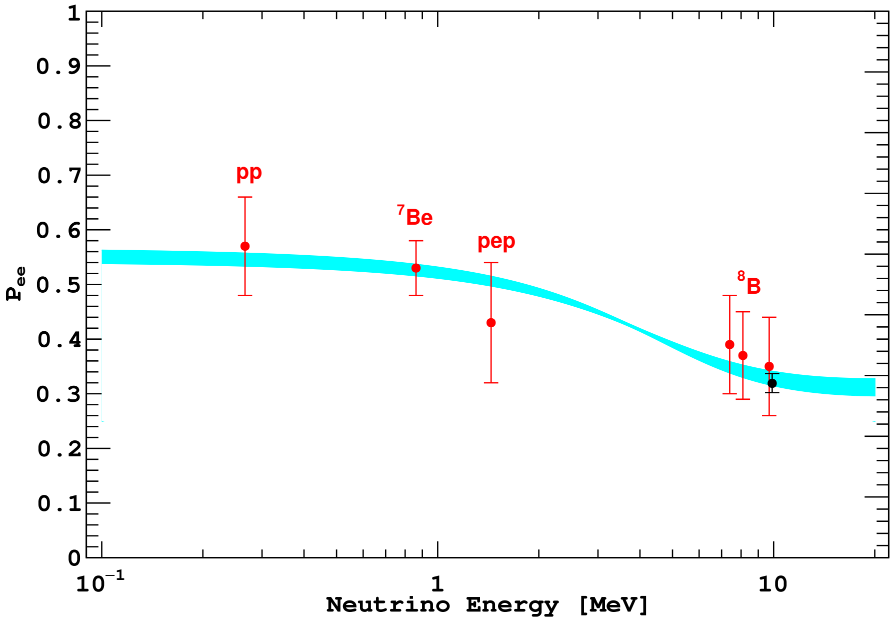
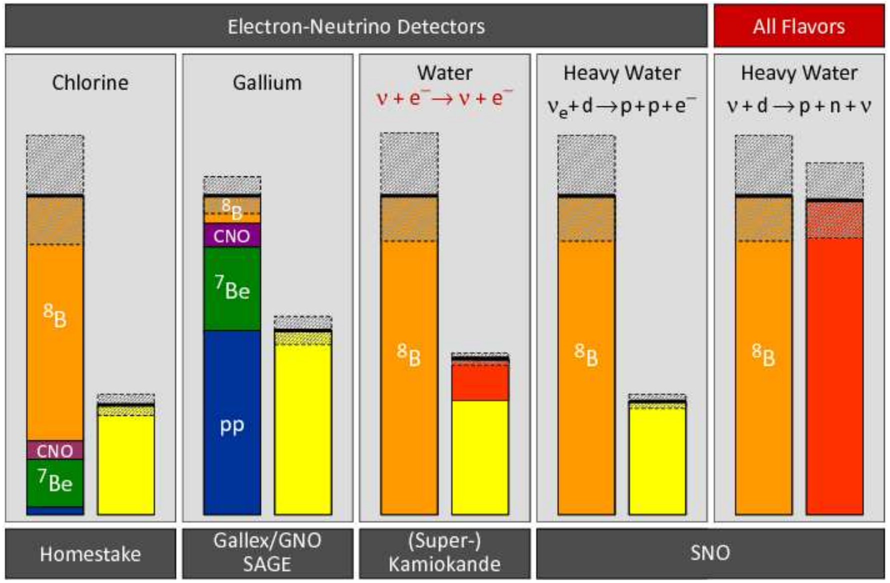
A puzzle: Solar Neutrinos





- Experiments measuring ν_e observe a deficit
- Deficit disappears in NC $\Rightarrow$ Solar Model Independent Effect
- Deficit is energy dependent $\Rightarrow P_{ee} \sim 30\% \; (< 0.5)$!!! for $E_\nu \gtrsim 8$ MeV

  But $\Delta m^2_{21} L_{\text{sun}-\text{Earth}}/E_\nu \sim 10^5 \; \Rightarrow$ averaged oscillations ($\langle P_{ee}\rangle = 1 - \frac{1}{2}\sin^2 2\theta$)

  How is it possible to have $\langle P_{ee}\rangle < \frac{1}{2}$ in averaged oscillation regime???

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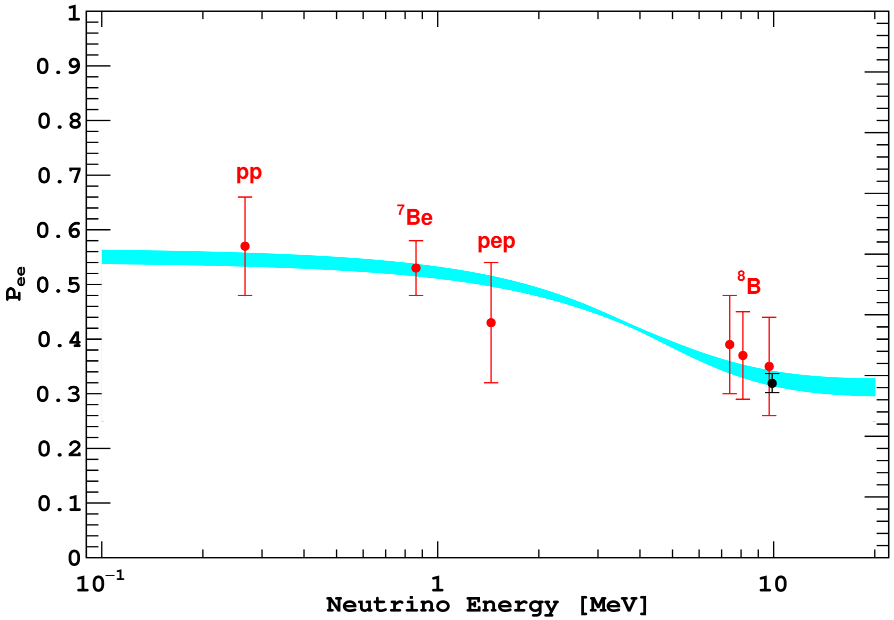

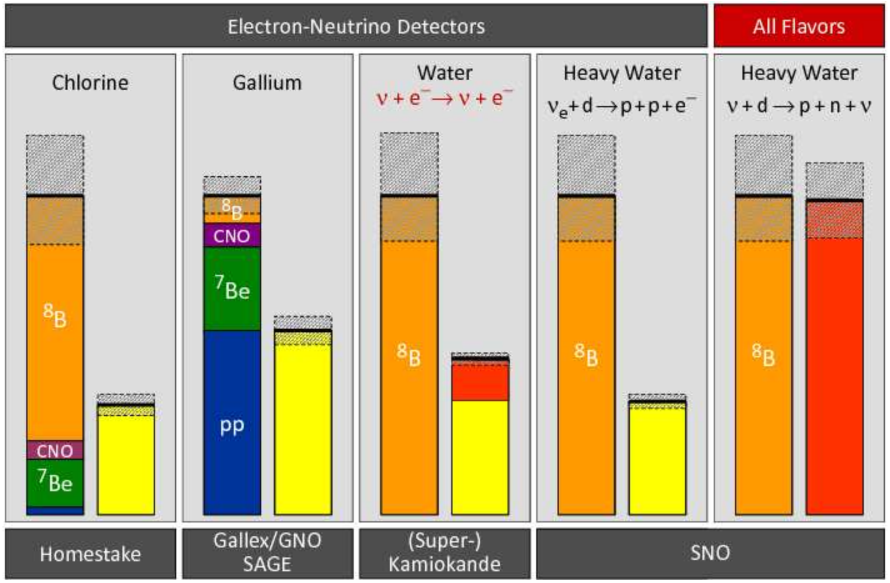

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  ANSWER: Matter effects

INTRO TO PHENOMENOLOGY WITH MASSIVE NEUTRINOS: LECTURE III

Concha Gonzalez-Garcia

(ICREA-University of Barcelona & YITP-Stony Brook)

OUTLINE

- Propagation in Matter: Effective Potentials
- Flavour Transitions in Matter: MSW
- Global 3ν picture

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Neutrinos in Matter: Effective Potentials

- In SM the characteristic ν-p interaction cross section

$$\sigma \sim \frac{G_F^2 E^2}{\pi} \sim 10^{-43}\mathrm{cm}^2 \quad \text{at } \mathrm{E}_\nu \sim \mathrm{MeV}$$

- So if a beam of $\Phi_\nu \sim 10^{10}\nu' s$ was aimed at the Earth only 1 would be deflected

  so it seems that for neutrinos *matter does not matter*

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  Does not contain forward elastic *coherent* scattering

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- In *coherent* interactions $\Rightarrow$ ν and medium momentum remain unchanged

  Interference of scattered and unscattered ν waves

- Coherence $\Rightarrow$ decoupling of ν evolution equation from eqs of medium.

- The effect of the medium is described by an effective potential depending on density and composition of matter

- Lets consider ν_e in a medium with e, p, and n. The low-energy Hamiltonian density:

$$H_W = \frac{G_F}{\sqrt{2}}[J^{(+)\alpha}(x)J^{(-)}_\alpha(x) + \frac{1}{4}J^{(N)\alpha}(x)J^{(N)}_\alpha(x)]$$

CC Int $\quad J^{(+)}_\alpha(x) = \overline{\nu_e}(x)\gamma_\alpha(1-\gamma_5)e(x) \qquad J^{(-)}_\alpha(x) = \overline{e}(x)\gamma_\alpha(1-\gamma_5)\nu_e(x)$

NC Int $\quad J^{(N)}_\alpha(x) = \overline{\nu_e}(x)\gamma_\alpha(1-\gamma_5)\nu_e(x) - \overline{e}(x)[\gamma_\alpha(1-\gamma_5) - s_W^2\gamma_\alpha]e(x)$

$$+\overline{p}(x)[\gamma_\alpha(1-g_A^{(p)}\gamma_5) - 4s_W^2\gamma_\alpha]p(x) - \overline{n}(x)[\gamma_\alpha(1-g_A^{(n)}\gamma_5) - 4s_W^2\gamma_\alpha]n(x)$$

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- Example: The effect of CC with the e medium. The effective CC Hamiltonian density:

$$H_{CC}^{(e)} = \frac{G_F}{\sqrt{2}}\int d^3p_e f(E_e)\left\langle \langle e(s,p_e)|\overline{e}\gamma^\alpha(1-\gamma_5)\nu_e\overline{\nu_e}\gamma_\alpha(1-\gamma_5)|e(s,p_e)\rangle \right\rangle$$

Fierz rearrange

$$= \frac{G_F}{\sqrt{2}}\overline{\nu_e}\gamma_\alpha(1-\gamma_5)\nu_e\int d^3p_e f(E_e)\left\langle \langle e(s,p_e)|\overline{e}\gamma_\alpha(1-\gamma_5)e|e(s,p_e)\rangle \right\rangle$$

$f(E_e)$ statistical energy distribution of e in *homogeneous and isotropic* medium.

$$\int d^3p_e f(E_e) = 1$$

$\left\langle \ldots \right\rangle \equiv$ summing over all e of momentum p_e.

coherence $\Rightarrow s, p_e$ same for initial and final e

• Expanding the electron fields e in plane waves (quantized in a volume $\mathcal{V}$)

$$\langle e(s,p_e)|\overline{e}\gamma_\alpha(1-\gamma_5)e|e(s,p_e)\rangle = \frac{1}{2E_e\,\mathcal{V}}\langle e(s,p_e)|\overline{u_s}(p_e)a_s^\dagger(p_e)\gamma_\alpha(1-\gamma_5)a_s(p_e)u_s(p_e)|e(s,p_e)\rangle$$

• Expanding the electron fields e in plane waves (quantized in a volume $\mathcal{V}$)

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• Since $a_s^\dagger(p_e)a_s(p_e) = \mathcal{N}_e^{(s)}(p_e)$ (number operator) and assuming that there are the same number of electrons with spin 1/2 and -1/2

$$\frac{1}{\mathcal{V}}\Big\langle\langle e(s,p_e)|a_s^\dagger(p_e)a_s(p_e)|e(s,p_e)\rangle\Big\rangle \equiv \sum_s N_e^s(p_e) = N_e(p_e)\frac{1}{2}\sum_s$$

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$$\Big\langle \langle e(s,p_e)|\overline{e}\gamma_\alpha(1-\gamma_5)e|e(s,p_e)\rangle \Big\rangle = \frac{N_e(p_e)}{4E_e}\sum_s \overline{u_s}(p_e)\gamma_\alpha(1-\gamma_5)u_s(p_e)$$

$$= \frac{N_e(p_e)}{4E_e}\sum_s Tr\Big[\overline{u_s}(p_e)\gamma_\alpha(1-\gamma_5)u_s(p_e)\Big] = \frac{N_e(p_e)}{4E_e}\sum_s Tr\Big[u_s(p_e)\overline{u_s}(p_e)\gamma_\alpha(1-\gamma_5)\Big]$$

$$= \frac{N_e(p_e)}{4E_e}Tr\sum_s \Big[u_s(p_e)\overline{u_s}(p_e)\gamma_\alpha(1-\gamma_5)\Big] = \frac{N_e(p_e)}{4E_e}Tr\Big[(m_e+\not{p})\gamma_\alpha(1-\gamma_5)\Big] = N_e(p_e)\frac{p_e^\alpha}{E_e}$$

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• For isotropic medium $\Rightarrow \int d^3p_e \vec{p_e} f(E_e)\, N_e(p_e) = 0$

• By definition $\int d^3p_e f(E_e)\, N_e(p_e) = N_e$ electron number density

- The effective charged current Hamiltonian density due to electrons in matter is then:

$$H_{CC}^{(e)} = \frac{G_F N_e}{\sqrt{2}} \overline{\nu_e}(x)\gamma_0(1-\gamma_5)\nu_e(x)$$

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- Thus the effective potential than ν_e “feels” due to e’s

$$\begin{aligned} V_{CC} &= \langle \nu_e | \int d^3x H_{CC}^{(e)} | \nu_e \rangle \\ &= \frac{G_F N_e}{\sqrt{2}} \langle \nu_e | \int d^3x \overline{\nu_e}(x)\gamma_0(1-\gamma_5)\nu_e(x) | \nu_e \rangle \\ &= \frac{G_F N_e}{\sqrt{2}} \frac{1}{2E_\nu \mathcal{V}} 2 \int d^3x \, u_{\nu_L}^\dagger u_{\nu_L} = \sqrt{2} G_F N_e \end{aligned}$$

$$V_{CC} = \sqrt{2} G_F N_e$$

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$$V_{CC} = \sqrt{2} G_F N_e$$

- for $\overline{\nu_e}$ the sign of V_{CC} is reversed

- Other potentials for ν_e $(\overline{\nu}_e)$ due to different particles in medium

medium	V_{CC}	V_{NC}
e^+ and e^-	$\pm\sqrt{2}G_F(N_e - N_{\bar{e}})$	$\mp\frac{G_F}{\sqrt{2}}(N_e - N_{\bar{e}})(1 - 4\sin^2\theta_W)$
p and $\bar{p}$	0	$\mp\frac{G_F}{\sqrt{2}}(N_p - N_{\bar{p}})(1 - 4\sin^2\theta_W)$
n and $\bar{n}$	0	$\mp\frac{G_F}{\sqrt{2}}(N_n - N_{\bar{n}})$
Neutral $(N_e = N_p)$	$\pm\sqrt{2}G_F N_e$	$\mp\frac{G_F}{\sqrt{2}}N_n$

For ν_μ and ν_τ: V_{NC} are the same as for ν_e BUT $V_{CC} = 0$ for any of these media

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• Estimating typical values:

$$V_{CC} = \sqrt{2}G_F N_e \simeq 7.6\, Y_e \,\frac{\rho}{10^{14}\mathrm{g/cm^3}}\ \mathrm{eV}$$

$Y_e = \frac{N_e}{N_p+N_n} \equiv$ relative number density

$\rho \equiv$ matter density

– At the solar core $\rho \sim 100$ g/cm^3 $\Rightarrow V \sim 10^{-12}$ eV

– At supernova $\rho \sim 10^{14}$ g/cm^3 $\Rightarrow V \sim$ eV

Neutrinos in Matter: Evolution Equation

Evolution Eq. for $|\nu\rangle = \nu_1|\nu_1\rangle + \nu_2|\nu_2\rangle \equiv \nu_\alpha|\nu_\alpha\rangle + \nu_\beta|\nu_\beta\rangle$

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(a) In vacuum in the mass basis: $$-i\frac{\partial}{\partial x}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix} = \left\{E \times I - \begin{pmatrix}\frac{m_1^2}{2E} & 0\\ 0 & \frac{m_2^2}{2E}\end{pmatrix}\right\}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix}$$

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(b) In vacuum in the weak basis

$$-i\frac{\partial}{\partial x}\begin{pmatrix}\nu_\alpha\\ \nu_\beta\end{pmatrix} = \left\{\left[E - \frac{m_1^2+m_2^2}{4E}\right] \times I - \begin{pmatrix}-\frac{\Delta m^2}{4E}\cos 2\theta & \frac{\Delta m^2}{4E}\sin 2\theta\\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta\end{pmatrix}\right\}\begin{pmatrix}\nu_\alpha\\ \nu_\beta\end{pmatrix}$$

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(c) In matter (e, p, n) in weak basis

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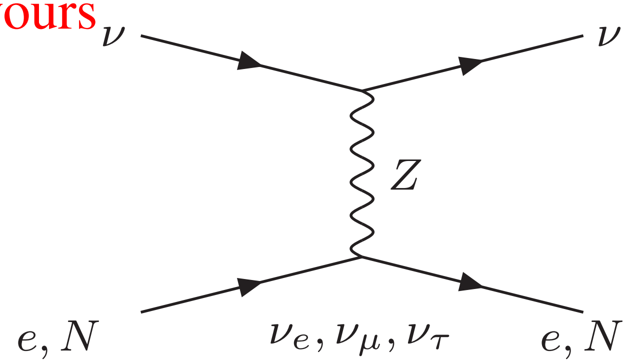
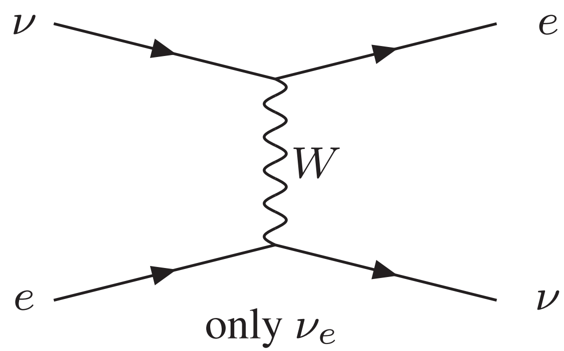
(c)$\neq$ (b) because different flavours have different interactions

For example $\alpha = e$,$\beta = \mu, \tau$:

$V_{CC} = V_\alpha - V_\mu = \sqrt{2}G_F N_e$

(opposite sign for $\overline{\nu}$)





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$$-i\frac{\partial}{\partial x}\begin{pmatrix}\nu_\alpha\\ \nu_\beta\end{pmatrix} = \left\{\left[E - \frac{V_\alpha+V_\beta}{2} - \frac{m_1^2+m_2^2}{4E}\right]\times I - \begin{pmatrix}\frac{V_\alpha-V_\beta}{2} - \frac{\Delta m^2}{4E}\cos 2\theta & \frac{\Delta m^2}{4E}\sin 2\theta\\ \frac{\Delta m^2}{4E}\sin 2\theta & -\frac{V_\alpha-V_\beta}{2} + \frac{\Delta m^2}{4E}\cos 2\theta\end{pmatrix}\right\}\begin{pmatrix}\nu_\alpha\\ \nu_\beta\end{pmatrix}$$

Diagonalizing:

$$-i\frac{\partial}{\partial x}\begin{pmatrix}\nu_\alpha\\ \nu_\beta\end{pmatrix} \equiv \left\{\left[E - \frac{\mu_1^2+\mu_2^2}{4E}\right]\times I - \begin{pmatrix}-\frac{\Delta \mu^2}{4E}\cos 2\theta_m & \frac{\Delta \mu^2}{4E}\sin 2\theta_m\\ \frac{\Delta \mu^2}{4E}\sin 2\theta_m & \frac{\Delta \mu^2}{4E}\cos 2\theta_m\end{pmatrix}\right\}\begin{pmatrix}\nu_\alpha\\ \nu_\beta\end{pmatrix}$$

Effective masses and mixing are different than in vacuum

⇒ Effective masses and mixing are different than in vacuum

– The effective masses: $(A = 2E(V_\alpha - V_\beta))$

$$\mu^2_{1,2}(x) = \frac{m_1^2 + m_2^2}{2} + E(V_\alpha + V_\beta) \mp \frac{1}{2}\sqrt{(\Delta m^2 \cos 2\theta - A)^2 + (\Delta m^2 \sin 2\theta)^2}$$

$$\Delta\mu^2(x) = \sqrt{(\Delta m^2 \cos 2\theta - A)^2 + (\Delta m^2 \sin 2\theta)^2}$$

– The mixing angle in matter

$$\tan 2\theta_m = \frac{\Delta m^2 \sin 2\theta}{\Delta m^2 \cos 2\theta - A}$$

⇒ Effective masses and mixing are different than in vacuum

– The effective masses: $(A = 2E(V_\alpha - V_\beta))$

$$\mu^2_{1,2}(x) = \frac{m_1^2 + m_2^2}{2} + E(V_\alpha + V_\beta) \mp \frac{1}{2}\sqrt{(\Delta m^2 \cos 2\theta - A)^2 + (\Delta m^2 \sin 2\theta)^2}$$

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- Dependence on relative sign between A and $\Delta m^2 \cos(2\theta)$

  ⇒ Information on sign Δm^2 or Octant of θ

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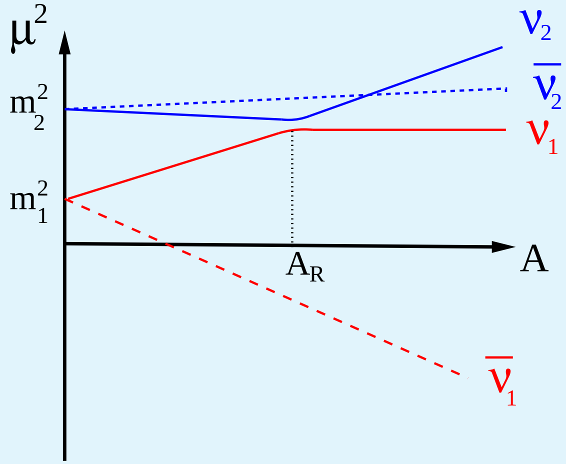

At *resonant* potential: $A_R = \Delta m^2 \cos 2\theta$

Minimum $\Delta\mu^2 = \mu_2^2 - \mu_1^2$

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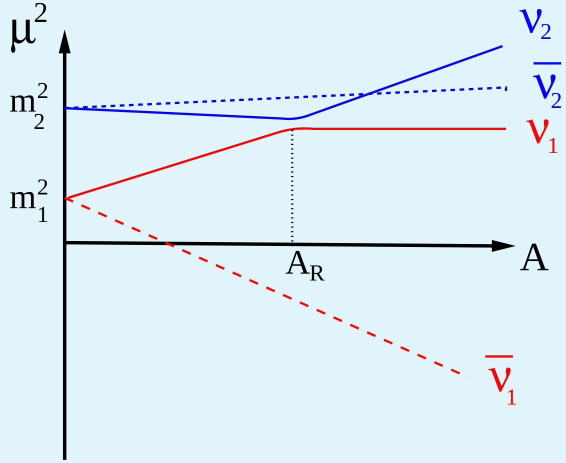

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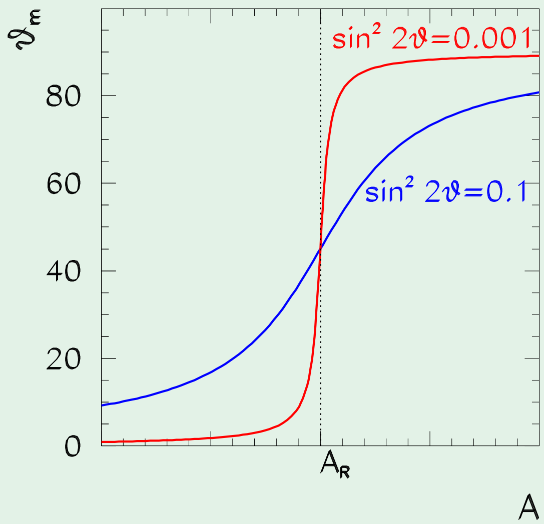

- $*$ At $A = 0$ (vacuum) $\Rightarrow \theta_m = \theta$
- $*$ At $A = A_R \Rightarrow \theta_m = \frac{\pi}{4}$
- $*$ At $A > A_R \Rightarrow \theta_m = \frac{\pi}{2} - \theta$
- $*$ At $A >> A_R \Rightarrow \theta_m = \frac{\pi}{2}$

The oscillation length in vacuum

$$L_0^{osc} = \frac{4\pi E}{\Delta m^2}$$

The oscillation length in matter

$$L^{osc} \equiv \frac{4\pi E}{\Delta\mu^2} = \frac{L_0^{osc}}{\sqrt{(\Delta m^2 \cos 2\theta - A)^2 + (\Delta m^2 \sin 2\theta)^2}}$$

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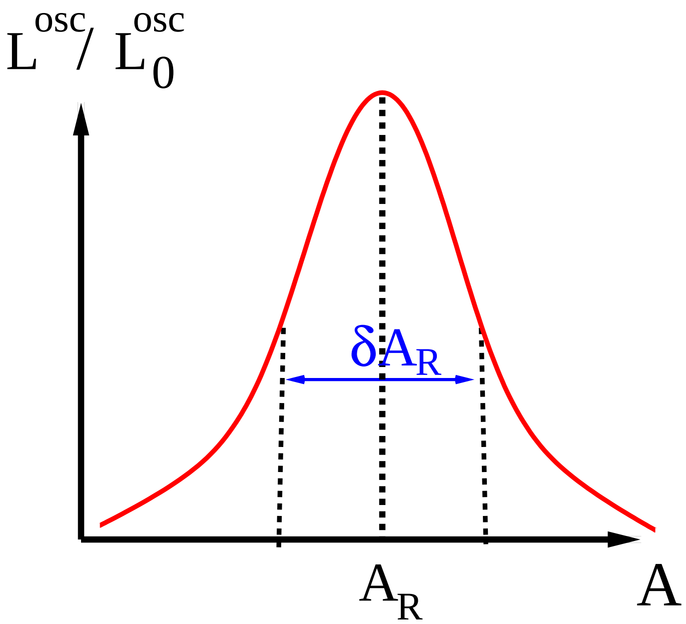
L^{osc} presents a resonant behaviour



At the resonant density $A_R = \Delta m^2 \cos\theta$

$$L_R^{osc} = \frac{L_0^{osc}}{\sin 2\theta}$$

The width of the resonance in potential:

$$\delta V_R \equiv \frac{\delta A_R}{E} = \frac{\Delta m^2 \sin 2\theta}{E}$$

The width of the resonance in distance:

$$\delta r_R = \frac{\delta V_R}{|\frac{dV}{dr}|_R}$$

• In terms of the instantaneous mass eigenstates in matter: $$\begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix} = U[\theta_m(x)] \begin{pmatrix} \nu_1^m(x) \\ \nu_2^m(x) \end{pmatrix}$$

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$\Rightarrow$ the evolution equation in flavour basis (removing diagonal part)

$$i \begin{pmatrix} \dot{\nu}_\alpha \\ \dot{\nu}_\beta \end{pmatrix} = \frac{1}{4E} \begin{pmatrix} A - \Delta m^2 \cos 2\theta & \Delta m^2 \sin 2\theta \\ \Delta m^2 \sin 2\theta & -A + \Delta m^2 \cos 2\theta \end{pmatrix} \begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix}$$

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$$i \begin{pmatrix} \dot\nu_1^m \\ \dot\nu_2^m \end{pmatrix} = \frac{1}{4E} U^\dagger(\theta_m) \begin{pmatrix} A - \Delta m^2 \cos 2\theta & \Delta m^2 \sin 2\theta \\ \Delta m^2 \sin 2\theta & -A + \Delta m^2 \cos 2\theta \end{pmatrix} U(\theta_m) \begin{pmatrix} \nu_1^m \\ \nu_2^m \end{pmatrix} - i\, U^\dagger \dot U(\theta_m) \begin{pmatrix} \nu_1^m \\ \nu_2^m \end{pmatrix}$$

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$\Rightarrow$ ν_i^m behave approximately as *evolution eigenstates*

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The adiabaticity condition

$$\frac{1}{V}\frac{dV}{dx}\Big|_R \ll \frac{\Delta m^2}{2E}\frac{\sin^2 2\theta}{\cos 2\theta} \equiv \delta r_R \gg L_R^{osc}/2\pi$$

$\Rightarrow$ Many oscillations take place in the resonant region

Neutrinos in The Sun : MSW Effect

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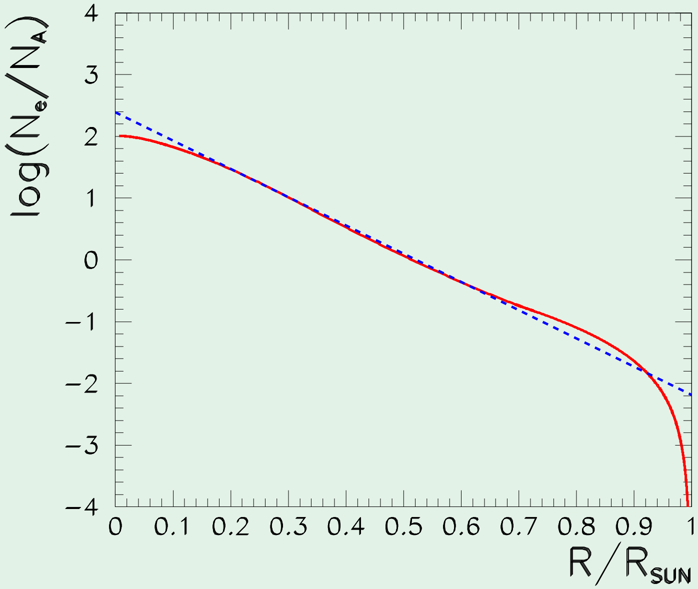
The solar matter density



$V_{CC} = \sqrt{2} G_F N_e \sim 10^{-14} \frac{N_e}{N_A}$ eV

At core: $V_{CC,0} \sim 10^{-14}$–10^{-12} eV

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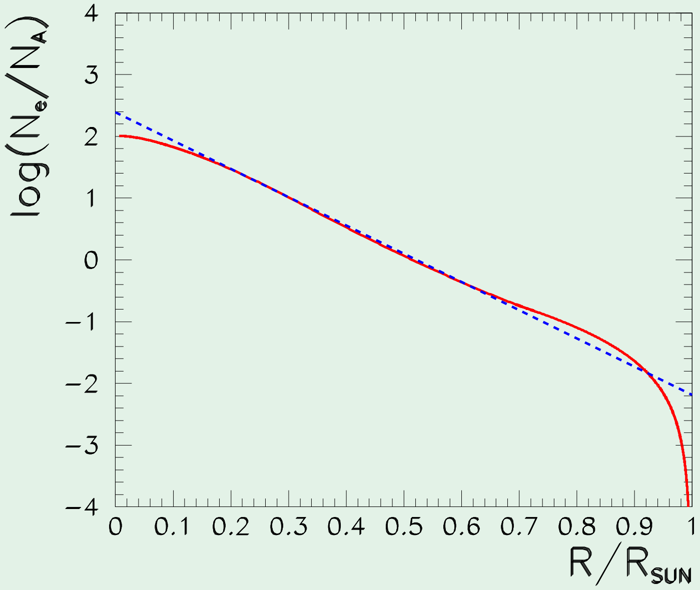

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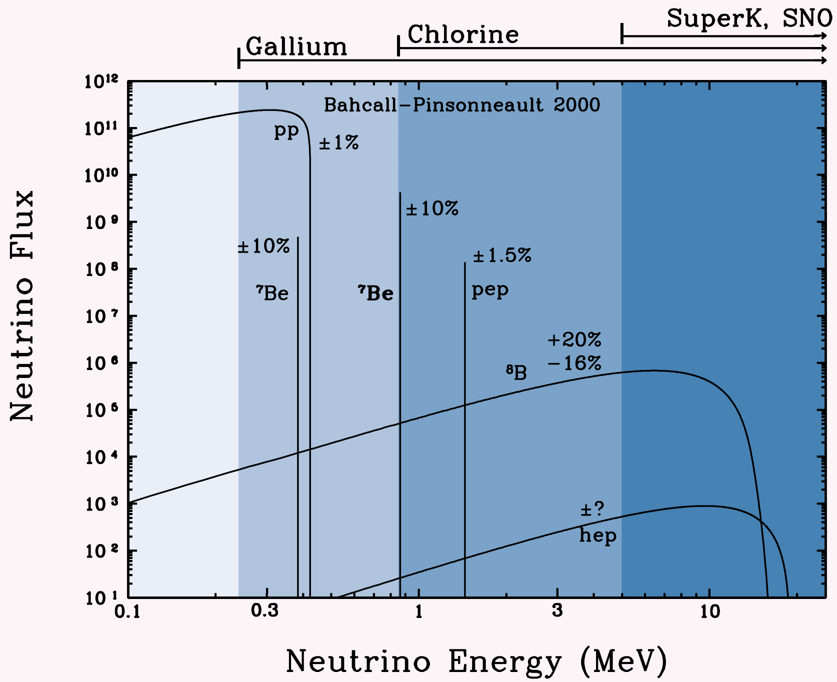
The energy spectrum of solar $\nu_e' s$



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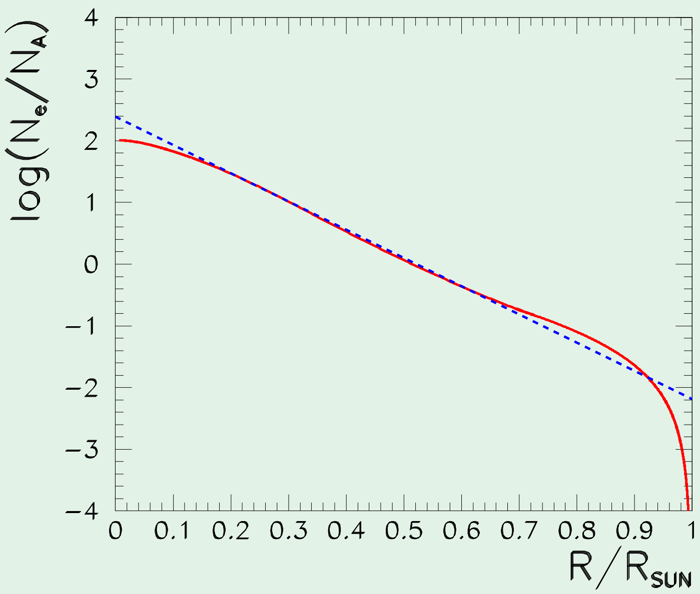

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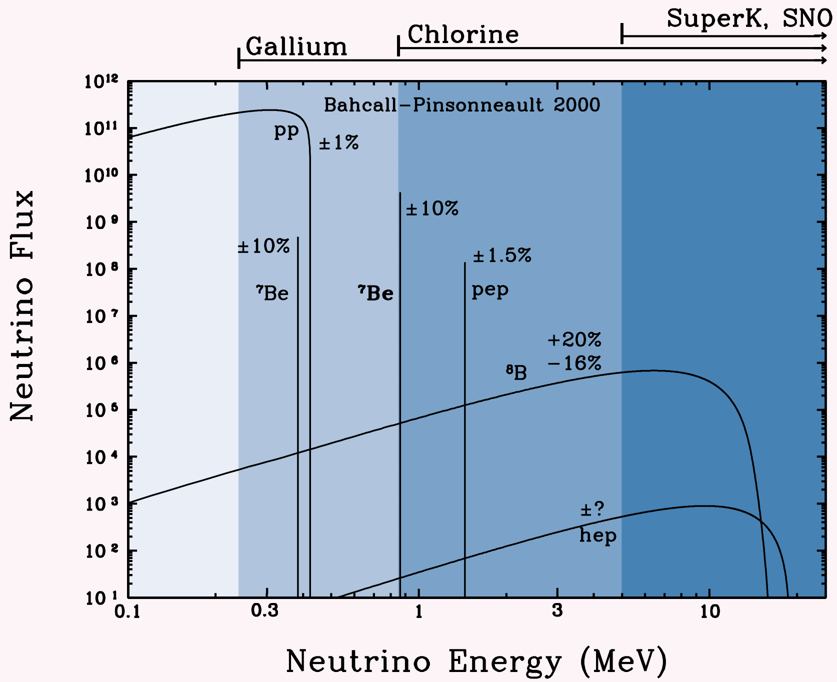

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- For $\nu_e \leftrightarrow \nu_{\mu(\tau)}$, in vacuum $\nu_e = \cos\theta\, \nu_1 + \sin\theta\, \nu_2$
- For 10^{-9} eV$^2 \lesssim \Delta m^2 \lesssim 10^{-4}$ eV^2 $\Rightarrow$ $2E_\nu V_{CC,0} > \Delta m^2 \cos 2\theta$

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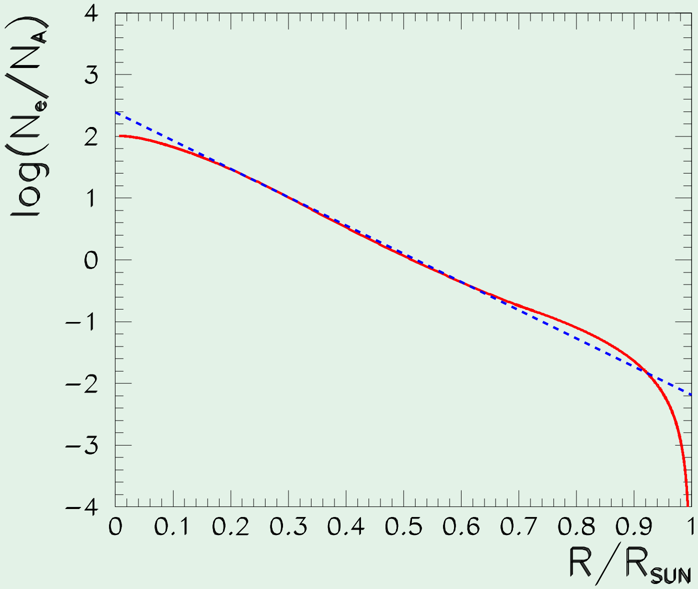

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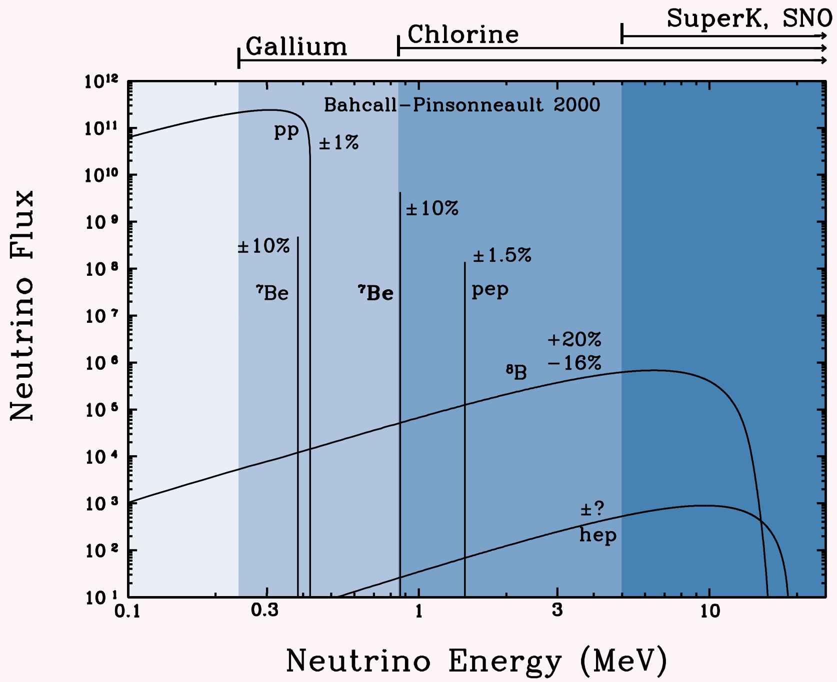

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$\Rightarrow$ ν can cross resonance condition in its way out of the Sun

For $\theta \ll \frac{\pi}{4}$: In vacuum $\nu_e = \cos\theta\,\nu_1 + \sin\theta\,\nu_2$ is mostly ν_1

In Sun core $\nu_e = \cos\theta_{m,0}\,\nu_1 + \sin\theta_{m,0}\,\nu_2$ is mostly ν_2

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If $\frac{(\Delta m^2/\mathrm{eV}^2)\sin^2 2\theta}{(E/\mathrm{MeV})\cos 2\theta} \gg 3 \times 10^{-9}$

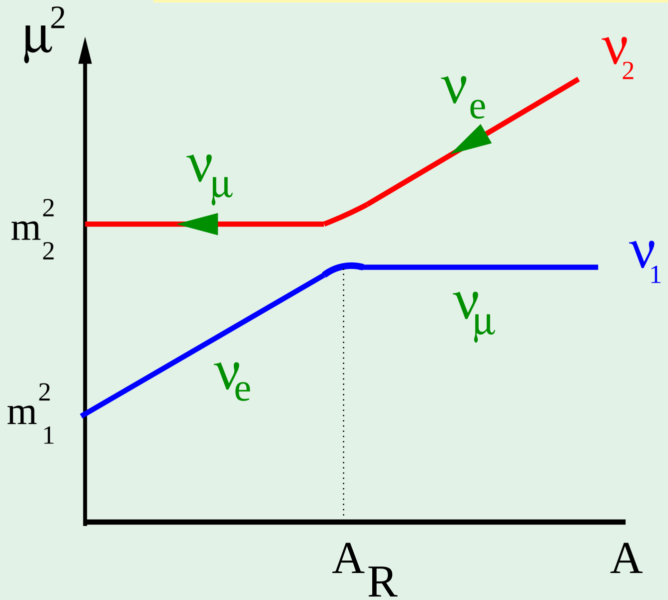
$\Rightarrow$ Adiabatic transition

$*$ ν is mostly ν_2 before and after resonance

$*$ $\theta_m \downarrow$ *dramatically* at resonance

$\Rightarrow \nu_e$ component $\downarrow \Rightarrow P_{ee} \downarrow$

This is the MSW effect



$$P_{ee} = \frac{1}{2}\left[1 + \cos 2\theta_{m,0}\cos 2\theta\right] \simeq \sin^2\theta$$

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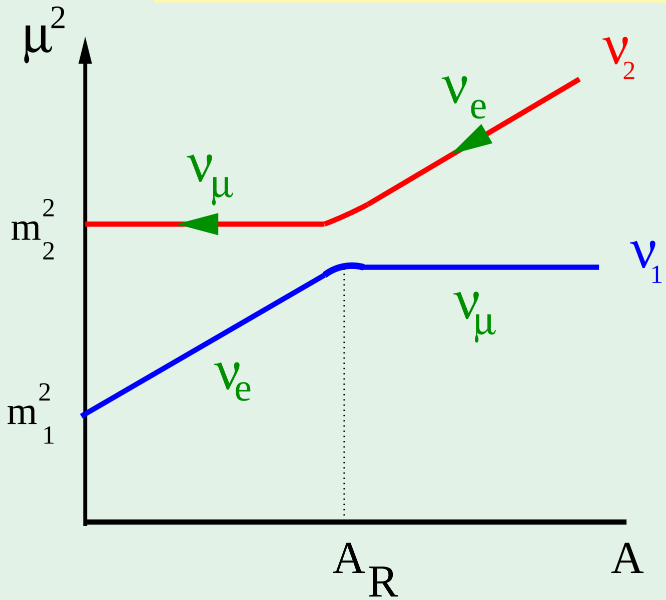

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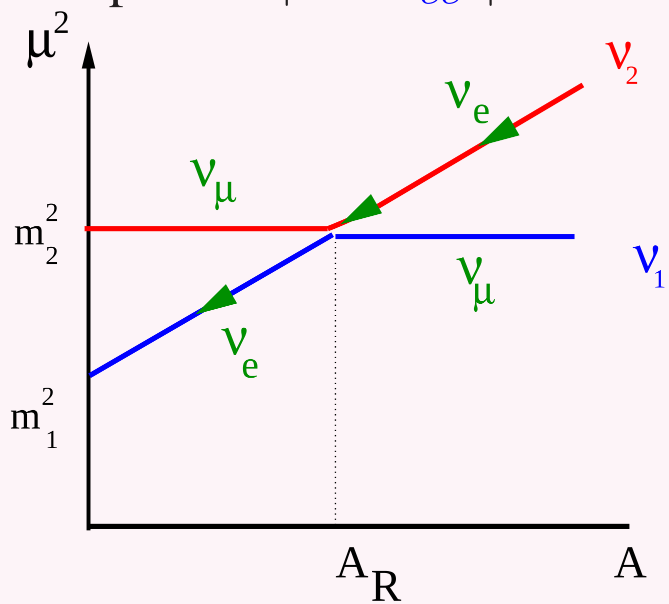
$\Rightarrow$ Non-Adiabatic transition

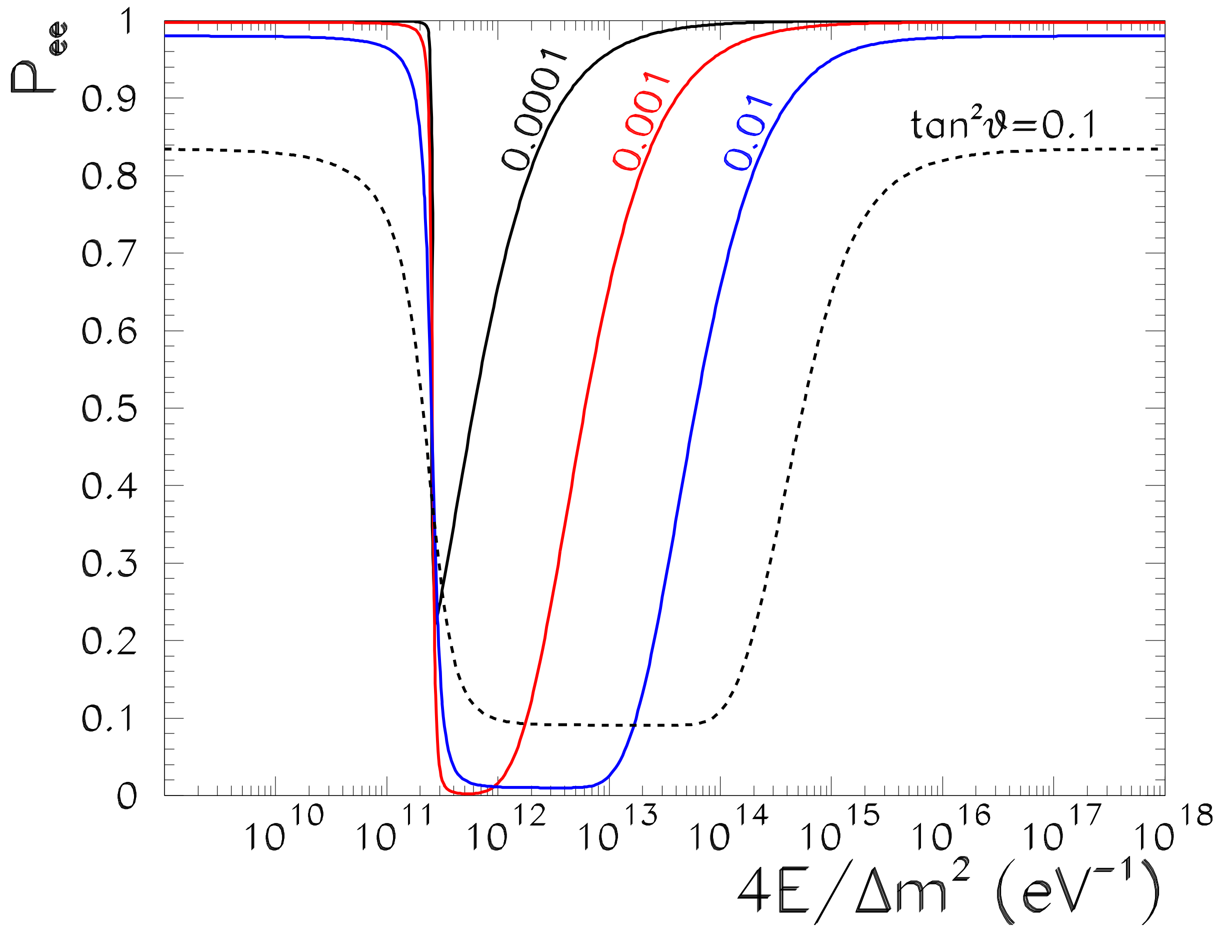
* ν is mostly ν_2 till the resonance
* At resonance the state can jump into ν_1 (with probability P_{LZ})

$\Rightarrow \nu_e$ component $\uparrow \Rightarrow P_{ee} \uparrow$



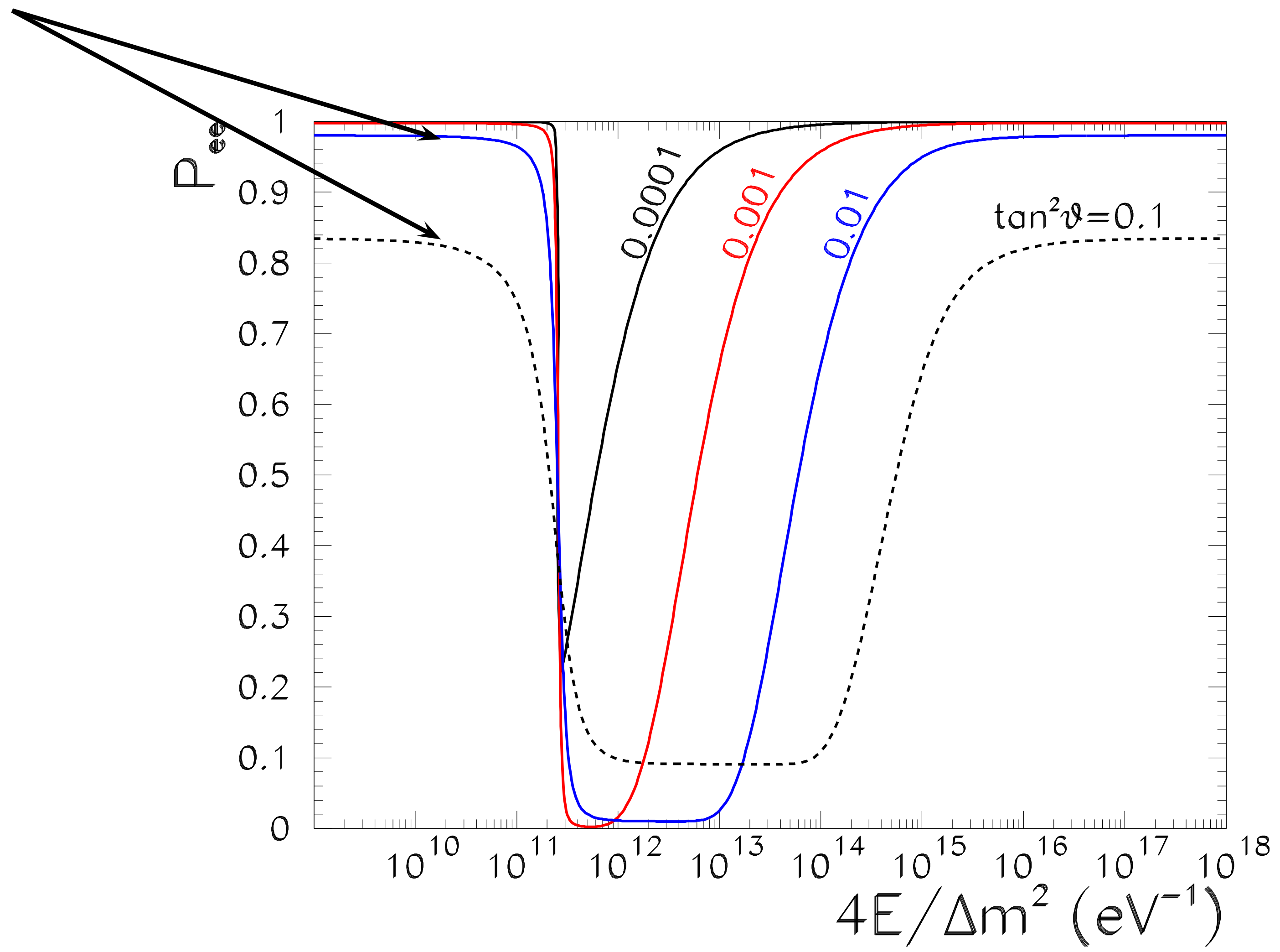
$$P_{ee} = \frac{1}{2}\left[1 + (1-2P_{LZ})\cos 2\theta_{m,0}\cos 2\theta\right]$$

Neutrinos in The Sun : MSW Effect



Neutrinos in The Sun : MSW Effect

ν does not cross resonance: $P_{ee} = 1 - \frac{1}{2}\sin^2 2\theta > \frac{1}{2}$



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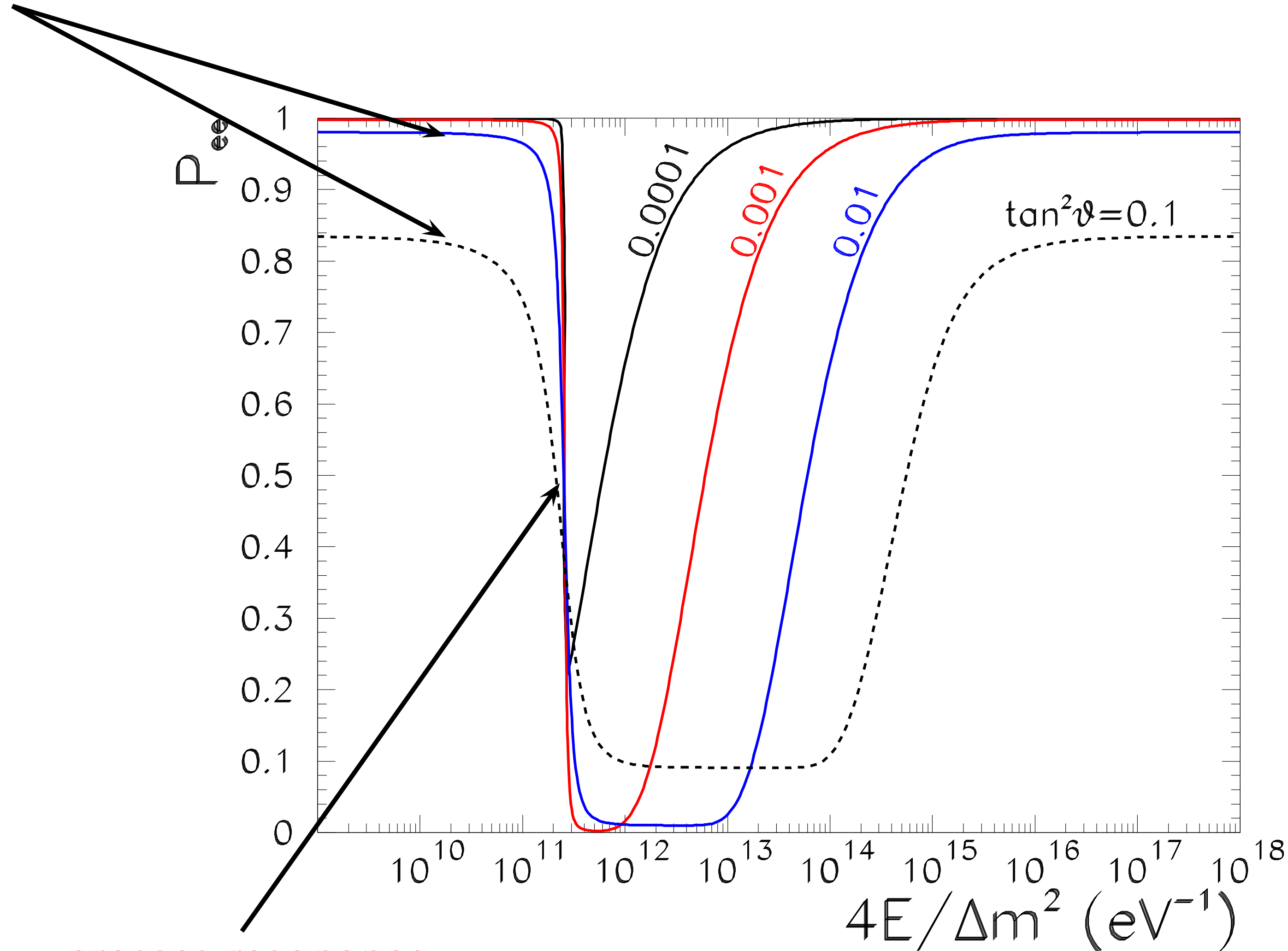

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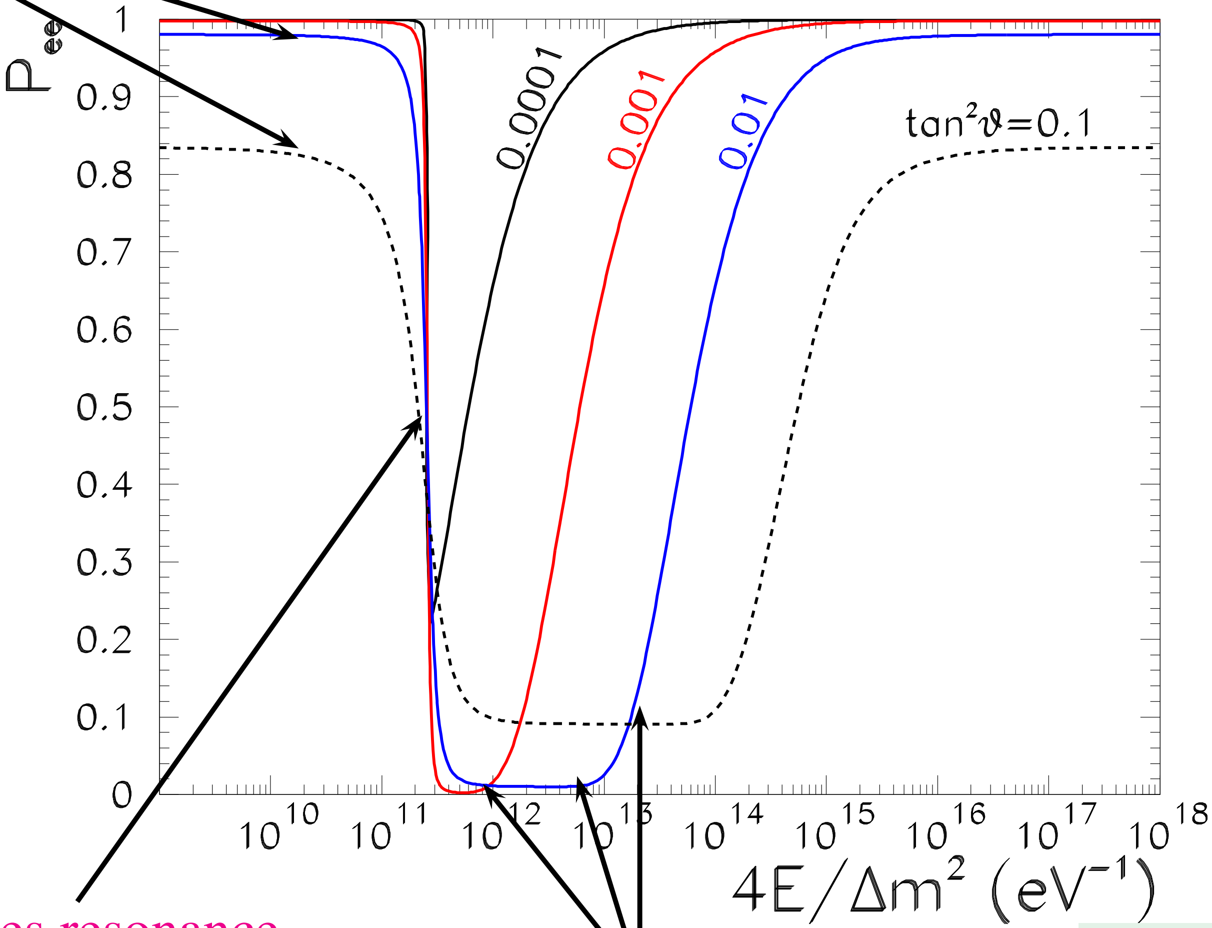

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Adiabatic MSW transition $P_{ee} = \sin^2\theta < \frac{1}{2}$

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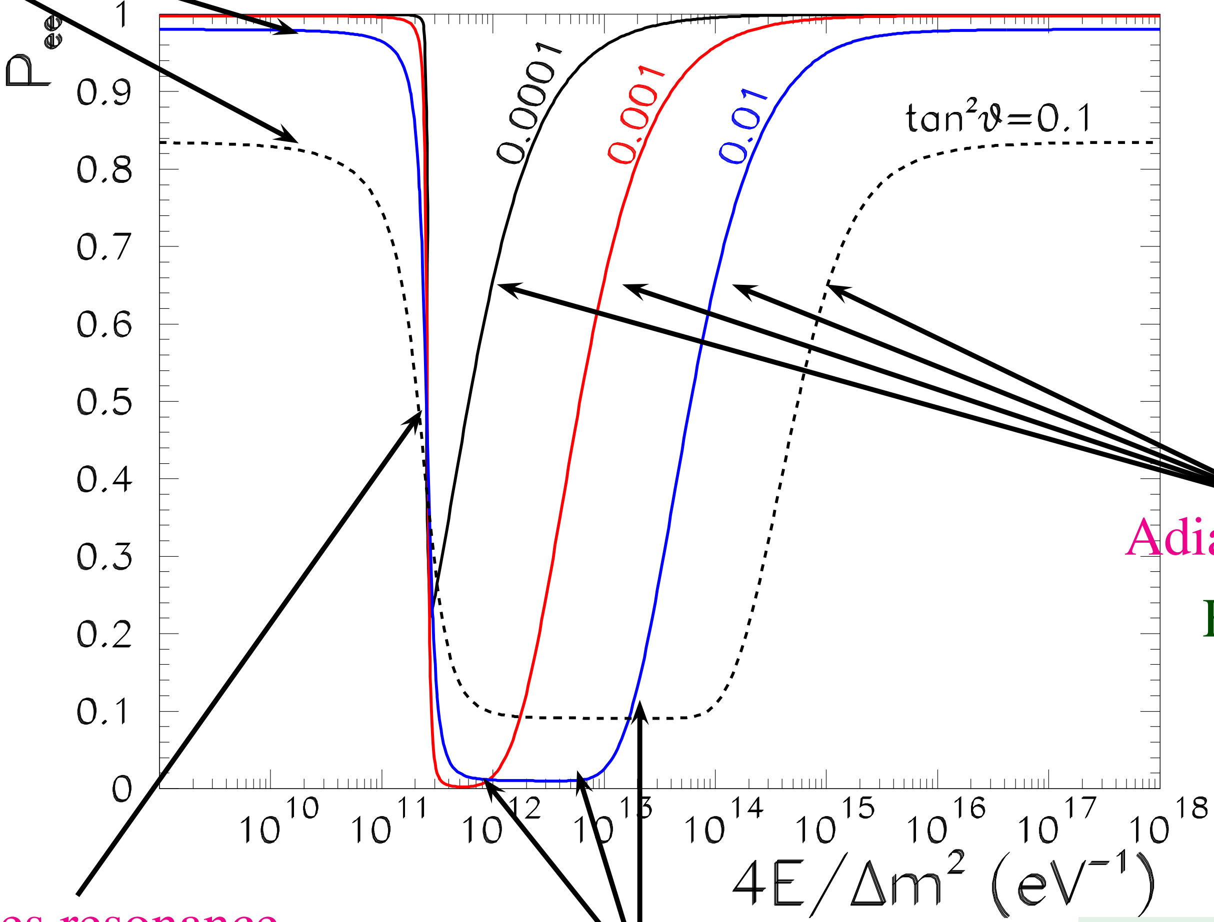


Adiabacity breaking

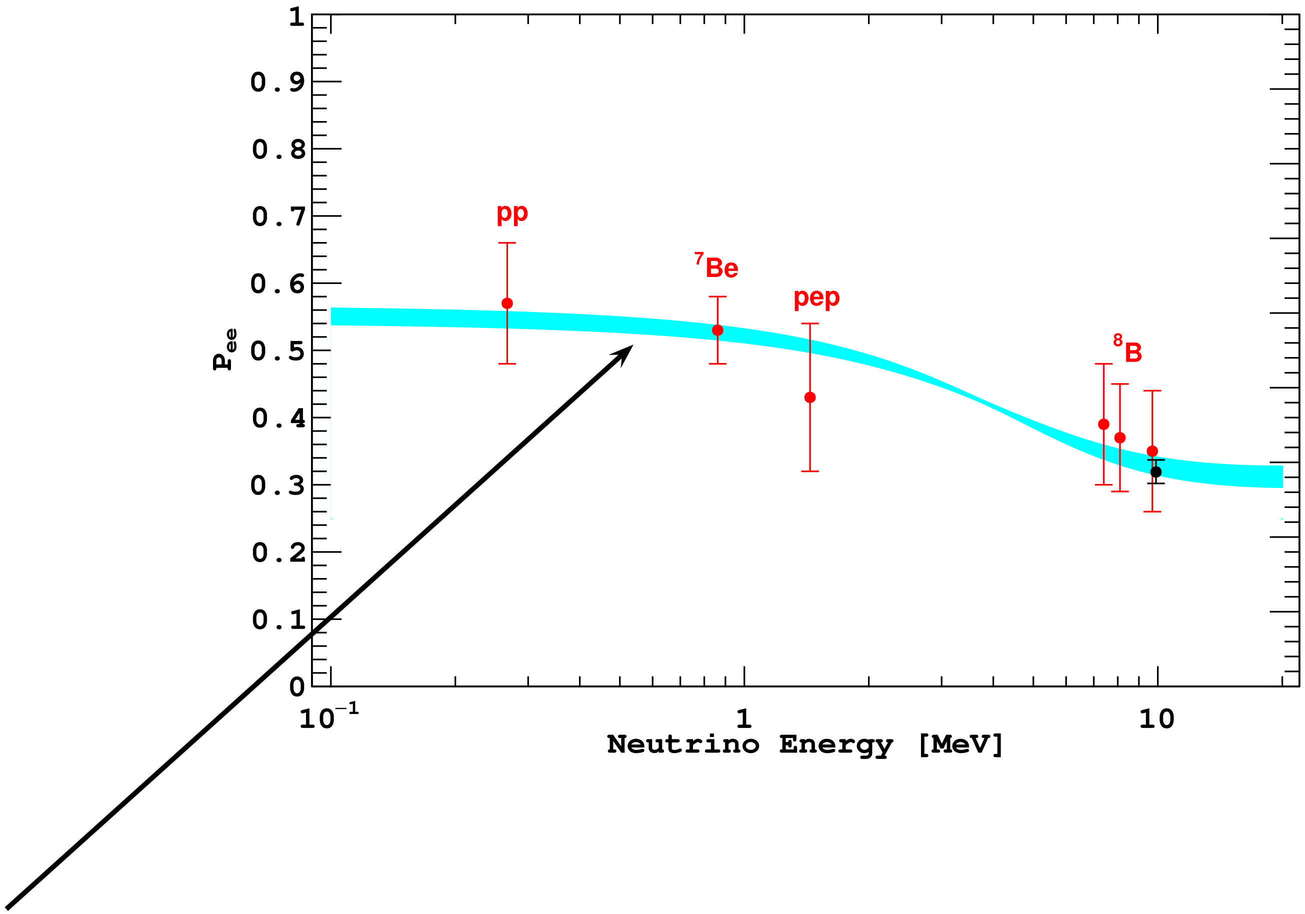
Effect of P_{LZ}

ν crosses resonance

MSW effect

Adiabatic MSW transition $P_{ee} = \sin^2\theta < \frac{1}{2}$

Neutrinos in The Sun : The answer



P_{ee} for $\Delta m^2_{21} = (7.41^{+0.21}_{-0.20}) \times 10^{-5}$ eV^2 and $\theta_{12} = 33.41^{\circ}{}^{+0.78}_{-0.75}$

$\Rightarrow$ Effective masses and mixing are different than in vacuum

– The effective masses: ($A = 2E(V_\alpha - V_\beta)$)

$$\mu^2_{1,2}(x) = \frac{m_1^2 + m_2^2}{2} + E(V_\alpha + V_\beta) \pm \frac{1}{2}\sqrt{(\Delta m^2 \cos 2\theta - A)^2 + (\Delta m^2 \sin 2\theta)^2}$$

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$$\tan 2\theta_m = \frac{\Delta m^2 \sin 2\theta}{\Delta m^2 \cos 2\theta - A}$$

- For constant matter density $\Rightarrow$ θ_m and μ_i are constant along ν evolution

  $\Rightarrow$ the evolution is determined by masses and mixing in matter so

$$P_{\alpha\neq\beta} = \sin^2(2\theta_m)\,\sin^2\left(\frac{\Delta\mu^2 L}{2E}\right)$$

- Dependence on relative sign between A and $\Delta m^2 \cos(2\theta)$

  $\Rightarrow$ Information on sign Δm^2 and Octant of θ

- Constant matter potential is a good approximation for LBL experiments.

Matter effects in LBL

- In the 3ν scenario one must solve: $i\frac{d\vec{\nu}}{dt} = H\,\vec{\nu}$ $\quad H = U \cdot H_0^d \cdot U^\dagger + V$

$$H_0^d = \frac{1}{2E_\nu}\text{diag}\left(-\Delta m^2_{21}, 0, \Delta m^2_{32}\right) \qquad V = \text{diag}\left(\pm\sqrt{2}G_F N_e, 0, 0\right)$$

$\Rightarrow$ $H = \tilde{U} \cdot H_m^d \cdot \tilde{U}^\dagger$ $\quad \tilde{U}$= effective mixing matrix in matter

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• At LBL: $\sqrt{2}G_F N_e \equiv V_{\oplus,\mathrm{CRUST}} \sim 5\times 10^{-14}\,\mathrm{eV}$ $\sim$ constant at ν trajectory

• The oscillation probability at L

$$P_{\alpha\beta} = \delta_{\alpha\beta} - 4\sum_{j<i}^{3}\mathrm{Re}[\tilde{U}^\star_{\alpha i}\tilde{U}_{\beta i}\tilde{U}_{\alpha j}\tilde{U}^\star_{\beta j}]\sin^2\left(\frac{\Delta\mu^2_{ij}L}{4E_\nu}\right) + 2\sum_{j<i}\mathrm{Im}[\tilde{U}^\star_{\alpha i}\tilde{U}_{\beta i}\tilde{U}_{\alpha j}\tilde{U}^\star_{\beta j}]\sin\left(\frac{\Delta\mu_{ij}L}{2E_\nu}\right)$$

$\Rightarrow$ Exact numerically computed probabilities

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$\Rightarrow$ Exact numerically computed probabilities

- Using: $\Delta m^2_{21} \ll \Delta m^2_{31}$ and θ_{13} relatively small

$\Rightarrow$ Approximate analitical expresions expanded in the small parameters

Matter effects in LBL

- Most relevant for $\nu_\mu \to \nu_e$

$$
\begin{aligned}
P_{\mu e(\bar\mu\bar e)} \simeq\ & s_{23}^2 \sin^2 2\theta_{13} \left(\frac{\Delta_{31}}{\Delta_{31} \mp V_\oplus}\right)^2 \sin^2\left(\frac{(\Delta_{31} \mp V_\oplus)\, L}{2}\right) \\
&+ \tilde J\, \frac{\Delta_{21}}{V_\oplus} \frac{\Delta_{31}}{\Delta_{31} \mp V_\oplus} \sin\left(\frac{V_\oplus L}{2}\right) \sin\left(\frac{(\Delta_{31} \mp V_\oplus) L}{2}\right) \cos\delta \cos\left(\frac{\Delta_{31}\, L}{2}\right) \\
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\end{aligned}
$$

$$\Delta_{ij} = \frac{\Delta m_{ij}^2}{2E_\nu}$$

$$\tilde J = c_{13} \sin^2 2\theta_{13} \sin^2 2\theta_{23} \sin^2 2\theta_{12}$$

$\Rightarrow$ Sensitivity to θ_{13}, octant of θ_{23}, δ_{CP}, sign$\Delta m_{31}^2 \equiv$ Ordering

Matter effects in LBL

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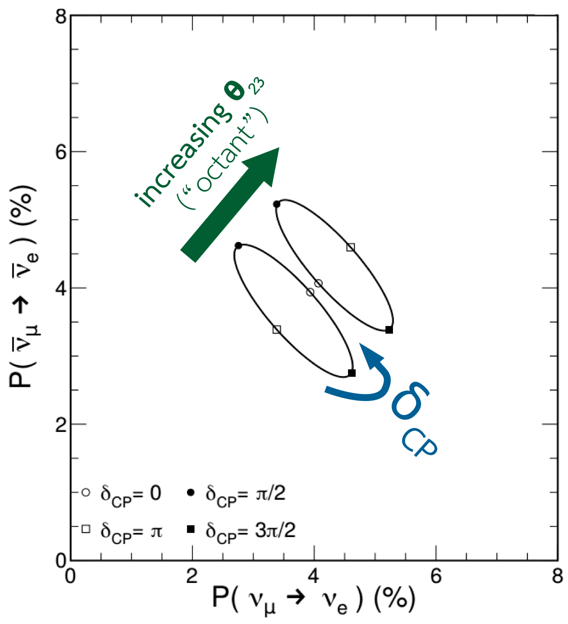
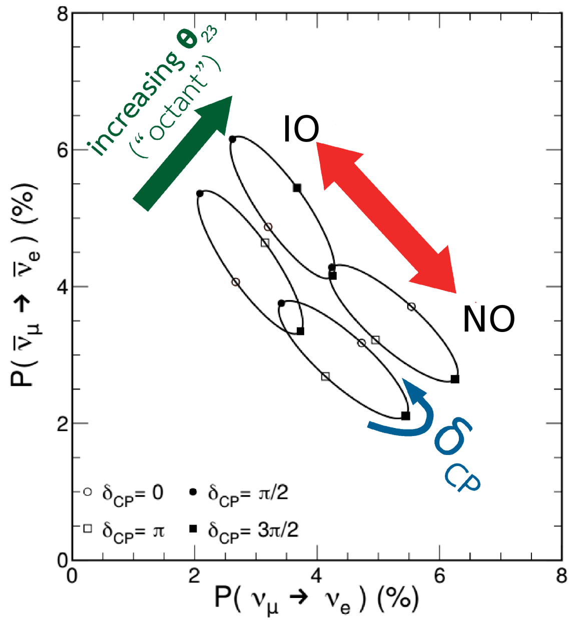




In plots: $\theta_{13} \sim 8^\circ$ fix

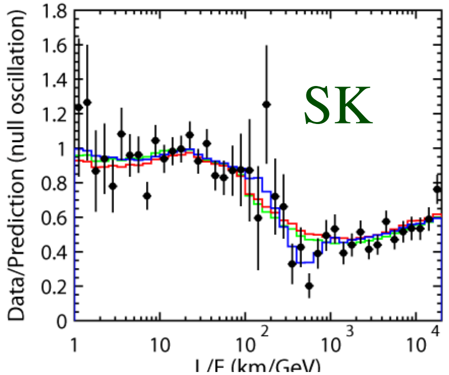
In plots: $\Delta_{31} L \sim \pi$ (osc max)

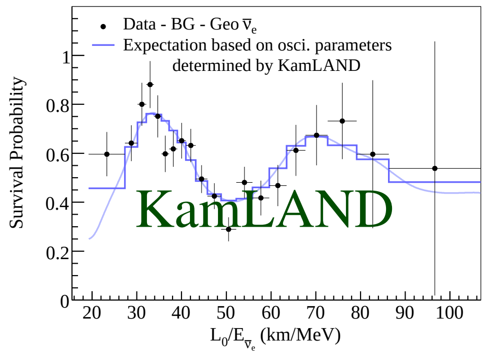
Left: $V_\oplus \ll \Delta_{31}$ (no matter)

Right: $V_\oplus L \sim 0.2$ (NOνA)

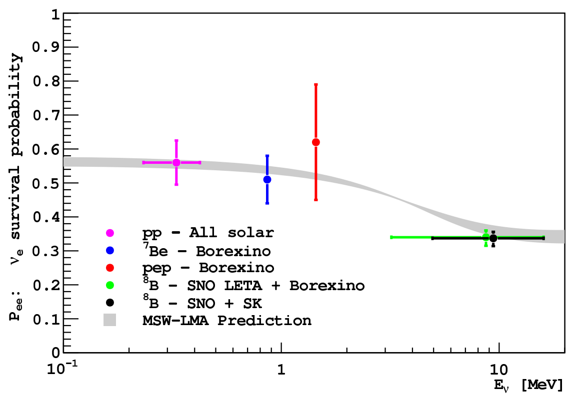
Plots taken from J. Wolcott 52nd FNAL users meeting talk

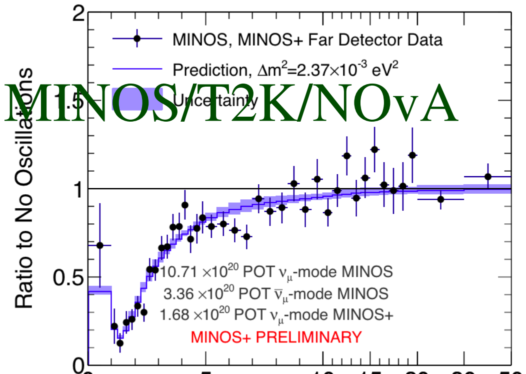
- We have observed with high (or good) precision:
  - $*$ Atmospheric ν_μ & $\bar\nu_\mu$ disappear most likely to ν_τ (**SK**,MINOS, ICECUBE)
  - $*$ Accel. ν_μ & $\bar\nu_\mu$ disappear at $L \sim 300/800$ Km (K2K, **T2K, MINOS, NOνA**)
  - $*$ Some accelerator ν_μ appear as ν_e at $L \sim 300/800$ Km (**T2K**, MINOS,NOνA)
  - $*$ Solar ν_e convert to ν_μ/ν_τ (Cl, Ga, **SK, SNO, Borexino**)
  - $*$ Reactor $\overline{\nu_e}$ disappear at $L \sim 200$ Km (**KamLAND**)
  - $*$ Reactor $\overline{\nu_e}$ disappear at $L \sim 1$ Km (D-Chooz, **Daya Bay, Reno**)
- Confirmed: Vacuum oscillation L/E pattern with 2 frequencies

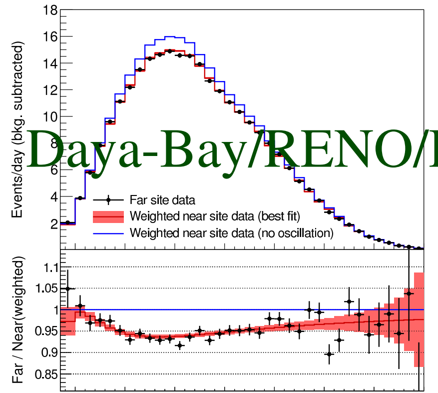




MSW conversion in Sun





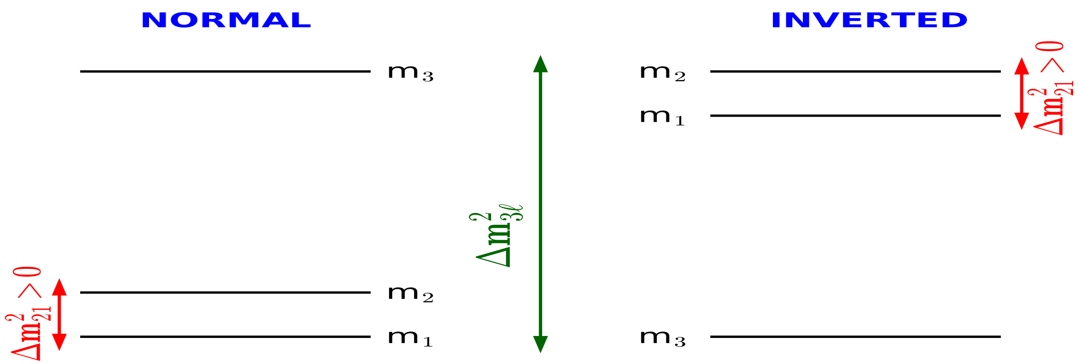


3ν Flavour Parameters

- For **for 3 ν's** : 3 Mixing angles + 1 Dirac Phase + 2 Majorana Phases

$$U_{\rm LEP} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta_{\rm cp}} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta_{\rm cp}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{21} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\eta_1} & 0 & 0 \\ 0 & e^{i\eta_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Convention: $0 \leq \theta_{ij} \leq 90°$ $\quad 0 \leq \delta \leq 360°$ $\quad \Rightarrow$ 2 Orderings



$\Delta m^2_{21} > 0$ by convention

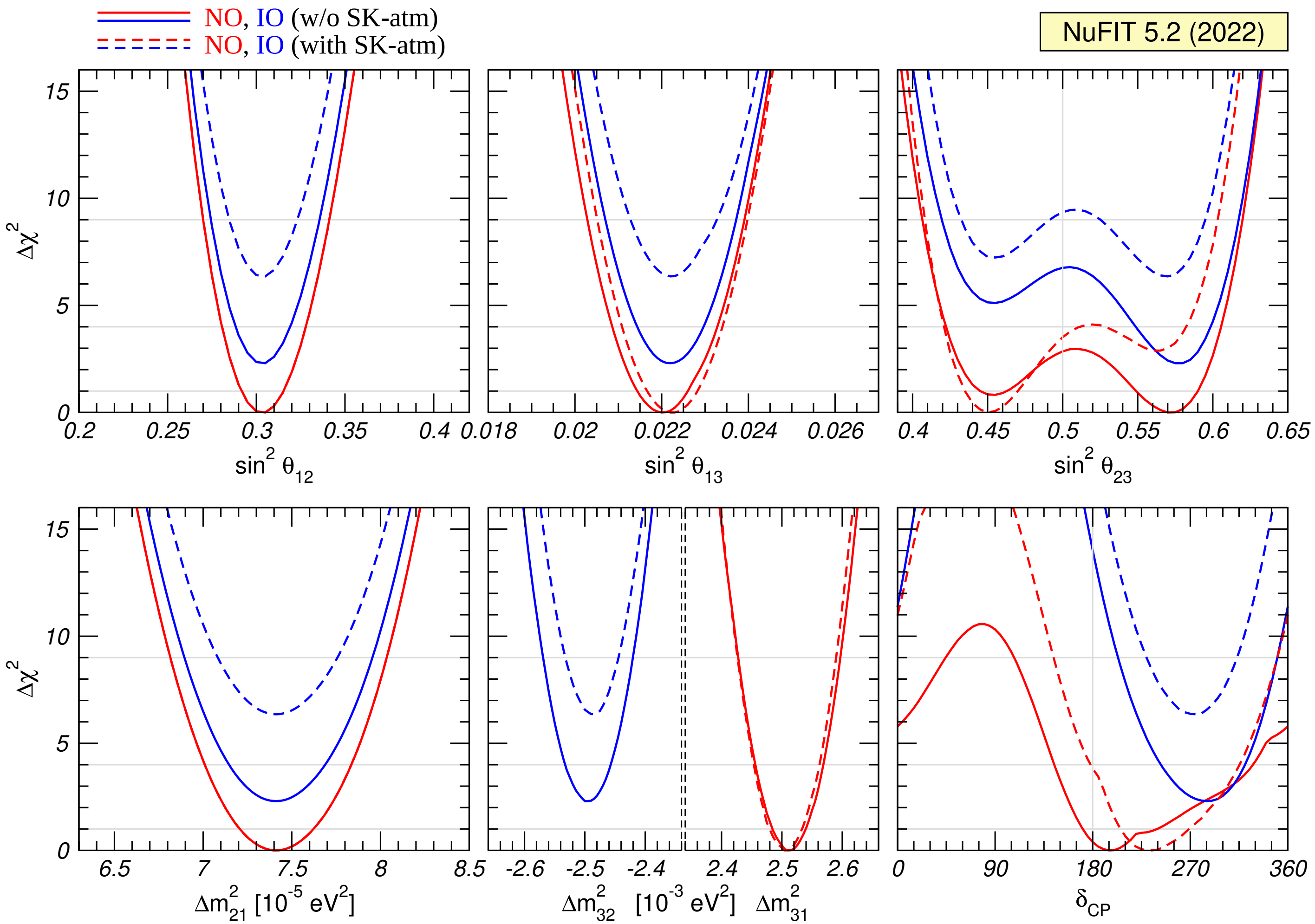
$$\Delta m^2_{3\ell} = \begin{cases} \Delta m^2_{31} > 0 \text{ for NO} \\ \Delta m^2_{32} < 0 \text{ for IO} \end{cases}$$

Experiment	Dominant Dependence	Important Dependence
Solar Experiments	θ_{12}	Δm^2_{21} , θ_{13}
Reactor LBL (KamLAND)	Δm^2_{21}	θ_{12} , θ_{13}
Reactor MBL (Daya Bay, Reno, D-Chooz)	θ_{13} $\Delta m^2_{3\ell}$	
Atmospheric Experiments (SK,IC)		θ_{23}, $\Delta m^2_{3\ell}$, θ_{13} ,$\delta_{\rm cp}$
Acc LBL ν_μ Disapp (Minos,T2K,NOvA)	$\Delta m^2_{3\ell}$ θ_{23}	
Acc LBL ν_e App (Minos,T2K,NOvA)	$\delta_{\rm cp}$	θ_{13}, θ_{23}

Summary: Global 3 ν Flavour Parameters

Global 6-parameter fit http://www.nu-fit.org

Esteban, Gonzalez-Garcia, Maltoni, Schwetz, Zhou, JHEP'20 [2007.14792]



SK-atm $\equiv \chi^2$ table from SK1-4

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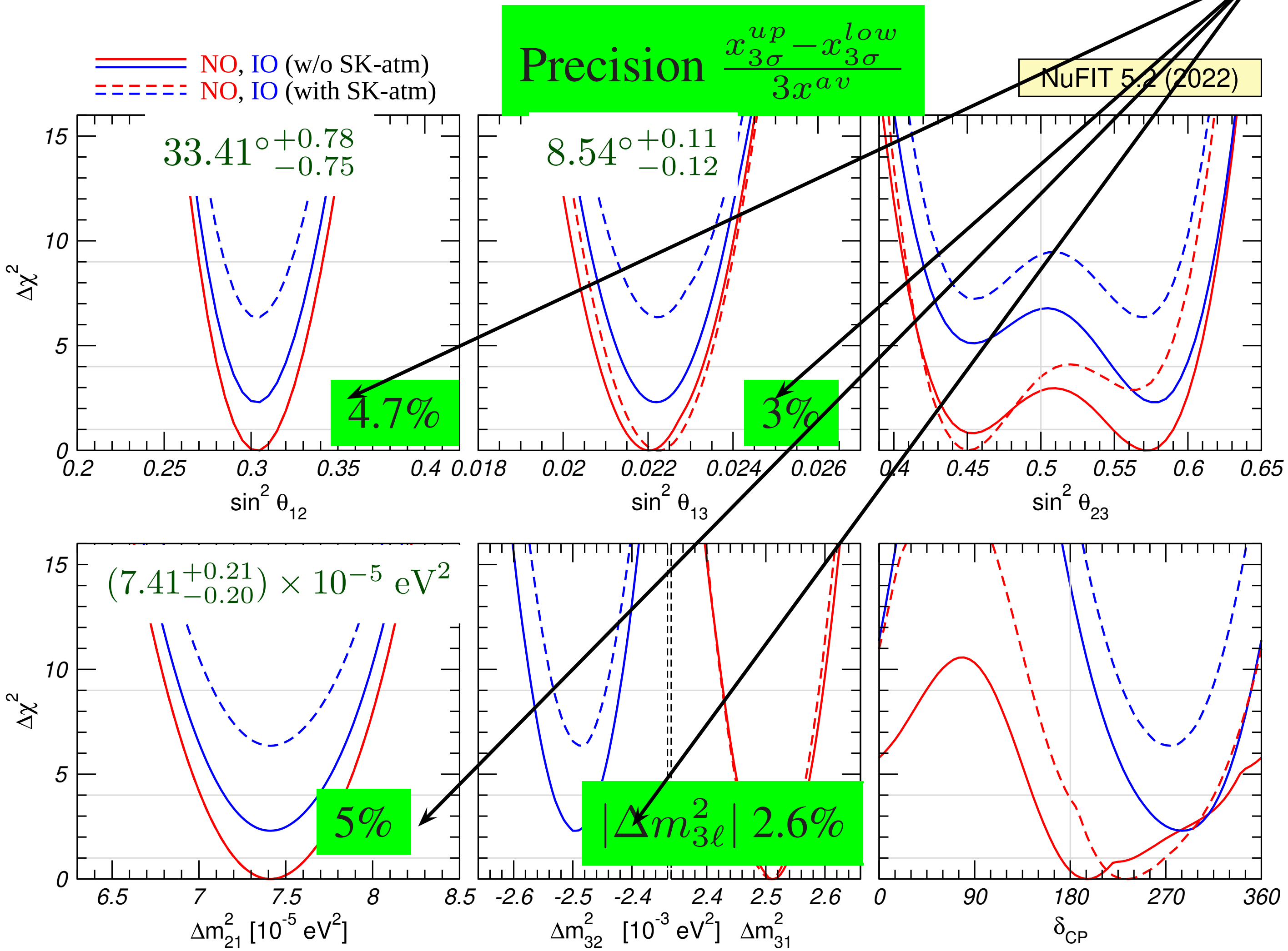
- 4 well-known parameters:

$\theta_{12}, \theta_{13}, \Delta m^2_{21}, |\Delta m^2_{3\ell}|$



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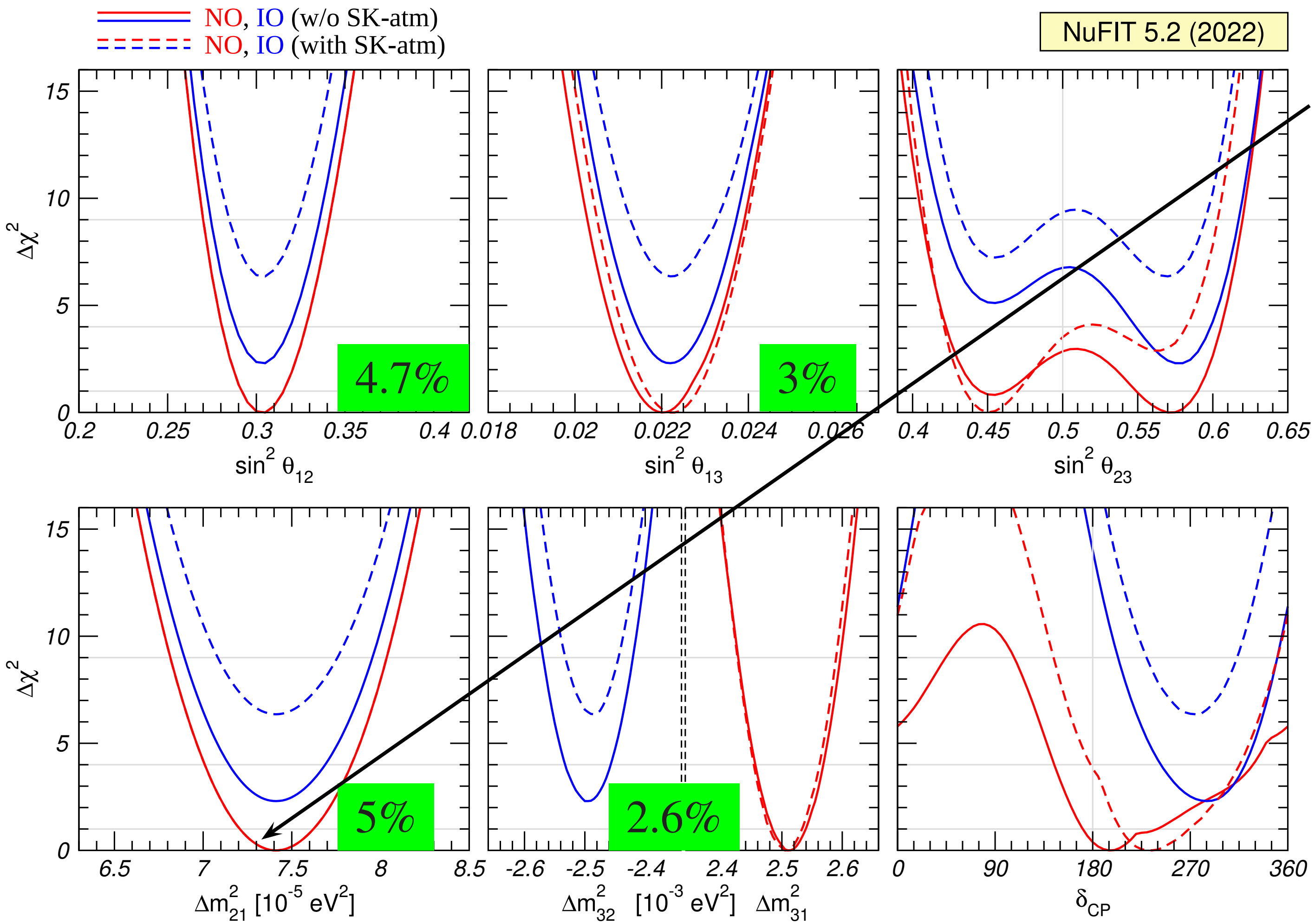

- 4 well-known parameters:

  θ_{12}, θ_{13}, Δm^2_{21}, $|\Delta m^2_{3\ell}|$

  Δm^2_{21} Solar vs KLAND

  Tension Resolved

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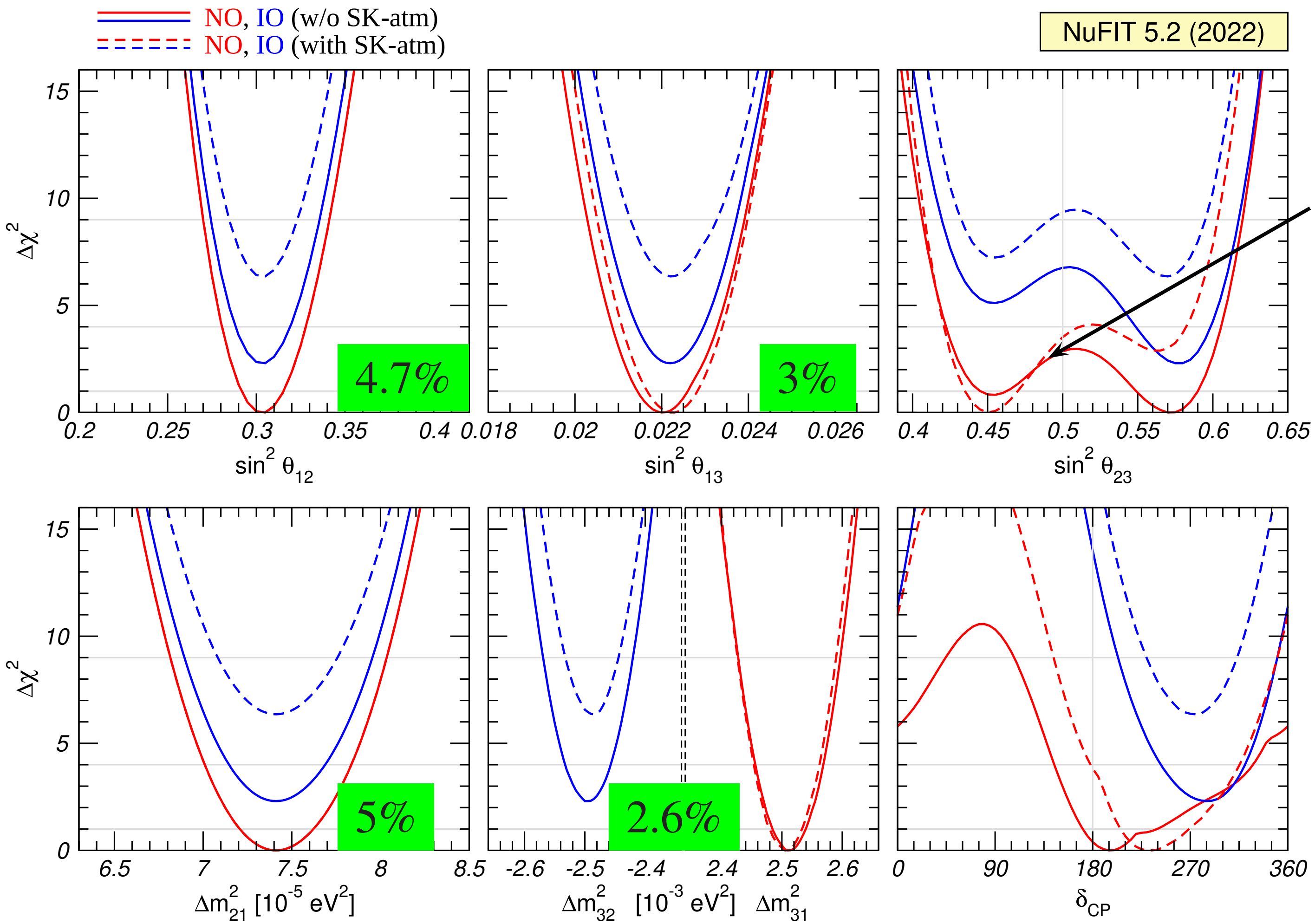

- 4 well-known parameters: $\theta_{12}, \theta_{13}, \Delta m^2_{21}, |\Delta m^2_{3\ell}|$

  Δm^2_{21} Solar vs KLAND

  Tension Resolved
- θ_{23}: Least known angle

  Maximal? Octant?

  non-robust wrt ATM

Flavour Parameters: Mixing Matrix

- We have the three leptonic mixing angles determined (at $\pm 3\sigma/6$)

$$|U|_{3\sigma} = \begin{pmatrix} 0.80 \to 0.85 & 0.51 \to 0.56 & 0.14 \to 0.16 \\ 0.23 \to 0.51 & 0.46 \to 0.69 & 0.63 \to 0.78 \\ 0.26 \to 0.53 & 0.47 \to 0.70 & 0.61 \to 0.76 \end{pmatrix}$$

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$$|V|_{\rm CKM} = \begin{pmatrix} 0.97427 \pm 0.00015 & 0.22534 \pm 0.0065 & (3.51 \pm 0.15) \times 10^{-3} \\ 0.2252 \pm 0.00065 & 0.97344 \pm 0.00016 & (41.2^{+1.1}_{-5}) \times 10^{-3} \\ (8.67^{+0.29}_{-0.31}) \times 10^{-3} & (40.4^{+1.1}_{-0.5}) \times 10^{-3} & 0.999146^{+0.000021}_{-0.000046} \end{pmatrix}$$

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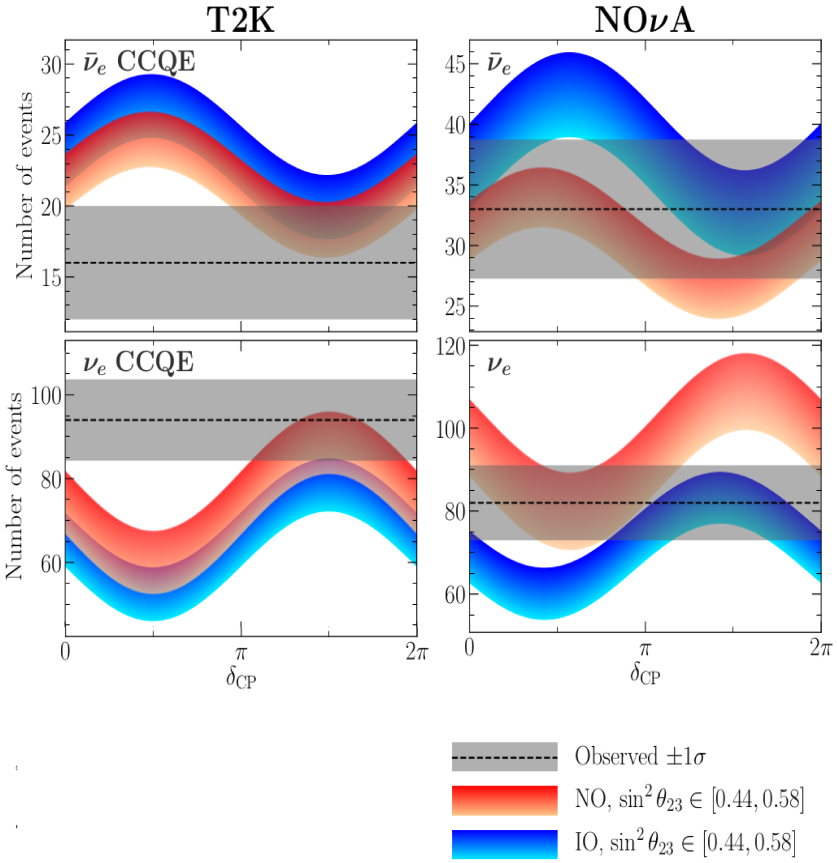
- Also very different flavour mixing of leptons vs quarks

CPV and MO in LBL

ν_e and $\overline{\nu}_e$ apperance events



Each T2K and NOνA favour NO

But tension in values of δ_{CP} in NO

$\Rightarrow$ IO best fit in LBL combination

CPV and MO in LBL

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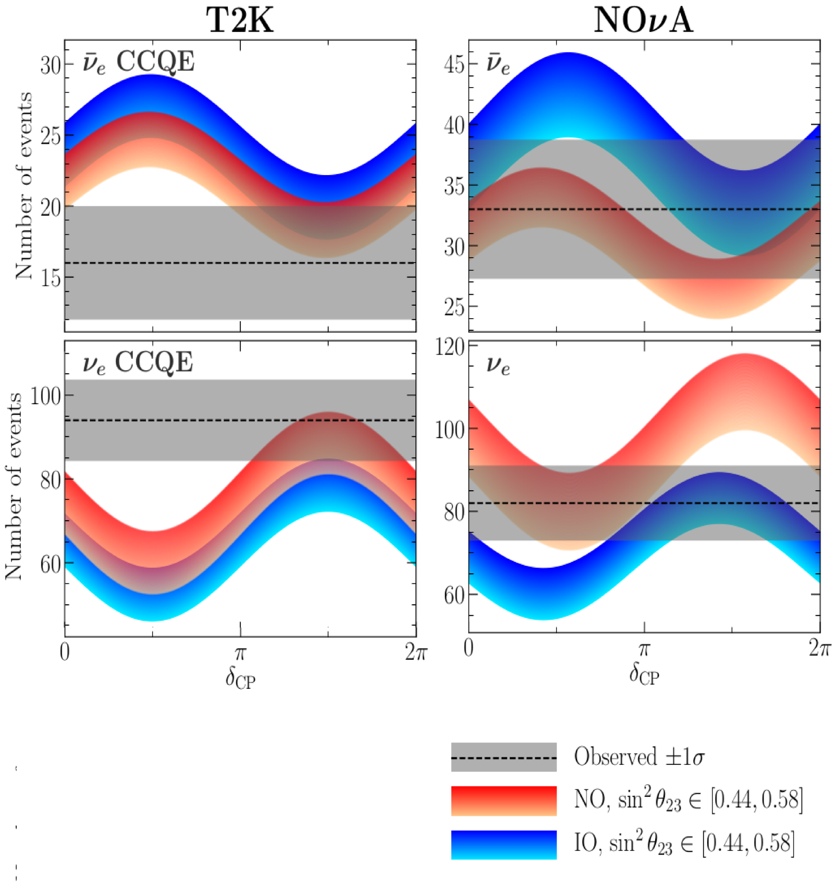

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CPV and MO in LBL+Reactors

At LBL determined in ν_μ and $\overline{\nu}_\mu$ disapp

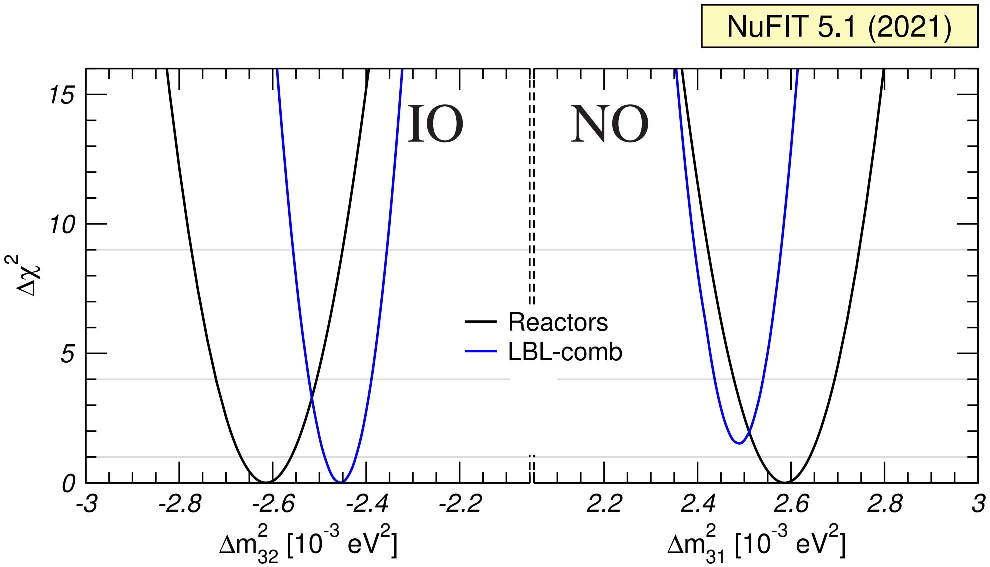
$$\Delta m^2_{\mu\mu} \simeq \Delta m^2_{3l} + \begin{matrix} c^2_{12}\Delta m^2_{21} & \text{NO} \\ s^2_{12}\Delta m^2_{21} & \text{IO} \end{matrix} + \dots$$

At reactors Daya-Bay, Reno in $\bar{\nu}_e$ disapp

$$\Delta m^2_{ee} \simeq \Delta m^2_{3l} + \begin{matrix} s^2_{12}\Delta m^2_{21} & \text{NO} \\ c^2_{12}\Delta m^2_{21} & \text{IO} \end{matrix}$$

Nunokawa,Parke,Zukanovich (2005)

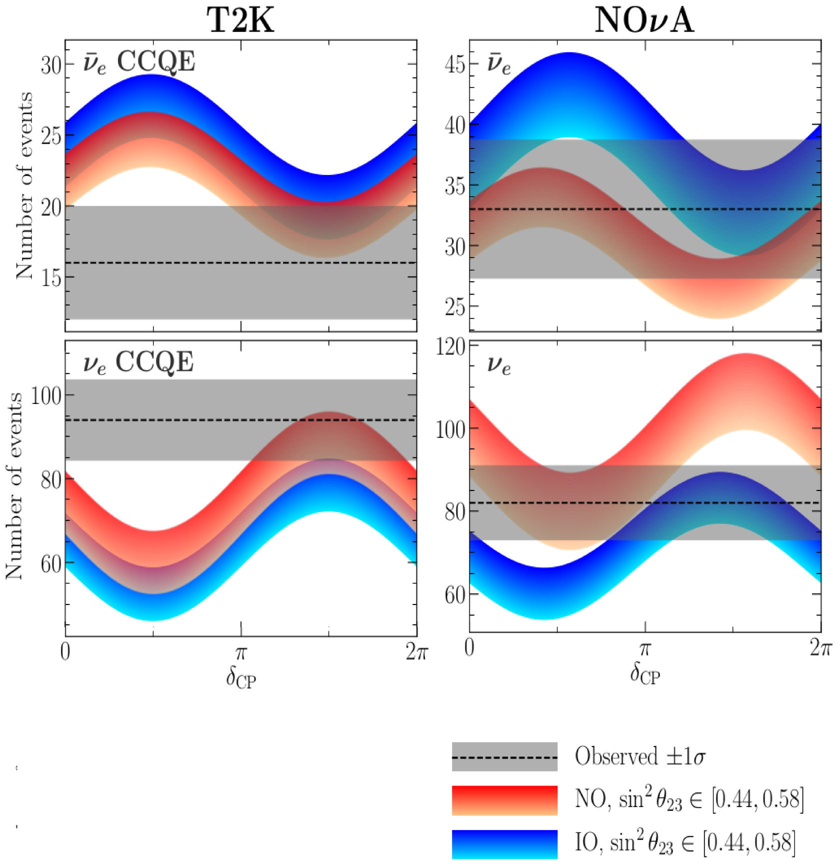
$\Rightarrow$ Contribution to MO from combination



$\Rightarrow$ NO best fit in LBL+Reactors

CPV and MO in LBL

ν_e and $\overline{\nu}_e$ apperance events



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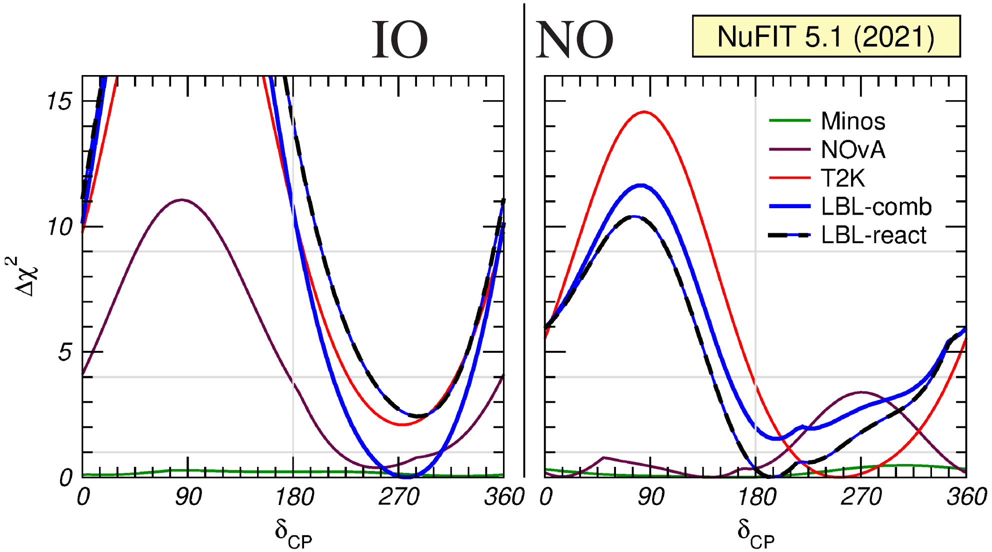

- in NO: b.f $\delta_{\rm CP} = 195°$ $\Rightarrow$ CPC allowed at 0.6 σ
- in IO: b.f $\delta_{\rm CP} \sim 270°$ $\Rightarrow$ CPC disfav. at 3 σ

Questions, Implications, Lessons . . .

- Still missing in the minimal 3ν scenario:

  Majorana or Dirac? Absolute values of ν mass scale

  CP violation in leptons? Normal or Inverted Ordering?

  Other Standing Experimental Puzzles:

  LSND-MiniBooNE $\nu_\mu \to \nu_e$ and others signals at SBL $\Rightarrow$ light sterile ν's?

  $\Rightarrow$ More data needed

- Still have no fundamental understanding of:

  Why are neutrinos so light?

  The Origin of Neutrino Mass

  Why are lepton mixing so different from quark's?

  The Flavour Puzzle

  $\Rightarrow$ More data needed

Summary II

- If $m_\nu \neq 0 \rightarrow$ Lepton Mixing $\equiv$ breaking of $L_e \times L_\mu \times L_\tau$
- Neutrino masses and mixing $\Rightarrow$ Flavour oscillations in ν propagation
- Experiments observing oscillations $\Rightarrow$ measurement of Δm^2_{ij} and θ_{ij}
- ν traveling through matter $\Rightarrow$ Modification of oscillation pattern
- Matter effect is crucial to interpretation of solar data
- Matter effect is allows to resolve angle octant and mass ordering
- 3ν mixing consistently *describes* all confirmed signals. But is that all there is?

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Young people with fresh new ideas needed!!!
AND HERE YOU ARE!!

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$\Delta_{ij} = \frac{\Delta m_{ij}^2}{2E_\nu}$

$\tilde{J} = c_{13} \sin^2 2\theta_{13} \sin^2 2\theta_{23} \sin^2 2\theta_{12}$

- Without independent determination of θ_{13}

Matter effects in LBL

- Most relevant for $\nu_\mu \to \nu_e$

$$\begin{aligned} P_{\mu e(\bar\mu\bar e)} \simeq\ & s^2_{23}\sin^2 2\theta_{13}\left(\frac{\Delta_{31}}{\Delta_{31}\mp V_\oplus}\right)^2 \sin^2\left(\frac{(\Delta_{31}\mp V_\oplus)\,L}{2}\right) \\ & + \tilde J\,\frac{\Delta_{21}}{V_\oplus}\,\frac{\Delta_{31}}{\Delta_{31}\mp V_\oplus}\,\sin\left(\frac{V_\oplus L}{2}\right)\sin\left(\frac{(\Delta_{31}\mp V_\oplus)L}{2}\right)\cos\delta\,\cos\left(\frac{\Delta_{31}\,L}{2}\right) \\ & \pm \tilde J\,\frac{\Delta_{21}}{V_\oplus}\,\frac{\Delta_{31}}{\Delta_{31}\mp V_\oplus}\,\sin\left(\frac{V_\oplus L}{2}\right)\sin\left(\frac{(\Delta_{31}\mp V_\oplus)L}{2}\right)\sin\delta\,\sin\left(\frac{\Delta_{31}\,L}{2}\right) + \ldots (2) \end{aligned}$$

$\Delta_{ij} = \frac{\Delta m^2_{ij}}{2E_\nu}$

$\tilde J = c_{13}\sin^2 2\theta_{13}\sin^2 2\theta_{23}\sin^2 2\theta_{12}$

- Without independent determination of θ_{13}

(a) $\theta_{23} \leftrightarrow \frac{\pi}{2} - \theta_{23}$ ambiguity:

$$P_{\mu\mu} \propto \sin^2 2\theta_{23} \text{ and } P_{\mu e(\bar\mu\bar e)}(\theta_{23},\theta_{13},\delta) = P_{\mu e(\bar\mu\bar e)}(\tfrac{\pi}{2}-\theta_{23},\theta'_{13},\delta')$$

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$$+\ \tilde J\,\frac{\Delta_{21}}{V_\oplus}\,\frac{\Delta_{31}}{\Delta_{31}\mp V_\oplus}\sin\left(\frac{V_\oplus L}{2}\right)\sin\left(\frac{(\Delta_{31}\mp V_\oplus)L}{2}\right)\cos\delta\,\cos\left(\frac{\Delta_{31}\,L}{2}\right)$$
$$\pm\ \tilde J\,\frac{\Delta_{21}}{V_\oplus}\,\frac{\Delta_{31}}{\Delta_{31}\mp V_\oplus}\sin\left(\frac{V_\oplus L}{2}\right)\sin\left(\frac{(\Delta_{31}\mp V_\oplus)L}{2}\right)\sin\delta\,\sin\left(\frac{\Delta_{31}\,L}{2}\right) + . \quad (2)$$

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(b) (θ_{13},δ) ambiguity: $P_{\mu e(\bar\mu\bar e)}(\theta_{13},\delta) = P_{\mu e(\bar\mu\bar e)}(\theta'_{13},\delta')$

Matter effects in LBL

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$$+ \tilde J \frac{\Delta_{21}}{V_\oplus} \frac{\Delta_{31}}{\Delta_{31} \mp V_\oplus} \sin\left(\frac{V_\oplus L}{2}\right) \sin\left(\frac{(\Delta_{31} \mp V_\oplus) L}{2}\right) \cos\delta \cos\left(\frac{\Delta_{31} L}{2}\right)$$
$$\pm \tilde J \frac{\Delta_{21}}{V_\oplus} \frac{\Delta_{31}}{\Delta_{31} \mp V_\oplus} \sin\left(\frac{V_\oplus L}{2}\right) \sin\left(\frac{(\Delta_{31} \mp V_\oplus) L}{2}\right) \sin\delta \sin\left(\frac{\Delta_{31} L}{2}\right) + .(2)$$

$\Delta_{ij} = \frac{\Delta m_{ij}^2}{2E_\nu}$

$\tilde J = c_{13} \sin^2 2\theta_{13} \sin^2 2\theta_{23} \sin^2 2\theta_{12}$

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(b) (θ_{13}, δ) ambiguity: $P_{\mu e(\bar\mu\bar e)}(\theta_{13}, \delta) = P_{\mu e(\bar\mu\bar e)}(\theta'_{13}, \delta')$

(c) (ordering, δ) ambiguity: $P_{\mu e(\bar\mu\bar e)}(\Delta m_{31}^2, \delta) = P_{\mu e(\bar\mu\bar e)}(-\Delta m_{31}^2, \delta')$

Matter effects in LBL

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$$\Delta_{ij} = \frac{\Delta m^2_{ij}}{2E_\nu}$$
$$\tilde J = c_{13} \sin^2 2\theta_{13} \sin^2 2\theta_{23} \sin^2 2\theta_{12}$$

- Without independent determination of θ_{13}

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$$\Downarrow$$

If only total number of ν_e, ν_μ, $\overline{\nu}_e$ and $\overline{\nu}_\mu$ at given L are measured $\Rightarrow$ 8-fold degeneracy

Matter effects in LBL

- Most relevant for $\nu_\mu \to \nu_e$

$$
\begin{aligned}
P_{\mu e(\bar\mu\bar e)} \simeq\; & s_{23}^2 \sin^2 2\theta_{13} \left(\frac{\Delta_{31}}{\Delta_{31} \mp V_\oplus}\right)^2 \sin^2\left(\frac{(\Delta_{31} \mp V_\oplus)\, L}{2}\right) \\
+\; & \tilde J \,\frac{\Delta_{21}}{V_\oplus}\, \frac{\Delta_{31}}{\Delta_{31} \mp V_\oplus} \sin\left(\frac{V_\oplus L}{2}\right) \sin\left(\frac{(\Delta_{31} \mp V_\oplus) L}{2}\right) \cos\delta \cos\left(\frac{\Delta_{31}\, L}{2}\right) \\
\pm\; & \tilde J \,\frac{\Delta_{21}}{V_\oplus}\, \frac{\Delta_{31}}{\Delta_{31} \mp V_\oplus} \sin\left(\frac{V_\oplus L}{2}\right) \sin\left(\frac{(\Delta_{31} \mp V_\oplus) L}{2}\right) \sin\delta \sin\left(\frac{\Delta_{31}\, L}{2}\right) + \ldots
\end{aligned}
$$

$\Delta_{ij} = \frac{\Delta m^2_{ij}}{2E_\nu}$

$\tilde J = c_{13} \sin^2 2\theta_{13} \sin^2 2\theta_{23} \sin^2 2\theta_{12}$

- If θ_{13} known and some E_ν information and large L

(a) Partial $\theta_{23} \leftrightarrow \frac{\pi}{2} - \theta_{23}$ ambiguity if θ_{23} not very non-maximal

$P_{\mu\mu} \propto \sin^2 2\theta_{23}$ and $P_{\mu e(\bar\mu\bar e)}(\theta_{23}, \theta_{13}, \delta) \simeq P_{\mu e(\bar\mu\bar e)}(\frac{\pi}{2} - \theta_{23}, \theta_{13}, \delta')$

(b) (θ_{13}, δ) ambiguity: $P_{\mu e(\bar\mu\bar e)}(\theta_{13}, \delta) = P_{\mu e(\bar\mu\bar e)}(\theta'_{13}, \delta')$

(c) Partial (ordering, δ) ambiguity: $P_{\mu e(\bar\mu\bar e)}(\Delta m^2_{31}, \delta) = P_{\mu e(\bar\mu\bar e)}(-\Delta m^2_{31}, \delta')$

if not long enough L