Neutrinos in the Standard Model

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Standard Model ingredients

- Renormalizable Quantum Field Theory in *flat* 3+1 spacetime
  → no gravity.
- Ingredients:
  - Gauge group $SU(3)_C xSU(2)_L xU(1)_Y$ → spin-1 bosons (G, W, B).
  - One complex scalar field in representation (**1**,**2**,1/2) → "Higgs" H.
  - Chiral spin-1/2 fermions in representation
  - $(\mathbf{3},\mathbf{2},1/6)+(\bar{\mathbf{3}},\mathbf{1},-4/6)+(\bar{\mathbf{3}},\mathbf{1},2/6)+(\mathbf{1},\mathbf{2},-1/2)+(\mathbf{1},\mathbf{1},1)+$
    $(\mathbf{3},\mathbf{2},1/6)+(\bar{\mathbf{3}},\mathbf{1},-4/6)+(\bar{\mathbf{3}},\mathbf{1},2/6)+(\mathbf{1},\mathbf{2},-1/2)+(\mathbf{1},\mathbf{1},1)+$
    $(\mathbf{3},\mathbf{2},1/6)+(\bar{\mathbf{3}},\mathbf{1},-4/6)+(\bar{\mathbf{3}},\mathbf{1},2/6)+(\mathbf{1},\mathbf{2},-1/2)+(\mathbf{1},\mathbf{1},1)$
    → Quarks and leptons.
- Now just write down all singlets (gauge and Lorentz) with mass dimension ≤ 4 to get SM Lagrangian.

Kinetic terms

- Fermions: $\underbrace{(\mathbf{3},\mathbf{2},1/6)}_{Q_L}+\underbrace{(\bar{\mathbf{3}},\mathbf{1},-4/6)}_{(u_R)^c}+\underbrace{(\bar{\mathbf{3}},\mathbf{1},2/6)}_{(d_R)^c}+\underbrace{(\mathbf{1},\mathbf{2},-1/2)}_{L_L}+\underbrace{(\mathbf{1},\mathbf{1},1)}_{(e_R)^c}$

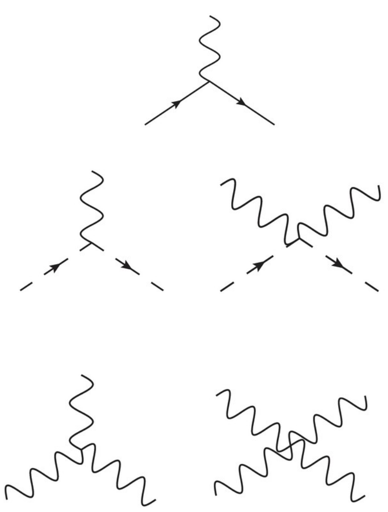
$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \qquad L_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

(in $SU(2)_L$ space)

- Abuse of notation and hindsight. Three copies of this.
- Gauge invariant kinetic terms $\left(D_\mu = \partial_\mu + i \sum g_j A^j_\mu T_j\right)$:

$$L_{kin} = i \sum_f \bar{f}\gamma^\mu D_\mu f$$
$$+ \ (D_\mu H)^\dagger (D^\mu H)$$
$$- \frac{1}{4} G^a_{\mu\nu} G^{a\,\mu\nu} - \frac{1}{4} W^a_{\mu\nu} W^{a\,\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

Kinetic terms include couplings to gauge bosons:



- All of these interactions just depend on 3 parameters:

$$g_1 \simeq 0.36\,, \quad g_2 \simeq 0.65\,, \quad g_3 \simeq 1.2\,.$$

Mass terms

- Gauge invariant mass terms (quadratic pieces without derivatives):

$$\mathrm{L}_{\mathsf{mass}} = \mu^2 \mathrm{H}^\dagger \mathrm{H}$$

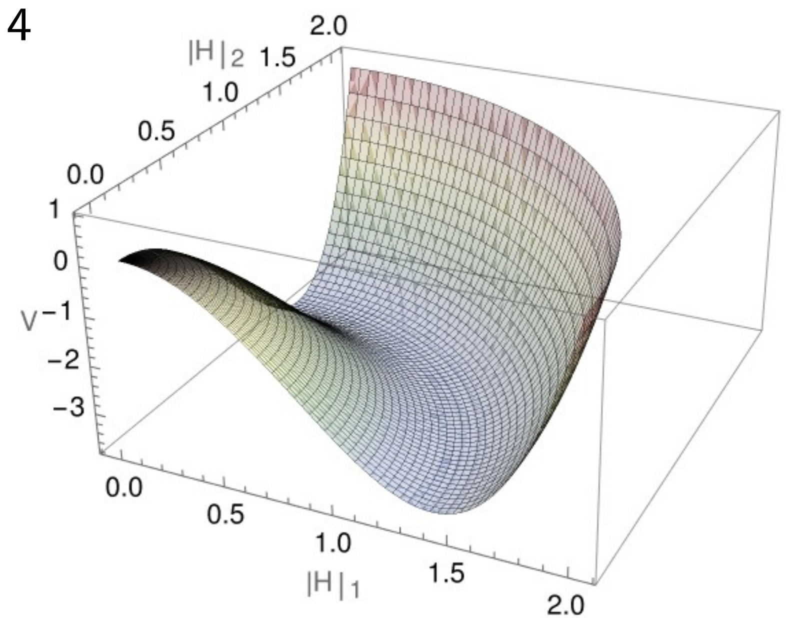
- Only the scalar has mass, all fermions are massless?!
- Gauge group SU(2)xU(1) forbids mass terms like $\bar{e}_L e_R$ or $\overline{e_R}^c e_R$.
- But we *know* fermions have mass, so SU(2)xU(1) must be broken!
- Break it explicitely? I.e. just write down $m_e \bar{e}_L e_R$ etc.?
  - Leads to inconsistencies like photon mass
  - ...unless one arranges the explicit breaking terms *very* carefully.
  - Not all breaking terms allowed, all controlled by *one* parameter.

Mass terms

- Gauge invariant mass terms (quadratic pieces without derivatives):

$$\mathrm{L}_{\text{mass}} = \mu^2 \mathrm{H}^\dagger \mathrm{H}$$

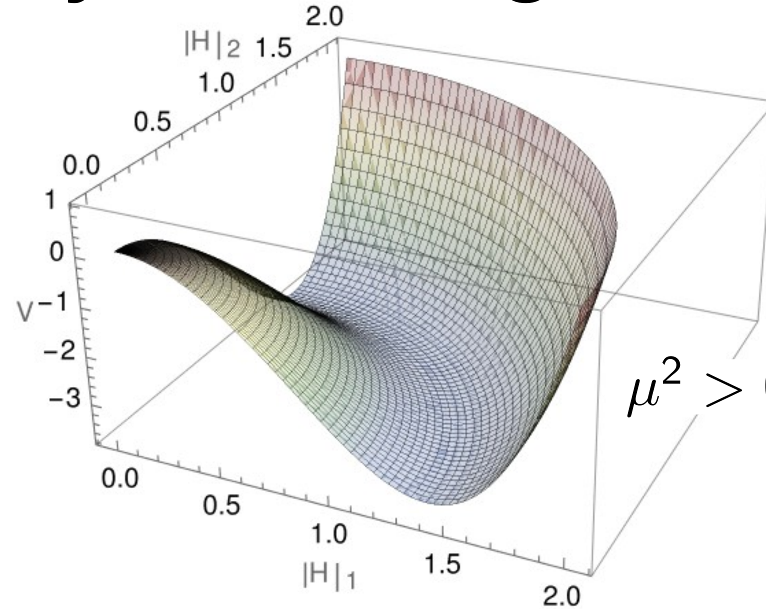
- Only the scalar has mass, all fermions are massless?!
- Gauge group SU(2)xU(1) forbids mass terms like $\bar{e}_L e_R$ or $\overline{e_R}^c e_R$.
- But we *know* fermions have mass, so SU(2)xU(1) must be broken!
- Nature: it is broken spontaneously (the reason we need the Higgs).
- Full potential for H: $\mathrm{V} = -\mu^2 |\mathrm{H}|^2 + \lambda |\mathrm{H}|^4$
  - If $\mu^2 > 0$, the minimum (vacuum state) is not at H = 0 but at $|\mathrm{H}|^2 = \dfrac{\mu^2}{2\lambda}$.
  - Vacuum value picks preferred direction in SU(2) space → broken.



Spontaneous symmetry breaking

- Scalar potential $\mathrm{V} = -\mu^2|\mathrm{H}|^2 + \lambda|\mathrm{H}|^4$ minimized by

$$|\mathrm{H}|^2 = \frac{\mu^2}{2\lambda}.$$



$\mu^2 > 0$

- Without loss of generality pick vacuum direction

$$\mathrm{H} = \begin{pmatrix} 0 \\ \mathrm{v}/\sqrt{2} \end{pmatrix} \quad \text{with real} \quad \mathrm{v} = \mu/\sqrt{\lambda} \simeq 246\,\mathrm{GeV}.$$

- Since H = (**1**,**2**,1/2), both SU(2) and $\mathrm{U}(1)_\mathrm{Y}$ are broken, but a U(1) subgroup survives!

$$\underbrace{\exp(\mathrm{i}\alpha \mathrm{T}_3)}_{\mathrm{SU}(2)}\,\underbrace{\exp(\mathrm{i}\alpha \mathrm{Y}_\mathrm{H})}_{\mathrm{U}(1)_\mathrm{Y}} \begin{pmatrix} 0 \\ \mathrm{v}/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 \\ \mathrm{v}/\sqrt{2} \end{pmatrix}$$

Spontaneous symmetry breaking

- H = (**1**,**2**,1/2) breaks SU(2)xU(1)$_Y$ to U(1)$_{electrodynamics}$! [Q = Y + T_3]
- Particles are excitations around the vacuum, should expand H as

$$\mathsf{H} = \begin{pmatrix} \mathsf{H}^+ \\ \mathsf{v}/\sqrt{2} + \mathsf{h}/\sqrt{2} + \mathsf{i}\mathsf{A}/\sqrt{2} \end{pmatrix}$$

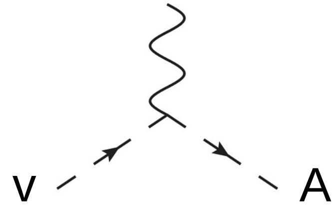
- Shifting by v generates SU(2)xU(1) breaking terms in Lagrangian.
- Find

$$\mathsf{V} = \frac{1}{2}(2\mu^2)\mathsf{h}^2 + \lambda \mathsf{v}\mathsf{h}^3 + \frac{1}{4}\lambda \mathsf{h}^4 + \text{other interactions.}$$

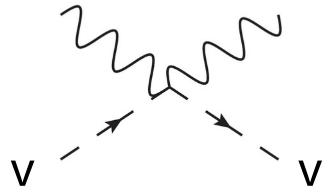
- The Higgs particle h has mass $\mathsf{m_h} = \sqrt{2}\mu = \sqrt{2\lambda}\mathsf{v} \simeq 125\,\text{GeV}$,
  A and H^+ are massless Goldstone bosons.
- Under U(1)$_{EM}$, h and A have charge 0, H^+ has charge 1.
- Now expand the H kinetic terms around the true vacuum.

Massive gauge bosons

- Kinetic term $(D_\mu H)^\dagger (D^\mu H)$ with $D_\mu = \partial_\mu + i \sum g_j A^j_\mu T_j$ contains terms



$\propto gv(\partial_\mu A)B^\mu$ A converts into B?



$\propto (gv)^2 B_\mu B^\mu$ Mass term for B?

- Rather than a massless Goldstone scalar + a massless vector we end up with a massive vector boson (3 degrees of freedom)!

  => **Higgs mechanism**.

- In the SM, the $U(1)_{EM}$ gauge boson (photon) remains massless, the other three get mass (A is swallowed by Z boson, H^+ by W^+).
- Can pick ‘unitary gauge’ that sets $A = 0 = H^+$.

Massive gauge bosons

- Find

$$\mathsf{m}_\mathsf{W} = \mathsf{g}_2\mathsf{v}/2 \simeq 80\,\mathsf{GeV}\,, \quad \mathsf{m}_\mathsf{Z} = \sqrt{\mathsf{g}_1^2 + \mathsf{g}_2^2}\,\mathsf{v}/2 \simeq 91\,\mathsf{GeV}.$$

  and

$$\begin{pmatrix} \mathsf{A}_\mu \\ \mathsf{Z}_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_\mathsf{W} & \sin\theta_\mathsf{W} \\ -\sin\theta_\mathsf{W} & \cos\theta_\mathsf{W} \end{pmatrix} \begin{pmatrix} \mathsf{B}_\mu \\ \mathsf{W}^3_\mu \end{pmatrix}$$

  with weak mixing angle $\theta_\mathsf{W} \simeq 30^\circ$:

$$\tan\theta_\mathsf{W} = \mathsf{g}_1/\mathsf{g}_2 \quad \text{or} \quad \sin^2\theta_\mathsf{W} = 1 - \mathsf{m}_\mathsf{W}^2/\mathsf{m}_\mathsf{Z}^2 \simeq 0.23.$$

- Electric charge $\mathsf{e} = \mathsf{g}_2 \sin\theta_\mathsf{W} \simeq 0.3$.
- Now we have massive **bosons**.

Kinetic fermion terms in this basis

$$\mathcal{L}_{\mathsf{kin}} \supset \sum_{\mathsf{f=u,d,e}} \mathsf{e}\mathsf{Q}_{\mathsf{f}}\, \bar{\mathsf{f}}\gamma^{\mu}\mathsf{f}\, \mathsf{A}_{\mu}$$

$$- \sum_{\mathsf{f=u,d,e,}\nu} \frac{\mathrm{g}}{\cos\theta_{\mathsf{W}}}(\mathsf{T}_3^{\mathsf{f}} - \mathsf{Q}^{\mathsf{f}}\sin^2\theta_{\mathsf{W}})\, \bar{\mathsf{f}_{\mathsf{L}}}\gamma^{\mu}\mathsf{f}_{\mathsf{L}}\, \mathsf{Z}_{\mu}$$

$$- \sum_{\mathsf{f=u,d,e}} \frac{\mathrm{g}}{\cos\theta_{\mathsf{W}}}(-\mathsf{Q}^{\mathsf{f}}\sin^2\theta_{\mathsf{W}})\, \bar{\mathsf{f}_{\mathsf{R}}}\gamma^{\mu}\mathsf{f}_{\mathsf{R}}\, \mathsf{Z}_{\mu}$$

$$- \left(\frac{\mathrm{g}}{\sqrt{2}}\, \bar{\mathsf{u}_{\mathsf{L}}}\gamma^{\mu}\mathsf{d}_{\mathsf{L}}\, \mathsf{W}_{\mu}^{+} + \frac{\mathrm{g}}{\sqrt{2}}\, \bar{\nu_{\mathsf{L}}}\gamma^{\mu}\mathsf{e}_{\mathsf{L}}\, \mathsf{W}_{\mu}^{+} + \mathsf{h.c.}\right)$$

$$\left(\mathsf{T}_3^{\mathsf{u}} = \mathsf{T}_3^{\nu} = +\tfrac{1}{2}\,,\ \mathsf{T}_3^{\mathsf{d}} = \mathsf{T}_3^{\mathsf{e}} = -\tfrac{1}{2}\,,\ \mathsf{Q}^{\nu} = 0\,,\ \mathsf{Q}^{\mathsf{e}} = -1\,,\ \mathsf{Q}^{\mathsf{u}} = \tfrac{2}{3}\,,\ \mathsf{Q}^{\mathsf{d}} = -\tfrac{1}{3}\right)$$

Kinetic fermion terms in this basis

$$\mathcal{L}_{\text{kin}} \supset \sum_{\text{f}=\text{u,d,e}} e Q_f \, \bar{f} \gamma^\mu f \, A_\mu$$

Quantum Electrodynamics

$$- \sum_{\text{f}=\text{u,d,e},\nu} \frac{g}{\cos\theta_W} (T_3^f - Q^f \sin^2\theta_W) \, \bar{f}_L \gamma^\mu f_L \, Z_\mu$$

$$- \sum_{\text{f}=\text{u,d,e}} \frac{g}{\cos\theta_W} (-Q^f \sin^2\theta_W) \, \bar{f}_R \gamma^\mu f_R \, Z_\mu$$

Neutral current

$$- \left(\frac{g}{\sqrt{2}} \bar{u}_L \gamma^\mu d_L \, W_\mu^+ + \frac{g}{\sqrt{2}} \bar{\nu}_L \gamma^\mu e_L \, W_\mu^+ + \text{h.c.} \right)$$

Charged current

$$\left(T_3^u = T_3^\nu = +\tfrac{1}{2} \, , \; T_3^d = T_3^e = -\tfrac{1}{2} \, , \; Q^\nu = 0 \, , \; Q^e = -1 \, , \; Q^u = \tfrac{2}{3} \, , \; Q^d = -\tfrac{1}{3} \right)$$

Massive fermions

- There are more terms we can write down in Lagrangian:

$$-\mathcal{L}_{\mathrm{Yukawa}} = y_e \overline{L}_L H e_R + y_d \overline{Q}_L H d_R + y_u \overline{Q}_L \tilde{H} u_R + \mathrm{h.c.}$$

where $\tilde{H} \equiv \begin{pmatrix} -H^* \\ H^- \end{pmatrix} \sim$ (**1**,**2**,-1/2).

- Can pick the y_f to be real, absorb phase into f_R.
- Expand H around vacuum:

$$y_e \overline{L}_L H e_R = \underbrace{y_e \frac{v}{\sqrt{2}}}_{m_e!} \bar{e}_L e_R + \cdots = m_e \bar{e} e + \frac{m_e}{v} \bar{e} e\, h$$

Higgs couplings proportional to mass

- Electron becomes massive and forms Dirac spinor $e = e_L + e_R$.
- Same for quarks, only neutrinos remain massless since no ν_R.

SM is shockingly efficient: same H gives mass to everything!

Three generations

- For whatever reason we have 3 fermion copies:

$$-\mathcal{L}_{\mathrm{Yukawa}} = y_e^{ij}\overline{L}_{L,i} H e_{R,j} + y_d^{ij}\overline{Q}_{L,i} H d_{R,j} + y_u^{ij}\overline{Q}_{L,i}\tilde{H} u_{R,j} + \mathrm{h.c.}$$
$$= \left(\bar{e}_{L,i} y_e^{ij} e_{R,j} + \bar{d}_{L,i} y_d^{ij} d_{R,j} + \bar{u}_{L,i} y_u^{ij} u_{R,j}\right)\left(\frac{v+h}{\sqrt{2}}\right) + \mathrm{h.c.}$$

- y_f are now 3x3 complex matrices.
- Linear algebra: use singular value decomposition to write

Diagonal matrix with positive entries

$$y_f = V_L^f \, y_f^{\mathrm{diag}} \, V_R^f$$

Unitary 3x3 matrices

- Now redefine $f_R \to (V_R^f)^\dagger f_R$, just removes V_R!
- Now redefine $f_L \to V_L^f f_L$,
  Yukawa/mass terms are diagonal: $y_e^{\mathrm{diag}} = \frac{\sqrt{2}}{v}\begin{pmatrix} m_e & & \\ & m_\mu & \\ & & m_\tau \end{pmatrix}$

...but

- Rotating components of doublets (u d) by different amounts gives

$$-\left(\frac{g}{\sqrt{2}}\,\bar{u}_L\,\boxed{(V_L^u)^\dagger V_L^d}\,\gamma^\mu d_L\,W_\mu^+ + \frac{g}{\sqrt{2}}\,\bar{\nu}_L\,\underbrace{\boxed{(V_L^\nu)^\dagger V_L^e}}\,\gamma^\mu e_L\,W_\mu^+ + \text{h.c.}\right)$$

V^ν arbitrary in SM, pick $V_L^\nu = V_L^e$

...but

- Rotating components of doublets (u d) by different amounts gives

$$-\left(\frac{g}{\sqrt{2}}\,\bar{u}_L\,\underbrace{\boxed{(V_L^u)^\dagger V_L^d}}\,\gamma^\mu d_L\,W_\mu^+ + \frac{g}{\sqrt{2}}\,\bar{\nu}_L\,\underbrace{\boxed{(V_L^\nu)^\dagger V_L^e}}\,\gamma^\mu e_L\,W_\mu^+ + \text{h.c.}\right)$$

This is just another unitary matrix, call it V; Cabibbo-Kobayashi-Maskawa matrix

V^ν arbitrary in SM, pick $V_L^\nu = V_L^e$

- Linear algebra: can write any unitary matrix as

$$\begin{pmatrix} e^{-i\phi(u)} & 0 & 0 \\ 0 & e^{-i\phi(c)} & 0 \\ 0 & 0 & e^{-i\phi(t)} \end{pmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} e^{-i\phi(d)} & 0 & 0 \\ 0 & e^{i\phi(s)} & 0 \\ 0 & 0 & e^{i\phi(b)} \end{pmatrix}$$

Redefine u_L and d_L once more to absorb/remove these phases.

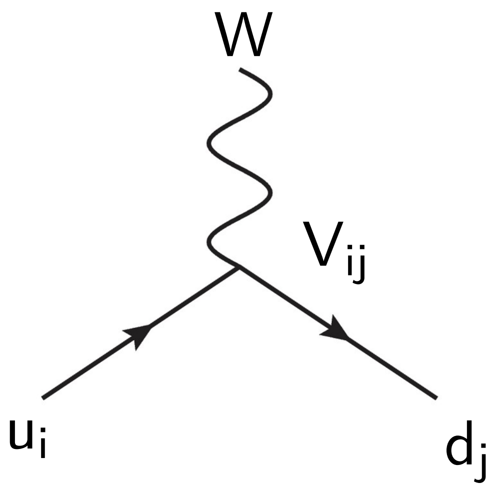
- Left with 3 angles & 1 phase:

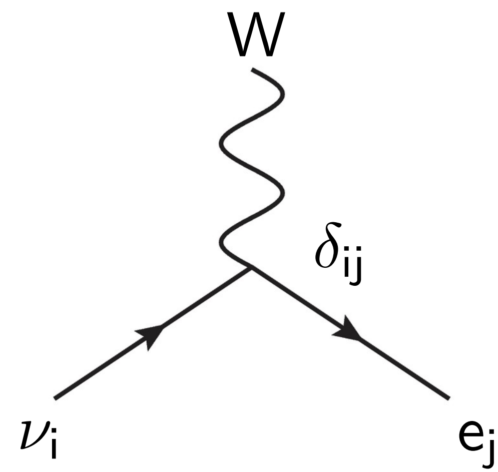
$$\theta_{12} \simeq 13^\circ\,,\ \ \theta_{13} \simeq 0.2^\circ\,,\ \ \theta_{23} \simeq 2.4^\circ\,,\ \ \delta \simeq 69^\circ\,.$$

...but

- Rotating components of doublets (u d) by different amounts gives

$$-\left(\frac{g}{\sqrt{2}}\,\bar{u}_L\,V\,\gamma^\mu d_L\,W_\mu^+ + \frac{g}{\sqrt{2}}\,\bar{\nu}_L\,\gamma^\mu e_L\,W_\mu^+ + \text{h.c.}\right)$$





- Quark-flavor changing interactions. Neutrino belongs to lepton.
- (There are no flavor-changing neutral currents.)

And that's the Standard Model of particle physics!

Number of parameters

- 3 g couplings, v, m_h, 9 fermion masses, 3 CKM angles & 1 phase.
- All measured by now (as of 2012), SM is now fully predictive!

Symbol	Description	Value
m_e	Electron mass	510.9989461(31) keV
m_μ	Muon mass	105.6583745(24) MeV
m_τ	Tau mass	1.77686(12) GeV
m_u	Up quark mass	2.16 MeV
m_d	Down quark mass	4.67 MeV
m_s	Strange quark mass	93.4 MeV
m_c	Charm quark mass	1.27 GeV
m_b	Bottom quark mass	4.18 GeV
m_t	Top quark mass	172.69 GeV
θ_{12}	CKM 12-mixing angle	13.1°
θ_{23}	CKM 23-mixing angle	2.4°
θ_{13}	CKM 13-mixing angle	0.2°
δ	CKM CP-violating Phase	0.995
g_1 or g'	U(1) gauge coupling	0.357
g_2 or g	SU(2) gauge coupling	0.652
g_3 or g_s	SU(3) gauge coupling	1.221
θ_{QCD}	QCD vacuum angle	~0
v	Higgs vacuum expectation value	246.2196(2) GeV
m_H	Higgs mass	125.18 GeV

18 parameters that determine all of particle physics.

Sneaky omission of last allowed term in SM Lagrangian: θGG.
Why 0? Strong CP problem.

Final SM Lagrangian

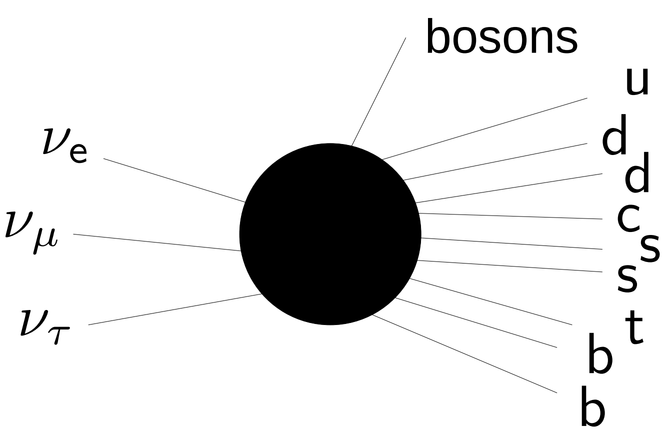
$$
\begin{aligned}
\mathcal{L}_{\mathrm{SM}} = {} & \mathrm{i}\sum_{\mathrm{f}} \bar{\mathrm{f}}\gamma^{\mu}\partial_{\mu}\mathrm{f} + \tfrac{1}{2}\partial_{\mu}\mathrm{h}\,\partial^{\mu}\mathrm{h} - \frac{1}{4}\mathrm{G}^{\mathrm{a}}_{\mu\nu}\mathrm{G}^{\mathrm{a}\,\mu\nu} - \frac{1}{4}\mathrm{W}^{\mathrm{a}}_{\mu\nu}\mathrm{W}^{\mathrm{a}\,\mu\nu} - \frac{1}{4}\mathrm{B}_{\mu\nu}\mathrm{B}^{\mu\nu} \\
& - \left(\bar{\mathrm{e}}\mathrm{m_e}\mathrm{e} + \bar{\mathrm{d}}\mathrm{m_d}\mathrm{d} + \bar{\mathrm{u}}\mathrm{m_u}\mathrm{u}\right)\left(1 + \frac{\mathrm{h}}{\mathrm{v}}\right) \\
& - \left(\mathrm{m}^2_{\mathrm{W}}\mathrm{W}^{\mu\,+}\mathrm{W}^{-}_{\mu} + \tfrac{1}{2}\mathrm{m}^2_{\mathrm{Z}}\mathrm{Z}_{\mu}\mathrm{Z}^{\mu}\right)\left(1 + \frac{\mathrm{h}}{\mathrm{v}}\right)^2 - \frac{1}{2}\mathrm{m}^2_{\mathrm{h}}\mathrm{h}^2\left(1 + \frac{\mathrm{h}}{2\mathrm{v}}\right)^2 \\
& + \sum_{\mathrm{f=u,d,e}} \mathrm{eQ_f}\,\bar{\mathrm{f}}\gamma^{\mu}\mathrm{f}\,\mathrm{A}_{\mu} + \sum_{\mathrm{q=u,d}} \mathrm{g_3}\,\bar{\mathrm{q}}\gamma^{\mu}\mathrm{T^a}\mathrm{q}\,\mathrm{G}^{\mathrm{a}}_{\mu} \\
& - \sum_{\mathrm{f=u,d,e},\nu} \frac{\mathrm{g}}{\cos\theta_{\mathrm{W}}}\left[\left(\mathrm{T}^{\mathrm{f}}_{3} - \mathrm{Q}^{\mathrm{f}}\sin^2\theta_{\mathrm{W}}\right)\bar{\mathrm{f}}_{\mathrm{L}}\gamma^{\mu}\mathrm{f}_{\mathrm{L}} + \left(-\mathrm{Q}^{\mathrm{f}}\sin^2\theta_{\mathrm{W}}\right)\bar{\mathrm{f}}_{\mathrm{R}}\gamma^{\mu}\mathrm{f}_{\mathrm{R}}\right]\mathrm{Z}_{\mu} \\
& - \left(\frac{\mathrm{g}}{\sqrt{2}}\,\bar{\mathrm{u}}_{\mathrm{L}}\,\mathrm{V}\,\gamma^{\mu}\mathrm{d}_{\mathrm{L}}\,\mathrm{W}^{+}_{\mu} + \frac{\mathrm{g}}{\sqrt{2}}\,\bar{\nu}_{\mathrm{L}}\gamma^{\mu}\mathrm{e}_{\mathrm{L}}\,\mathrm{W}^{+}_{\mu} + \mathrm{h.c.}\right)
\end{aligned}
$$

Accidental symmetries

- SM Lagrangian has more (global) symmetries than we imposed:
- $q \to e^{i\alpha} q$: U(1) symmetry → conserved quark/baryon number.
- $\ell_j \to e^{i\alpha}\ell_j$, $\nu_j \to e^{i\alpha}\nu_j$: U(1) symmetry → conserved lepton number.

$$G_{\mathrm{global}} = \underbrace{U(1)_B}_{\text{Forbids proton decay.}} \times \underbrace{U(1)_{L_e} \times U(1)_{L_\mu} \times U(1)_{L_\tau}}_{\text{Forbids lepton flavor violation, e.g. } \mu \to e\gamma}$$

- ‘t Hooft: at quantum level, non-perturbative instanton/sphaleron processes break the subgroup $U(1)_{B+L_e+L_\mu+L_\tau}$:



$$\propto e^{-(4\pi)^2/g^2}$$

Too suppressed to ever see this, but relevant in hot early universe, esp. for leptogenesis.

Accidental symmetries

- Actual global symmetry group after 't Hooft:

$$G_{\text{global}} = \underbrace{U(1)_{B-L}}_{\text{Broken if neutrinos are Majorana, might be conserved if neutrinos are Dirac.}} \times \underbrace{U(1)_{L_e - L_\mu} \times U(1)_{L_\mu - L_\tau}}_{\text{Violated by neutrino oscillations.}}$$

- Models for neutrino oscillation need to break last two U(1), generically expect charged lepton flavor violation as well:

$$\mu \to e\gamma\,,\;\; \mu \to 3e\,,\;\; \tau \to e\mu\mu\,,\;\; Z \to e\mu\,,\;\; \text{etc.}$$

- Rich experimental program for these processes, e.g. Mu2e @ FNAL.

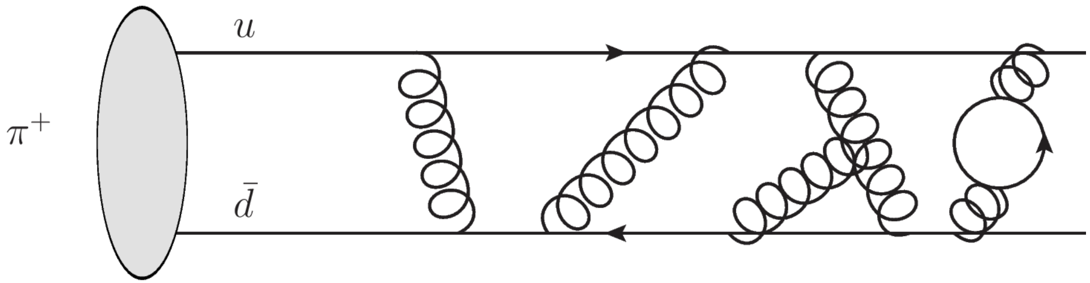
Quantum Chromodynamics

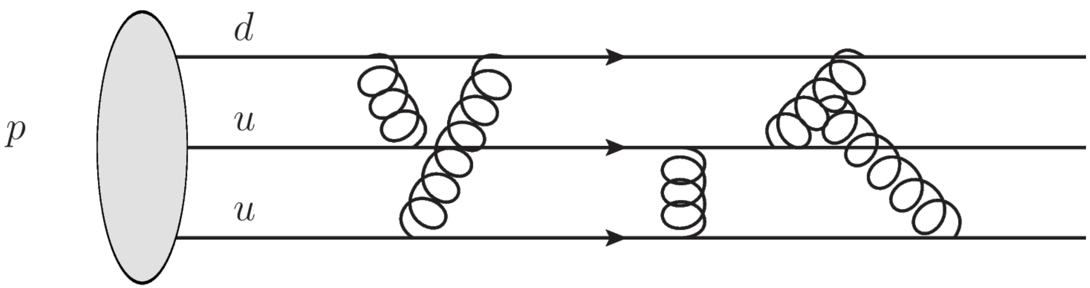
- Everything simple so far, all coupling constants are small enough for perturbation theory (Feynman diagrams), except for g_3.
- At low energies, g_3 is large and strongly binds quarks and gluons.
- Should not talk about quarks/gluons but rather their color-neutral bound states.
- Mesons are $\bar{q}q$ or gg states (bosons), baryons qqq (fermions).
- E.g.

$$\pi^+ = \bar{\mathrm{d}}\mathrm{u}\,,\, \pi^0 = \bar{\mathrm{u}}\mathrm{u} \text{ or } \bar{\mathrm{d}}\mathrm{d}\,,\ldots$$

$$\mathrm{p} = \mathrm{uud}\,,\,\mathrm{n} = \mathrm{udd}\,,\ldots$$

- Most complicated SM part.
- Either *Chiral Perturbation Theory* or *Lattice QCD*.





LIGHT UNFLAVORED ($S = C = B = 0$)

	$I^G(J^{PC})$		$I^G(J^{PC})$
• $\pi^\pm$	$1^-(0^-)$	• $\rho(1700)$	$1^+(1^{--})$
• π^0	$1^-(0^{-+})$	• $a_2(1700)$	$1^-(2^{++})$
• η	$0^+(0^{-+})$	$a_0(1710)$	$1^-(0^{++})$
• $f_0(500)$	$0^+(0^{++})$	• $f_0(1710)$	$0^+(0^{++})$
aka σ; was $f_0(600)$, f_0(400-1200)		$X(1750)$	$?^-(1^{--})$
		$\eta(1760)$	$0^+(0^{-+})$
• $\rho(770)$	$1^+(1^{--})$	$f_0(1770)$	$0^+(0^{++})$
• $\omega(782)$	$0^-(1^{--})$	• $\pi(1800)$	$1^-(0^{-+})$
• $\eta'(958)$	$0^+(0^{-+})$	$f_2(1810)$	$0^+(2^{++})$
• $f_0(980)$	$0^+(0^{++})$	$X(1835)$	$?^?(0^{-+})$
• $a_0(980)$	$1^-(0^{++})$	• $\phi_3(1850)$	$0^-(3^{--})$
• $\phi(1020)$	$0^-(1^{--})$	$\eta_1(1855)$	$0^+(1^{-+})$
• $h_1(1170)$	$0^-(1^{+-})$	• $\eta_2(1870)$	$0^+(2^{-+})$
• $b_1(1235)$	$1^+(1^{+-})$	• $\pi_2(1880)$	$1^-(2^{-+})$
• $a_1(1260)$	$1^-(1^{++})$	$\rho(1900)$	$1^+(1^{--})$
• $f_2(1270)$	$0^+(2^{++})$	$f_2(1910)$	$0^+(2^{++})$
• $f_1(1285)$	$0^+(1^{++})$	$a_0(1950)$	$1^-(0^{++})$
• $\eta(1295)$	$0^+(0^{-+})$	• $f_2(1950)$	$0^+(2^{++})$
• $\pi(1300)$	$1^-(0^{-+})$	• $a_4(1970)$	$1^-(4^{++})$
• $a_2(1320)$	$1^-(2^{++})$	was $a_4(2040)$	
• $f_0(1370)$	$0^+(0^{++})$	$\rho_3(1990)$	$1^+(3^{--})$
• $\pi_1(1400)$	$1^-(1^{-+})$	$\pi_2(2005)$	$1^-(2^{-+})$

STRANGE ($S = \pm1$, $C = B = 0$)

	$I(J^P)$
• $K^\pm$	$1/2(0^-)$
• K^0	$1/2(0^-)$
• K^0_S	$1/2(0^-)$
• K^0_L	$1/2(0^-)$
• $K_0^*(700)$	$1/2(0^+)$
aka κ; was $K_0^*(800)$	
• $K^*(892)$	$1/2(1^-)$
• $K_1(1270)$	$1/2(1^+)$
• $K_1(1400)$	$1/2(1^+)$
• $K^*(1410)$	$1/2(1^-)$
• $K_0^*(1430)$	$1/2(0^+)$
• $K_2^*(1430)$	$1/2(2^+)$
• $K(1460)$	$1/2(0^-)$
$K_2(1580)$	$1/2(2^-)$
$K(1630)$	$1/2(?^?)$
• $K_1(1650)$	$1/2(1^+)$
• $K^*(1680)$	$1/2(1^-)$
• $K_2(1770)$	$1/2(2^-)$
• $K_3^*(1780)$	$1/2(3^-)$
• $K_2(1820)$	$1/2(2^-)$
$K(1830)$	$1/2(0^-)$
$K_0^*(1950)$	$1/2(0^+)$

CHARMED, STRANGE ($C = \pm1$, $S = \pm1$) (including possibly non- $q\bar{q}$ states)

	$I(J^P)$
• $D_s^\pm$	$0(0^-)$
• $D_s^{*\pm}$	$0(?^?)$
• $D_{s0}^*(2317)^\pm$	$0(0^+)$
• $D_{s1}(2460)^\pm$	$0(1^+)$
• $D_{s1}(2536)^\pm$	$0(1^+)$
• $D_{s2}^*(2573)$	$0(2^+)$
$D_{s0}(2590)^+$	$0(0^-)$
• $D_{s1}^*(2700)^\pm$	$0(1^-)$
$D_{s1}^*(2860)^\pm$	$0(1^-)$
was D_sJ^*(2860)	
• $D_{s3}^*(2860)^\pm$	$0(3^-)$
$D_{sJ}(3040)^\pm$	$0(?^?)$

BOTTOM ($B = \pm1$)

• $B^\pm$	$1/2(0^-)$
• B^0	$1/2(0^-)$
• $B^\pm/B^0$ ADMIXTURE	
• $B^\pm/B^0/B^0_s/b$-baryon ADMIXTURE	
V_{cb} and V_{ub} CKM Matrix Elements	
• B^*	$1/2(1^-)$
• $B_1(5721)$	$1/2(1^+)$

$b\bar{b}$ (including possibly non- $q\bar{q}$ states)

	$I^G(J^{PC})$
• $\eta_b(1S)$	$0^+(0^{-+})$
• $\Upsilon(1S)$	$0^-(1^{--})$
• $\chi_{b0}(1P)$	$0^+(0^{++})$
• $\chi_{b1}(1P)$	$0^+(1^{++})$
• $h_b(1P)$	$0^-(1^{+-})$
• $\chi_{b2}(1P)$	$0^+(2^{++})$
$\eta_b(2S)$	$0^+(0^{-+})$
• $\Upsilon(2S)$	$0^-(1^{--})$
• $\Upsilon_2(1D)$	$0^-(2^{--})$
was $\Upsilon(1D)$	
• $\chi_{b0}(2P)$	$0^+(0^{++})$
• $\chi_{b1}(2P)$	$0^+(1^{++})$
• $h_b(2P)$	$0^-(1^{+-})$
• $\chi_{b2}(2P)$	$0^+(2^{++})$
• $\Upsilon(3S)$	$0^-(1^{--})$
• $\chi_{b1}(3P)$	$0^+(1^{++})$
• $\chi_{b2}(3P)$	$0^+(2^{++})$
• $\Upsilon(4S)$	$0^-(1^{--})$
aka $\Upsilon(10580)$	
$\Upsilon(10753)$	$?^?(1^{--})$
• $\Upsilon(10860)$	$0^-(1^{--})$
• $\Upsilon(11020)$	$0^-(1^{--})$

p	$1/2^+$	****
n	$1/2^+$	****
$N(1440)$	$1/2^+$	****
$N(1520)$	$3/2^-$	****
$N(1535)$	$1/2^-$	****
$N(1650)$	$1/2^-$	****
$N(1675)$	$5/2^-$	****
$N(1680)$	$5/2^+$	****
$N(1700)$	$3/2^-$	***
$N(1710)$	$1/2^+$	****
$N(1720)$	$3/2^+$	****
$N(1860)$	$5/2^+$	**
$N(1875)$	$3/2^-$	***
was N(2080)		
$N(1880)$	$1/2^+$	***
$N(1895)$	$1/2^-$	****
was N(2090)		
$N(1900)$	$3/2^+$	****
$N(1990)$	$7/2^+$	**
$N(2000)$	$5/2^+$	**
was N(1900)		
$N(2040)$	$3/2^+$	*
$N(2060)$	$5/2^-$	***
was N(2200)		
$N(2100)$	$1/2^+$	***
$N(2120)$	$3/2^-$	***
$N(2190)$	$7/2^-$	****

$\Delta(1232)$	$3/2^+$	****
$\Delta(1600)$	$3/2^+$	****
$\Delta(1620)$	$1/2^-$	****
$\Delta(1700)$	$3/2^-$	****
$\Delta(1750)$	$1/2^+$	*
$\Delta(1900)$	$1/2^-$	***
$\Delta(1905)$	$5/2^+$	****
$\Delta(1910)$	$1/2^+$	****
$\Delta(1920)$	$3/2^+$	***
$\Delta(1930)$	$5/2^-$	***
$\Delta(1940)$	$3/2^-$	**
$\Delta(1950)$	$7/2^+$	****
$\Delta(2000)$	$5/2^+$	**
$\Delta(2150)$	$1/2^-$	*
$\Delta(2200)$	$7/2^-$	***
$\Delta(2300)$	$9/2^+$	**
$\Delta(2350)$	$5/2^-$	*
$\Delta(2390)$	$7/2^+$	*
$\Delta(2400)$	$9/2^-$	**
$\Delta(2420)$	$11/2^+$	****
$\Delta(2750)$	$13/2^-$	**
$\Delta(2950)$	$15/2^+$	**
Λ	$1/2^+$	****
$\Lambda(1380)$	$1/2^-$	**
$\Lambda(1405)$	$1/2^-$	****
$\Lambda(1520)$	$3/2^-$	****

Σ^+	$1/2^+$	****
Σ^0	$1/2^+$	****
Σ^-	$1/2^+$	****
$\Sigma(1385)$	$3/2^+$	****
$\Sigma(1580)$	$3/2^-$	*
$\Sigma(1620)$	$1/2^-$	*
$\Sigma(1660)$	$1/2^+$	***
$\Sigma(1670)$	$3/2^-$	****
$\Sigma(1750)$	$1/2^-$	***
$\Sigma(1775)$	$5/2^-$	****
$\Sigma(1780)$	$3/2^+$	*
was $\Sigma(1730)$		
$\Sigma(1880)$	$1/2^+$	**
$\Sigma(1900)$	$1/2^-$	**
$\Sigma(1910)$	$3/2^-$	***
was $\Sigma(1940)$		
$\Sigma(1915)$	$5/2^+$	****
$\Sigma(1940)$	$3/2^+$	*
$\Sigma(2010)$	$3/2^-$	*
was Sigma(2000)		
$\Sigma(2030)$	$7/2^+$	****
$\Sigma(2070)$	$5/2^+$	*
$\Sigma(2080)$	$3/2^+$	*
$\Sigma(2100)$	$7/2^-$	*
$\Sigma(2110)$	$1/2^-$	*
was $\Sigma(2160)$		
$\Sigma(2230)$	$3/2^+$	*

Ξ^0	$1/2^+$	****
Ξ^-	$1/2^+$	****
$\Xi(1530)$	$3/2^+$	****
$\Xi(1620)$		*
$\Xi(1690)$		***
$\Xi(1820)$	$3/2^-$	***
$\Xi(1950)$		***
$\Xi(2030)$	$\frac{5}{2}^?$	***
$\Xi(2120)$		*
$\Xi(2250)$		**
$\Xi(2370)$		**
$\Xi(2500)$		*
Ω^-	$3/2^+$	****
$\Omega(2012)^-$	$?^-$	***
$\Omega(2250)^-$		***
$\Omega(2380)^-$		**
$\Omega(2470)^-$		**

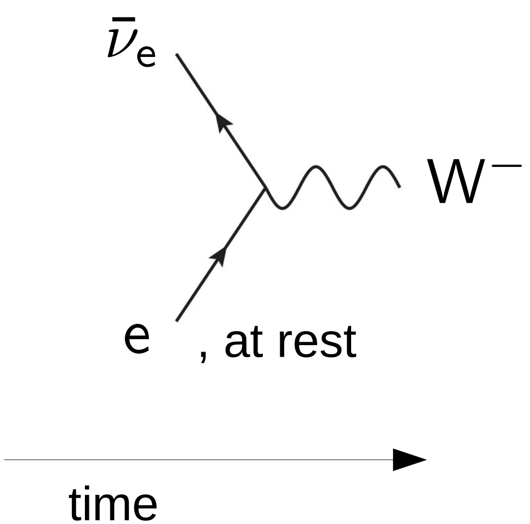
Λ_c^+	$1/2^+$	****
$\Lambda_c(2595)^+$	$1/2^-$	***
$\Lambda_c(2625)^+$	$3/2^-$	***
$\Lambda_c(2765)^+$ or $\Sigma_c(2765)$		*
$\Lambda_c(2860)^+$	$3/2^+$	***
$\Lambda_c(2880)^+$	$5/2^+$	***
$\Lambda_c(2940)^+$	$3/2^-$	***
$\Sigma_c(2455)$	$1/2^+$	****
$\Sigma_c(2520)$	$3/2^+$	***
$\Sigma_c(2800)$		***
Ξ_c^+	$1/2^+$	***
Ξ_c^0	$1/2^+$	****
$\Xi_c'^+$	$1/2^+$	***
$\Xi_c'^0$	$1/2^+$	***
$\Xi_c(2645)$	$3/2^+$	***
$\Xi_c(2790)$	$1/2^-$	***
$\Xi_c(2815)$	$3/2^-$	***
$\Xi_c(2923)$		**
$\Xi_c(2930)$		**
$\Xi_c(2970)$	$1/2^+$	***
was $\Xi_c(2980)$		
$\Xi_c(3055)$		***
$\Xi_c(3080)$		***
$\Xi_c(3123)$		*
Ω_c^0	$1/2^+$	***

Neutrinos in the SM (finally!)

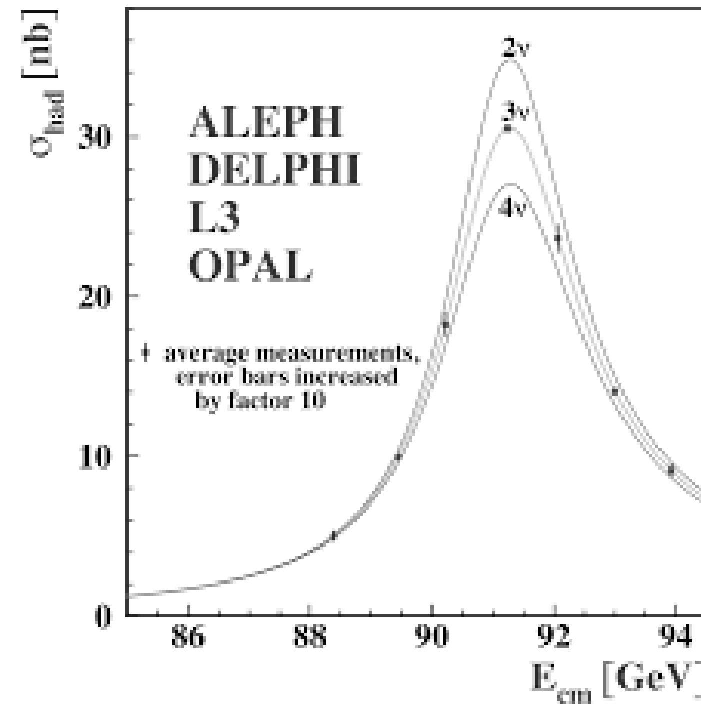
- Neutrinos only couple to Z and W:

$$-\frac{g}{2\cos\theta_W}\,\bar{\nu}_L\gamma^\mu\nu_L\, Z_\mu - \left(\frac{g}{\sqrt{2}}\,\bar{\nu}_L\gamma^\mu e_L\, W^+_\mu + \text{h.c.}\right)$$

- Flavor universal, couplings are not small.
- Confirmed via Z and W decays @ LEP.
- But, since Z & W are very heavy and unstable, we rarely deal with processes with them in initial/final state.



Requires a ridiculous
$E_\nu \sim 6\times 10^6$ GeV!
(Glashow resonance).

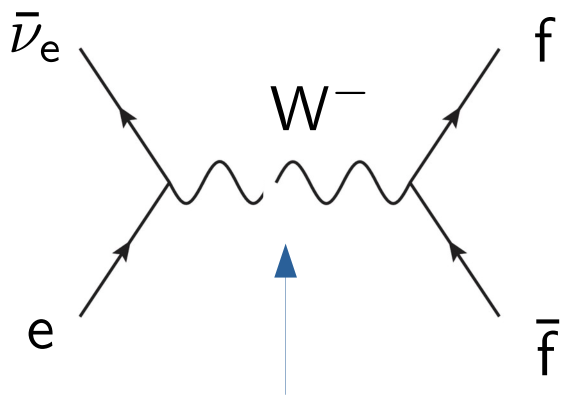


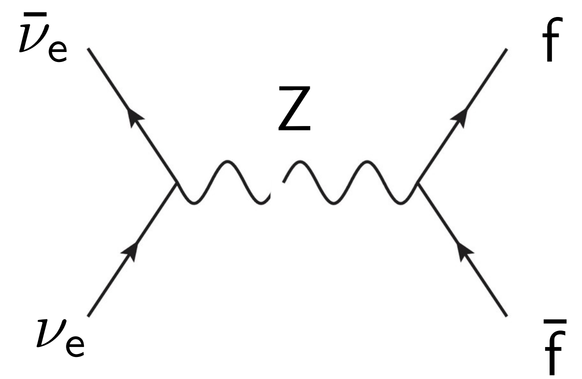
Fermi interactions

- Neutrinos only couple to Z and W:

$$- \frac{g}{2\cos\theta_W} \bar{\nu}_L \gamma^\mu \nu_L Z_\mu - \left(\frac{g}{\sqrt{2}} \bar{\nu}_L \gamma^\mu e_L W^+_\mu + \text{h.c.} \right)$$

- More important: off-shell Z & W:





$$\propto \frac{1}{m_W^2 - s} \simeq \frac{1}{m_W^2}$$

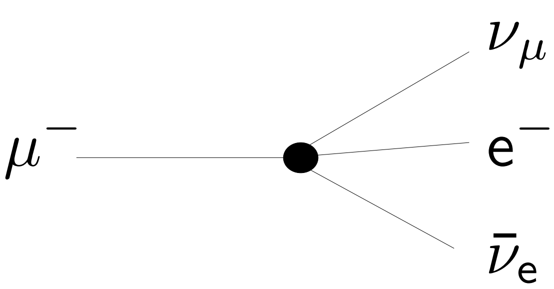
- At low energies, we can *integrate out* Z & W to get simpler Lagrangian. (Solve classical equation of motion for W/Z neglecting kinetic energy.)

Fermi Lagrangian

- Current-current interactions:

$$\underbrace{\left(\frac{g}{\sqrt{2}}\right)^2 \frac{1}{m_W^2}}_{\equiv 4\frac{G_F}{\sqrt{2}} = \frac{1}{(v/\sqrt{2})^2}} \left(\bar{u_L}\, V\, \gamma^\mu d_L + \bar{\nu_L}\gamma^\mu e_L\right) \left(\bar{d_L}\, V^\dagger\, \gamma_\mu u_L + \bar{e_L}\gamma_\mu \nu_L\right)$$

- Historically this was the starting point, UV completion via W much later.
- Similar for Z interactions; prefactor again $\left(\frac{g}{\sqrt{2}\cos\theta_W}\right)^2 \frac{1}{m_Z^2} = 4\frac{G_F}{\sqrt{2}}$
- At low energies, all amplitudes involving ν come with factor G_F.

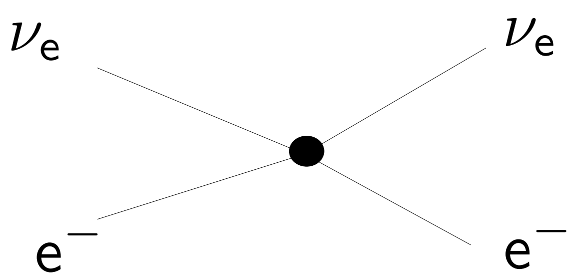


$$\mathcal{M} \propto G_F \qquad \Rightarrow \Gamma \propto G_F^2 m_\mu^5.$$

Three-body kinematics explains Michel spectrum.
Similar for tauon decays.

Neutrino scattering

- Neutrino-electron scattering:



$$\mathcal{M} \propto G_F \qquad \Rightarrow \sigma \propto G_F^2 s.$$

- A bit more careful with kinematics: $\sigma \propto G_F^2\, 2m_e E_\nu \quad \text{for} \quad E_\nu \gg m_e,$

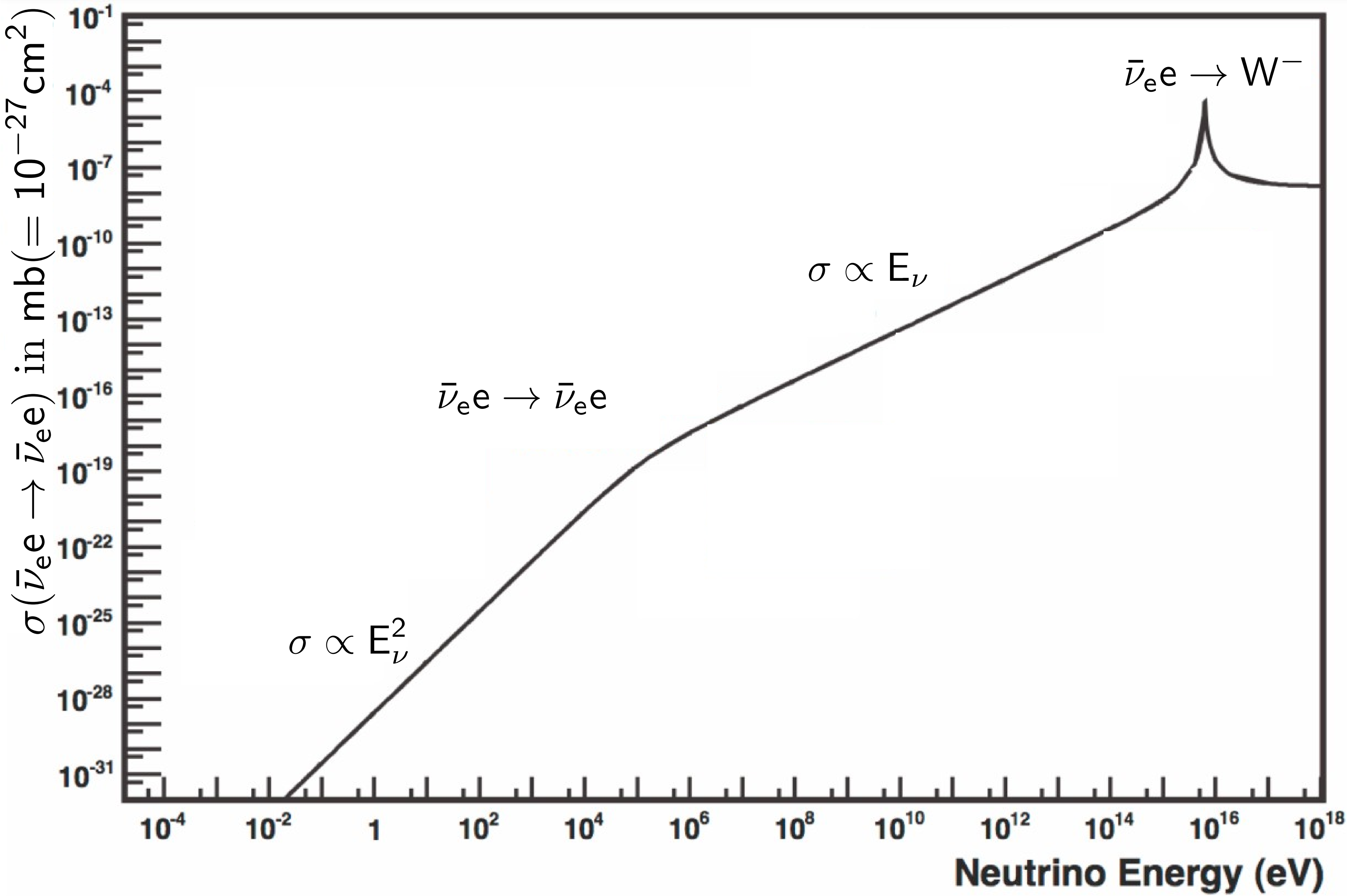
$$\sigma \propto G_F^2\, 4E_\nu^2 \quad \text{for} \quad E_\nu \ll m_e$$

- Good enough for estimates, O(1) prefactors depend on process.
- By assumption $E_\nu \ll m_W$, so cross sections are *tiny*:

$$\sigma \sim 10^{-44}\text{cm}^2 \left(\frac{E_\nu}{\text{MeV}}\right)$$

- Tiny not because of a coupling, but because of large Z & W mass!

[Formaggio & Zeller: arXiv:1305.7513]

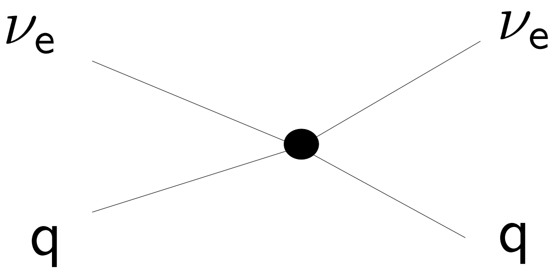


Process $1+2 \longrightarrow 3+4$	$3\pi G_F^{-2}\sigma_{12\to34}$
$\nu_e+\overline{\nu}_e \longrightarrow \nu_e+\overline{\nu}_e$	s
$\nu_e+\nu_e \longrightarrow \nu_e+\nu_e$	$\frac{3}{2}s$
$\nu_e+\overline{\nu}_e \longrightarrow \nu_{\mu(\tau)}+\overline{\nu}_{\mu(\tau)}$	$\frac{1}{4}s$
$\nu_e+\overline{\nu}_{\mu(\tau)} \longrightarrow \nu_e+\overline{\nu}_{\mu(\tau)}$	$\frac{1}{4}s$
$\nu_e+\nu_{\mu(\tau)} \longrightarrow \nu_e+\nu_{\mu(\tau)}$	$\frac{3}{4}s$
$\nu_e+\overline{\nu}_e \longrightarrow e^++e^-$	$\frac{4}{\sqrt{s}}\sqrt{s-4m_e^2}[(s-m_e^2)(\tilde{g}_L^{l2}+g_R^{l2})+6m_e^2\tilde{g}_L^l g_R^l]$
$e^++e^- \longrightarrow \nu_e+\overline{\nu}_e$	$\frac{\sqrt{s}}{2\sqrt{s-4m_e^2}}[(s-m_e^2)(\tilde{g}_L^{l2}+g_R^{l2})+6m_e^2\tilde{g}_L^l g_R^l]$
$\nu_e+e^- \longrightarrow \nu_e+e^-$	$(3\,s\,\tilde{g}_L^{l2}+s\,g_R^{l2}-3\,m_e^2\,\tilde{g}_L^l\,g_R^l)$
$\overline{\nu}_e+e^- \longrightarrow \overline{\nu}_e+e^-$	$(3\,s\,g_R^{l2}+s\,\tilde{g}_L^{l2}-3\,m_e^2\,\tilde{g}_L^l\,g_R^l)$
$\nu_\mu+\overline{\nu}_\mu \longrightarrow \nu_\mu+\overline{\nu}_\mu$	s
$\nu_\mu+\nu_\mu \longrightarrow \nu_\mu+\nu_\mu$	$\frac{3}{2}s$
$\nu_\mu+\overline{\nu}_\mu \longrightarrow \nu_{e(\tau)}+\overline{\nu}_{e(\tau)}$	$\frac{1}{4}s$
$\nu_\mu+\overline{\nu}_{e(\tau)} \longrightarrow \nu_\mu+\overline{\nu}_{e(\tau)}$	$\frac{1}{4}s$
$\nu_\mu+\nu_{e(\tau)} \longrightarrow \nu_\mu+\nu_{e(\tau)}$	$\frac{3}{4}s$
$\nu_\mu+\overline{\nu}_\mu \longrightarrow e^++e^-$	$\frac{4}{\sqrt{s}}\sqrt{s-4m_e^2}[(s-m_e^2)(g_L^{l2}+g_R^{l2})+6m_e^2 g_L^l g_R^l]$
$e^++e^- \longrightarrow \nu_\mu+\overline{\nu}_\mu$	$\frac{\sqrt{s}}{2\sqrt{s-4m_e^2}}[(s-m_e^2)(g_L^{l2}+g_R^{l2})+6m_e^2 g_L^l g_R^l]$
$\nu_\mu+e^- \longrightarrow \nu_\mu+e^-$	$(3\,s\,g_L^{l2}+s\,g_R^{l2}-3\,m_e^2\,g_L^l\,g_R^l)$
$\overline{\nu}_\mu+e^- \longrightarrow \overline{\nu}_\mu+e^-$	$(3\,s\,g_R^{l2}+s\,g_L^{l2}-3\,m_e^2\,g_L^l\,g_R^l)$

[Lesgourges et al, *Neutrino Cosmology,* Cambridge books, See also arXiv:1305.7513]

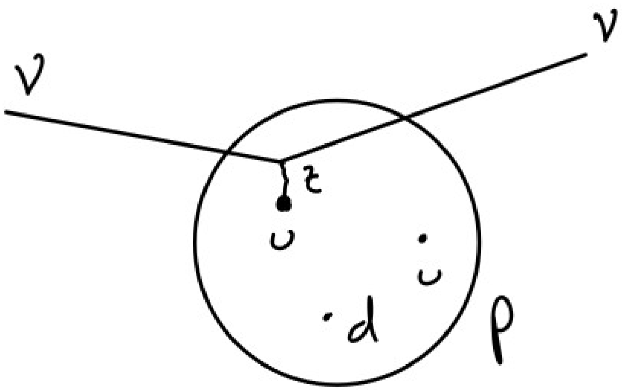
Neutrino-quark scattering

- Neutrino-quark scattering:



$$\mathcal{M} \propto G_F \qquad \Rightarrow \sigma \propto G_F^2 s.$$

- Same as for electrons, but difficult to apply: no free quarks.
- But: shoot neutrinos into nuclei with enough energy (~10 GeV) to "see" quarks: Deep Inelastic Scattering

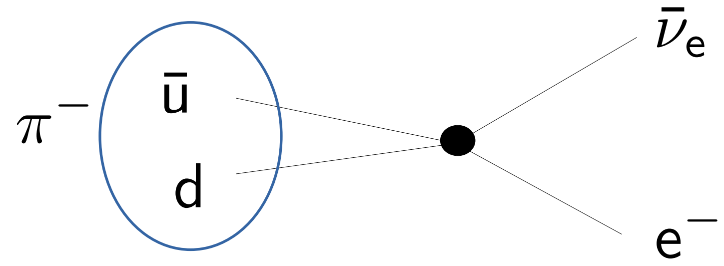


- Important to confirm quark picture.
- At lower energies we need to know how neutrinos couple to hadrons.

Neutrino-meson coupling

- Quarks sit inside mesons:

$$2\sqrt{2}G_F\ \underbrace{\bar{u}_L\, V\, \gamma^\mu d_L}_{V_{ud}\frac{f_\pi}{\sqrt{2}}\,\partial^\mu\pi^-}\ \bar{e}_L\gamma_\mu\nu_L$$



- Constant f that describes QCD binding.
- Find $\mathcal{M} \propto p^\mu_{\pi^-}\,\bar{u}(p_e)\gamma_\mu P_L v(p_\nu) = m_e\,\bar{u}(p_e)P_L v(p_\nu)$

$$\Rightarrow\ \Gamma(\pi^- \to e^-\bar{\nu}_e) \propto G_F^2 f_\pi^2 V_{ud}^2 m_e^2 m_\pi$$

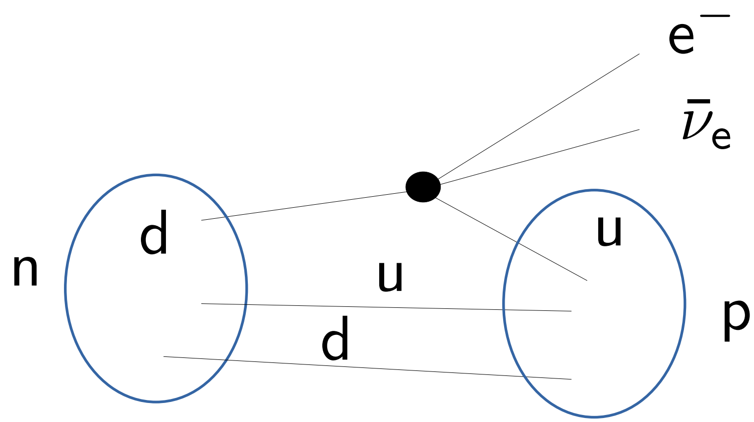
  so the pion decay into muons dominates by $m_\mu^2/m_e^2 \sim 4\times10^4$!
- Can also argue lepton mass flip from spin/helicity (homework).
- Replace d by s for kaons, heavier mesons can make τ: D_s or B → τν.

Meson decays are good sources of muon and tauon neutrinos.

Neutrino-baryon coupling

- Quarks sit inside baryons:

$$2\sqrt{2}G_F \underbrace{\bar{u_L}\, V \gamma^\mu d_L}_{\frac{1}{2}V_{ud}\,\bar{p}\gamma^\mu(g_V - g_A\gamma_5)n} \bar{e_L}\gamma_\mu \nu_L$$



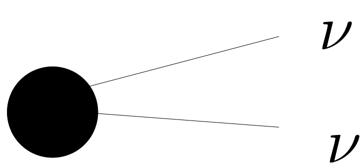
- Constants $g_V = 1$ & $g_A \simeq 1.3$ that describe QCD binding.
- This was Fermi's & Pauli's starting point to explain beta decays!
- Describes neutron decay and all beta decays of nuclei. Three-body decay gives continuous electron energy spectrum.
- $m_n - m_p \simeq 1.3\,\text{MeV}$ too small to make muons or tauons.
- Small phase space makes beta decays sensitive to neutrino mass.
- (At lower energies neutrinos couple to entire nucleon/nucleus.)

Beta decays are good sources of electron neutrinos.

Neutrino coupling to gravity

- Einstein's gravity couples to everything, including neutrinos.
- Neutrinos bend in gravitational fields, just like photons.
- Still the best pointers possible as they barely see matter, EM fields etc.
- Neutrino production via gravity? Hawking radiation!
  - Black holes have a temperature and emit thermal particles

$$T_{\mathrm{Hawking}} = \frac{\hbar c^3}{8\pi G_N M k_B} \simeq \frac{10^{19}\,\mathrm{kg}}{M}\,\mathrm{eV}$$



  - Need small black holes, not observed yet.
  - (Would also emit ν_R or any other BSM particle!)

[see Lunardini & Perez-Gonzalez, arXiv:1910.07864]

Extra homework: do these Hawking neutrinos oscillate?

Neutrino force

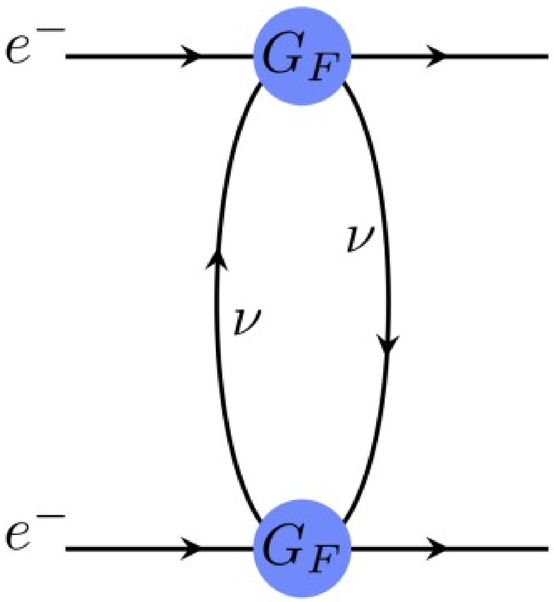
- SM neutrinos are massless.
- Massless particles give long-range interactions?
- Need two to form boson.
- Potential:

$$V_{ee}(r) = \left(2\sin^2\theta_W + \frac{1}{2}\right)^2 \frac{G_F^2}{4\pi^3}\frac{1}{r^5}$$

- Dominates over gravity's 1/r potential for

  r < 10^{-8} m.
- Neutrino masses change the r dependence.
- Not yet observed experimentally.

  [see Xu, arXiv:2305.08032]



Fifth, sixth, and seventh force?

Take home messages

- Standard Model is a mathematically consistent quantum field theory with ~18 free parameters, all measured now.
- Neutrino interactions well understood, only difficulties come from QCD: neutrino cross sections on hadrons...
- Standard Model very successful, but wrong:
  - Neutrino masses
  - Dark matter
  - (Finetuning: strong CP, baryon asymmetry, inflation)
  - Quantum gravity?

Need more experimental data to build
Standard Model II

Backup

Aside 1: Chiral fermions

- Notation confusing at times…
- First off: what is spin?
  - Behavior under rotations, i.e. SO(3) representation?
  - Only works for integer spins, not spin ½!
  - Can show that the Poincare symmetry (Lorentz + boost) is isomorphic to Lie algebra SU(2)xSU(2).
  - Spin is SU(2)xSU(2) representation theory:
    - Scalar = (1,1)
    - Vector = (2,2)
    - Spin ½ = (1,2) or (2,1).

Aside 1: Chiral fermions

- Simplest fermion in QFT: Weyl fermion
  - Two-component complex object with Lagrangian

$$\mathcal{L} = i\psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi - \frac{1}{2} m\psi\psi - \frac{1}{2} m^* \psi^\dagger \psi^\dagger$$

  - Mass term only allowed if fermion has no charges!
  - (Can also write this as a four-component Majorana spinor.)
- Usually used: Dirac fermion
  - Take two Weyl fermions with opposite U(1) charge:

$$\mathcal{L} = i\chi^\dagger \bar{\sigma}^\mu \partial_\mu \chi + i\xi^\dagger \bar{\sigma}^\mu \partial_\mu \xi - m\chi\xi - m\xi^\dagger \chi^\dagger$$

  - Put them in four-component Dirac spinor

$$\Psi \equiv \begin{pmatrix} \chi \\ \xi^\dagger \end{pmatrix} \quad \Rightarrow \quad \begin{aligned} &\mathcal{L} = i\overline{\Psi}\gamma^\mu \partial_\mu \Psi - m\overline{\Psi}\Psi \\ &\Rightarrow \; (-i\not{\partial} + m)\Psi = 0 \end{aligned}$$

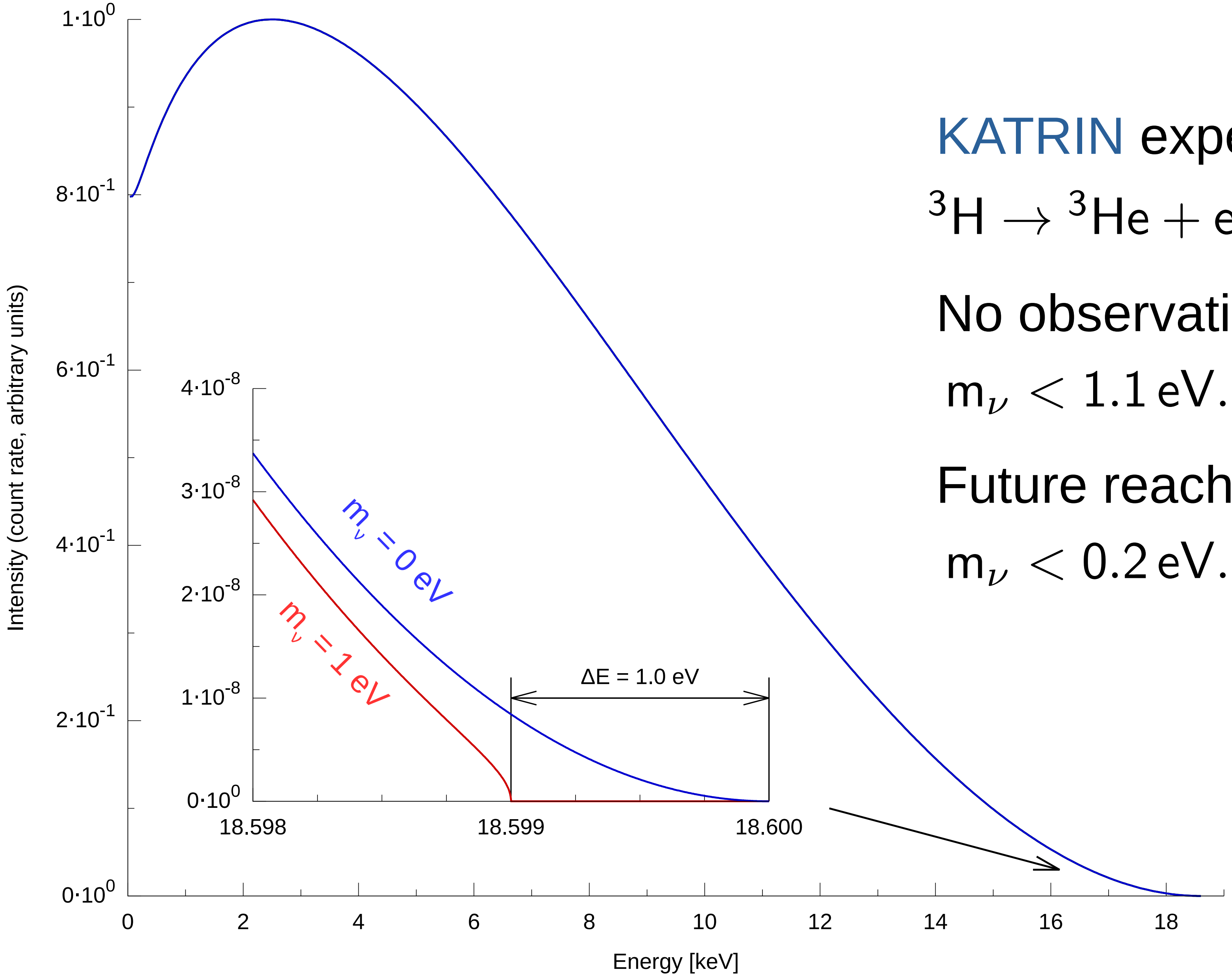
Three-neutrino parametrization

- Three neutrinos: unitary mixing matrix U =

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & e^{-i\delta} s_{13} \\ 0 & 1 & 0 \\ -e^{i\delta} s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

- $s_{ij} \equiv \sin\theta_{ij}$, $c_{ij} \equiv \cos\theta_{ij}$.
- CP violation via δ.
- Two mass-squared differences: $\Delta m^2_{ij} = m^2_i - m^2_j$.

Beta decay endpoint



KATRIN experiment:

$$^3\mathrm{H} \to {}^3\mathrm{He} + e^- + \bar{\nu}_e$$

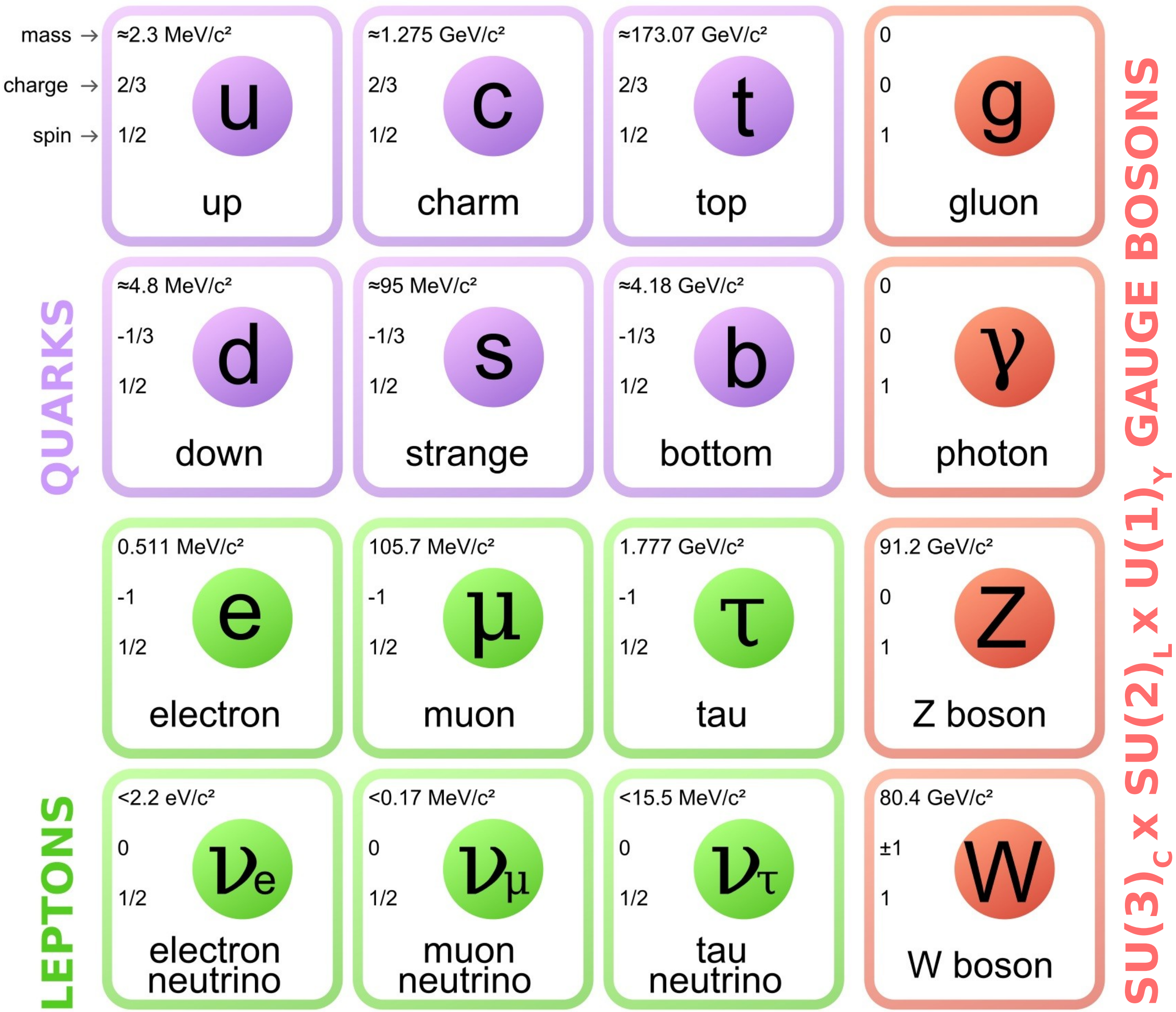
No observation:

$m_\nu < 1.1\,\mathrm{eV}$. [PRL '19]

Future reach down to

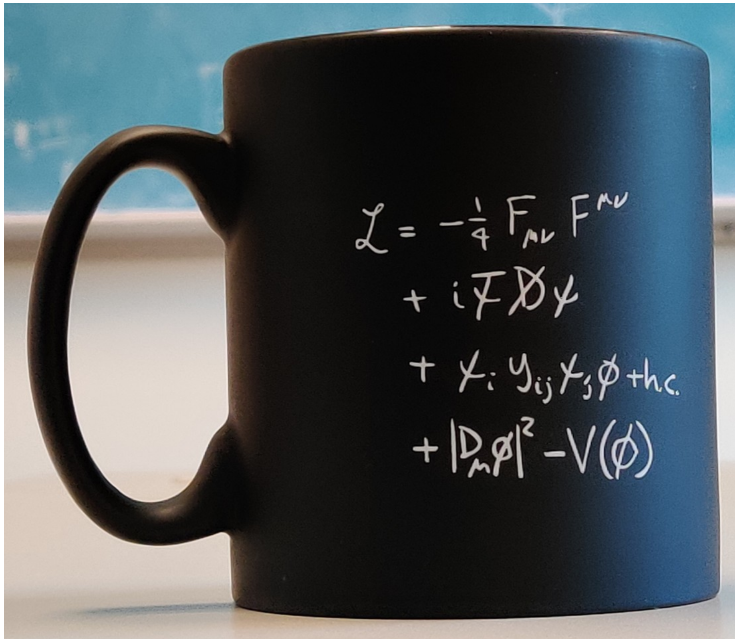
$m_\nu < 0.2\,\mathrm{eV}$.

Standard Model of Particle Physics



Englert & Higgs '13

[wikipedia]

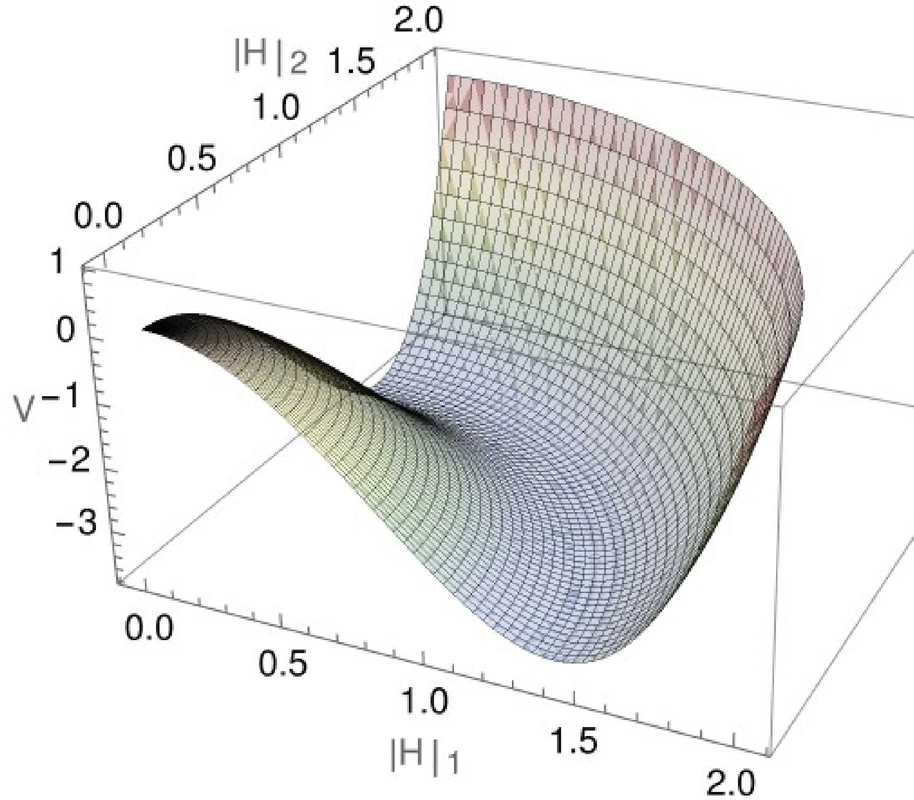


Masses in the Standard Model

- $\mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y}$ gauge symmetry forbids mass terms.
- Masses via spontaneous symmetry breaking $\rightarrow \mathrm{U}(1)_{\mathrm{EM}}$.
- Higgs-fermion couplings:

$$\mathcal{L}_{\mathrm{SM}} \supset y_f \, \bar{f}_L \, H \, f_R + \mathrm{h.c.}$$

$$\rightarrow \underbrace{y_f \, \langle H \rangle}_{m_f \,=\, y_f \,\times\, 174\,\mathrm{GeV}} \bar{f}_L \, f_R + \mathrm{h.c.}$$



- For neutrinos: no ν_R!

The 3 neutrinos $\nu_{e,\mu,\tau}$ in the SM are massless.