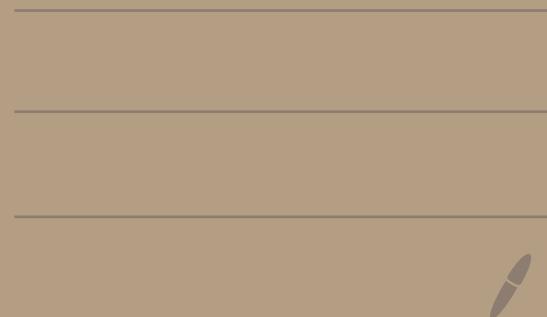


Probe of the origin of neutrino mass

Lecture II

16/8/2023

Fermilab



L R S M \rightarrow seesaw mechanism

SM = theory of origin of mass

\uparrow (e, \bar{e})

δ maximal $\leftrightarrow \phi$



$$\boxed{m_\nu = 0}$$



$$\mathcal{K} = S \otimes \mathbb{R}$$

$$\Downarrow W_L^\pm, z_L \leftrightarrow W_R^\pm, z_R$$

- $G_{LR} = SU(2)_L \times SU(2)_R \times U(1)$

self-contained theory

(Feynman) :

1. guess = principle

equivivalence \Leftrightarrow Einstein

gauge \Leftrightarrow Glashow - - -

2. minimal framework

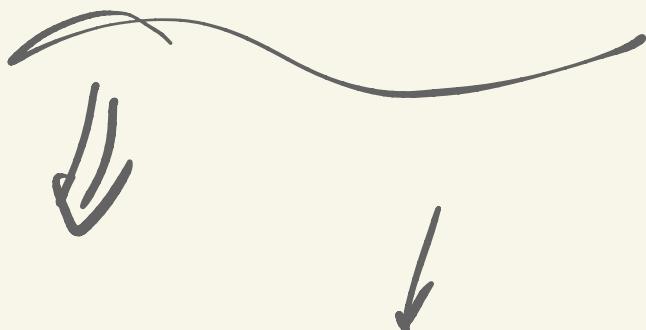
3. leave it

4. "we" make predictions

5. predictions = good theory

6. exp. decides

SM = predictive theory
of masses

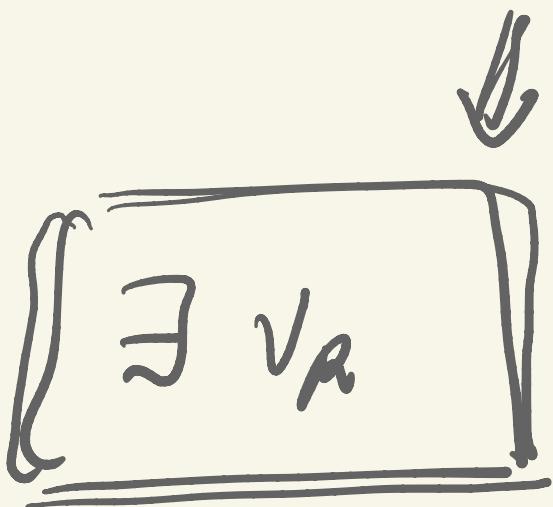


$L_{RSM} = -1/r$ of v mass

- matter

$$\varrho_L = \begin{pmatrix} u \\ d \end{pmatrix}_L \xleftrightarrow{P} \begin{pmatrix} u \\ d \end{pmatrix}_R = \varrho_R$$

$$l_L = \begin{pmatrix} \circlearrowleft \\ e \end{pmatrix}_L \xleftrightarrow{P} \begin{pmatrix} \circlearrowleft \\ e \end{pmatrix}_R = l_R$$



- Higgs sector (later)

$G_{LR} \longrightarrow G_{SM}$

$$\langle \Delta_R \rangle = v_R (M_R)$$

$$M_R = M_W$$

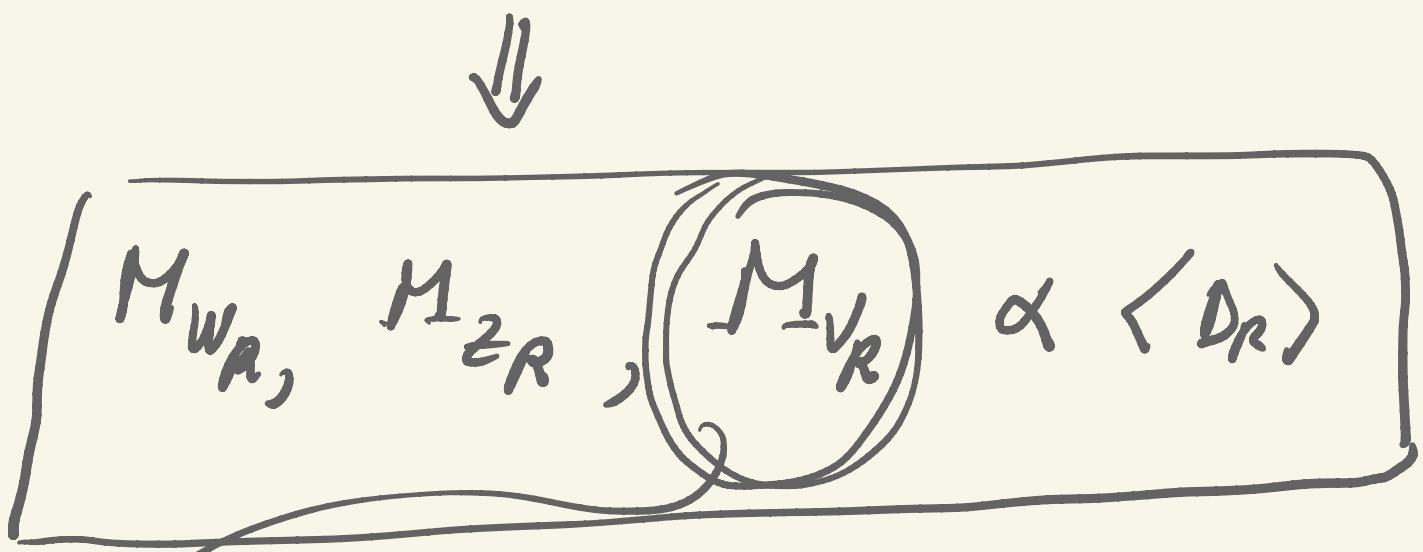
$$M_R \gg M_W$$

LHC: $M_R \gtrsim 4 \text{ TeV}$

Higgs sector: \therefore

$$M_{\text{new}}'' \propto M_R \propto \langle \Delta_R \rangle = v_R$$

$$M_{\text{Hd}}''(G_M) \propto M_L \propto M_W \propto \langle \phi \rangle$$



$M_{\text{new Higgs}} \propto \langle \Delta_R \rangle$

$$C \equiv i \partial^2 \partial^0 = \begin{pmatrix} i \sigma_2 & 0 \\ 0 & -i \sigma_2 \end{pmatrix}$$

$$v_R^T C M_{V_R} v_R$$

$$v_R = \begin{pmatrix} 0 \\ u_R \end{pmatrix}$$

$$\Rightarrow v_R^T C v_R = u_R^T i \sigma_2 u_R$$

↗ (inv)

$$u_R \rightarrow e^{i \vec{\sigma}_k \cdot \vec{P}} u_R$$

$$\vec{P} = \vec{\Theta} + i \vec{\chi}$$

ROT BOOST



$$\mathcal{L}(v_R) = i \bar{v}_R^\dagger \partial^\mu \partial_\mu v_R -$$

$$- \frac{1}{2} m_H \underbrace{\left(\bar{v}_R^\dagger C v_R + h.c. \right)}_{\Delta L=2}$$



$$N_L \equiv C \bar{v}_R^\dagger = i \gamma^2 \gamma^0 \gamma^0 v_R^*$$

$$= \begin{pmatrix} 0 & i\alpha_2 \\ -i\alpha_2 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ u_R^* \end{pmatrix}$$

$$N_L = \begin{pmatrix} i\sigma_2 u_R^* \\ 0 \end{pmatrix}$$



$$\begin{aligned} N &= \nu_R + (c \bar{\nu}_R^\top = N) \\ &= N_L + \underline{c \bar{N}_L^\top} \end{aligned}$$

$$N = \begin{pmatrix} i\sigma_2 u_R^* & \text{anti-lepton} \\ u_R & \text{lepton} \end{pmatrix}$$

$$\nu_R \rightarrow c \bar{\nu}_R^\top \equiv c (\nu_R^+ r^0)^\top$$

$$= \underline{\underline{c \gamma_0 \nu_R^*}}$$



$$N_H = N_L + C \bar{N}_L^T$$

RH neutrino

 heavy (Majorana) neutral lepton

 heavy (Majorana) neutral lepton

$$\bar{N}N = (\bar{N}_L + \bar{C}\bar{N}_L^T)(N_L + C\bar{N}_L^T)$$

$$\bar{N}_L N_L = N_L^+ \gamma^0 N_L = N^+ L \gamma^0 L N$$

$$= N^+ \underbrace{\gamma^0}_{R \cdot L} N$$

$$\begin{array}{ccc}
 \Downarrow & & \overline{\partial} \\
 \bar{N}N = \bar{N}_L C \bar{N}_L^T + \overline{C \bar{N}_L^T} & & N_L \\
 & & \parallel
 \end{array}$$

$$N_L^T C N_L$$



$$(i) \bar{N} N = N_L^T C N_L + h.c.$$

$$(ii) \bar{N} \gamma^\mu \partial_\mu N = (\bar{N}_L + \bar{C} \bar{N}_L^T) \gamma^\mu \partial_\mu$$

$(N_L + \dots)$

$$= \bar{N}_L \gamma^\mu \partial_\mu N_L + \underbrace{\bar{N}_L \gamma^\mu \partial_\mu N_L}_{\text{show}}$$

$$= 2 \bar{N}_L \gamma^\mu \partial_\mu N_L$$



$$\mathcal{L}(v_R) = i \bar{v}_R \gamma^\mu \partial_\mu v_R -$$

$$- \frac{m_u}{2} (v_R^T C v_R + h.c)$$

$$= i \bar{N}_L \gamma^\mu \partial_\mu N_L - \text{---} (N_L^T C N_L + h.c)$$

$$= \left(\frac{1}{2} \right) \left[i \bar{N} \gamma^\mu \partial_\mu N - m_N \bar{N} N \right]$$

↓

$p^2 = m_N^2$

$$(ii) \quad \mathcal{L}_{SM} = \dots \bar{\ell}_L \phi \ell_R \gamma_e$$

$$+ \bar{\nu}_R \tilde{\phi}^+ \gamma_5 \ell_L + h.c.$$

$$(\tilde{\phi} \equiv \text{15}_2 \phi^*)$$

$\bar{\nu}_R \tilde{\phi}^+ \gamma_5 \ell_L$

$$\phi_{un} = \left(\begin{matrix} 0 \\ v_{SM} + h \end{matrix} \right)$$

$$\bar{V}_R \gamma_0 \tilde{\phi}_m^+ l_i = \bar{V}_R M_D v_L$$

$$\boxed{M_D = \gamma_0 v_{sy}}$$

$$N_L = C \bar{V}_R^T$$

$$\Rightarrow \bar{V}_R M_D v_L = \boxed{N_L^T C M_D v_L}$$



$$Y(v_L, N_L) = N_L^T C M_D v_L +$$

$$+ \frac{1}{2} N_L^T C M_N N_L + h.c.$$

$$= \frac{1}{2} N_L^T C M_D V_L + \frac{1}{2} N_L^T C M_D V_L$$

$$+ \frac{1}{2} N_L^T C M_N N_L + h.c.$$

$$= \frac{1}{2} \left[N_L^T C M_D V_L + (-) V_L^T C^T M_D^T N_L \right]$$

$$+ \frac{1}{2} N_L^T C M_N N_L$$

$$\boxed{C^T = -C}$$

$$= \frac{1}{2} \left[N_L^T C M_D V_L + V_L^T C M_D^T N_L \right]$$

$$+ N_L^T C M_N N_L]$$



$$(M_N = M_N^T)$$



$$N_L \begin{pmatrix} 0 & M_D^T \\ M_D & M_N \end{pmatrix} N_L$$

ν_L

1 gen.

↓

$M_N \gg M_D$

$$M_{\nu_N} = \begin{pmatrix} 0 & m_D \\ m_D & m_N \end{pmatrix}$$

↓

$$m_H \approx m_N$$

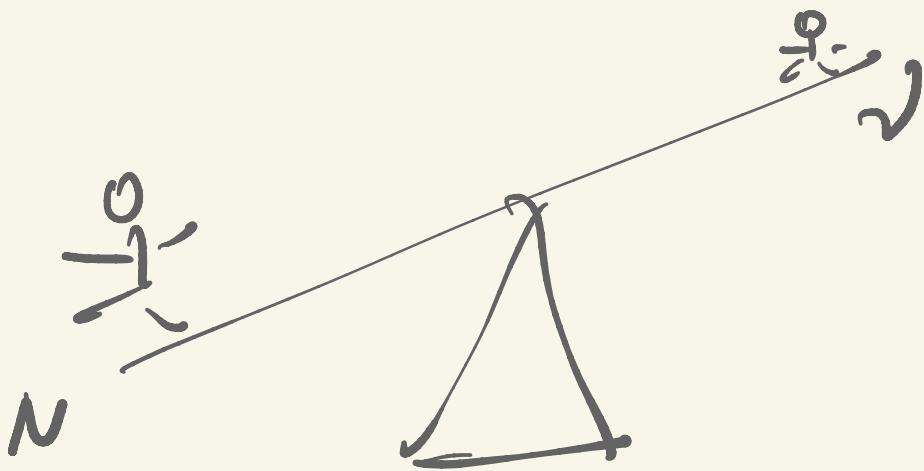
Jee saw
assumption

$$m_L \approx - \frac{m_D^2}{m_N}$$

$(m_L \approx m_\nu)$

$$\left. \det M_{VN} = -\mu_0^2 = \mu_L \mu_H \right\} \pi$$

Jeersaw



$$\Rightarrow \boxed{\mu_0 = i \sqrt{\mu_N \mu_V}}$$

↓ generators (u)



$$\underline{M}_{\nu N} = \begin{pmatrix} 0 & M_D^\top \\ M_D & M_N \end{pmatrix}$$

Majorana mass matrix =

= symmetric

$$(\underline{M} = \underline{M}^\top)$$

$$\underline{M}_{\nu N}^\top = \underline{M}_{\nu N}$$

- $\mathcal{H} = \mathcal{H}^+ \Rightarrow U^+ H V = h \text{ (diag)}$

- $S = S^\top \Rightarrow U^\top S V = 1 \text{ (-1-)}$

$$U^T M_{VN} U = D_{VN} \cong \begin{pmatrix} M_V & 0 \\ 0 & M_N \end{pmatrix}$$

$$U = \begin{pmatrix} 1 & \Theta^+ \\ -\Theta & 1 \end{pmatrix}, (\Theta \ll 1)$$

$$UU^+ = \mathbb{1} + \underbrace{O(\Theta\Theta^+)}_y$$

$$\begin{pmatrix} 1 & -\Theta^T \\ \Theta^* & 1 \end{pmatrix} \begin{pmatrix} 0 & M_D^T \\ M_D & M_N \end{pmatrix} U$$

$$\cong \begin{pmatrix} -\Theta^T M_D & M_D^T - \Theta^T M_N \\ M_D & M_N \end{pmatrix} U$$

$$\underset{\Theta \neq 0}{\approx} \begin{pmatrix} -\Theta^T M_D & M_D^T - \Theta^T M_N \\ M_D - M_N \Theta & M_N \end{pmatrix} = 0$$

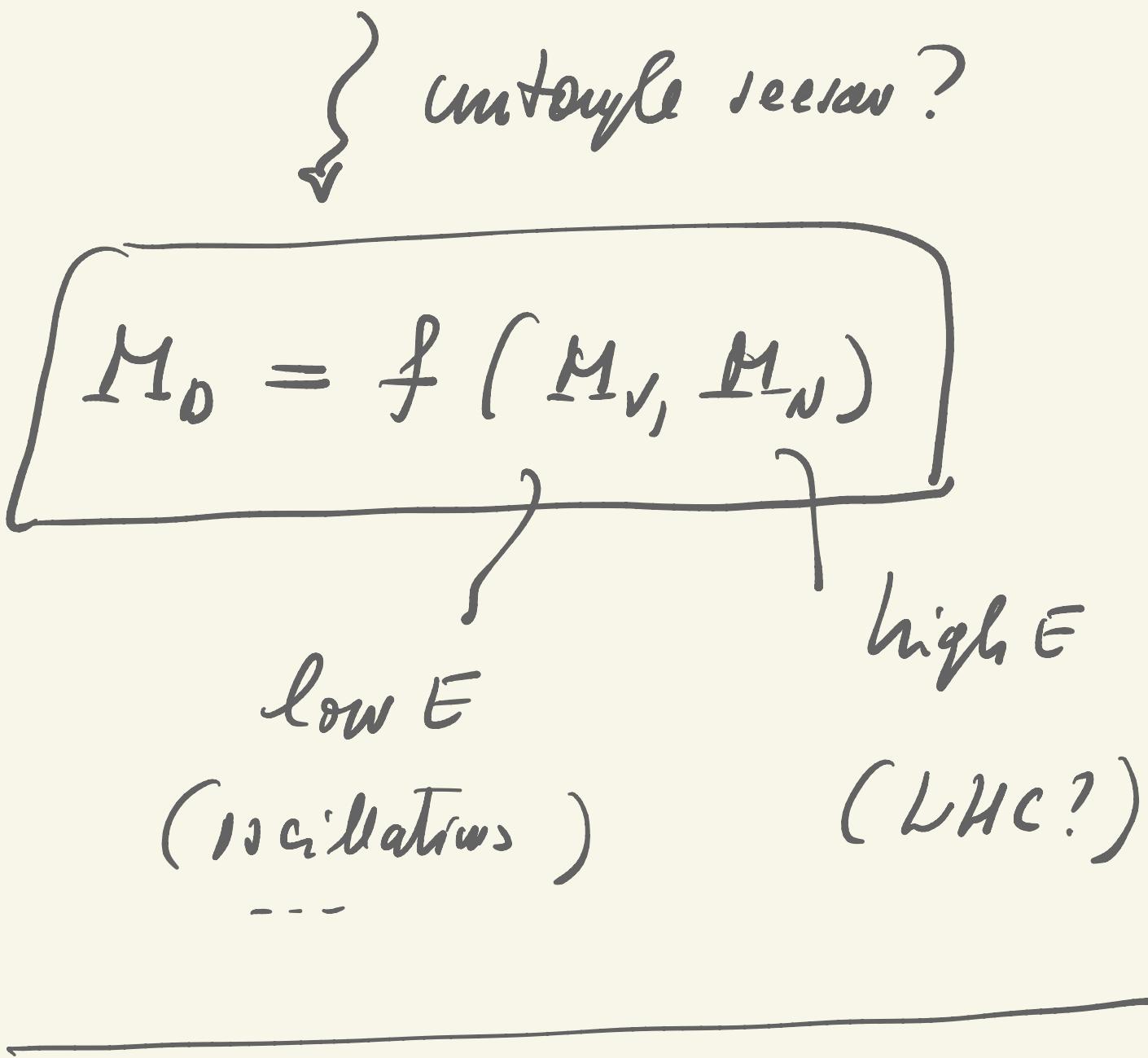
$$\boxed{\Theta = \frac{1}{M_N} M_D}$$

$$\boxed{M_V = -\Theta^T M_D = -M_D^T \frac{1}{M_N} M_D}$$

(i) $M_V^T = M_V$

(ii) $M_N \rightarrow \infty \Rightarrow M_V \rightarrow 0$

(iii) $M_D \rightarrow 0 \Rightarrow M_V \rightarrow 0$



$$\begin{array}{ccc}
 SU(2)_R \times U(1) & \xrightarrow{\quad} & |\overline{B-L}| \\
 \downarrow & \langle \Delta_R \rangle & \\
 V_{Y^{(1)}}
 \end{array}$$

$\Delta_R = SU(2)_R$ triplet

$$(B-L) \Delta_R = 2$$



$$\mathcal{L}_Y(\Delta_R) = \ell_R^T C i\sigma_2 \Delta_R \ell_R + h.c.$$

$$\langle \Delta_R \rangle \neq 0$$

$$\Rightarrow \boxed{M_{\nu_R} = Y_D \langle \Delta_R \rangle}$$

~~$\ell_R^T C \ell_R$~~ charge!

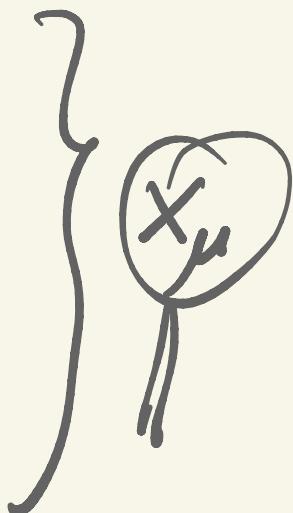
Boosts

$$u_L \rightarrow e^{i\vec{\sigma}/2} (\vec{\theta} + i\vec{x}) u_L$$

vector

$$z' = \frac{z - vt}{\sqrt{1 - v^2}}$$

$$t' = \frac{t - vx}{\sqrt{1 - v^2}}$$



Locals
vector

$$A_\mu \underbrace{\bar{\psi} \gamma^\mu \psi}_{\text{vector}}$$

$$\bar{\psi}_L \gamma^\mu \psi_L = \text{vector}$$

$$\psi_L = \begin{pmatrix} u_L \\ 0 \end{pmatrix} \Rightarrow$$

$$\bar{\psi}_L \gamma^\mu \psi_L = u_L^+ \sigma^\mu u_L .$$

$$\sigma^\mu = (1; \vec{\sigma})$$

$$u_L^+ \sigma^\mu u_L \leftrightarrow A^\mu (x^\mu)$$

$$u_L^+ \sigma^0 u_L \leftrightarrow t$$

$$u_L^+ \sigma^3 u_L \leftrightarrow z$$

$$t \leftrightarrow u_L^+ u_L \quad z \leftrightarrow u_L^+ \sigma_3 u_L \quad \boxed{\tan \chi_3 = \alpha}$$

