Lepton-Nucleus Interactions — Part II

International Neutrino Summer School
Fermilab, August 10, 2023

Noemi Rocco

Summary of electron-nucleon scattering

- We consider the process:

$$\ell^-(k) + N(p) \to \ell^-(k') + N(p')$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Mott}} = \frac{\alpha^2}{4E_k^2 \sin^4 \theta/2} \cos^2 \frac{\theta}{2}$$

Scattering on a point-like spinless target

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Mott}} \left[1 - \frac{q^2}{2M^2} \tan^2 \frac{\theta}{2}\right]$$

Scattering on a point-like 1/2 spin target

- Protons and neutrons have an internal structure: described by electric and magnetic form factors

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Mott}} \left[\frac{G_E^2 - \frac{q^2}{4M^2}G_M^2}{1 - \frac{q^2}{4M^2}} - \frac{q^2}{2M^2}G_M^2 \tan^2 \frac{\theta}{2}\right]$$

Rosenbluth separation

Fermilab

Noemi Rocco, nrocco@fnal.gov

Determination of nucleon form factors

- A reduced cross section can be defined as

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \times \frac{\epsilon G_E^2 + \tau G_M^2}{\epsilon(1+\tau)}$$
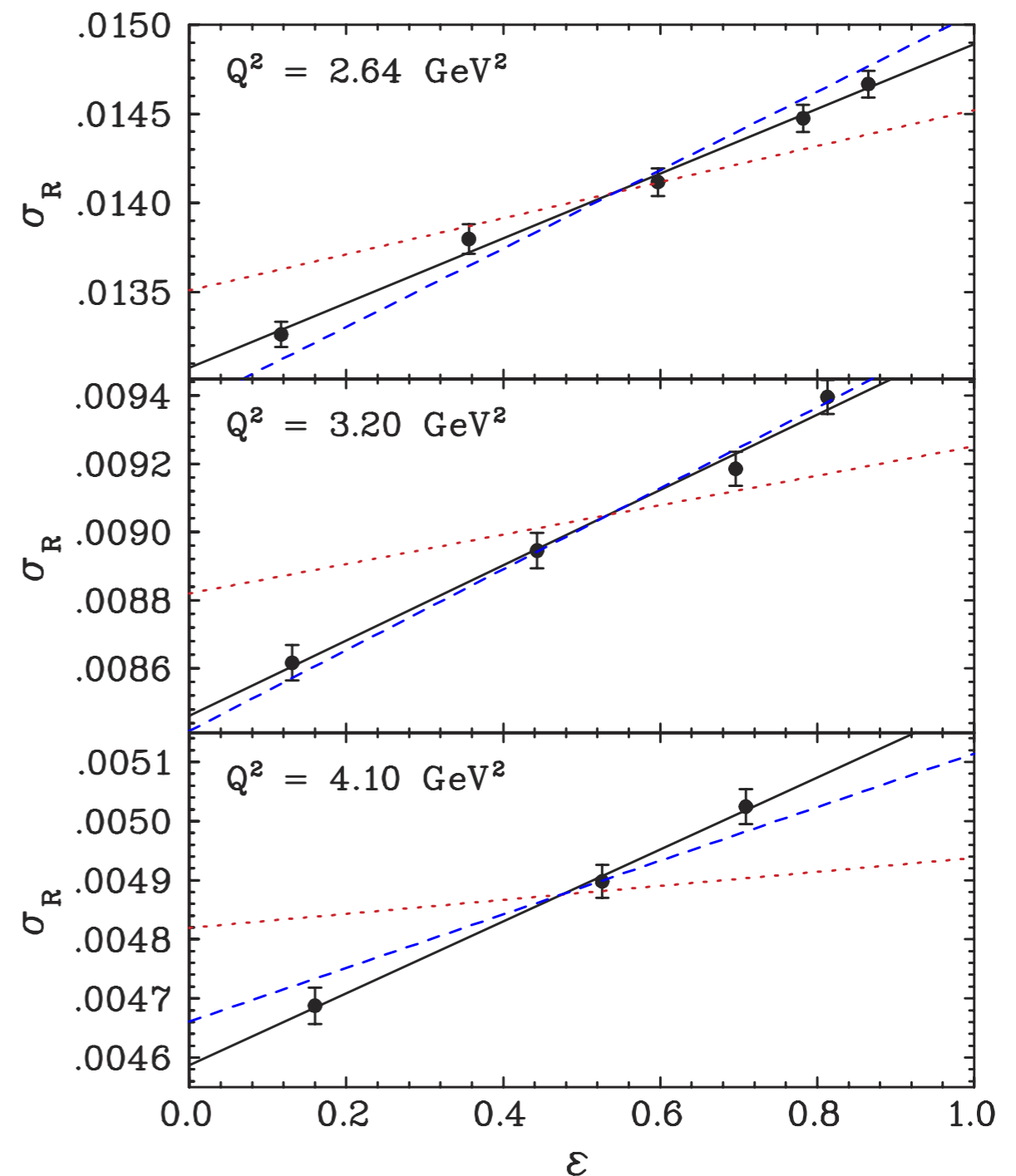
- The virtual photon polarization parameter is

$$\epsilon = \left[1 + 2(\tau+1)\tan^2\frac{\theta}{2}\right]^1$$

- Measuring angular dependence of the cross section at fixed Q^2

$$\sigma_R = \epsilon(1+\tau)\frac{\sigma}{\sigma_{\text{Mott}}} = \epsilon G_E^2 + \tau G_M^2$$

- In Born approximation: G_E^2 is the slope and the intercept is $\tau\, G_M^2$

✎ Qattan et al.,PRL 94, 142301 (2005)

$\cdots$ polarization transfer experiment

$---$ previous Rosenbluth
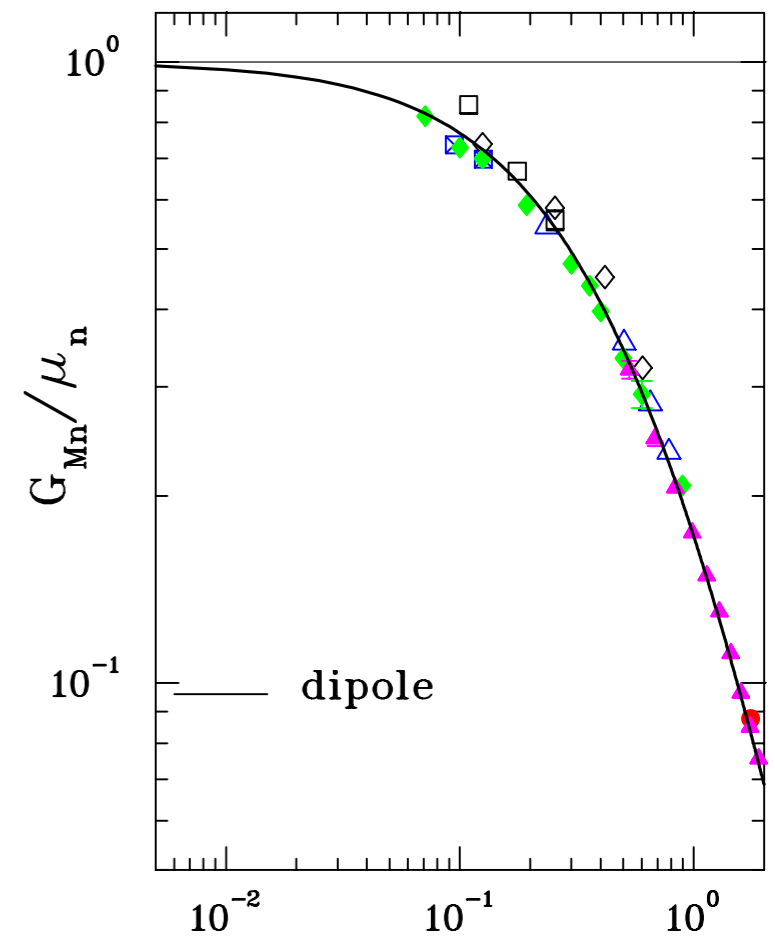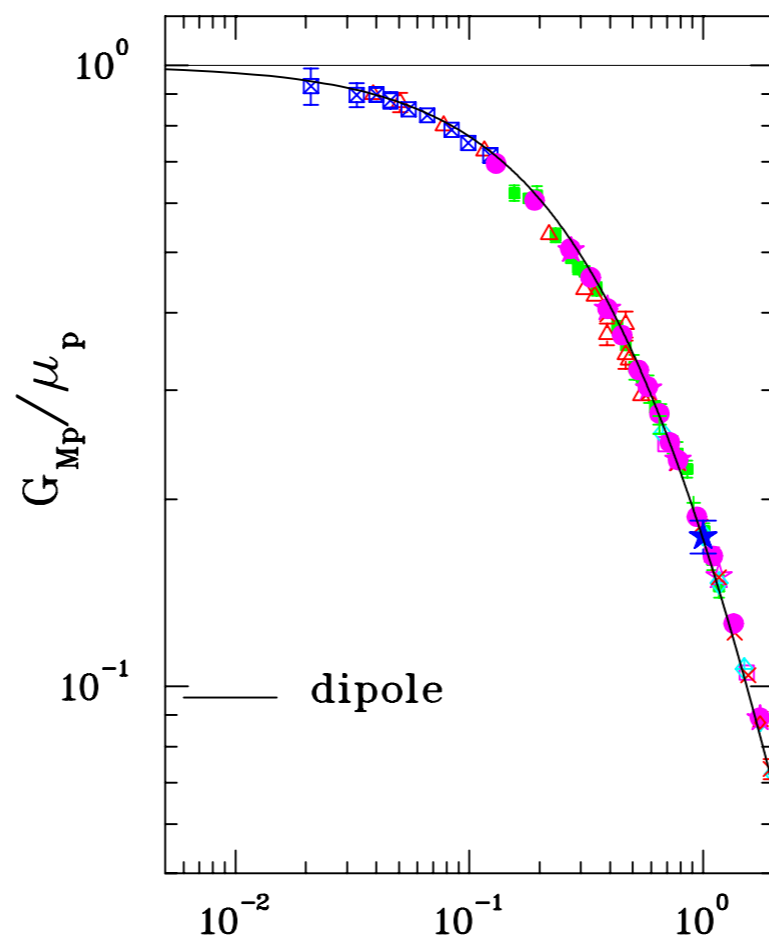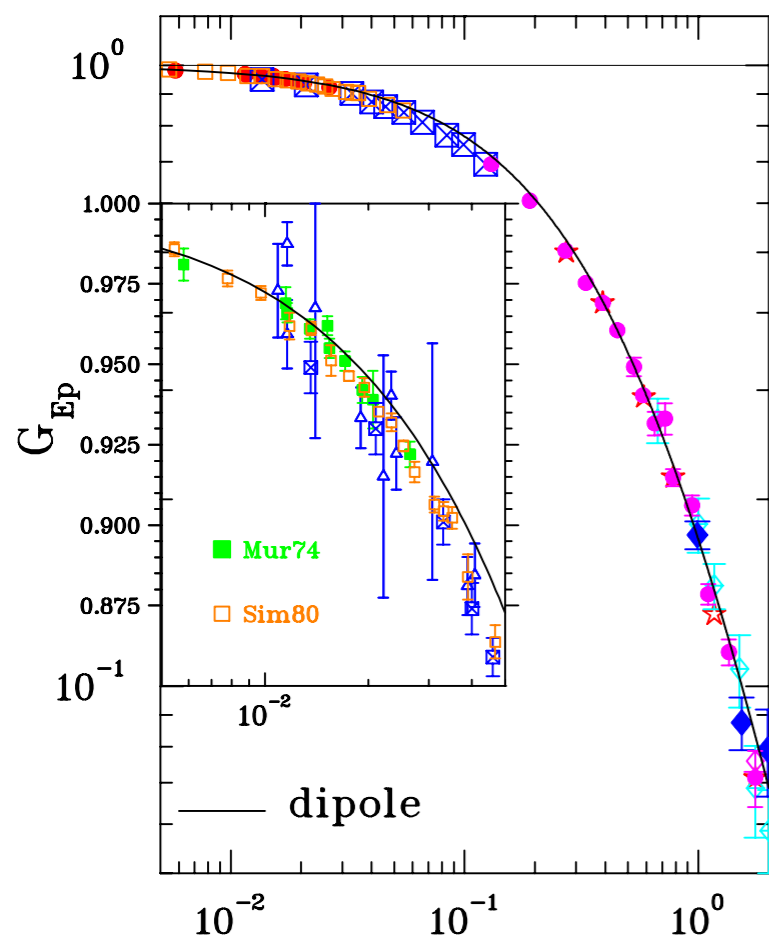
Noemi Rocco, nrocco@fnal.gov

Determination of nucleon form factors

$$G_E^p(q^2) = G_D(q^2) \ , \ G_M^p(q^2) = \mu_p G_D(q^2) \ , \ G_M^n(q^2) = \mu_n G_D(q^2) \ , \ G_D = \left(1 - \frac{q^2}{M_V^2}\right)^{-2}$$

$$\mu_p = 2.793, \ \mu_n = -1.913,$$
$$M_V^2 = 0.71 \ \text{GeV}^2$$

✎ Perdrisat et al., Prog.Part.Nucl.Phys. 59 (2007) 694–764

Lattice QCD in a Nutshell

- Lattice QCD is a technique in which space-time is discretized into a four-dimensional grid

- The QCD path integral over the quark and gluon fields at each point in the grid is performed in Euclidean space-time using Monte Carlo methods

- The quark mass is a parameter of the calculation : m_π

  - Space time is discretized, step a: UV cutoff

  - Finite volume L : infrared cutoff

- The physical limit is recovered by imposing that

$$m_\pi \to m_\pi{}^{phys} \, , \, a \to 0, \, L \to \infty$$

quark field

gluon field

x, y, z

t

a

L
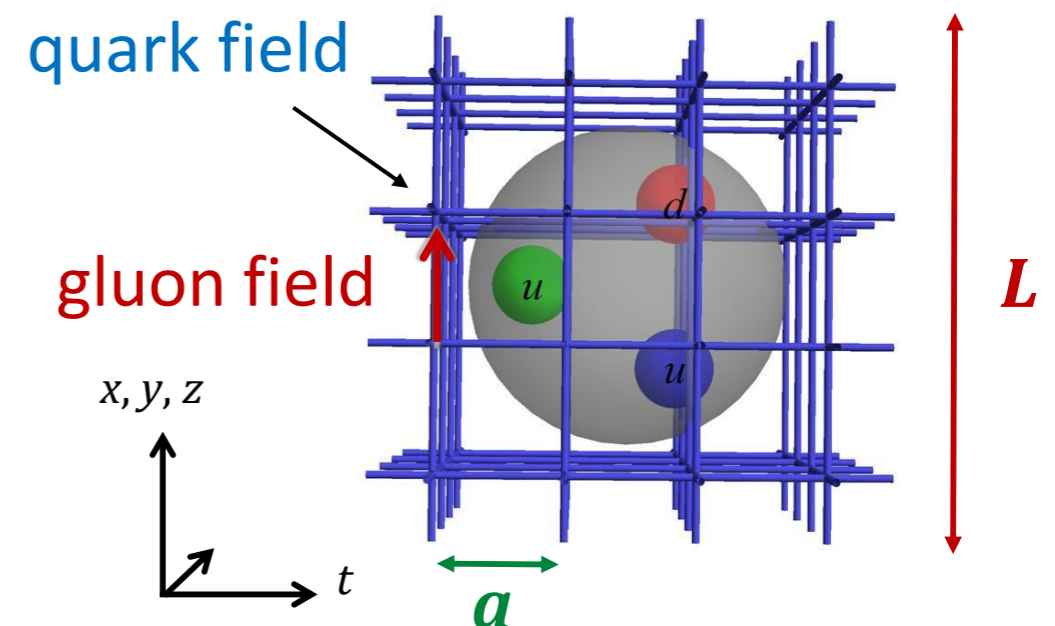
Figure by Huey-Wen Lin

Noemi Rocco, nrocco@fnal.gov

‡ Fermilab

Lattice QCD form factors
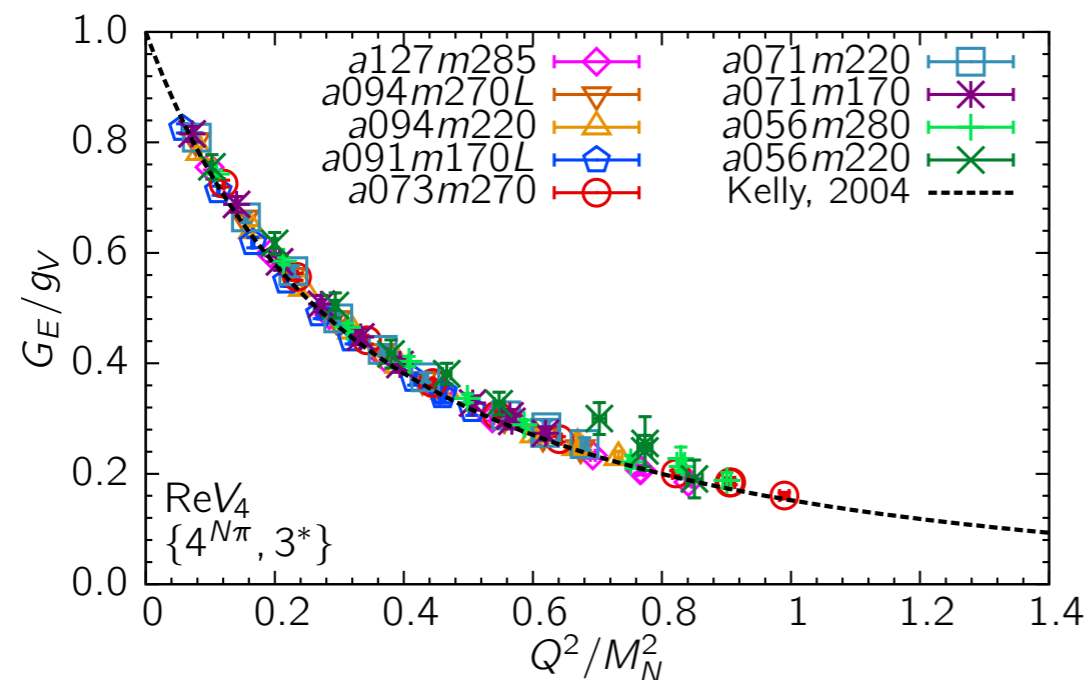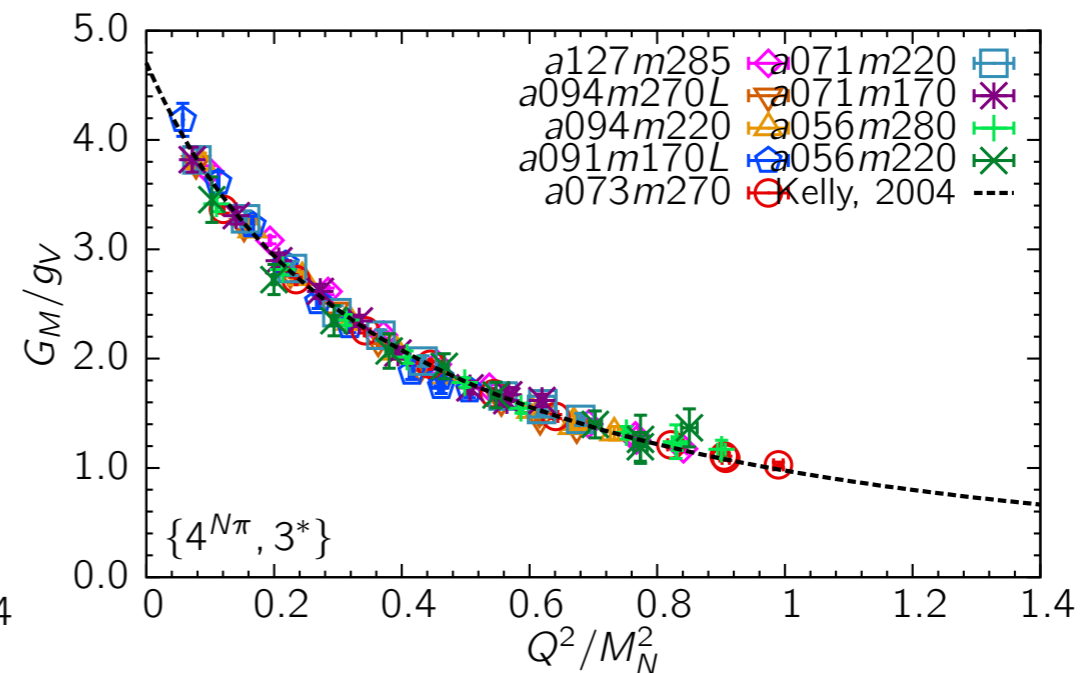
Electric

Magnetic

- Isovector electric and magnetic form factors

- The form factors do not show significant dependence on the lattice spacing or the quark mass

- Good agreement with the Kelly curve. Validates the lattice methodology

R. Gupta, Introduction to Lattice QCD, arXiv:hep-lat/9807028

Noemi Rocco, nrocco@fnal.gov

Fermilab

Neutrino-Nucleon scattering

Charge Current (CC)

Neutral Current (NC)
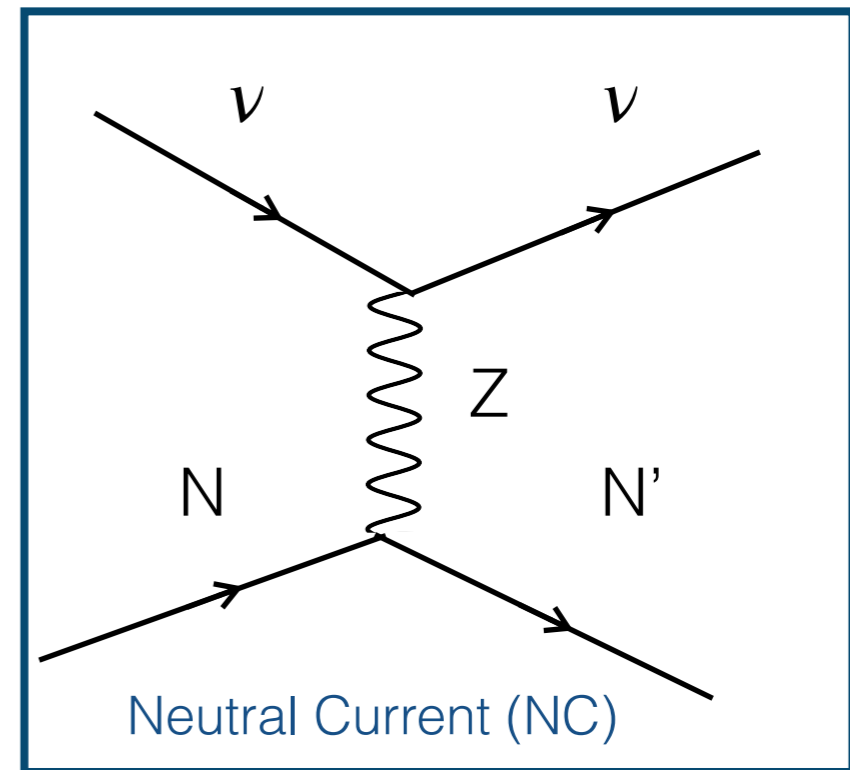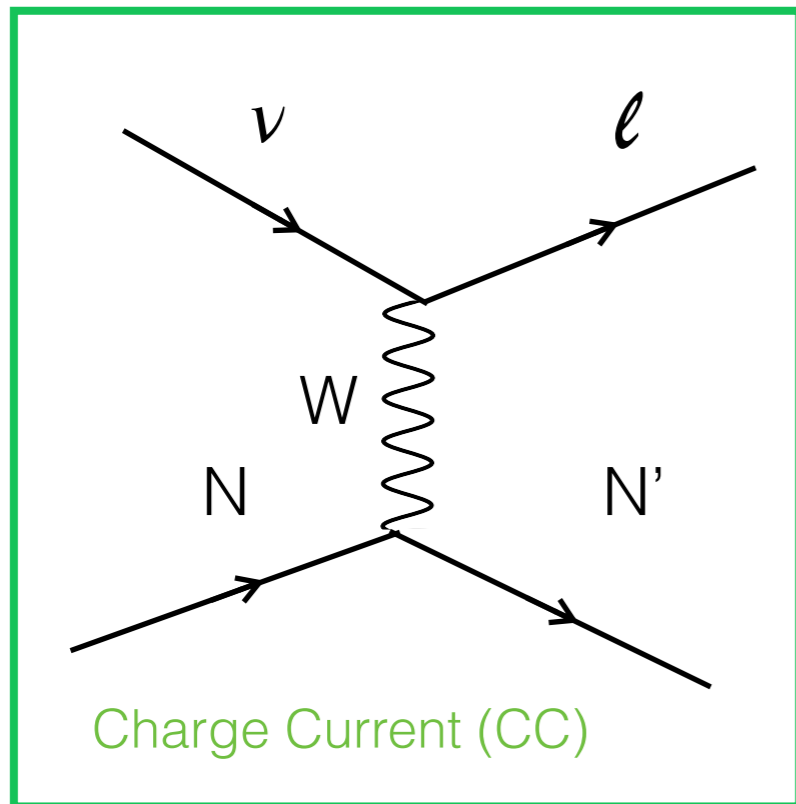
- Exchange of the W boson

- Lepton produced has the same flavor of the neutrino

- Initial and final nucleon have different isospin

- Exchange of the Z boson

- Independent of the neutrino flavor

- Initial and final nucleon have same isospin

F. Close, An Introduction to quark and partons

Fermilab

Noemi Rocco, nrocco@fnal.gov

Neutrino-Nucleon scattering

- Differential cross section for CC and NC processes

$$\frac{d^2\sigma}{dE' d\Omega'} = \frac{1}{16\pi^2}\frac{G^2}{2}L_{\mu\nu}W^{\mu\nu}$$
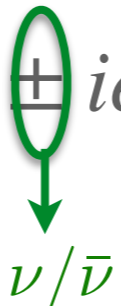
- For NC

$$G = G_F$$

- For CC

$$G = G_F \cos\theta_c$$

$$G_F = 1.1803 \times 10^{-5}~\mathrm{GeV}^{-2}~,~\cos\theta_c = 0.97425$$

- Leptonic Tensor:

$$L_{\mu\nu} = 2[k_\mu k'_\nu + k'_\mu k_\nu - g_{\mu\nu} k\cdot k' \pm i\epsilon_{\mu\nu\alpha\beta}k'^\alpha k^\beta]$$
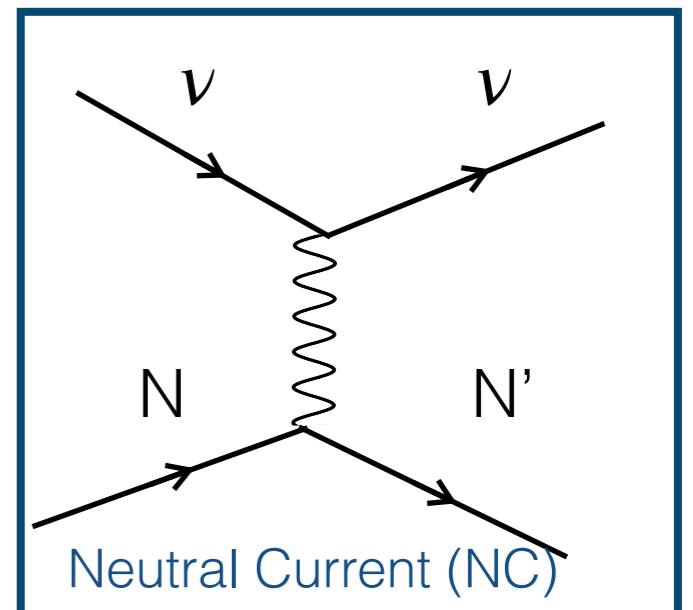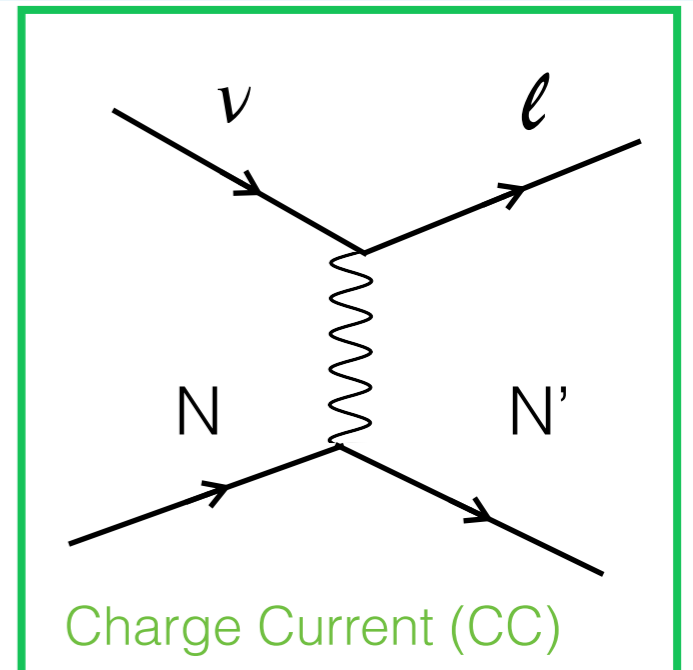
$$\nu/\bar\nu$$

- Hadronic Tensor:

$$W_{\mu\nu} = \sum_{\sigma_i\sigma_f}\frac{1}{2E_p}\int\frac{d^3p'}{2E_{p'}}\langle N(p)|J^\mu|N'(p')\rangle\langle N'(p')|J^\nu|N(p)\rangle$$

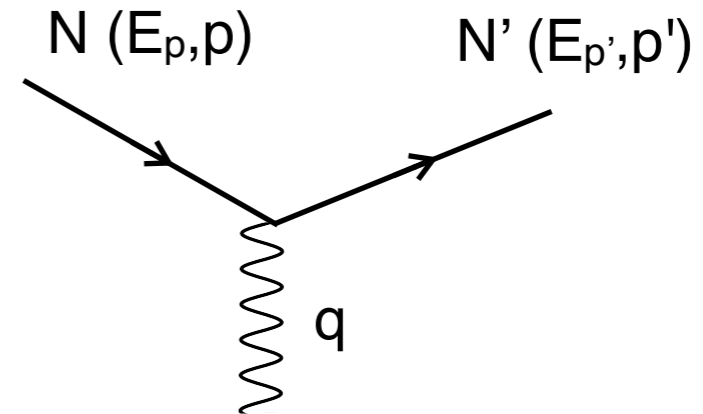$$\times\,\delta^{(4)}(p' + k' - p - k)$$

Charge Current (CC)

Neutral Current (NC)

Noemi Rocco, nrocco@fnal.gov

Neutrino-Nucleon scattering

- Electroweak current operator:

$$\langle N'|J^\mu|N\rangle = \bar{u}(p')\Gamma^\mu u(p) = J_V^\mu + J_A^\mu$$

Vector   Axial

N (E_p, p) N' $(E_{p'}, p')$

q

- The vector contribution is given by:

$$J_V^\mu = \mathcal{F}_1 \gamma^\mu + i\sigma^{\mu\nu} q_\nu \frac{\mathcal{F}_2}{2M}$$

- The axial contribution is given by:

$$J_A^\mu = -\gamma^\mu \gamma_5 \mathcal{F}_A - q^\mu \gamma_5 \frac{\mathcal{F}_P}{M}$$

- General expression for both neutral- and charge current processes. The iso-spin dependence of these form factors is different (see next slide).

- The Vector current is the same of the electromagnetic: Conserved Vector Current hypothesis

T. Leitner, O. Buss, L. Alvarez-Ruso, and U. Mosel, Electron- and neutrino-nucleus scattering from the quasielastic to the resonance region, Phys. Rev. C 79, 034601 (2009).
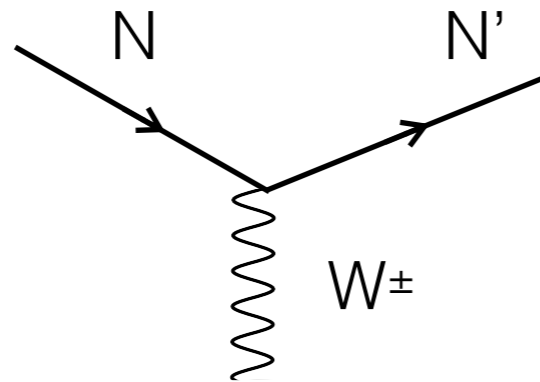
Noemi Rocco, nrocco@fnal.gov

Neutrino-Nucleon scattering
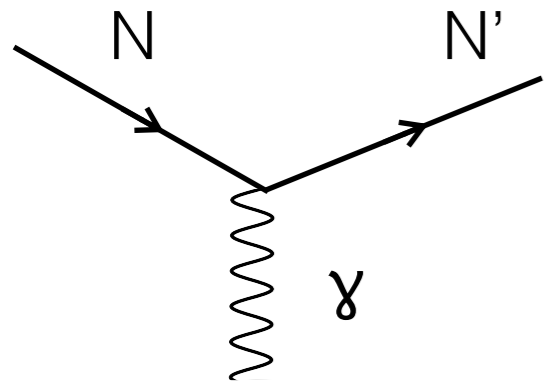
- EM

$$\mathcal{F}_1 = \frac{1}{2}[F_1^S + F_1^V \tau_z]$$
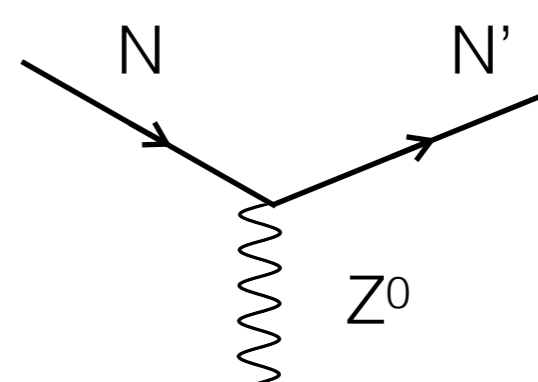
$$\mathcal{F}_2 = \frac{1}{2}[F_2^S + F_2^V \tau_z]$$

- CC

$$\mathcal{F}_1 = F_1^V \tau_\pm$$

$$\mathcal{F}_2 = F_2^V \tau_\pm$$

$$\mathcal{F}_A = F_A \tau_\pm$$

$$\mathcal{F}_P = F_P \tau_\pm$$

- NC

$$\mathcal{F}_1 = \frac{1}{2}[-2sin^2\theta_W F_1^S + (1 - 2sin^2\theta_W)F_1^V \tau_z]$$

$$\mathcal{F}_2 = \frac{1}{2}[-2sin^2\theta_W F_2^S + (1 - 2sin^2\theta_W)F_2^V \tau_z]$$

$$\mathcal{F}_A = \frac{1}{2}F_A \tau_z$$

$$\mathcal{F}_P = \frac{1}{2}F_p \tau_z$$

- We used the Conserved Vector Current hypothesis:

$$F_1^V \tau_z \rightarrow F_1^V \tau_\pm \ , \ F_2^V \tau_z \rightarrow F_2^V \tau_\pm$$

- PCAC:

$$F_P = \frac{2m_N^2}{(m_\pi^2 - q^2)}F_A$$

Axial form factor determination

- The axial form-factor has been fit to the dipole form

$$F_A(q^2) = \frac{g_A}{(1 - q^2/m_A^2)^2}$$

- The intercept $g_A = -1.2723$ is known from neutron β decay

- Different values of m_A from experiments
  - $m_A = 1.02$ GeV q.e. scattering from deuterium
  - $m_A = 1.35$ GeV @ MiniBooNE

- Alternative derivation based on **z-expansion** —model independent parametrization

$$F_A(q^2) = \sum_{k=0}^{k_{max}} a_k z(q^2)^k,$$

known functions

free parameters

Bhattacharya, Hill, and Paz PRD 84 (2011) 073006

A.S.Meyer et al, Phys.Rev.D 93 (2016) 11, 113015
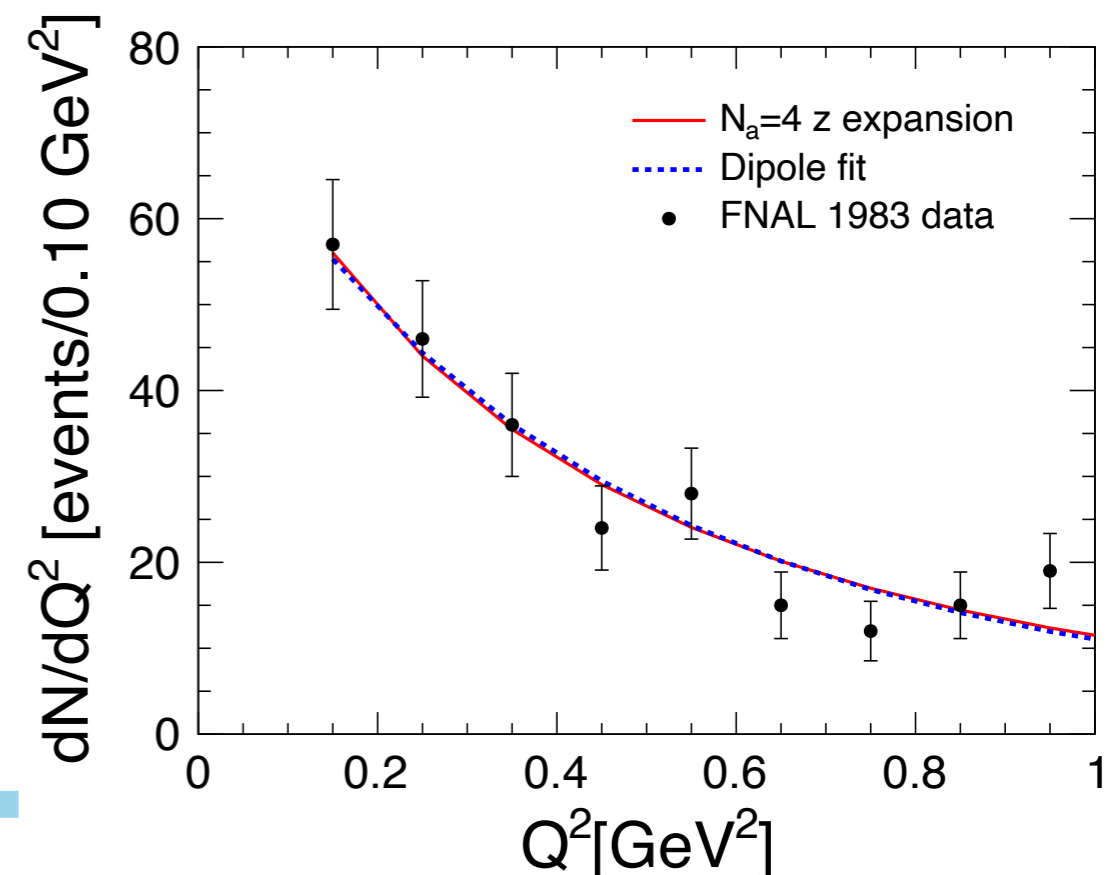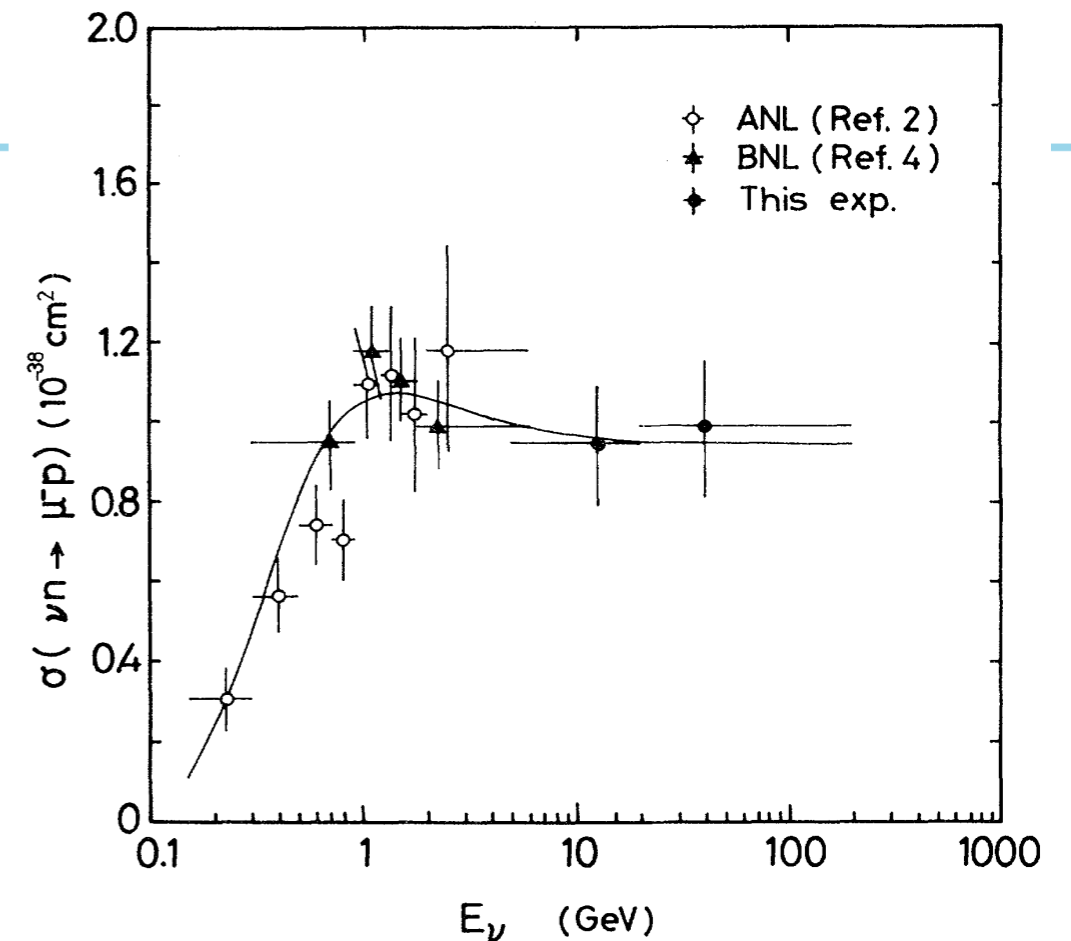
Neutrino-Nucleon scattering
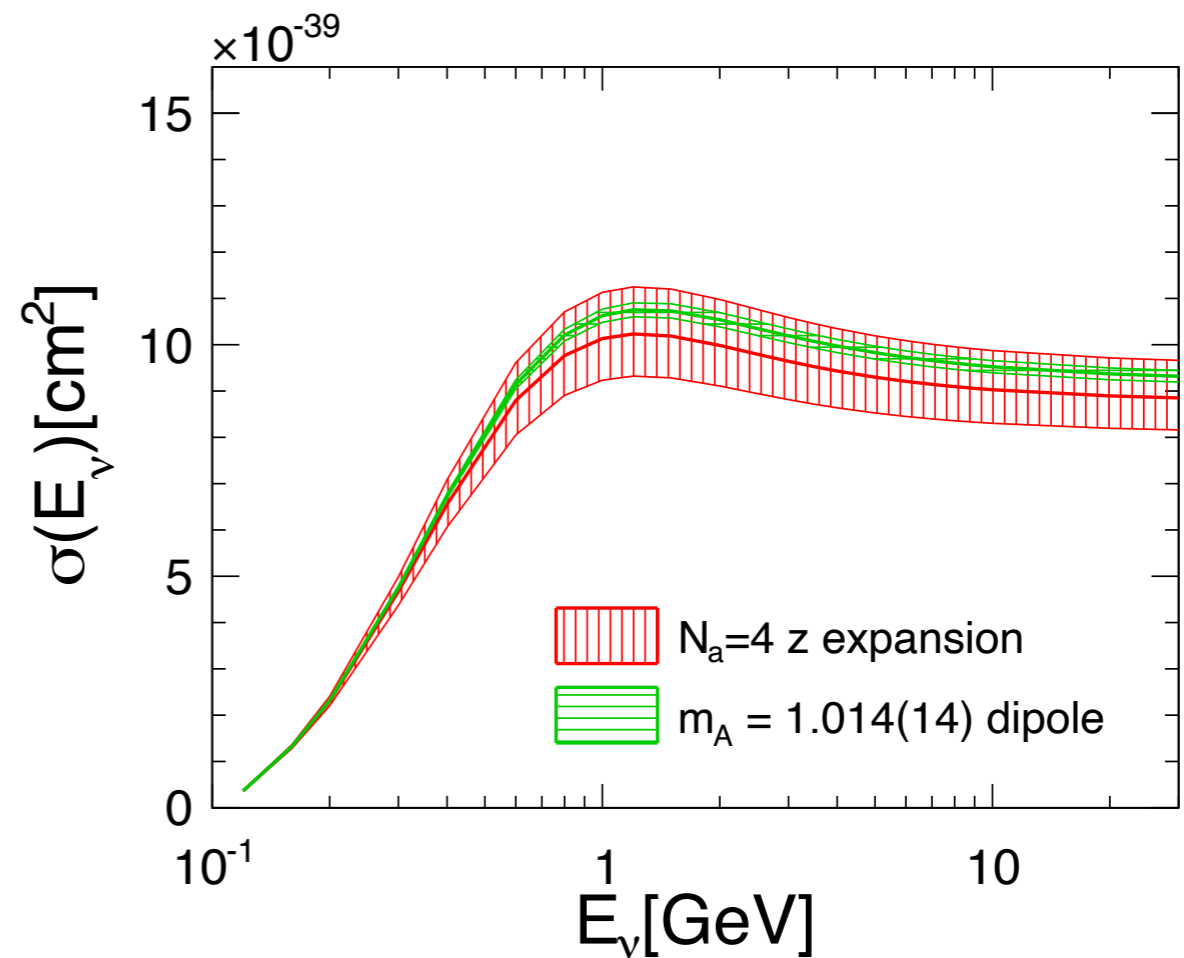
- Sum rule can be enforced ensuring that the form factor falls smoothly to zero at large Q^2

$$z(q^2, t_{\text{cut}}, t_0) = \frac{\sqrt{t_{\text{cut}} - q^2} - \sqrt{t_{\text{cut}} - t_0}}{\sqrt{t_{\text{cut}} - q^2} + \sqrt{t_{\text{cut}} - t_0}} \,, \qquad \sum_{k=n}^{\infty} k(k-1)\cdots(k-n+1)a_k = 0 \,, \quad n = 0, 1, 2, 3$$

Fit deuteron data replacing dipole axial form factor with z-expansion, enforce the sum rule constraints
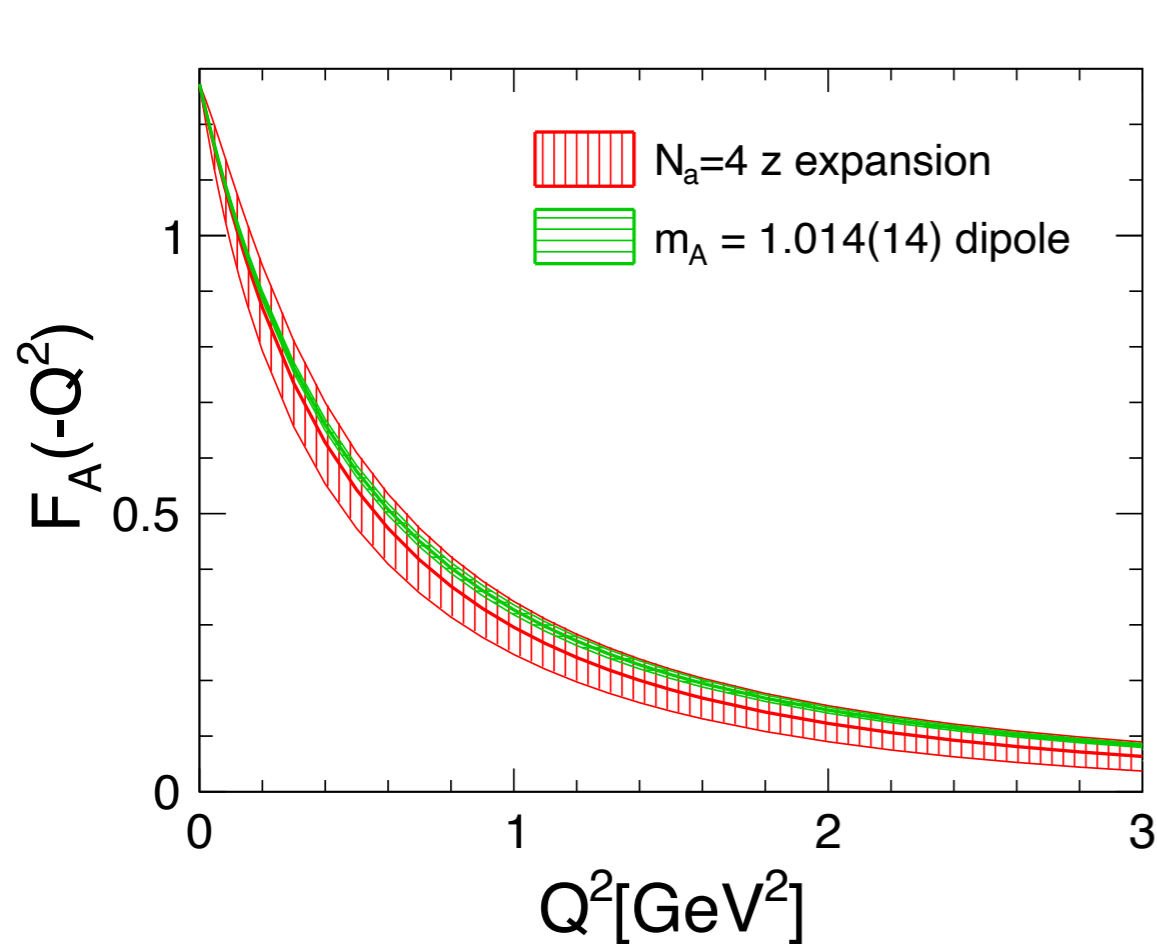
A.S.Meyer, Phys.Rev.D 93 (2016) 11, 113015

Axial form factor determination

D.Simons, N. Steinberg et al, 2210.02455

Legend:
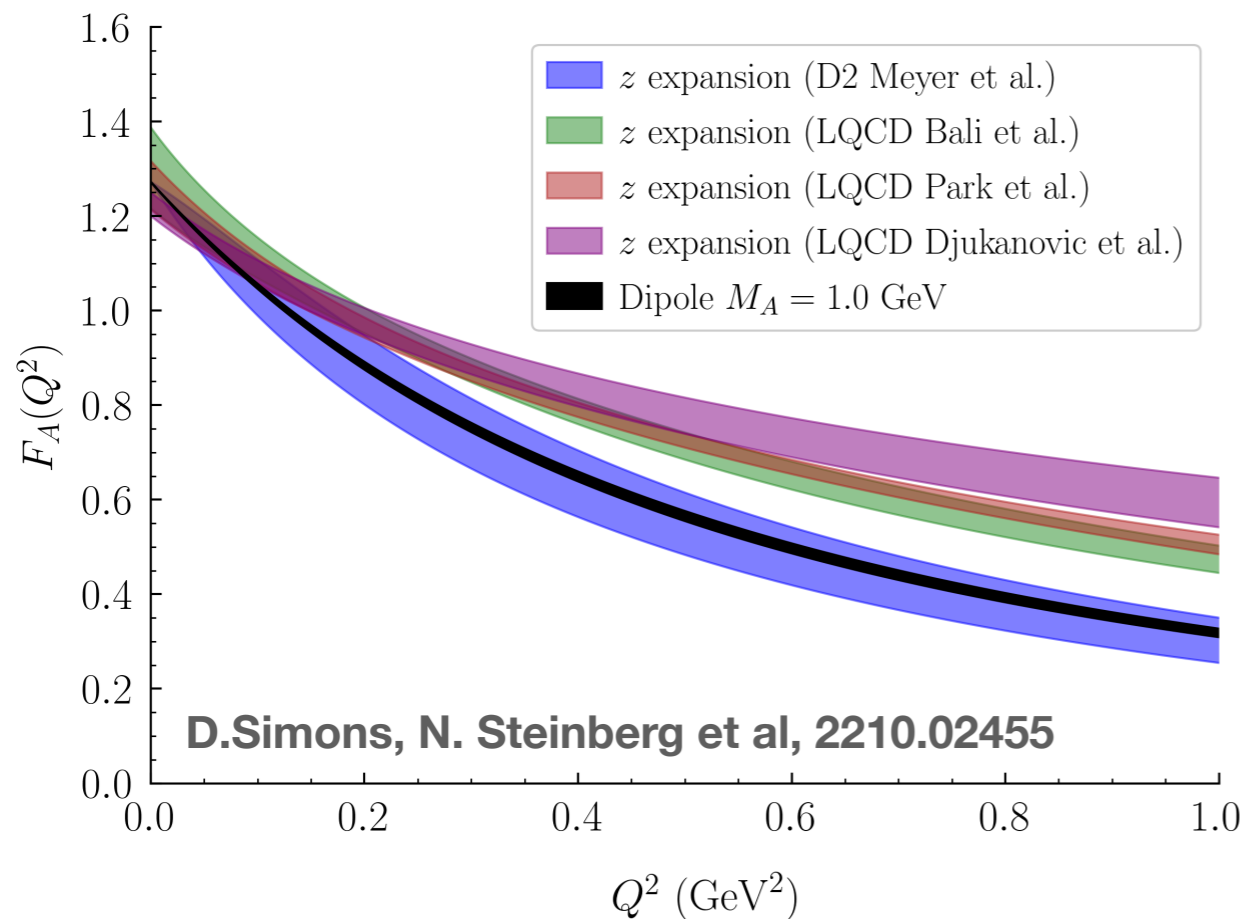- z expansion (D2 Meyer et al.)
- z expansion (LQCD Bali et al.)
- z expansion (LQCD Park et al.)
- z expansion (LQCD Djukanovic et al.)
- Dipole $M_A = 1.0$ GeV

Axes: $F_A(Q^2)$ vs Q^2 (GeV^2)

A. Meyer, A. Walker-Loud, C. Wilkinson, 2201.01839

D2 Meyer et al: fits to neutrino-deuteron scattering data

LQCD result: general agreement between the different calculations

LQCD results are 2-3σ larger than D2 Meyer ones for $Q^2 > 0.3$ GeV^2

O. Tomalak, R. Gupta, T. Battacharaya, 2307.14920

Legend:
- deuterium fit
- hydrogen fit
- PNDME

Axes: $G_A(Q^2)$ vs Q^2 [GeV^2]

Comparison with recent MINERvA antineutrino-hydrogen charged-current measurements

1-2σ agreement with MINERvA data and LQCD prediction by PNDME Collaboration

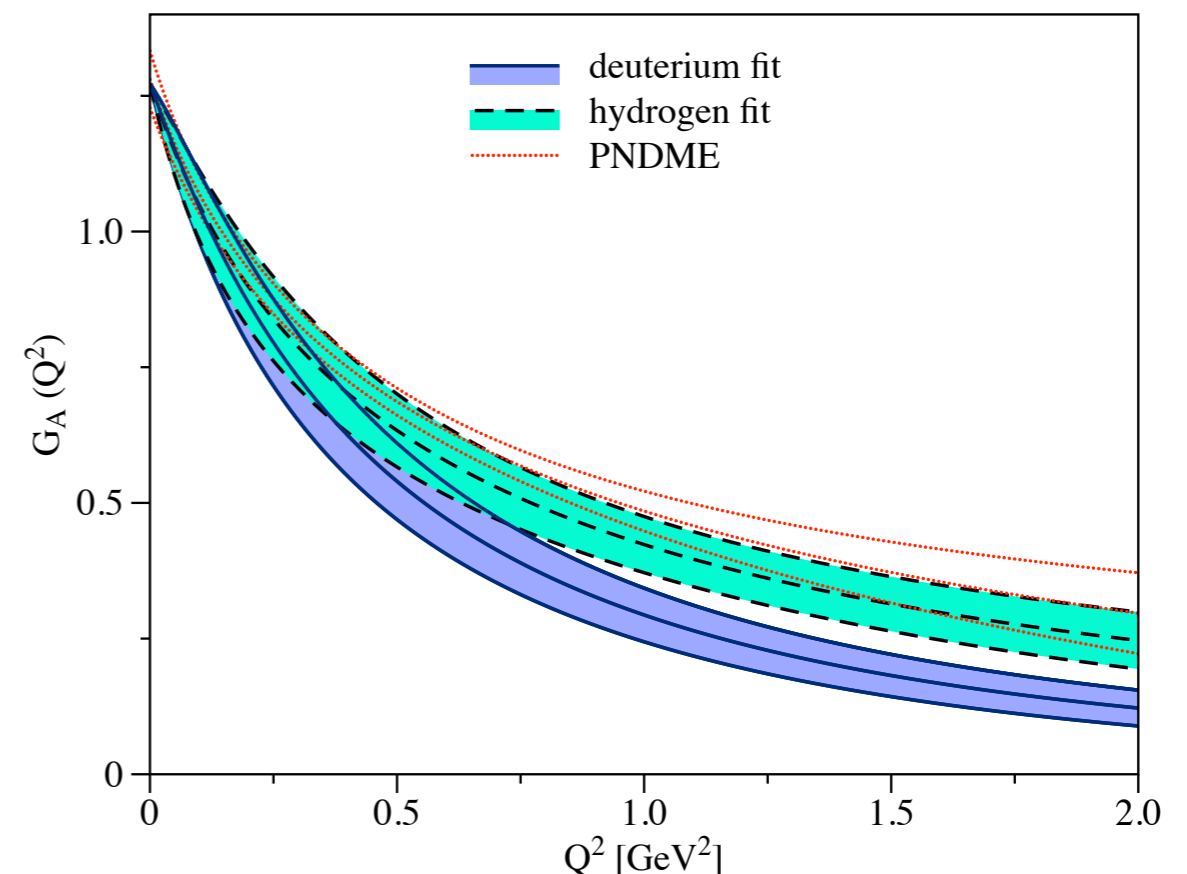Novel methods are needed to remove excited-state contributions and discretization errors

Fermilab
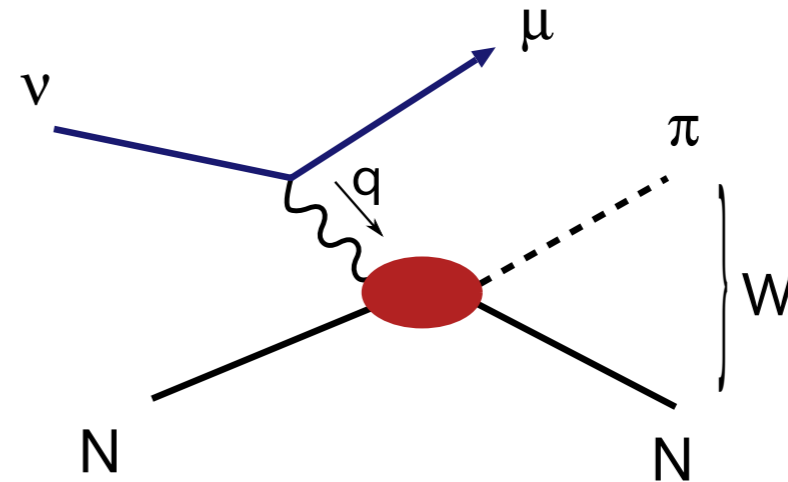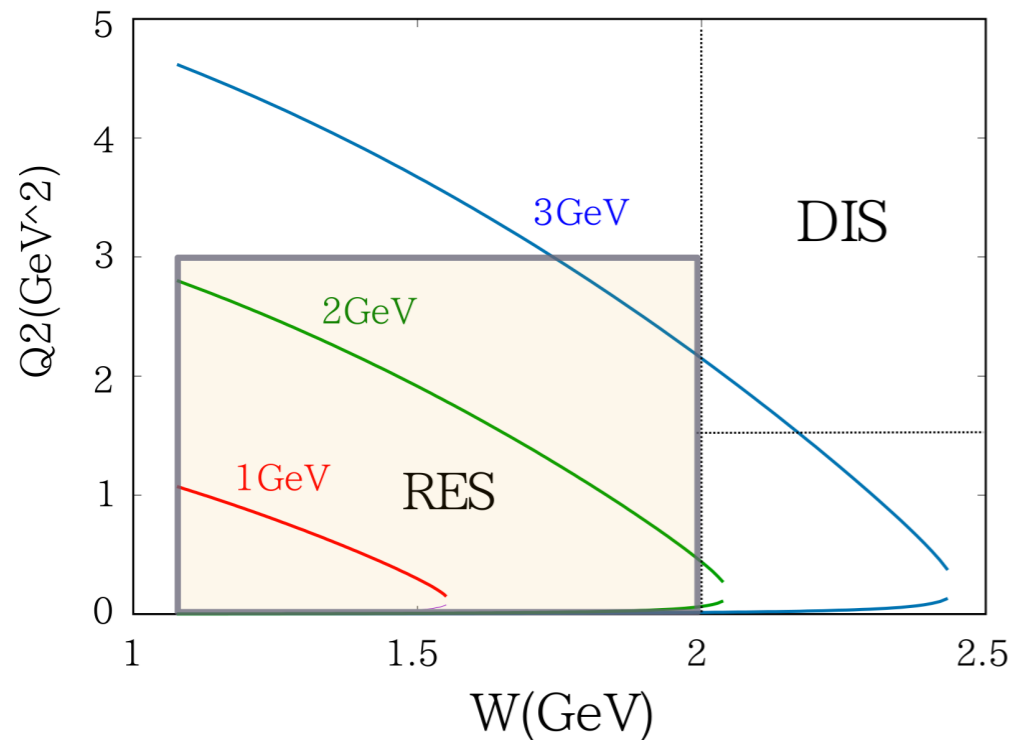
Pion Production

$$W^2 = (p_N(f) + p_\pi)^2 = (p_\nu + p_N(i) - p_\mu)^2$$

$$Q^2 = -q^2 = -(p_\nu - p_\mu)^2$$
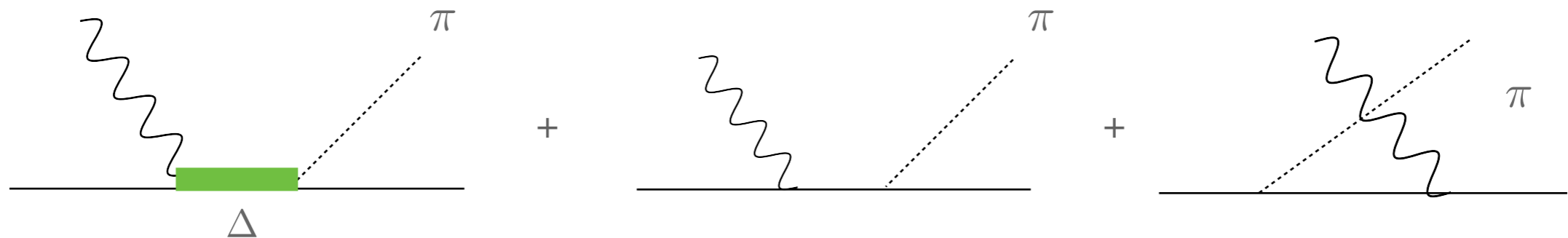
Contribution from both background and Δ-resonance states (but also higher resonances)

Pion Production
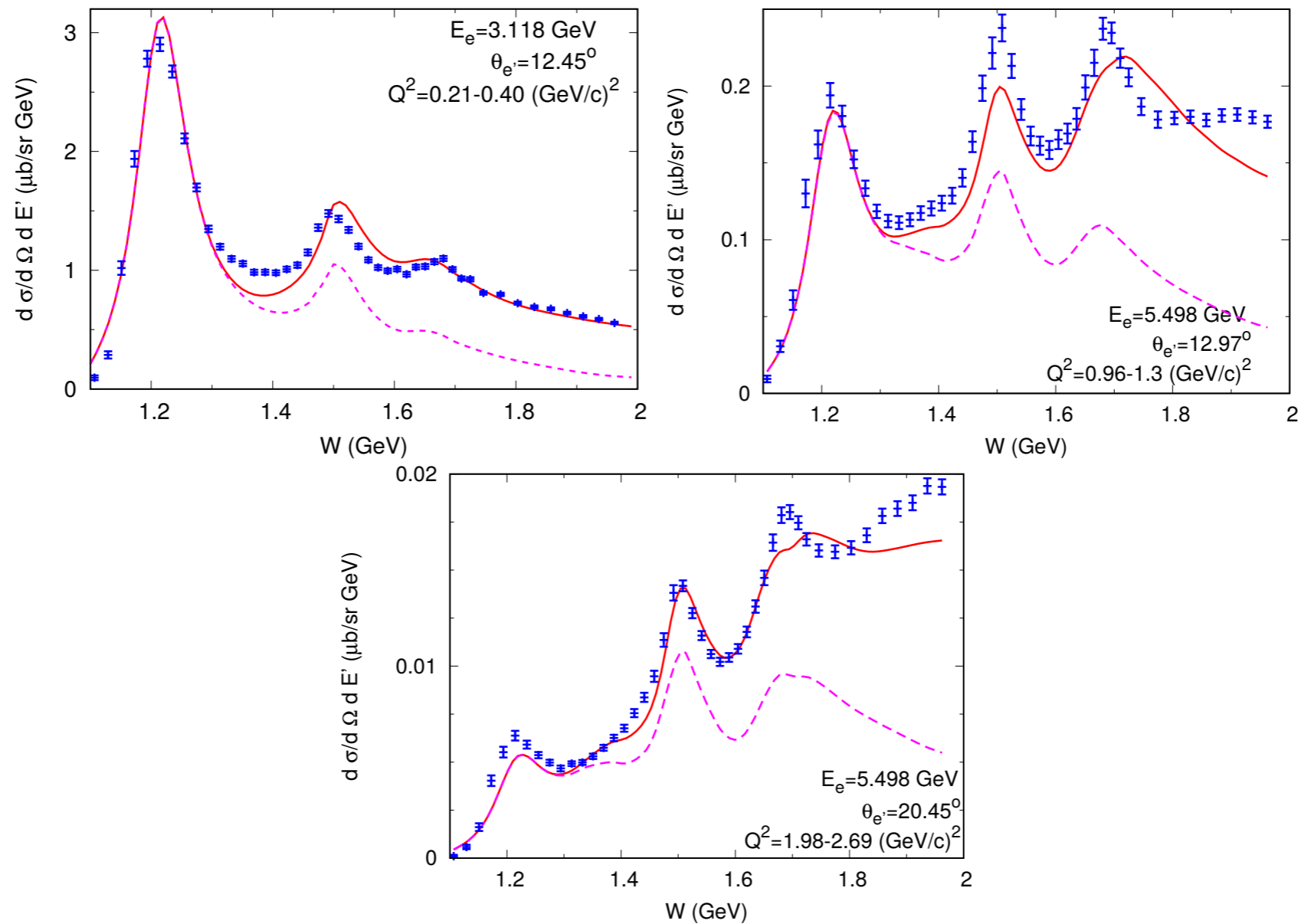
ANL-Osaka Partial-Wave Amplitudes (PWA) H.Kamano, T.-S. Lee, S.X. Nakamura, T.Sato, arXiv:1909.11935v1

Noemi Rocco, nrocco@fnal.gov

Pion Production

$\nu_\mu p \to \mu^- p \pi^+$, $W_{\pi N} < 1.4$ GeV

$\nu_\mu n \to \mu^- n \pi^+$, $W_{\pi N} < 1.4$ GeV

$\nu_\mu p \to \mu^- p \pi^+$, $W_{\pi N} < 2$ GeV

$\nu_\mu n \to \mu^- n \pi^+$, $W_{\pi N} < 2$ GeV
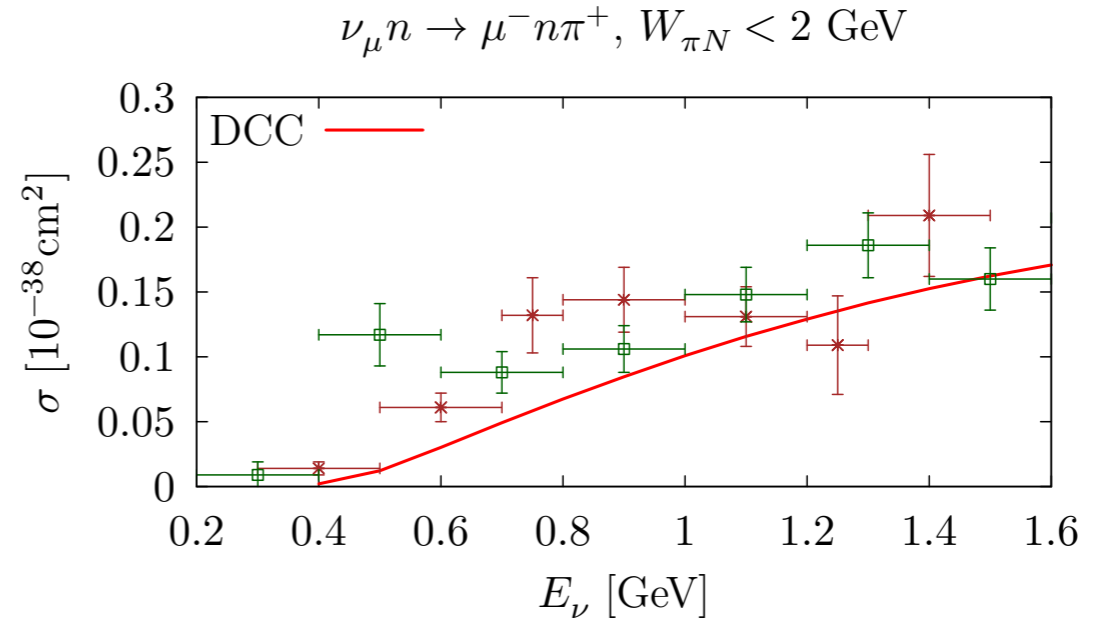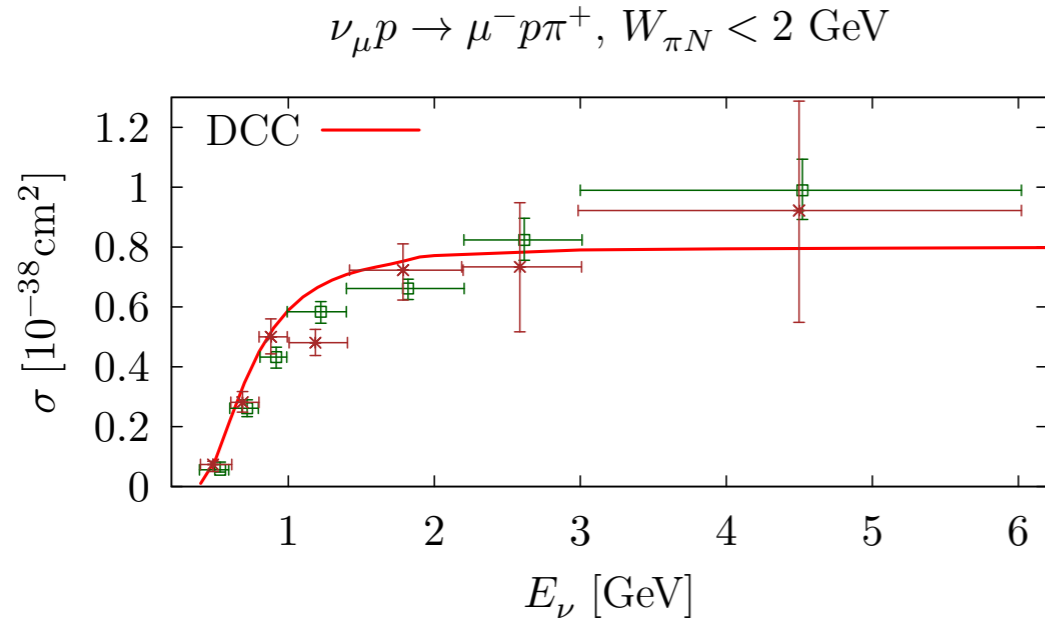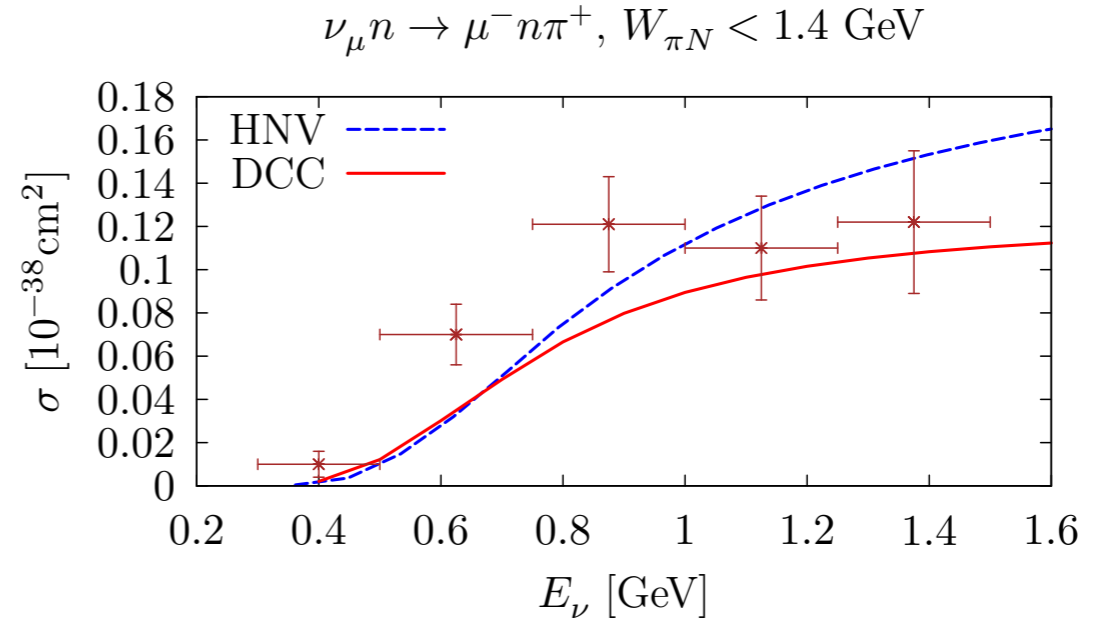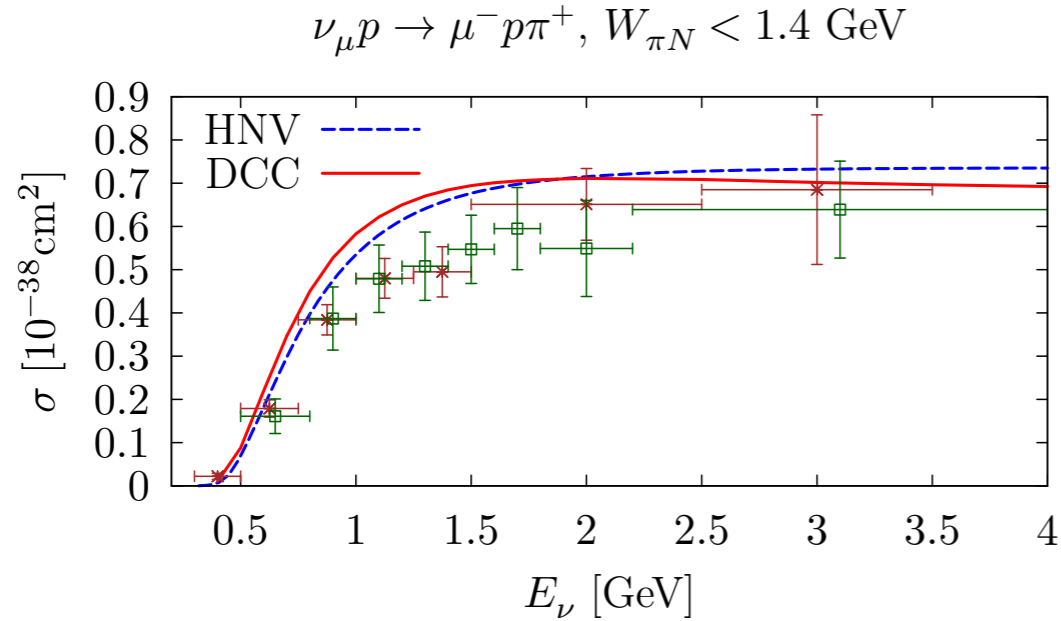
J. Sobczyk, E. Hernandez, S.X. Nakamura, J. Nieves, T. Sato PRD 98 (2018) 073001

Pion Production

Summary of models for neutrino reaction in RES

	Res	Non-res	Unit.	1pi	2pi	Tot
RS	Delta,N*	-	X	O		O
LPP	Delta,N*	X	X	O		O
HVM	Delta(1232)	chiral	O	O		
	Delta(1232)+N(1440)	chiral	X	O	O	
Giessen	Delta, N*	phen.	X	O		O
ANL-Osaka	Delta, N*	O	O	O	O	O

RS: D. Rein, L. M. Sehgal AP133(81), LPP: O. Lalakulich,E.A. Paschos,G. Piranlshvili,PRD74(2006)
HNV: E. Hernandez,J. Nieves,M. Valverde PRD76(2007) Giessen: T. Leitner,O.Buss,L.Alvarez-Ruso,U. Mosel,PRC79(2009)
ANL-Osaka DCC:S.X.Nakamura,H. Kamano,TS,PRD92(2015) ,TS,D. Uno,T.-S.H.Lee PRC67(2003)

R. Gonzales-Jimenes et al. PRD95,113007(2017)+Regge

Parton Structure of the nucleon

Above the pion production threshold W ≈ 1080 MeV the excitation of the Δ(1232) dominates, but at higher W the dynamics results from the interplay of overlapping baryon resonances, non-resonant amplitudes and their interference.

It is this region of W above the Δ(1232) and at moderate $Q^2 > 1$ GeV² that we refer to as **Shallow Inelastic Scattering (SIS)**. As Q2 grows, one approaches the onset of Deep Inleastic Scattering (DIS).

Transition from **strong interactions** described in terms of **hadronic degrees of freedom** to those among **quarks and gluons** described by perturbative QCD.
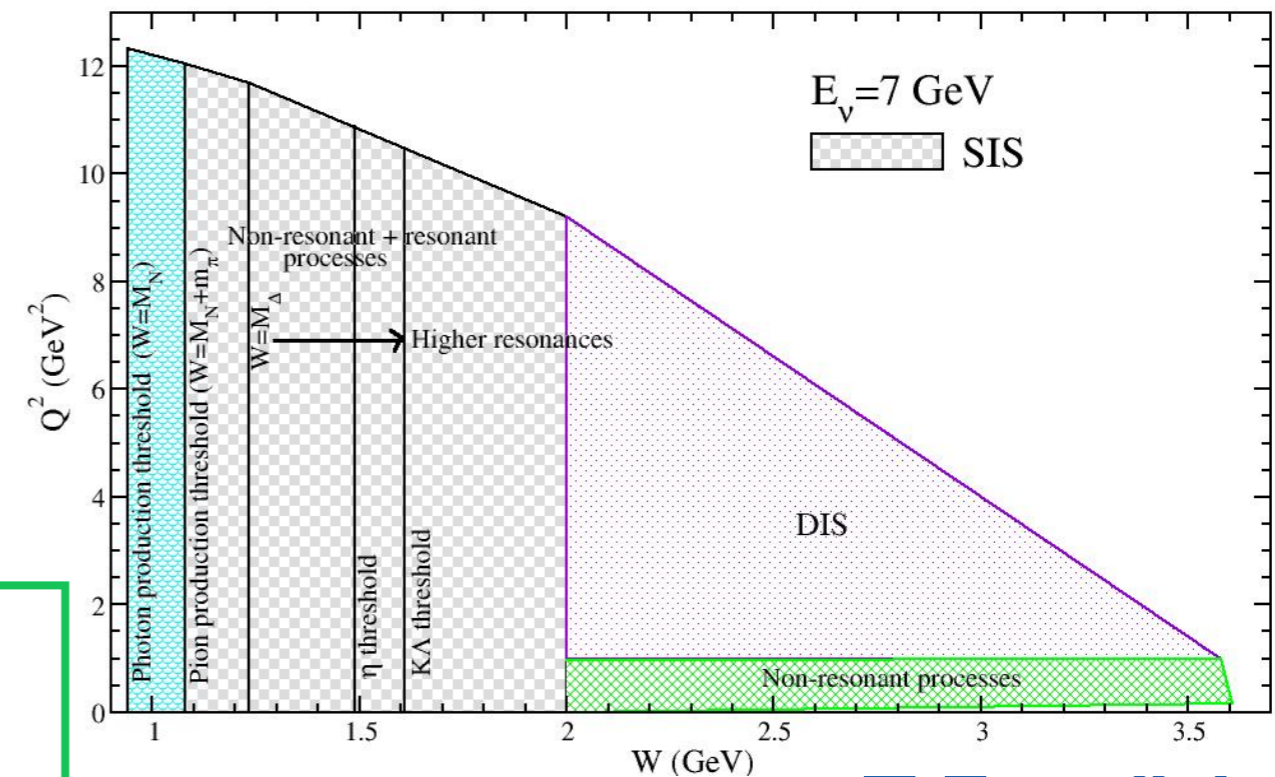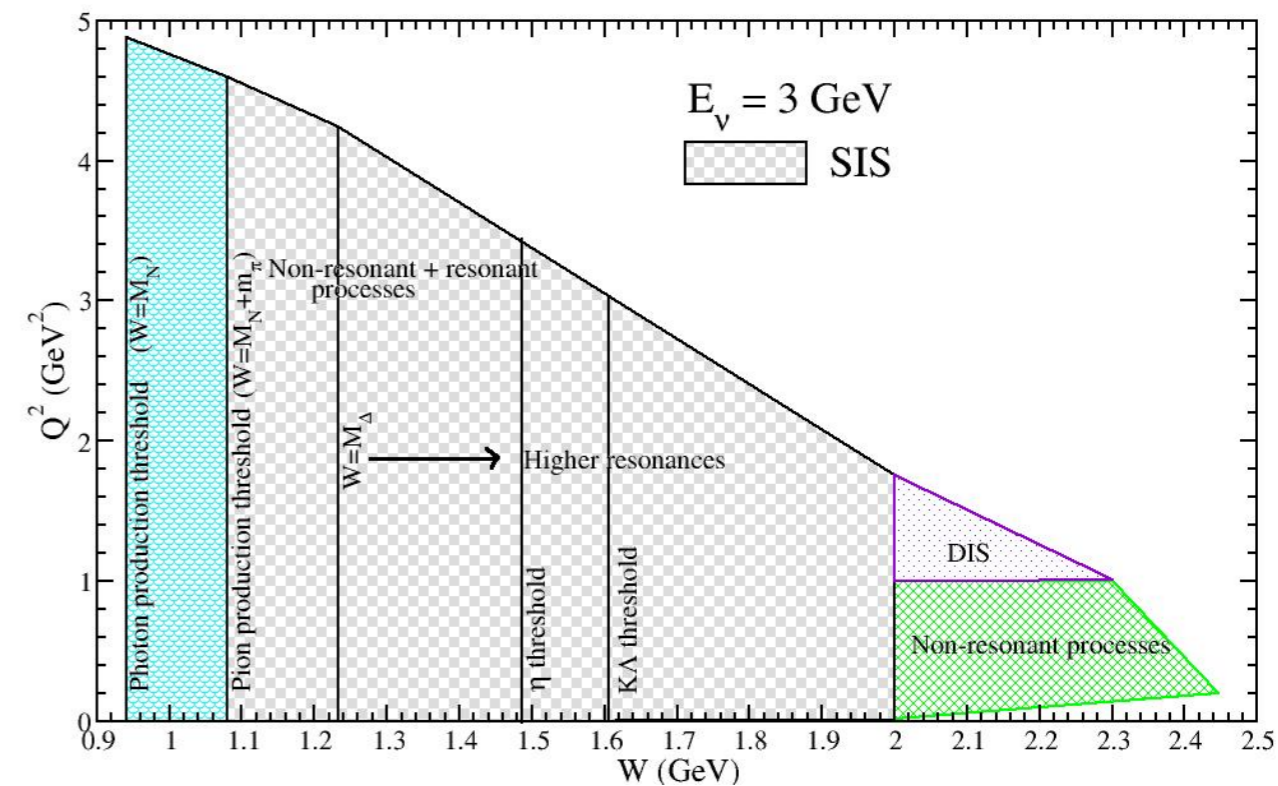
Theoretical tools for neutrino scattering, Contribution to: 2022 Snowmass Summer Study

Noemi Rocco, nrocco@fnal.gov

Parton Structure of the nucleon

- Nuclear level
- Nucleon level
- Quark level
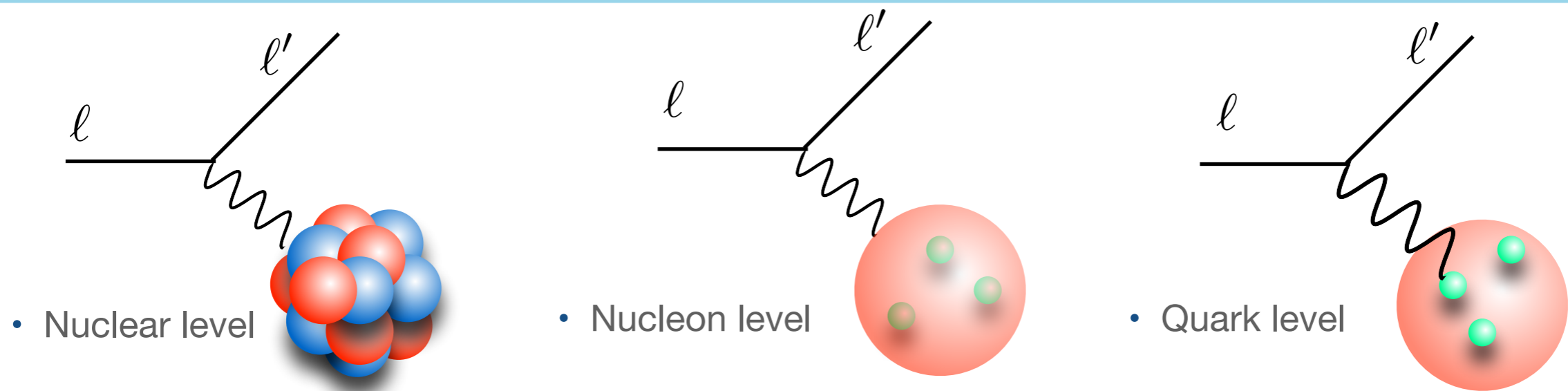
Parton model is analogous to the notion that a nucleus is a collection of noninteracting nucleons—but with a critical difference. Nucleons are rather easily liberated from nuclei, but the **division of a hadron into its constituent partons has never been observed**

$$\frac{d^2\sigma}{dQ^2 d\nu} = \frac{\pi}{E E'} \frac{d^2\sigma}{dE' d\Omega'} = \frac{4\pi\alpha^2}{Q^4} \frac{E'}{E} \left[2 W_1(Q^2, \nu) \sin^2\left(\frac{\theta}{2}\right) + W_2(Q^2, \nu) \cos^2\left(\frac{\theta}{2}\right) \right].$$

C. Quigg, Gauge Theories of the strong, weak, and electromagnetic interaction

‡ Fermilab

Noemi Rocco, nrocco@fnal.gov

Parton Structure of the nucleon

The transition to the parton model is made in the infinite- momentum frame, in which the longitudinal momentum of the proton is extremely large

Momentum of individual partons is:

$$p_i^{\mu} = x_i \, P^{\mu}$$

$P_{\parallel}$

$p_{1\parallel} = x_1 P_{\parallel}$

$p_{N\parallel} = x_N P_{\parallel}$

Elastic electron-proton

$$W_1 = \frac{Q^2}{4m_N^2}(F_1 + F_2)^2 \delta\left(\omega - \frac{Q^2}{2m_N}\right)$$

$$W_2 = \left(F_1^2 + F_2^2 \frac{Q^2}{4m_N^2}\right)\delta\left(\omega - \frac{Q^2}{2m_N}\right)$$

Electron-point-like particle

$$W_1^{\text{point}} = \frac{Q^2}{4m^2}\delta\left(\omega - \frac{Q^2}{2m}\right)$$

$$W_2^{\text{point}} = \delta\left(\omega - \frac{Q^2}{2m}\right)$$

✳ Let's move to the blackboard

🔷 Fermilab

Noemi Rocco, nrocco@fnal.gov

Parton Structure of the nucleon

- Nuclear level
- Nucleon level
- Quark level

$$\frac{d^2\sigma}{dQ^2 dv} = \frac{\pi}{E E'} \frac{d^2\sigma}{dE' d\Omega'} = \frac{4\pi\alpha^2}{Q^4} \frac{E'}{E} \left[2 W_1(Q^2, v) \sin^2\left(\frac{\theta}{2}\right) + W_2(Q^2, v) \cos^2\left(\frac{\theta}{2}\right) \right].$$
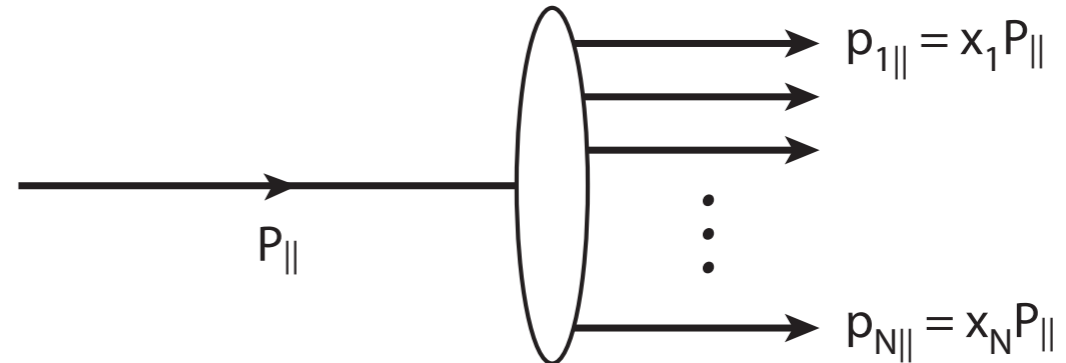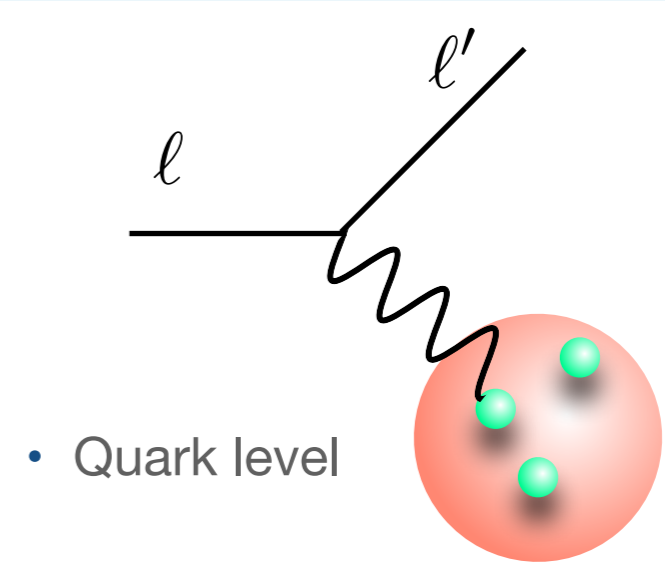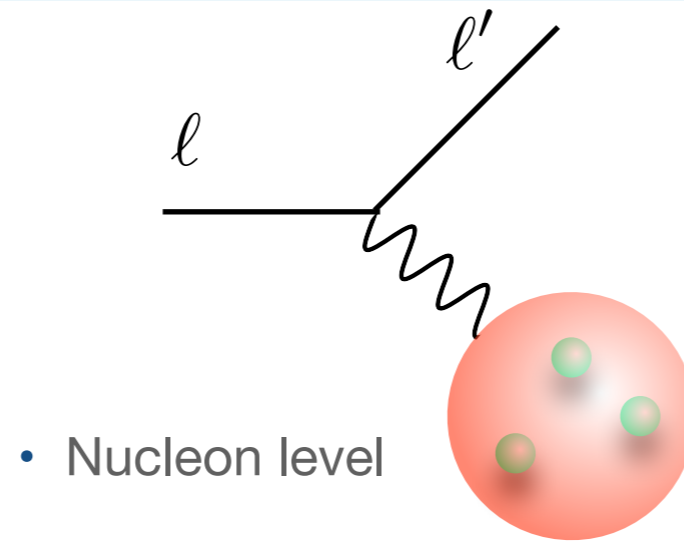
$$\frac{d^2\sigma^v}{dQ^2 dv} = \frac{G_F^2}{2\pi} E' E \left[2 W_1^v \sin^2\left(\frac{\theta}{2}\right) + W_2^v \cos^2\left(\frac{\theta}{2}\right) + W_3^v \frac{(E + E')}{M} \sin^2\left(\frac{\theta}{2}\right) \right]$$

$\nu + N$ has the contribution of an additional structure function $\mathcal{F}_3(x) \equiv v\, W_3^v(x)$

Quark-Hadron Duality

Duality can then be considered as a conceptual experimental bridge between **free** and **confined partons.**

Language of quarks/gluons in the DIS region and, as W decreases, one transitions to hadrons; in the SIS region that includes both resonant and non-resonant pion production.

- 1970 Bloom-Gilman duality in (e,e') experiments on p

DIS scaling curve extrapolated down into the resonance region passes through the average of the "peaks and valleys" of the resonant structure.

Comparison of F^p_2 from the **series of resonances** measured by E94-110 vs the **Nachtmann variable ξ** compared to the **extrapolated DIS measurement** from the NMC collaboration at 5 GeV²
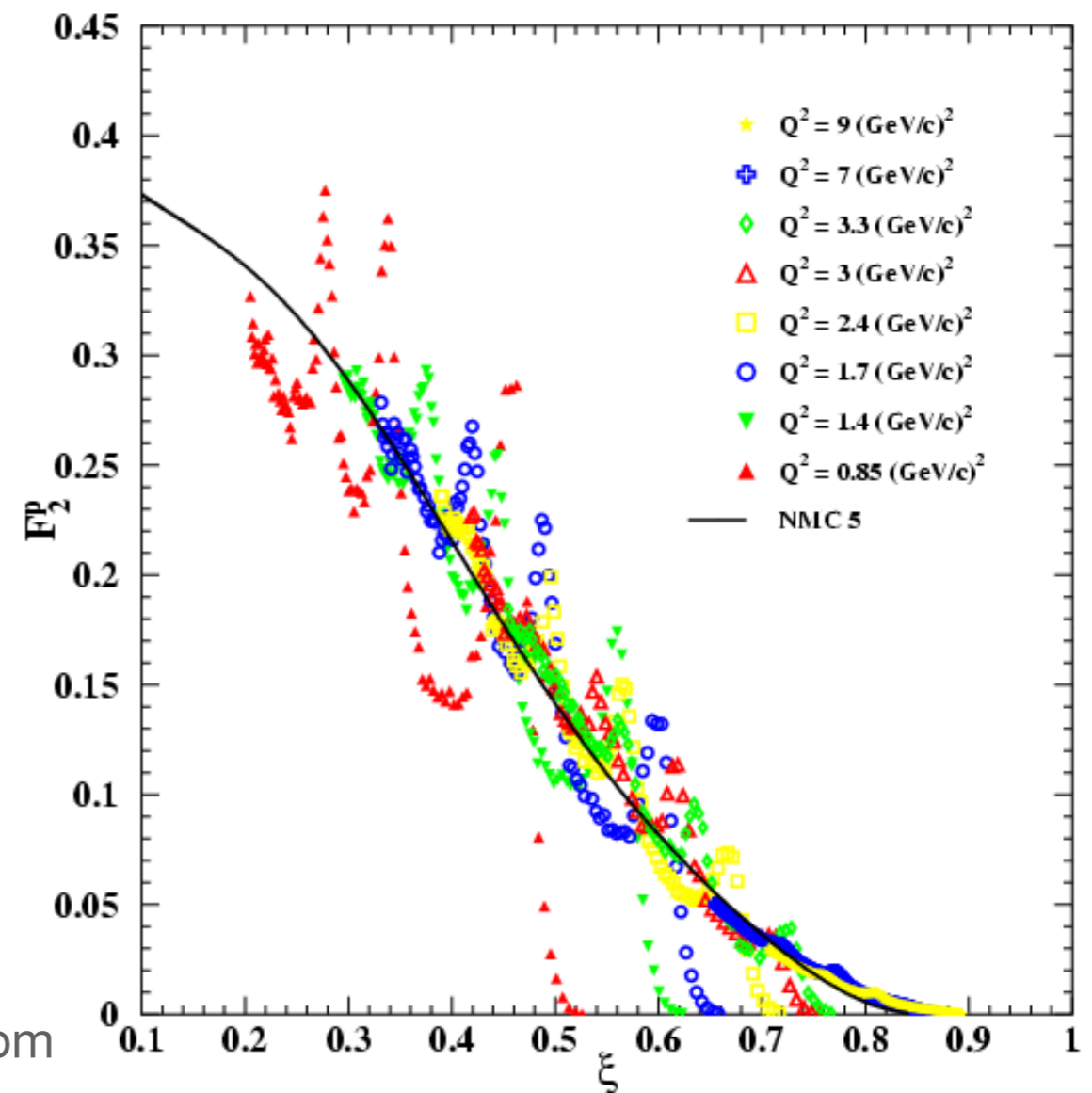
For Q²> 0.5 GeV² resonances follow the extrapolated DIS curve showing quark-hadron duality

Noemi Rocco, nrocco@fnal.gov

Quark-Hadron Duality in ν

The experimental study of duality with neutrinos is very restricted since the measurement of resonance production by v-N interactions is confined to **low-statistics data obtained in hydrogen and deuterium bubble chamber experiments from the 70's and 80's.**

For CC ν scattering

ANL-Osaka DCC model underestimates F_2 from DIS
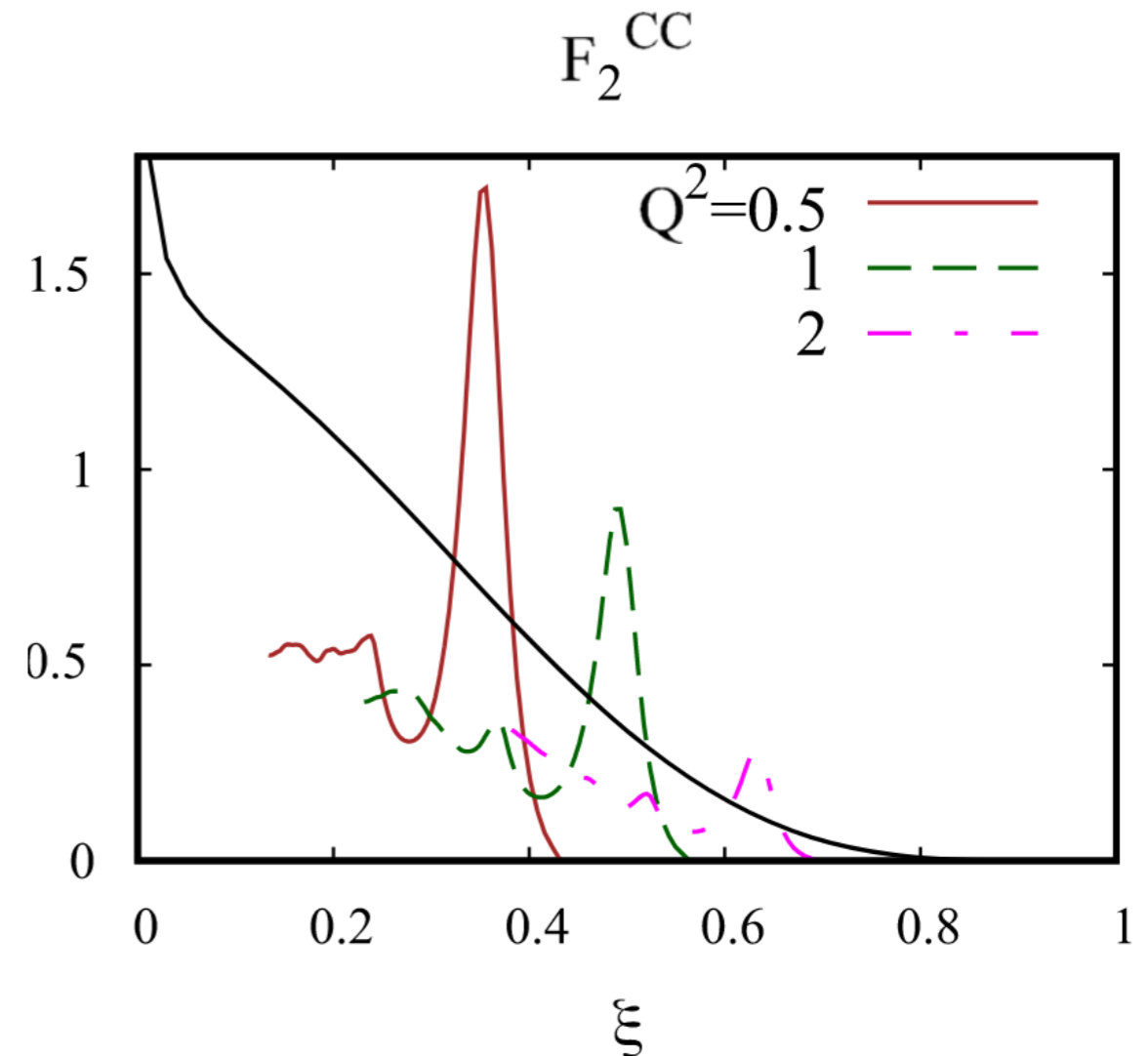
[T. Sato EPJ:ST (2021) 230:4409-4418 (2022)]

see also:
[O. Lalakulich et al. PRC79 015206 (2009)]

Quark-Hadron Duality in ν

The experimental study of duality with neutrinos is very restricted since the measurement of resonance production by v-N interactions is confined to **low-statistics data obtained in hydrogen and deuterium bubble chamber experiments from the 70's and 80's.**

For CC ν scattering

ANL-Osaka DCC model underestimates F_2 from DIS

Improvement with DIS upon modifying the Q2- dependence of the axial form factor:

$$A_\lambda^A(Q^2) = A_\lambda^A(0)\frac{A_\lambda^V(Q^2)}{A_\lambda^V(0)}$$

F_2^{CC}
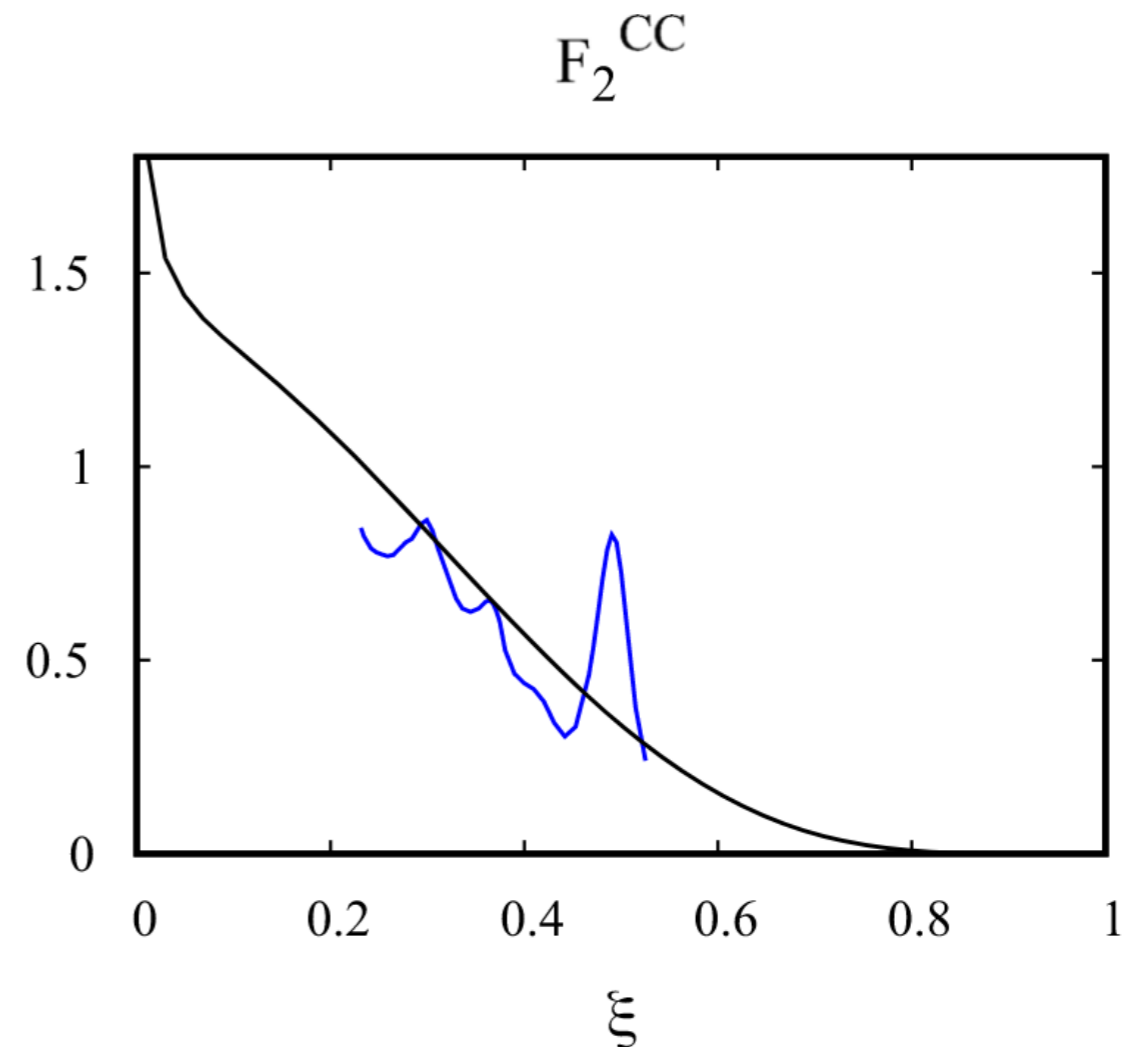
- **More data are needed to better constrain/understand ν-N scattering**

From theory to experiment

Free nuc

$\nu\mu$

p

Free nucleon scattering yields a single p in the final state

Short range correlations

Meson Exchange Currents

Final state interactions
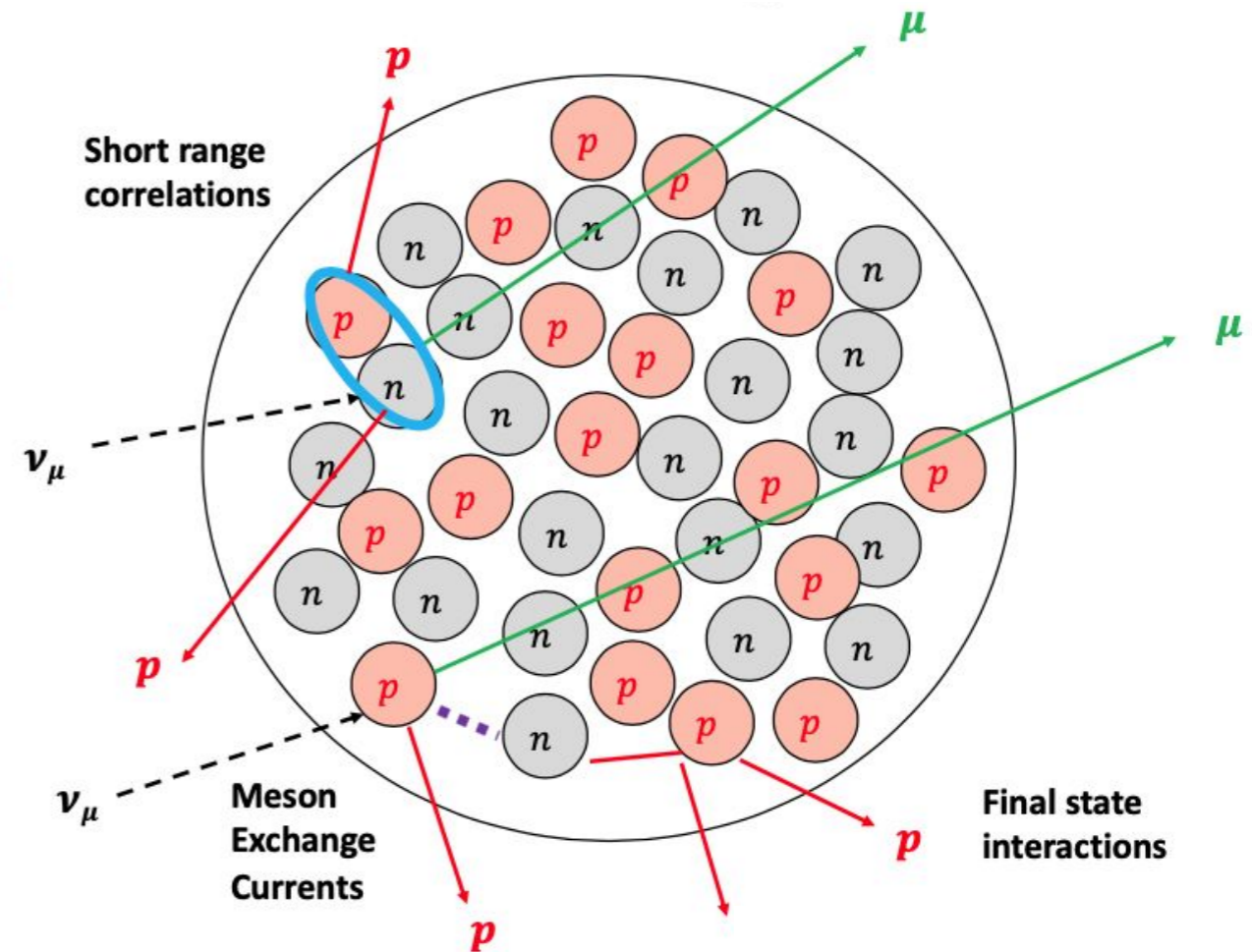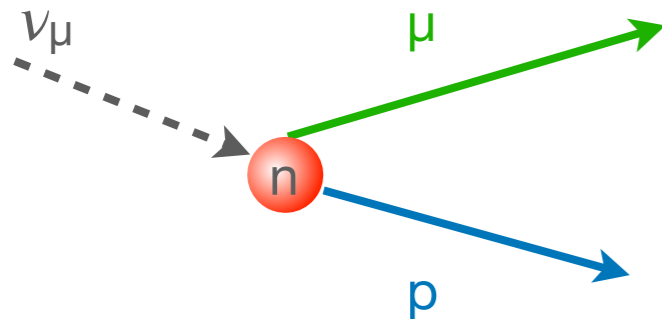
Nuclear model describing the target nucleus

Different reaction mechanisms depending on the momentum transferred to the the nucleus

Final state interactions: describe how the particles propagate through the nuclear medium

🎗 Fermilab

The Nucleus internal structure

Nuclei are **strongly interacting** many body systems exhibiting fascinating properties

Nuclear chart. **Magic numbers** N or Z= 2, 8, 20, 28, 50 and 126; <u>major shell complete</u> and are <u>more stable</u> than other elements

strong field
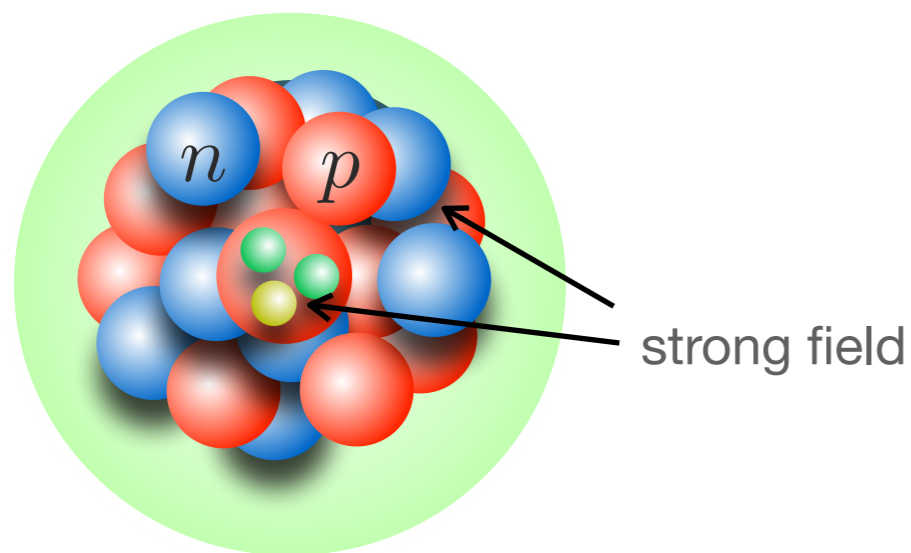
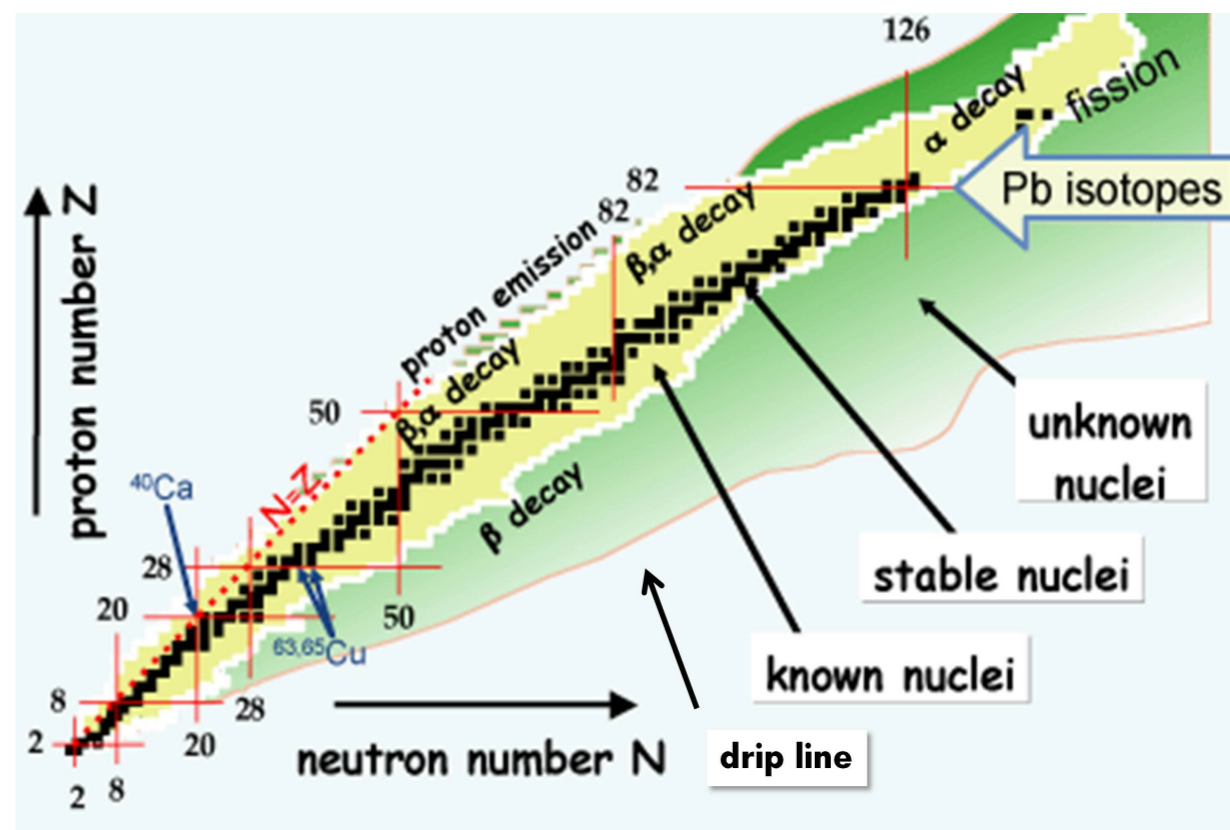The nucleus is formed by **protons** and **neutrons**: nucleons.

Each nucleon is made of three **quarks** held together by strong interactions→mediated by **gluons**

The nucleus is held together by the strong interactions between quark and gluons of neighboring nucleons

Nuclear Physicists **effectively** describe the interactions between protons and neutrons in terms of **exchange of pions**
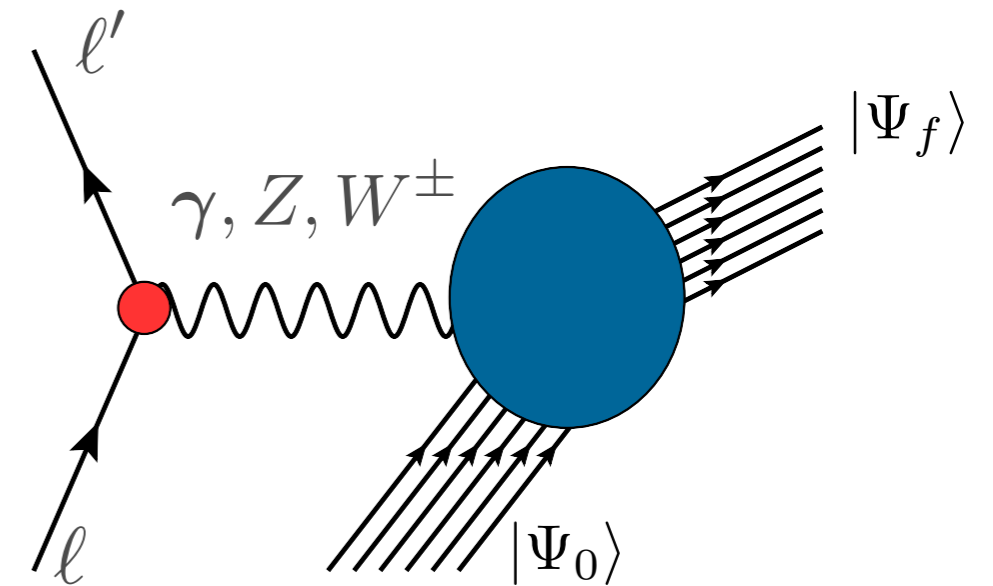
Theory of lepton-nucleus scattering

- The cross section of the process in which a lepton scatters off a nucleus is given by

$$d\sigma \propto L^{\alpha\beta} R_{\alpha\beta}$$

Leptonic Tensor: is the same as before, completely determined by lepton kinematics

Hadronic Tensor: nuclear response function

$$R_{\alpha\beta}(\omega, \mathbf{q}) = \sum_f \langle 0 | J_\alpha^\dagger(\mathbf{q}) | f \rangle \langle f | J_\beta(\mathbf{q}) | 0 \rangle \delta(\omega - E_f + E_0)$$

The initial and final wave functions describe many-body states:

$$|0\rangle = |\Psi_0^A\rangle \ , |f\rangle = |\Psi_f^A\rangle, |\psi_p^N, \Psi_f^{A-1}\rangle, |\psi_k^\pi, \psi_p^N, \Psi_f^{A-1}\rangle \dots$$

For inclusive reactions, the hadronic final state is not detected. We need to sum over all the possible ones

Comparing electron- and neutrino-nucleus

- We start by defining the nuclear response functions, for a given value of **q** and ω

$$W^{\mu\nu}(\mathbf{q}, \omega) = \sum_f \langle 0|(J^\mu)^\dagger(\mathbf{q}, \omega)|f\rangle \langle f|J^\nu(\mathbf{q}, \omega)|0\rangle \delta^{(4)}(p_0 + \omega - p_f)$$

- **Electron case** we write the **inclusive** double differential cross section as:

$$\frac{d\sigma}{dE'd\Omega} = \sigma_{\text{Mott}} \left[\left(\frac{q^2}{\mathbf{q}^2}\right)^2 R_L + \left(\frac{-q^2}{2\mathbf{q}^2} + tan^2\frac{\theta}{2}\right) R_T \right]$$

where: $\quad R_L = W_{00} \quad , \quad R_T = W_{xx} + W_{yy}$

- Neutrino case:

$$\left(\frac{d\sigma}{dE'd\Omega}\right)_{\nu/\bar{\nu}} = \frac{G^2}{4\pi^2}\frac{k'}{2E_\nu}\left[\hat{L}_{CC}R_{CC} + 2\hat{L}_{CL}R_{CL} + \hat{L}_{LL}R_{LL} + \hat{L}_T R_T \pm 2\hat{L}_{T'}R_{T'}\right],$$

- Where the nuclear responses are given by

$$R_{CC} = W^{00} \qquad\qquad R_{LL} = W^{33} \qquad , \quad R_{T'} = -\frac{i}{2}(W^{12} - W^{21})$$

$$R_{CL} = -\frac{1}{2}(W^{03} + W^{30}) \qquad R_T = W^{11} + W^{22}$$

Noemi Rocco, nrocco@fnal.gov

🎇 **Fermilab**

Liquid Drop Model

- The nucleus is treated as a drop of incompressible nuclear fluid. The fluid is made of nucleons which are held together by the strong nuclear forces.
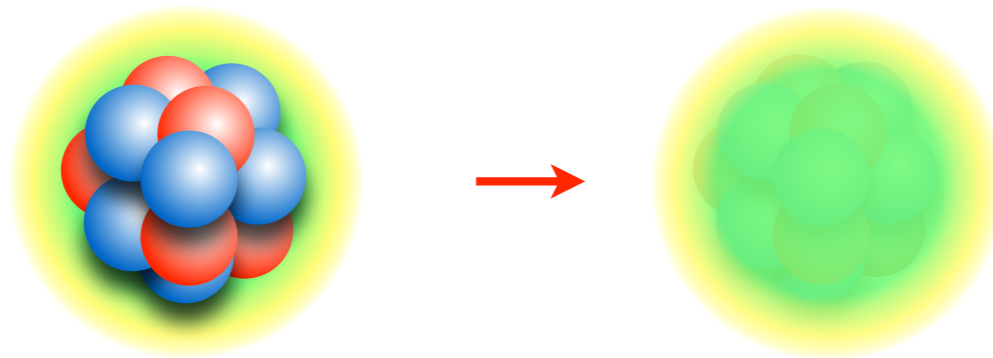
- This model explains the spherical shape and the binding energy of nuclei.

- The nuclear binding is given by the Weizsäcker formula

$$B(A, Z) = a_V A + a_S A^{2/3} + a_C \frac{Z^2}{A^{1/3}} + S_N \frac{(N - Z)^2}{A} + a_P \frac{(-1)^Z + (-1)^N}{2A^{1/2}}$$

Volume   Surface   Coulomb   Symmetry   Pairing

Liquid Drop Model

- The nuclear binding is given by the Weizsäcker formula

$$B(A,Z) = a_V A + a_S A^{2/3} + a_C \frac{Z^2}{A^{1/3}} + S_N \frac{(N-Z)^2}{A} + a_P \frac{(-1)^Z + (-1)^N}{2A^{1/2}}$$

Volume   Surface   Coulomb   Symmetry   Pairing

- **Volume Term:** This term represents the attractive nuclear force that acts over the entire volume of the nucleus

- **Surface Term:** This term accounts for the fact that nucleons at the surface of the nucleus have fewer neighboring nucleons than those in the interior

- **Coulomb Term:** This term represents the electrostatic repulsion between protons in the nucleus due to their positive charges.

- **Asymmetry Term:** It reflects the preference for equal numbers of protons and neutrons, which contribute to greater nuclear stability. Deviations from this balance result in less binding energy.

- **Pairing Term:** This term considers the additional binding energy due to the pairing of nucleons (protons with protons and neutrons with neutrons) in even numbers.

🔹 Fermilab

Initial state: global Fermi gas
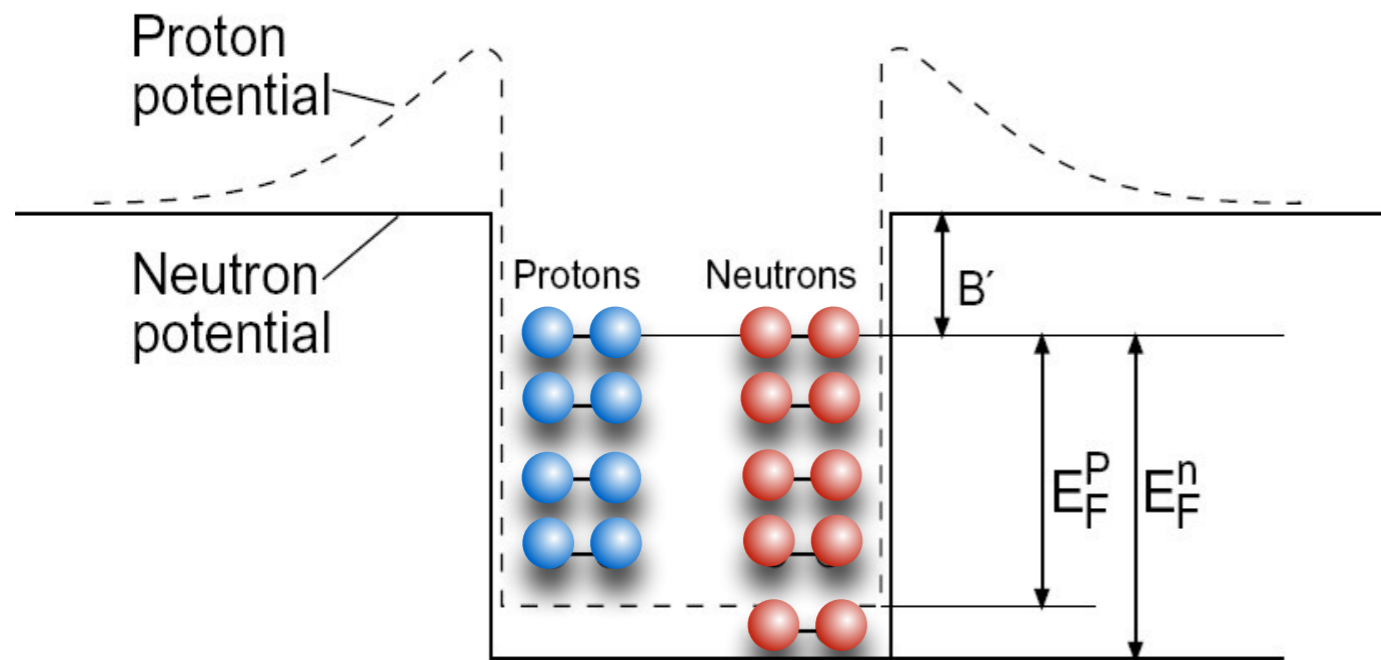
- Simple picture of the nucleus: only statistical correlations are retained (Pauli exclusion principle)

- Protons and neutrons are considered as moving freely within the nuclear volume

- The nuclear potential wells are rectangular: constant inside the nucleus and goes sharply to zero at its edge

- The energy of the highest occupied state is the Fermi energy: E_F

- The difference B' between the top of the well and the Fermi level is constant for most nuclei and is just the average binding energy per nucleon B'/A ~ 7-8 MeV

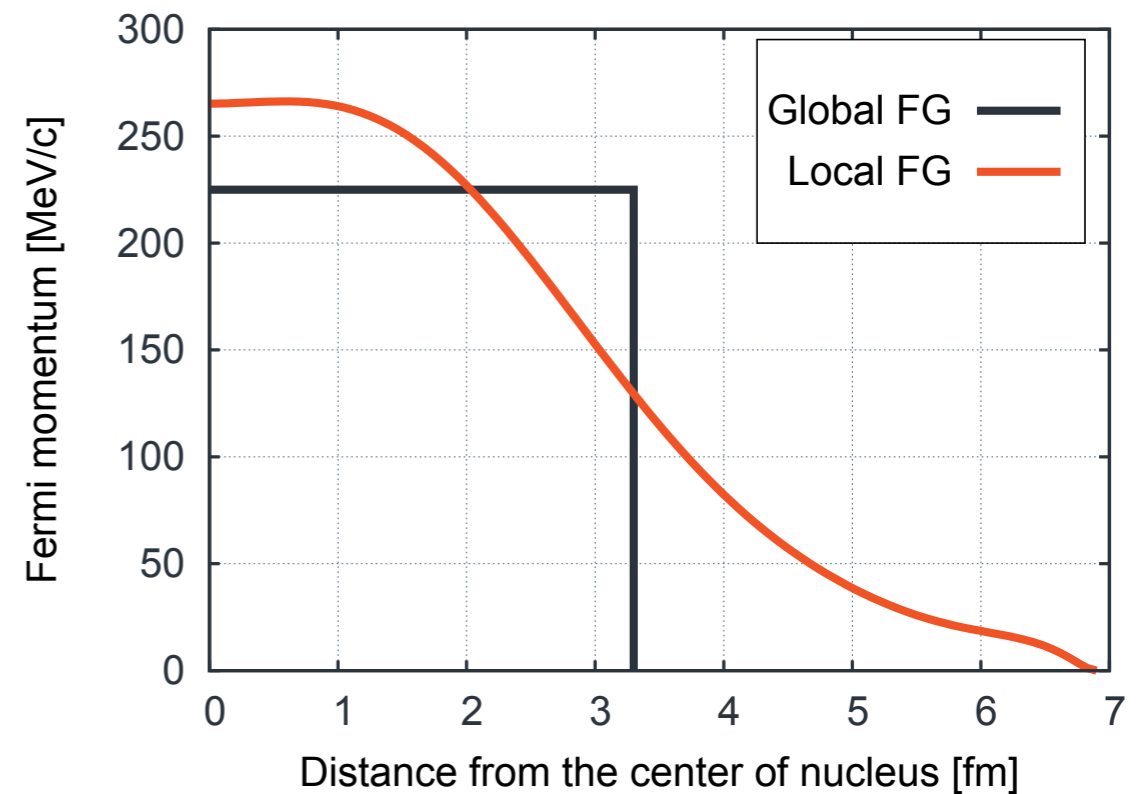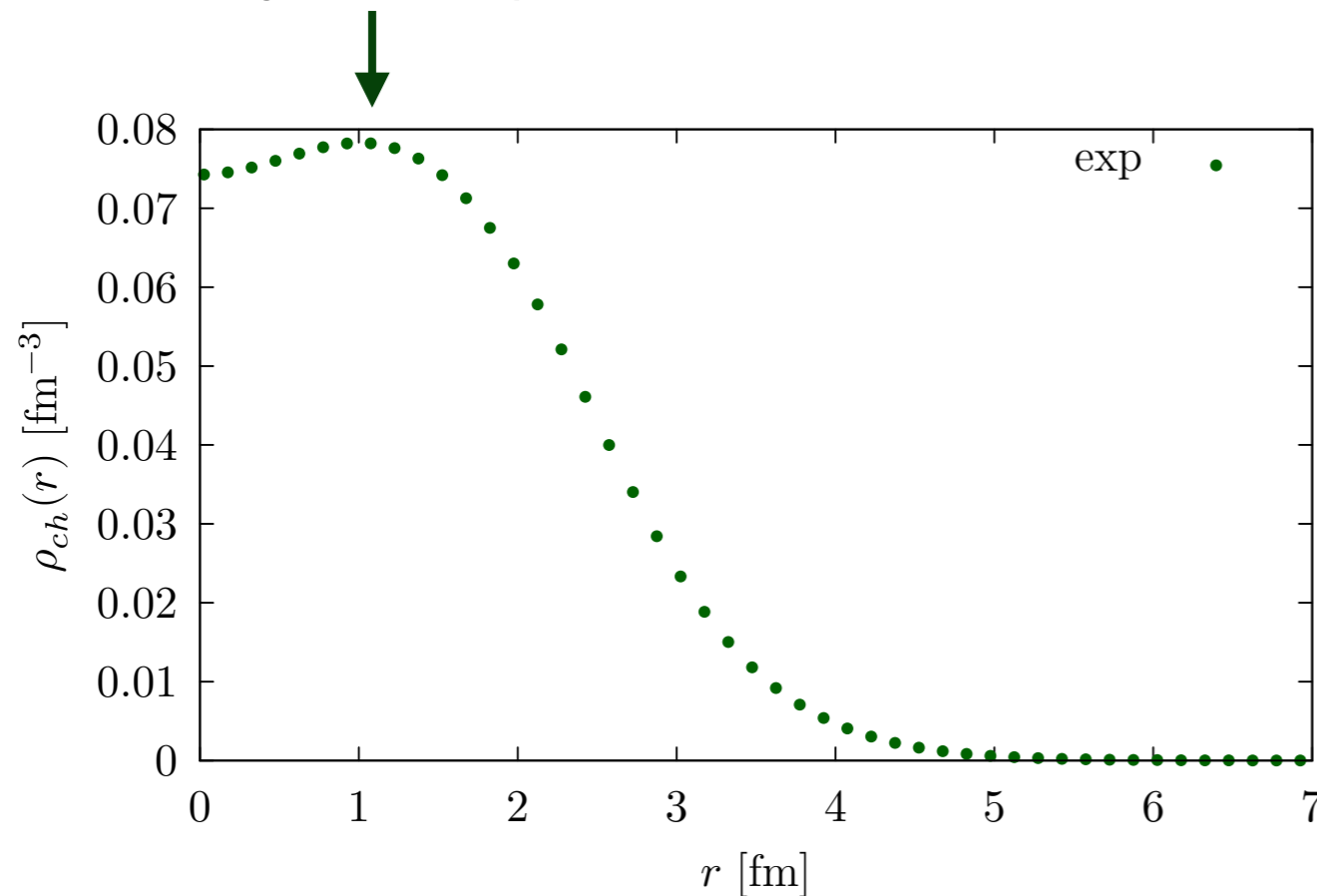C. Bertulani, Nuclear Physics in a Nutshell

Noemi Rocco, nrocco@fnal.gov

Initial state: Local Fermi gas

• A spherically symmetric nucleus can be approximated by concentric spheres of a constant density.
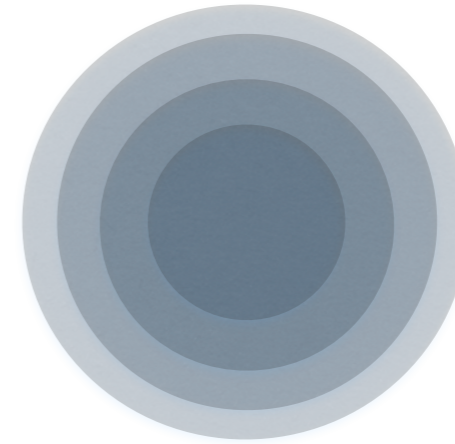
More likely to find a particle r ~ r_{ch} ~ 2.5 fm

Figure by T. Golan

• **Global Fermi Gas**

$$p_F = \left(\frac{9\pi \cdot n}{4A}\right)^{1/3} \cdot \frac{}{R_0}$$

• **Local Fermi Gas**

$$p_F = \hbar\left(3\pi^2 \rho(r)\frac{n}{A}\right)^{1/3}$$

Noemi Rocco, nrocco@fnal.gov

Initial state: shell Model

- As in the Fermi Gas model: the nucleons move within the nucleus independently of each other

- Difference: the nucleons are not free: subject to a central potential

- Each nucleon moves in an average potential created by the other nucleons, the potential should be chosen to best reproduce the experimental results

$$H = \sum_i \frac{p_i^2}{2m} + \sum_{i<j} v_{ij} + \dots \quad \longrightarrow \quad H = \sum_i \frac{p_i^2}{2m} + \sum_i^A U_i + H_{\text{res}}$$

- We solve the Schrödinger Equation:

$$H\,\psi = E\,\psi$$

$$\begin{cases} E = E_1 + E_2 + \dots + E_A \\ \psi(1,\dots,A) = \mathcal{A}[\phi_1(1)\dots\phi_A(A)] \end{cases}$$

Initial state: shell Model

- Example: Particles are subject to an harmonic oscillator potential

$$U(r) = \frac{1}{2} m \omega^2 r^2$$

The frequency should be adapted to the mass number A

- We will seek solutions of the type

$$\psi(r) = \frac{u(r)}{r} Y_l^m(\theta, \phi)$$

← Spherical Harmonics

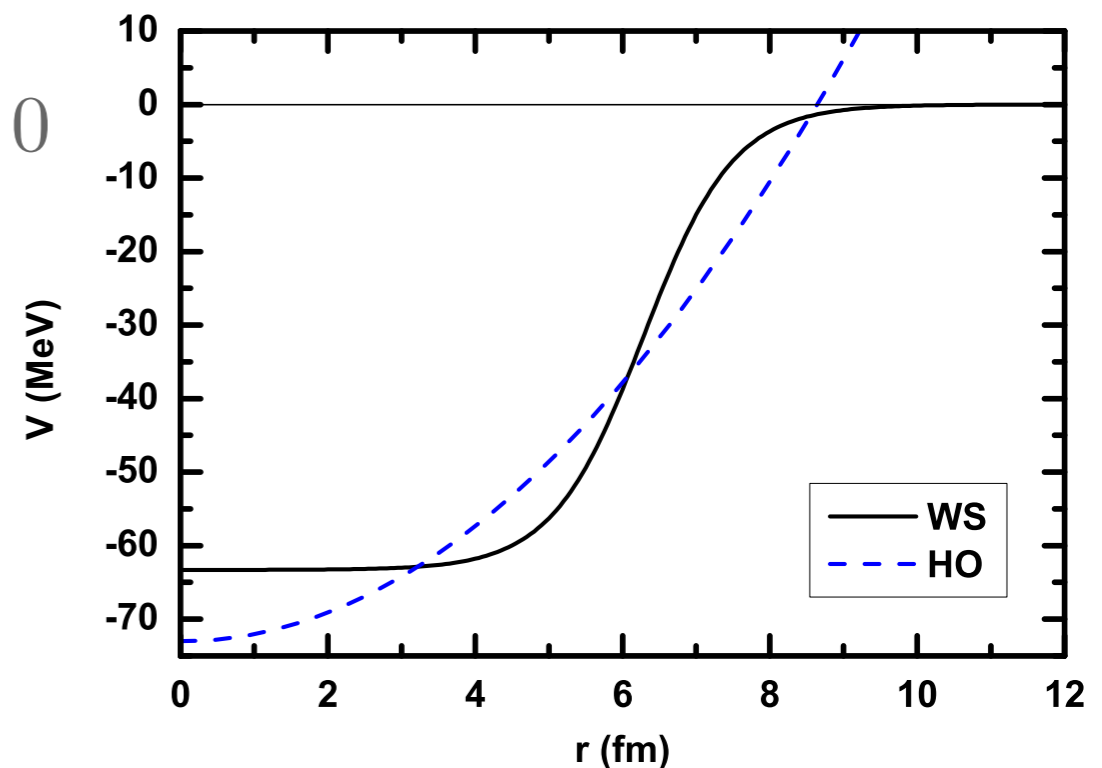- Solving the Schrödinger Equation reduces to a solution of u:

$$\frac{d^2 u}{dr^2} + \left\{ \frac{2m}{\hbar^2}[E - U(r)] - \frac{l(l+1)}{r^2} \right\} u(r) = 0$$

$$E_{nl} = \hbar\omega \left(2n + l - \frac{1}{2} \right) \quad \text{Eigenvalues}$$

- A more realistic potential is the Wood Saxon:
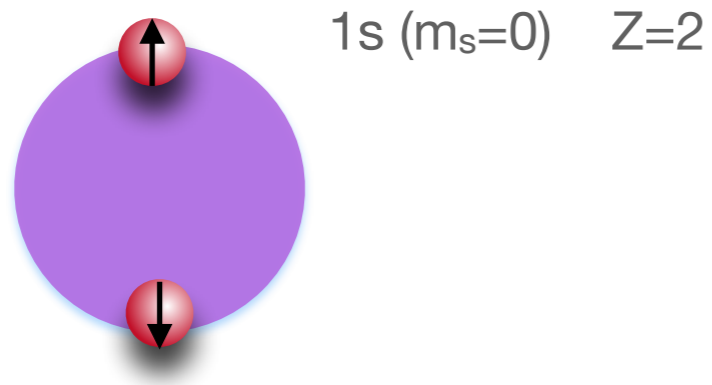
$$V = V_0/[1 + exp[(r - R)/a]$$

V_0, r, a, are adjustable parameters chosen to best reproduce the **experimental results**

Physical Review C 87(1):014334

Noemi Rocco, nrocco@fnal.gov

Nuclear Shell Model

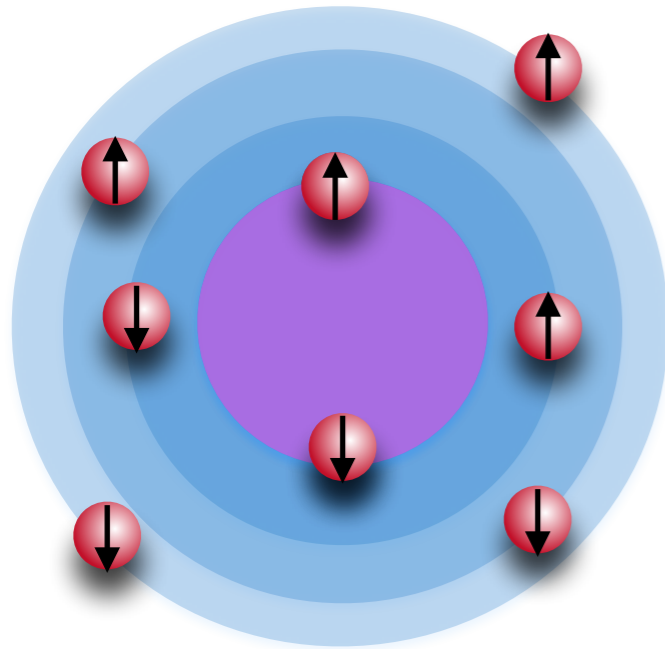The lowest level, s shell, can contain 2 protons

Our assumption: central potential

1s (m_s=0)   Z=2

$$H = \sum_i \frac{p_i^2}{2m} + \sum_i V(r_i)$$

n is the principal quantum number, **l** orbital momentum, **m** magnetic quantum number

Nuclear Shell Model

The p shell can contain up to 6 protons

1s (m$_s$=0)
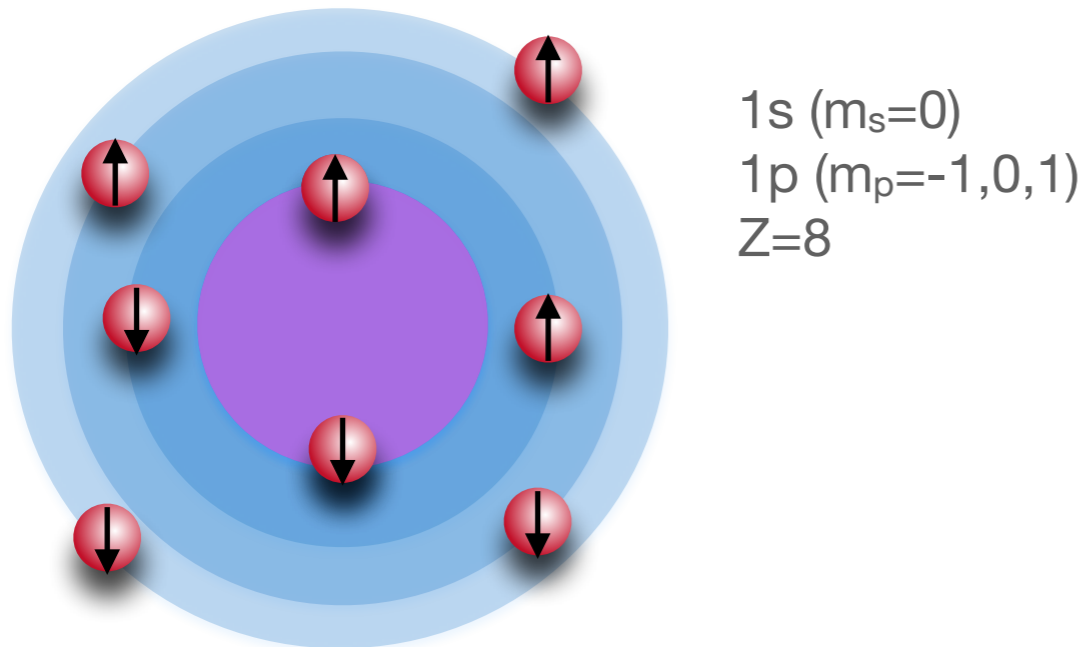1p (m$_p$=-1,0,1)
Z=8

Our assumption: central potential

$$H = \sum_i \frac{p_i^2}{2m} + \sum_i V(r_i)$$

We explained the first two magic numbers: 2 and 8.
We can follow the same strategy for the Z=20 case;
but at the next step we obtain Z=40 **while
experimentally Z=50**

n is the principal quantum number, **l** orbital
momentum, **m** magnetic quantum number

Nuclear Shell Model

The p shell can contain up to 6 protons

1s (m_s=0)
1p (m_p=-1,0,1)
Z=8

Maria Goeppert Mayer poses with her colleagues in front of Argonne's Physics building.

Our assumption: central potential

$$H = \sum_i \frac{p_i^2}{2m} + \sum_i V(r_i)$$

We explained the first two magic numbers: 2 and 8. We can follow the same strategy for the Z=20 case; but at the next step we obtain Z=40 **while experimentally Z=50**

In 1963, Goeppert Mayer, Jensen, and Wigner shared the Nobel Prize for Physics "for their discoveries concerning nuclear shell structure."

The solution to the puzzle lies in the **spin-orbit coupling**. This effect in the nuclear potential is 20 times larger then in Atomic Physics

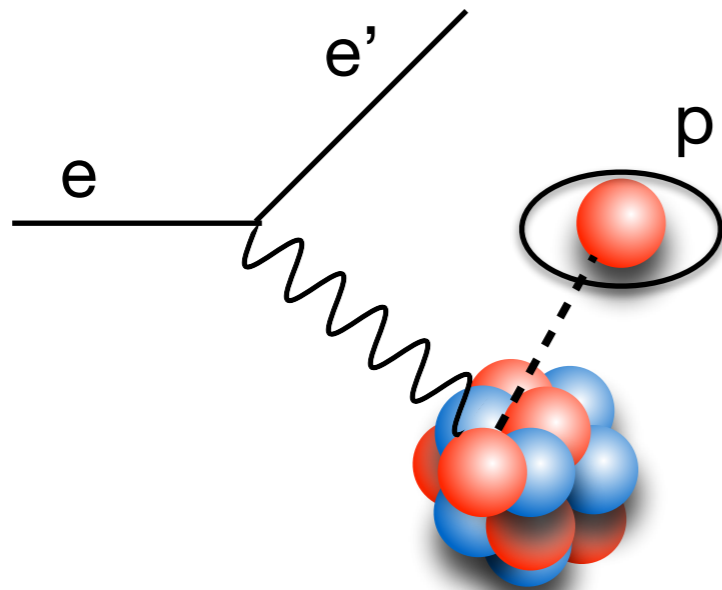$$V(r) \rightarrow V(r) + W(r)\mathbf{L} \cdot \mathbf{S}$$

The spin-orbit introduces an energy split which modifies the shell structure and reproduces magic number up to Z=126
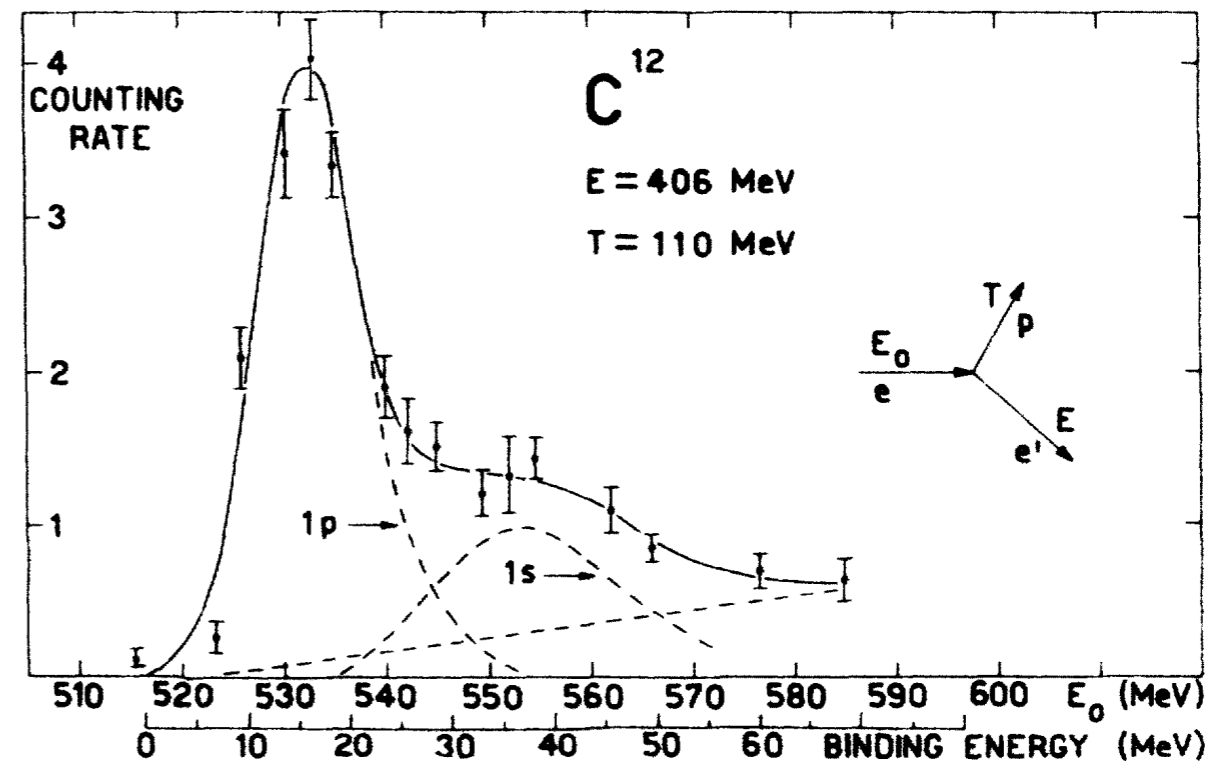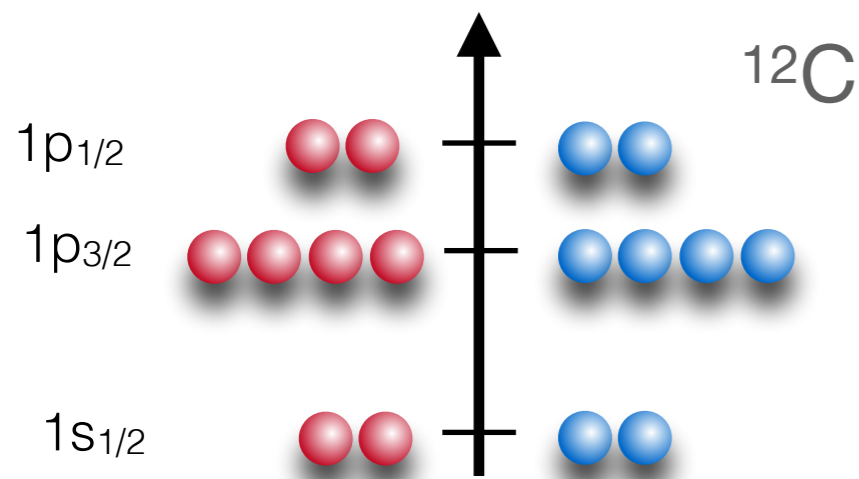
(e,e'p) scattering experiments

- (e,e'p) experiments are extremely important to investigate the internal structure of the nucleus

U.Amaldi et al, Phys. Rev. Lett. 13, 10 (1964)

- Assuming NO FSI the energy and momentum of the initial nucleon can be identified with the measured p_{miss} and E_{miss}
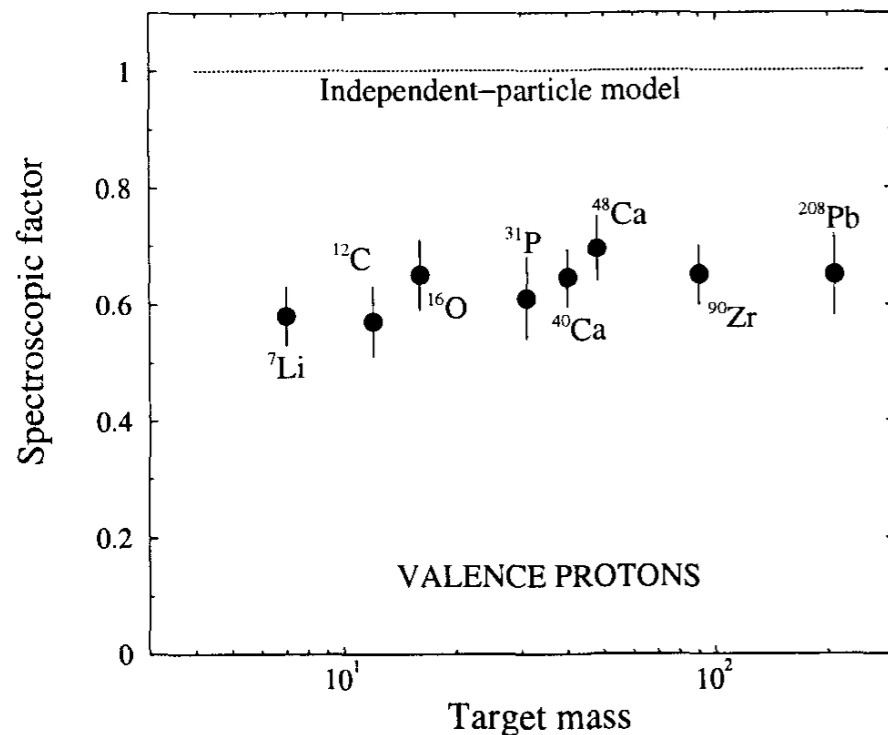

$1p_{1/2}$

$1p_{3/2}$

$1s_{1/2}$

^{12}C

- The peak coming from four 1p protons is visible

- The contribution of the two 1s protons is not clearly separated with this resolution

(e,e'p) scattering experiments

- Electron and proton experiments also pinned down the limitations of MF approaches

- Quenching of the spectroscopic factors of valence states has been confirmed by a number of high resolution (e,e'p) experiments
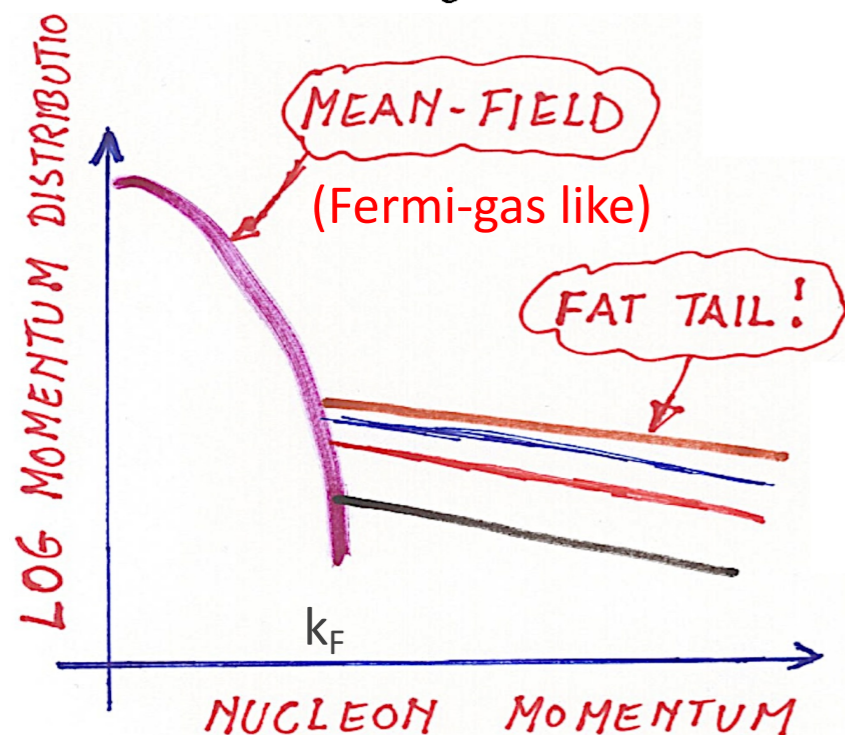
Subedi et al., Science 320, 1476 (2008)

- **Semi-exclusive 2N-SRC** experiments at $x>1$ allows to detect both nucleons and reconstruct the initial state
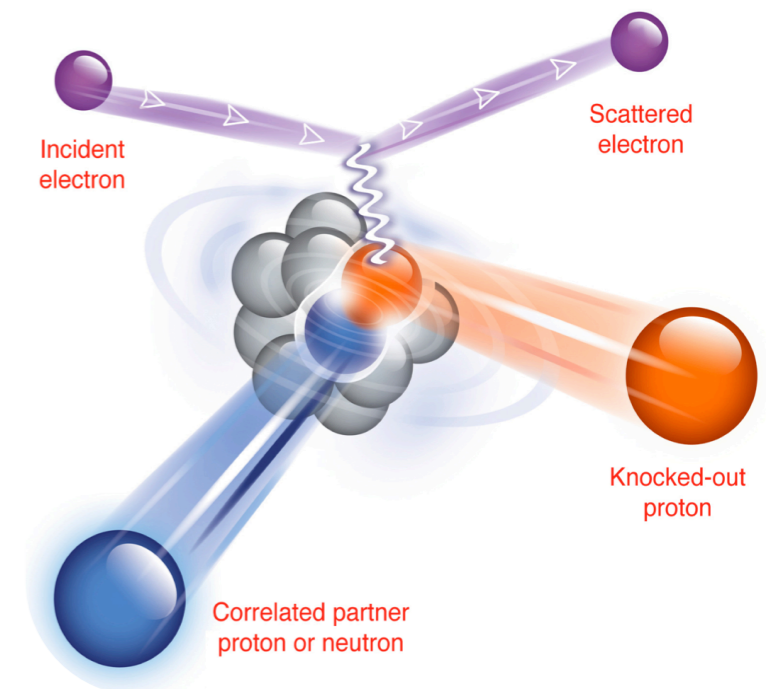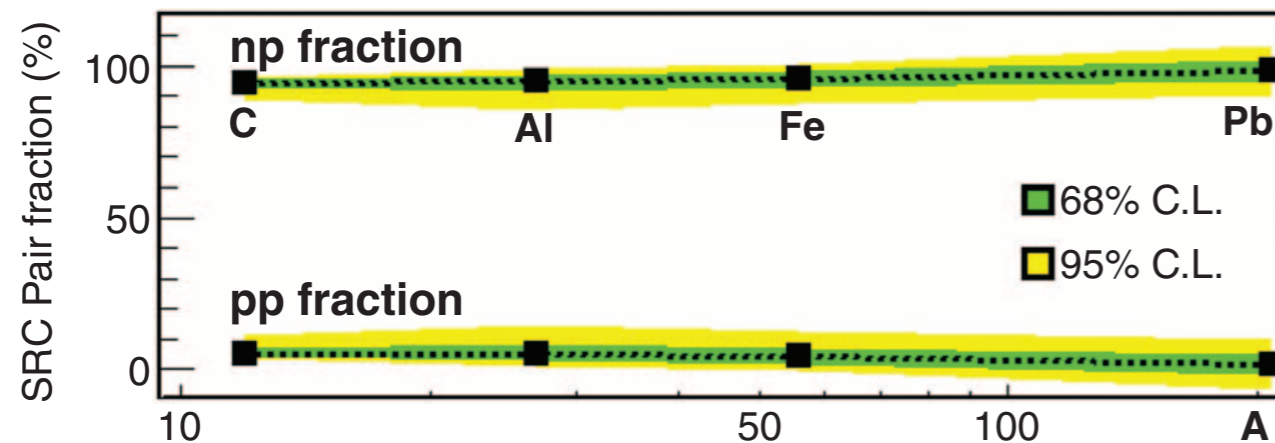

- Confirmed that the **high momentum tail** of the nuclear wave function consists mainly of 2N-SRC

- The large-momentum (short-range) component of the wave function is dominated by the presence of Short Range Correlated (SRC) pairs of nucleons

(e,e'p) scattering experiments

- Observed dominance of np-over-pp pairs for a variety of nuclei

Subedi et al., Science 320, 1476 (2008)

- SRC pairs are in spin-triplet state, a consequence of the tensor part of the nucleon-nucleon interaction

Bottom Line

- Two-body Physics can not be neglected:
  - ~20% of the nucleons in nuclei
  - ~100% of the high k (>pF) nucleons

- Have large relative momentum and low center of mass momentum

- Universality of high-momentum component
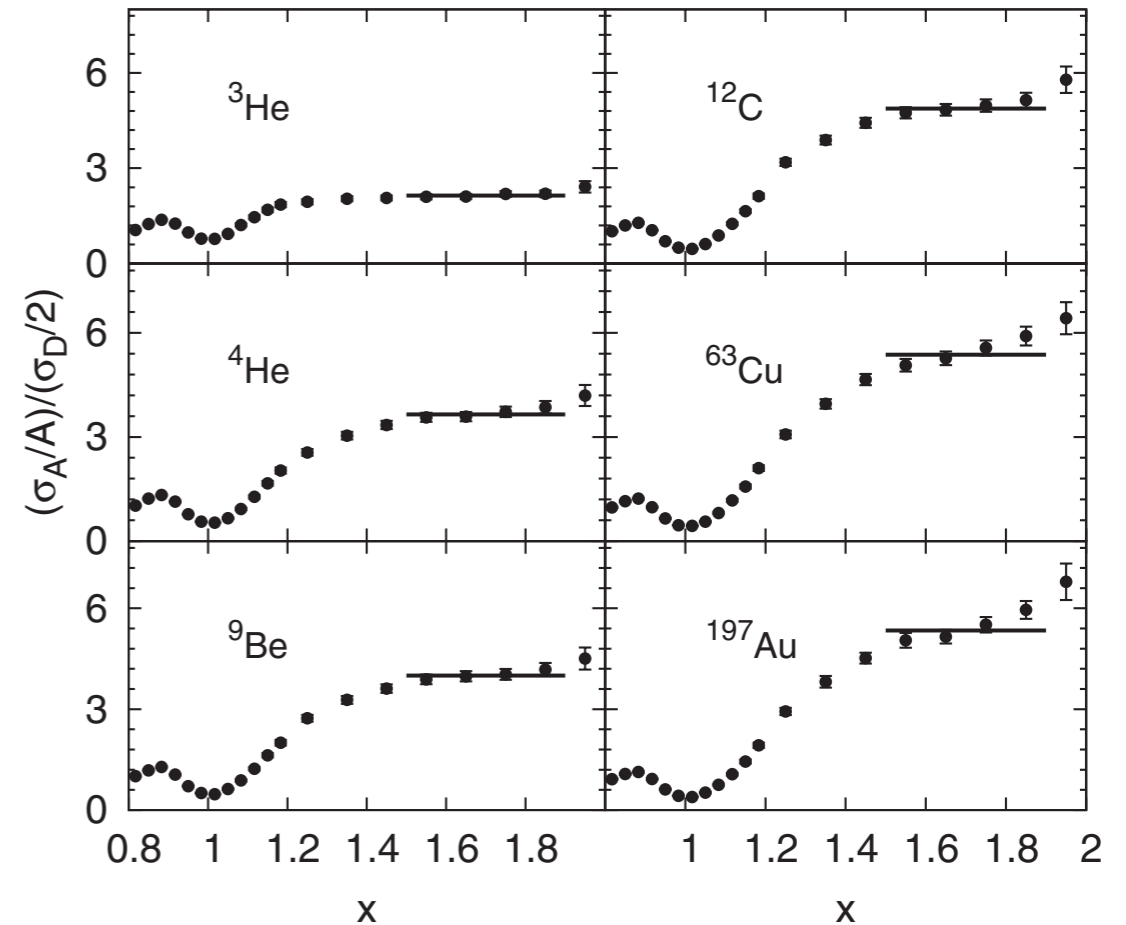
N. Fomin et al., PRL 108, 092502 (2012)

- The cross section ratio: A/d, sensitive to $n_A(k)/n_d(k)$

- Observed scaling for x>1.5

$$n_A(k > p_F) = a_2(A) \times n_d(k)$$

The basic model of nuclear theory

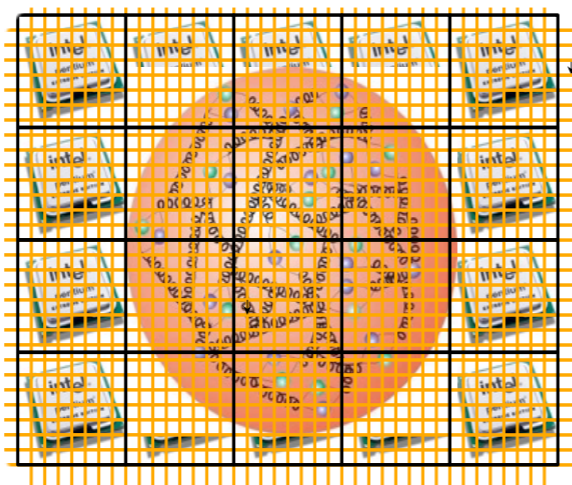### Effective Hamiltonians and consistent currents
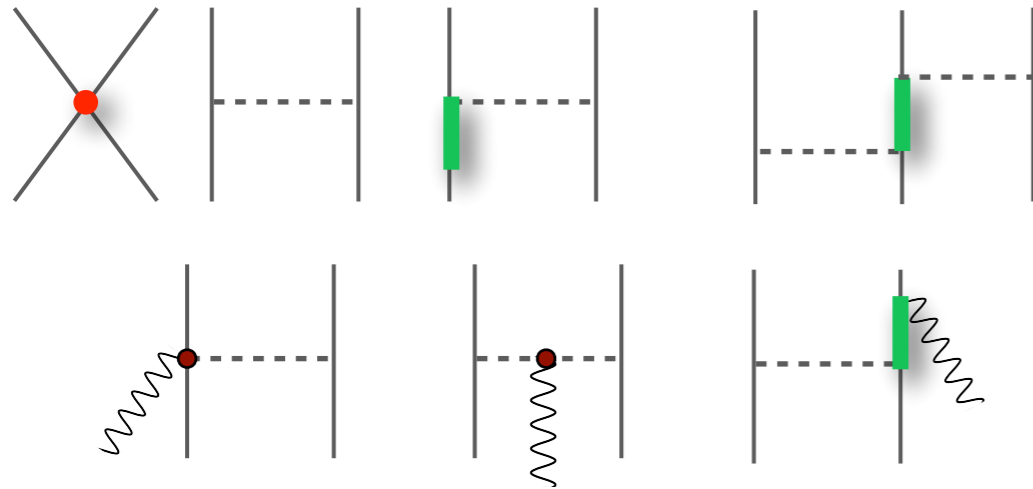
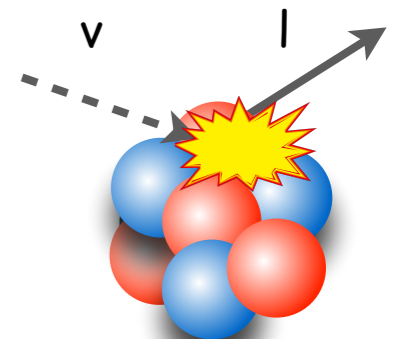### Accurate nuclear many-body methods

$$H|\Psi_n\rangle = E_n|\Psi_n\rangle$$

$$J_{mn} = \langle\Psi_m|J|\Psi_n\rangle$$

Quantum Chromodynamcs

Nuclei and electroweak interactions
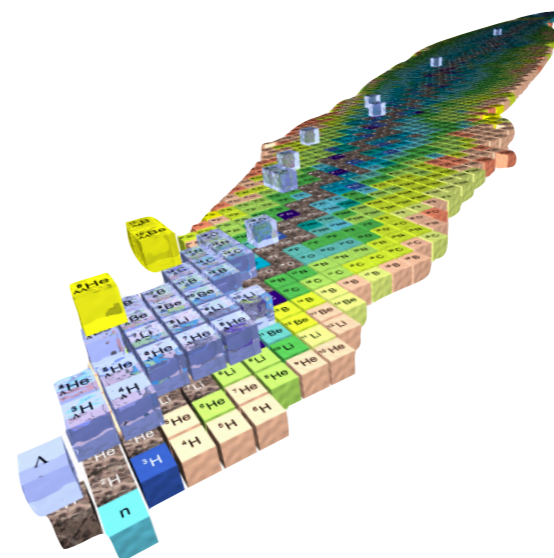
The basic model of nuclear theory

Effective field theories are the link between QCD and nuclear observables. At low energy, the effective degrees of freedom are pions and nucleons:

$$H = \sum_i \frac{\mathbf{p}_i^2}{2m} + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \dots$$

1-body   **2-body**   **3-body**

R. Machleidt , D. R. Entem,
Phys.Rept.503:1-75,2011

E. Epelbaum, (Lectures), arXiv:1001.3229
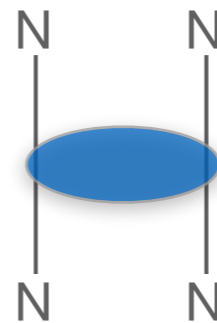
Different strategies to construct two- and three-body interactions

✤ Chiral Effective Field Theory interactions

✤ Phenomenological potentials

Nucleon-nucleon potential

Aoki et al. Comput.Sci.Disc. 1(2008)015009

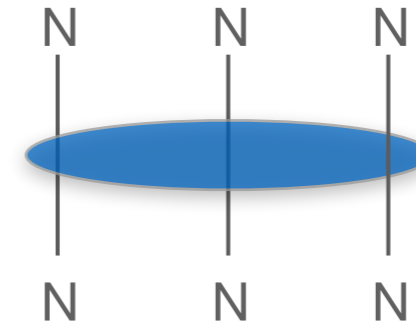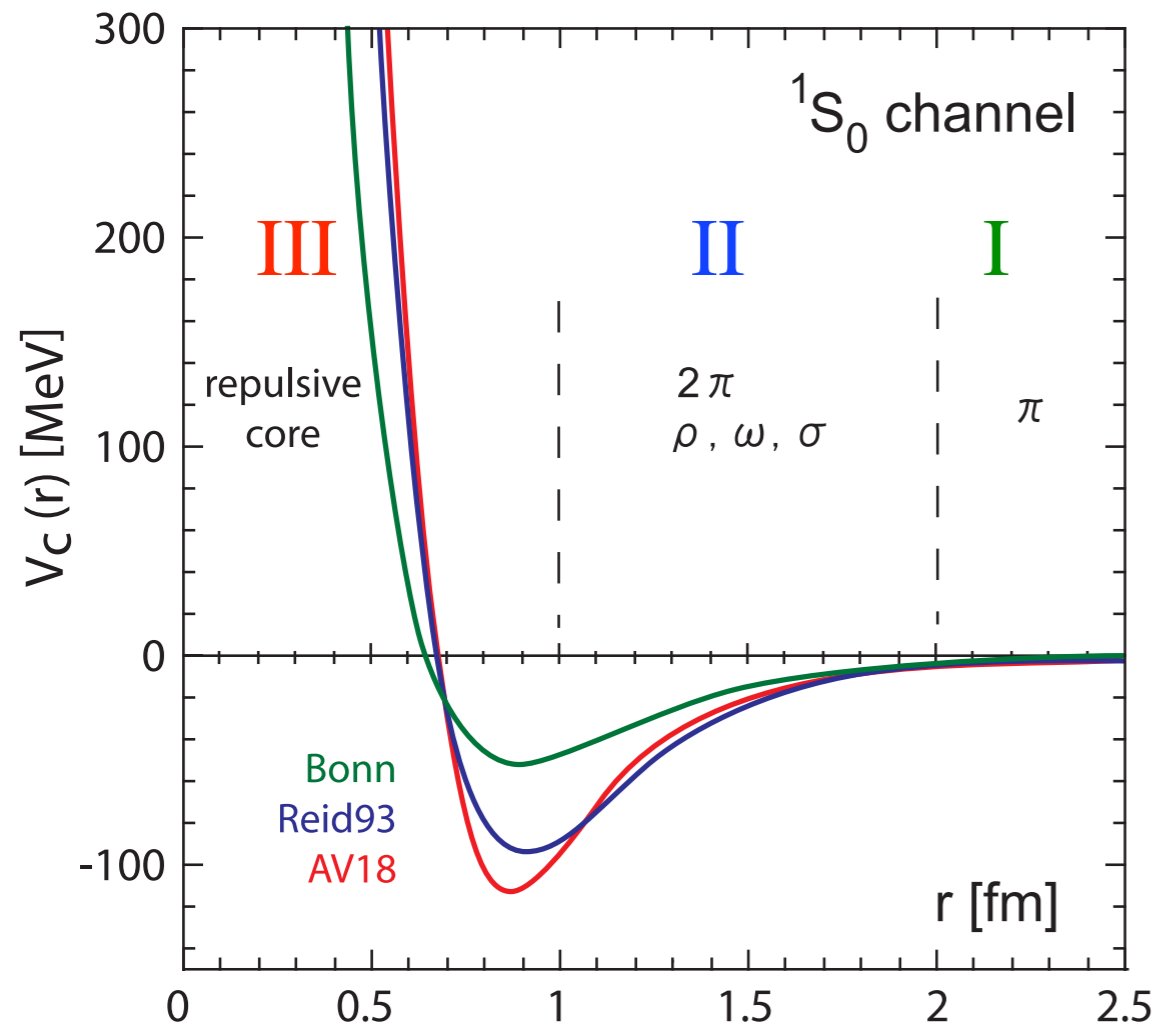Long range part: One π exchange

Range: $\dfrac{1}{m_\pi} \sim 1.4$ fm

Medium range part: Two π exchange

Range: $\dfrac{1}{2m_\pi} \sim 0.7$ fm

σ, ρ, ω exchange

2π exchange

Short range part: Repulsive core

I : OPEP
II : σ, ρ, ω
III : quark ?

(Taketani)

Bonn PRC 63, 024001, 2001

Reid93 PRC 49, 2950, 1994

AV18: Wiringa PRC 51, 38, 1995

Phenomenological potential: av18 + IL7

Phenomenological potentials explicitly include the **long-range one-pion exchange interaction** and a set of **intermediate- and short-range phenomenological terms**

- **Argonne v₁₈** is a finite, local, configuration-space potential controlled by ~4300 np and pp scattering data below 350 MeV of the Nijmegen database

$$v_{ij} = \sum_{p=1}^{18} v^p(r_{ij}) O^p_{ij}$$

- Phenomenological three-nucleon interactions, like the **Illinois 7**, effectively include the lowest nucleon excitation, the Δ(1232) resonance, end other nuclear effects

$$V^{3N}_{ijk} = A^{PW}_{2\pi} O^{2\pi,PW}_{ijk} + A^{SW}_{2\pi} O^{2\pi,SW}_{ijk} + A^{\Delta R}_{3\pi} O^{3\pi,\Delta R}_{ijk} + A^R O^R_{ijk}$$

The parameters of the AV18 + IL7 are fit to properties of **exactly solvable light nuclear systems**.

Chiral effective field theory

Chiral Hamiltonians exploits the (approximate) broken chiral symmetry of QCD

Identify the soft and hard scale of the problem $\quad \mathcal{L}^{(n)} \sim \left(\dfrac{q}{\Lambda_b} \right)^n$ ~ 100 MeV soft scale

~ 1 GeV hard scale

Design an organizational scheme that can distinguish between more and less important terms:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}^{(0)} + \mathcal{L}^{(1)} + \mathcal{L}^{(2)}$$

Contact interactions lead to LEC:

Short range two-nucleon interaction fit to deuteron and NN scattering

Three nucleon interactions fitted on light nuclei

Long-range LEC are determined from π-nucleon scattering

H. Hergert, Front. in Phys. 8, 379 (2020)

	NN	**3N**
LO $(Q/\Lambda_\chi)^0$		
NLO $(Q/\Lambda_\chi)^2$		
NNLO $(Q/\Lambda_\chi)^3$		
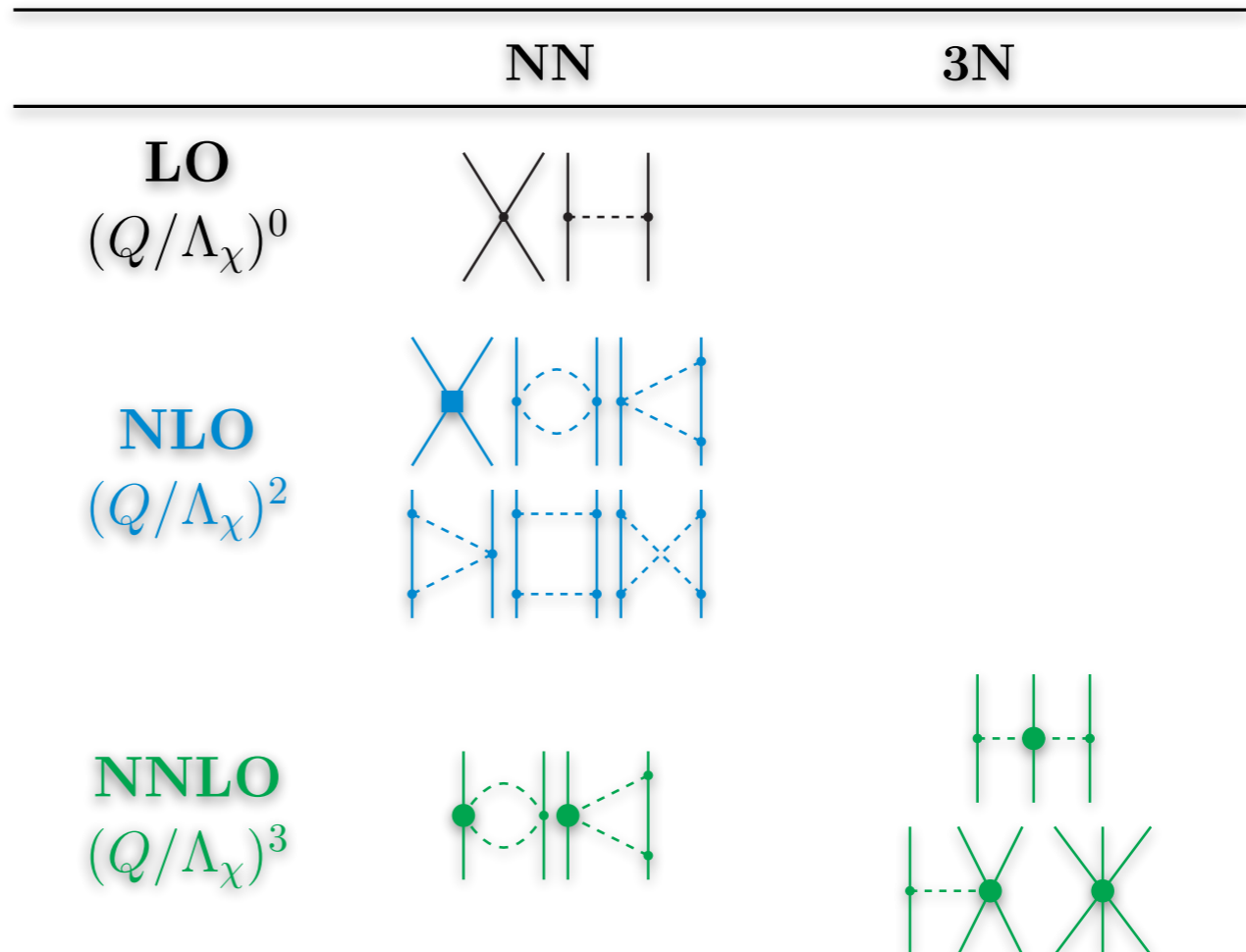
Formulate statistical models for uncertainties: Bayesian estimates of EFT errors

S. Wesolowski, et al, PRC 104, 064001 (2021)

Noemi Rocco, nrocco@fnal.gov

Fermilab

The basic model of nuclear theory
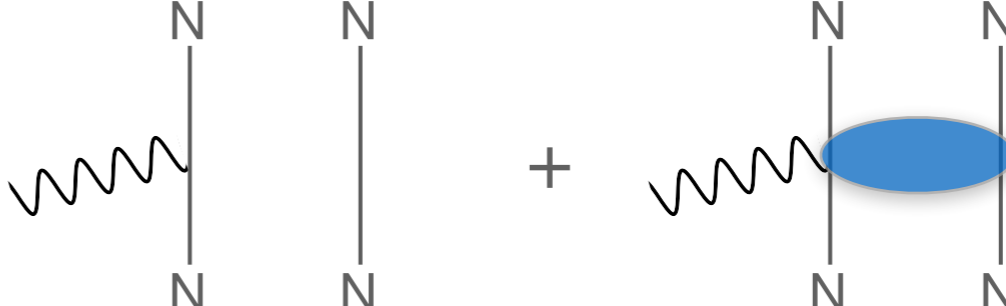
The current operator describes how the external probe interacts with nucleon, nucleons pairs, create new particles …

The structure of the current operator is constrained by the Hamiltonian through the continuity equation

$$\nabla \cdot \mathbf{J}_{\text{EM}} + i[H, J^0_{\text{EM}}] = 0 \qquad \qquad \left[v_{ij}, j^0_i\right] \neq 0$$

The Hamiltonian structure implies that the current operator includes one and two-body contributions

$$J^\mu(q) = \sum_i j^\mu_i + \sum_{i<j} j^\mu_{ij} + \dots$$

- ❖ Chiral Effective Field Theory Electroweak many-body currents

- ❖ "Phenomenological" Electroweak many-body currents

Variational Monte Carlo

In variational Monte Carlo, one **assumes a suitable form for the trial wave function**

$$|\Psi_T\rangle = \mathcal{F}|\Phi\rangle$$

Φ : Mean field component; slater determinant of single-particle orbitals

$\mathcal{F}$: correlations (2b & 3b) induced by H

The correlation operator reflects the spin-isospin dependence of the nuclear interaction

$$\mathcal{F} \equiv \left(\mathcal{S} \prod_{i<j} F_{ij} \right) \qquad F_{ij} \equiv \sum_p f_{ij}^p O_{ij}^p$$

The best parameters are found by **optimizing the variational energy** $\quad \dfrac{\langle \Psi_T | H | \Psi_T \rangle}{\langle \Psi_T | \Psi_T \rangle} = E_T \geq E_0$
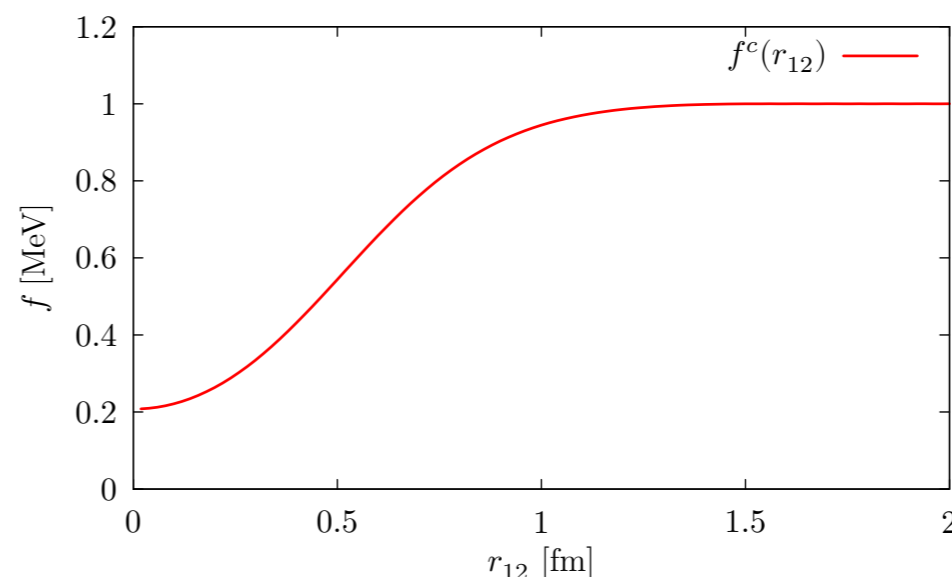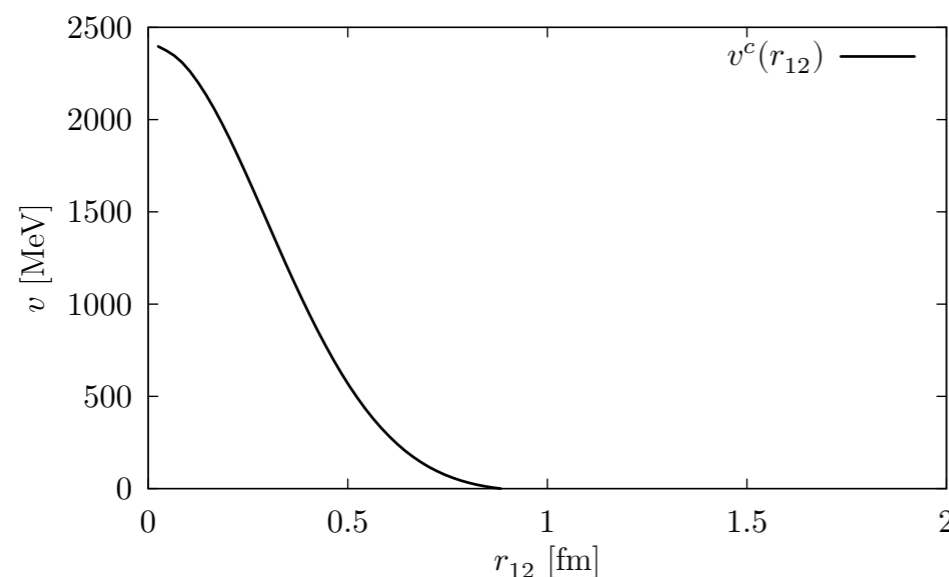
Noemi Rocco, nrocco@fnal.gov

Green's Function Monte Carlo

Any trial wave function can be expanded in the complete set of eigenstates of the the Hamiltonian according to

$$|\Psi_T\rangle = \sum_n c_n |\Psi_n\rangle \qquad\qquad H|\Psi_n\rangle = E_n|\Psi_n\rangle$$

GFMC overcomes the limitations of the variational wave-function by using an imaginary-time projection technique to **projects out the exact lowest-energy state**

$$\lim_{\tau\to\infty} e^{-(H-E_0)\tau}|\Psi_T\rangle = \lim_{\tau\to\infty} \sum_n c_n\, e^{-(E_n-E_0)\tau}|\Psi_n\rangle = c_0|\Psi_0\rangle$$

The direct calculation of the imaginary-time propagator for strongly-interacting systems involves prohibitive difficulties

J. Carlson , et al. Rev. Mod. Phys. 87 (2015) 1067

The imaginary-time evolution is broken into N small imaginary-time steps, and complete sets of states are inserted

$$e^{-(H-E_0)\tau}|\Psi_V\rangle = \int dR_1 \dots dR_N |R_N\rangle\langle R_N|e^{-(H-E_0)\Delta\tau}|R_{N-1}\rangle \dots \langle R_2|e^{-(H-E_0)\Delta\tau}|R_1\rangle \Psi_V(R_1)$$

Short Time Propagator

Noemi Rocco, nrocco@fnal.gov

Green's Function Monte Carlo

Any trial wave function can be expanded in the complete set of eigenstates of the the Hamiltonian according to

$$|\Psi_T\rangle = \sum_n c_n|\Psi_n\rangle \qquad\qquad H|\Psi_n\rangle = E_n|\Psi_n\rangle$$

GFMC overcomes the limitations of the variational wave-function by using an imaginary-time projection technique to **projects out the exact lowest-energy state**

$$\lim_{\tau\to\infty} e^{-(H-E_0)\tau}|\Psi_T\rangle = \lim_{\tau\to\infty} \sum_n c_n\, e^{-(E_n-E_0)\tau}|\Psi_n\rangle = c_0|\Psi_0\rangle$$

^{4}He

E_0=-28.3 MeV
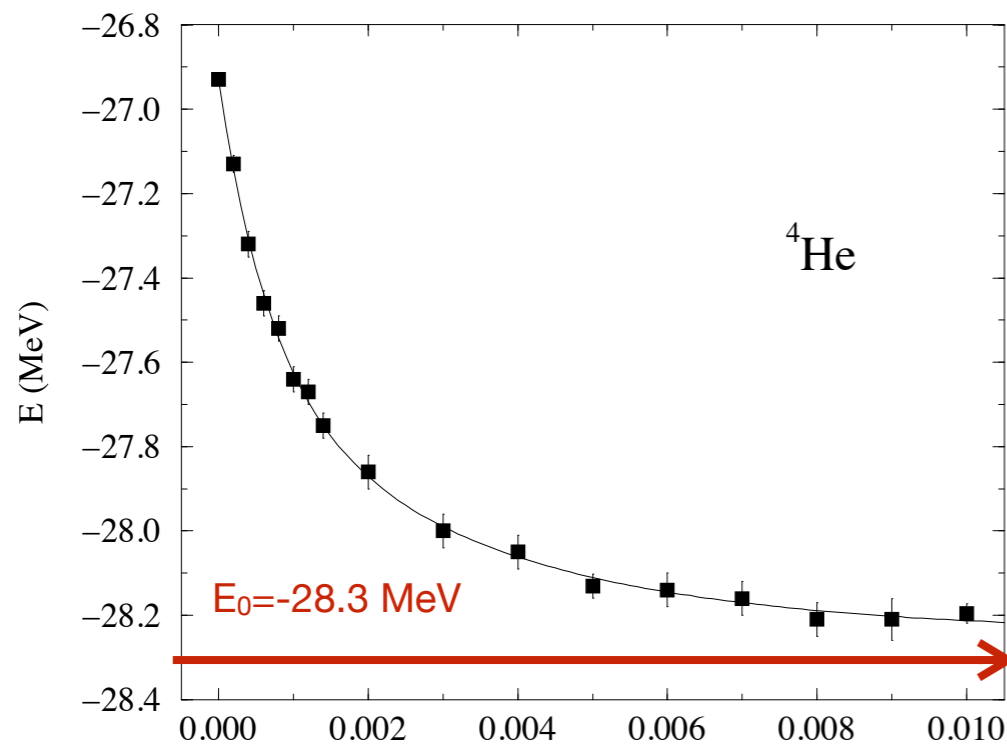
B. Pudliner et al., PRC **56**, 1720 (1997)

The computational cost of the calculation is
2^A x A!/(Z!(A-Z)!)

$$|S\rangle = \begin{pmatrix} s\uparrow\uparrow\uparrow \\ s\uparrow\uparrow\downarrow \\ s\uparrow\downarrow\uparrow \\ s\uparrow\downarrow\downarrow \\ s\downarrow\uparrow\uparrow \\ s\downarrow\uparrow\downarrow \\ s\downarrow\downarrow\uparrow \\ s\downarrow\downarrow\downarrow \end{pmatrix}$$

Noemi Rocco, nrocco@fnal.gov

🎇 Fermilab

Solve the Many Body Nuclear problem

Develop Computational Methods to solve numerically

$$H\Psi(\mathbf{R}; s_1 \ldots s_A, \tau_1 \ldots \tau_A) = E\Psi(\mathbf{R}; s_1 \ldots s_A, \tau_1 \ldots \tau_A)$$

Quantum Monte Carlo techniques are suitable to solve the Schroedinger equation of medium nuclei
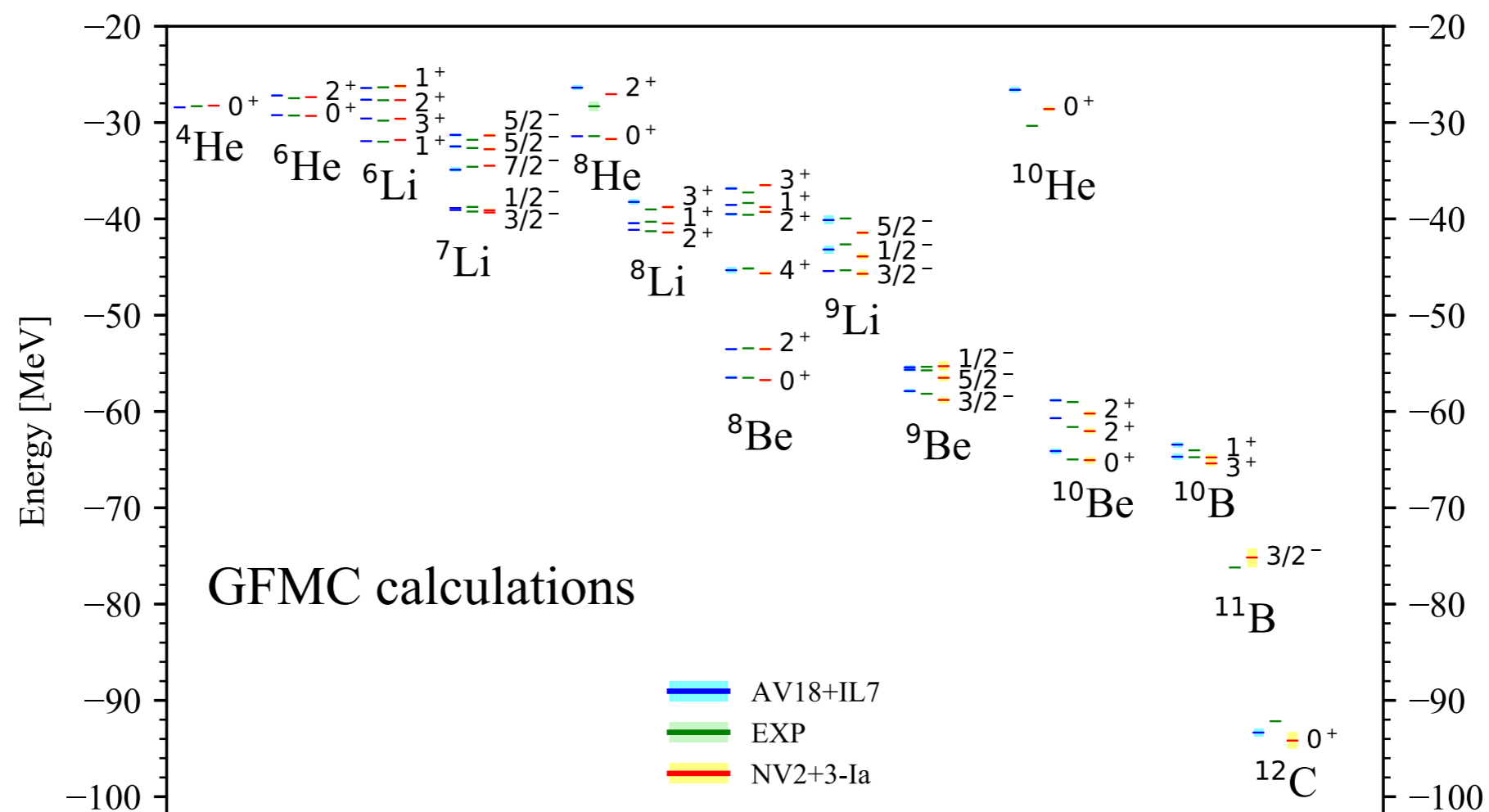
M. Piarulli, et al. Phys.Rev.Lett. 120 (2018) 5, 052503

Probing the target structure

The interaction depends on the **mediator** energy (dubbed as energy transfer ω)

Higher energy transfer = smaller de Broglie wavelength = the probe can resolve the structure inside the nucleon

Halzen & Martin

Lepton-nucleus cross section

$$\frac{d\sigma}{d\Omega dE'}$$

QE

MEC

RES

DIS

Coherent

QE

MEC

RES

DIS

$$\omega_e \sim \frac{q^2}{2m}$$

Energy transfer ω

Different reaction mechanisms contributing to lepton-nucleus cross section
—fixed value of the beam energy (monochromatic)

In neutrino experiments these contributions are not nicely separated

🔷 Fermilab
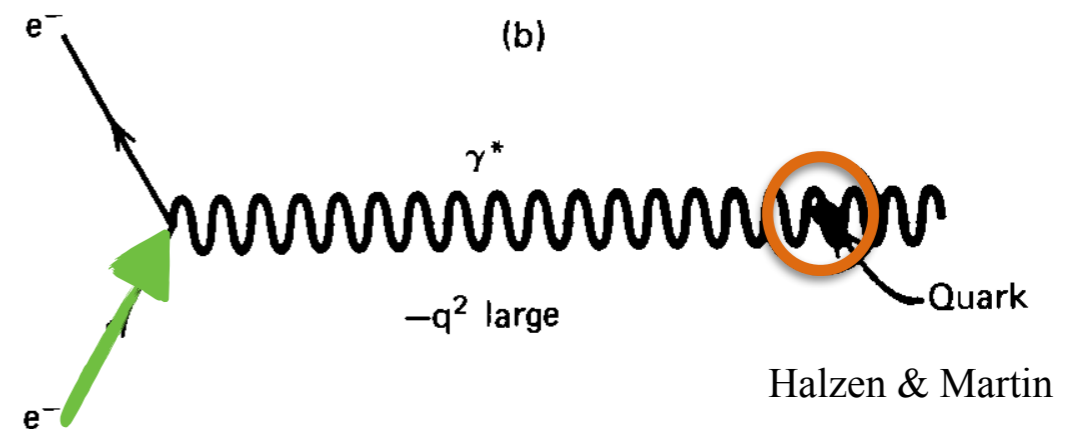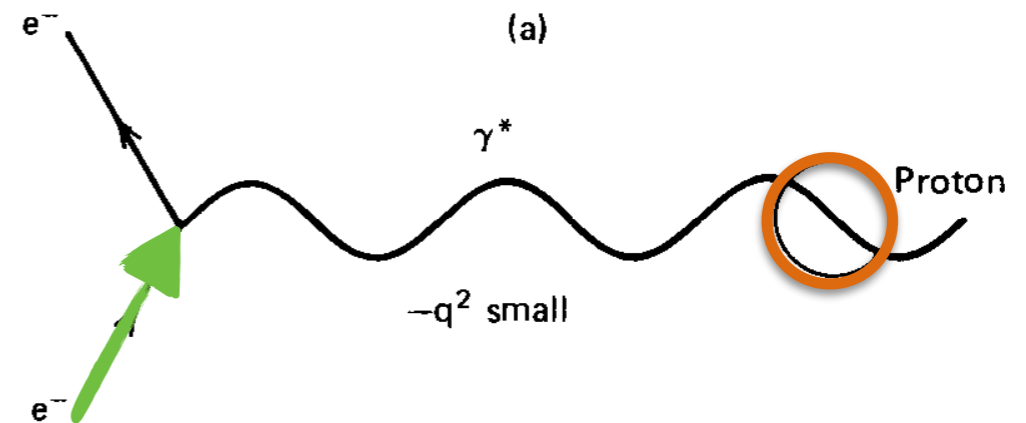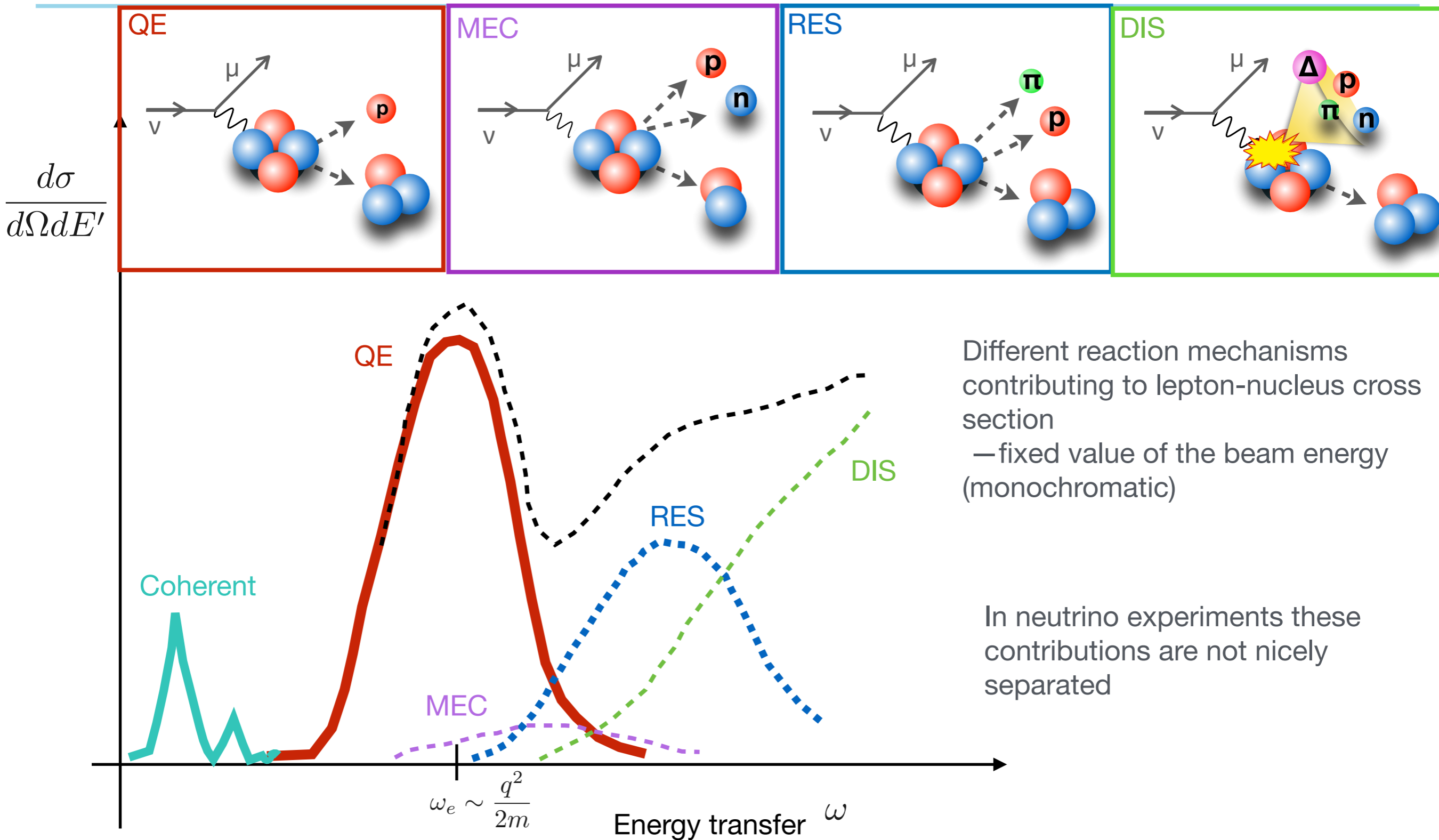
Coherent elastic ν-Nucleus scattering (CEvNS)

- The first detection of CEvNS was published by the COHERENT collaboration in 2017, and opened an exciting chapter of using CEvNS to test the Standard Model and search for new physics

- Neutral-current process that arises when the momentum transfer in the neutrino-nucleus interaction is less than the inverse of the size of the nucleus Ev ~ 50 MeV.

- The interaction is mediated by the Z-boson, with its vector component leading to the coherent enhancement

CEvNS Cross Section:

$$\frac{d\sigma}{dT} = \frac{G_F^2}{\pi} M_A \left[1 - \frac{T}{E_i} - \frac{M_A T}{2E_i^2} \right] \frac{Q_W^2}{4} F_W^2(q)$$

Nuclear recoil: $\quad T \in \left[0, \dfrac{2E_i^2}{(M_A + 2E_i)} \right]$

- Final state nucleus stays in its ground state
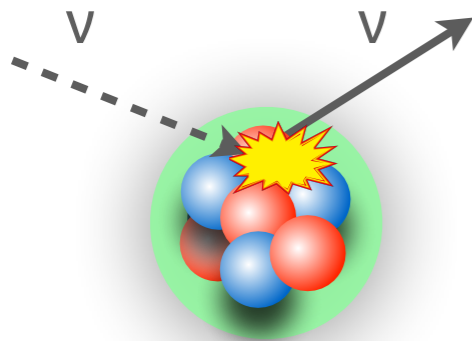- Tiny recoil energy, large cross section
- Signal: keV energy nuclear recoil

Weak nuclear charge: $\quad Q_W^2 = [g_n^V N + g_p^V Z]^2 = [N - \left(1 - 4\sin^2\theta_W\right) Z]^2$

The proton contribution is suppressed with respect to the neutron one

🔷 **Fermilab**

Noemi Rocco, nrocco@fnal.gov

Coherent elastic ν-Nucleus scattering (CEvNS)

- Due to the suppression of the proton weak charge, main contribution comes from the neutron $d\sigma \propto N^2$

Weak Form Factor:
$$F_W(q^2) = \frac{1}{Q_W}\left[ZQ_W^p F_p(q^2) + NQ_W^n F_n(q^2)\right]$$

$$F_p(q^2) = \frac{4\pi}{Z}\int dr\ r^2\ \frac{\sin(qr)}{qr}\ \rho_p(r) \qquad\qquad F_n(q^2) = \frac{4\pi}{N}\int dr\ r^2\ \frac{\sin(qr)}{qr}\ \rho_n(r)$$

- $\rho_{p(n)}(r)$ is the proton (neutron) density distribution for a given nucleus

- Helm phenomenological parametrization: $\quad F_{\text{Helm}}(q^2) = \frac{3j_1(qR_0)}{qR_0}e^{-q^2s^2/2}$
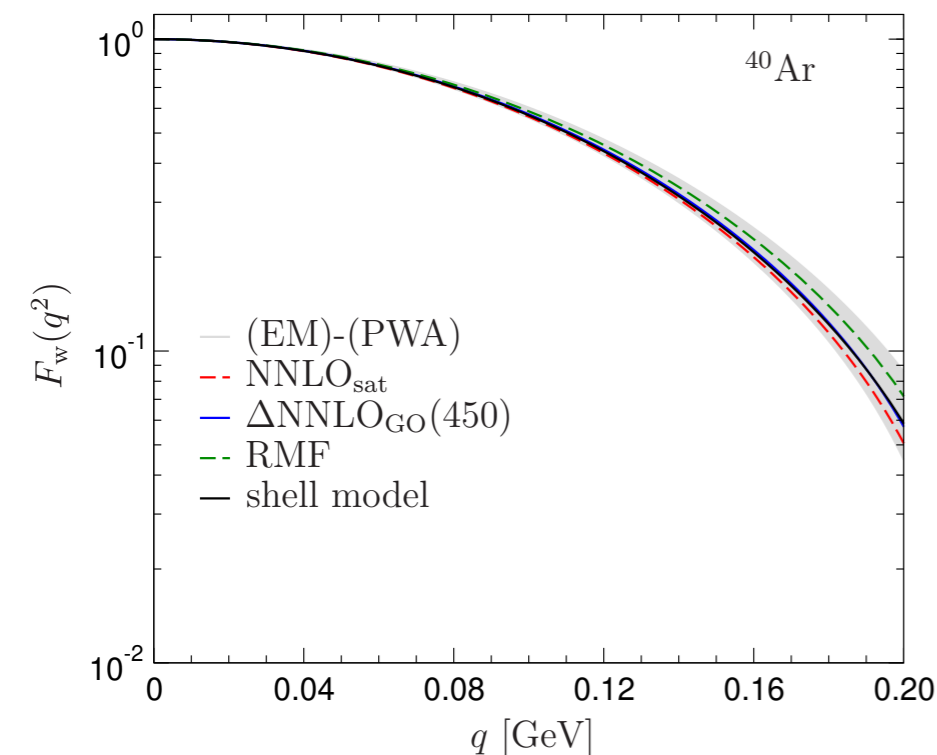
The nucleon distribution is given by the convolution of a uniform density with radius R_0 and a Gaussian profile with width s, the surface thickness.

This parameterization needs to assume a value for the neutron radius, related to R_0 and only try to capture the leading nuclear responses, with the neutron distribution largely unconstrained

Noemi Rocco, nrocco@fnal.gov

🎇 Fermilab

Coherent elastic ν-Nucleus scattering (CEvNS)

Theoretical tools for neutrino scattering, Contribution to: 2022 Snowmass Summer Study

Retaining all responses that at least display coherent enhancement, $F_W(q^2)$ also receives the contribution of **spin-orbit corrections**, which add coherently for nucleons with spin aligned with the orbital angular momentum.

Theoretical predictions for the weak form factor of ^{40}Ar from Relativistic Mean Field, coupled-cluster, and shell-model [20] calculations.

O. Tomalak, P. Machado, V. Pandey, R. Plestid, JHEP 02, 097 (2021)
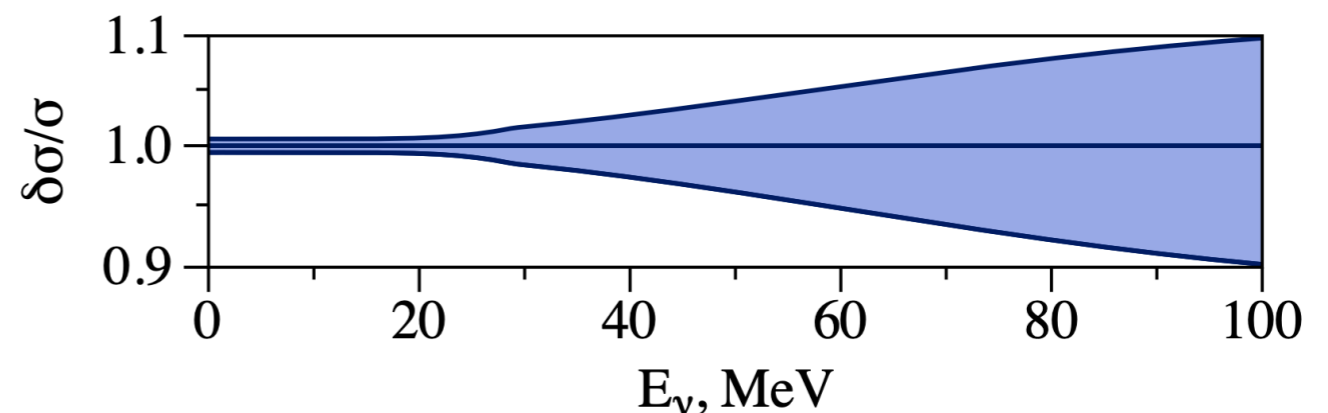
CEvNS cross section needs to be known at percent level precision to allows resolving degeneracies in the standard and non-standard physics observables in CEvNS experiments.

Relative CEvNS cross section theoretical uncertainty

Fermilab

Ab initio Methods

Ab-initio methods (CC, IMSRG, SCGF, QMC, etc) are systematically improvable many-body approaches.

A. Ekström et al, Front. Phys.11 (2023) 29094
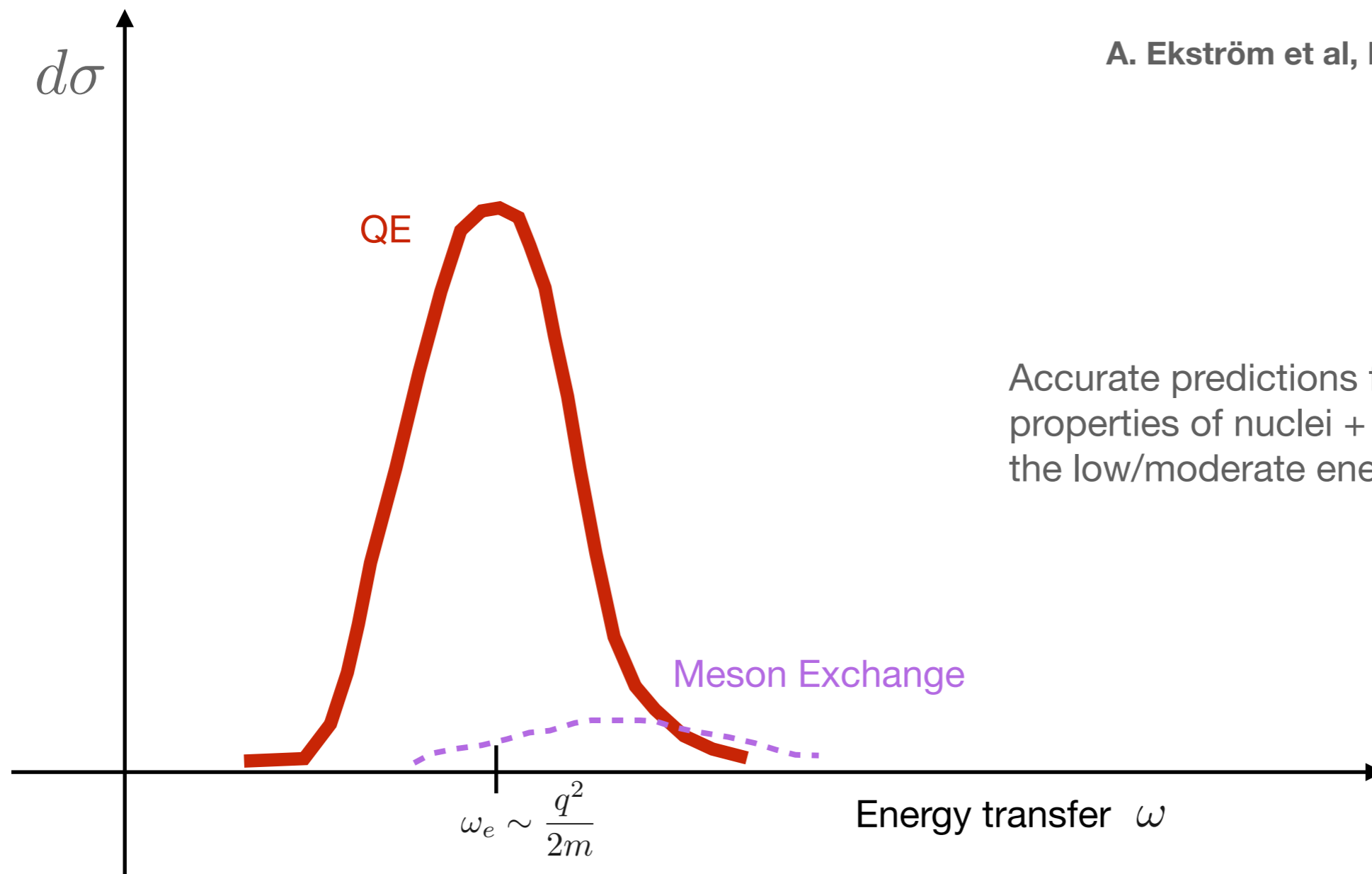
Accurate predictions for ground state properties of nuclei + response functions in the low/moderate energy region

$d\sigma$

QE

Meson Exchange

$\omega_e \sim \dfrac{q^2}{2m}$

Energy transfer ω

Nuclear response function involves evaluating a number of transition amplitudes.
Valuable information can be obtained from the **integral transform of the response function**

$$E_{\alpha\beta}(\sigma, \mathbf{q}) = \int d\omega K(\sigma, \omega) R_{\alpha\beta}(\omega, \mathbf{q}) = \langle \psi_0 | J_\alpha^\dagger(\mathbf{q}) K(\sigma, H - E_0) J_\beta(\mathbf{q}) | \psi_0 \rangle$$

Inverting the integral transform is a complicated problem   A. Lovato et al, PRL117 (2016), 082501, PRC97 (2018), 022502
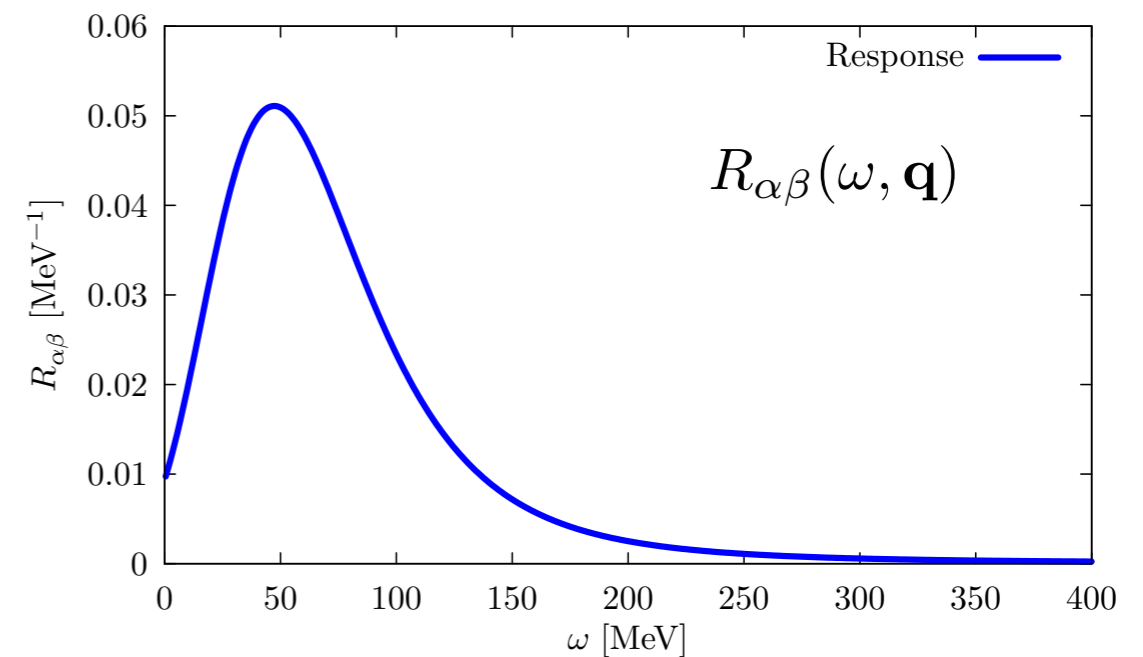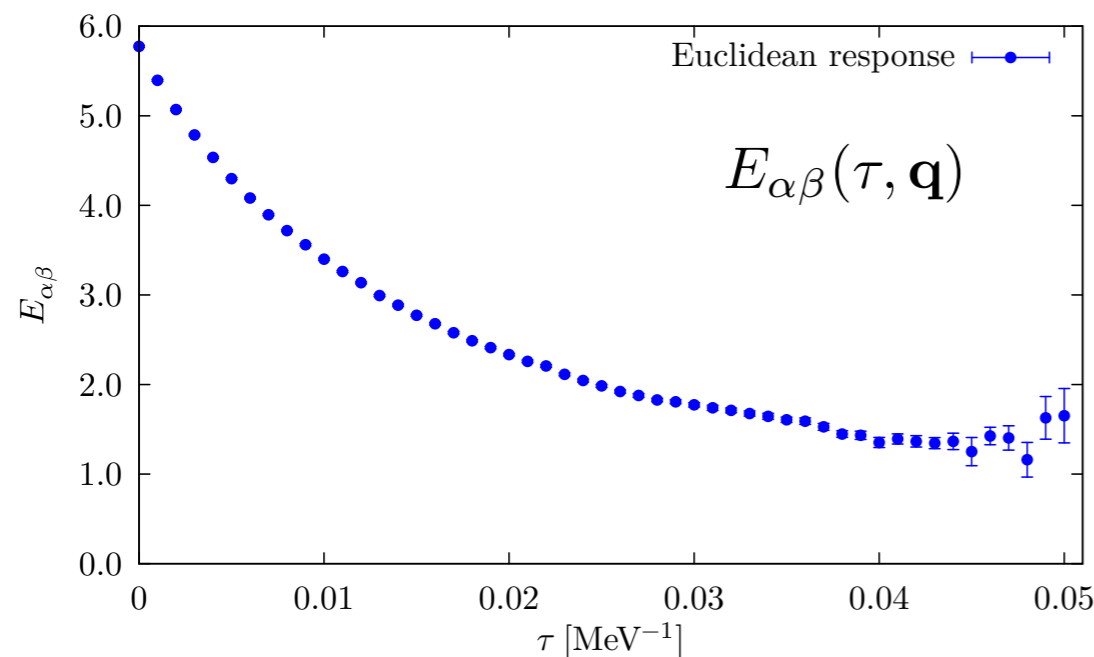
Same problem applies to different realm physics for example lattice QCD

Green's Function Monte Carlo

^{12}C, q=570 MeV

- – – GFMC O_{1b}
- —— GFMC O_{1b+2b}
- –·–· PWIA
- –○– World data

Alessandro Lovato et al. PRL 117 082501 (2016)
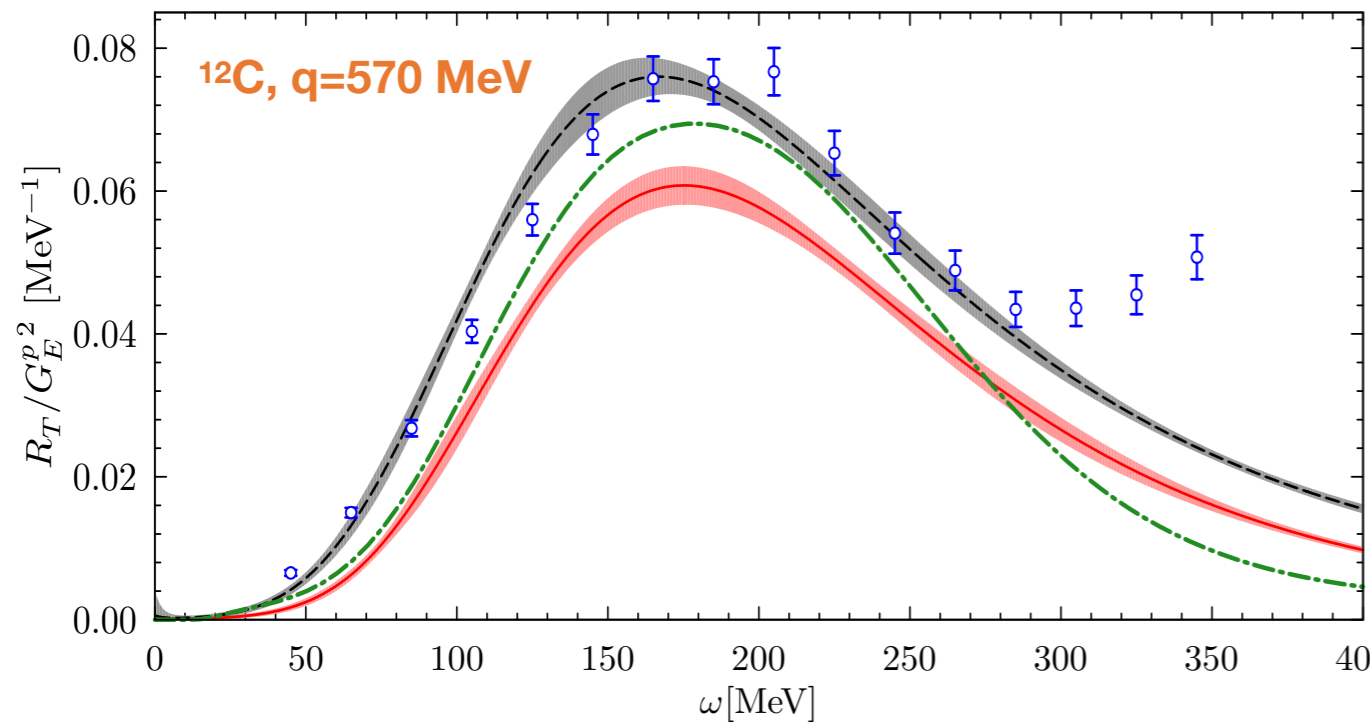

^{12}C(e,e') E_e=620 MeV $\theta = 60°$

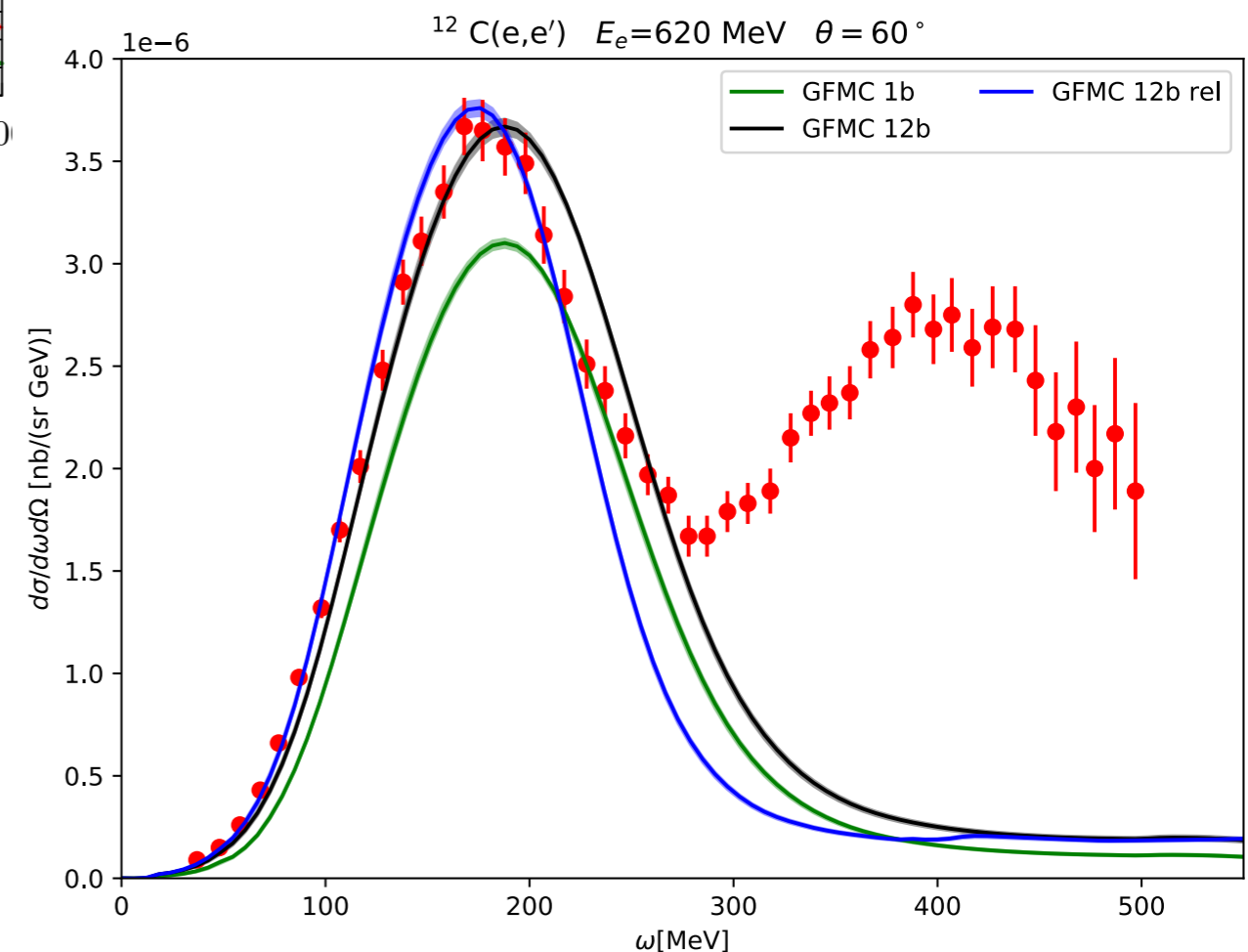— GFMC 1b   — GFMC 12b rel
— GFMC 12b

Limitations:

Medium mass nuclei A < 13

Inclusive results which are virtually correct in the QE

Relies on non-relativistic treatment of the kinematics

Can not handle explicit pion degrees of freedom

A.Lovato, NR, et al, *submitted to Universe*

Fermilab

Study of model dependence in neutrino predictions

MiniBooNE results; study of the dependence on the axial form factor:

Legend:
- GFMC Dipole ($M_A = 1$ GeV)
- GFMC z expansion (D2)
- GFMC z expansion (LQCD)
- MB
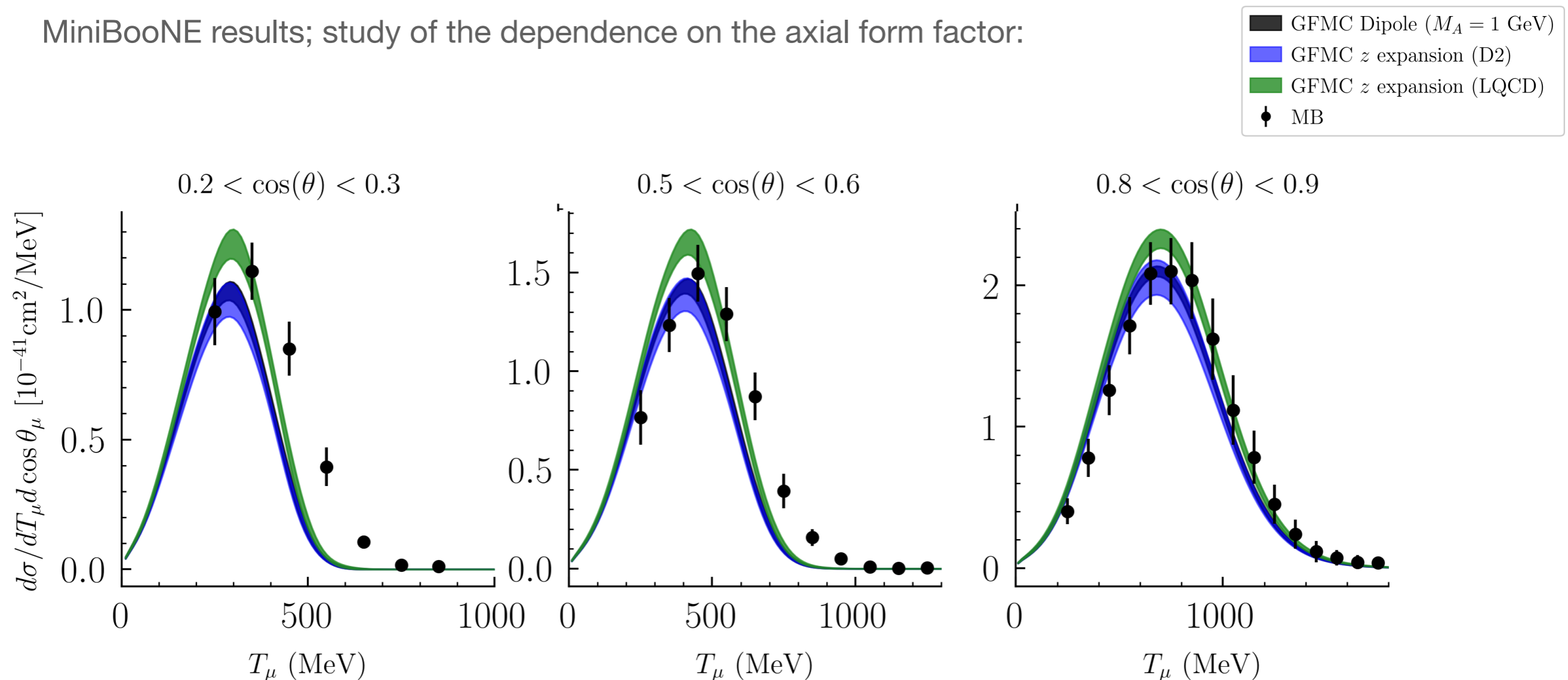
D.Simons, N. Steinberg et al, 2210.02455

MiniBooNE	$0.2 < \cos\theta_\mu < 0.3$	$0.5 < \cos\theta_\mu < 0.6$	$0.8 < \cos\theta_\mu < 0.9$
GFMC Difference in $d\sigma_{\text{peak}}$ (%)	18.6	17.1	12.2

Noemi Rocco, nrocco@fnal.gov

Study of model dependence in neutrino predictions

T2K results; study of the dependence on the axial form factor:

Legend:
- GFMC Dipole ($M_A = 1$ GeV)
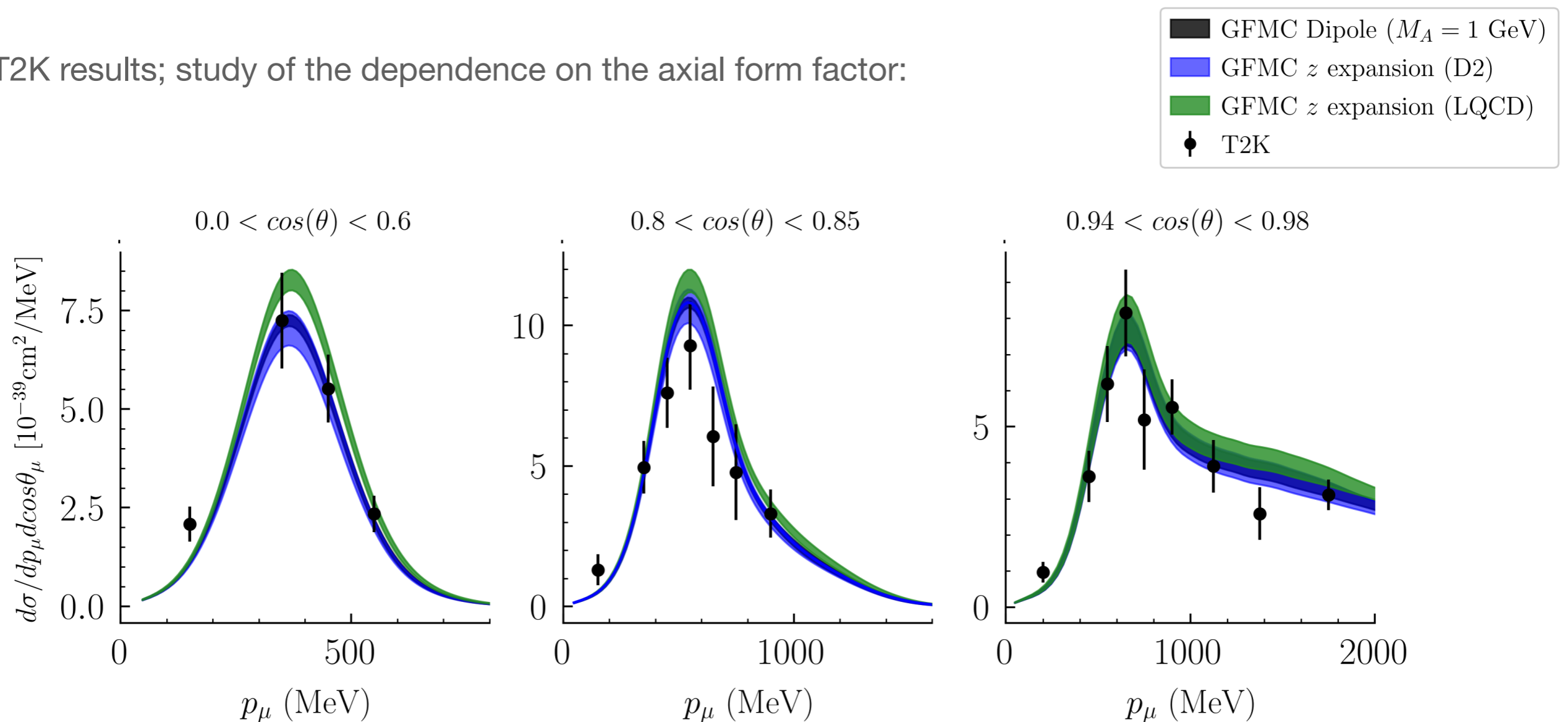- GFMC z expansion (D2)
- GFMC z expansion (LQCD)
- T2K

Panel titles: $0.0 < cos(\theta) < 0.6$, $0.8 < cos(\theta) < 0.85$, $0.94 < cos(\theta) < 0.98$

y-axis: $d\sigma/dp_\mu dcos\theta_\mu$ $[10^{-39}\text{cm}^2/\text{MeV}]$

x-axis: p_μ (MeV)

D.Simons, N. Steinberg et al, 2210.02455

T2K	$0.0 < \cos\theta_\mu < 0.6$	$0.80 < \cos\theta_\mu < 0.85$	$0.94 < \cos\theta_\mu < 0.98$
GFMC difference in $d\sigma_{\text{peak}}$ (%)	15.8	8.0	4.6

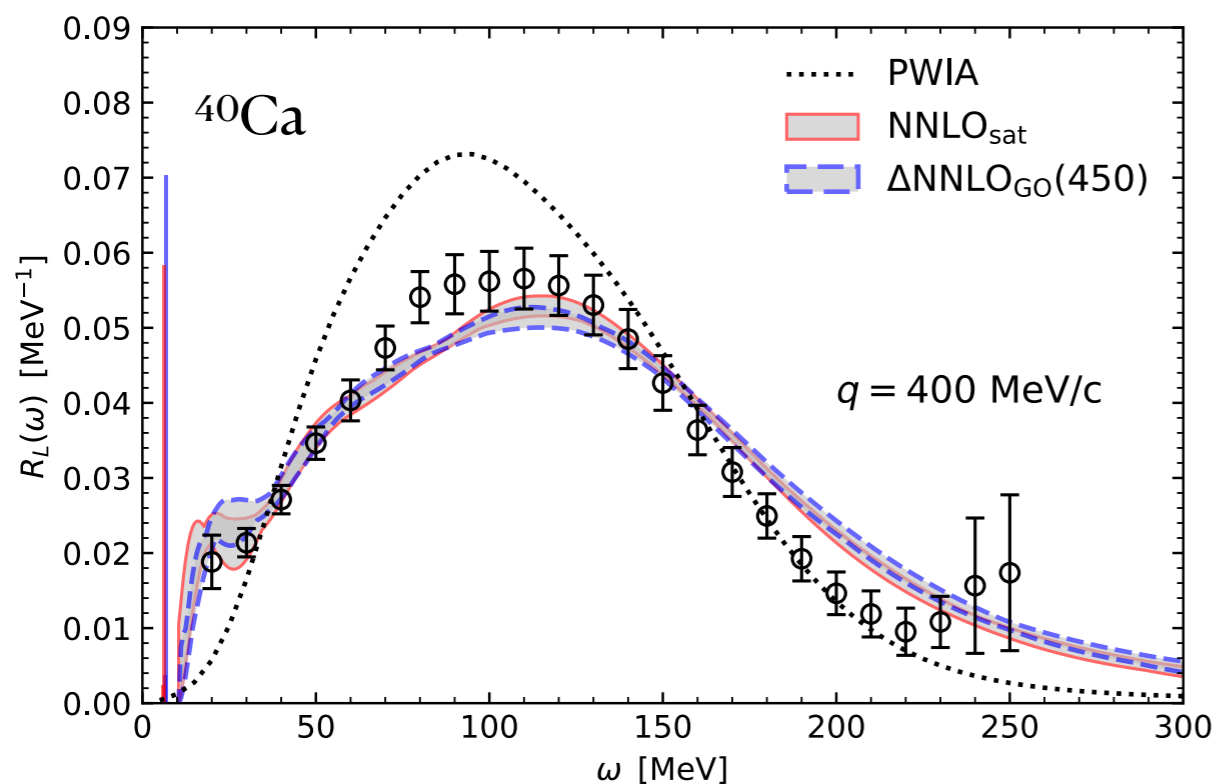🎗 Fermilab

Coupled Cluster Method

Reference state Hartree Fock: $|\Psi\rangle$ Include correlations through e^T operator

Similarity transformed Hamiltonian $e^{-T} H e^{T} |\Psi\rangle = \bar{H} |\Psi\rangle = E |\Psi\rangle$

Expansion in second quantization single + doubles:

$$T = \sum t_a^i a_a^\dagger a_i + \sum t_{ab}^{ij} a_a^\dagger a_b^\dagger a_i a_j + \dots$$

Polynomial scaling with the number of nucleons (predictions for ^{132}Sn and ^{208}Pb)
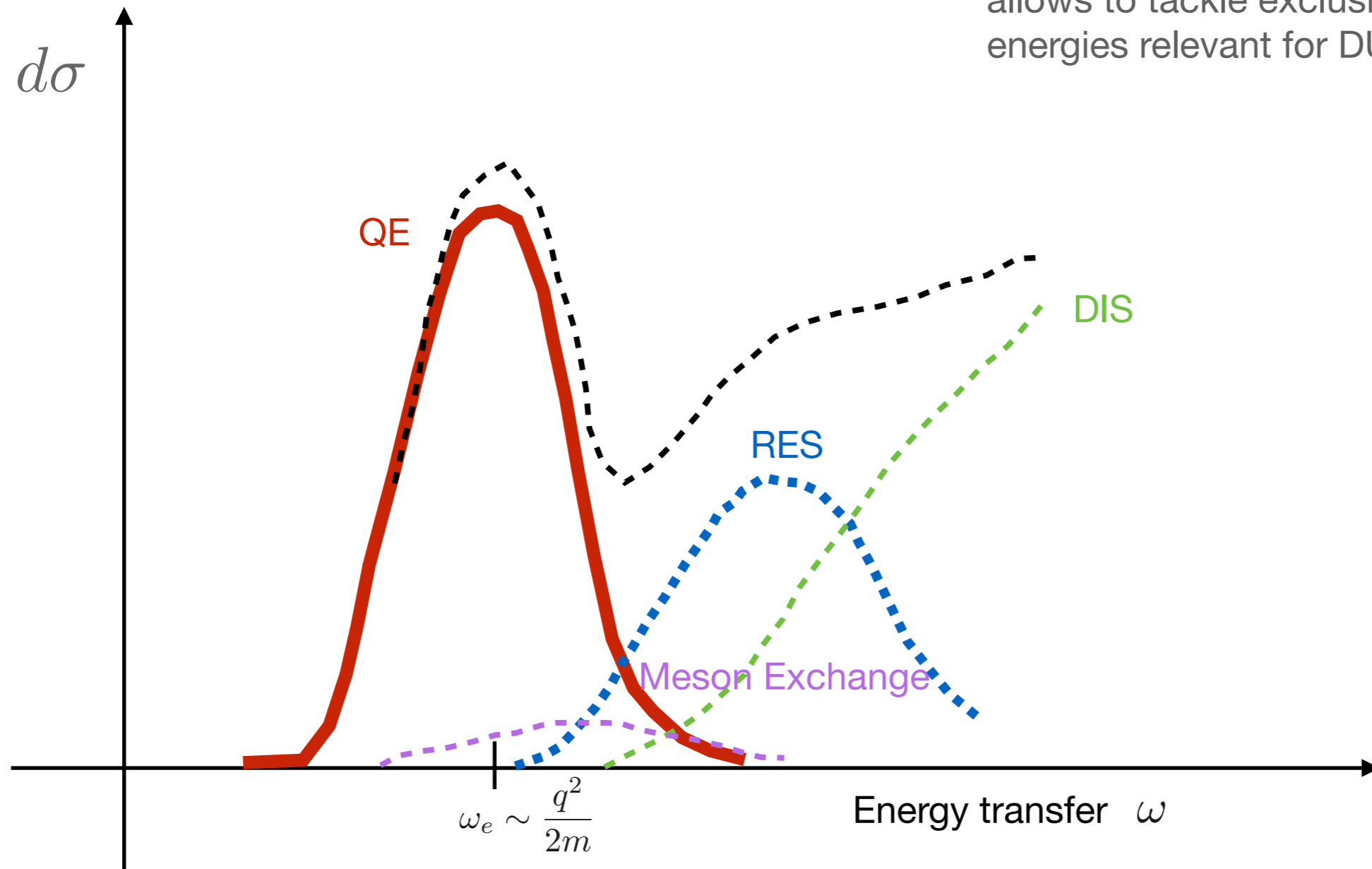
Electroweak response functions obtained using **LIT**

$$K_\Gamma(\omega, \sigma) = \frac{1}{\pi} \frac{\Gamma}{\Gamma^2 + (\omega - \sigma)^2}$$

JES, B. Acharya, S. Bacca, G. Hagen; PRL 127 (2021) 7, 072501

     Noemi Rocco, nrocco@fnal.gov

Factorization Based Approaches

Factorization of the hadronic final states: allows to tackle exclusive channels + higher energies relevant for DUNE

$d\sigma$

QE

DIS

RES

Meson Exchange

$\omega_e \sim \dfrac{q^2}{2m}$

Energy transfer ω

Noemi Rocco, nrocco@fnal.gov

🎗 Fermilab

Short-Time Approximation

❖ Based on Factorization
❖ Retains two-body physics
❖ Response functions are given by the scattering from pairs of fully interacting nucleons that propagate into a correlated pair of nucleons
❖ Allows to retain both two-body correlations and currents at the vertex
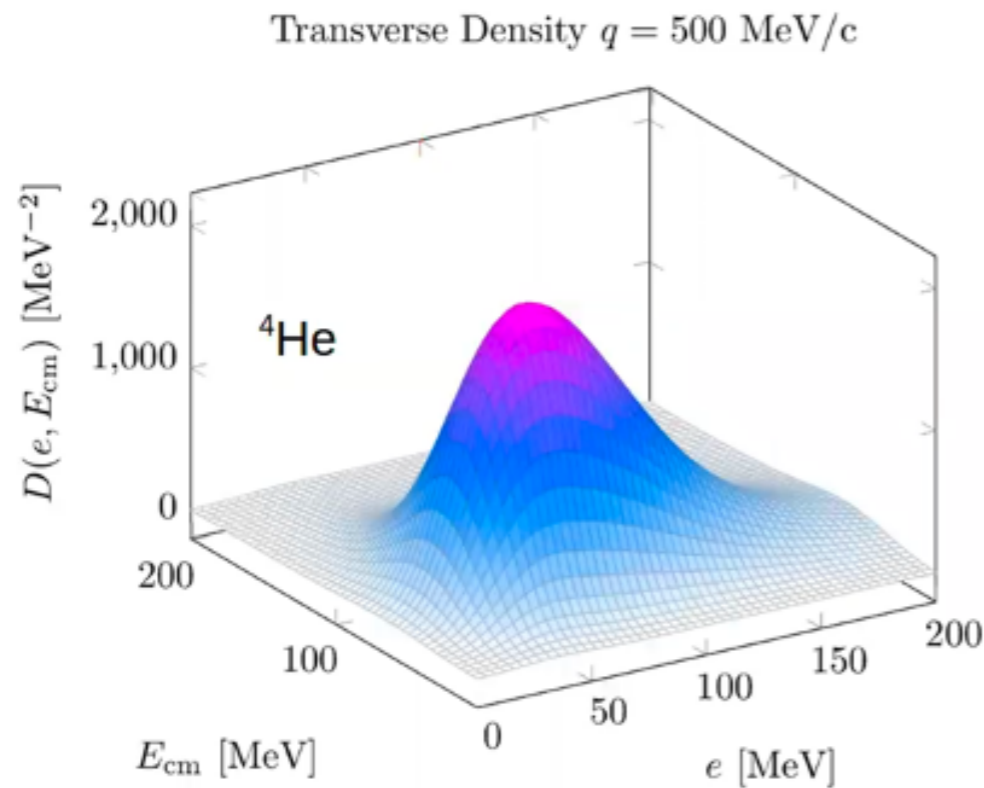❖ Provides "more" exclusive information in terms of nucleon-pair kinematics via the Response Densities
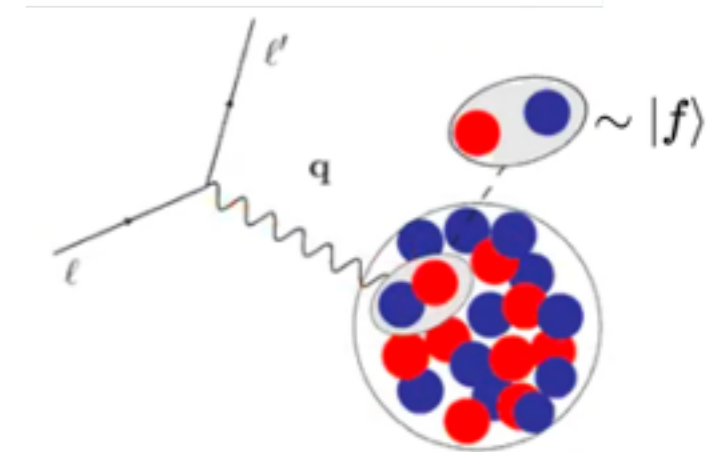
The sum over all final states is replaced by a two nucleon propagator

$$R_\alpha(q,\omega) = \int_{-\infty}^{\infty} \frac{dt}{2\pi} e^{i(\omega+E_i)t} \left\langle \Psi_i \left| O_\alpha^\dagger(\mathbf{q}) e^{-iHt} O_\alpha(\mathbf{q}) \right| \Psi_i \right\rangle$$

The STA restricts the propagation to two active nucleons and allows to compute density functions of the CoM and relative momentum of the pair

$$R^{\mathrm{STA}}(q,\omega) \sim \int \delta(\omega + E_0 - E_f)\, de\, dE_{cm} \mathcal{D}(e, E_{cm}; q)$$

Noemi Rocco, nrocco@fnal.gov

🔷 **Fermilab**

Short-Time Approximation
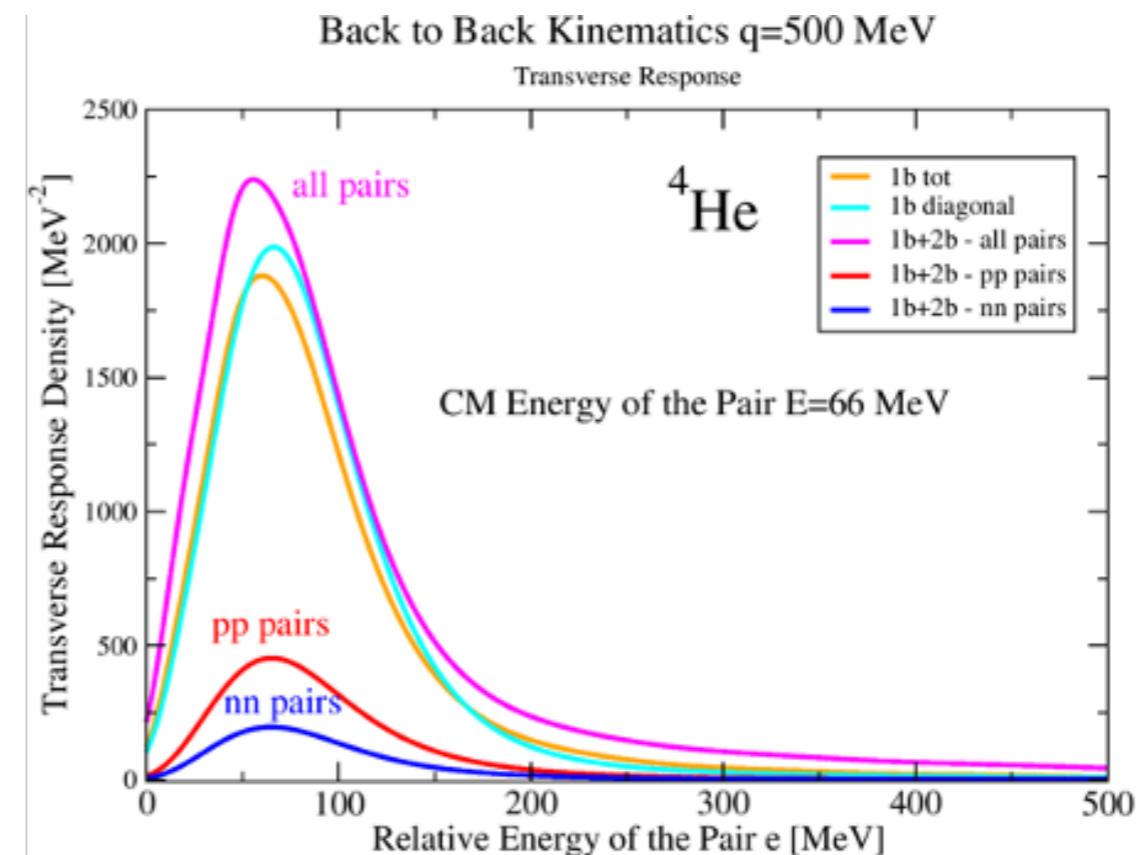

Transverse Density $q = 500$ MeV/c

Pastore et al. PRC101(2020)044612

- ● pp pairs
- ● nn pairs
- ● 1 body tot
- ● all pairs tot

$$R^{STA}(q,\omega) \sim \int \delta(\omega + E_0 - E_f) de \, dE_{cm} \mathcal{D}(e, E_{cm}; q)$$

Electron scattering from 4He:

- ❖ Response density as a function of (E,e)
- ❖ Give access to particular kinematics for the struck nucleon pair
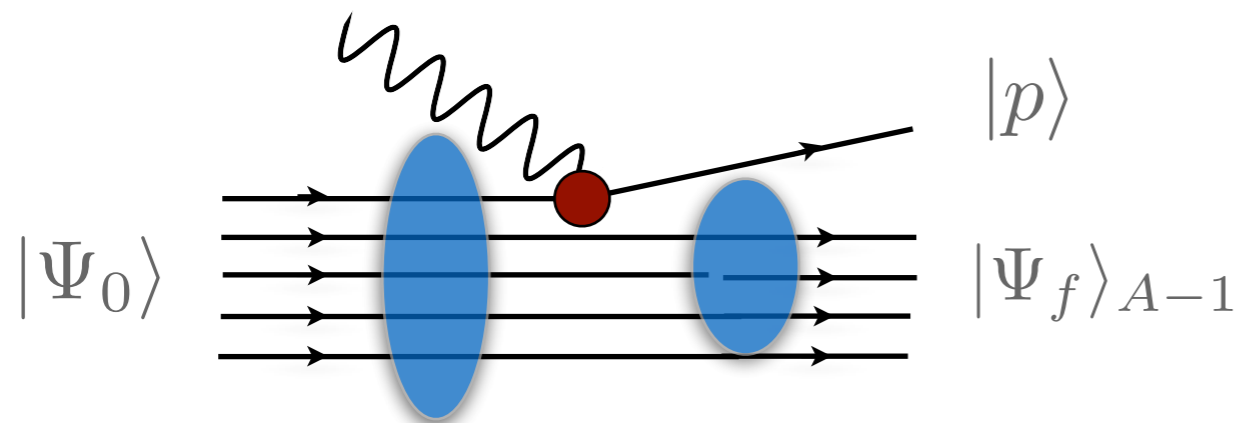

Back to Back Kinematics q=500 MeV

🎇 **Fermilab**

Spectral function approach

At large momentum transfer, the scattering reduces to the sum of individual terms

$$J_\alpha = \sum_i j_\alpha^i \qquad\qquad |\Psi_f\rangle \to |p\rangle \otimes |\Psi_f\rangle_{A-1}$$

$$J^\mu \to \sum_i j_i^\mu \qquad |\psi_f^A\rangle \to |p\rangle \otimes |\psi_f^{A-1}\rangle \qquad E_f = E_f^{A-1} + e(\mathbf{p})$$

The incoherent contribution of the one-body response reads

$$R_{\alpha\beta} \simeq \int \frac{d^3k}{(2\pi)^3} dE P_h(\mathbf{k}, E) \sum_i \langle k|j_\alpha^{i\,\dagger}|k+q\rangle\langle k+q|j_\beta^i|k\rangle\delta(\omega + E - e(\mathbf{k}+\mathbf{q}))$$

The Spectral Function is the imaginary part of the two point Green's Function

Different many-body methods can be adopted to determine it

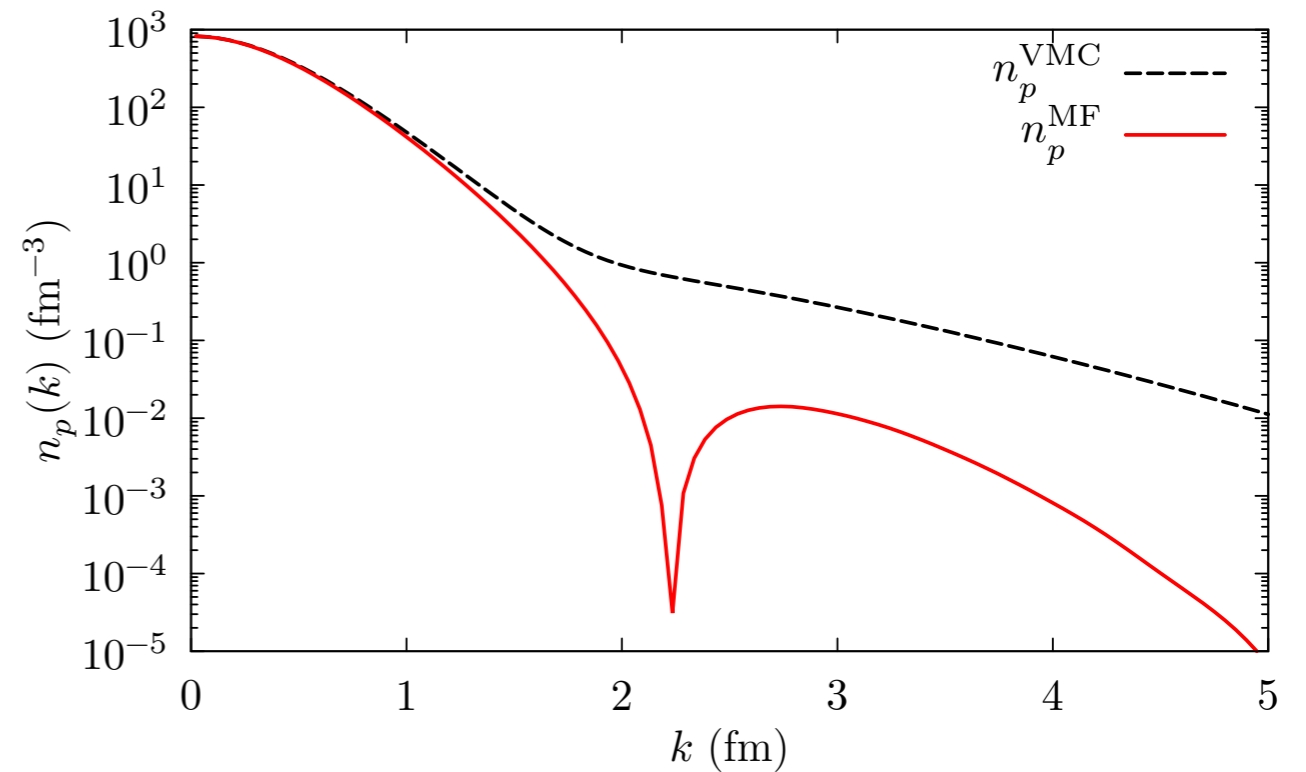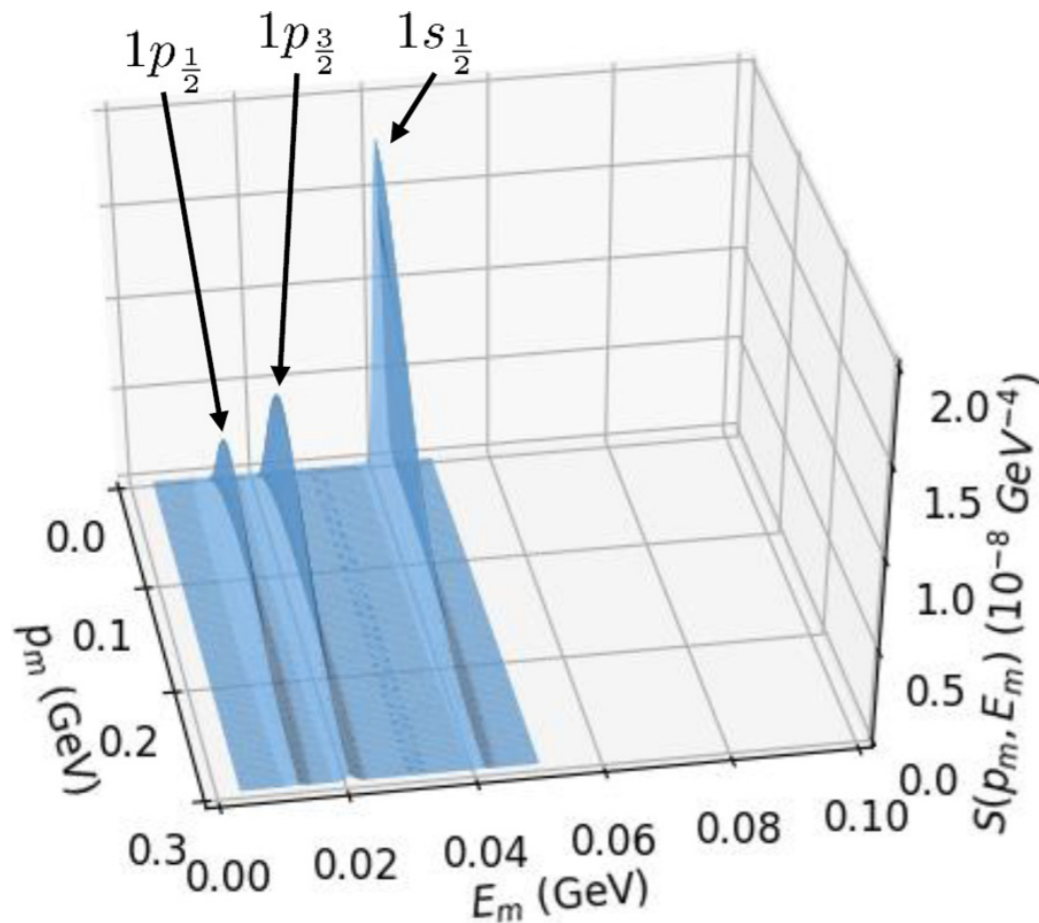**I. Korover, et al Phys.Rev.C 107 (2023) 6, L061301**

L. Andreoli, AL, et al. PRC 105, 014002 (2022)

NR, Frontiers in Phys. 8 (2020) 116

Noemi Rocco, nrocco@fnal.gov

🎇 Fermilab

Spectral function approach

- Single-nucleon spectral function:

$$P_h(\mathbf{k}, E) = P_{\mathrm{MF}}(\mathbf{k}, E) + P_{\mathrm{corr}}(\mathbf{k}, E)$$

$$P_{\mathrm{MF}}(\mathbf{k}, E) = \left| \langle \Psi_0^A | \left[|k\rangle \otimes | \Psi^{A-1}\rangle \right] \right|^2$$

$$\times \delta \left(E - B_0^A + B_0^{A-1} - \frac{k^2}{2M_{A-1}} \right)$$

- Can be derived from IPSM calculations + experiments or computed using ab-initio
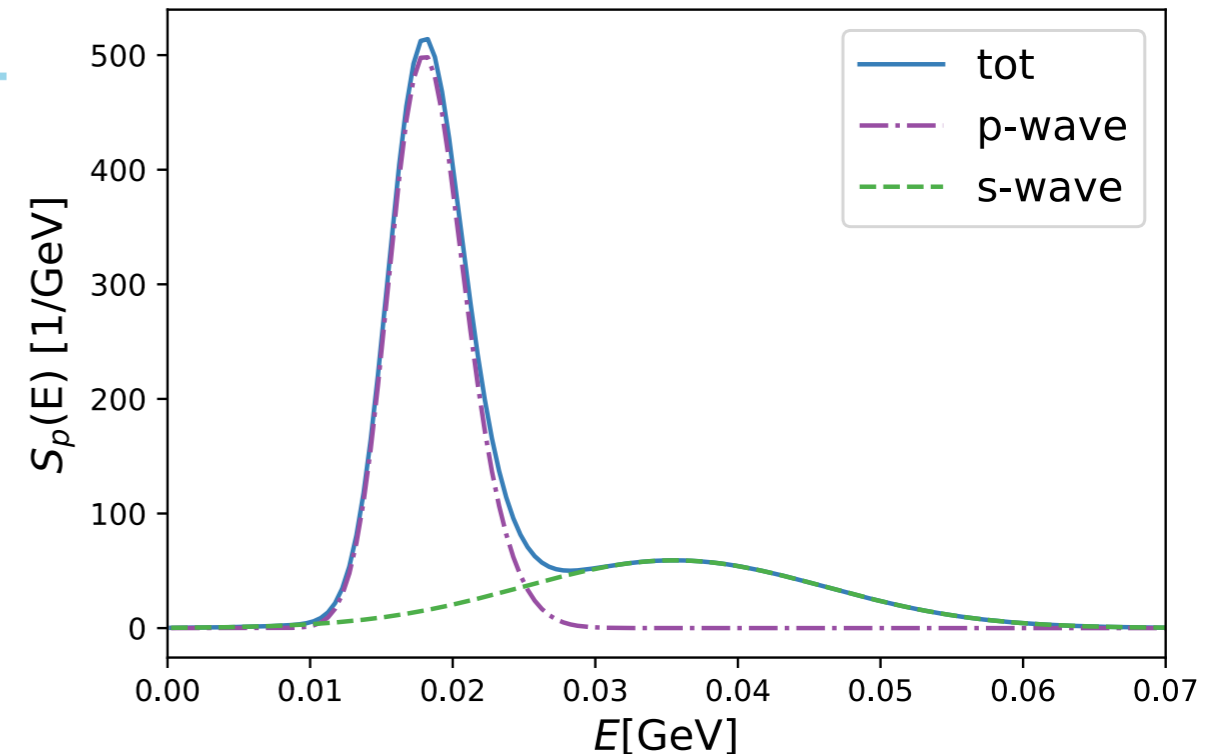
QMC Spectral Fu[nction]

- The p-shell contribution has [been obtained] by FT the radial overlaps:

$$^{12}\mathrm{C}(0^+) \to {}^{11}\mathrm{B}(3/2^-)$$

$$^{12}\mathrm{C}(0^+) \to {}^{11}\mathrm{B}(1/2^-) + p$$

$$^{12}\mathrm{C}(0^+) \to {}^{11}\mathrm{B}(3/2^-)^* + p \,.$$

R. Crespo, et al, Phys.Lett.B 803 (2020) 135355

- The quenching of the spectroscopic factors automatically emerges from the VMC calculations

Computing the s-shell contribution is non trivial within VMC. We explored different alternatives:

- Quenched Harmonic Oscillator

- Quenched Wood Saxon

- VMC overlap associated for the $^4\mathrm{He}(0^+) \to {}^3\mathrm{H}(1/2^+) + p$ transition

Korover, et al, CLAS collaboration submitted (2021)
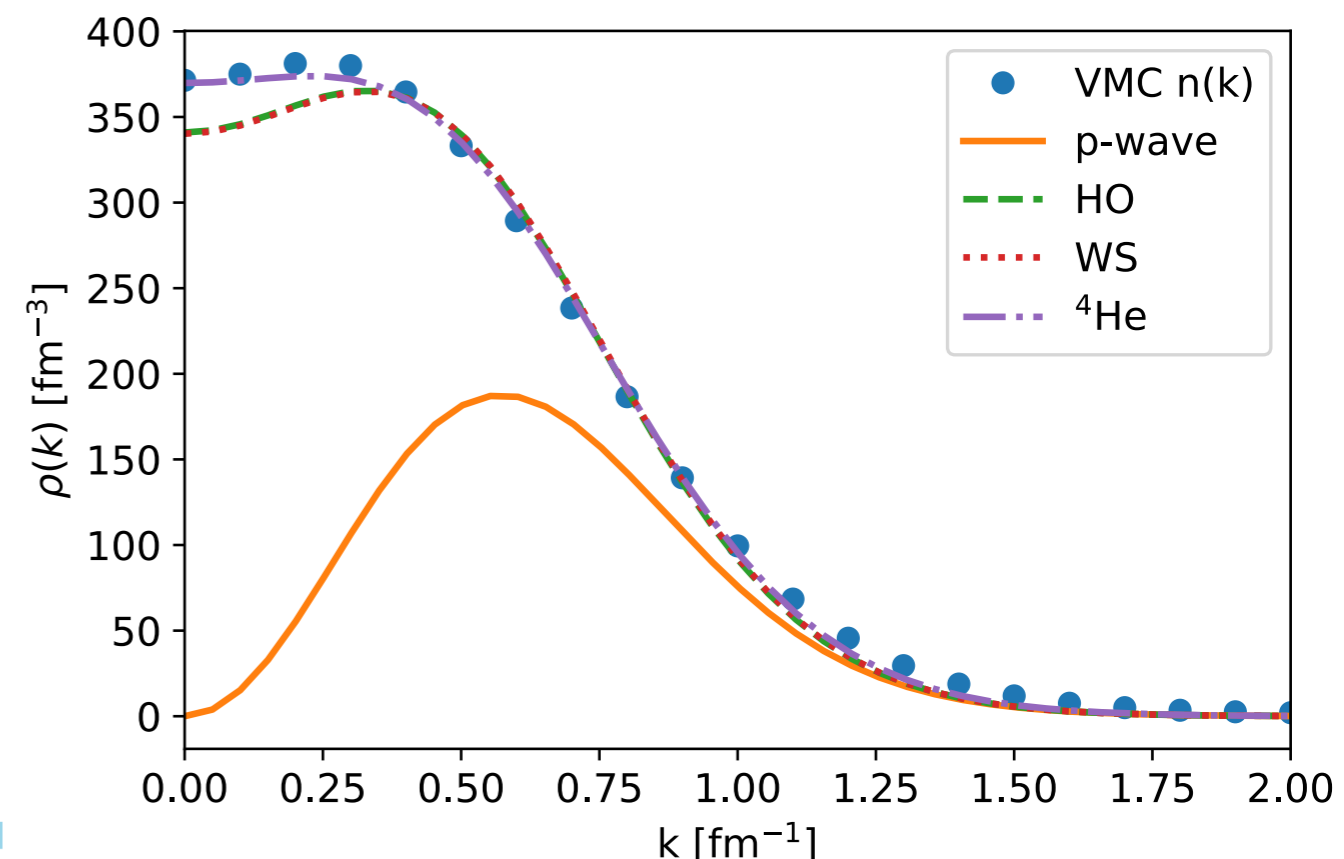
Spectral function approach

$$P_p^{\text{corr}}(\mathbf{k}, E) = \sum_n \int \frac{d^3 k'}{(2\pi)^3} |\langle \Psi_0^A |[|k\rangle \, |k'\rangle \, |\Psi_n^{A-2}\rangle]|^2 \delta(E + E_0^A - e(\mathbf{k}') - E_n^{A-2})$$
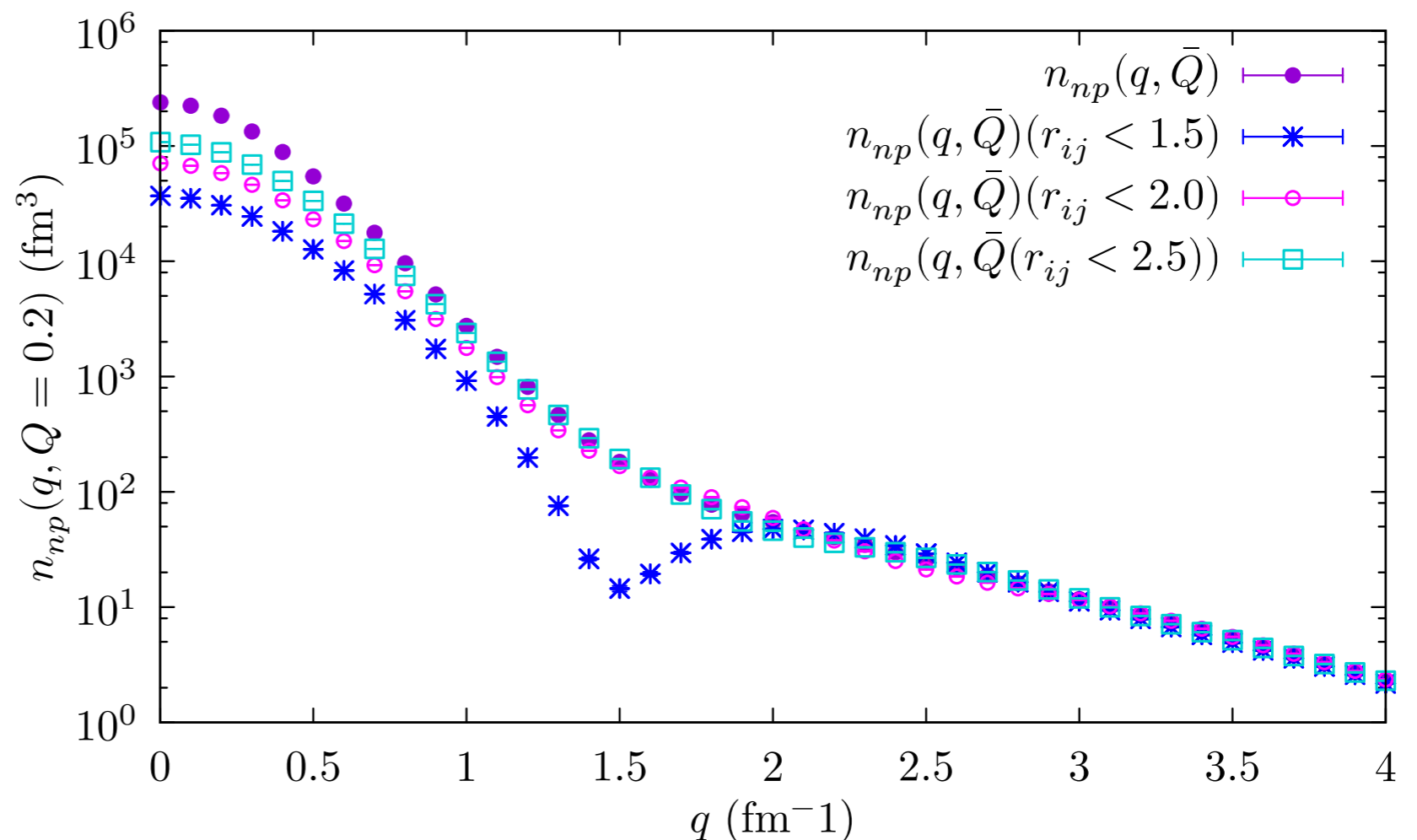
Using QMC techniques

$$\sum_{\tau_{k'}=p,n} n_{p,\tau_{k'}}(\mathbf{k}, \mathbf{k}') \delta\left(E - B_A \quad - e(\mathbf{k}') + B_{A-2} - \frac{(\mathbf{k} + \mathbf{k}')^2}{2m_{A-2}} \right)$$
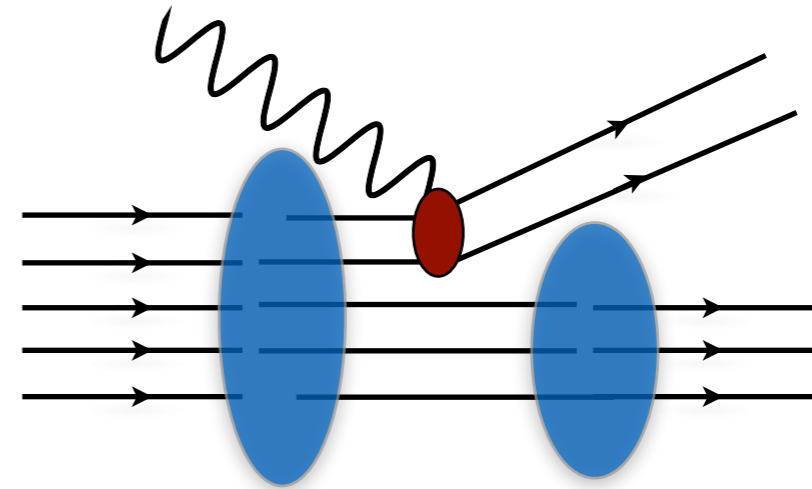
Only SRC pairs should be considered: $\left| \Psi_0^{A-1} \right\rangle$ and $|k'\rangle |\psi_n^{A-2}\rangle$ be orthogonalized

One can introduce **cuts** on the **relative distance** between the particles in the two-body momentum distribution

Noemi Rocco, nrocco@fnal.gov

Two-body currents

$$|f\rangle \rightarrow |pp'\rangle_a \otimes |f_{A-2}\rangle \quad \Rightarrow$$

$$R_{2\mathrm{b}}^{\mu\nu}(\mathbf{q}, \omega) = \frac{V}{4} \int dE \, d^3k \, d^3k' \, d^3p \, d^3p' \, P_{2\mathrm{b}}(\mathbf{k}, \mathbf{k}', E) 2\langle kk'|j_{2\mathrm{b}}^{\mu}|pp'\rangle_a \langle pp'|j_{2\mathrm{b}}^{\nu}{}^{\dagger}|kk'\rangle$$

$$\times \, \delta(\omega - E + 2m - e(\mathbf{p}) - e(\mathbf{p}'))\delta^{(3)}(\mathbf{p} + \mathbf{q} - \mathbf{p}')$$

The normalization volume for the nuclear wave functions $V = \rho/A$ with $\rho = 3\pi^2 k_F^3/2$ depends on the Fermi momentum of the nucleus

Factor 1/4 accounts for the fact that we sum over indistinguishable pairs of particles, while the factor 2 stems from the equality of the product of the direct terms and the product of the two exchange terms after interchange of indices

Two-body currents - pion contribution
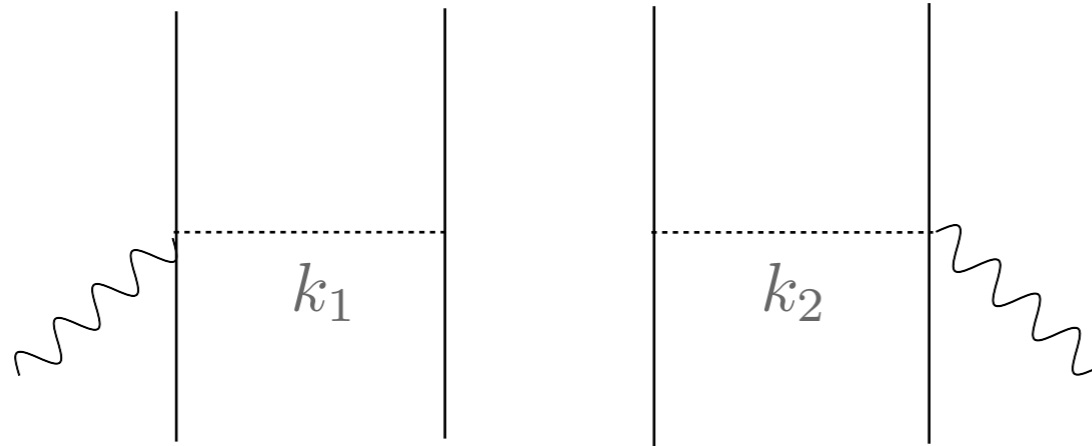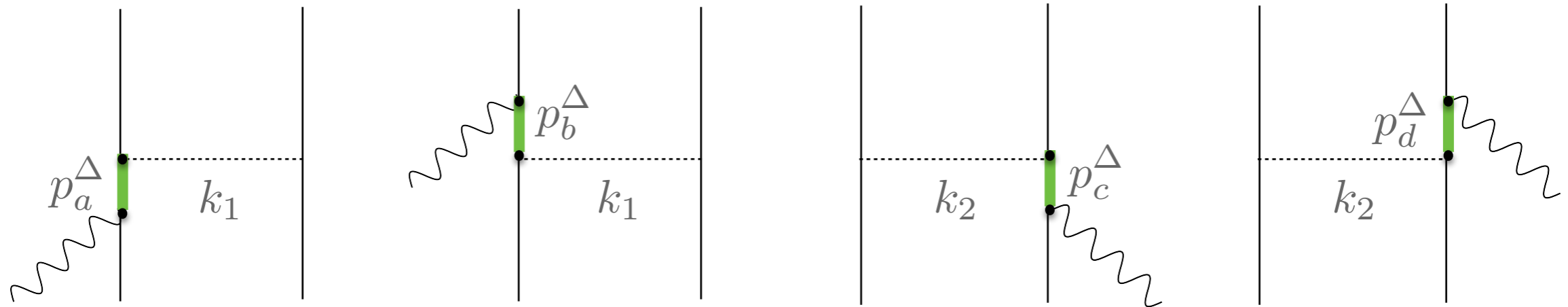

Seagull
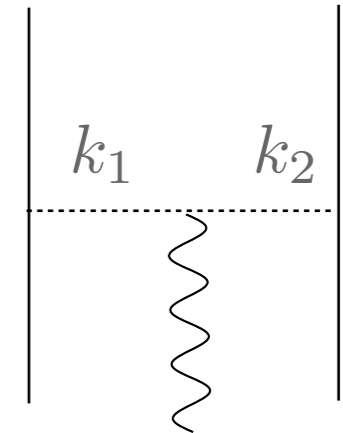
Pion in flight

$$(J_{\text{sea}}^\mu) = (I_V)_z \frac{f_{\pi NN}^2}{m_\pi^2} F_1^V(q) F_{\pi NN}^2(k_1) \Pi(k_1)_{(1)} \left(\gamma_5 \gamma^\mu\right)_{(2)} - (1 \leftrightarrow 2)$$

Isospin structure: $\qquad (I_V)_z = \left(\tau^{(1)} \times \tau^{(2)}\right)_z$

$$J_{\text{pif}}^\mu = \frac{f_{\pi NN}^2}{m_\pi^2} F_1^V(q) F_{\pi NN}(k_1) F_{\pi NN}(k_2) \Pi(k_1)_{(1)} \Pi(k_2)_{(2)} (k_1^\mu - k_2^\mu)$$

Pion propagation and absorption: $\qquad \Pi(k) = \dfrac{\gamma_5 \slashed{k}}{k^2 - m_\pi^2}$

Noemi Rocco, nrocco@fnal.gov

Fermilab

Two-body currents - Delta contribution

$$j^\mu_\Delta = \frac{3}{2}\frac{f_{\pi NN}f^*}{m^2_\pi}\left\{\Pi(k_2)_{(2)}\left[\left(-\frac{2}{3}\tau^{(2)}+\frac{I_V}{3}\right)_z F_{\pi NN}(k_2)F_{\pi N\Delta}(k_2)(J^\mu_a)_{(1)}\right.\right.$$

$$\left.\left.-\left(\frac{2}{3}\tau^{(2)}+\frac{I_V}{3}\right)_z F_{\pi NN}(k_2)F_{\pi N\Delta}(k_2)(J^\mu_b)_{(1)}\right]+(1\leftrightarrow 2)\right\}$$
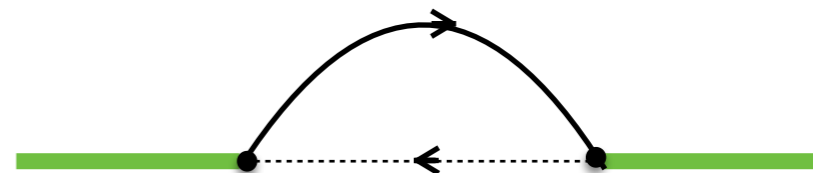
where   Rarita Schwinger propagator

$$(J^\mu_a)_V = (k_1)^\alpha G_{\alpha\beta}(p_\Delta)\left[\frac{C^V_3}{m_N}\left(g^{\beta\mu}\slashed{q}-q^\beta\gamma^\mu\right)+\frac{C^V_4}{m^2_N}\left(g^{\beta\mu}q\cdot p_\Delta - q^\beta p^\mu_\Delta\right)\right.$$

$$\left.+\frac{C^V_5}{m^2_N}\left(g^{\beta\mu}q\cdot k - q^\beta k^\mu + C^V_6\, g^{\beta\mu}\right)\right]\gamma_5$$

Two-body currents - Delta contribution

Rarita-Schwinger propagator

$$G^{\alpha\beta}(p_\Delta) = \frac{P^{\alpha\beta}(p_\Delta)}{p_\Delta^2 - M_\Delta^2}$$

The spin 3/2 projection operator reads

$$P^{\alpha\beta}(p_\Delta) = (\not{p}_\Delta + M_\Delta)\left[g^{\alpha\beta} - \frac{1}{3}\gamma^\alpha\gamma^\beta - \frac{2}{3}\frac{p_\Delta^\alpha p_\Delta^\beta}{M_\Delta^2} + \frac{1}{3}\frac{p_\Delta^\alpha\gamma^\beta - p_\Delta^\beta\gamma^\alpha}{M_\Delta}\right].$$

To account for the resonant behavior of the Δ: $\qquad M_\Delta \to M_\Delta - i\Gamma(p_\Delta)/2$

$$\Gamma(p_\Delta) = -2\mathrm{Im}\Sigma_{\pi N}(s) = \frac{(4f_{\pi N\Delta})^2}{12\pi m_\pi^2}\frac{|\mathbf{d}|^3}{\sqrt{s}}(m_N + E_d)R(\mathbf{r}^2)$$
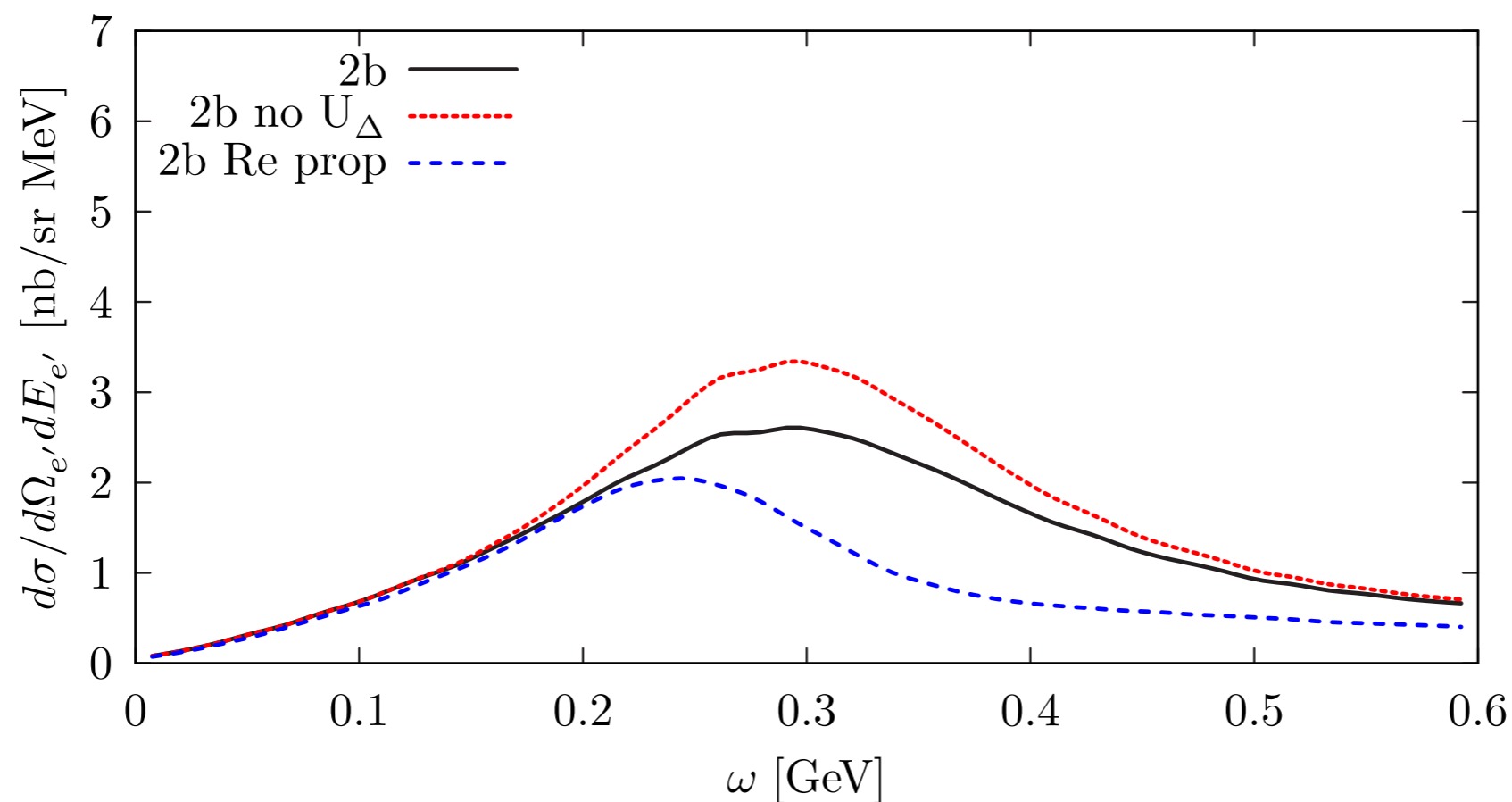
d is the decay three-momentum in the πN center of mass frame

In medium effects of the Δ $\qquad \Gamma_\Delta(p_\Delta) \to \Gamma_\Delta(p_\Delta) - 2\mathrm{Im}[U_\Delta(p_\Delta, \rho = \rho_0)]$

Noemi Rocco, nrocco@fnal.gov

Two-body currents - Delta contribution

E_e=730 MeV, θ_e=37.0°

Legend:
- 2b (solid black)
- 2b no U_Δ (red dotted)
- 2b Re prop (blue dashed)
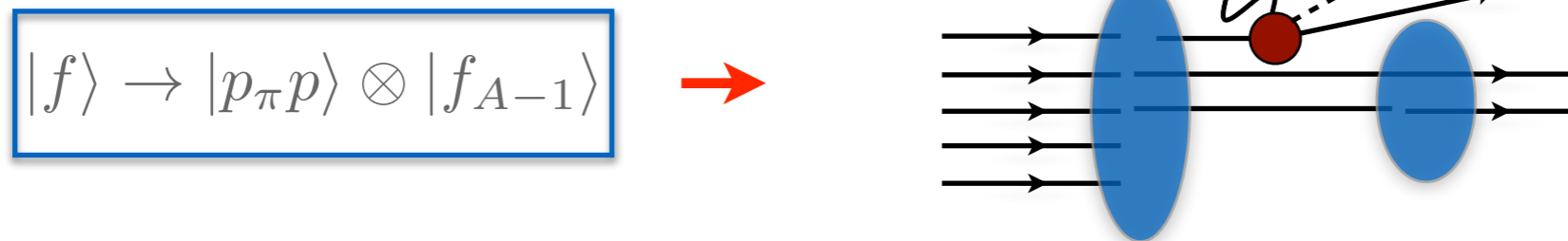
Axes: $d\sigma/d\Omega_{e'}dE_{e'}$ [nb/sr MeV] vs ω [GeV]

$$\Gamma(p_\Delta) = -2\text{Im}\Sigma_{\pi N}(s) = \frac{(4f_{\pi N\Delta})^2}{12\pi m_\pi^2}\frac{|\mathbf{d}|^3}{\sqrt{s}}(m_N + E_d)R(\mathbf{r}^2)$$

d is the decay three-momentum in the πN center of mass frame

In medium effects of the Δ
$$\Gamma_\Delta(p_\Delta) \rightarrow \Gamma_\Delta(p_\Delta) - 2\text{Im}[U_\Delta(p_\Delta, \rho = \rho_0)]$$

Noemi Rocco, nrocco@fnal.gov
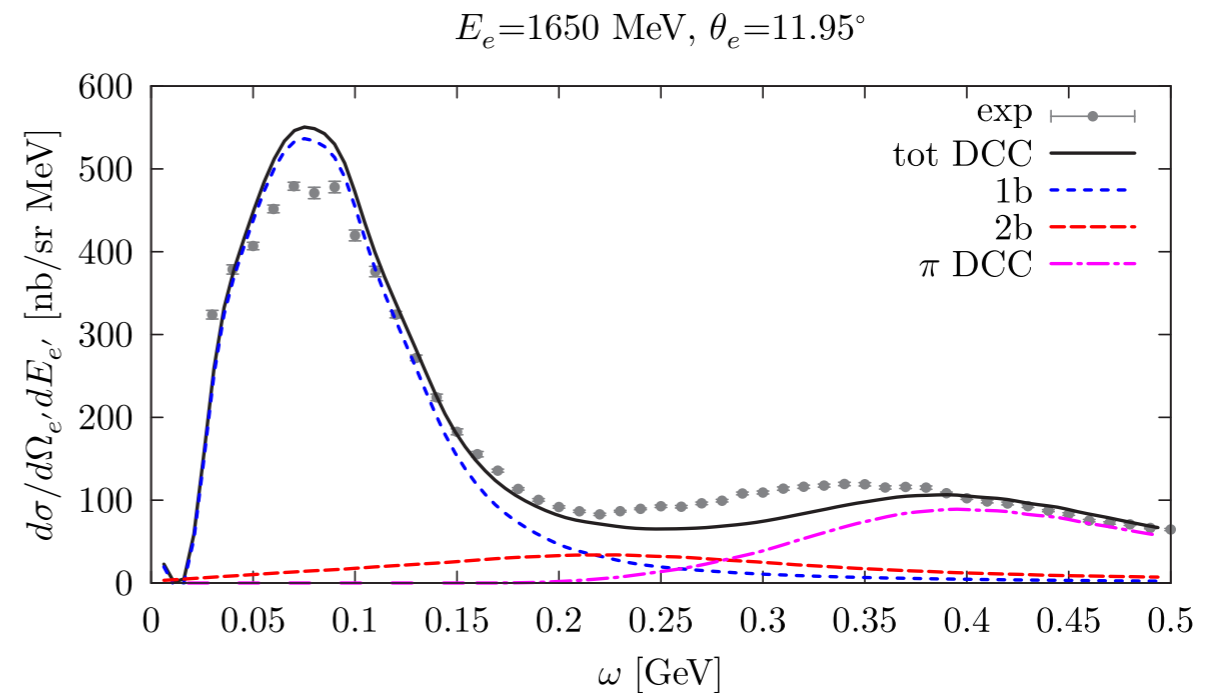
Fermilab

Spectral function approach

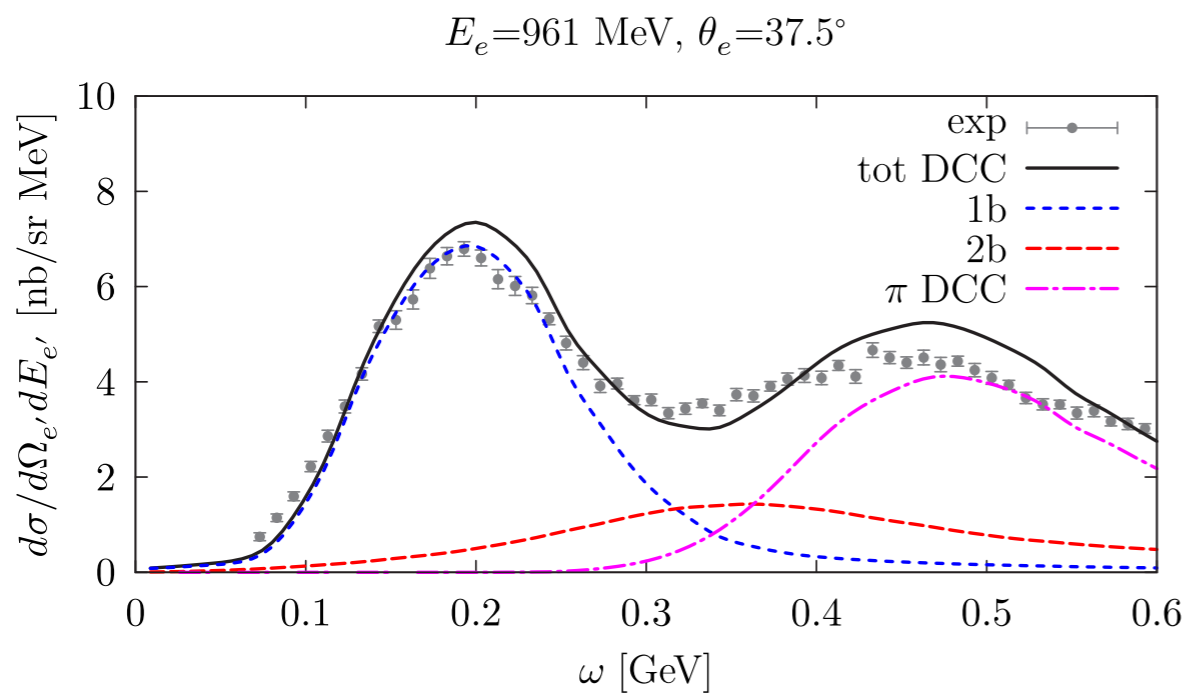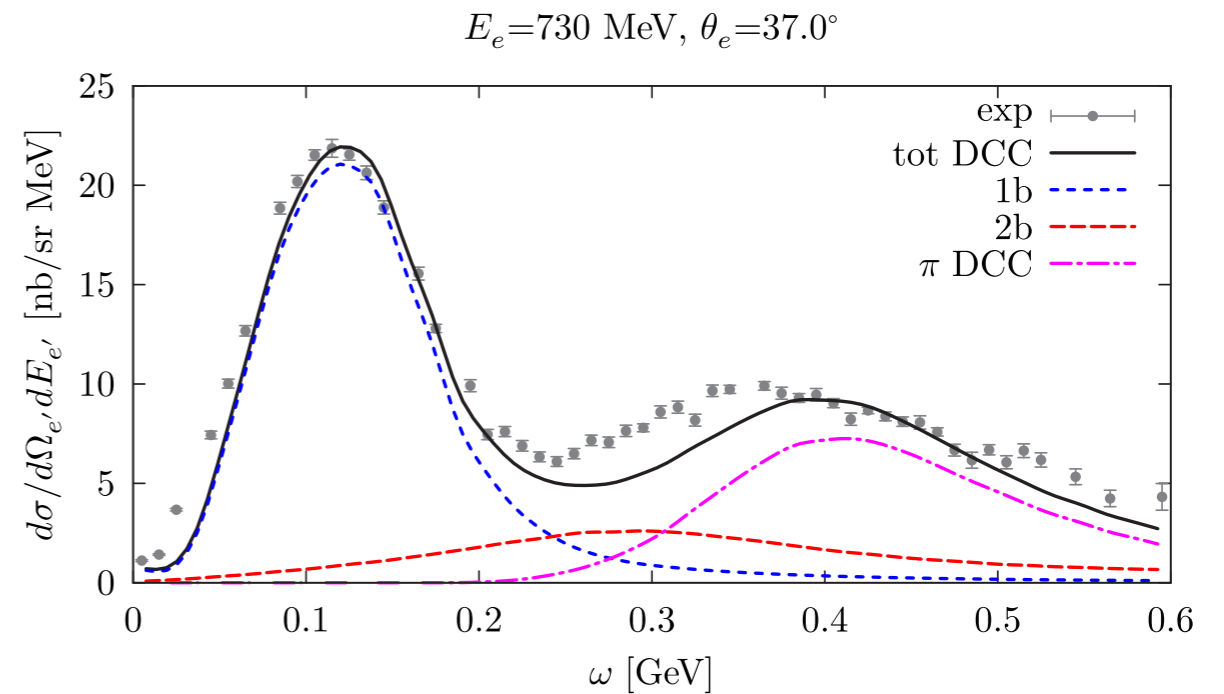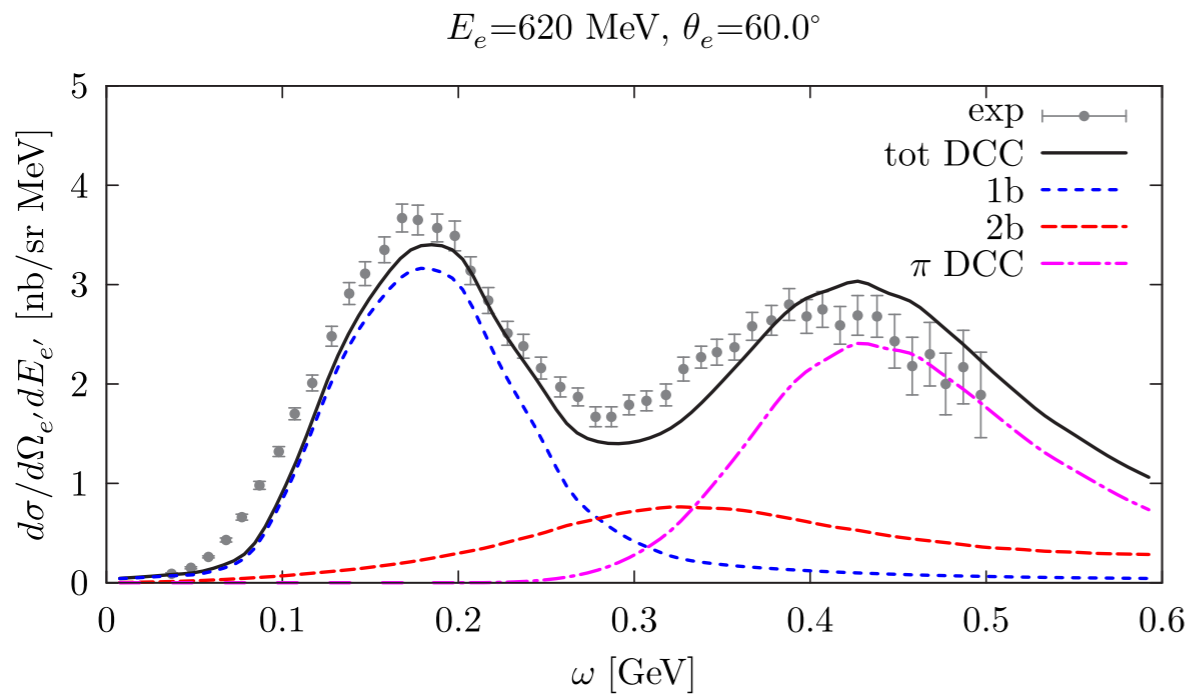$$|f\rangle \to |p_\pi p\rangle \otimes |f_{A-1}\rangle$$

Production of real π in the final state:

$$R^{\mu\nu}_{1b1\pi}(\mathbf{q}, \omega) = \int \frac{d^3 k}{(2\pi)^3} dE P_h(\mathbf{k}, E) \frac{d^3 p_\pi}{(2\pi)^3} \sum_I \langle k|j_I^{\mu\dagger}|p_\pi p\rangle \langle p_\pi p|j_I^\nu|k\rangle \Big|_{\mathbf{p}=\mathbf{k}+\mathbf{q}-\mathbf{p}_\pi}$$
$$\times \delta(\omega - E + m_N - e(\mathbf{k} + \mathbf{q} - \mathbf{p}_\pi) - e_\pi(\mathbf{p}_\pi)) ,$$

✳ Pion production elementary amplitudes currently derived within the **Dynamic Couple Chanel approach**;
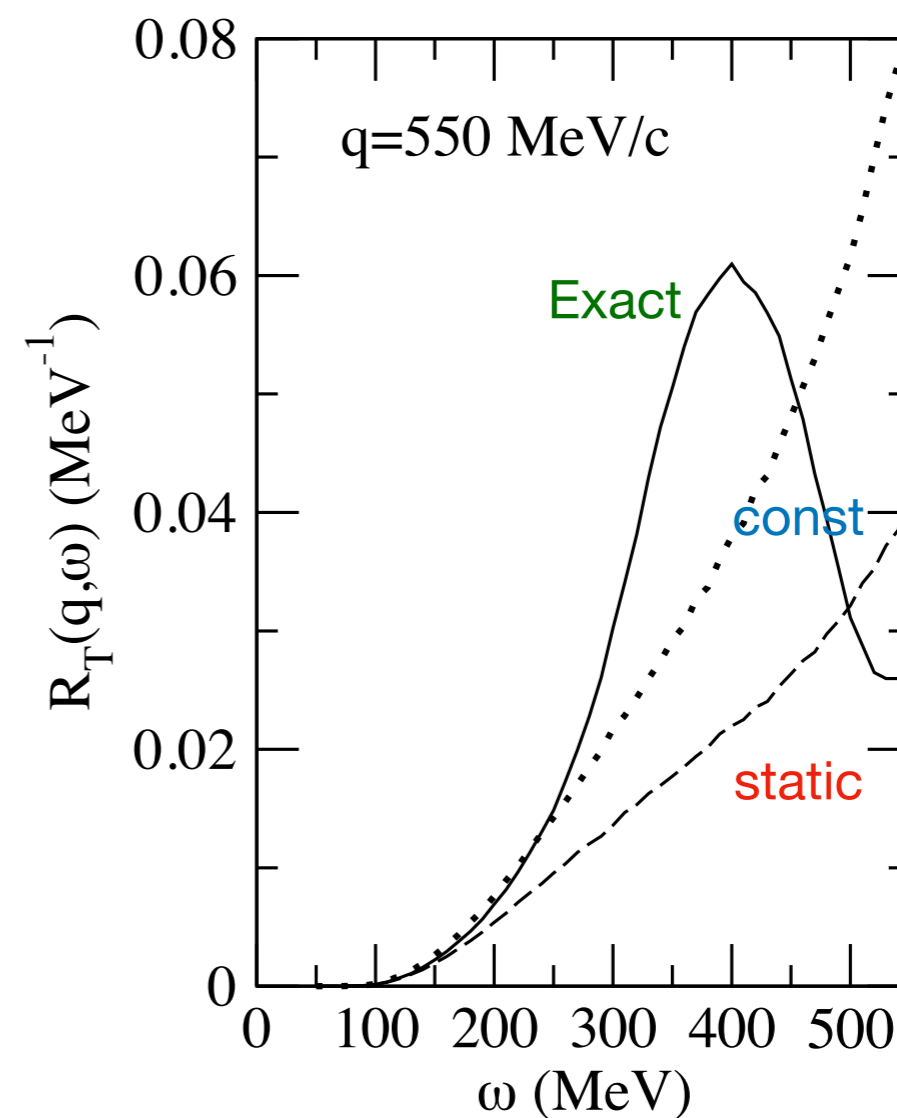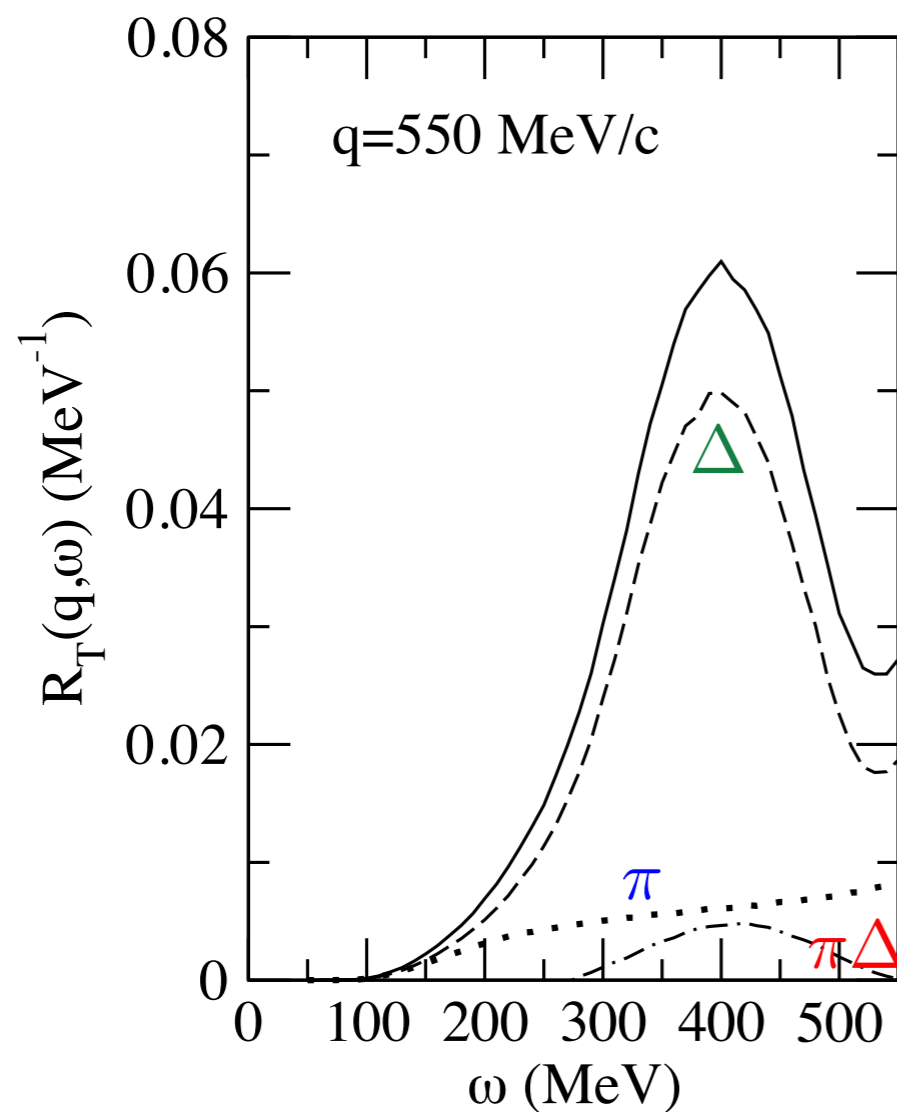
Noemi Rocco, nrocco@fnal.gov

≉ Fermilab

Spectral function approach

E_e=620 MeV, θ_e=60.0°

E_e=730 MeV, θ_e=37.0°

E_e=961 MeV, θ_e=37.5°

E_e=1650 MeV, θ_e=11.95°

Electron scattering results

Noemi Rocco, nrocco@fnal.gov

🔹 Fermilab

Two-body currents - Delta contribution

A. De Pace, et al, Nucl.Phys.A 726 (2003) 303-326

RGFG — electromagnetic response of A=56 k_F= 1.3 fm^{-1}

Noemi Rocco, nrocco@fnal.gov

Axial Form Factors Uncertainty needs

$0.2 < \cos(\theta) < 0.3$

D.Simons, N. Steinberg et al, 2210.02455

Legend (top plot):
- —— SF Dipole ($M_A = 1$ GeV)
- SF z expansion (D2)
- SF z expansion (LQCD)
- MB

Legend (bottom plot):
- SF 1b
- SF 2b
- SF 12b
- MB

y-axis: $d\sigma/dT_\mu d\cos\theta_\mu$ $[10^{-41}\text{cm}^2/\text{MeV}]$

x-axis: T_μ (MeV)
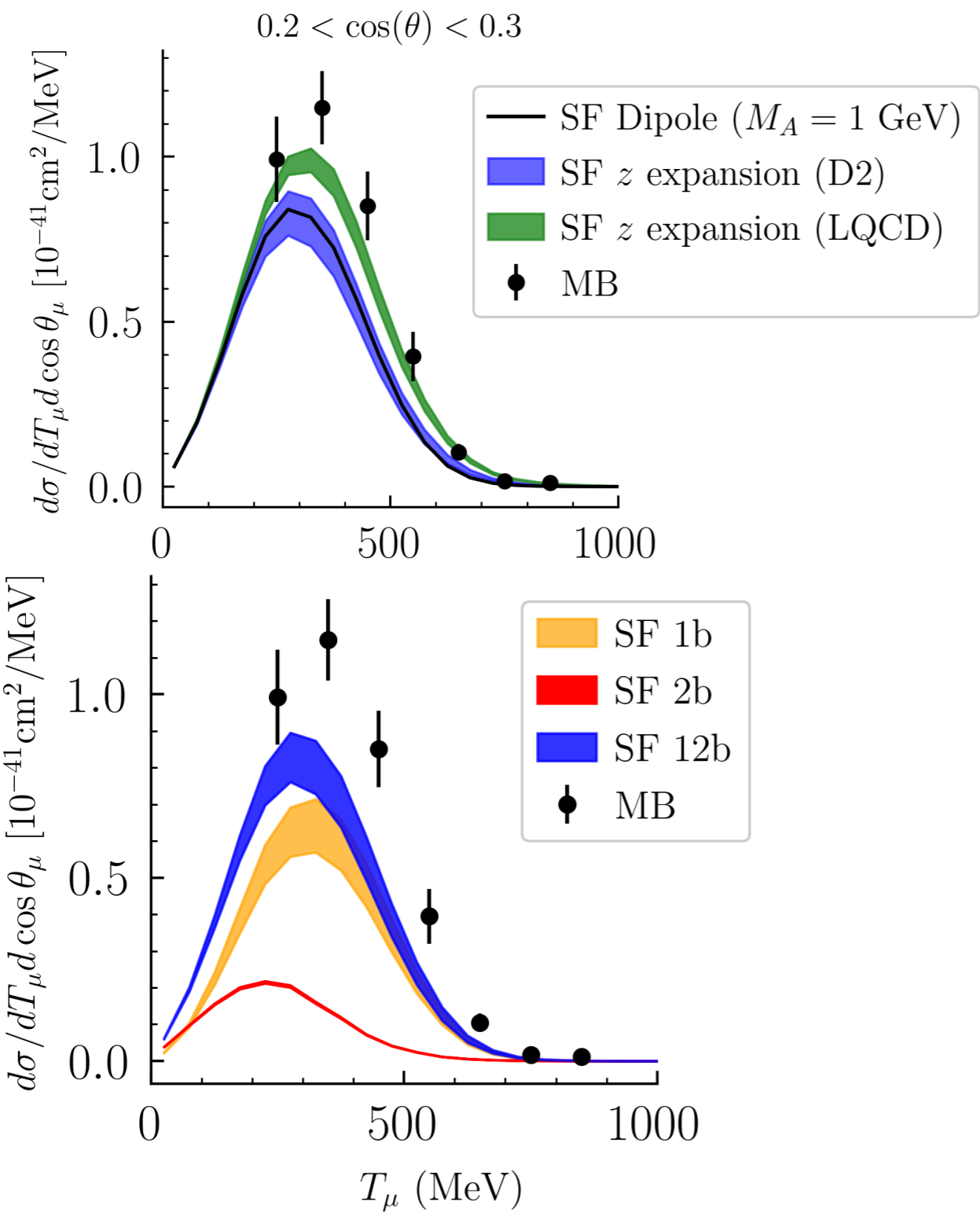
✳ Axial form factor dependence:

MiniBooNE	$0.2 < \cos\theta_\mu < 0.3$
SF Difference in $d\sigma_{\text{peak}}$ (%)	16.3

✳ Many-body method dependence:

MiniBooNE	$0.2 < \cos\theta_\mu < 0.3$
GFMC/SF difference in $d\sigma_{\text{peak}}$ (%)	22.8

Noemi Rocco, nrocco@fnal.gov

🔷 Fermilab

Axial Form Factors Uncertainty needs

$0.0 < cos(\theta) < 0.6$

Legend (top plot):
- SF Dipole ($M_A = 1$ GeV)
- SF z expansion (D2)
- SF z expansion (LQCD)
- T2K

Axes: $d\sigma/dp_\mu dcos\theta_\mu$ [10^{-39}cm^2/MeV], p_μ (MeV)

Legend (bottom plot):
- SF 1b
- SF 2b
- SF 12b
- T2K
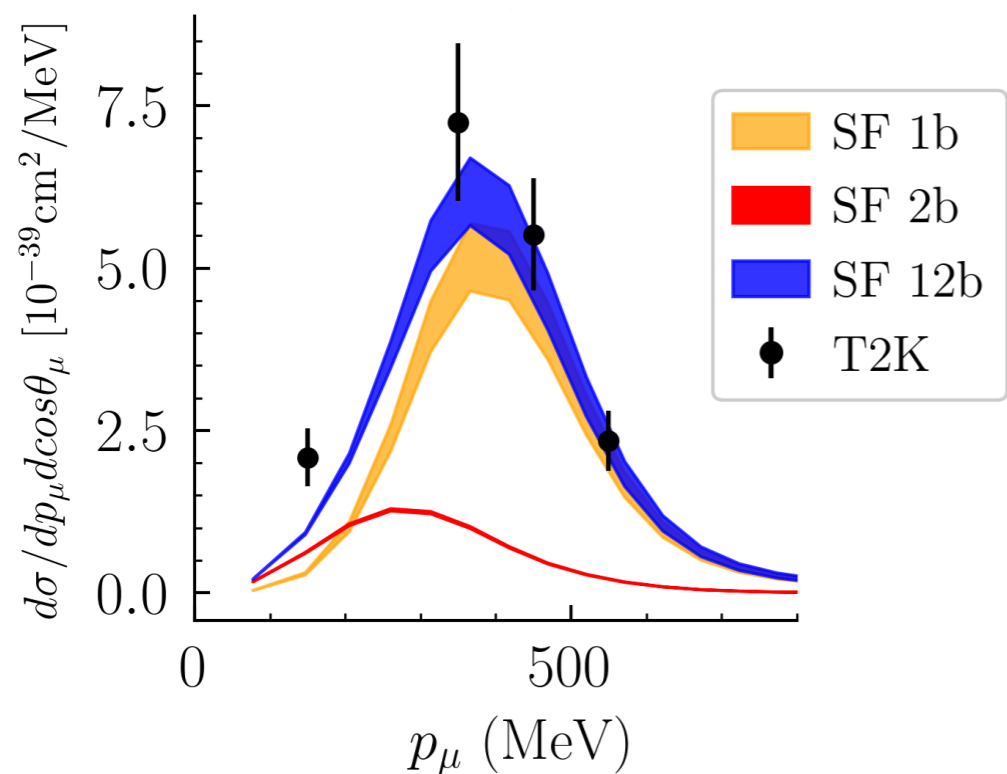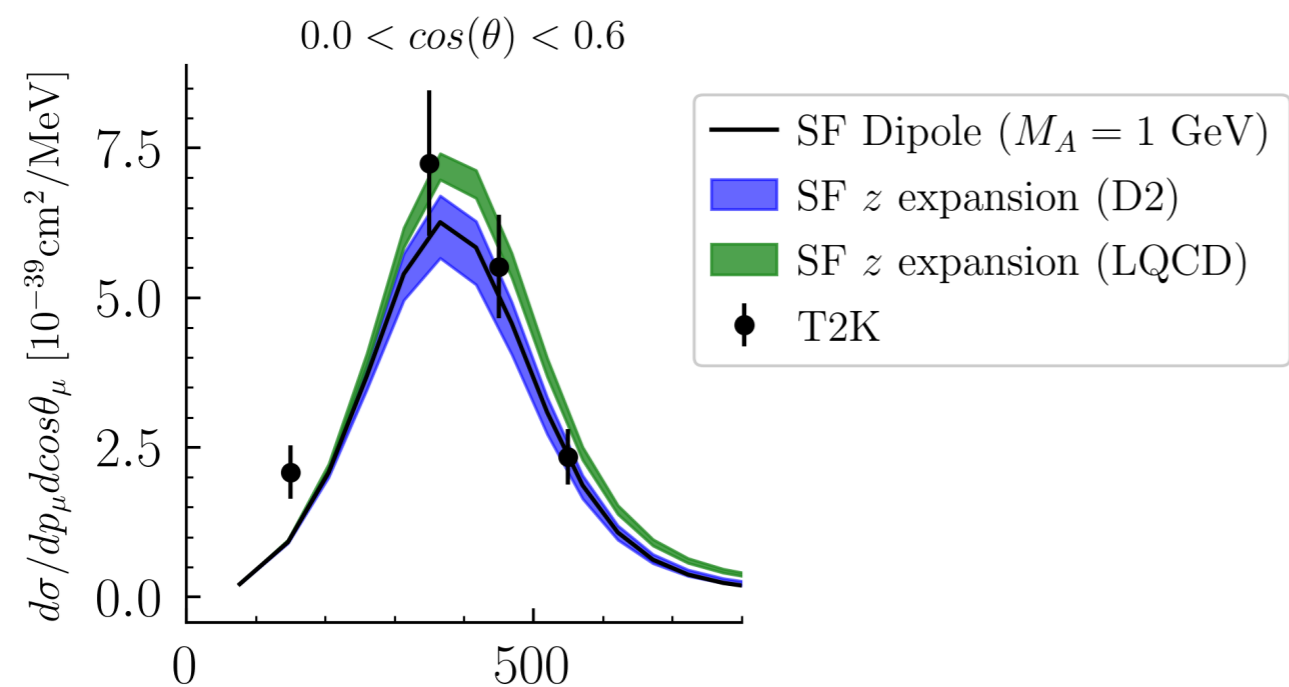
D.Simons, N. Steinberg et al, 2210.02455

✴ **Axial form factor dependence:**

T2K	$0.0 < \cos\theta_\mu < 0.6$
SF difference in $d\sigma_{\text{peak}}$ (%)	15.3

✴ **Many-body method dependence:**

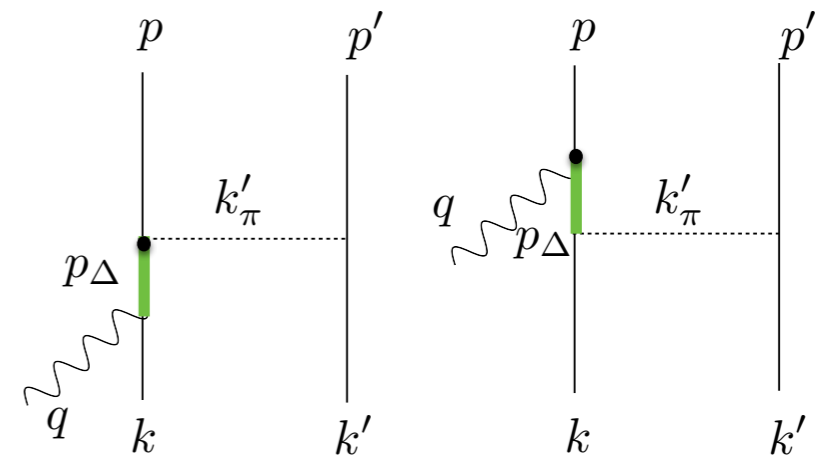| T2K	$0.0 < \cos\theta_\mu < 0.6$
GFMC/SF difference in $d\sigma_{\text{peak}}$ (%)	13.4

🎗 **Fermilab**

Resonance Uncertainty needs

The largest contributions to two-body currents arise from resonant $N \rightarrow \Delta$ transitions yielding pion production
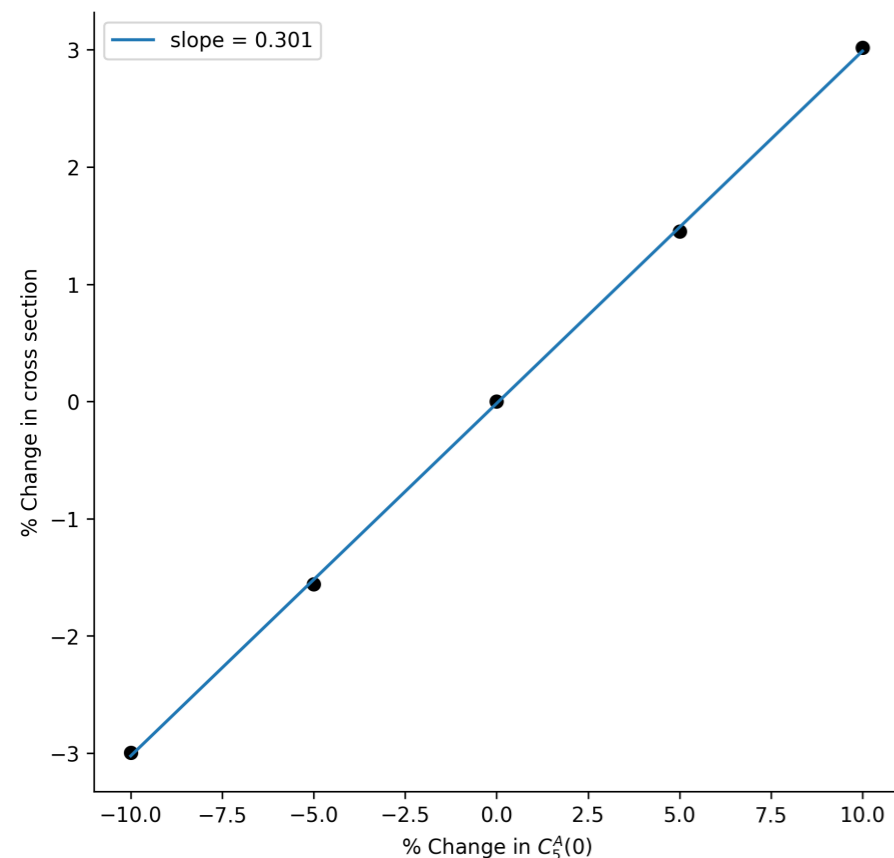
The normalization of the dominant $N \rightarrow \Delta$ transition form factor needs be known to 3% precision to achieve 1% cross-section precision for MiniBooNE kinematics

State-of-the-art determinations of this form factor from experimental data on pion electroproduction achieve 10-15% precision (under some assumptions)

D.Simons, N. Steinberg et al, 2210.02455

Hernandez et al, PRD 81 (2010)

Further constraints on $N \rightarrow \Delta$ transition relevant for two-body currents and π production will be necessary to achieve few-percent cross-section precision

Noemi Rocco, nrocco@fnal.gov

Scaling in the Fermi gas model

- <u>Scaling of the first kind:</u> the nuclear electromagnetic responses divided by an appropriate function describing the single-nucleon physics no longer depend on two variables ω and $\mathbf{q}$, but only upon $\psi(\mathbf{q}, \omega)$
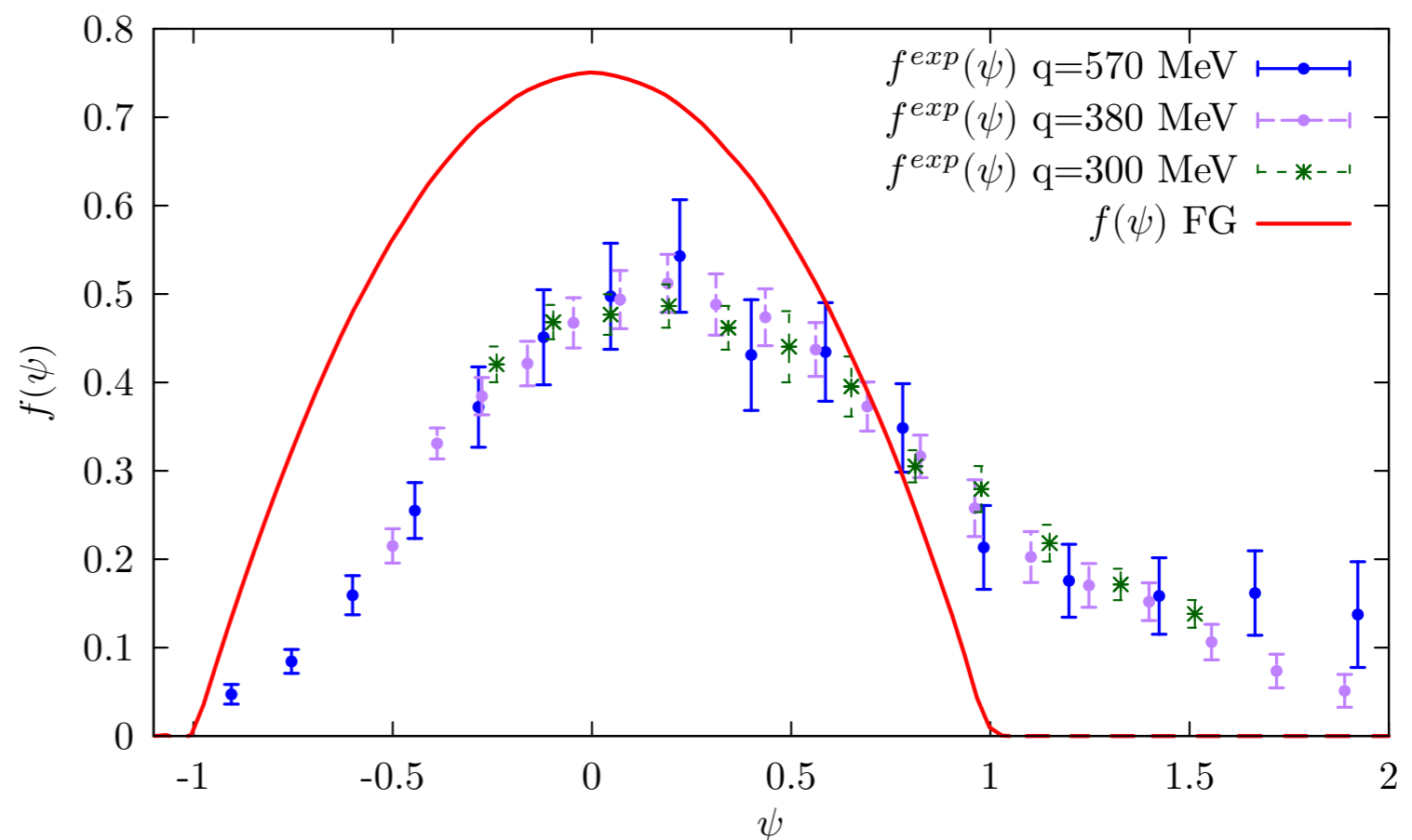
$$f_{L,T}(\mathbf{q}, \omega) = p_F \frac{R_{L,T}}{G_{L,T}} = \frac{\text{nuclear response function}}{\text{elem single nucleon function}}$$

- For sufficiently large values of $\mathbf{q}$

$$f(\mathbf{q}, \omega) \to f(\psi)$$

- Within the Relativistic Fermi gas scaling of the first kind is exactly fulfilled
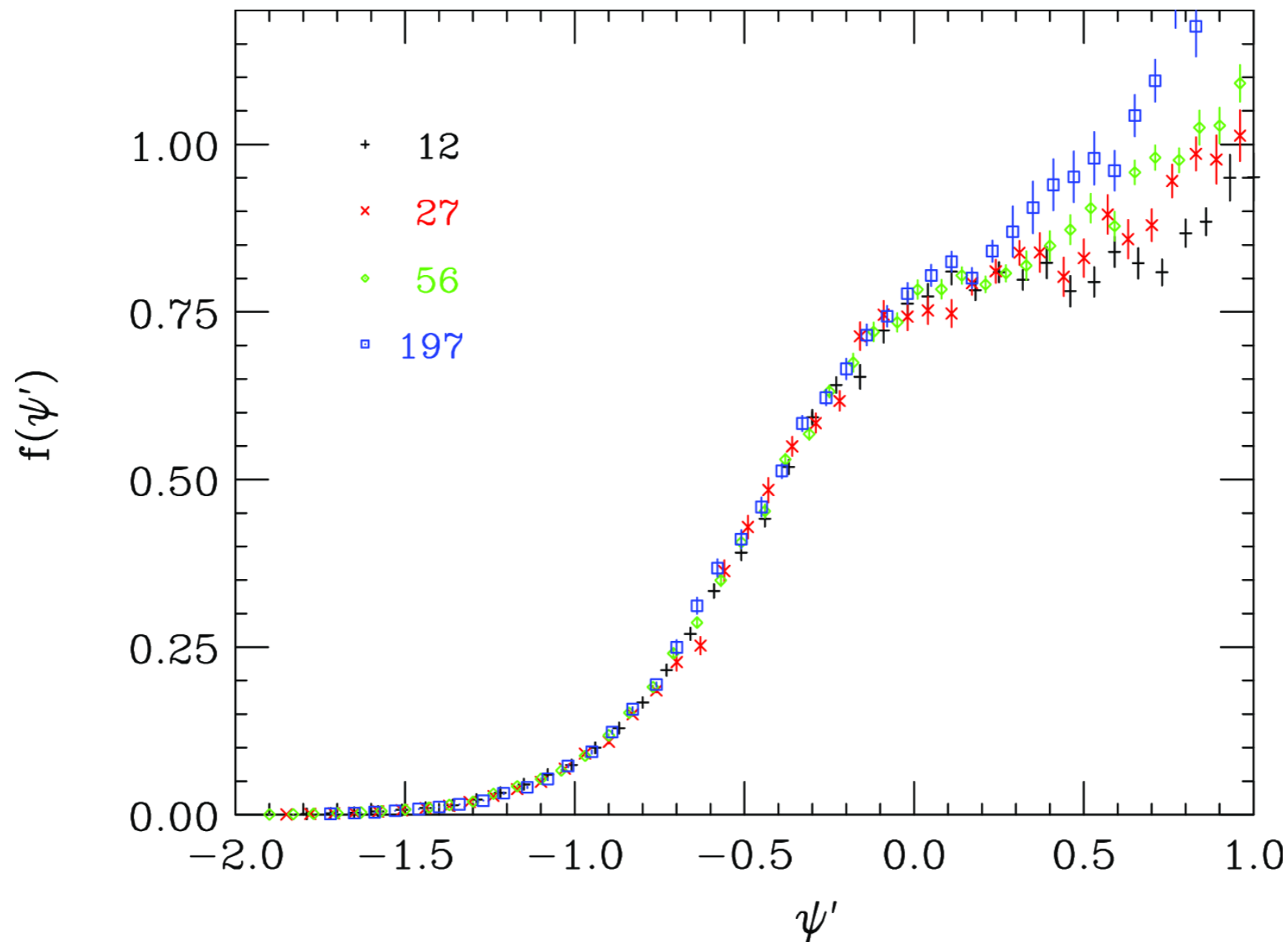
$$f_{\text{RFG}} = \frac{3}{4}(1 - \psi^2)$$

- Scaling function of ^{12}C, obtained dividing the R_L by the single nucleon functions for different $\mathbf{q}$

     Noemi Rocco, nrocco@fnal.gov

Scaling in the Fermi gas model

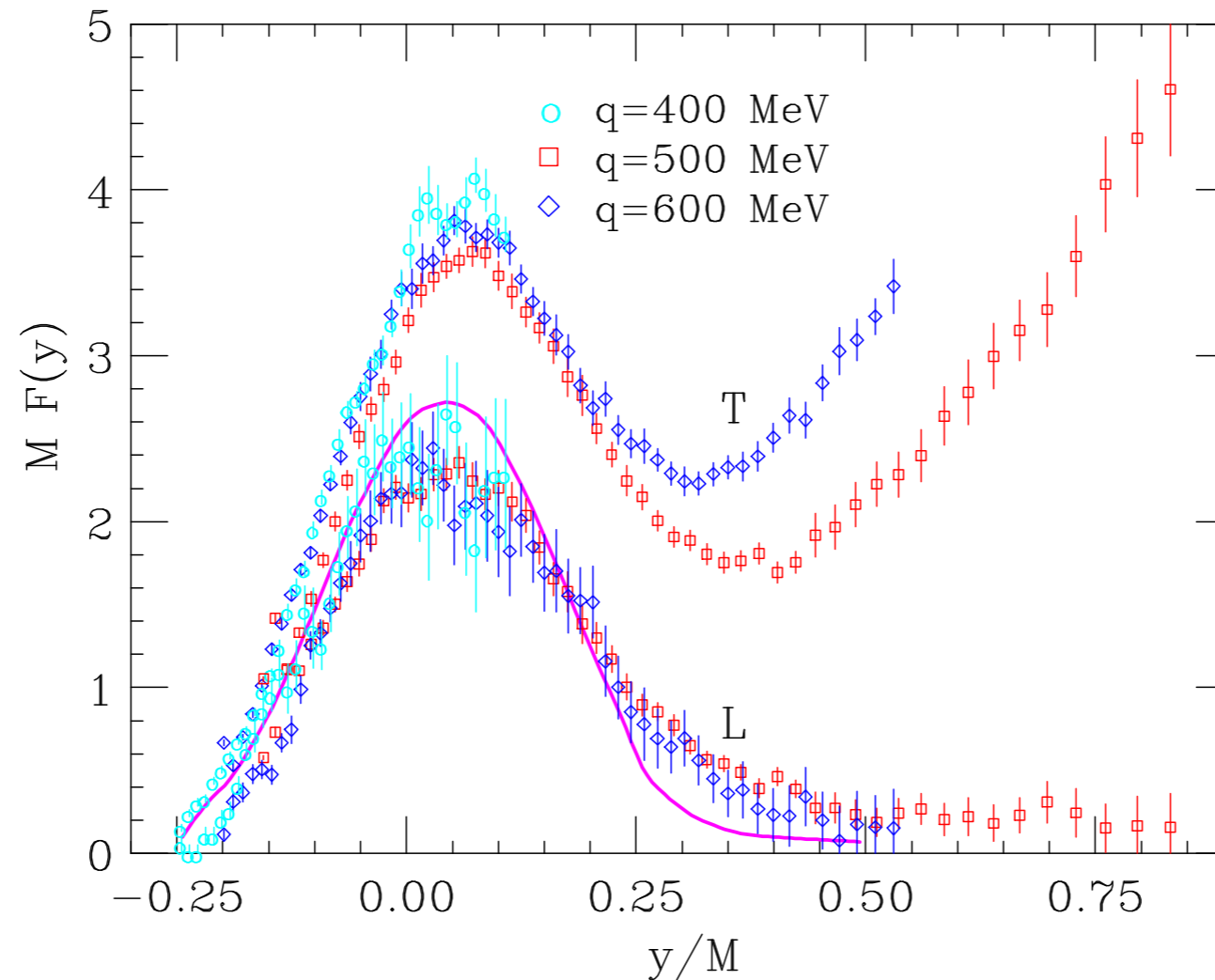- Scaling of the second kind: independence of the nuclear species A

Two-body currents violate scaling

- Within the FG model we have this simple expression:

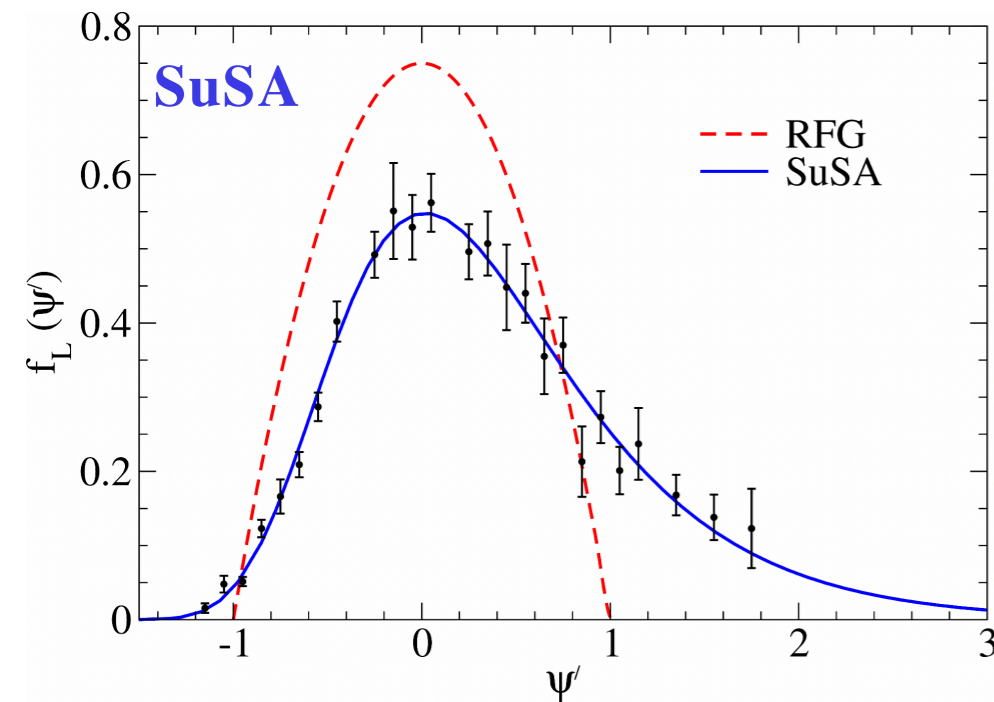$$R_{L,T} = (1 - \psi^2)\theta(1 - \psi^2) \times G_{L,T}$$

- Scaling functions associated with the longitudinal (L) and transverse (T) response of Carbon,

- The onset of scaling is clearly visible in the quasi-elastic peak

- Large scaling violations appear in $F_T(y)$ at y>0

Noemi Rocco, nrocco@fnal.gov

Super-Scaling (SuSA) model

Amaro *et al.*, PRC71 (2005)

SuSA

- - - RFG
—— SuSA

Basic idea is to use the <u>scaling function</u> extracted from <u>longitudinal (e,e')</u> data to predict ν-scattering cross sections

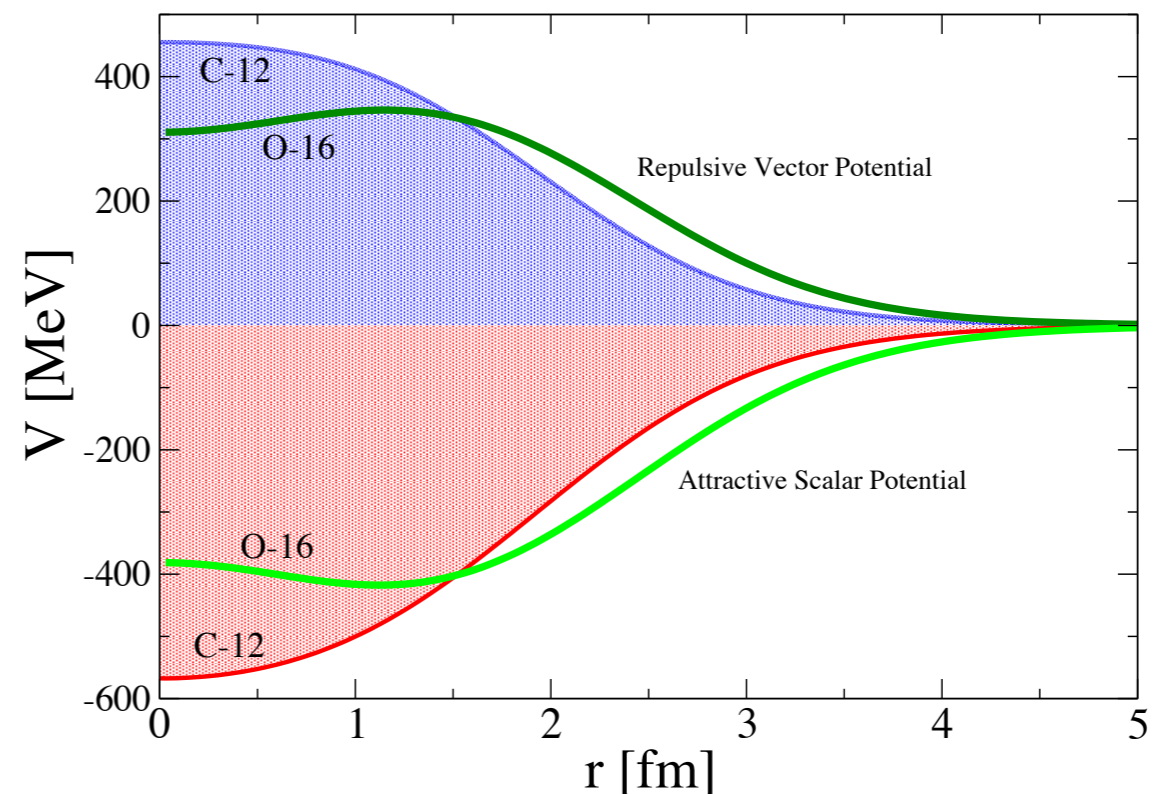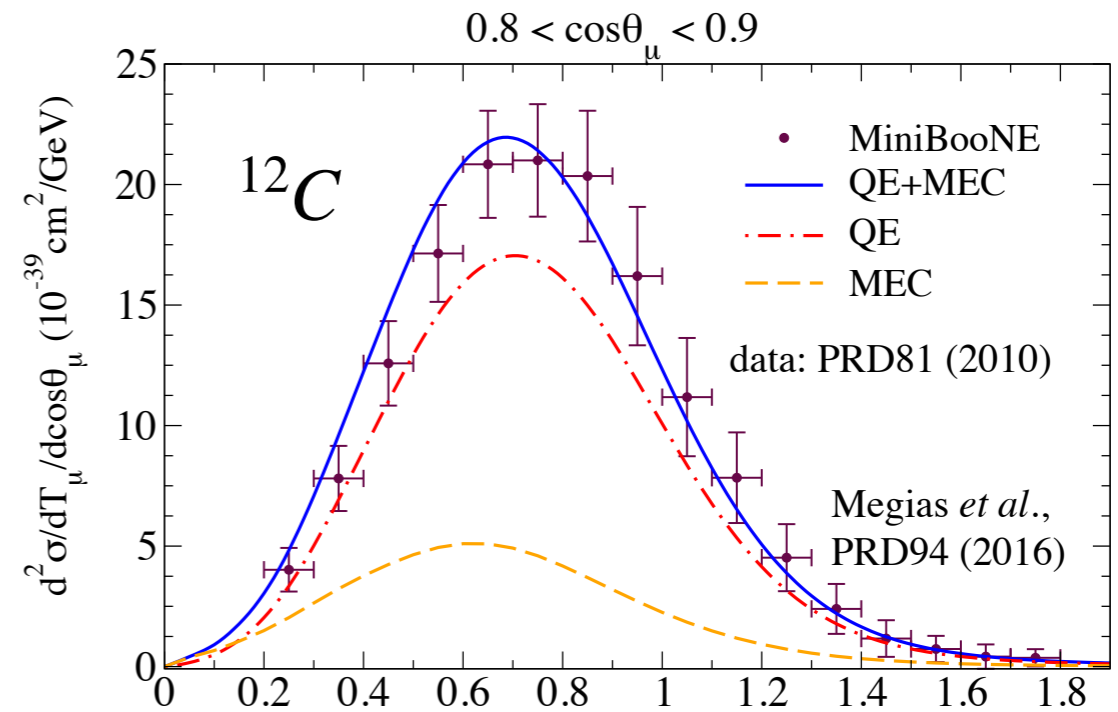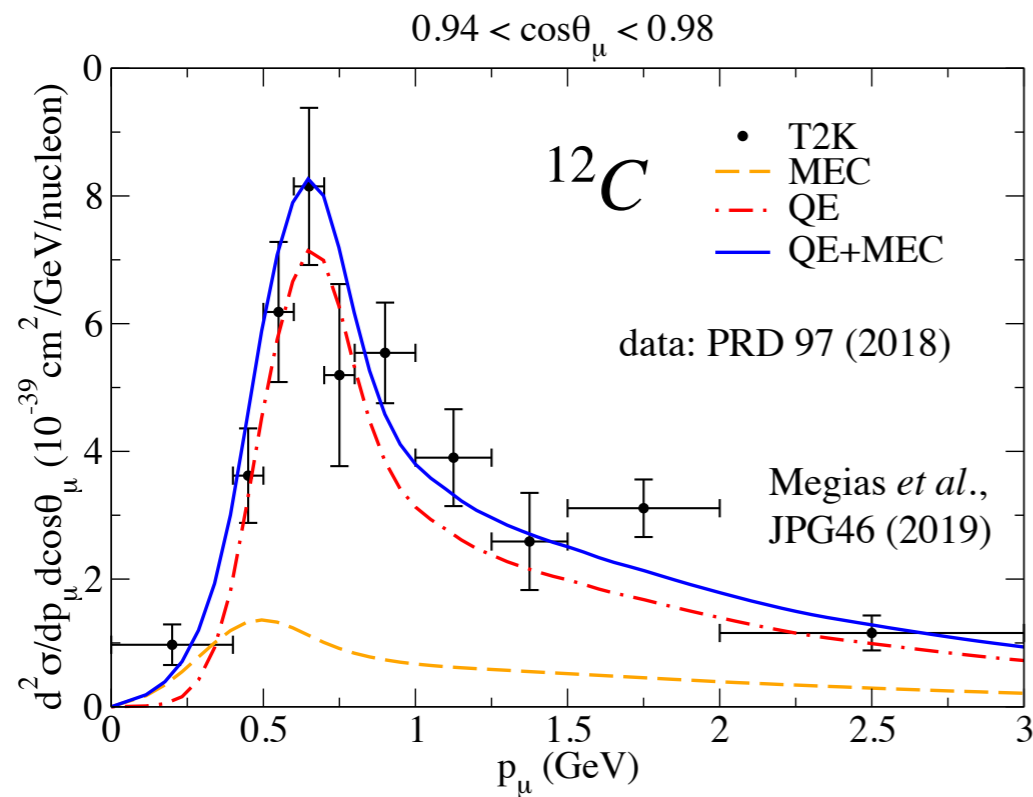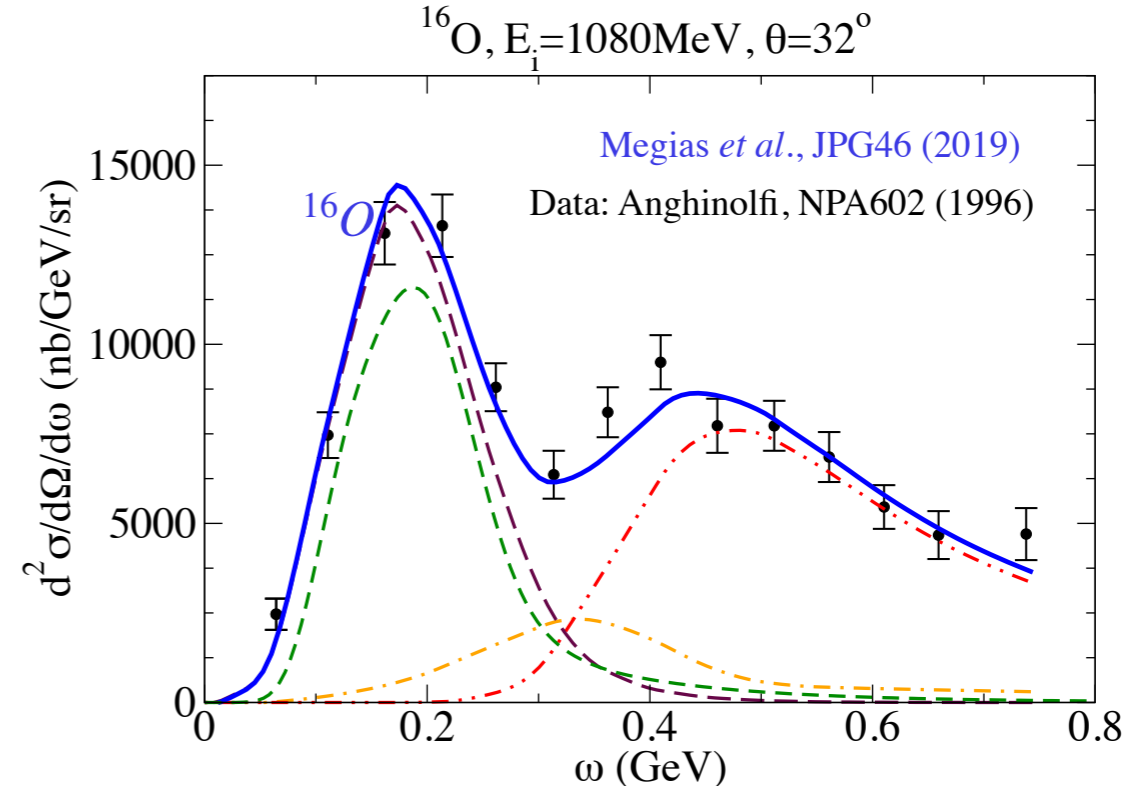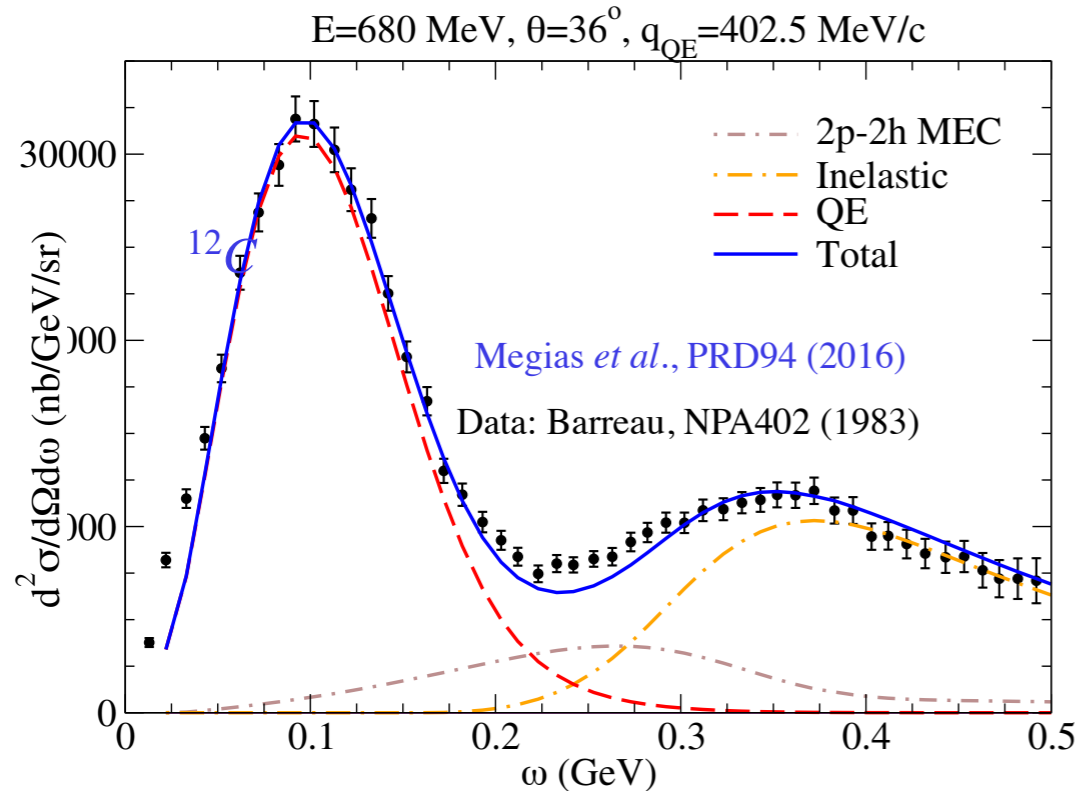**Gonzalez-Jimenez *et al.*, PRC90(2014)**

The nucleon wave functions are solutions of the Dirac equation with phenomenological relativistic scalar and vector mean field potentials

$$(i\gamma^\mu \partial_\mu - M - S + V)\,\psi(\vec{r}, t) = 0$$

SuSAv2 : uses scaling functions extracted from Relativistic Mean Field calculations.

$f_T > f_L$ in agreement with L/T separated *(e,e')* data

QuSAV2 predictions

$0.80 < \cos\theta_\mu < 0.85$

E=680 MeV, θ=36°, q_{QE}=402.5 MeV/c

- — · — 2p-2h MEC
- — · — Inelastic
- — — QE
- —— Total

^{12}C

Megias *et al.*, PRD94 (2016)

Data: Barreau, NPA402 (1983)

$0.85 < \cos\theta_\mu < 0.90$

^{16}O, E=1080MeV, θ=32°

Megias *et al.*, JPG46 (2019)

Data: Anghinolfi, NPA602 (1996)

^{16}O

ω (GeV)

$0.9 < \cos\theta_\mu < 1.0$

$0.8 < \cos\theta_\mu < 0.9$

^{12}C

- • MiniBooNE
- —— QE+MEC
- — · — QE
- — — MEC

data: PRD81 (2010)

Megias *et al.*, PRD94 (2016)

$0.6 < \cos\theta_\mu < 0.7$

$0.70 < \cos\theta_\mu < 0.80$

$0.6 < \cos\theta_\mu < 0.7$

$0.80 < \cos\theta_\mu < 0.85$

Fern

Neutrino Simulation

Different **Monte Carlo event generators** on the market
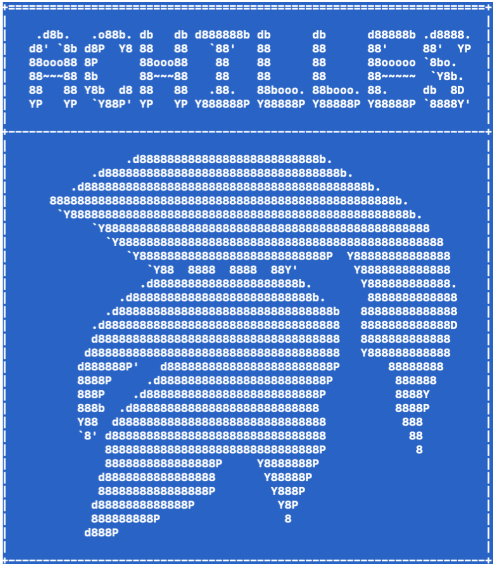
Event generators also used to predict **backgrounds** and **efficiency**

Using a factorization scheme

Many generators also tune theoretical models to data
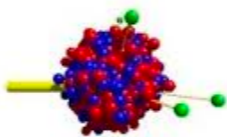
Noemi Rocco, nrocco@fnal.gov

From neutrino-nucleon to neutrino-nucleus

Factorization Scheme:

$$|\mathcal{M}(\{k\} \to \{p\})|^2 \simeq \sum\!\!\!\!\!\!\!\!\int_{p'} |\mathcal{V}(\{k\} \to \{p'\})|^2 \times |\mathcal{P}(\{p'\} \to \{p\})|^2$$

• Nuclear Model

• Primary Interaction
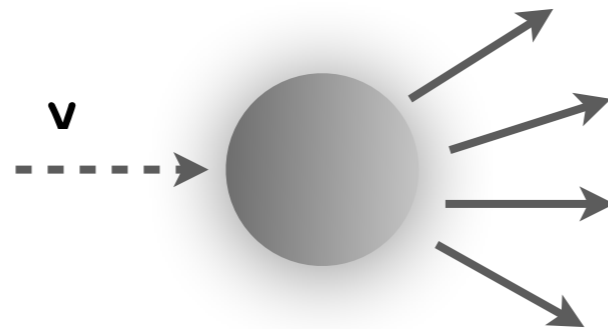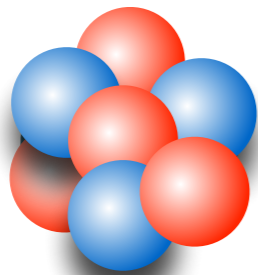
• Evolution out of the nucleus (intra-nuclear) cascade

The propagation of **nucleons** through the **nuclear medium** is crucial in the analysis of neutrino oscillation experiments.

• Elastic scattering

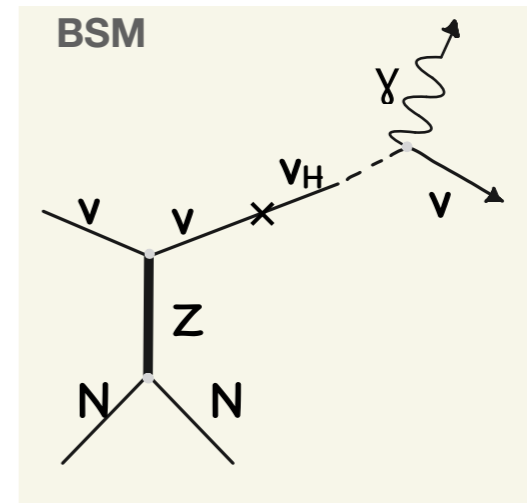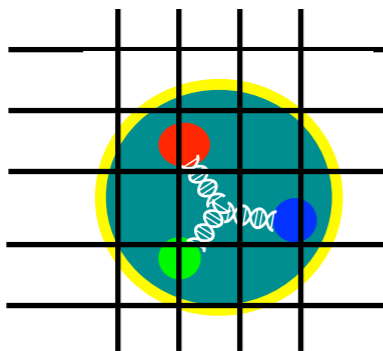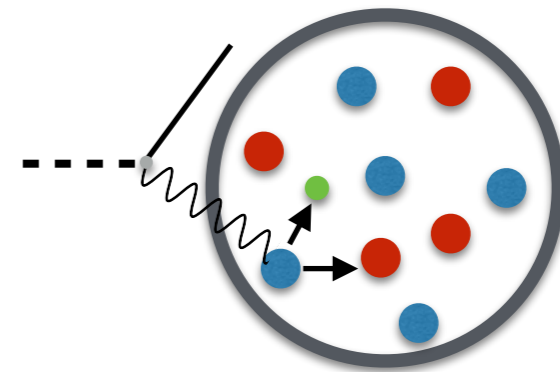• Charge exchange

• Pion Production

• Absorption

‡ Fermilab

Summary

$$\nu A$$

(2019)

Nuclear Physics:

BSM
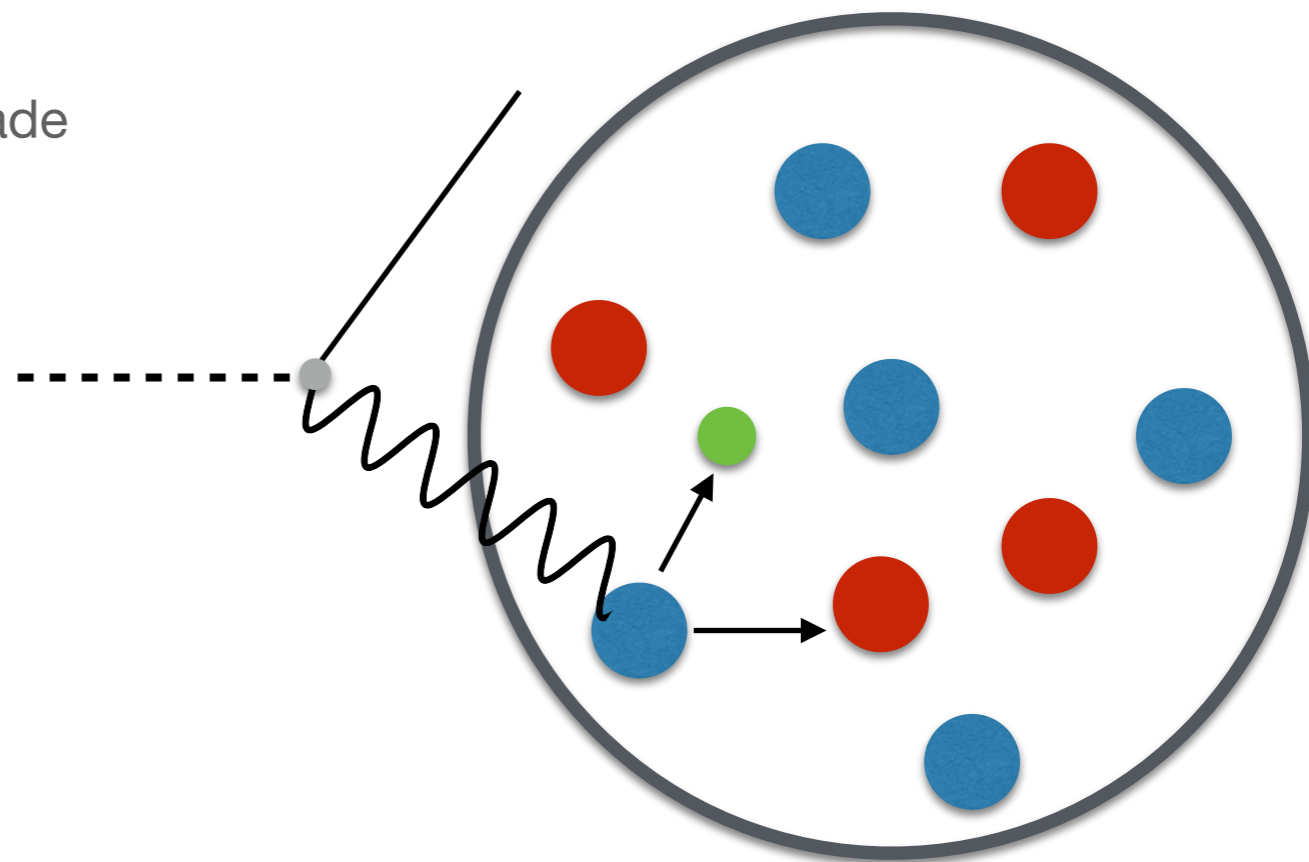
Simulate neutrino-nucleus cross sections to untangle neutrino oscillations from the measured interactions

$$N, \pi, \Delta$$

Lattice QCD :

Event Generator :

Fermilab

Thank you for your attention!