Lepton-Nucleus Interactions —Part I

International Neutrino Summer School
Fermilab, August 9, 2023

Noemi Rocco

Something about me

- In October 2008, I started my Physics studies at "La Sapienza University of Rome". I obtained my Bachelor and my Master degree there

- In 2012, I started my PhD focusing on Neutrino Physics

- Between 2016 to 2018 I have been traveling a lot: Canada, Spain, and UK

- In 2018 I moved to Chicago and started working in the Theory Group at Fermilab.

(Fermilab, 2019)

- Modeling neutrino-nucleus interactions: Quantum Monte Carlo, Spectral Function

Noemi Rocco, nrocco@fnal.gov

Practical Info

- Some of the Figures have been taken from papers published in peer reviews. The references are reported as:

  Authors, Journal's name and # of the paper

  If there is anything you find interesting, I strongly encourage you to download the paper and read it!

- I also included some suggestions for more 'pedagogical' readings. The references are indicated as

Author, Title of the Book/Journal

- Please, ask question! Now or later: nrocco@fnal.gov

Introduction

? Which are the topics that have been in the previous lectures?

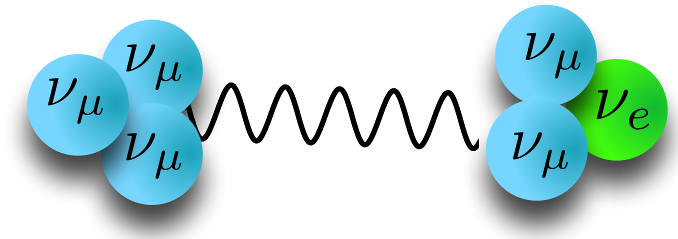
- Neutrinos are extremely elusive particles, many open questions: how they oscillate, what is their mass hierarchy…
- Neutrinos can be produced by different sources, different energies and very different physics
- Neutrinos can not be directly measured: we study the interactions that take place in the detectors and "extract" neutrino properties

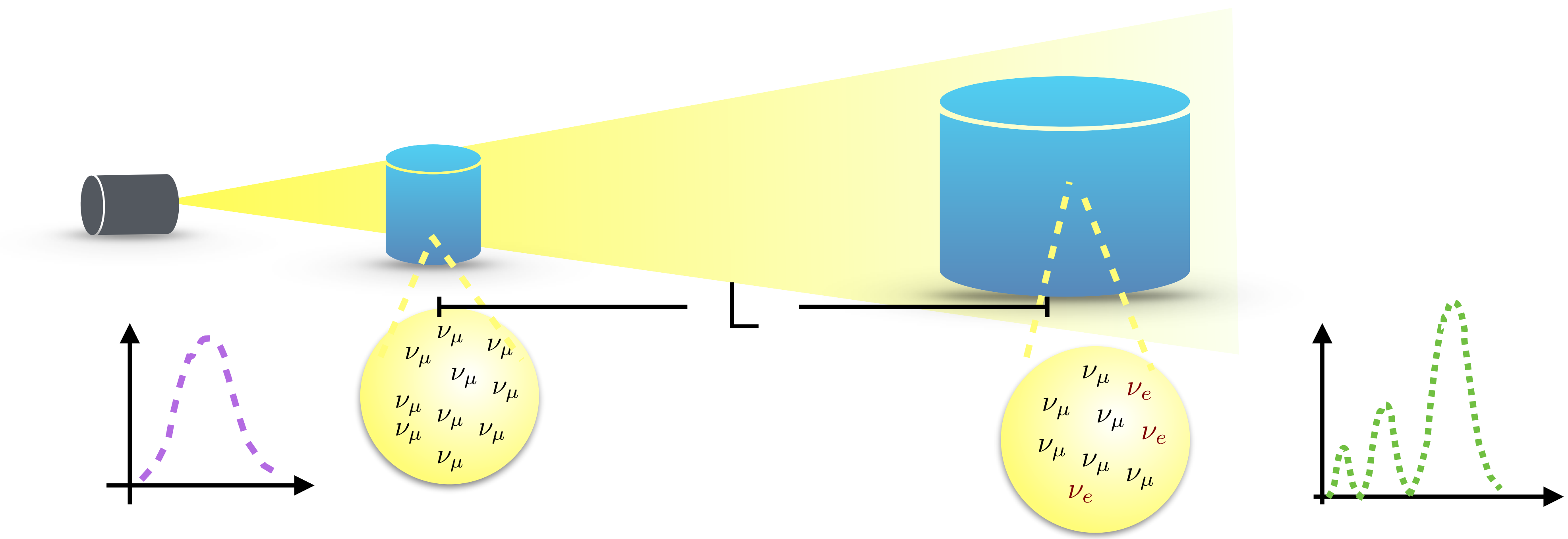
! Understanding neutrino interactions means understanding their cross sections

- What is a cross section?
- What did we learn from electron scattering that can be used to better understand neutrinos?
- What are the different contributions to neutrino-nucleus cross sections

Addressing Neutrino-Oscillation Physics

$$P_{\nu_\mu \to \nu_e}(E, L) \sim \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4E}\right) \to \Phi_e(E, L)/\Phi_\mu(E, 0)$$





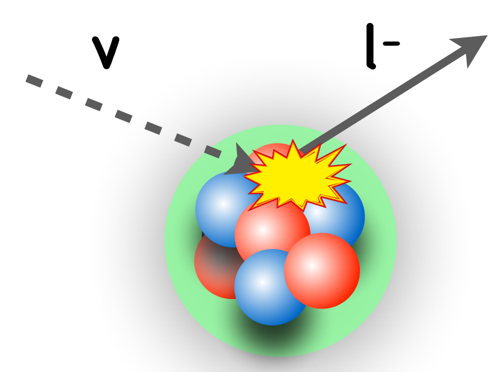
Detectors measure the **neutrino interaction rate:**

$$N_e(E_{\rm rec}, L) \propto \sum_i \Phi_e(E, L)\sigma_i(E) f_{\sigma_i}(E, E_{\rm rec})dE$$

Reconstructed
ν energy

Cross Section

Smearing
matrix



A precise determination of σ(E) is crucial to extract ν oscillation parameters

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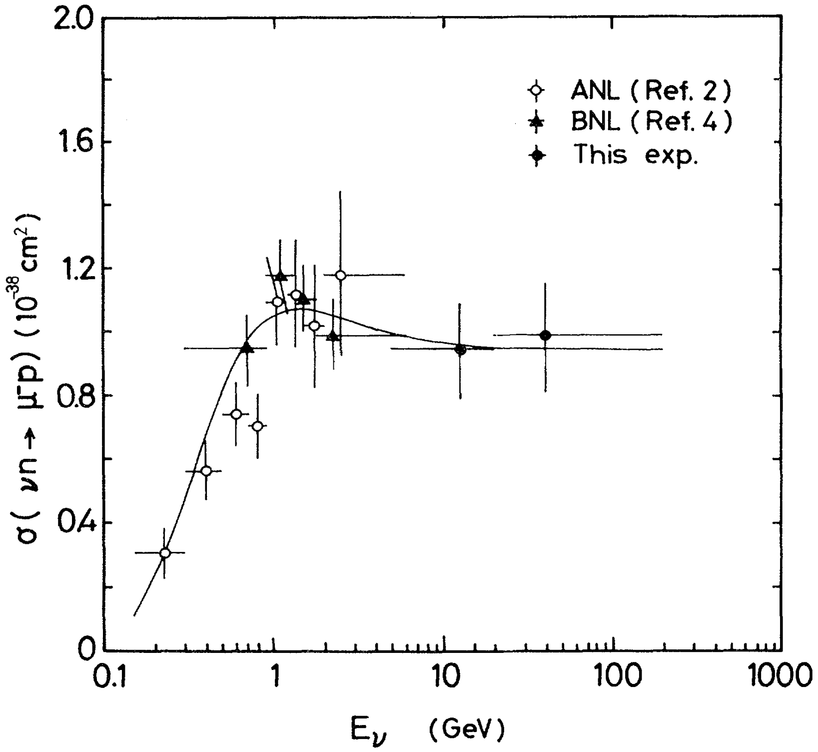
To study neutrinos we use nuclei



T. Kitagaki et al, Phys. Rev. D 28, 436 (1983)

Neutrino scattering extensively studied 1970-90's using deuterium-filled **bubble chambers**

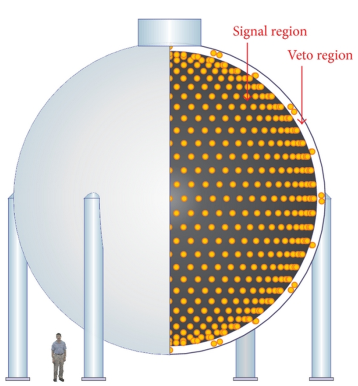
$$\mathcal{N}_{\text{hits}} = \sigma \times \Phi \times N$$

Targets

Bubble Chamber experiment at Fermilab

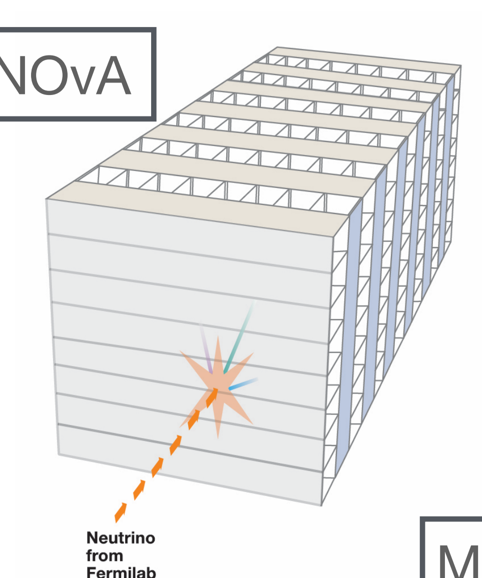
Utilize **heavy target** in neutrino detectors to maximize interactions→ understand **nuclear structure**

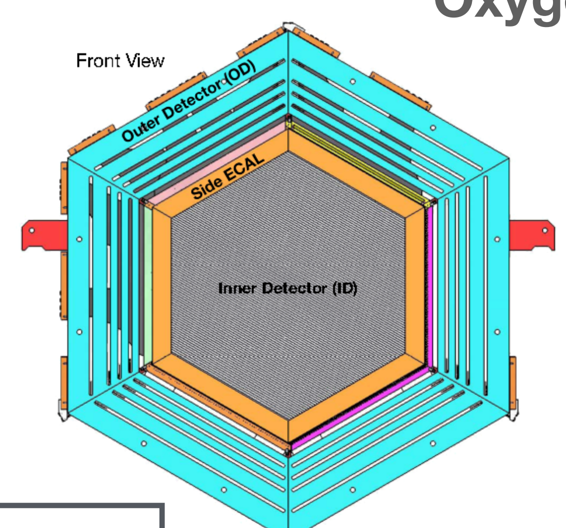
Carbon



MiniBooNE

NOvA



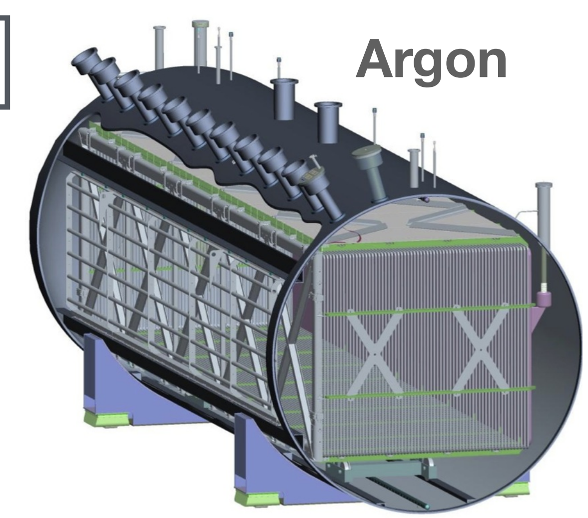


MINERvA

Oxygen

S-K

Argon



MicroBooNE

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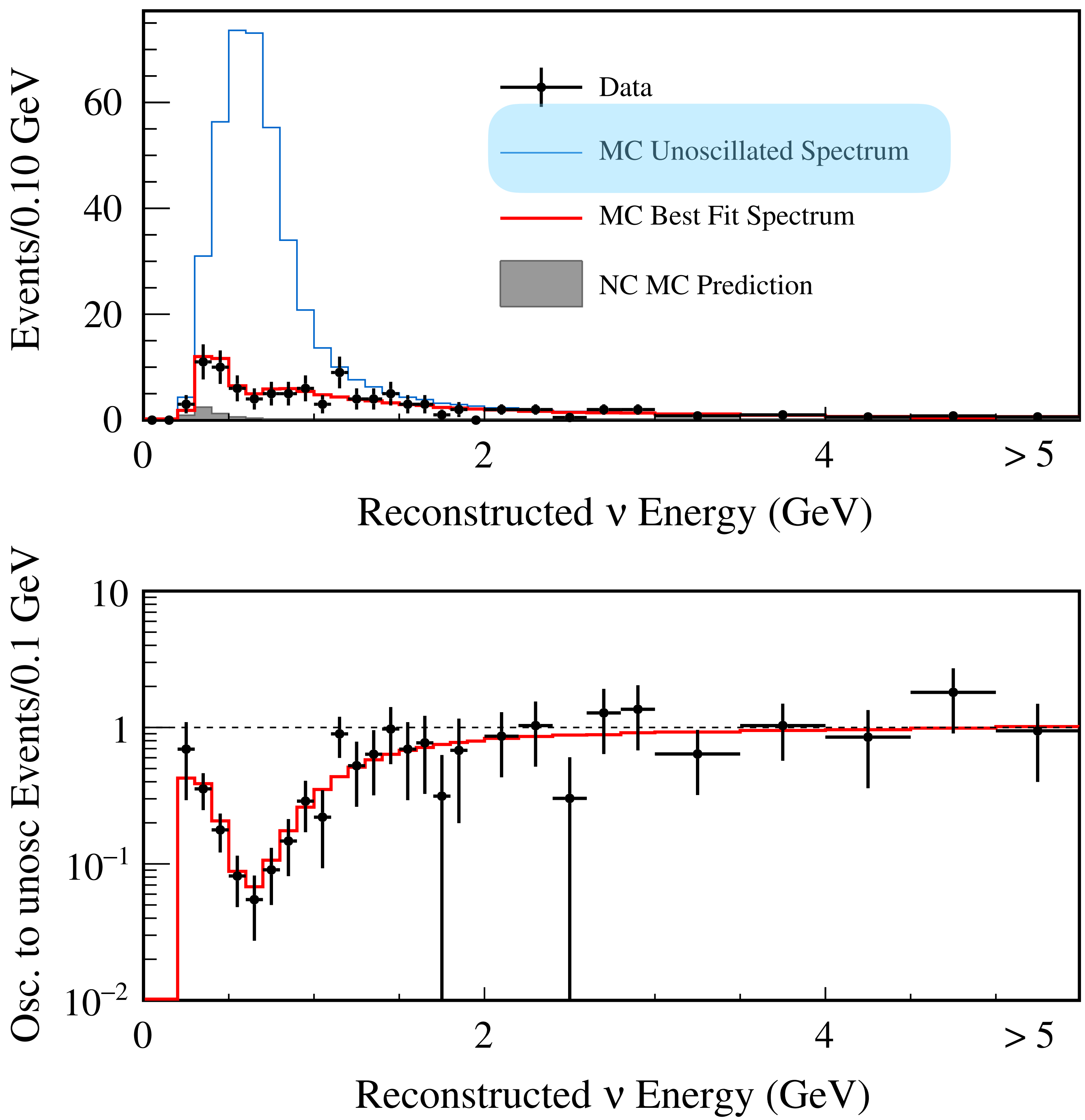
Oscillations Require E_ν reconstruction

T2K, Phys. Rev. D 91, 072010 (2015)



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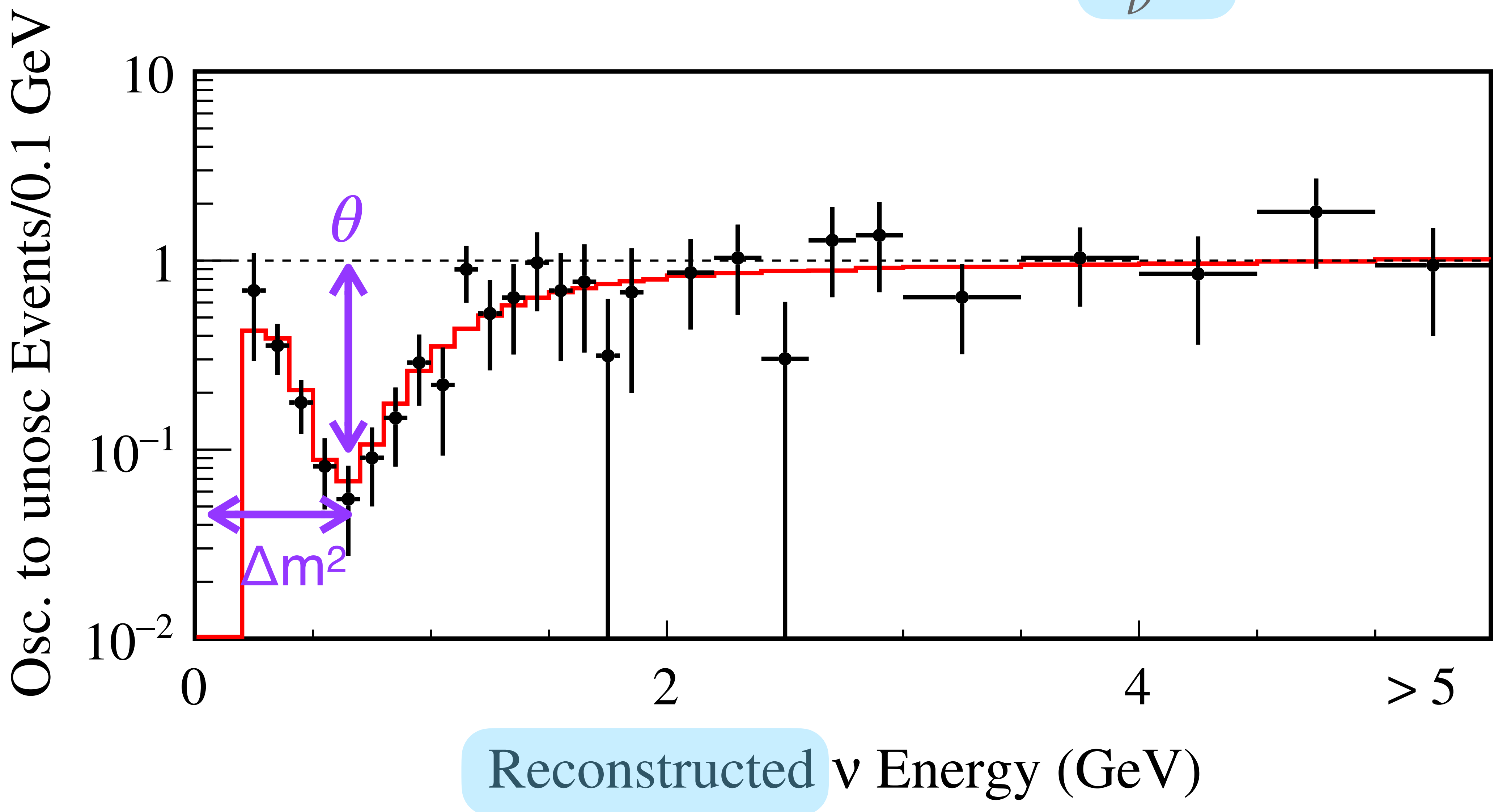
Oscillations Require E_ν reconstruction

T2K, Phys. Rev. D 91, 072010 (2015)

$$P(\nu_\mu \to \nu_x) \sim \sin^2 2\theta \, \sin^2\left(\frac{\Delta m^2 L}{4E_\nu^{true}}\right)$$



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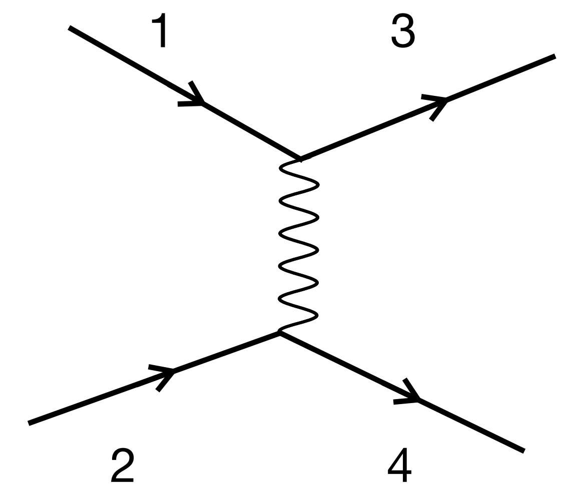
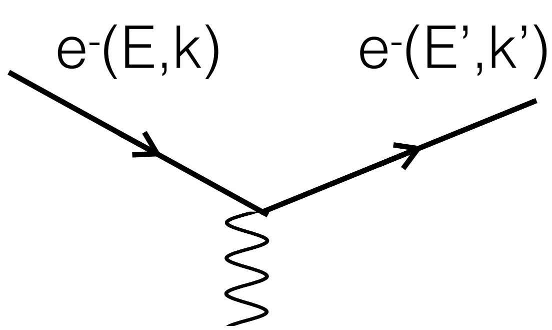
How do we compute a cross section?



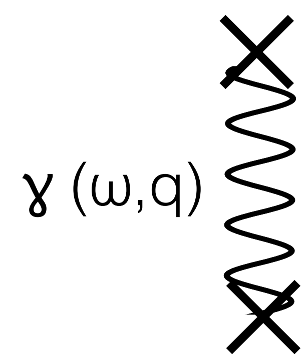
- We start from a generic process: 1+2 ➜ 3+4
- The cross section can be written as

$$d\sigma = \frac{1}{\text{flux}}\frac{1}{2E_1}\frac{1}{2E_2}|\mathcal{A}|^2\frac{d^3p_3}{(2\pi)^3 2E_3}\frac{d^3p_4}{(2\pi)^3 2E_4}(2\pi)^4\delta^{(4)}(p_1+p_2-p_3-p_4)$$

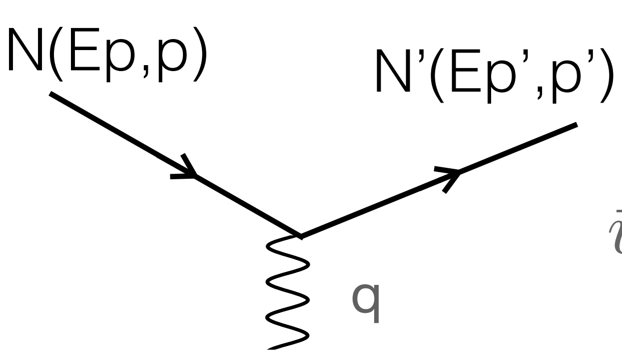
F. Close, An Introduction to quark and partons



$$\bar{u}(\mathbf{k}',\lambda')\gamma^\mu u(\mathbf{k},\lambda)$$



$$\frac{e^2\, g^{\mu\nu}}{q^2}$$



$$\bar{u}(\mathbf{p}',\alpha')\Gamma^\mu(q)u(\mathbf{p},\alpha) = \bar{u}(\mathbf{p}',\alpha')\left[F_1\gamma^\mu + i\frac{\sigma^{\mu\nu}q_\nu}{2M}F_2\right]u(\mathbf{p},\alpha)$$

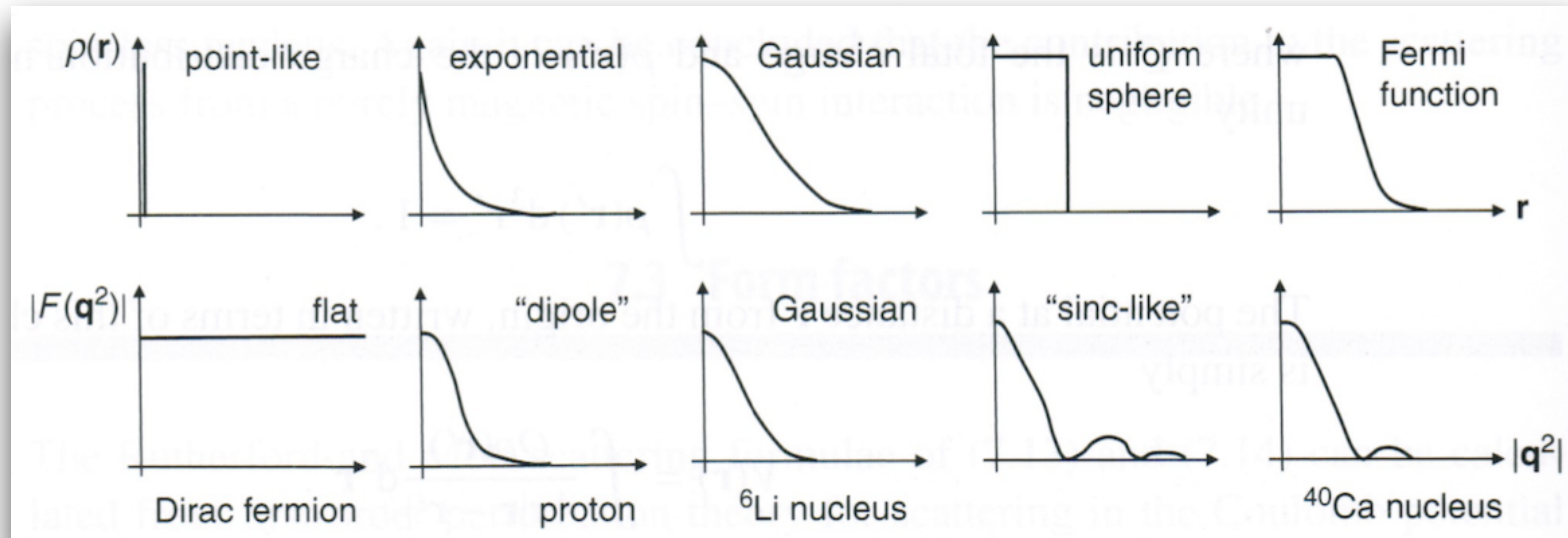
Nucleon Form Factor

- Accounts for the finite size of the nucleon

$$\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{exp}} = \left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Mott}} \cdot |F(\mathbf{q}^2)|^2$$

- Form factor and electric charge distribution are Fourier pairs

$$F(\mathbf{q}^2) = \int d^3\mathbf{r}\ \rho(\mathbf{r}) \cdot e^{i\mathbf{q}\cdot\mathbf{r}}$$



✐ Thomson, M.: Modern Particle Physics, Cambridge University Press, 2013

Summary of electron-nucleus scattering

- We consider the process:

$$\ell^-(k) + N(p) \to \ell^-(k') + N(p')$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\rm Mott} = \frac{\alpha^2}{4E_k^2 \sin^4\theta/2}\cos^2\frac{\theta}{2}$$

Scattering on a point-like spinless target

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{\rm Mott}\left[1 - \frac{q^2}{2M^2}\tan^2\frac{\theta}{2}\right]$$

Scattering on a point-like 1/2 spin target

- Protons and neutrons have an internal structure: described by electric and magnetic form factors

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{\rm Mott}\left[\frac{G_E^2 - \frac{q^2}{4M^2}G_M^2}{1 - \frac{q^2}{4M^2}} - \frac{q^2}{2M^2}G_M^2 \tan^2\frac{\theta}{2}\right]$$

Rosenbluth separation

Determination of nucleon form factors

- A reduced cross section can be defined as

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{\rm Mott} \times \frac{\epsilon G_E^2 + \tau G_M^2}{\epsilon(1+\tau)}$$

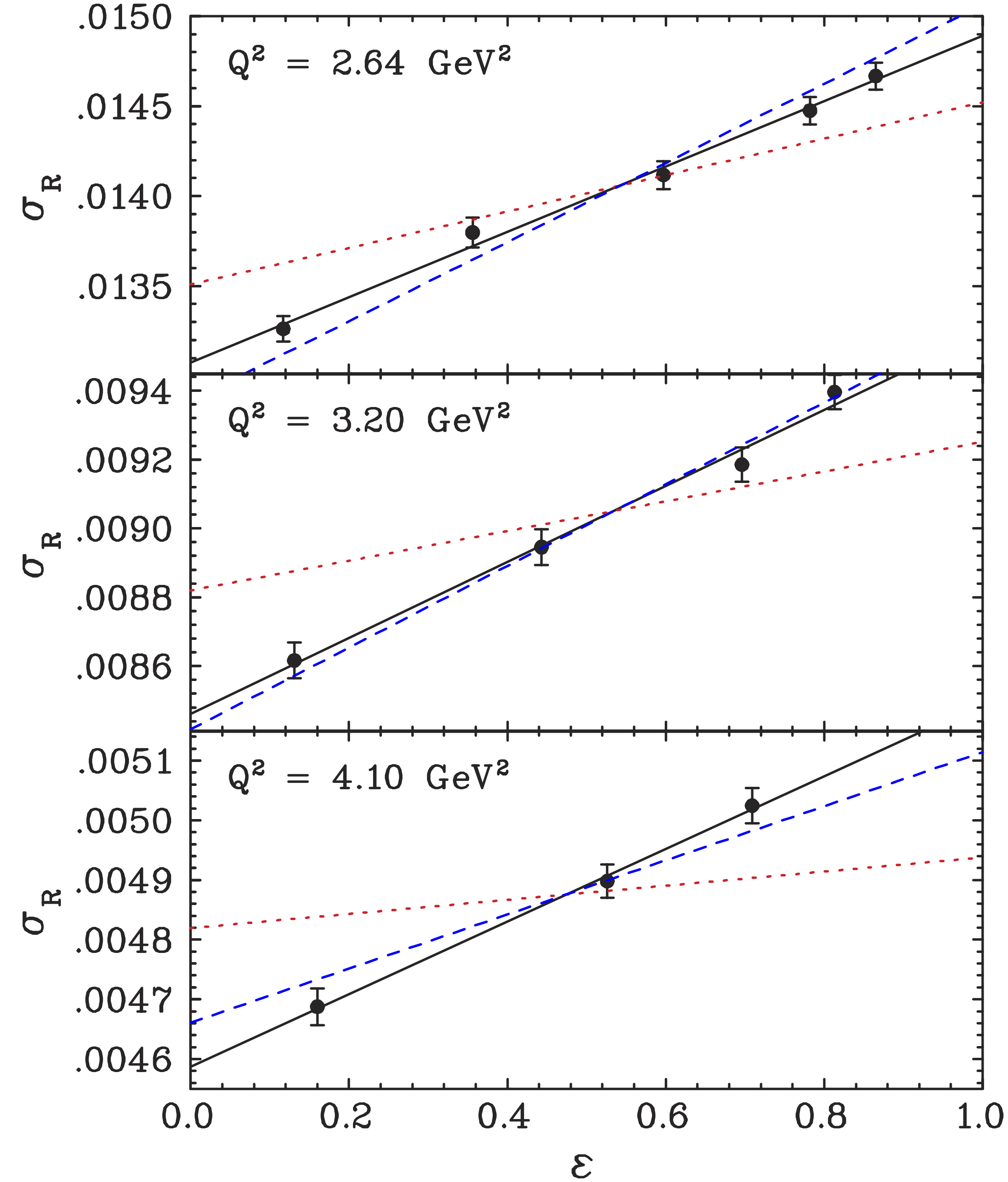
- The virtual photon polarization parameter is

$$\epsilon = \left[1 + 2(\tau+1)\tan^2\frac{\theta}{2}\right]^{1}$$

- Measuring angular dependence of the cross section at fixed Q^2

$$\sigma_R = \epsilon(1+\tau)\frac{\sigma}{\sigma_{\rm Mott}} = \epsilon G_E^2 + \tau G_M^2$$

✐ Qattan et al.,PRL 94, 142301 (2005)



··· polarization transfer experiment
- - - previous Rosenbluth

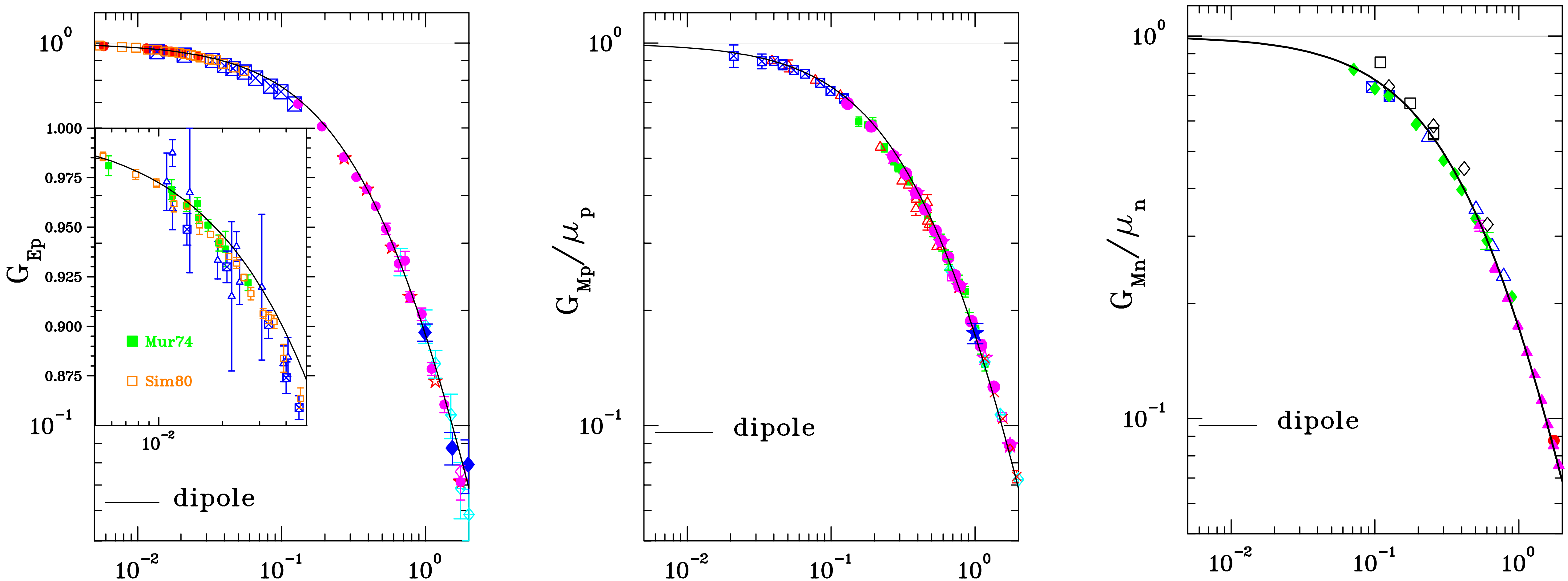
- In Born approximation: G_E^2 is the slope and the intercept is τG_M^2

Determination of nucleon form factors

$$G_E^p(q^2) = G_D(q^2)\ ,\ G_M^p(q^2) = \mu_p G_D(q^2)\ ,\ G_M^n(q^2) = \mu_n G_D(q^2)\ ,\ G_D = \left(1 - \frac{q^2}{M_V^2}\right)^{-2}$$

$$\mu_p = 2.793,\ \ \mu_n = -1.913,$$
$$M_V^2 = 0.71\ \mathrm{GeV}^2$$

Perdrisat et al., Prog.Part.Nucl.Phys. 59 (2007) 694-764



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Lattice QCD in a Nutshell

- Lattice QCD is a technique in which space-time is discretized into a four-dimensional grid

- The QCD path integral over the quark and gluon fields at each point in the grid is performed in Euclidean space-time using Monte Carlo methods

- The quark mass is a parameter of the calculation : m_π

  - Space time is discretized, step a: UV cutoff

  - Finite volume L : infrared cutoff

  - The physical limit is recovered by imposing that

$$m_\pi \to m_\pi^{phys}, \; a \to 0, \; L \to \infty$$

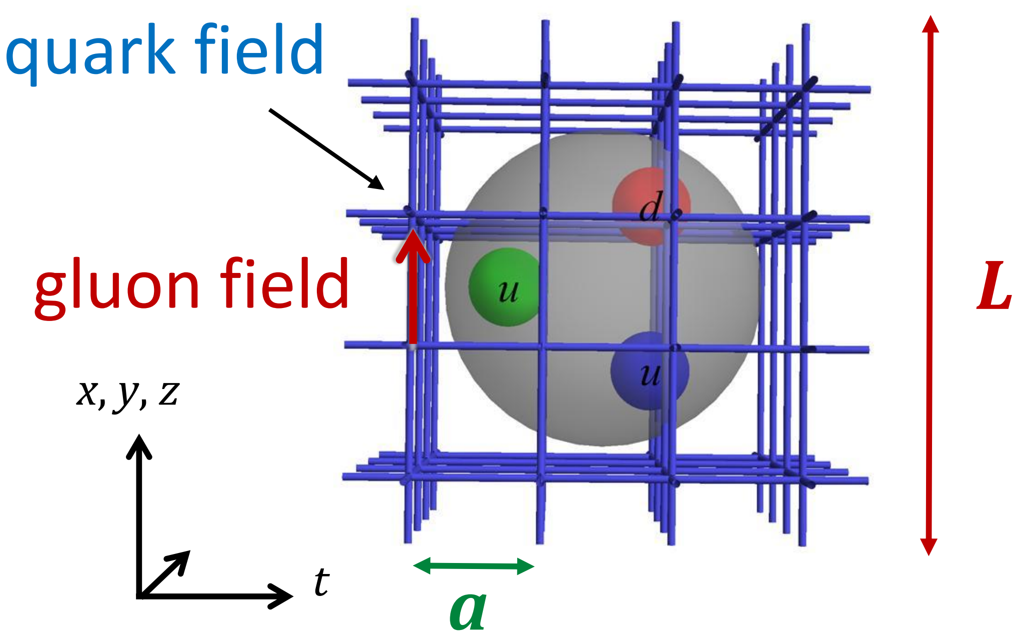

Figure by Huey-Wen Lin

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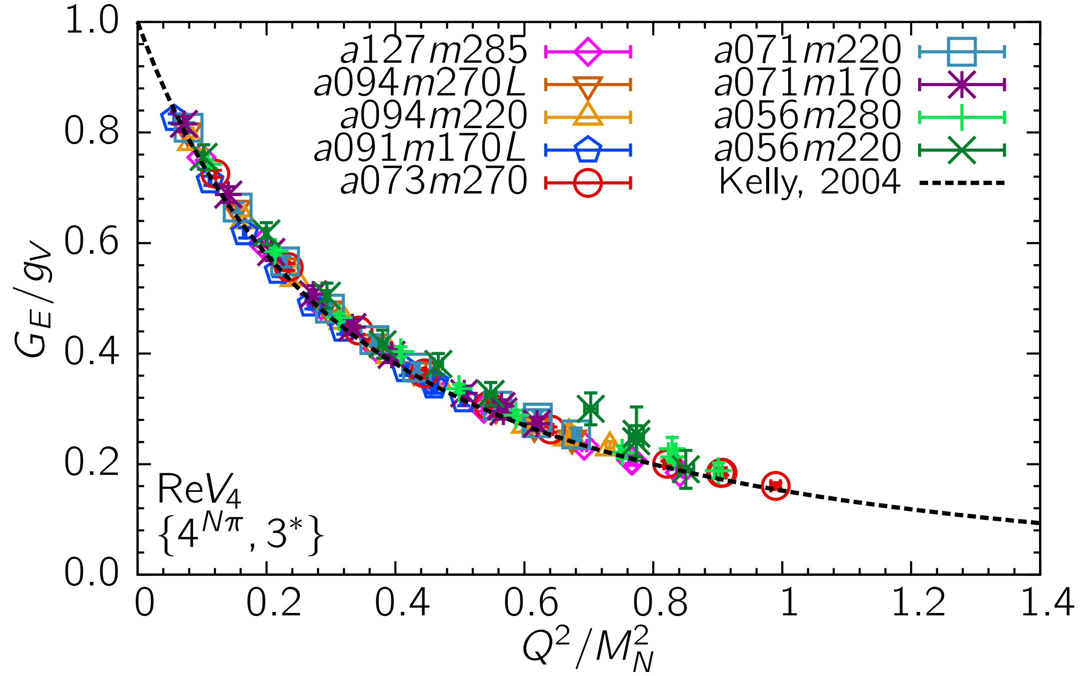
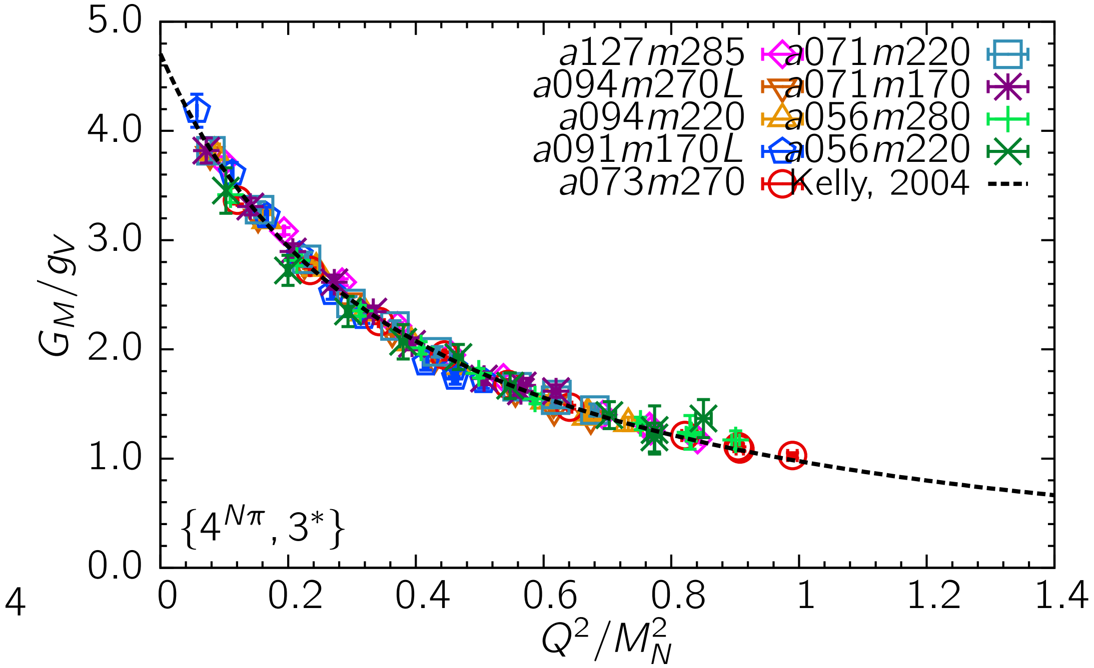
Lattice QCD form factors

Gupta's talk @Lattice2023



Electric

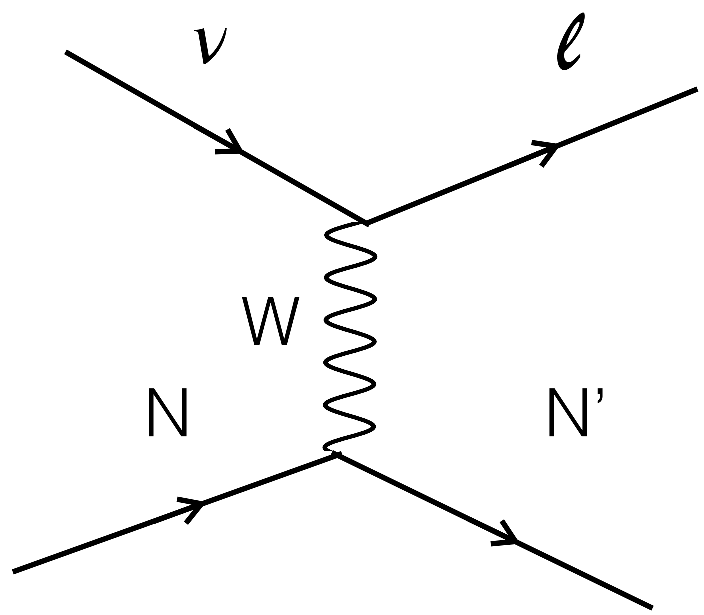


Magnetic

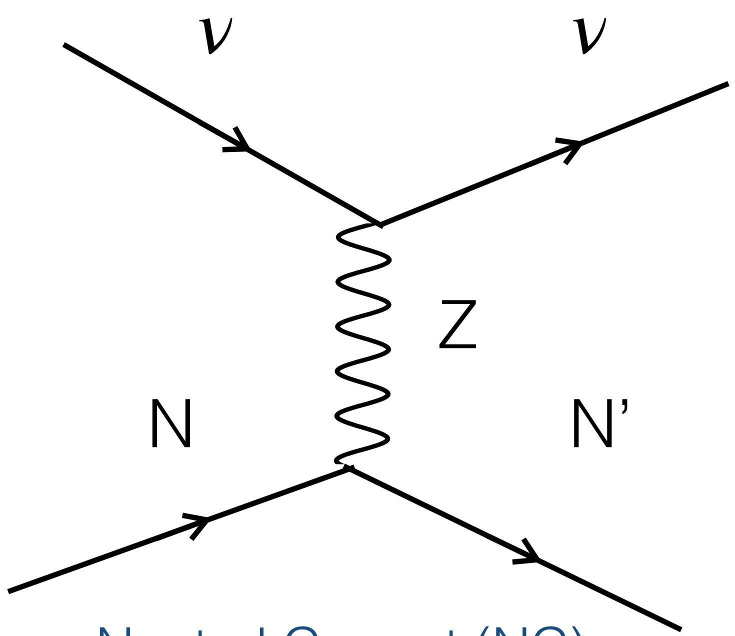
- Isovector electric and magnetic form factors
- The form factors do not show significant dependence on the lattice spacing or the quark mass
- Good agreement with the Kelly curve. Validates the lattice methodology

R. Gupta, Introduction to Lattice QCD, arXiv:hep-lat/9807028

Neutrino-Nucleon scattering



Charge Current (CC)



Neutral Current (NC)

- Exchange of the W boson
- Lepton produced has the same flavor of the neutrino
- Initial and final nucleon have different isospin

- Exchange of the Z boson
- Independent of the neutrino flavor
- Initial and final nucleon have same isospin

F. Close, An Introduction to quark and partons

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Neutrino-Nucleon scattering

- Differential cross section for CC and NC processes

$$\frac{d^2\sigma}{dE'd\Omega'} = \frac{1}{16\pi^2}\frac{G^2}{2}L_{\mu\nu}W^{\mu\nu}$$

- For NC

$$G = G_F$$

- For CC

$$G = G_F \cos\theta_c$$

$$G_F = 1.1803 \times 10^{-5}\ \mathrm{GeV}^{-2}\ ,\ \cos\theta_c = 0.97425$$

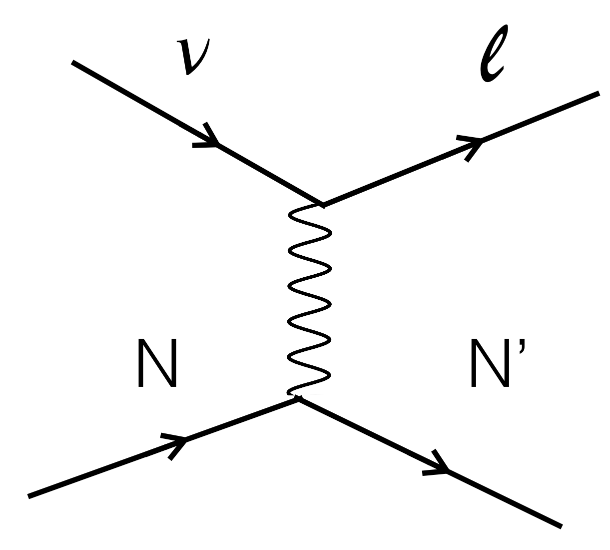
- Leptonic Tensor:

$$L_{\mu\nu} = 8[k_\mu k'_\nu + k'_\mu k_\nu - g_{\mu\nu} k\cdot k' \pm i\epsilon_{\mu\nu\alpha\beta}k'^\alpha k^\beta]$$

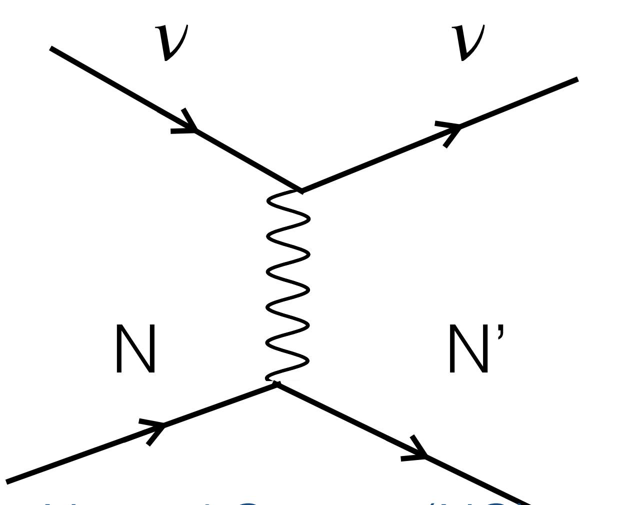
$\nu/\bar\nu$

- Hadronic Tensor:

$$W_{\mu\nu} = \sum_{\sigma_i\sigma_f}\frac{1}{2E_p}\int\frac{d^3p'}{2E_{p'}}\langle N(p)|J^\mu|N'(p')\rangle\langle N'(p')|J^\nu|N(p)\rangle$$
$$\times\ \delta^{(4)}(p'+k'-p-k)$$



Charge Current (CC)



Neutral Current (NC)

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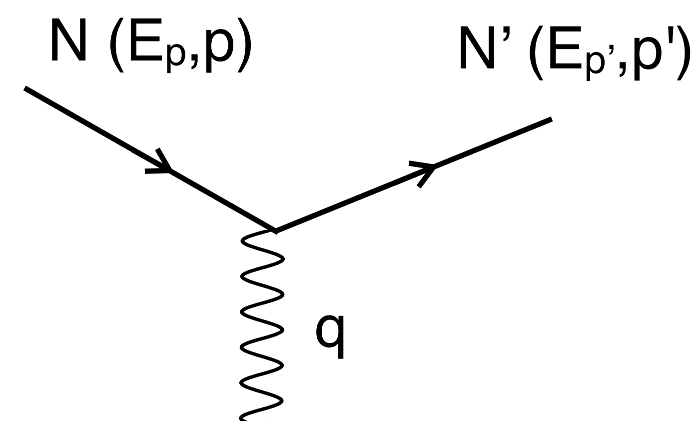
Neutrino-Nucleon scattering

- Electroweak current operator:

$$\langle N'|J^\mu|N\rangle = \bar{u}(p')\Gamma^\mu u(p) = J^\mu_V + J^\mu_A$$

Vector Axial



- The vector contribution is given by:

$$J^\mu_V = \mathcal{F}_1\gamma^\mu + i\sigma^{\mu\nu}q_\nu\frac{\mathcal{F}_2}{2M}$$

- The axial contribution is given by:

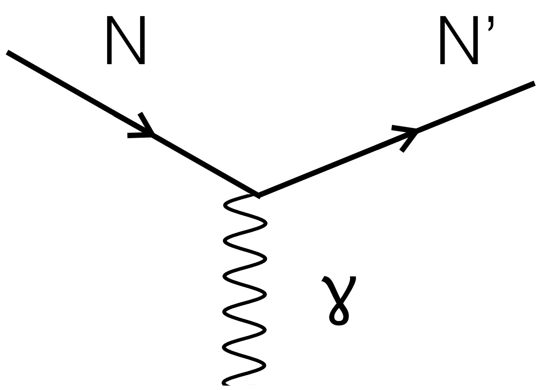
$$J^\mu_A = -\gamma^\mu\gamma_5\mathcal{F}_A - q^\mu\gamma_5\frac{\mathcal{F}_P}{M}$$

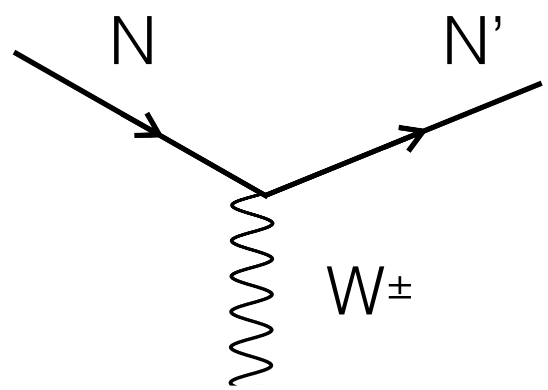
- General expression for both neutral- and charge current processes. The iso-spin dependence of these form factors is different (see next slide).

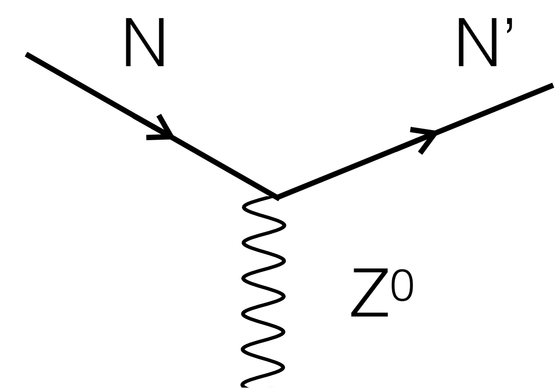
- The Vector current is the same of the electromagnetic: Conserved Vector Current hypothesis

T. Leitner, O. Buss, L. Alvarez-Ruso, and U. Mosel, Electron- and neutrino-nucleus scattering from the quasielastic to the resonance region, Phys. Rev. C 79, 034601 (2009).

Neutrino-Nucleon scattering







- EM

$$\mathcal{F}_1 = \frac{1}{2}[F_1^S + F_1^V \tau_z]$$

$$\mathcal{F}_2 = \frac{1}{2}[F_2^S + F_2^V \tau_z]$$

- CC

$$\mathcal{F}_1 = F_1^V \tau_\pm$$

$$\mathcal{F}_2 = F_2^V \tau_\pm$$

$$\mathcal{F}_A = F_A \tau_\pm$$

$$\mathcal{F}_P = F_P \tau_\pm$$

- NC

$$\mathcal{F}_1 = \frac{1}{2}[-2sin^2\theta_W F_1^S + (1-2sin^2\theta_W)F_1^V \tau_z]$$

$$\mathcal{F}_2 = \frac{1}{2}[-2sin^2\theta_W F_2^S + (1-2sin^2\theta_W)F_2^V \tau_z]$$

$$\mathcal{F}_A = \frac{1}{2}F_A \tau_z$$

$$\mathcal{F}_P = \frac{1}{2}F_p \tau_z$$

- We used the Conserved Vector Current hypothesis:

$$F_1^V \tau_z \to F_1^V \tau_\pm \ , \ F_2^V \tau_z \to F_2^V \tau_\pm$$

- PCAC:

$$F_P = \frac{2m_N^2}{(m_\pi^2 - q^2)} F_A$$

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Axial form factor determination

- The axial form-factor has been fit to the dipole form

$$F_A(q^2) = \frac{g_A}{(1 - q^2/m_A^2)^2}$$

- The intercept g_A=-1.2723 is known from neutron β decay

- Different values of m_A from experiments
  - m_A =1.02 GeV q.e. scattering from deuterium
  - m_A =1.35 GeV @ MiniBooNE

- Alternative derivation based on **z-expansion** —model independent parametrization

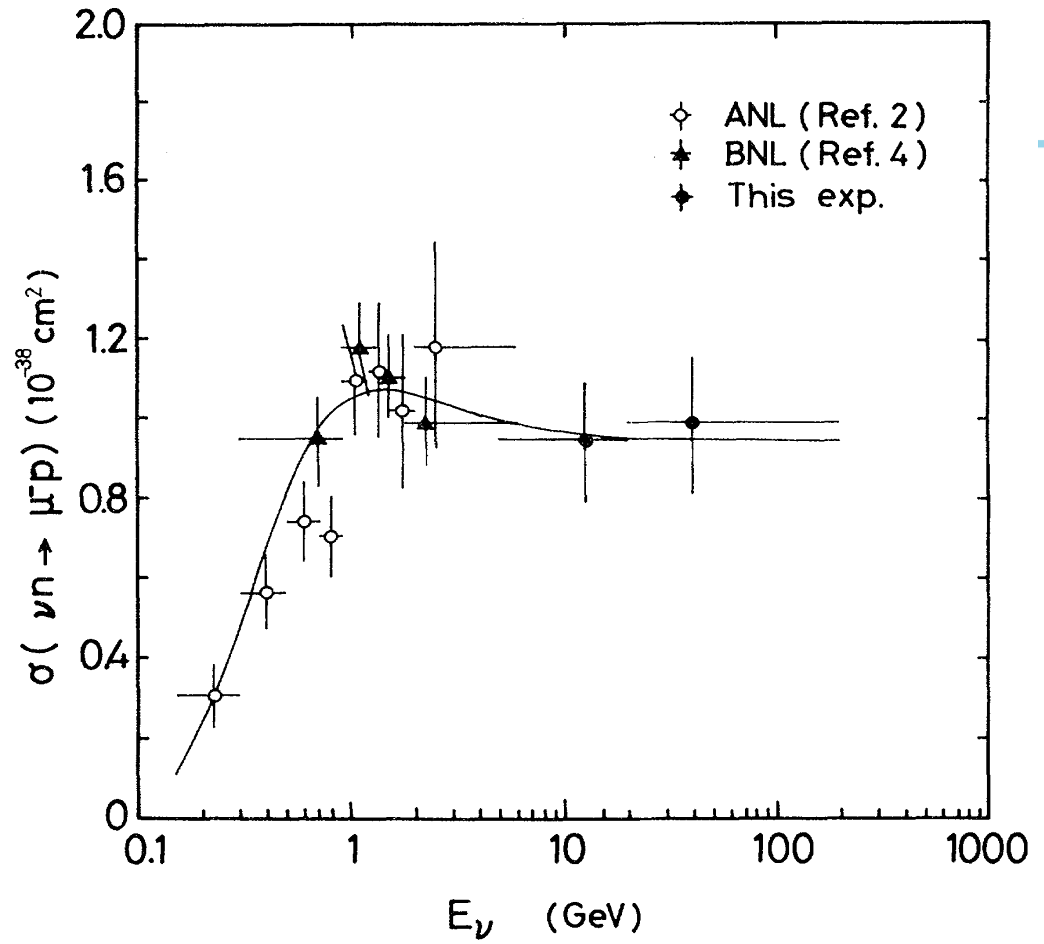
$$F_A(q^2) = \sum_{k=0}^{k_{\max}} a_k z(q^2)^k,$$

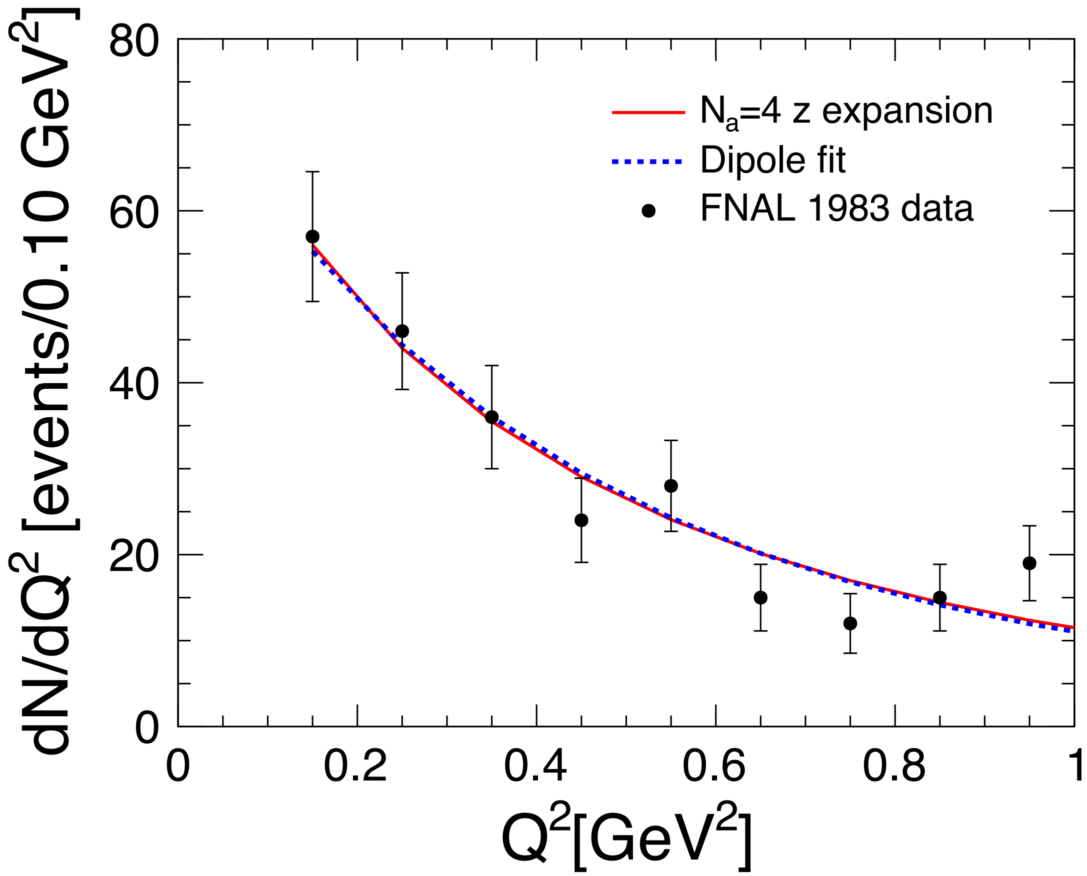
known functions

free parameters

Bhattacharya, Hill, and Paz PRD 84 (2011) 073006

A.S.Meyer et al, Phys.Rev.D 93 (2016) 11, 113015



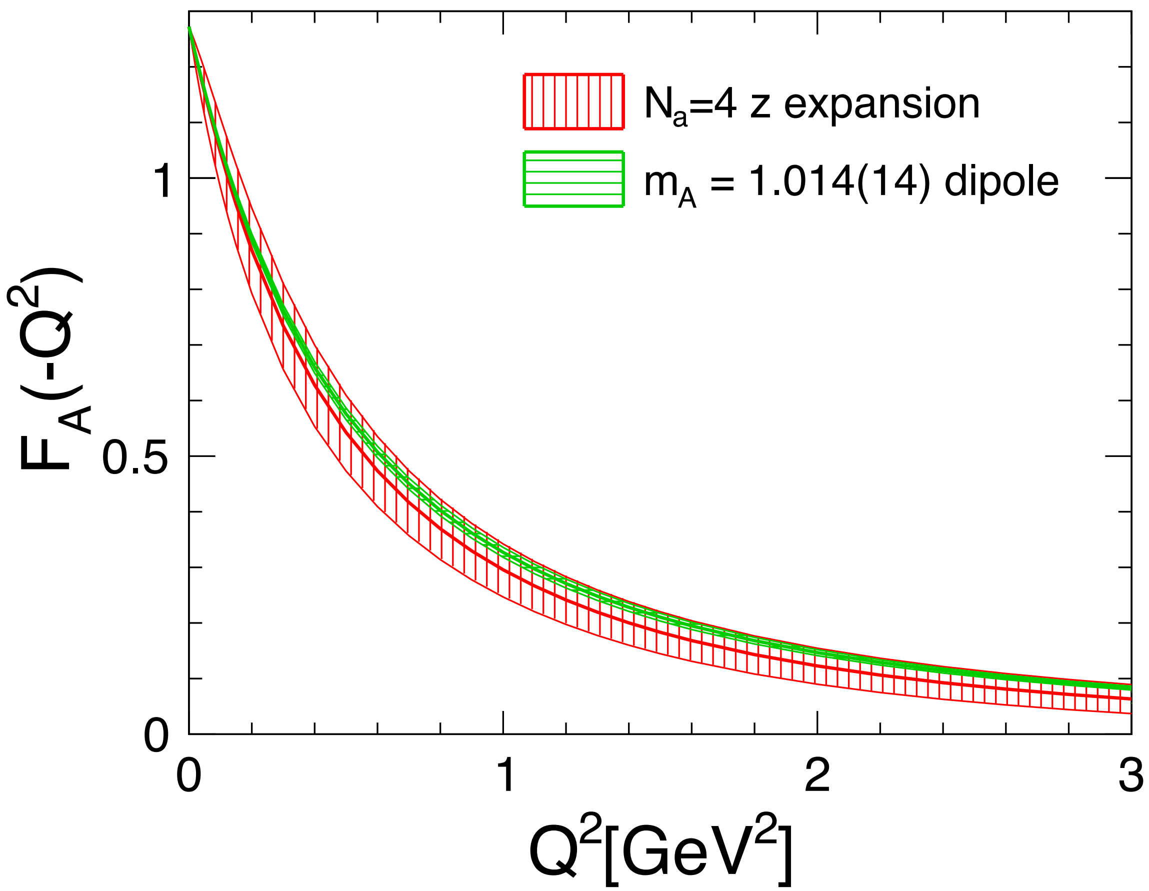


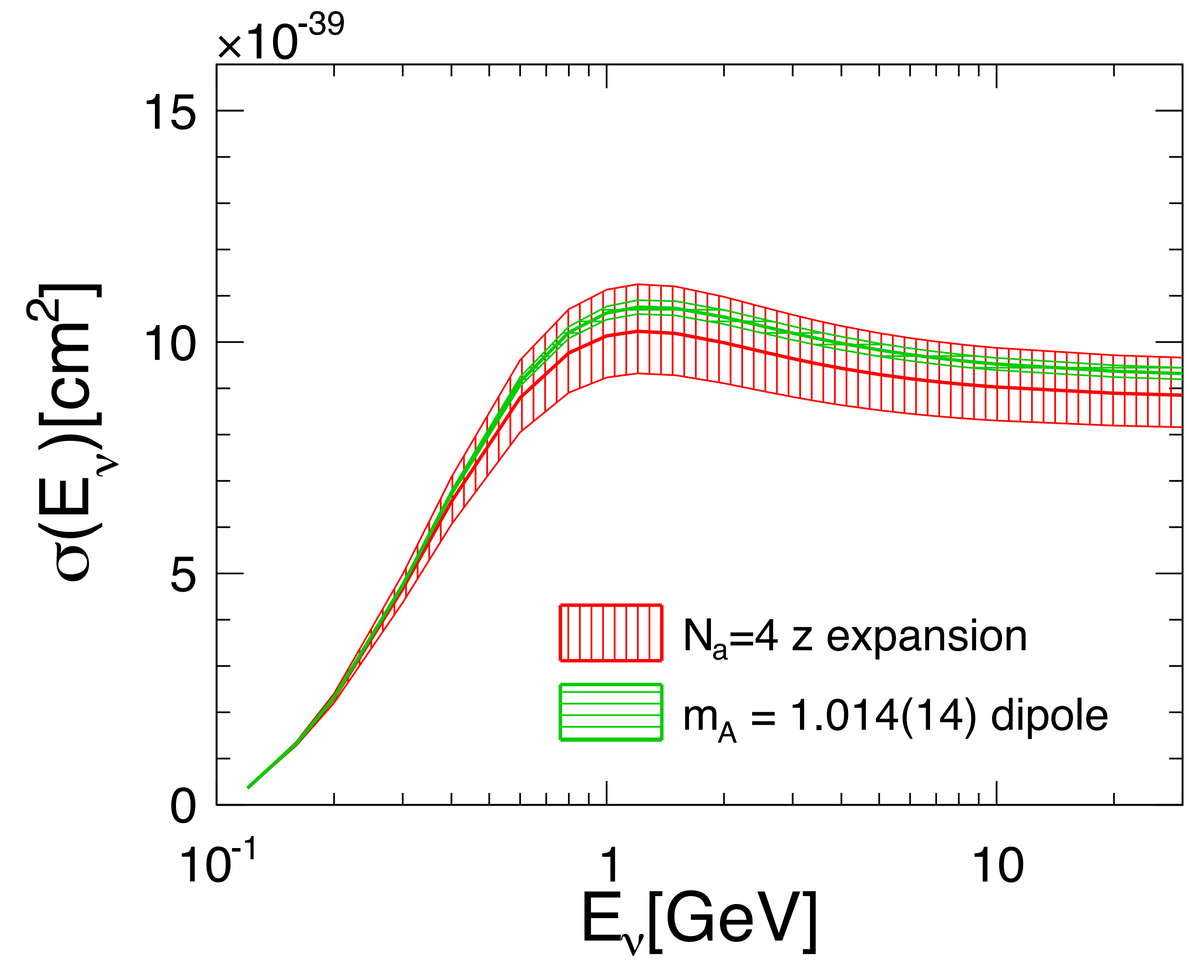
Neutrino-Nucleon scattering

- Sum rule can be enforced ensuring that the form factor falls smoothly to zero at large Q^2

$$z(q^2, t_{\rm cut}, t_0) = \frac{\sqrt{t_{\rm cut} - q^2} - \sqrt{t_{\rm cut} - t_0}}{\sqrt{t_{\rm cut} - q^2} + \sqrt{t_{\rm cut} - t_0}}\,, \qquad \sum_{k=n}^{\infty} k(k-1)\cdots(k-n+1)a_k = 0\,, \quad n = 0, 1, 2, 3$$

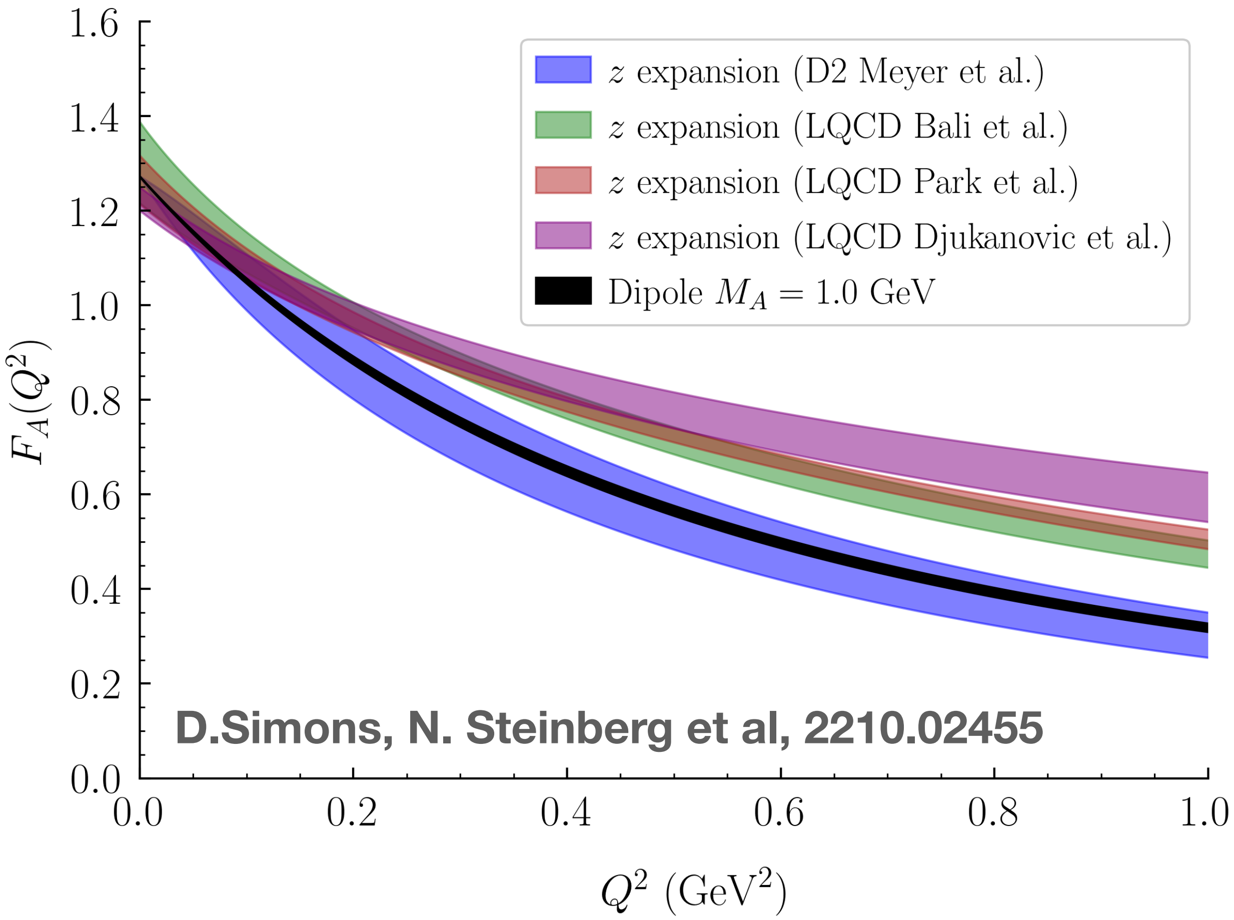
Fit deuteron data replacing dipole axial form factor with z-expansion, enforce the sum rule constraints





A.S.Meyer, Phys.Rev.D 93 (2016) 11, 113015

Axial form factor determination



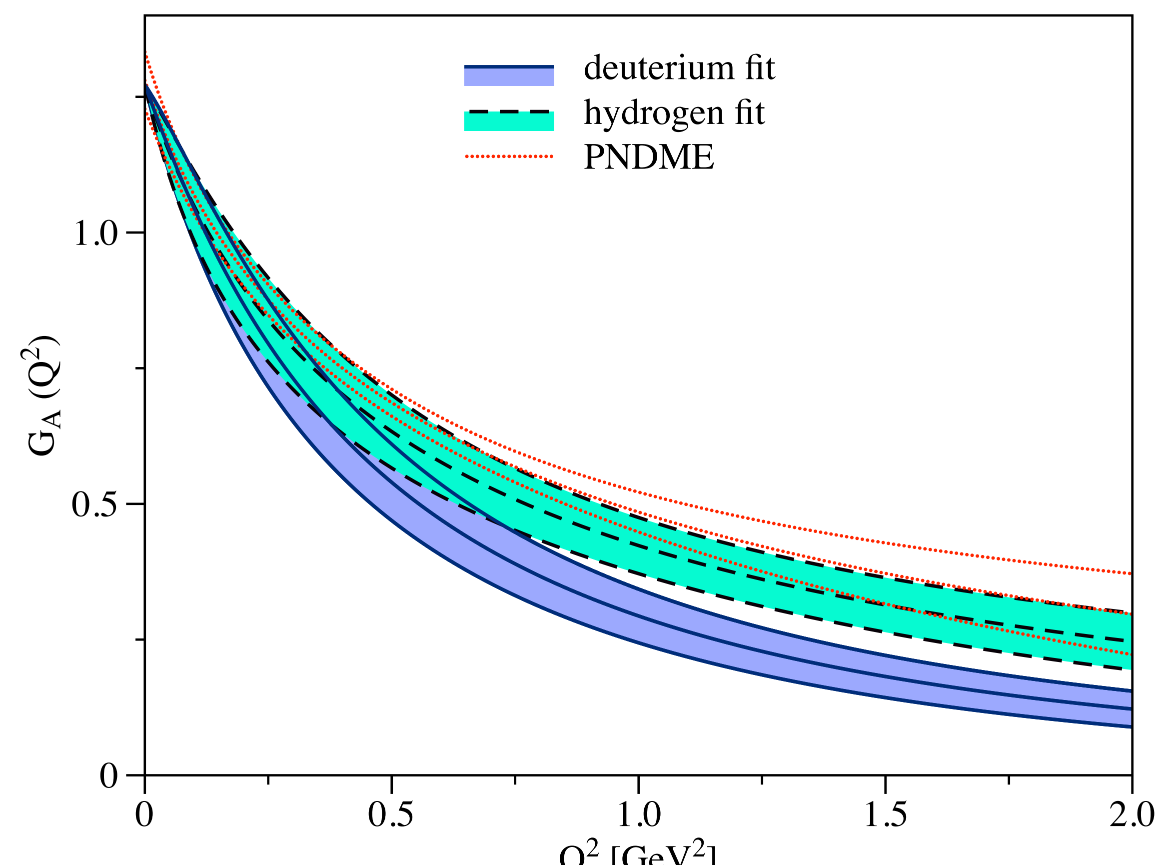
A. Meyer, A. Walker-Loud, C. Wilkinson, 2201.01839

D2 Meyer et al: fits to neutrino-deuteron scattering data

LQCD result: general agreement between the different calculations

LQCD results are 2-3σ larger than D2 Meyer ones for $Q^2 > 0.3$ GeV^2

O. Tomalak, R. Gupta, T. Battacharaya, 2307.14920



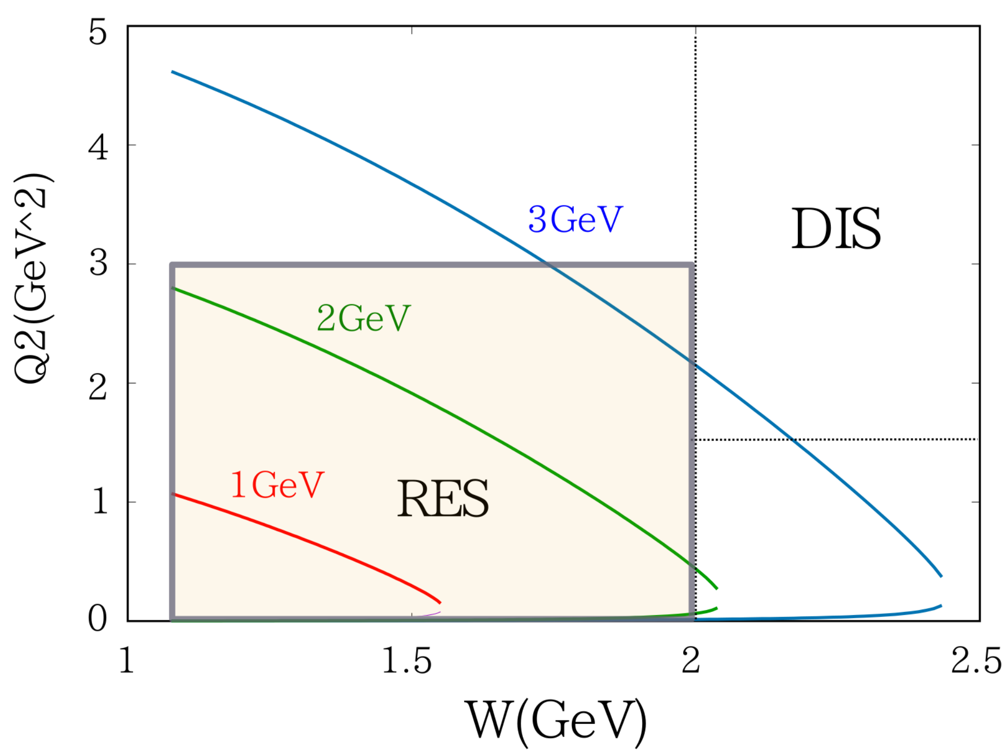
Comparison with recent MINERvA antineutrino-hydrogen charged-current measurements

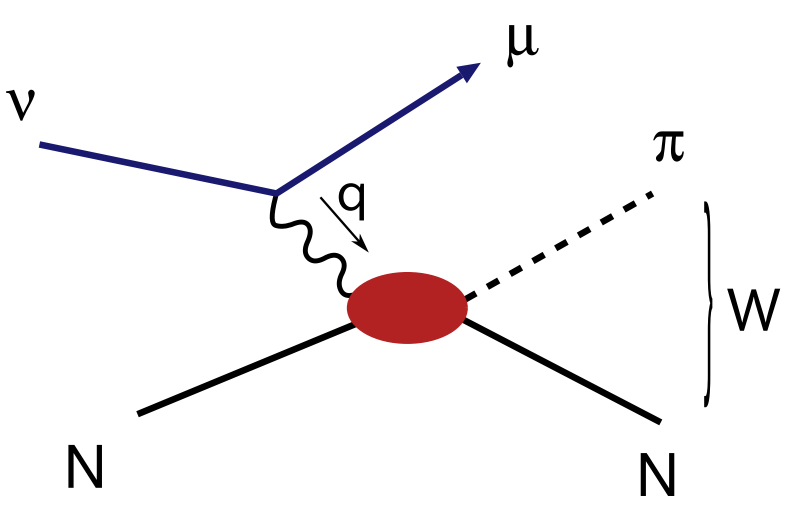
1-2σ agreement with MINERvA data and LQCD prediction by PNDME Collaboration

Novel methods are needed to remove excited-state contributions and discretization errors

Pion Production

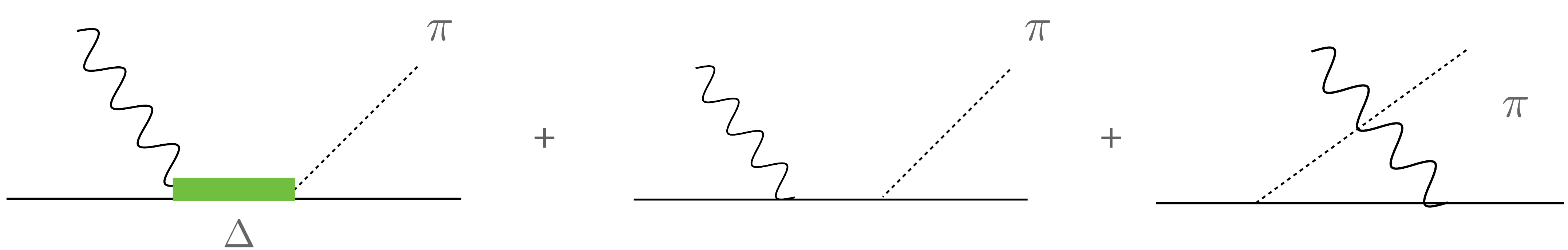
T.Sato talks @ NuSTEC Workshop on Neutrino-Nucleus Pion Production in the Resonance Region





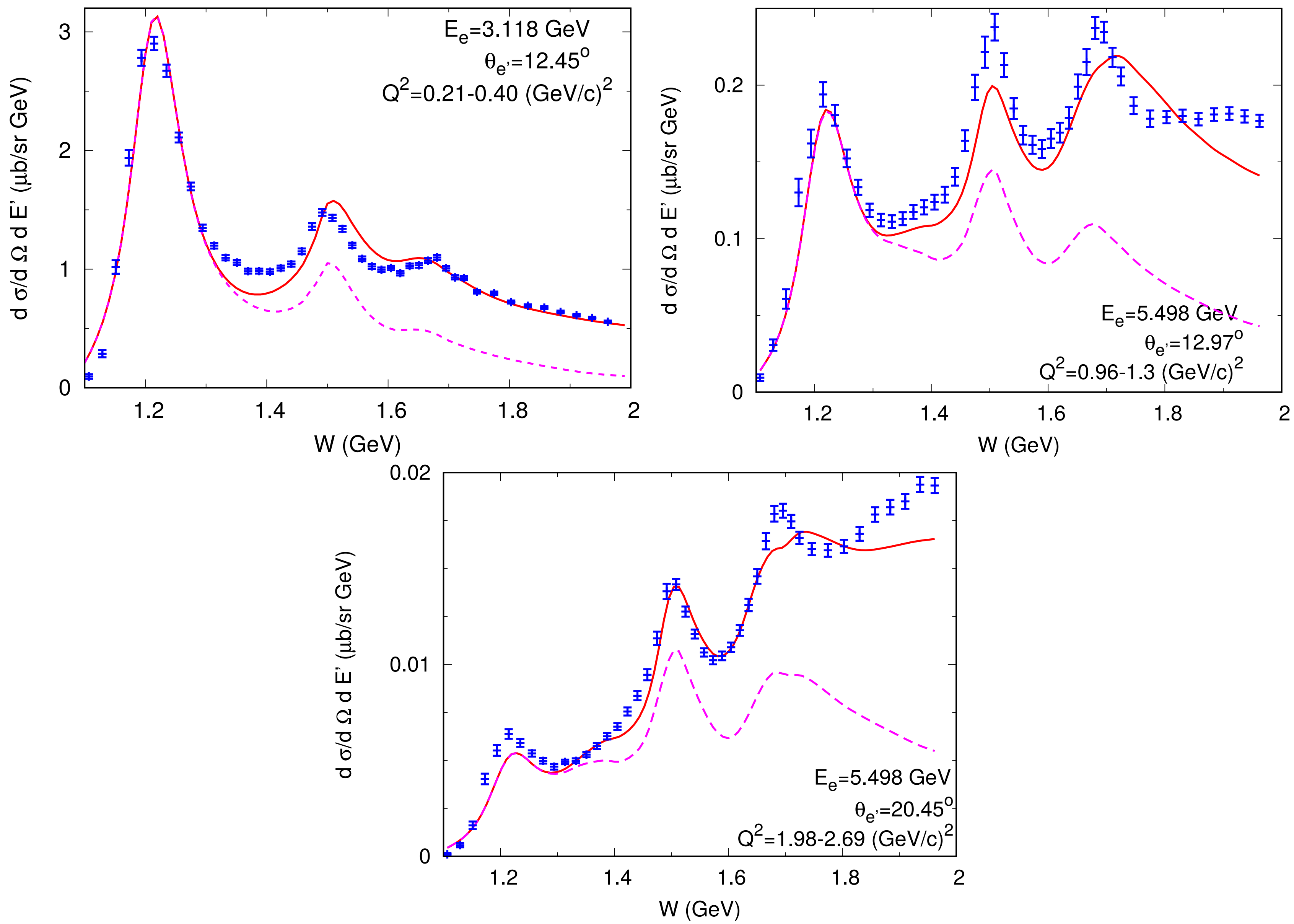
$$W^2 = (p_N(f) + p_\pi)^2 = (p_\nu + p_N(i) - p_\mu)^2$$

$$Q^2 = -q^2 = -(p_\nu - p_\mu)^2$$



Contribution from both background and Δ-resonance states (but also higher resonances)

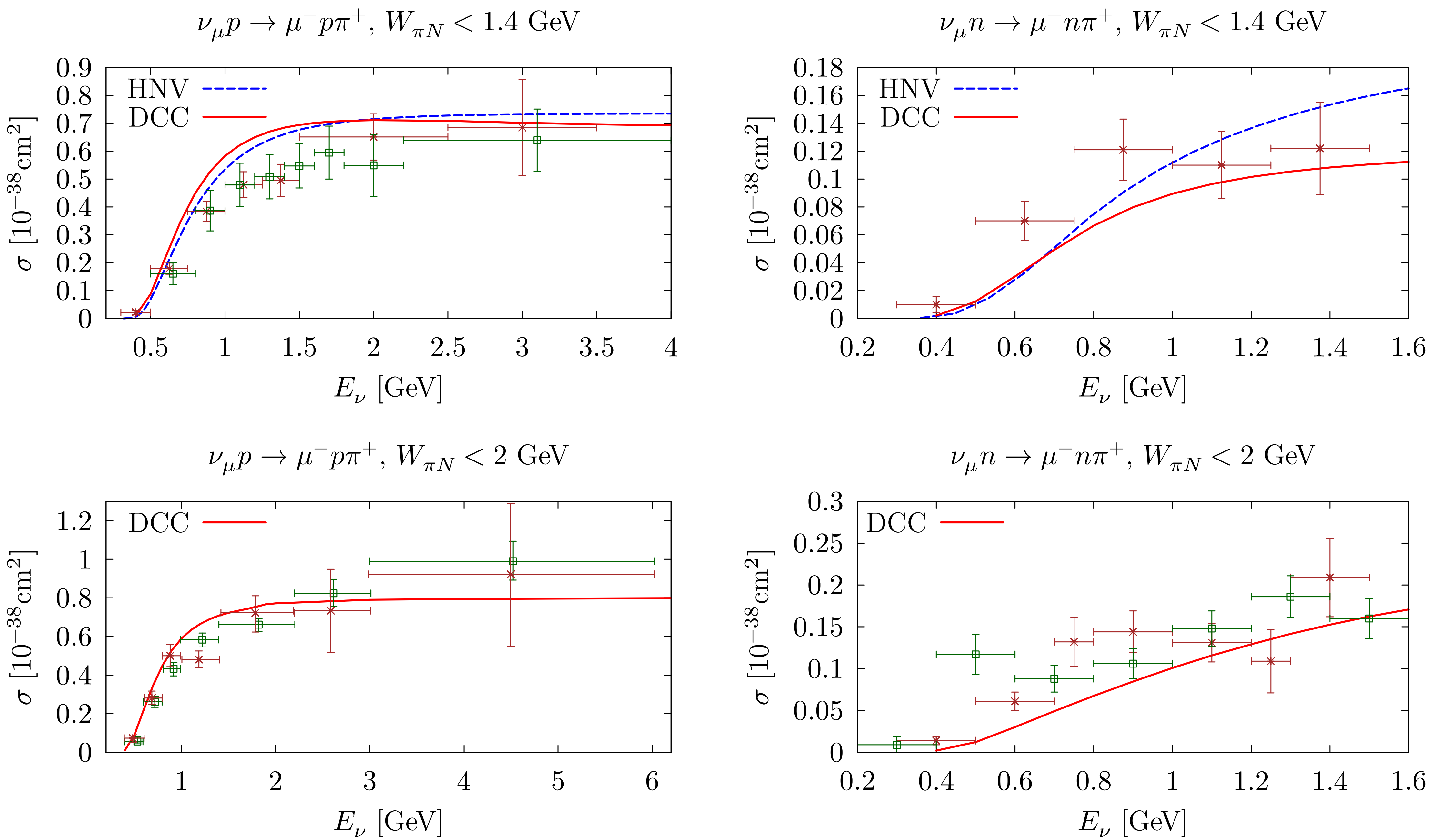
Pion Production



ANL-Osaka Partial-Wave Amplitudes (PWA) H.Kamano, T.-S. Lee, S.X. Nakamura, T.Sato, arXiv:1909.11935v1

Noemi Rocco, nrocco@fnal.gov

Pion Production



J. Sobczyk, E. Hernandez, S.X. Nakamura, J. Nieves, T. Sato PRD 98 (2018) 073001

Pion Production

Summary of models for neutrino reaction in RES

	Res	Non-res	Unit.	1pi	2pi	Tot
RS	Delta,N*	-	X	O		O
LPP	Delta,N*	X	X	O		O
HVM	Delta(1232)	chiral	O	O		
	Delta(1232)+N(1440)	chiral	X	O	O	
Giessen	Delta, N*	phen.	X	O		O
ANL-Osaka	Delta, N*	O	O	O	O	O

RS: D. Rein, L. M. Sehgal AP133(81), LPP: O. Lalakulich,E.A. Paschos,G. Piranlshvili,PRD74(2006)
HNV: E. Hernandez,J. Nieves,M. Valverde PRD76(2007) Giessen: T. Leitner,O.Buss,L.Alvarez-Ruso,U. Mosel,PRC79(2009)
ANL-Osaka DCC:S.X.Nakamura,H. Kamano,TS,PRD92(2015) ,TS,D. Uno,T.-S.H.Lee PRC67(2003)

R. Gonzales-Jimenes et al. PRD95,113007(2017)+Regge

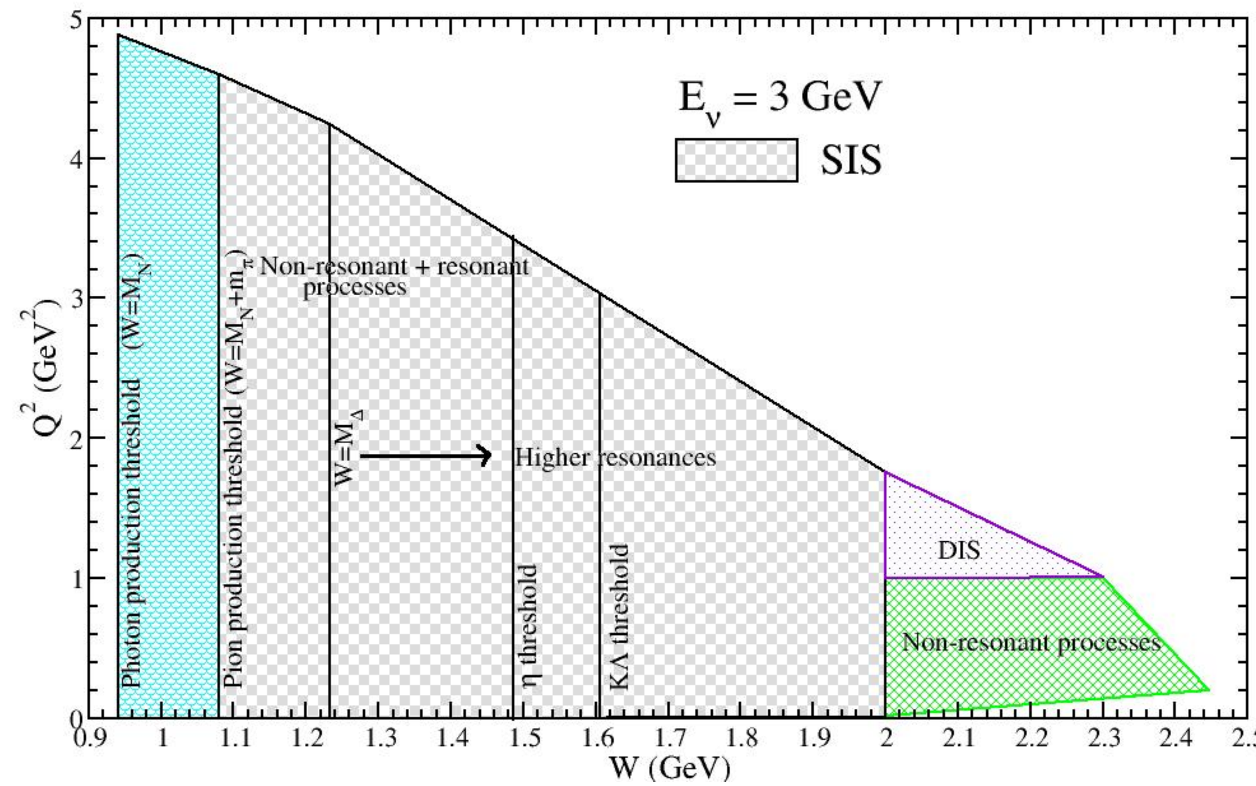
Noemi Rocco, nrocco@fnal.gov

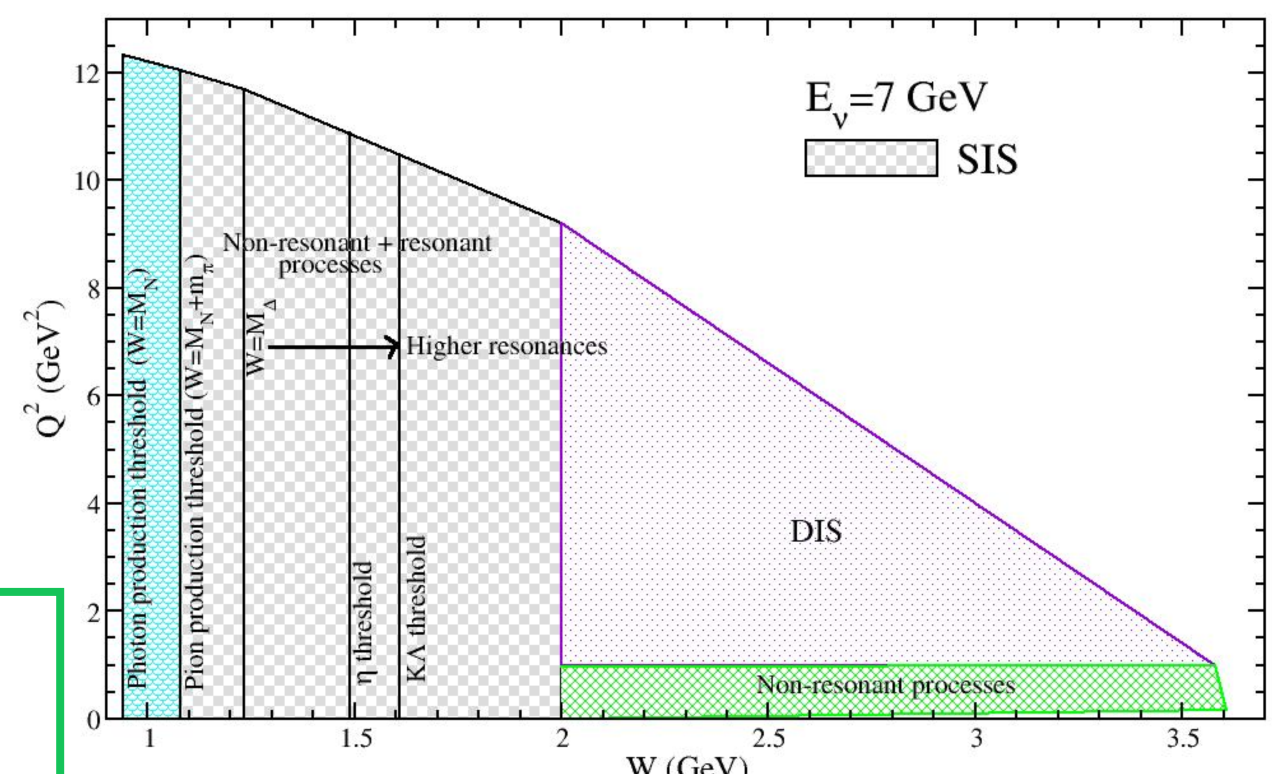
Parton Structure of the nucleon

Above the pion production threshold W ≈ 1080 MeV the excitation of the Δ(1232) dominates, but at higher W the dynamics results from the interplay of overlapping baryon resonances, non-resonant amplitudes and their interference.

It is this region of W above the Δ(1232) and at moderate $Q^2 > 1$ GeV^2 that we refer to as **Shallow Inelastic Scattering (SIS)**. As Q2 grows, one approaches the onset of Deep Inleastic Scattering (DIS).

Transition from **strong interactions** described in terms of **hadronic degrees of freedom** to those among **quarks and gluons** described by perturbative QCD.

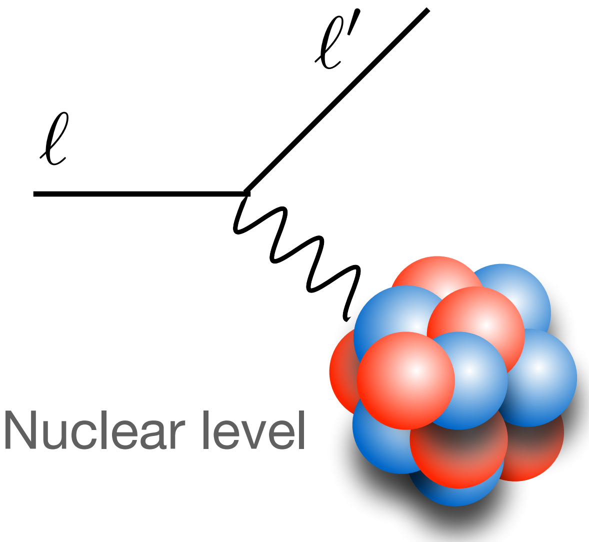




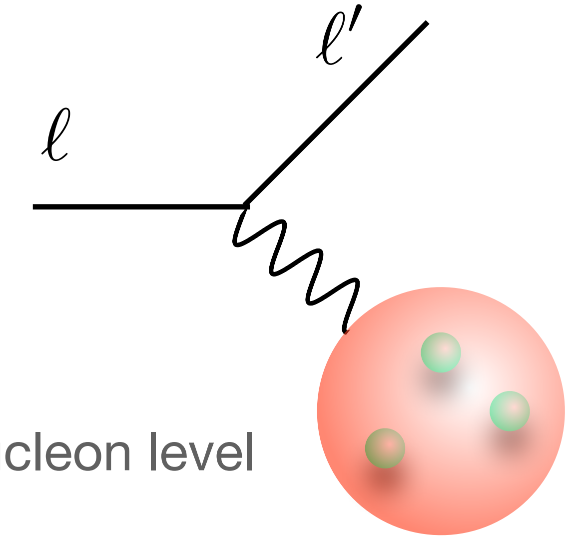
Theoretical tools for neutrino scattering,
Contribution to: 2022 Snowmass Summer Study

Noemi Rocco, nrocco@fnal.gov

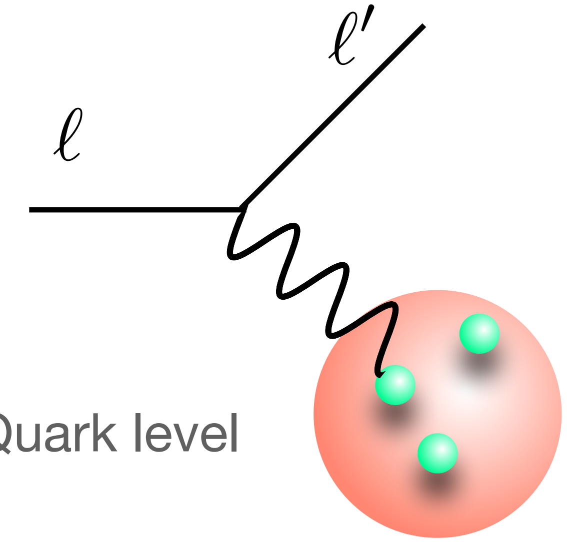
Parton Structure of the nucleon



- Nuclear level



- Nucleon level



- Quark level

Parton model is analogous to the notion that a nucleus is a collection of noninteracting nucleons—but with a critical difference. Nucleons are rather easily liberated from nuclei, but the **division of a hadron into its constituent partons has never been observed**

$$\frac{d^2\sigma}{dQ^2 d\nu} = \frac{\pi}{EE'}\frac{d^2\sigma}{dE'd\Omega'} = \frac{4\pi\alpha^2}{Q^4}\frac{E'}{E}\left[2W_1(Q^2,\nu)\sin^2\left(\frac{\theta}{2}\right) + W_2(Q^2,\nu)\cos^2\left(\frac{\theta}{2}\right)\right].$$

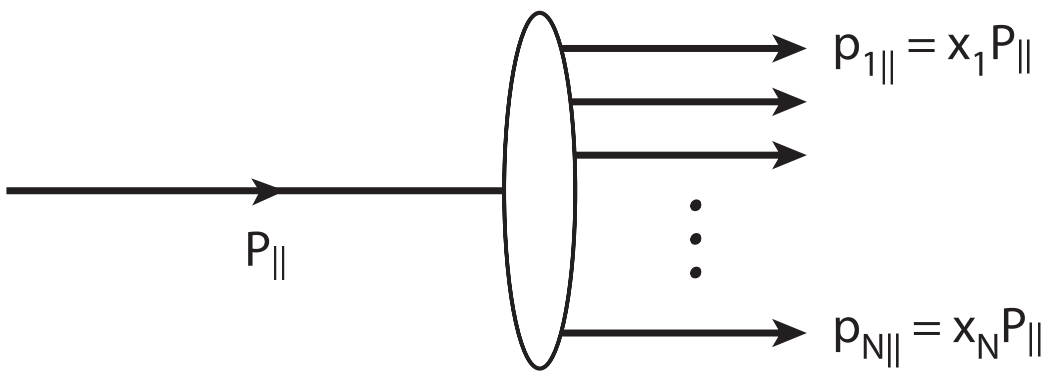
C. Quigg, Gauge Theories of the strong, weak, and electromagnetic interaction

Parton Structure of the nucleon

The transition to the parton model is made in the infinite- momentum frame, in which the longitudinal momentum of the proton is extremely large

Momentum of individual partons is:

$$p_i^\mu = x_i P^\mu$$



Elastic electron-proton

$$W_1 = \frac{Q^2}{4m_N^2}(F_1 + F_2)^2 \delta\Big(\omega - \frac{Q^2}{2m_N}\Big)$$

$$W_2 = \Big(F_1^2 + F_2^2 \frac{Q^2}{4m_N^2}\Big)\delta\Big(\omega - \frac{Q^2}{2m_N}\Big)$$

Electron-point-like particle

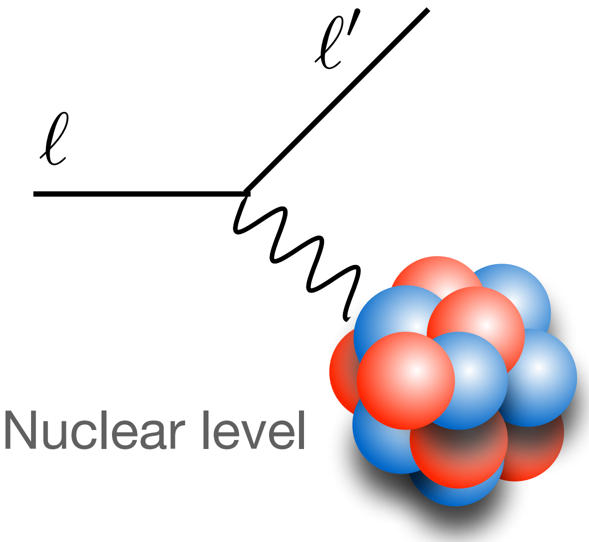
$$W_1^{\text{point}} = \frac{Q^2}{4m^2}\delta\Big(\omega - \frac{Q^2}{2m}\Big)$$

$$W_2^{\text{point}} = \delta\Big(\omega - \frac{Q^2}{2m}\Big)$$

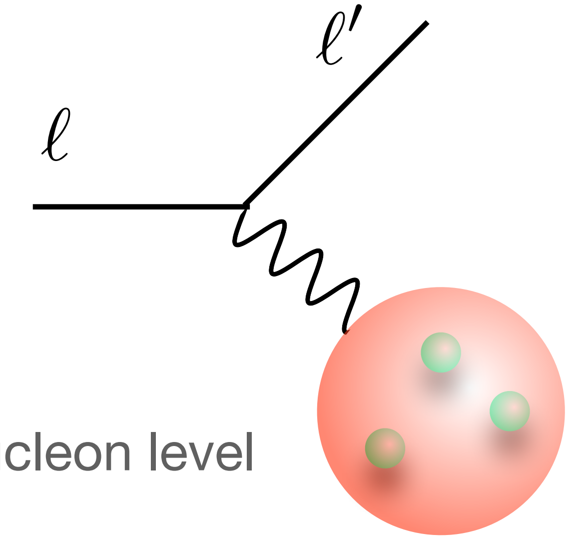
✳ Let's move to the blackboard

Parton Structure of the nucleon

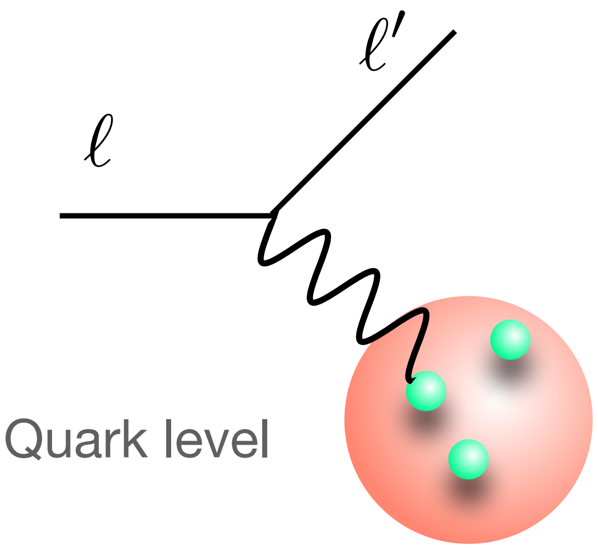
- Nuclear level



- Nucleon level



- Quark level



$$\frac{d^2\sigma}{dQ^2 d\nu} = \frac{\pi}{EE'} \frac{d^2\sigma}{dE' d\Omega'} = \frac{4\pi\alpha^2}{Q^4} \frac{E'}{E} \left[2 W_1(Q^2, \nu) \sin^2\left(\frac{\theta}{2}\right) + W_2(Q^2, \nu) \cos^2\left(\frac{\theta}{2}\right) \right].$$

$$\Downarrow$$

$$\frac{d^2\sigma^\nu}{dQ^2\, d\nu} = \frac{G_F^2}{2\pi} E' E \left[2 W_1^\nu \sin^2\left(\frac{\theta}{2}\right) + W_2^\nu \cos^2\left(\frac{\theta}{2}\right) + W_3^\nu \frac{(E+E')}{M} \sin^2\left(\frac{\theta}{2}\right) \right]$$

$\nu + N$ has the contribution of an additional structure function $\quad \mathcal{F}_3(x) \equiv \nu\, W_3^\nu(x)$

Quark-Hadron Duality

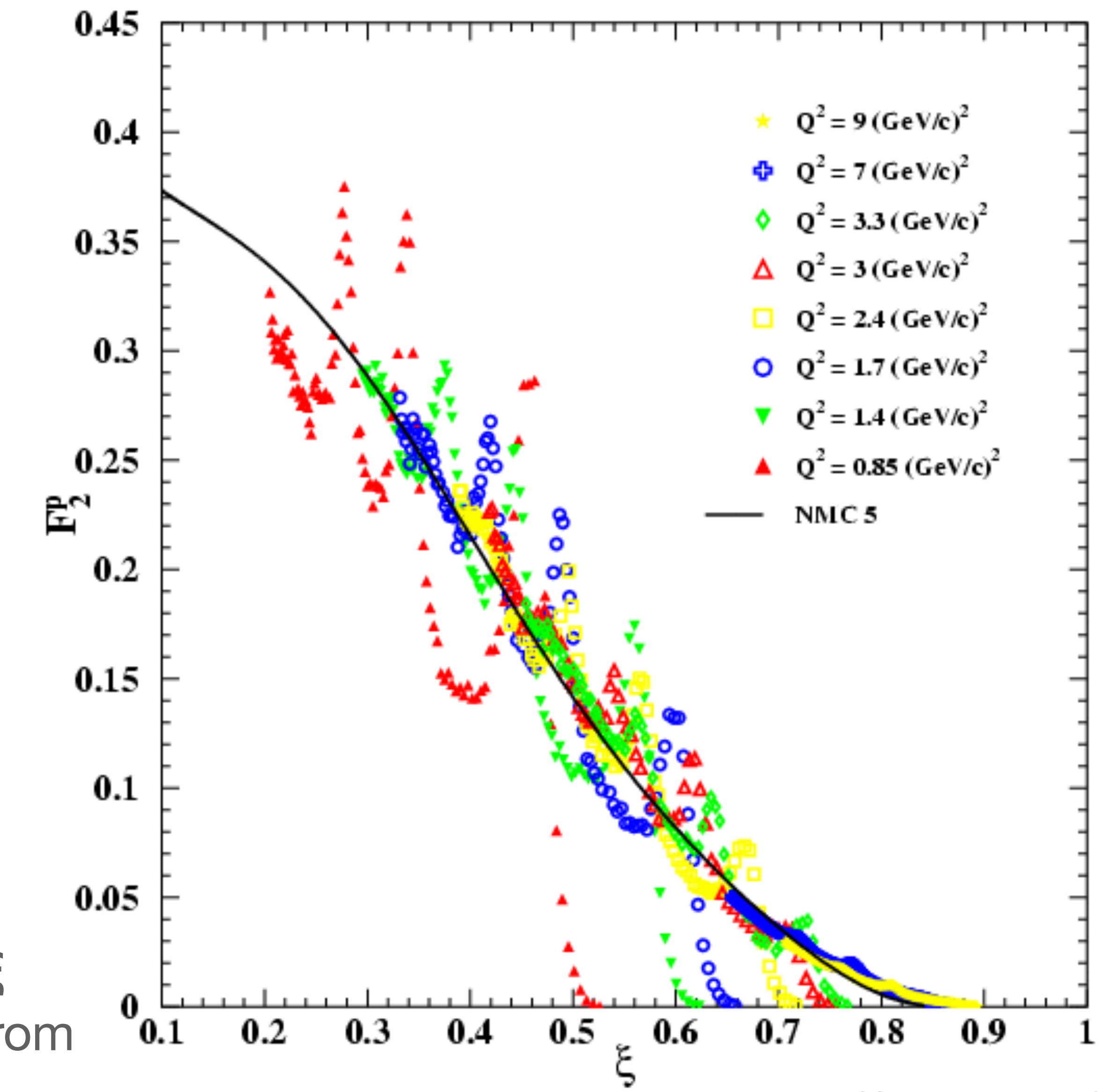
Duality can then be considered as a conceptual experimental bridge between **free** and **confined partons.**

Language of quarks/gluons in the DIS region and, as W decreases, one transitions to hadrons; in the SIS region that includes both resonant and non-resonant pion production.

- 1970 Bloom-Gilman duality in (e,e') experiments on *p*

DIS scaling curve extrapolated down into the resonance region passes through the average of the "peaks and valleys" of the resonant structure.

Comparison of F^p_2 from the **series of resonances** measured by E94-110 vs the **Nachtmann variable ξ** compared to the **extrapolated DIS measurement** from the NMC collaboration at 5 GeV^2



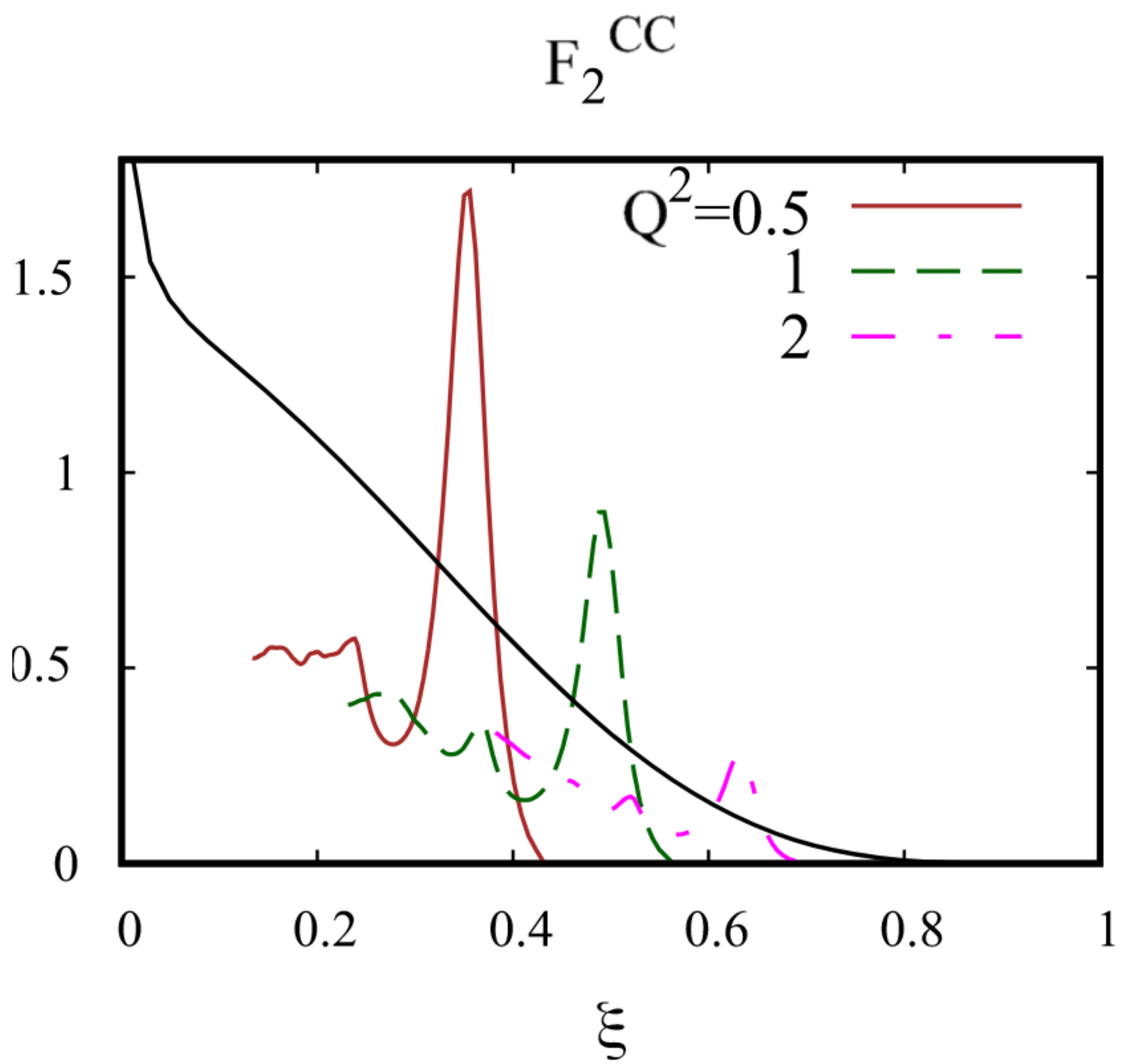
For $Q^2 > 0.5$ GeV^2 resonances follow the extrapolated DIS curve showing quark-hadron duality

Quark-Hadron Duality in ν

The experimental study of duality with neutrinos is very restricted since the measurement of resonance production by ν-N interactions is confined to **low-statistics data obtained in hydrogen and deuterium bubble chamber experiments from the 70's and 80's.**

For CC ν scattering

ANL-Osaka DCC model underestimates F_2 from DIS



[T. Sato EPJ:ST (2021) 230:4409-4418 (2022)]

see also:
[O. Lalakulich et al. PRC79 015206 (2009)]

Quark-Hadron Duality in ν

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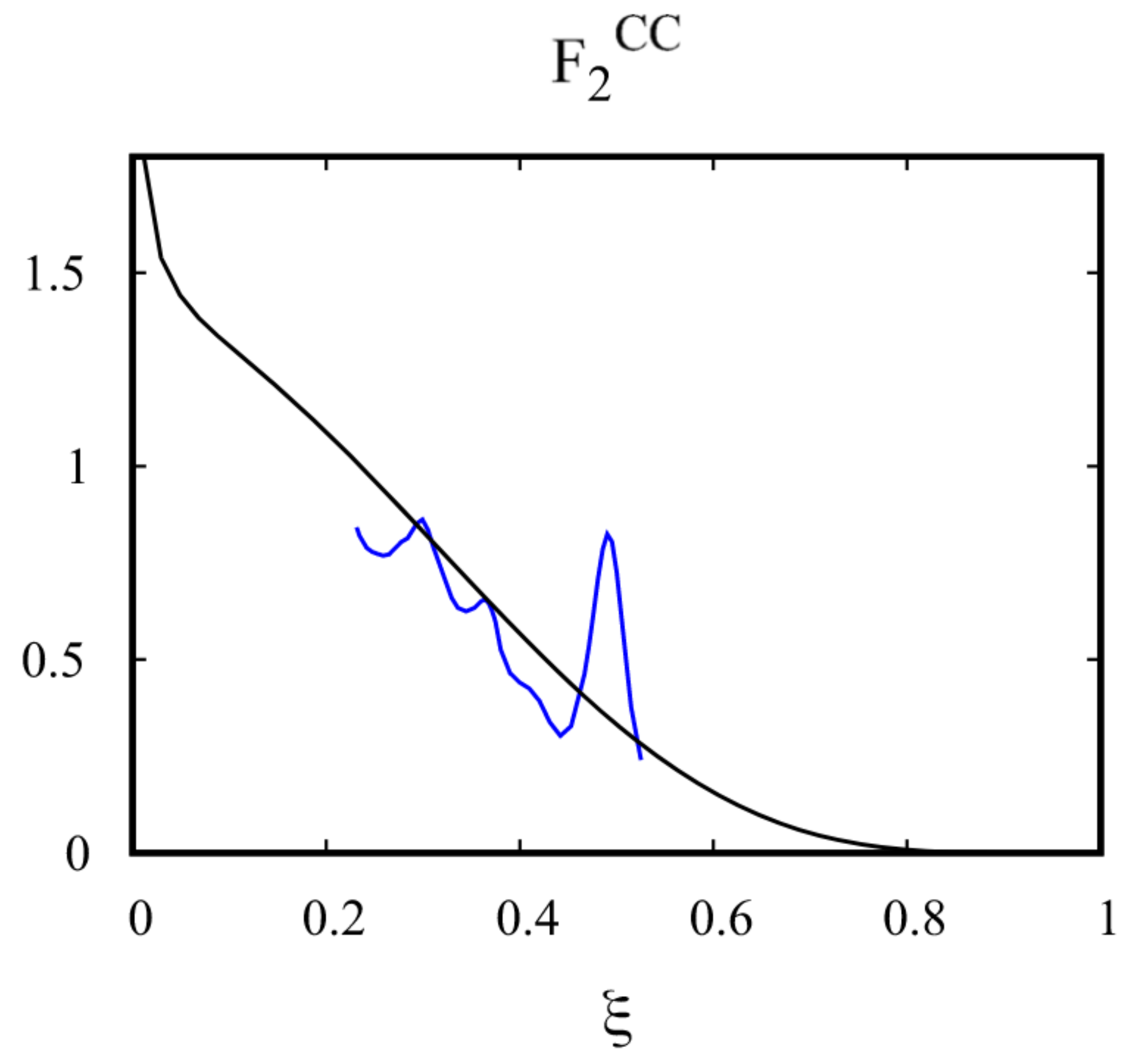
For CC ν scattering

ANL-Osaka DCC model underestimates F_2 from DIS

Improvement with DIS upon modifying the Q2- dependence of the axial form factor:

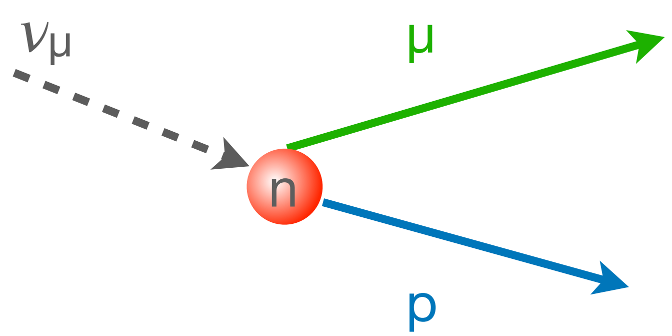
$$A_\lambda^A(Q^2) = A_\lambda^A(0)\frac{A_\lambda^V(Q^2)}{A_\lambda^V(0)}$$

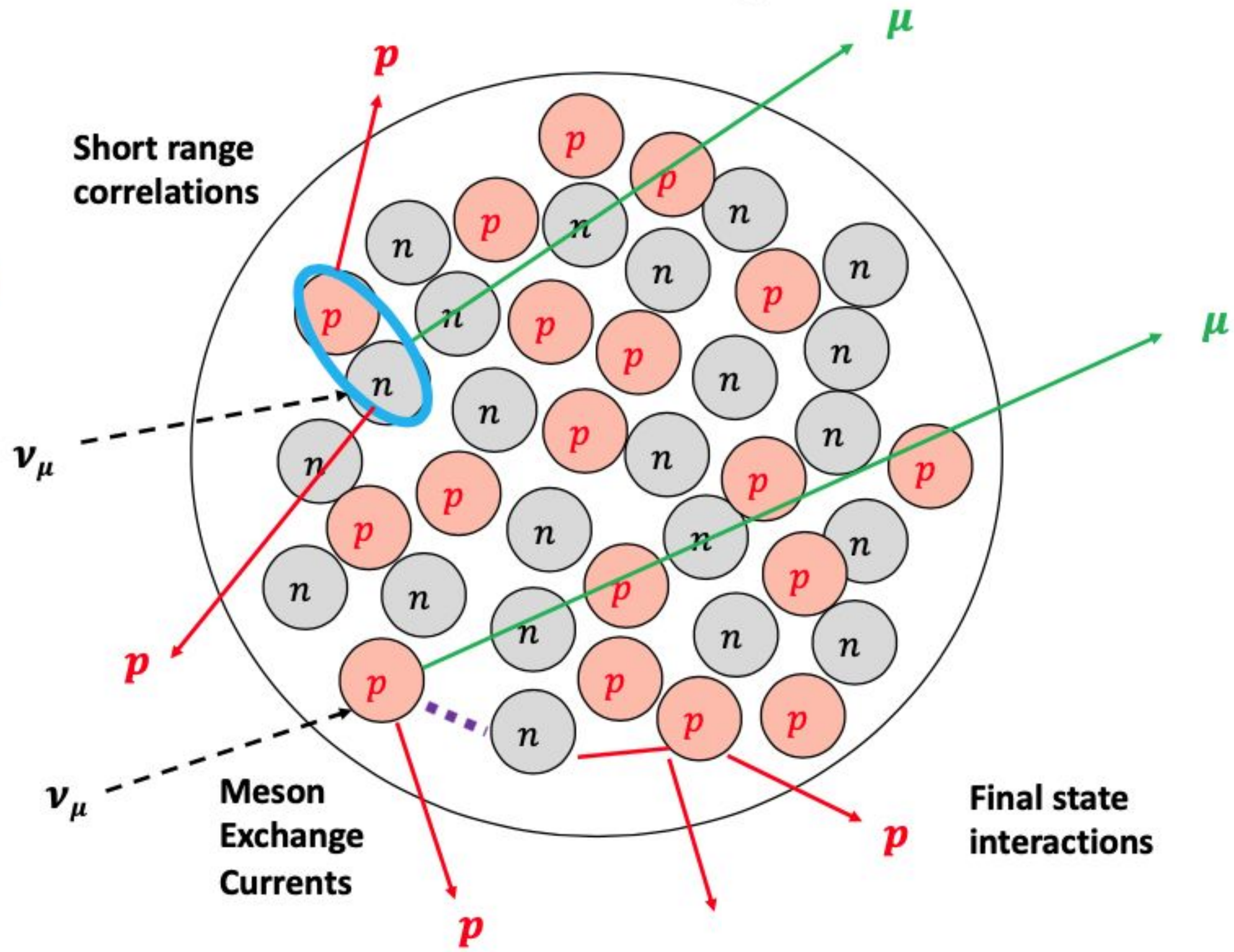


- **More data are needed to better constrain/understand ν-N scattering**

From theory to experiment

Free nucleon scattering case





Nuclear model describing the target nucleus

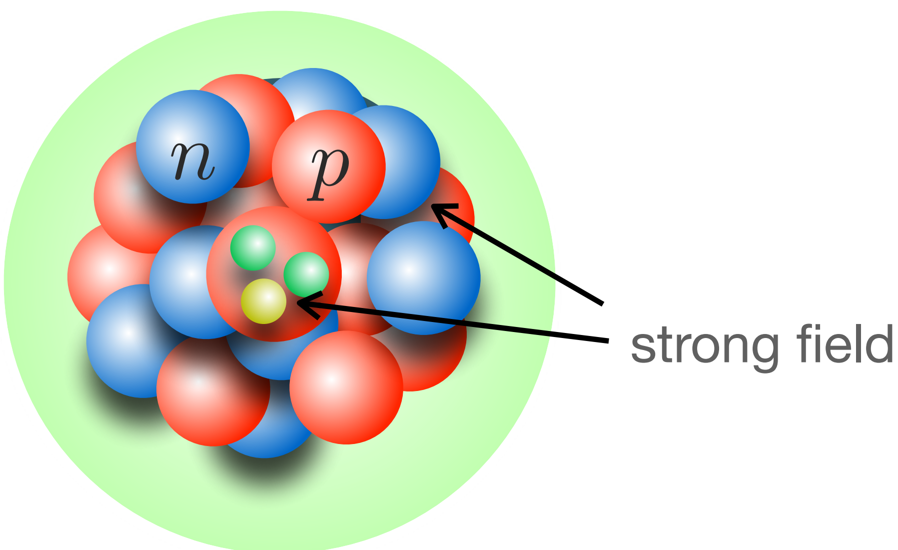
Different reaction mechanisms depending on the momentum transferred to the the nucleus

Final state interactions: describe how the particles propagate through the nuclear medium

Noemi Rocco, nrocco@fnal.gov
Fermilab

The Nucleus internal structure

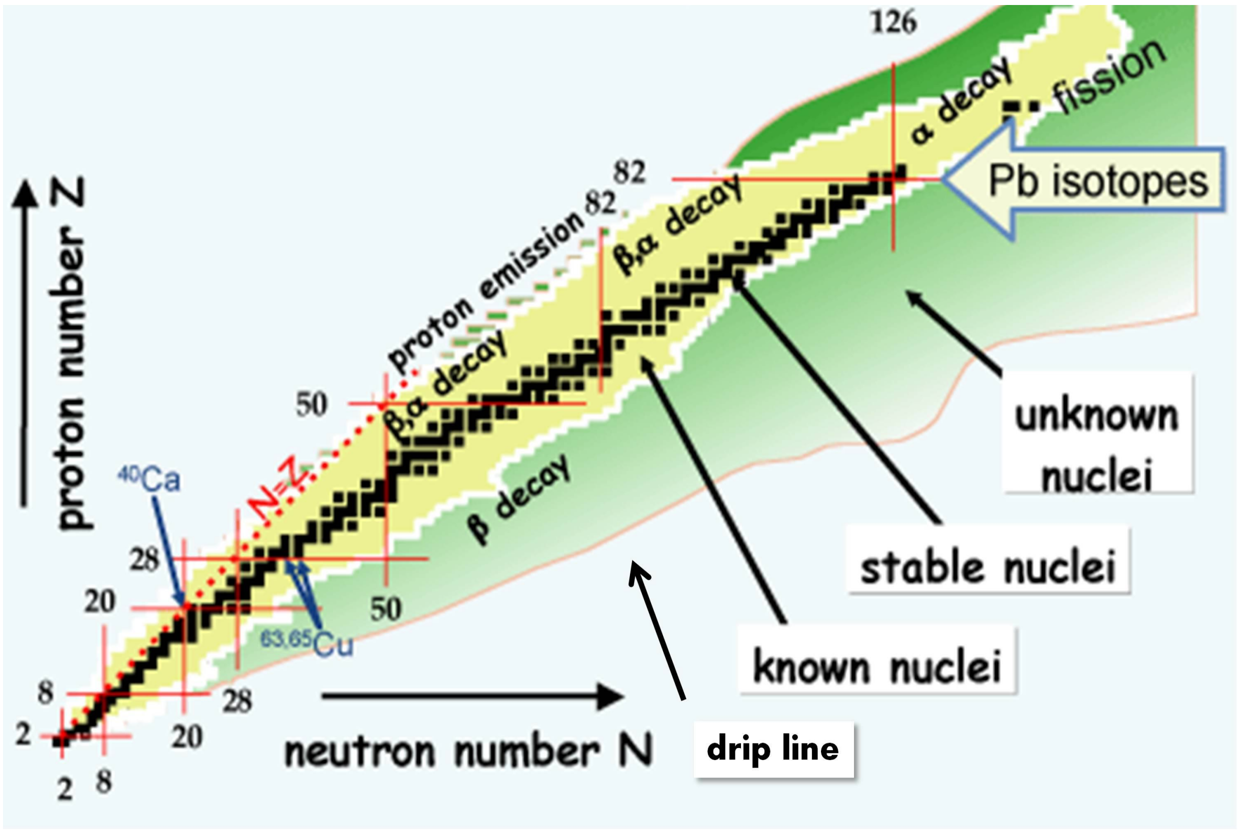
Nuclei are **strongly interacting** many body systems exhibiting fascinating properties



The nucleus is formed by **protons** and **neutrons**: nucleons.

Each nucleon is made of three **quarks** held together by strong interactions→mediated by **gluons**

Nuclear chart. **Magic numbers** N or Z= 2, 8, 20, 28, 50 and 126; major shell complete and are more stable than other elements



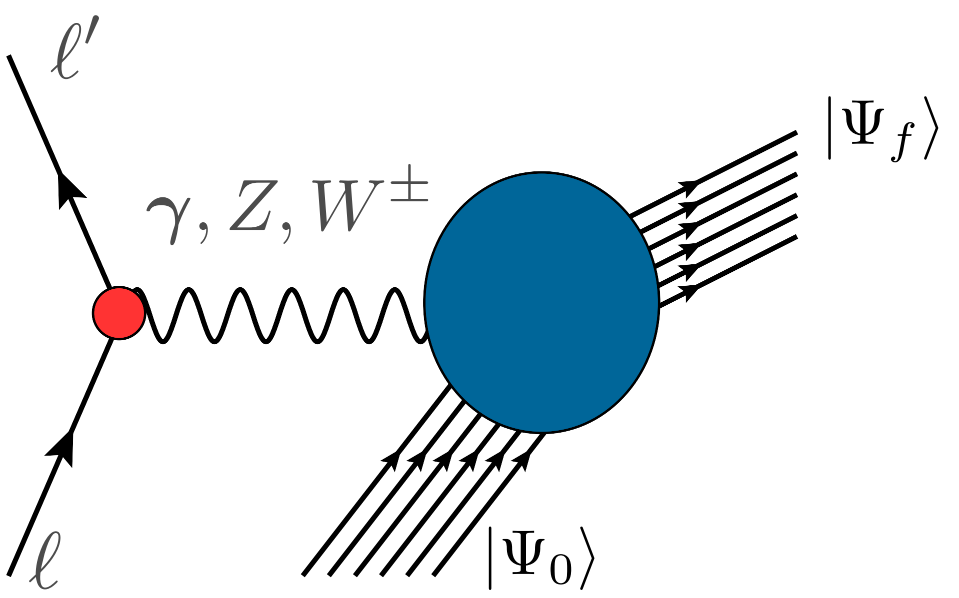
The nucleus is held together by the strong interactions between quark and gluons of neighboring nucleons

Nuclear Physicists **effectively** describe the interactions between protons and neutrons in terms of **exchange of pions**

Theory of lepton-nucleus scattering

- The cross section of the process in which a lepton scatters off a nucleus is given by

$$d\sigma \propto L^{\alpha\beta} R_{\alpha\beta}$$



Leptonic Tensor: is the same as before, completely determined by lepton kinematics

Hadronic Tensor: nuclear response function

$$R_{\alpha\beta}(\omega, \mathbf{q}) = \sum_f \langle 0|J^{\dagger}_{\alpha}(\mathbf{q})|f\rangle\langle f|J_{\beta}(\mathbf{q})|0\rangle\delta(\omega - E_f + E_0)$$

The initial and final wave functions describe many-body states:

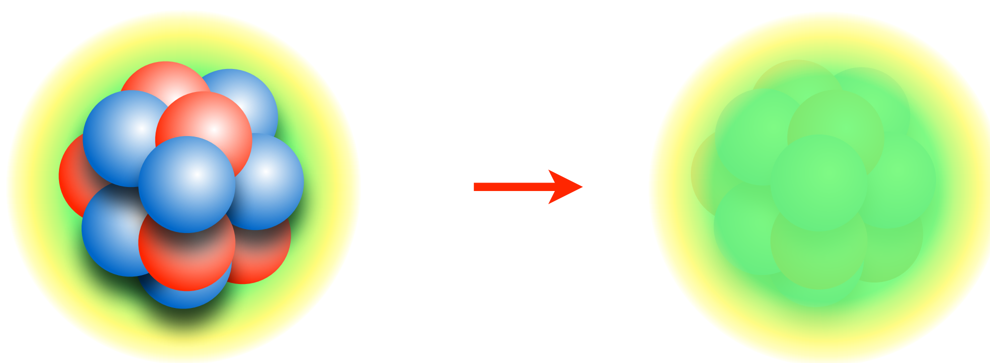
$$|0\rangle = |\Psi_0^A\rangle \ , |f\rangle = |\Psi_f^A\rangle, |\psi_p^N, \Psi_f^{A-1}\rangle, |\psi_k^{\pi}, \psi_p^N, \Psi_f^{A-1}\rangle \ldots$$

For inclusive reactions, the hadronic final state is not detected. We need to sum over all the possible ones

Noemi Rocco, nrocco@fnal.gov

Liquid Drop Model

- The nucleus is treated as a drop of incompressible nuclear fluid. The fluid is made of nucleons which are held together by the strong nuclear forces.



- This model explains the spherical shape and the binding energy of nuclei.

- The nuclear binding is given by the Weizsäcker formula

$$B(A,Z) = a_V A + a_S A^{2/3} + a_C \frac{Z^2}{A^{1/3}} + S_N \frac{(N-Z)^2}{A} + a_P \frac{(-1)^Z + (-1)^N}{2A^{1/2}}$$

Volume Surface Coulomb Symmetry Pairing

Liquid Drop Model

- The nuclear binding is given by the Weizsäcker formula

$$B(A,Z) = a_V A + a_S A^{2/3} + a_C \frac{Z^2}{A^{1/3}} + S_N \frac{(N-Z)^2}{A} + a_P \frac{(-1)^Z + (-1)^N}{2A^{1/2}}$$

Volume Surface Coulomb Symmetry Pairing

- **Volume Term:** This term represents the attractive nuclear force that acts over the entire volume of the nucleus

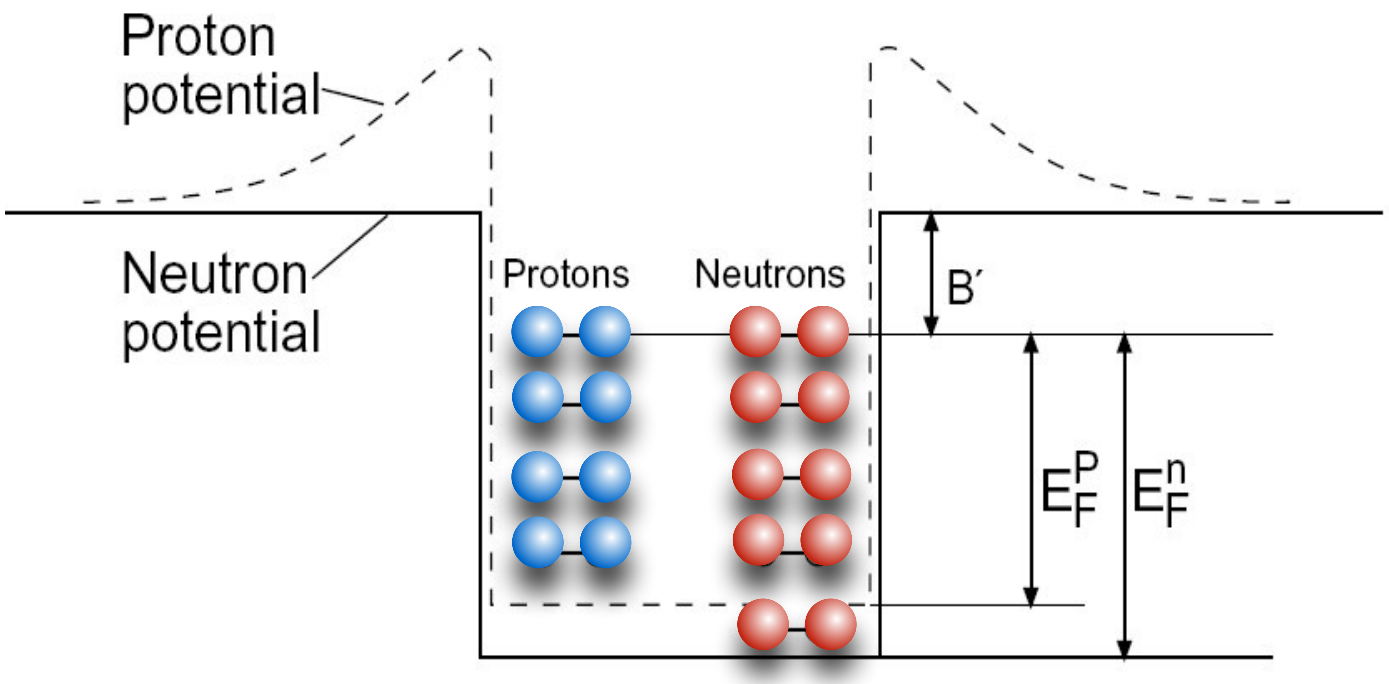
- **Surface Term:** This term accounts for the fact that nucleons at the surface of the nucleus have fewer neighboring nucleons than those in the interior

- **Coulomb Term:** This term represents the electrostatic repulsion between protons in the nucleus due to their positive charges.

- **Asymmetry Term:** It reflects the preference for equal numbers of protons and neutrons, which contribute to greater nuclear stability. Deviations from this balance result in less binding energy.

- **Pairing Term:** This term considers the additional binding energy due to the pairing of nucleons (protons with protons and neutrons with neutrons) in even numbers.

Initial state: global Fermi gas



- Simple picture of the nucleus: only statistical correlations are retained (Pauli exclusion principle)

- Protons and neutrons are considered as moving freely within the nuclear volume

- The nuclear potential wells are rectangular: constant inside the nucleus and goes sharply to zero at its edge

- The energy of the highest occupied state is the Fermi energy: E_F

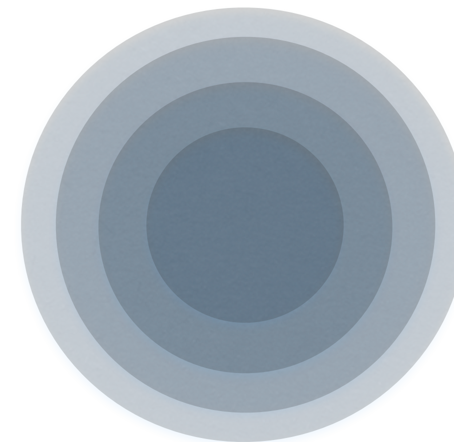
- The difference B' between the top of the well and the Fermi level is constant for most nuclei and is just the average binding energy per nucleon B'/A ~ 7-8 MeV

C. Bertulani, Nuclear Physics in a Nutshell

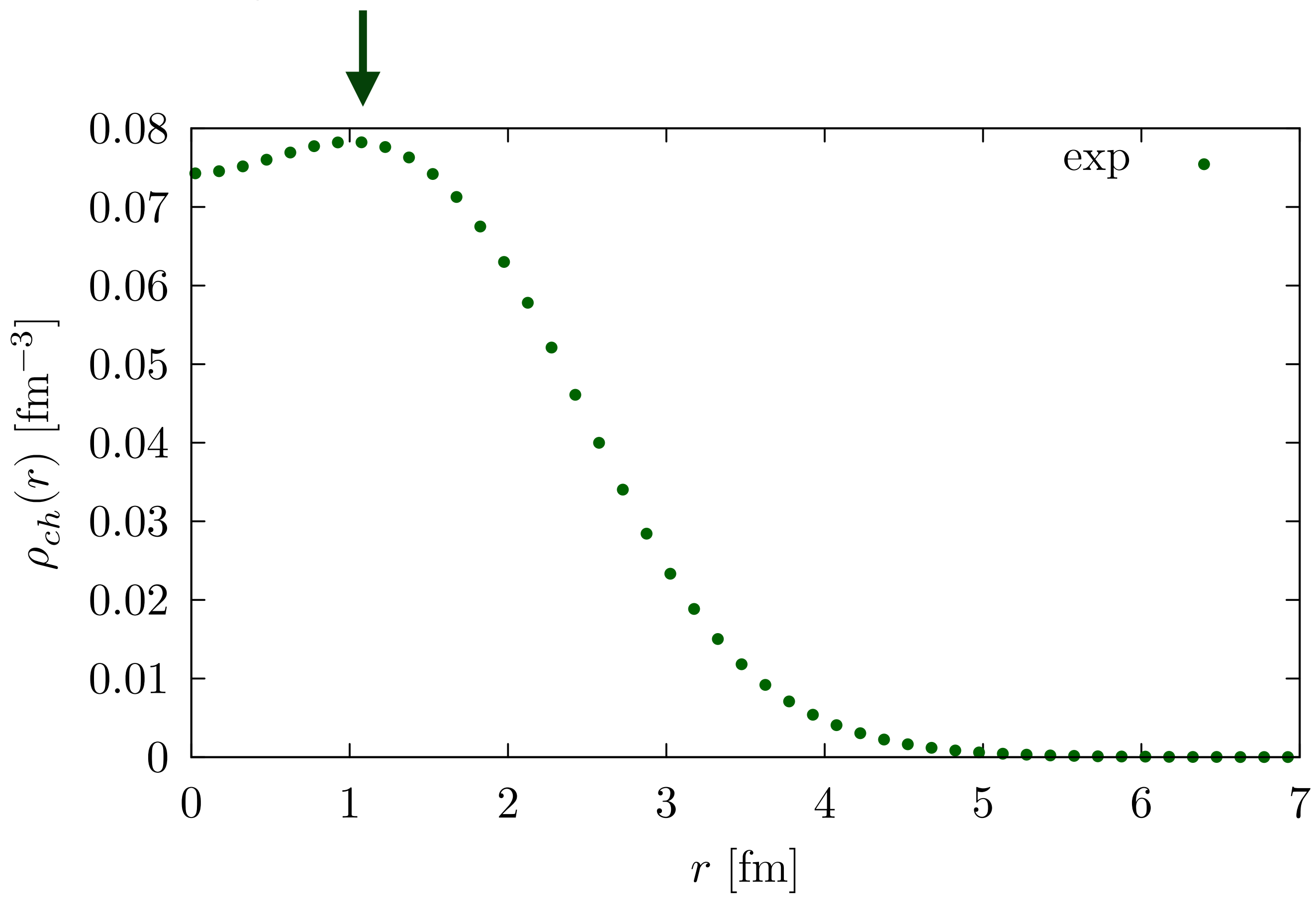
Noemi Rocco, nrocco@fnal.gov

Initial state: Local Fermi gas

- A spherically symmetric nucleus can be approximated by concentric spheres of a constant density.



More likely to find a particle r ~ r_{ch}~ 2.5 fm



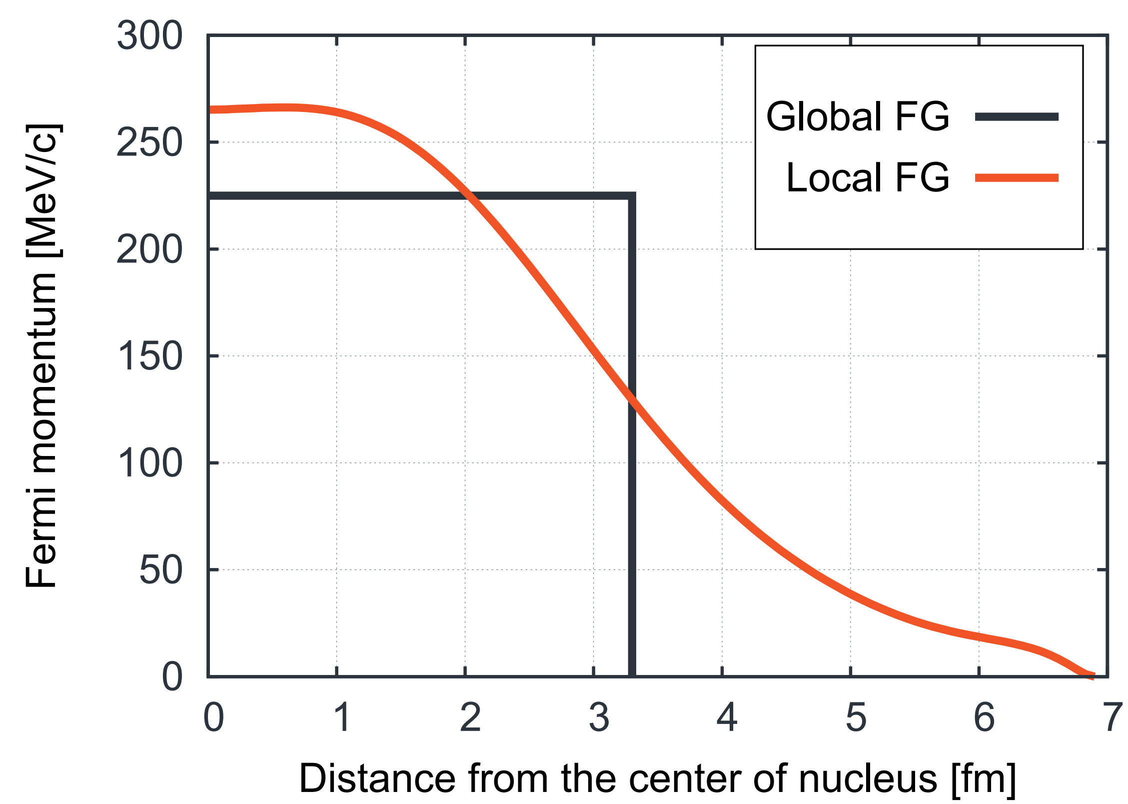


Figure by T. Golan

- **Global Fermi Gas**

$$p_F = \left(\frac{9\pi \cdot n}{4A}\right)^{1/3} \cdot \frac{\hbar}{R_0}$$

- **Local Fermi Gas**

$$p_F = \hbar\left(3\pi^2 \rho(r) \frac{n}{A}\right)^{1/3}$$

Noemi Rocco, nrocco@fnal.gov

Initial state: shell Model

- As in the Fermi Gas model: the nucleons move within the nucleus independently of each other
- Difference: the nucleons are not free: subject to a central potential

- Each nucleon moves in an average potential created by the other nucleons, the potential should be chosen to best reproduce the experimental results

$$H = \sum_i \frac{p_i^2}{2m} + \sum_{i<j} v_{ij} + \dots \longrightarrow H = \sum_i \frac{p_i^2}{2m} + \sum_i^A U_i + H_{\rm res}$$

- We solve the Schrödinger Equation:

$$H\,\psi = E\,\psi \qquad \begin{cases} E = E_1 + E_2 + \ldots + E_A \\ \psi(1, \ldots, A) = \mathcal{A}[\phi_1(1)\ldots\phi_A(A)] \end{cases}$$

Initial state: shell Model

- Example: Particles are subject to an harmonic oscillator potential

$$U(r) = \frac{1}{2} m \omega^2 r^2$$

The frequency should be adapted to the mass number A

- We will seek solutions of the type $\psi(r) = \frac{u(r)}{r} Y_l^m(\theta, \phi)$ ← Spherical Harmonics

- Solving the Schrödinger Equation reduces to a solution of u:

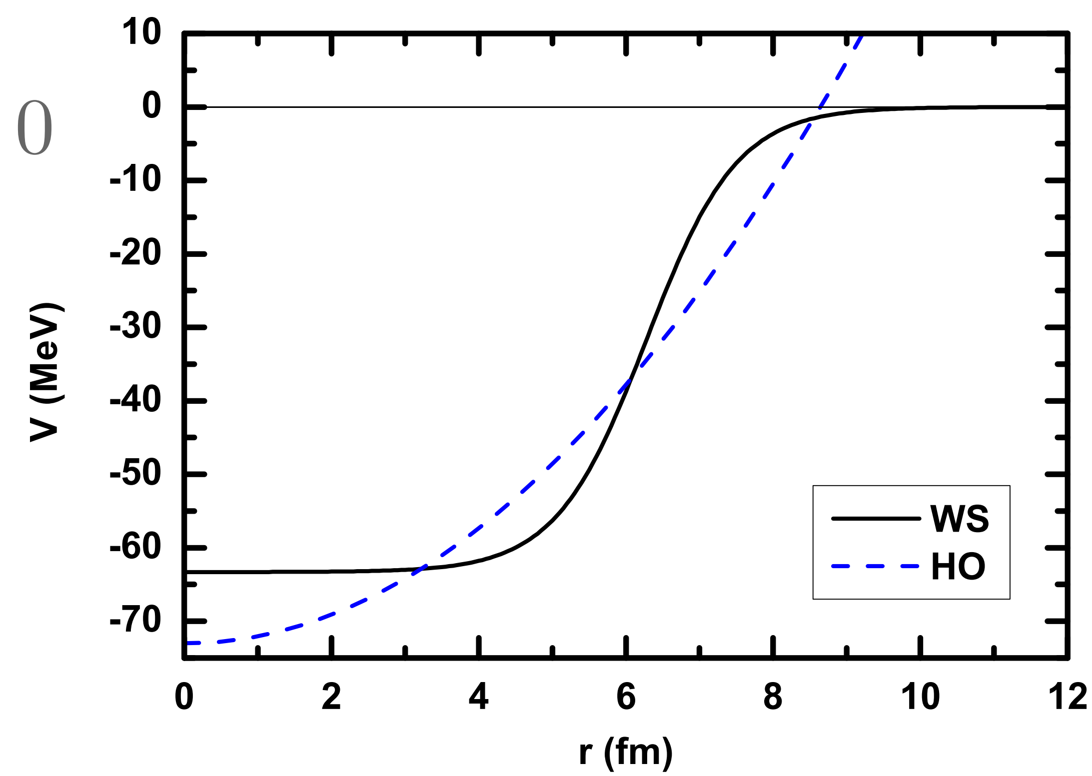
$$\frac{d^2 u}{dr^2} + \left\{ \frac{2m}{\hbar^2}[E - U(r)] - \frac{l(l+1)}{r^2} \right\} u(r) = 0$$

$$E_{nl} = \hbar\omega\left(2n + l - \frac{1}{2}\right)$$ Eigenvalues

- A more realistic potential is the Wood Saxon:

$$V = V_0/[1 + exp[(r - R)/a]$$

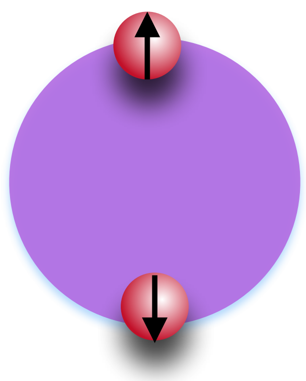
V_0, r, a, are adjustable parameters chosen to best reproduce the **experimental results**



Physical Review C 87(1):014334

Nuclear Shell Model

The lowest level, s shell, can contain 2 protons



1s (m_s=0)   Z=2

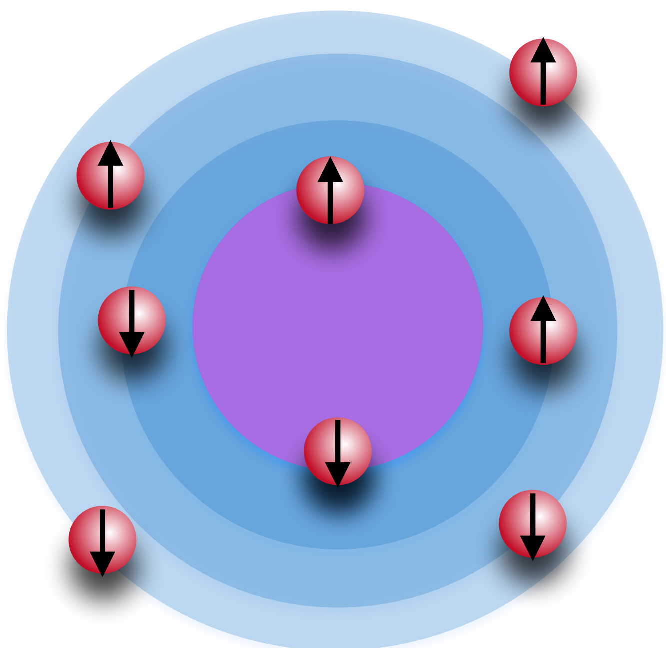
n is the principal quantum number, **l** orbital momentum, **m** magnetic quantum number

Our assumption: central potential

$$H = \sum_i \frac{p_i^2}{2m} + \sum_i V(r_i)$$

Nuclear Shell Model

The p shell can contain up to 6 protons



1s (m_s=0)
1p (m_p=-1,0,1)
Z=8

Our assumption: central potential

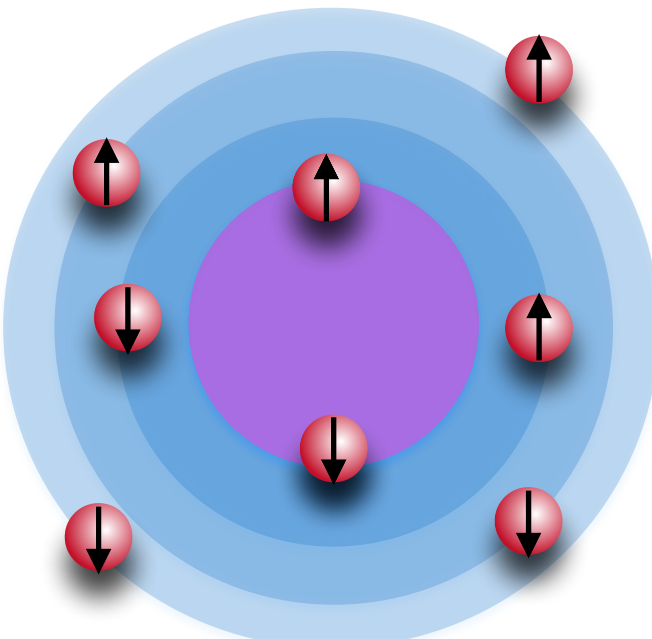
$$H = \sum_i \frac{p_i^2}{2m} + \sum_i V(r_i)$$

We explained the first two magic numbers: 2 and 8. We can follow the same strategy for the Z=20 case; but at the next step we obtain Z=40 **while experimentally Z=50**

n is the principal quantum number, **l** orbital momentum, **m** magnetic quantum number

Nuclear Shell Model

The p shell can contain up to 6 protons



1s (m_s=0)
1p (m_p=-1,0,1)
Z=8

✐ Maria Goeppert Mayer poses with her colleagues in front of Argonne's Physics building.

Our assumption: central potential

$$H = \sum_i \frac{p_i^2}{2m} + \sum_i V(r_i)$$

We explained the first two magic numbers: 2 and 8. We can follow the same strategy for the Z=20 case; but at the next step we obtain Z=40 **while experimentally Z=50**

In 1963, Goeppert Mayer, Jensen, and Wigner shared the Nobel Prize for Physics "for their discoveries concerning nuclear shell structure."

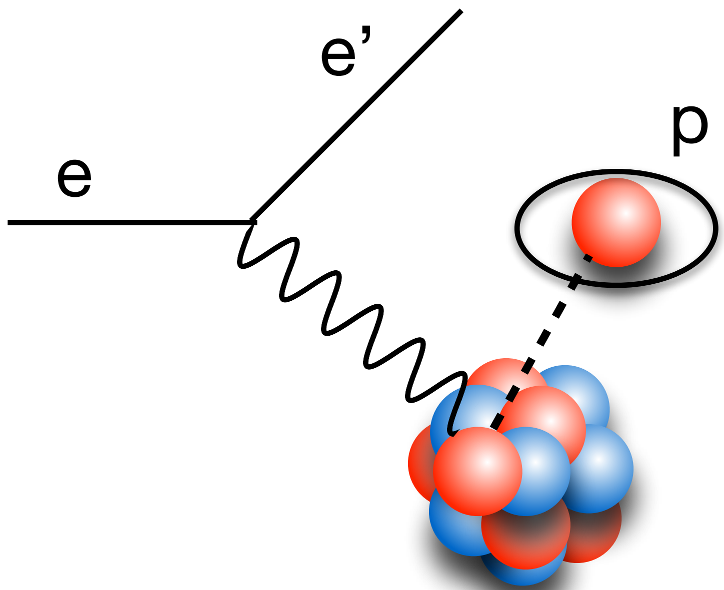
The solution to the puzzle lies in the **spin-orbit coupling**. This effect in the nuclear potential is 20 times larger then in Atomic Physics

$$V(r) \rightarrow V(r) + W(r)\mathbf{L}\cdot\mathbf{S}$$

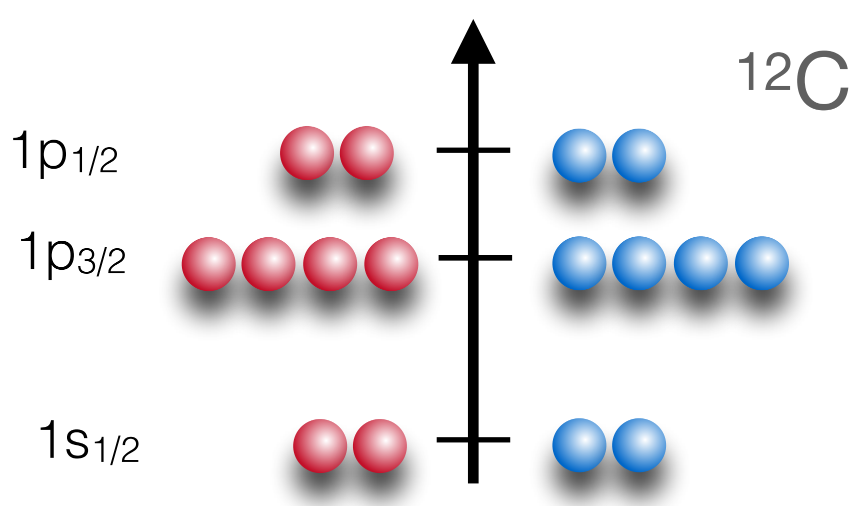
The spin-orbit introduces an energy split which modifies the shell structure and reproduces magic number up to Z=126

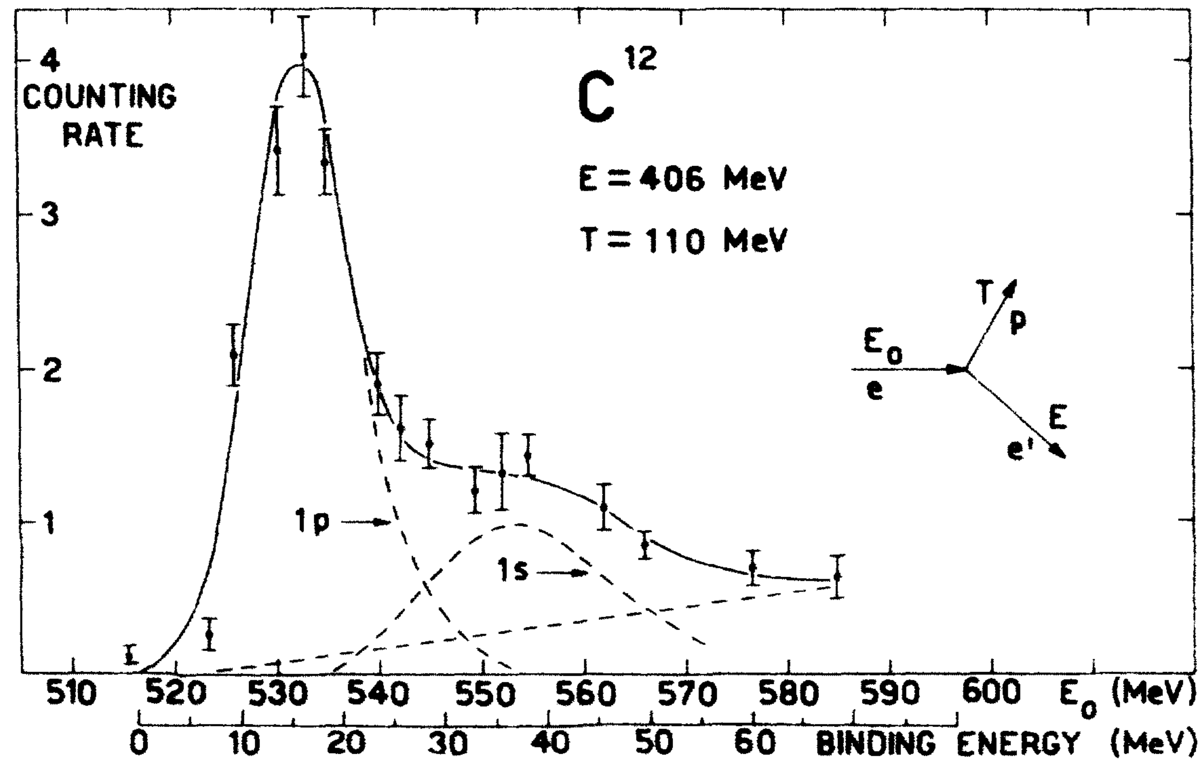
(e,e'p) scattering experiments

- (e,e'p) experiments are extremely important to investigate the internal structure of the nucleus



- Assuming NO FSI the energy and momentum of the initial nucleon can be identified with the measured p_{miss} and E_{miss}





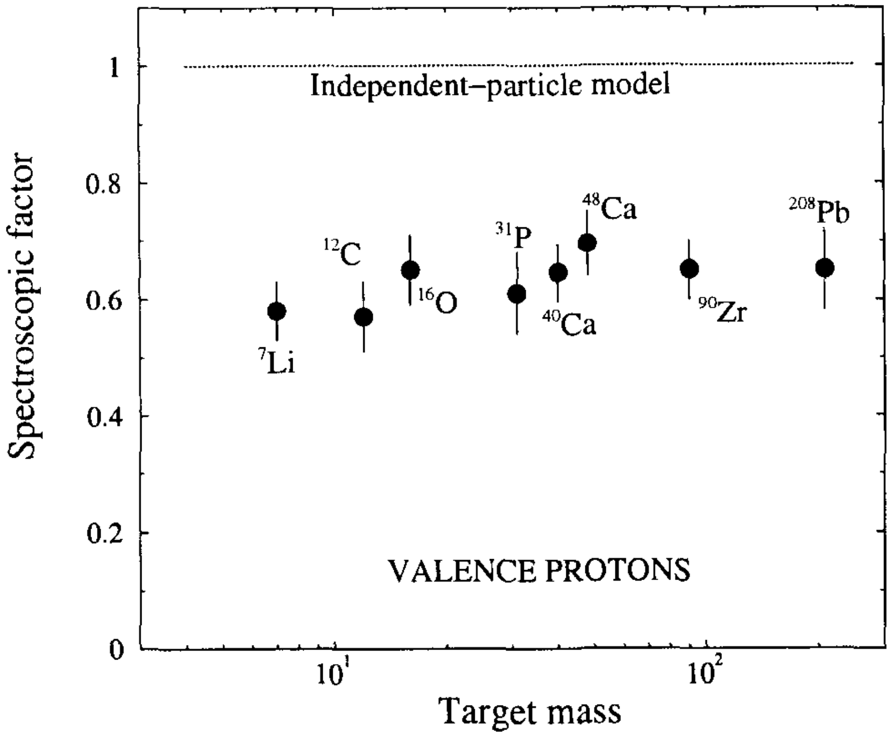
U.Amaldi et al, Phys. Rev. Lett. 13, 10 (1964)

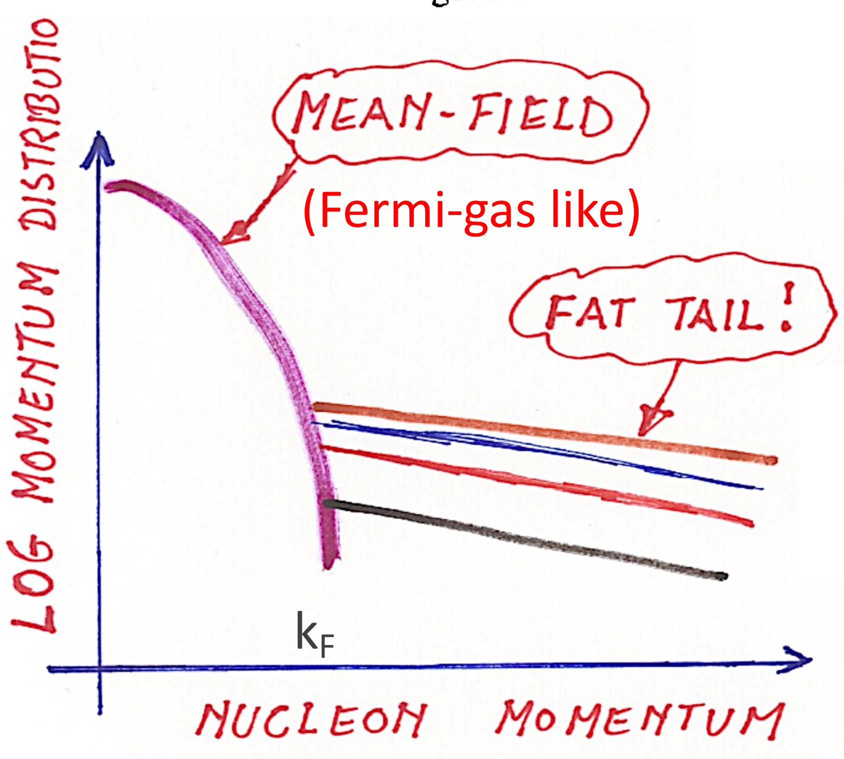
- The peak coming from four 1p protons is visible

- The contribution of the two 1s protons is not clearly separated with this resolution

(e,e'p) scattering experiments

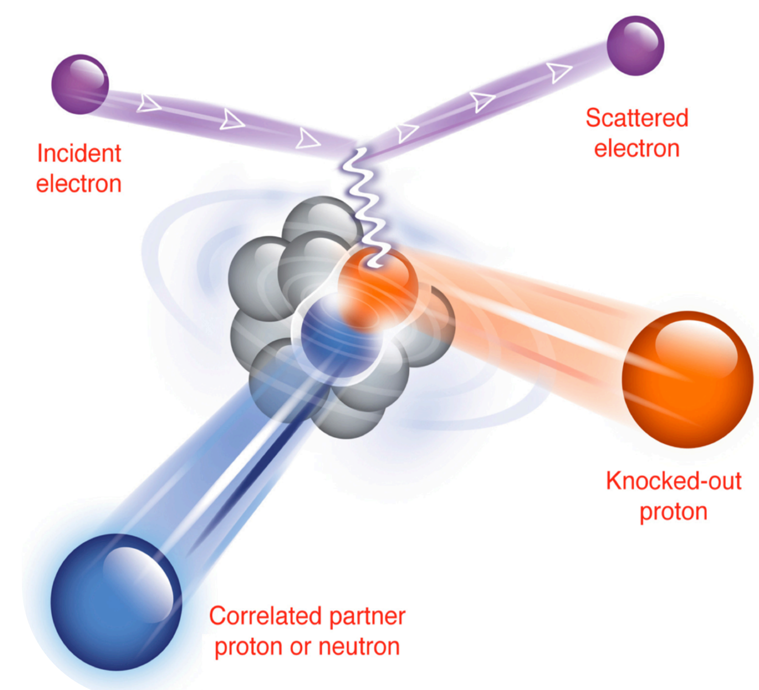
- Electron and proton experiments also pinned down the limitations of MF approaches





- Quenching of the spectroscopic factors of valence states has been confirmed by a number of high resolution (e,e'p) experiments

Subedi et al., Science 320, 1476 (2008)



- **Semi-exclusive 2N-SRC** experiments at x>1 allows to detect both nucleons and reconstruct the initial state

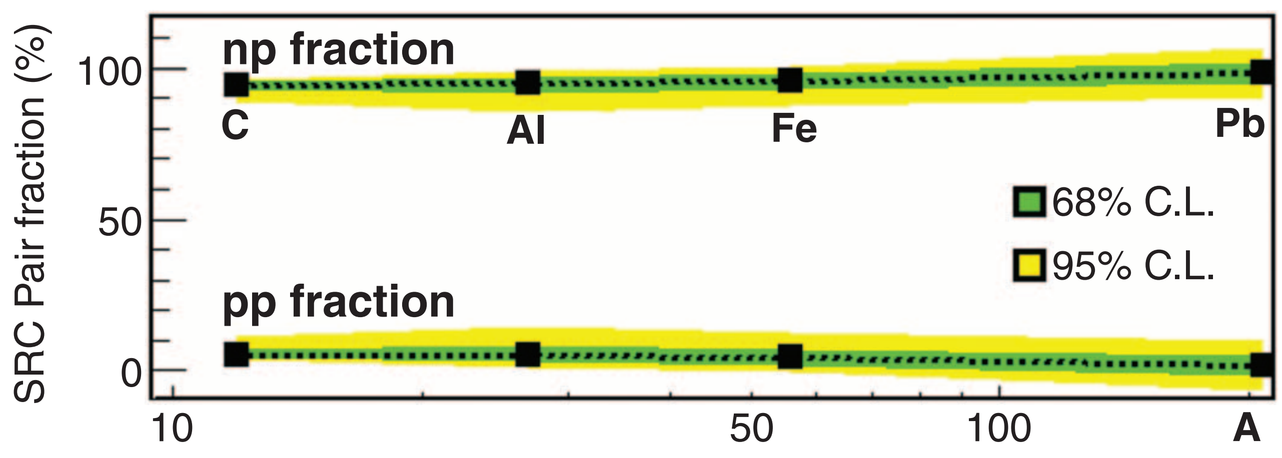
- Confirmed that the **high momentum tail** of the nuclear wave function consists mainly of 2N-SRC

- The large-momentum (short-range) component of the wave function is dominated by the presence of Short Range Correlated (SRC) pairs of nucleons

Figure by Or Hen

(e,e'p) scattering experiments

- Observed dominance of np-over-pp pairs for a variety of nuclei



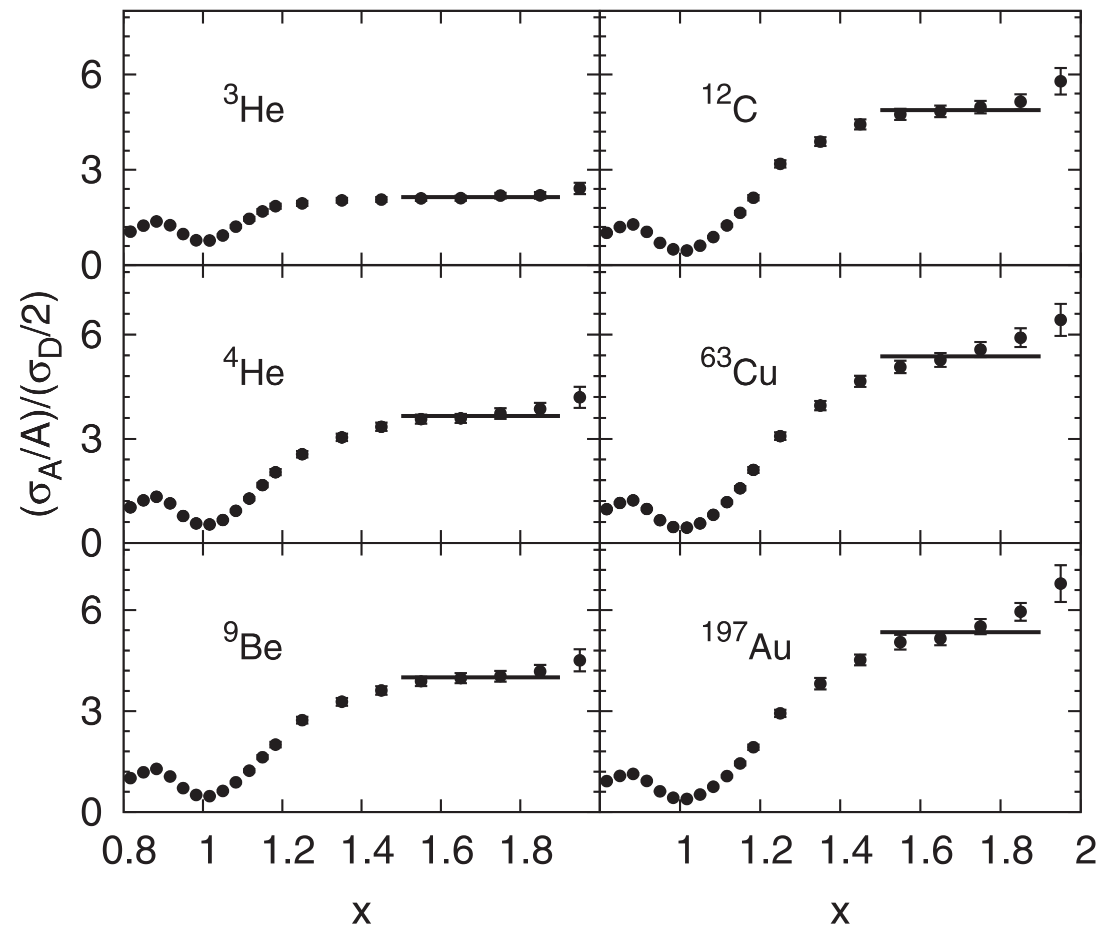
Subedi et al., Science 320, 1476 (2008)

- SRC pairs are in spin-triplet state, a consequence of the tensor part of the nucleon-nucleon interaction

Bottom Line

- Two-body Physics can not be neglected:
  - ~20% of the nucleons in nuclei
  - ~100% of the high k (>pF) nucleons
- Have large relative momentum and low center of mass momentum

- Universality of high-momentum component



N. Fomin et al., PRL 108, 092502 (2012)

- The cross section ratio: A/d, sensitive to $n_A(k)/n_d(k)$
- Observed scaling for x>1.5

$$n_A(k > p_F) = a_2(A) \times n_d(k)$$

Figure by Or Hen