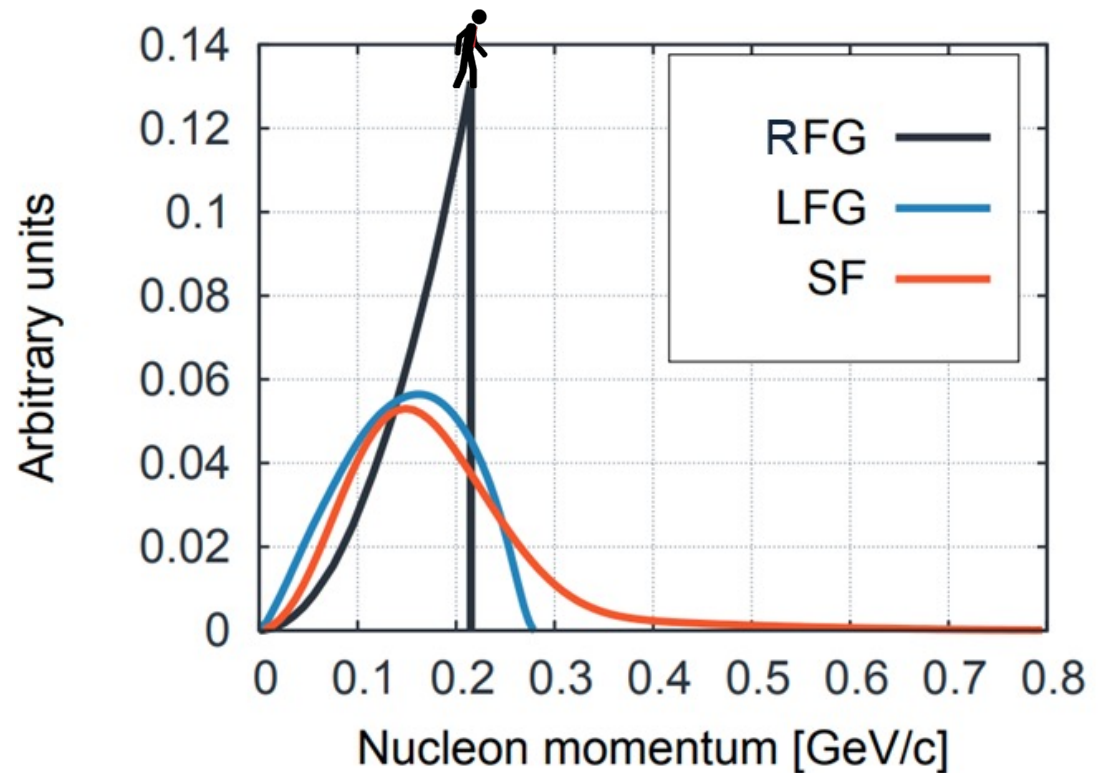
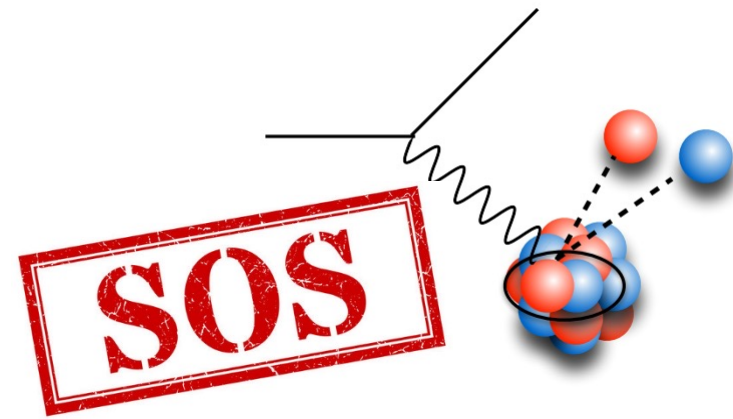
Example: 2p2h

- Theory give us: $\frac{d^2\sigma}{dq_0 dq_3}$
- GENIE predicts: $\frac{d^8\sigma}{dq_0 dq_3 d\mathbf{p}_1 d\mathbf{p}_2}$
- How!?



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Summary so far

- Weak Interactions with neutrinos
 - Point-like scattering is “easy” to calculate
 - Interactions with nucleons is more challenging due to their finite extent
- Neutrino-nucleon interactions
 - QE: almost calculable with some form factors
 - RES: much more difficult, lots of diagrams to consider
 - DIS: easy for *inclusive* high Q^2 , hard at low Q^2 , hadronic side a total guess
- Neutrino-nucleus interactions
 - Nuclear effects: there are lots of them, they **significantly change the cross section**
 - **Not all models can predict everything!**

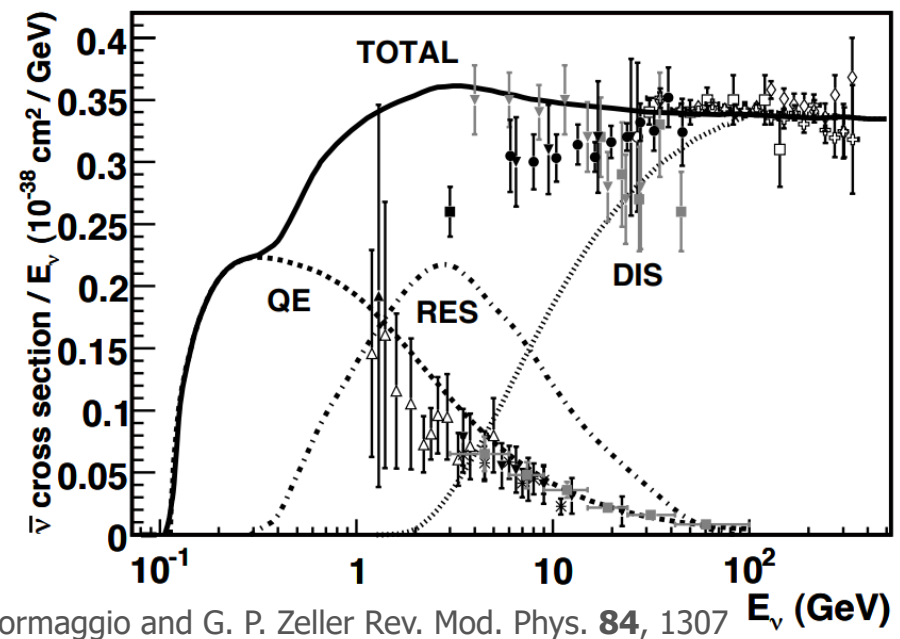
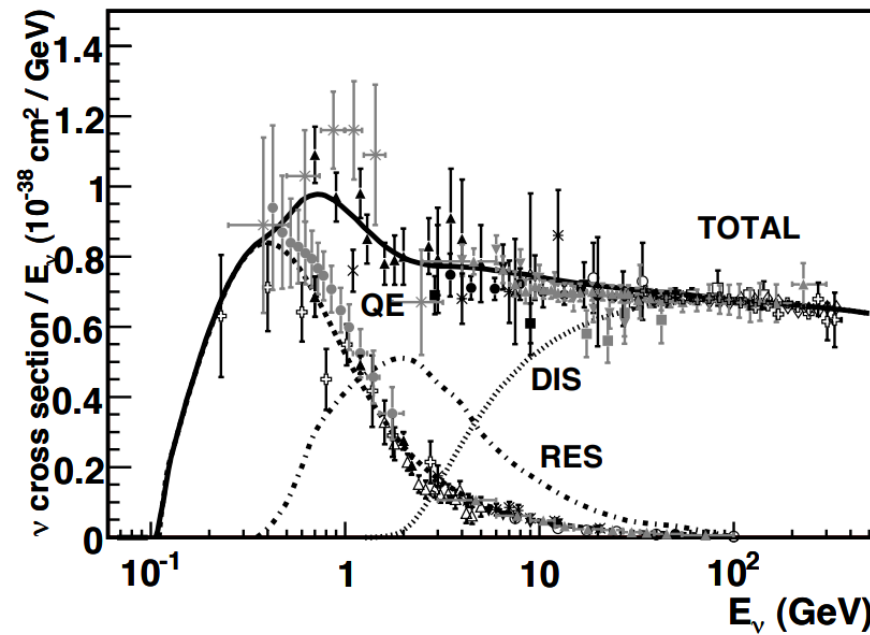
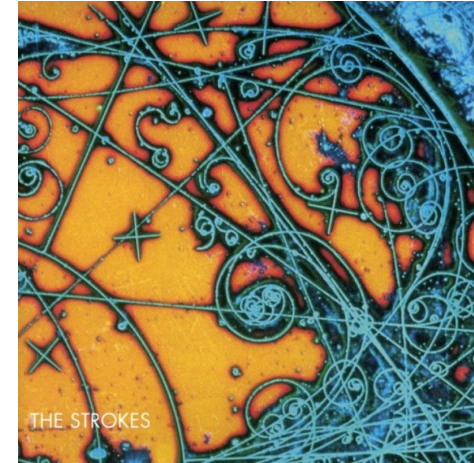
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 - Nuclear effects: there are lots of them, they significantly change the cross section
 - Not all models can predict everything!
- Neutrino event generators
 - Many generators on the market, each with different use cases
 - Take theory where possible, but **need to “fill the gaps”** for a complete calculation
 - **This limits generators predictive power** (details next lecture!)

Backups

Neutrino-nucleon cross sections

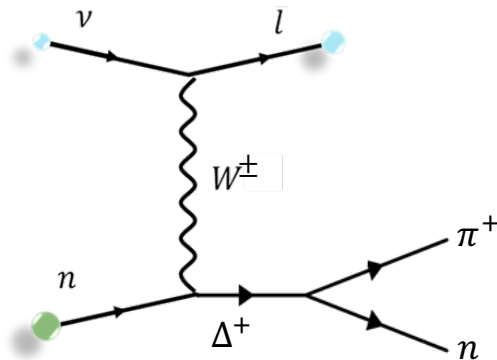
- Discussed neutrino-nucleon interactions
- But it's been a long time since we've measured this process!
- Almost all modern experiments use nuclear targets



J. A. Formaggio and G. P. Zeller Rev. Mod. Phys. **84**, 1307

Resonant Pion Production

CCRES



Current Matrix Elements from a Relativistic Quark Model*

R. P. Feynman, M. Kislinger, and F. Ravndal

Lauritsen Laboratory of Physics, California Institute of Technology, Pasadena, California 91109

(Received 17 December 1970)

- The model's used in today's neutrino experiments are based on an approximate model from the 1970s

gence of the axial-vector current matrix elements. Starting only from these two constants, the slope of the Regge trajectories, and the masses of the particles, 75 matrix elements are calculated, of which more than $\frac{3}{4}$ agree with the experimental values within 40%. The prob-

ficing theoretical adequacy for simplicity. We shall choose a relativistic theory which is naive and obviously wrong in its simplicity, but which is definite and in which we can calculate as many things as possible – not expecting the results to agree exactly with experiment, but to see how closely our “shadow of the truth” equation gives a partial reflection of reality. In our attempt to maintain simplicity, we shall evidently have to violate known principles of a complete relativistic field theory (for example, unitarity). We shall attempt to modify our calculated results in a general way to allow, in a vague way, for these errors.

- The model includes its own form factors, including an axial part with an analogous M_A (and an additional uncertainty in the form factor numerator)

$$f_A(q^2) = \frac{f_A(0)}{\left(1 - \frac{q^2}{M_A^2}\right)^2}$$

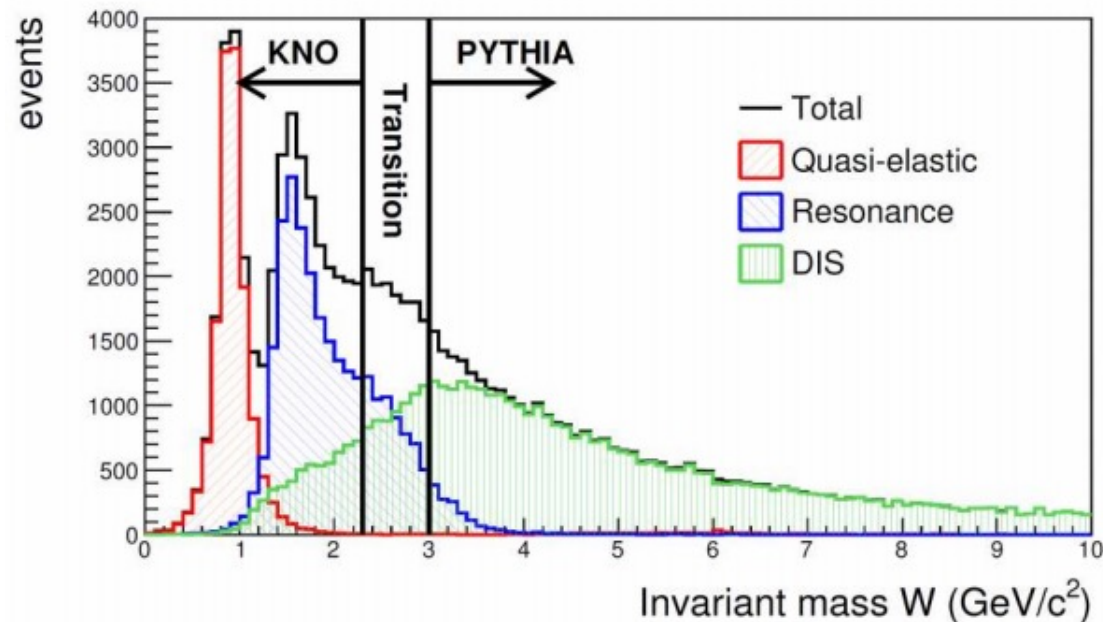
- Theoretical developments are underway but it's safe to say CCRES is less well understood than CCQE!

DIS-RES Transition Region

- There is no cut off where we better describe interactions in a DIS framework compared to In a RES framework
- In general we use models that extrapolate between regions which are definitely DIS (e.g. $W > 5$ GeV) and that are definitely RES (e.g. $W < 2$ GeV)

W = interaction invariant mass

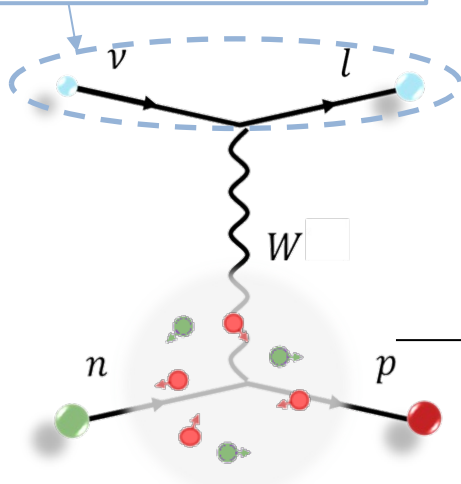
- Different simulations use different *ad-hoc* methods of dealing with this
- But this is a region that will be important for DUNE!



Inclusive calculations

Inclusive calculations come “pre-integrated” over hadron kinematics

Only predicts lepton kinematics!

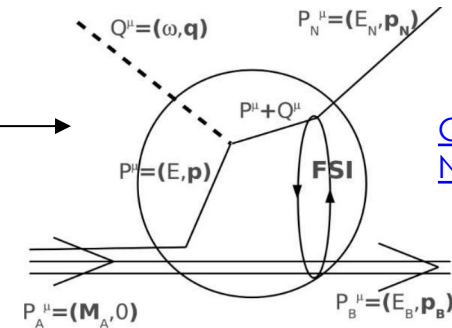


E.g. Inclusive quasielastic charged-current neutrino-nucleus reactions, J. Nieves et al, 2004

$$\frac{d^2\sigma_{\nu l}}{d\Omega(\hat{k}')dE'_l} = \frac{|\vec{k}'|}{|\vec{k}|} \frac{G^2}{4\pi^2} L_{\mu\sigma} (W^{\mu\sigma})$$

All of the nuclear dynamics lives in here

[G. Megias NuInt18 talk](#)



RMF-FSI: Scattered nucleon w.f. is solution of Dirac eq. in presence of the same potentials used to describe the bound nucleon w.f.

In some calculations, the nuclear effects considered **includes the impact of Final State Interactions (FSI)** with a QM treatment

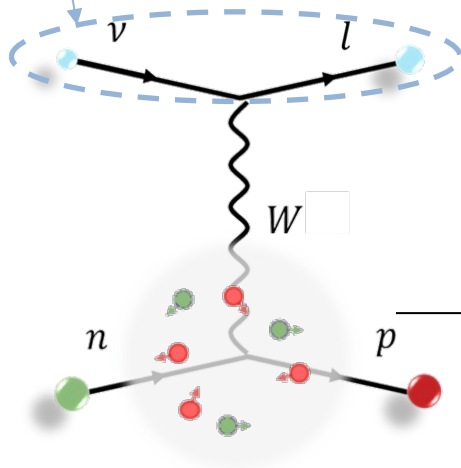
Like this, FSI changes the matrix element

- Affects cross section as a function of lepton *and* hadron kinematics!

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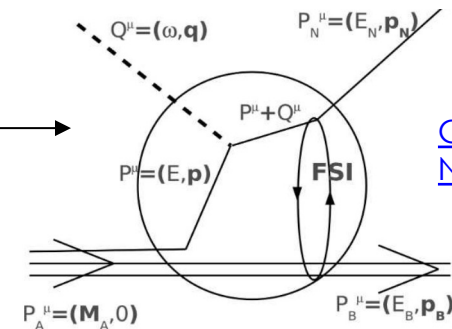
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FSI like this is included in QE models, but not 2p2h

SuSA or Valencia 2p2h – no consideration of FSI

SuSAv2 or CRPA in GENIE v3 – impact of FSI on inclusive cross section is considered