INTRO TO PHENOMENOLOGY WITH MASSIVE NEUTRINOS

Concha Gonzalez-Garcia

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Summary I

- In the SM:
  - Accidental global symmetry: $B \times L_e \times L_\mu \times L_\tau \leftrightarrow m_\nu \equiv 0$
  - neutrinos are left-handed ($\equiv$ helicity -1): $m_\nu = 0 \Rightarrow$ chirality $\equiv$ helicity
  - No distinction between Majorana or Dirac Neutrinos
- If $m_\nu \neq 0 \rightarrow$ Need to extend SM

  $\rightarrow$ different ways of adding m_ν to the SM
  - *breaking* total lepton number ($L = L_e + L_\mu + L_\tau$) $\rightarrow$ Majorana ν: $\nu = \nu^C$
  - *conserving* total lepton number $\rightarrow$ Dirac ν: $\nu \neq \nu^C$

  $\rightarrow$ Lepton Mixing$\equiv$ breaking of $L_e \times L_\mu \times L_\tau$
- From direct searches of ν-mass: $m_\nu \leq \mathcal{O}(eV)$

  Question: How to search for $m_\nu \ll \mathcal{O}(eV)$?

  Answer: Neutrino Oscillations

INTRO TO PHENOMENOLOGY WITH MASSIVE NEUTRINOS: LECTURES II-III

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OUTLINE

- Neutrino Flavour Oscillations in Vacuum
- Propagation in Matter: Effective Potentials
- Flavour Transitions in Matter: MSW
- Global 3ν picture

Effects of ν Mass: Flavour Transitions

- Flavour ($\equiv$ Interaction) basis (production and detection): ν_e, ν_μ and ν_τ
- Mass basis (free propagation in space-time): ν_1, ν_2 and ν_3 ...

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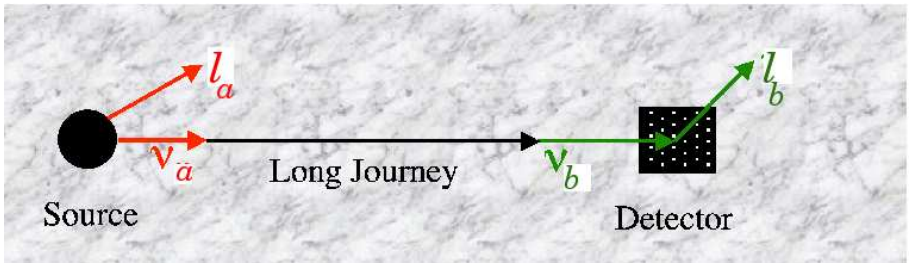
- In general interaction eigenstates $\neq$ propagation eigenstates

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{\rm LEP} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \dots \end{pmatrix} \Rightarrow$$

Flavour not conserved in propagation



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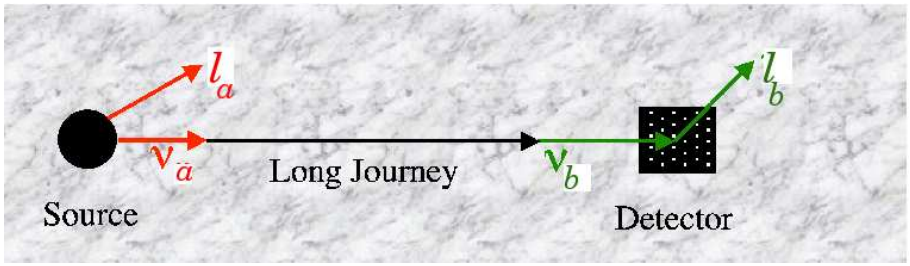

- The probability $P_{\alpha\beta}$ of producing neutrino with flavour α and detecting with flavour β has to depend on:

  – Misalignment between interaction and propagation states ($\equiv U$)
  – Difference between propagation eigenvalues
  – Propagation distance

Mass Induced Flavour Oscillations in Vacuum

- If neutrinos have mass, a weak eigenstate $|\nu_\alpha\rangle$ produced in $l_\alpha + N \to \nu_\alpha + N'$

  is a linear combination of the mass eigenstates ($|\nu_i\rangle$)

$$|\nu_\alpha\rangle = \sum_{i=1}^{n} U^*_{\alpha i} \, |\nu_i\rangle$$

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- it can be detected with flavour β with probability

$$P_{\alpha\beta} = |\langle\nu_\beta|\nu_\alpha(t)\rangle|^2 = |\sum_{j=1}^{n}\sum_{i=1}^{n} U^*_{\alpha i} U_{\beta j} \langle\nu_j|\nu_i(t)\rangle|^2$$

Mass Induced Flavour Oscillations in Vacuum

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- We call E_i the neutrino energy and m_i the neutrino mass

- Under the approximations:

(**1**) $|\nu\rangle$ is a *plane wave* $\Rightarrow$ $|\nu_i(t)\rangle = \mathrm{e}^{-i\,E_i t} |\nu_i(0)\rangle$ and using $\langle \nu_j | \nu_i \rangle = \delta_{ij}$

$$P_{\alpha\beta} = \delta_{\alpha\beta} - 4 \sum_{i<j}^{n} \mathrm{Re}[U_{\alpha i} U^*_{\beta i} U^*_{\alpha j} U_{\beta j}] \sin^2\left(\frac{\Delta_{ij}}{2}\right) + 2 \sum_{i<j} \mathrm{Im}[U_{\alpha i} U^*_{\beta i} U^*_{\alpha j} U_{\beta j}] \sin\left(\Delta_{ij}\right)$$

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(**2**) *relativistic* ν

$$E_i = \sqrt{p_i^2 + m_i^2} \simeq p_i + \frac{m_i^2}{2E_i}$$

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$$E_i = \sqrt{p_i^2 + m_i^2} \simeq p_i + \frac{m_i^2}{2E_i}$$

(**3**) Lowest order in mass $p_i \simeq p_j = p \simeq E$

$$\frac{\Delta_{ij}}{2} = \frac{(m_i^2 - m_j^2)\,L}{4\,E} = 1.27 \frac{m_i^2 - m_j^2}{\mathrm{eV}^2} \frac{L/E}{\mathrm{Km/GeV}}$$

Mass Induced Flavour Oscillations in Vacuum

- The oscillation probability:

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– The last piece $2\sum_{i<j}\mathrm{Im}[U_{\alpha i}U^*_{\beta i}U^*_{\alpha j}U_{\beta j}]\sin\left(\Delta_{ij}\right)$ opposite sign for $\overline{\nu}$

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– If $\alpha = \beta \Rightarrow$ $\mathrm{Im}[U_{\alpha i}U^*_{\alpha i}U^*_{\alpha j}U_{\alpha j}] = \mathrm{Im}[|U^\star_{\alpha i}|^2|U_{\alpha j}|^2] = 0$

$\Rightarrow$ CP violation observable only for $\beta \neq \alpha$

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- $P_{\alpha\beta}$ depends on Neutrino Parameters and on Two *set-up* Parameters:
  - $\Delta m^2_{ij} = m_i^2 - m_j^2$ The mass differences
  - $U_{\alpha j}$ The mixing angles (and Dirac phases)
  - E The neutrino energy
  - L Distance ν source to detector

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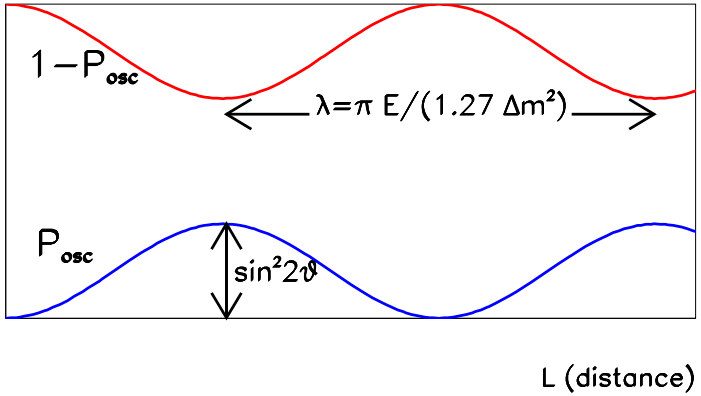
- No information on mass scale nor Majorana phases

Mass Induced 2-ν Oscillations

- When oscillations between 2-ν dominate: $U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

$$P_{osc} = \sin^2(2\theta)\,\sin^2\left(\frac{\Delta m^2 L}{4E}\right) \quad \text{Appear}$$

$$P_{\alpha\alpha} = 1 - P_{osc} \quad \text{Disappear}$$



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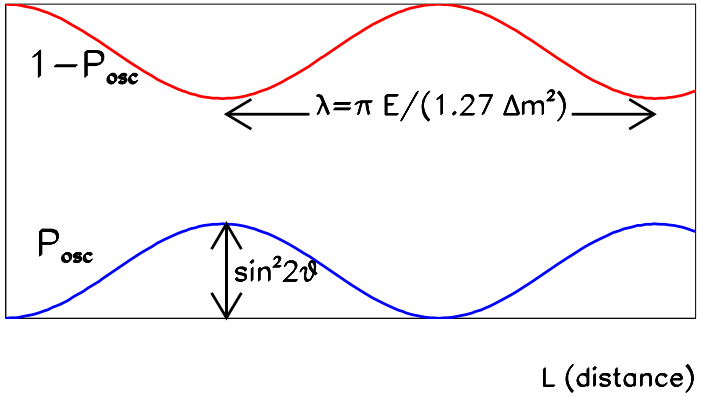

- P_{osc} is symmetric *independently* under $\Delta m^2 \to -\Delta m^2$ *or* $\theta \to \frac{\pi}{2} - \theta$

  $\Rightarrow$ No information on ordering ($\equiv$ signΔm^2) nor octant of θ

- U is real $\Rightarrow$ no CP violation

This only happens for 2ν vacuum oscillations

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- A state mixture of 2 neutrino species $|\nu_\alpha\rangle$ and $|\nu_\beta\rangle$ or equivalently of $|\nu_1\rangle$ and $|\nu_2\rangle$

$$\Phi(x) = \Phi_\alpha(x)|\nu_\alpha\rangle + \Phi_\beta(x)|\nu_\beta\rangle = \Phi_1(x)|\nu_1\rangle + \Phi_2(x)|\nu_2\rangle$$

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$$E\,\Phi_1 = \Big[-i\,\alpha_x\frac{\partial}{\partial x} + \beta\, m_1\Big]\Phi_1$$
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$$\Big(\alpha_x\big\{E^2 - m_i^2\big\}^{1/2} + \beta\,m_i\Big)\,\phi_i = E\phi_i \qquad (1)$$

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- Using (1) in Dirac Eq. we can factorize ϕ_i and α_x and get:

$$-i\frac{\partial\nu_1(x)}{\partial x} = \sqrt{E^2 - m_1^2}\;\nu_1(x)$$
$$-i\frac{\partial\nu_2(x)}{\partial x} = \sqrt{E^2 - m_2^2}\;\nu_2(x)$$

- In the relativistic limit $\sqrt{E^2 - m_i^2} \simeq E - \dfrac{m_i^2}{2E}$

$$-i\frac{\partial}{\partial x}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix} = \begin{pmatrix} E - \frac{m_1^2}{2E} & 0 \\ 0 & \frac{E-m_2^2}{2E}\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix} = \left[E - \frac{m_1^2+m_2^2}{4E}\right] I - \begin{pmatrix} -\frac{\Delta m^2}{4E} & 0 \\ 0 & \frac{\Delta m^2}{4E}\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix}$$

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- An overall phase: $\nu_\alpha \to e^{i\eta x}\nu_\alpha$ and $\nu_\beta \to e^{i\eta x}\nu_\beta$ is unobservable

  $\Rightarrow$ pieces proportional to $I = \begin{pmatrix} 1 & 0 \\ 0 & 1\end{pmatrix}$ do not affect evolution:

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$$-i\frac{\partial}{\partial x}\begin{pmatrix}\nu_\alpha\\ \nu_\beta\end{pmatrix} = \left[E - \frac{m_1^2+m_2^2}{4E}\right] I - \begin{pmatrix} -\frac{\Delta m^2}{4E}\cos 2\theta & \frac{\Delta m^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta\end{pmatrix}\begin{pmatrix}\nu_\alpha\\ \nu_\beta\end{pmatrix}$$

- An overall phase: $\nu_\alpha \to \mathrm{e}^{i\eta x}\nu_\alpha$ and $\nu_\beta \to \mathrm{e}^{i\eta x}\nu_\beta$ is unobservable

  $\Rightarrow$ pieces proportional to $I = \begin{pmatrix} 1 & 0 \\ 0 & 1\end{pmatrix}$ do not affect evolution:

$$\Rightarrow \quad -i\frac{\partial}{\partial x}\begin{pmatrix}\nu_\alpha\\ \nu_\beta\end{pmatrix} = -\begin{pmatrix} -\frac{\Delta m^2}{4E}\cos 2\theta & \frac{\Delta m^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta\end{pmatrix}\begin{pmatrix}\nu_\alpha\\ \nu_\beta\end{pmatrix}$$

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$$\nu_\alpha(x) = A_1 \, \mathrm{e}^{-i\,\omega x} + A_2 \, \mathrm{e}^{+i\,\omega x}$$
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- And the flavour transition probability

$$P_{\alpha\neq\beta} = |\nu_\beta(L)|^2 = B_1^2 + B_2^2 + 2B_1B_2\cos(2\omega L) = \sin^2(2\theta)\,\sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

ν Oscillations: Experimental Probes

- Generically there are two types of experiments to search for ν oscillations :

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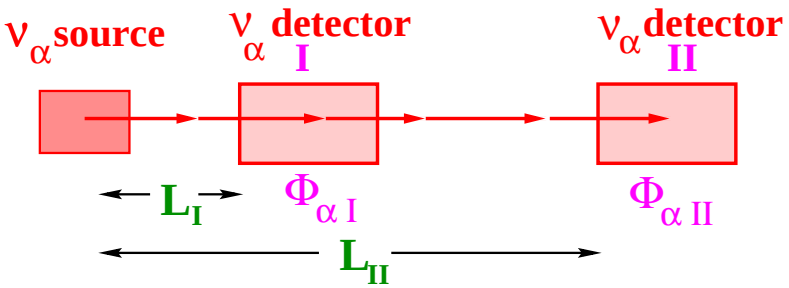
Disappearance Experiment



Compares $\Phi_{\alpha\, I}$ and $\Phi_{\alpha\, II}$ to look for loss

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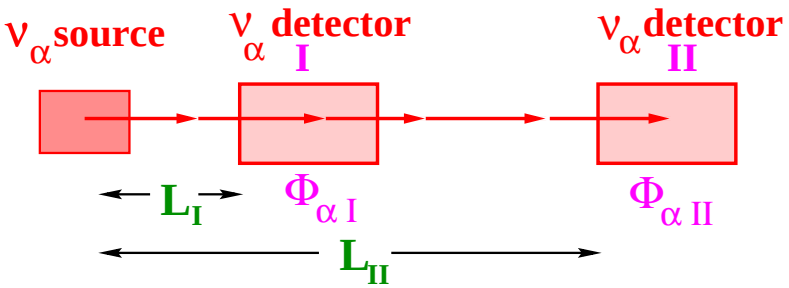

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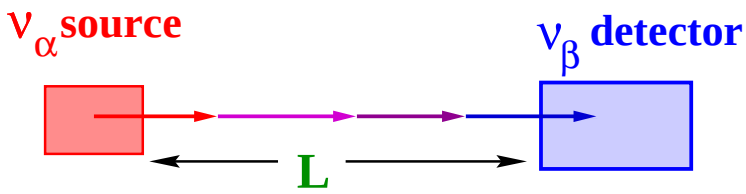

Searches for β diff α

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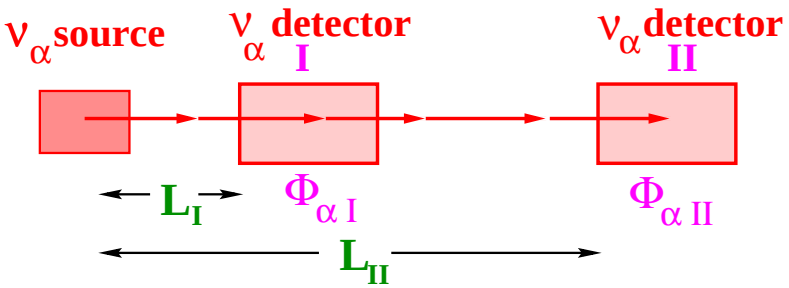

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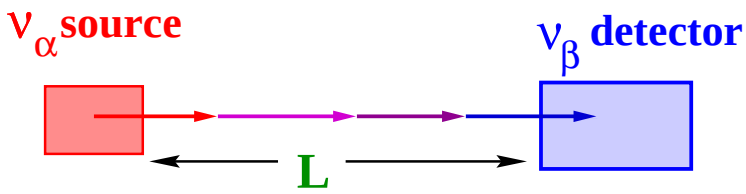

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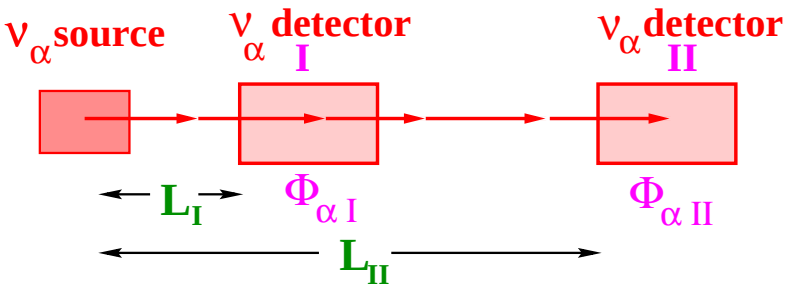

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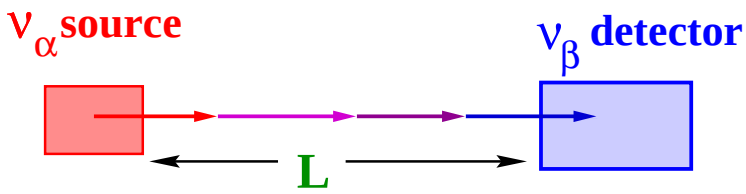
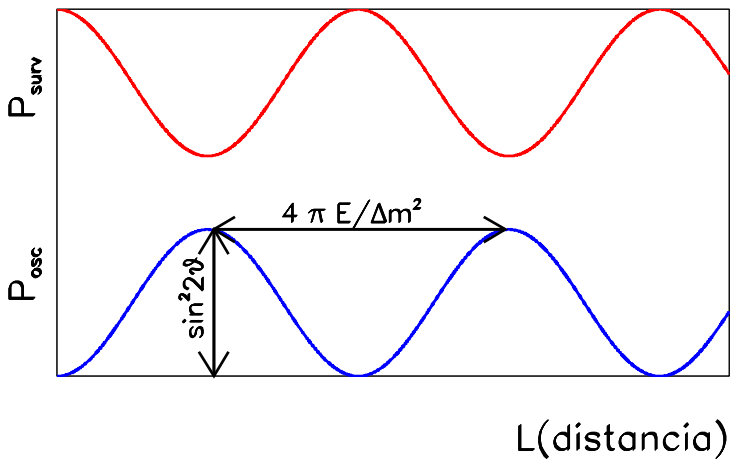


Searches for β diff α

- To detect oscillations we can study the neutrino flavour as function of the Distance to the source



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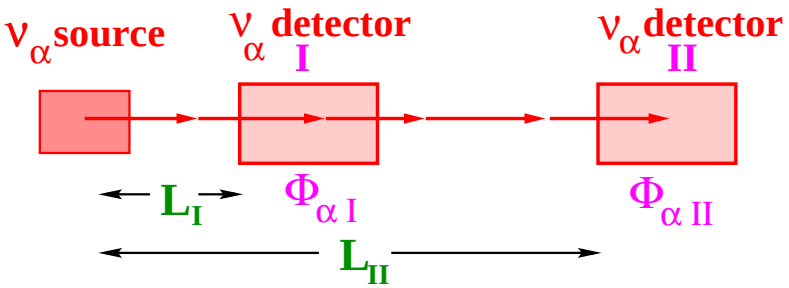

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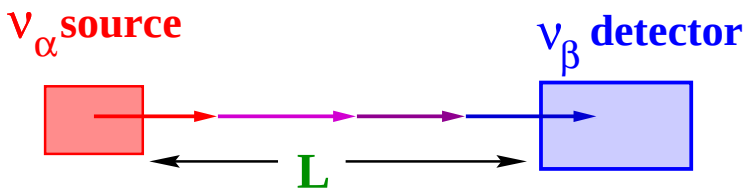

Searches for β diff α

- To detect oscillations we can study the neutrino flavour

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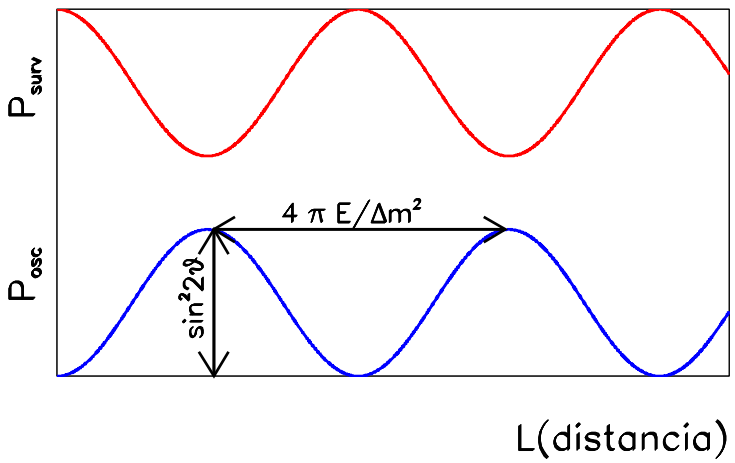
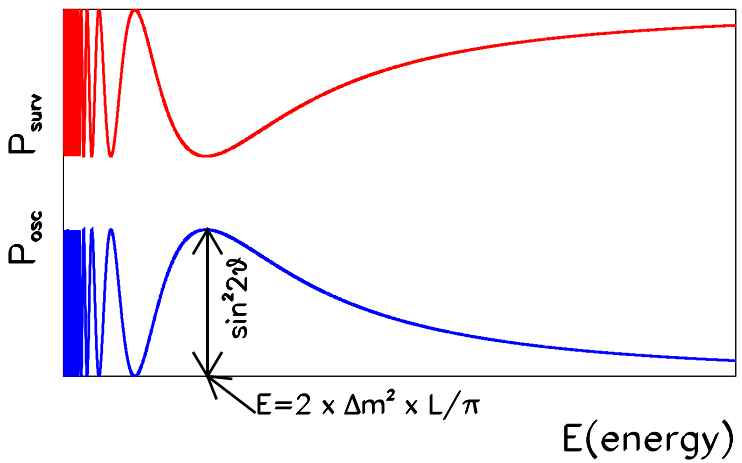


As function of the neutrino Energy



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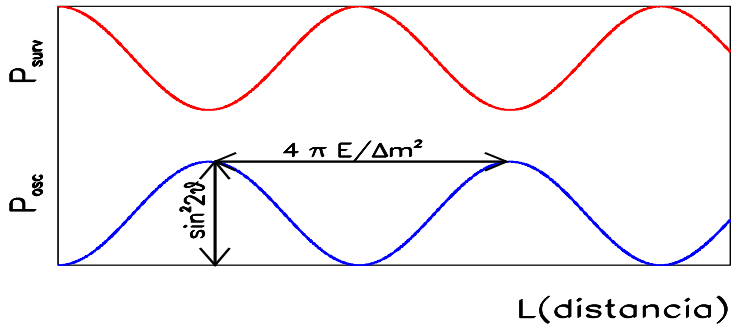

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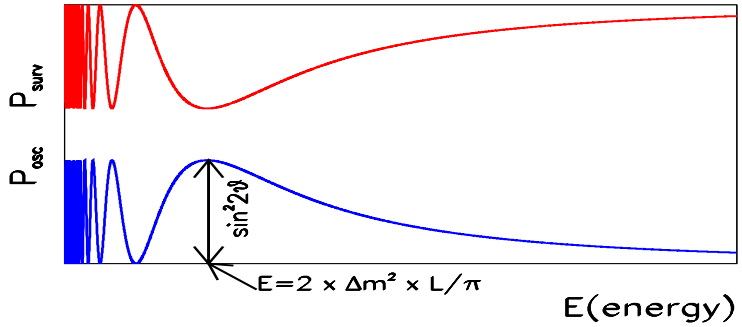

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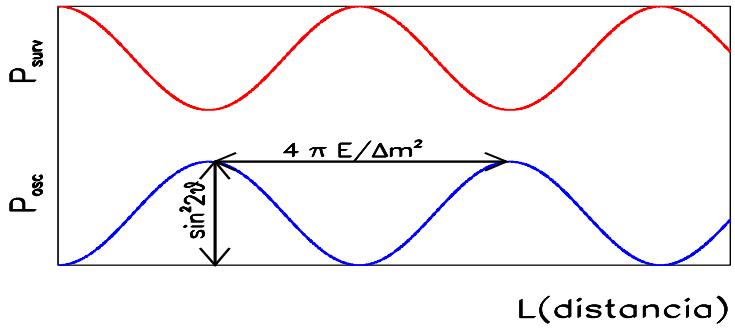

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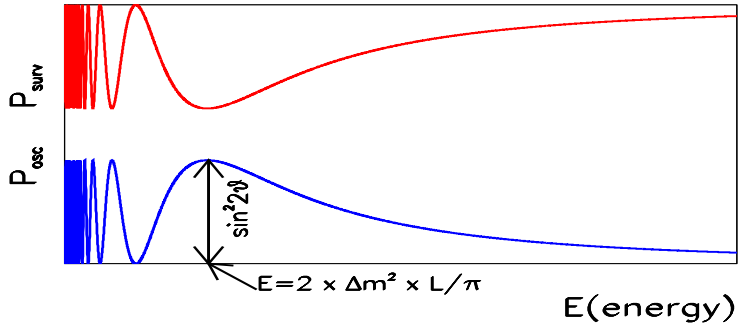
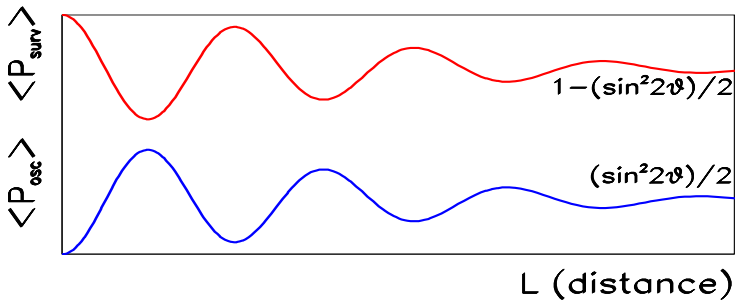
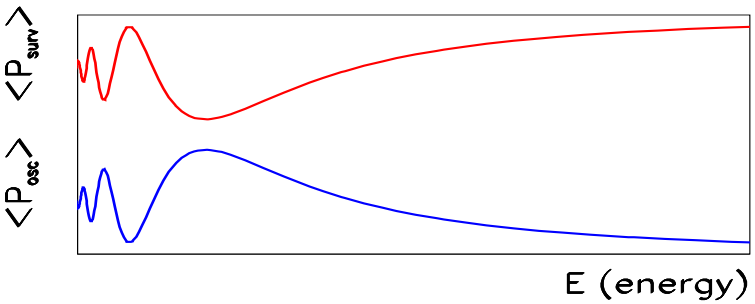


- In real experiments $\Rightarrow$ $\langle P_{\alpha\beta}\rangle = \int dE_\nu \frac{d\Phi}{dE_\nu} \sigma_{CC}(E_\nu) P_{\alpha\beta}(E_\nu)$





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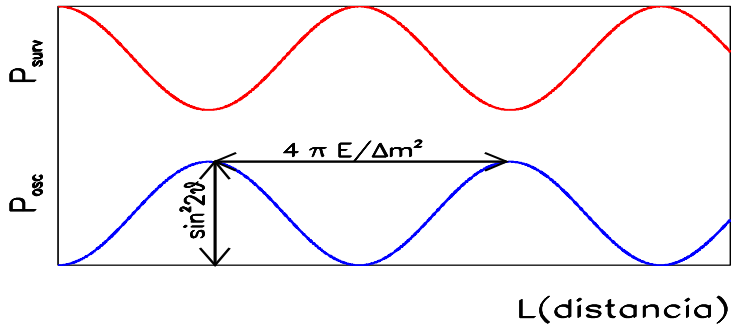

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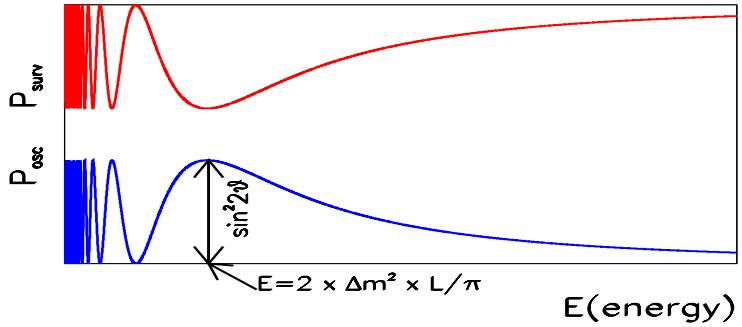

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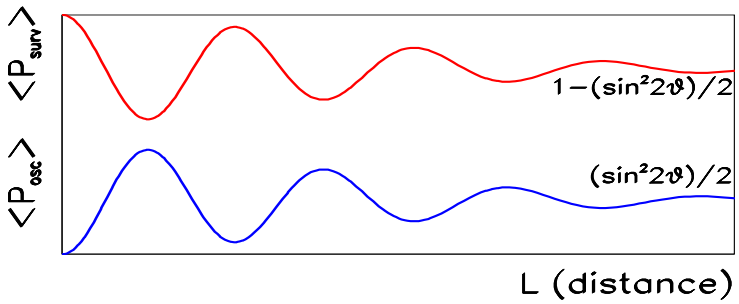

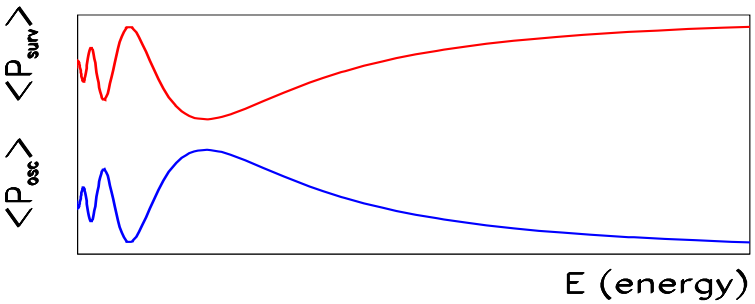

- Maximal sensitivity for $\Delta m^2 \sim E/L$

– $\Delta m^2 \ll E/L \;\Rightarrow \langle \sin^2\left(\Delta m^2 L/4E\right)\rangle \simeq 0 \rightarrow \langle P_{\alpha\neq\beta}\rangle \simeq 0$ & $\langle P_{\alpha\alpha}\rangle \simeq 1$

– $\Delta m^2 \gg E/L \;\Rightarrow \langle \sin^2\left(\Delta m^2 L/4E\right)\rangle \simeq \frac{1}{2} \rightarrow \langle P_{\alpha\neq\beta}\rangle \simeq \frac{\sin^2(2\theta)}{2} \leq \frac{1}{2}$ & $\langle P_{\alpha\alpha}\rangle \geq \frac{1}{2}$

Alternative Oscillation Mechanisms

- Oscillations are due to:
  - Misalignment between CC-int and propagation states: Mixing $\Rightarrow$ Amplitude
  - Difference phases of propagation states $\Rightarrow$ Wavelength. For Δm^2-OSC $\lambda = \dfrac{4\pi E}{\Delta m^2}$

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- ν masses are not the only mechanism for oscillations

Violation of Equivalence Principle (VEP): Gasperini 88, Halprin,Leung 01

Non universal coupling of neutrinos $\gamma_1 \neq \gamma_2$ *to gravitational potential* ϕ

$$\lambda = \frac{\pi}{E|\phi|\delta\gamma}$$

Violation of Lorentz Invariance (VLI): Coleman, Glashow 97

Non universal asymptotic velocity of neutrinos $c_1 \neq c_2 \Rightarrow E_i = \frac{m_i^2}{2p} + c_i p$

$$\lambda = \frac{2\pi}{E\Delta c}$$

Interactions with space-time torsion: Sabbata, Gasperini 81

Non universal couplings of neutrinos $k_1 \neq k_2$ *to torsion strength* Q

$$\lambda = \frac{2\pi}{Q\Delta k}$$

Violation of Lorentz Invariance (VLI) Colladay, Kostelecky 97; Coleman, Glashow 99

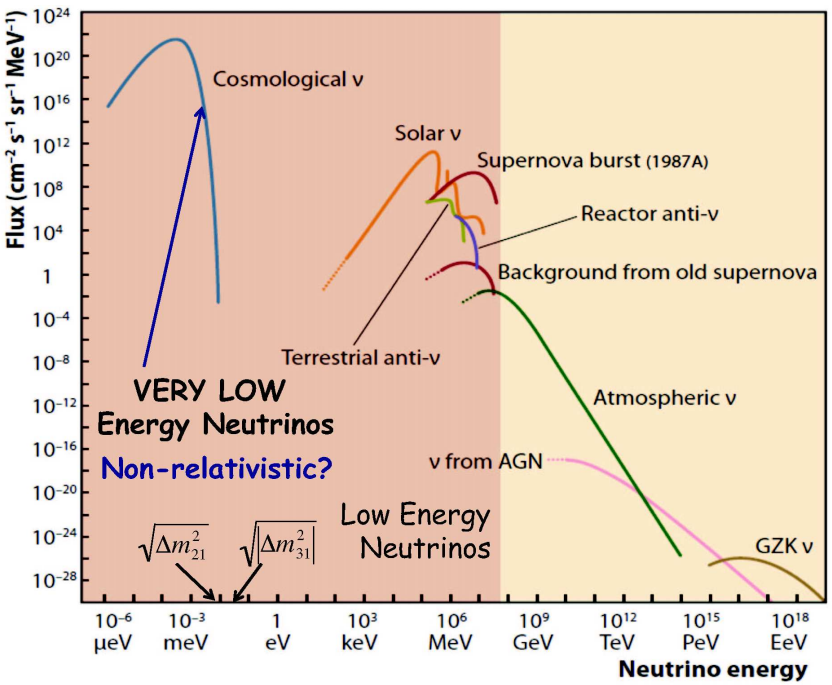
due to CPT violating terms: $\bar{\nu}_L^\alpha b_\mu^{\alpha\beta} \gamma_\mu \nu_L^\beta \Rightarrow E_i = \frac{m_i^2}{2p} \pm b_i$

$$\lambda = \pm\frac{2\pi}{\Delta b}$$

To allow observation of neutrino oscillations:

– Nature has to be good: mixing angles ($\equiv$ amplitudes) must be not too small

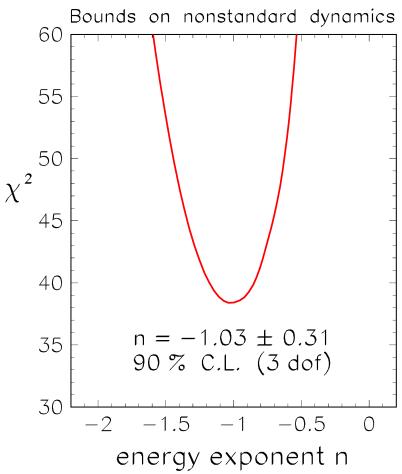
– Need the right set up ($\equiv$right L and E) to be sensitive to the phase



Source	E (GeV)	L (Km)	Δm^2 (eV^2)
Solar	10^{-3}	10^7	10^{-10}
Atmos	0.1–10^2	10–10^3	10^{-1}–10^{-4}
Reactor	10^{-3}	SBL: 0.1–1	10^{-2}–10^{-3}
		LBL: 10–10^2	10^{-4}–10^{-5}
Accel	10	SBL: 0.1	$\gtrsim 0.01$
		LBL: 10^2– 10^3	10^{-2}–10^{-3}

Alternative Mechanisms vs ATM ν's

- With early SK ATM's they could be rule out as dominant



Fogli, Lisi and Marrone hep-ph/0105139

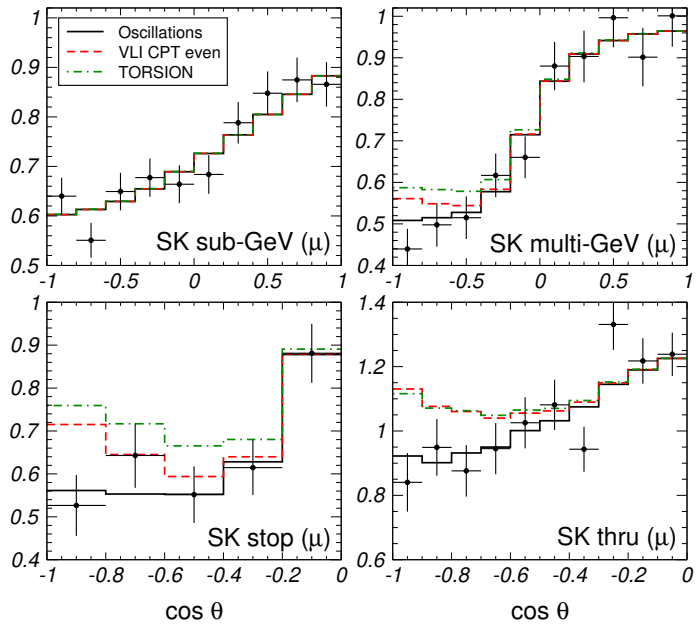
Different L/E dependence:

$$P_{\mu\tau} = \alpha \sin^2(\beta L E^n)$$

$n = -1$ oscillations
$n = 1$ Viol Equiv. Principle
$n = 1$ Viol Lorentz invariance

Fit : $n = -1.03 \pm 0.31$ 90%CL

- And soon after severely constrained (MCG-G, M. Maltoni PRD 04,07)



At 90% CL:

$$\frac{|\Delta c|}{c} \leq 1.2 \times 10^{-24}$$

$$|\phi\, \Delta\gamma| \leq 5.9 \times 10^{-25}$$

$$|Q\, \Delta k| \leq 4.8 \times 10^{-23}\ \text{GeV}$$

$$|\Delta b| \leq 3.0 \times 10^{-23}\ \text{GeV}$$

- We have observed with high (or good) precision:

  * Atmospheric ν_μ & $\bar{\nu}_\mu$ disappear most likely to ν_τ (SK,MINOS, ICECUBE)
  * Accel. ν_μ & $\bar{\nu}_\mu$ disappear at $L \sim 300/800$ Km (K2K, T2K, MINOS, NOνA)
  * Some accelerator ν_μ appear as ν_e at $L \sim 300/800$ Km (T2K, MINOS,NOνA)
  * Solar ν_e convert to ν_μ/ν_τ (Cl, Ga, SK, SNO, Borexino)
  * Reactor $\overline{\nu_e}$ disappear at $L \sim 200$ Km (KamLAND)
  * Reactor $\overline{\nu_e}$ disappear at $L \sim 1$ Km (D-Chooz, Daya Bay, Reno)

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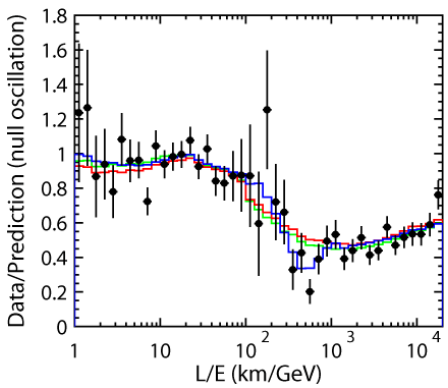
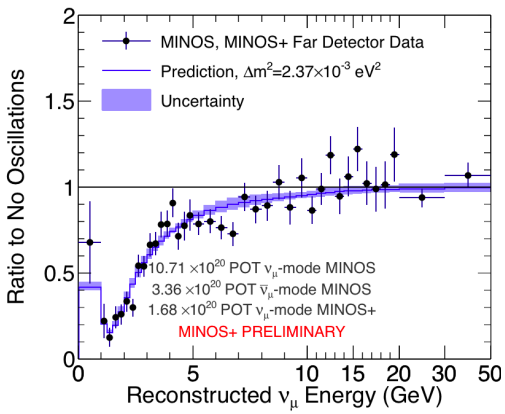
- Confirmed: Oscillation L/E pattern: 2 frequencies and 3 distinct amplitudes

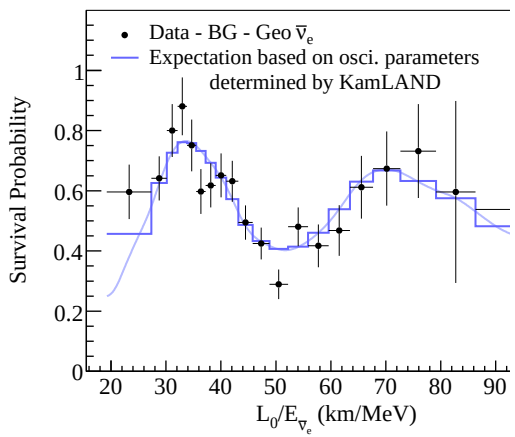
SK/IC



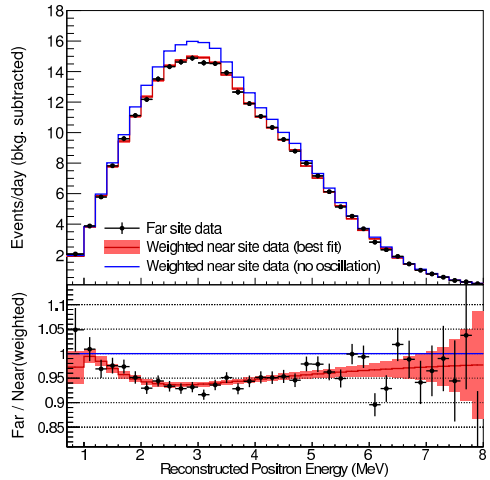
MINOS/T2K/NOvA



KamLAND



DB/RENO/D-CH



$$\frac{\Delta m^2}{\mathrm{eV}^2} \sim 2\,10^{-3}, \theta \sim 45^\circ$$

$$\frac{\Delta m^2}{\mathrm{eV}^2} \sim 10^{-5}$$
$$\theta \sim 30^\circ, 60^\circ$$

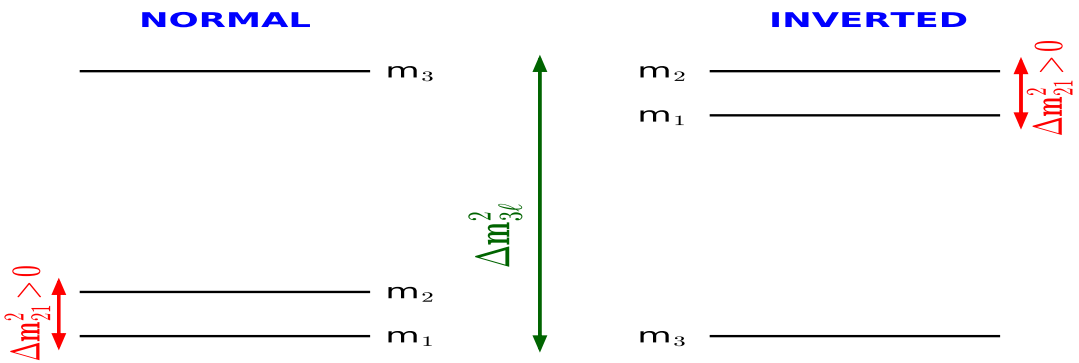
$$\frac{\Delta m^2}{\mathrm{eV}^2} \sim 2 \times 10^{-3}$$
$$\theta \sim 8^\circ$$

3ν Oscillations: Standard Parametrization

- **For 3 ν's** : 3 Mixing angles + 1 Dirac Phase + 2 Majorana Phases

$$U_{\rm LEP} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta_{\rm cp}} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta_{\rm cp}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{21} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\eta_1} & 0 & 0 \\ 0 & e^{i\eta_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Convention: $0 \leq \theta_{ij} \leq 90^\circ \quad 0 \leq \delta \leq 360^\circ \quad \Rightarrow$ 2 Orderings



$\Delta m^2_{21} > 0$ by convention

$$\Delta m^2_{3\ell} = \begin{cases} \Delta m^2_{31} > 0 \text{ for NO} \\ \Delta m^2_{32} < 0 \text{ for IO} \end{cases}$$

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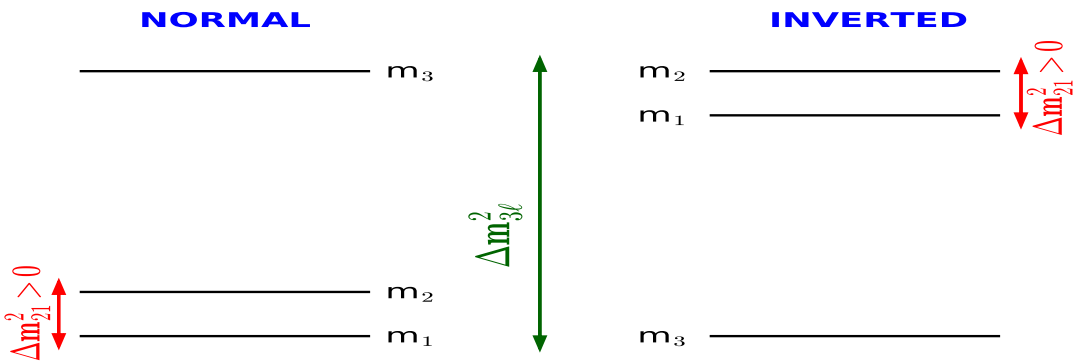

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- Well described by vacuum oscillations:

Experiment	
Reactor LBL (KamLAND)	$\rightarrow \Delta m^2_{21}$, θ_{12} , θ_{13}
Reactor MBL (Daya Bay, Reno, D-Chooz)	$\rightarrow \theta_{13}$, $\Delta m^2_{3\ell}$
Atmospheric Experiments	$\rightarrow \theta_{23}$, $\Delta m^2_{3\ell}$
Acc LBL ν_μ Disapp (Minos,T2K,NOvA)	$\rightarrow \Delta m^2_{3\ell}$, θ_{23}

- $\bar{\nu}_e$ disapp at KamLAND ($|\Delta m^2_{3\ell}| \gg E/L$): $P_{ee} = c^4_{13}\left[1 - \sin^2 2\theta_{12} \sin^2\left(\frac{\Delta m^2_{21} L}{4E}\right)\right] + s^4_{13}$

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- $\bar{\nu}_e$ disapp at MBL React (Daya-Bay, Reno, D-Chooz): $P_{ee} \simeq 1 - \sin^2 2\theta_{13} \sin^2 \frac{\Delta m^2_{ee} L}{4E}$

$$\Delta m^2_{ee} = c_{12}^2 \Delta m^2_{31} + s_{12}^2 \Delta m^2_{32} \simeq \Delta m^2_{3l} + {}^{s_{12}^2 \Delta m^2_{21} \text{ NO}}_{c_{12}^2 \Delta m^2_{21} \text{ IO}}$$

Nunokawa,Parke,Zukanovich (2005)

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- ν_μ and $\bar{\nu}_\mu$ disappearance at LBL: $P_{\mu\mu} \simeq 1 - \sin^2 2\theta_{\mu\mu} \sin^2 \frac{\Delta m^2_{\mu\mu} L}{4E}$

$$\begin{aligned}
\sin^2 \theta_{\mu\mu} &= c_{13}^2 \sin^2 \theta_{23} \\
\Delta m^2_{\mu\mu} &= s_{12}^2 \, \Delta m^2_{31} + c_{12}^2 \, \Delta m^2_{32} + \cos \delta_{\mathrm{CP}} \, s_{13} \sin 2\theta_{12} t_{23} \Delta m^2_{21} \\
&\simeq \Delta m^2_{3l} + {}^{c_{12}^2 \Delta m^2_{21} \ \mathrm{NO}}_{s_{12}^2 \Delta m^2_{21} \ \mathrm{IO}} + \ldots
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\end{aligned}$$

$$\Rightarrow \begin{cases} \text{Precise determination of } \Delta \mathrm{m}^2_{21} \text{ and } |\Delta \mathrm{m}^2_{3\ell}| \text{ and } \theta_{13} \\ \text{No determination of octant of } \theta_{12} \text{ nor of } \theta_{23} \\ \text{Potential sensitivity to ordering} \\ \text{Very subdominant sensitivity to } \delta_{\mathrm{CP}} \end{cases}$$

- ν_e and $\bar{\nu}_e$ appearance ($\nu_\mu \to \nu_e$ and $\bar{\nu}_\mu \to \bar{\nu}_e$) at LBL :

$$
\begin{aligned}
P_{\mu e(\bar{\mu}\bar{e})} \quad &\simeq s_{23}^2 \sin^2 2\theta_{13} \sin^2 \left(\frac{\Delta m_{31}^2 L}{4E}\right) \\
&+ c_{13} \sin^2 2\theta_{12} \sin^2 2\theta_{13} \sin^2 2\theta_{23} \cos\left(\pm\delta - \frac{\Delta m_{31}^2 L}{4E}\right) \left(\frac{\Delta m_{21}^2 L}{4E}\right) \sin\left(\frac{\Delta m_{31}^2 L}{4E}\right)
\end{aligned}
$$

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$$+ c_{13} \sin^2 2\theta_{12} \sin^2 2\theta_{13} \sin^2 2\theta_{23} \cos\left(\pm\delta - \frac{\Delta m_{31}^2 L}{4E}\right)\left(\frac{\Delta m_{21}^2 L}{4E}\right)\sin\left(\frac{\Delta m_{31}^2 L}{4E}\right)$$

Exactly the CPV piece:

$$P_{\mu e} - P_{(\bar{\mu}\bar{e})} = \frac{1}{2} c_{13} \sin^2 2\theta_{12} \sin^2 2\theta_{13} \sin^2 2\theta_{23} \sin\delta$$
$$\times \left[\sin\left(\frac{\Delta m_{13}^2 L}{4E}\right) + \sin\left(\frac{\Delta m_{32}^2 L}{4E}\right) + \sin\left(\frac{\Delta m_{21}^2 L}{4E}\right)\right]$$

$\Rightarrow$ Genuinely 3 ν effect: $\begin{cases} \text{it cancels if any mixing angle is zero} \\ \text{it cancels if any } \Delta \mathrm{m}_{\mathrm{ij}}^2 \text{ is zero} \end{cases}$

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Exactly the CPV piece:

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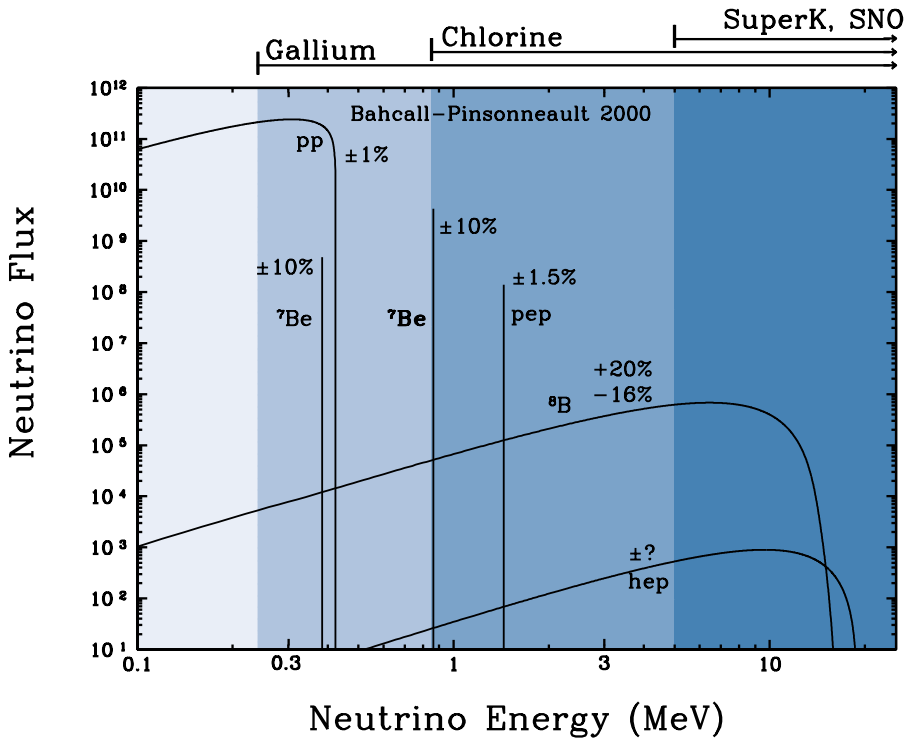
$\Rightarrow$ Genuinely 3 ν effect: $\begin{cases} \text{it cancels if any mixing angle is zero} \\ \text{it cancels if any } \Delta \mathrm{m}_{\mathrm{ij}}^2 \text{ is zero} \end{cases}$

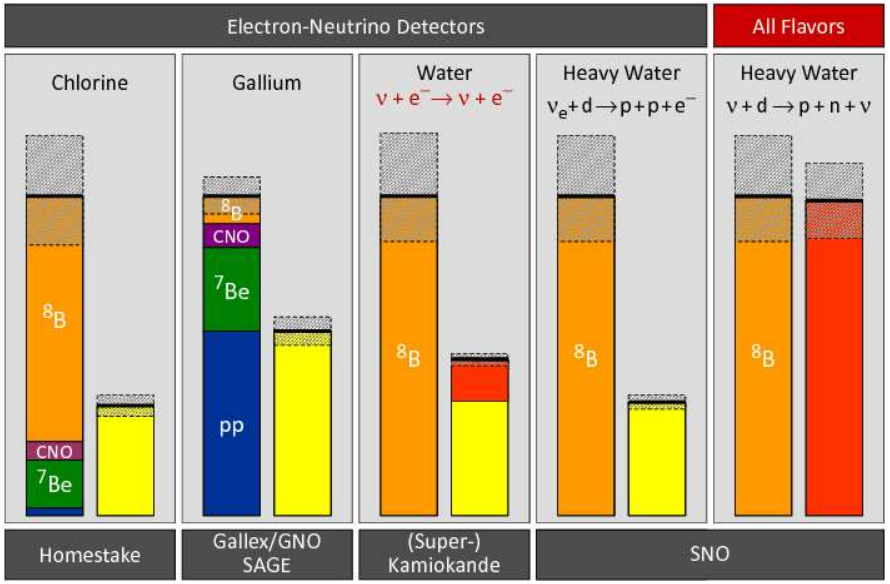
$\Rightarrow$ In 3ν oscillations CPV in any channel $\nu_\alpha \to \nu_{\beta\neq\alpha}$ always proportional to

$$\mathrm{Im}\left[U_{\alpha i}\, U_{\alpha j\neq i}^* U_{\beta i}^*\, U_{\beta j\neq i}\right] = \frac{1}{8}\, c_{13} \sin^2 2\theta_{12} \sin^2 2\theta_{13} \sin^2 2\theta_{23} \sin\delta$$

This is the Jarlskog invariant

A puzzle: Solar Neutrinos





- Experiments measuring ν_e observe a deficit
- Deficit disappears in NC $\Rightarrow$ Solar Model Independent Effect
- Deficit is energy dependent

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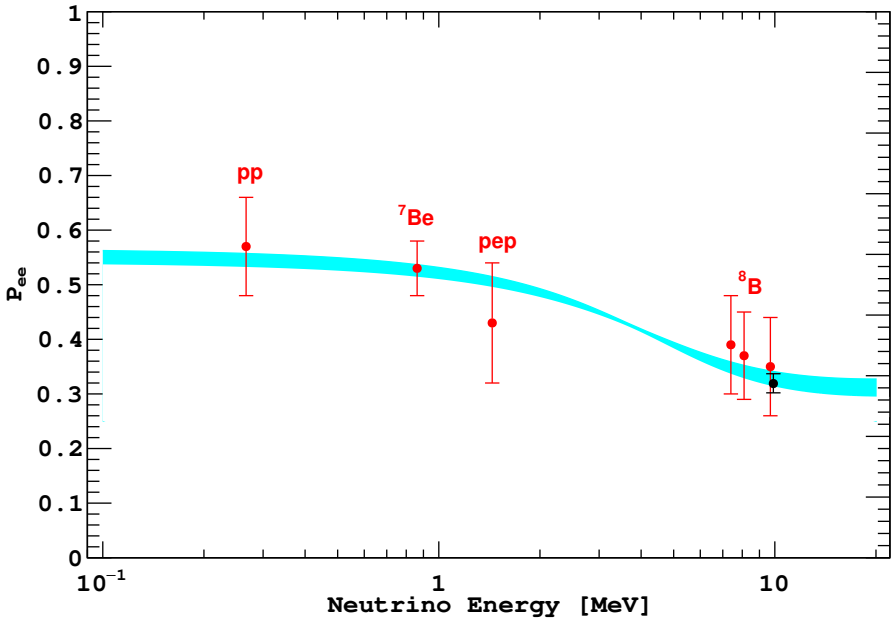

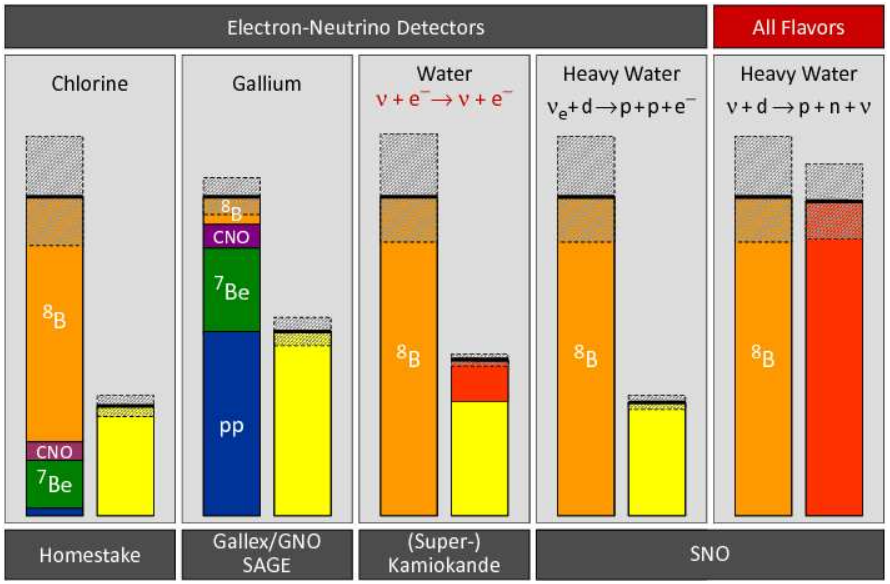

- Experiments measuring ν_e observe a deficit
- Deficit disappears in NC $\Rightarrow$ Solar Model Independent Effect
- Deficit is energy dependent $\Rightarrow$ $P_{ee} \sim 30\% \, (< 0.5)$**!!!** for $E_\nu \gtrsim 8$ MeV

  But $\Delta m^2_{21} L_{\rm sun-Earth}/E_\nu \sim 10^5$ $\Rightarrow$ averaged oscillations

  How is it possible to have $\langle P_{ee} \rangle < \frac{1}{2}$ in averaged oscillation regime???