

The background of the slide is a composite image. The upper portion shows a vibrant view of the Milky Way galaxy, with its characteristic spiral arms and dense star fields, appearing in shades of purple, blue, and white against a dark cosmic background. The lower portion of the image shows the skeletal metal framework of a large radio telescope dish, likely the Arecibo Observatory, silhouetted against the starry sky.

# Neutrinos and Cosmology I

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International Neutrino  
Summer School 2023  
8-17-2023

Image Credit: ACT / Princeton

# Outline

## Lecture I

- Cosmology Basics
- Thermal History
- Cosmic Neutrino Background
- Light Relics Beyond the Standard Model
- Big Bang Nucleosynthesis
- Cosmic Microwave Background Basics

## Lecture II

- Measuring Light Relics
- Massive Cosmic Neutrinos
- CMB Lensing and Neutrino Mass
- Other Cosmic Probes of Neutrino Mass
- Cosmology/Lab Complementarity

# Cosmology Basics

# The Universe is Expanding

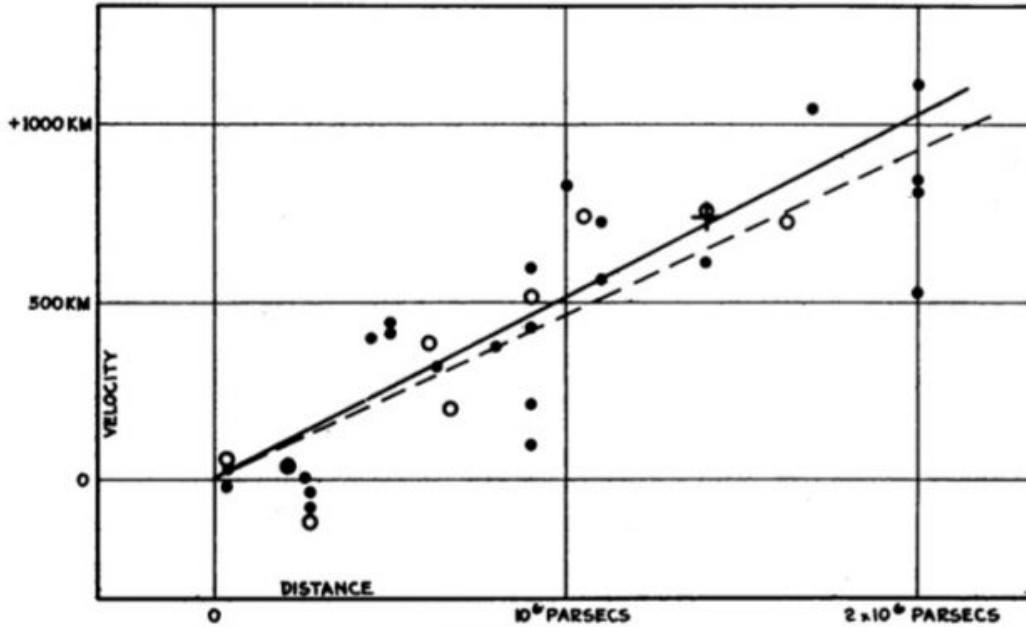


FIGURE 1  
Velocity-Distance Relation among Extra-Galactic Nebulae.



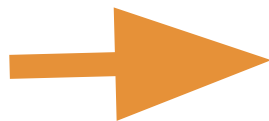
# Physics of Expansion

Einstein Field Equations

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

“Matter tells spacetime how to curve; spacetime tells matter how to move” - *Wheeler*

Homogeneous  
, Isotropic  
Universe



Friedmann Equation

$$H^2 \equiv \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho$$

Expansion Rate

Scale Factor

Energy Density

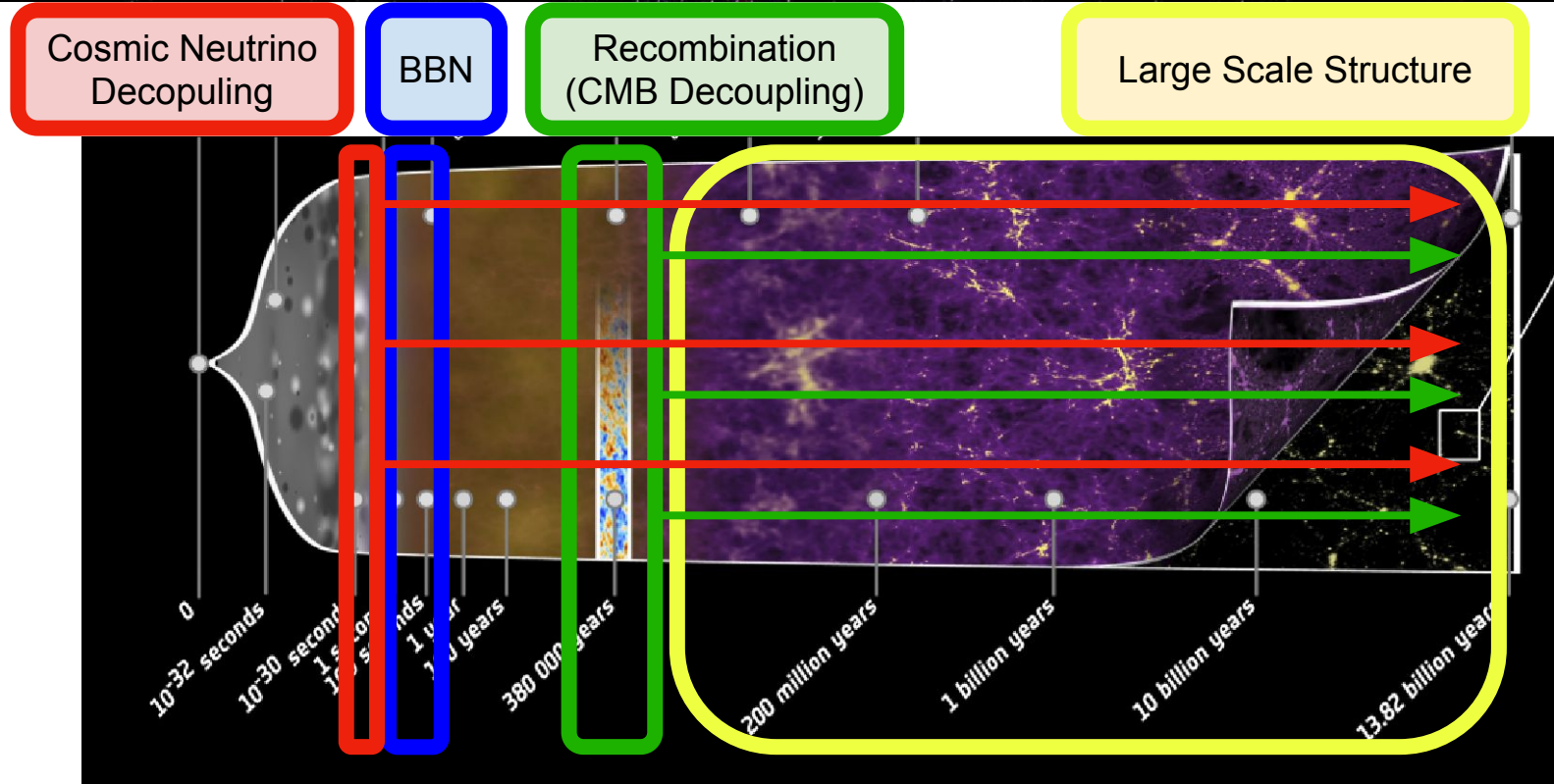
$$\rho_m \propto a^{-3}$$

Non-Relativistic Matter

$$\rho_r \propto a^{-4}$$

Relativistic Radiation

# History of the Universe



# Thermal History

# Thermal Plasma

- The early universe was filled with a relativistic neutral plasma
- For each species with  $m \ll T$ , the distribution function is

$$f(p) = \frac{1}{\exp(p/T) \mp 1}$$

- Species with  $m \gg T$  are Boltzmann suppressed:  $n \sim \exp(-m/T)$
- The energy density of each relativistic species is

$$\rho(T) = \frac{g}{(2\pi)^3} \int dp \frac{4\pi p^3}{\exp(p/T) \mp 1} = \begin{cases} g \frac{\pi^2}{30} T^4 \\ \frac{7}{8} g \frac{\pi^2}{30} T^4 \end{cases}$$

Bosons

Fermions

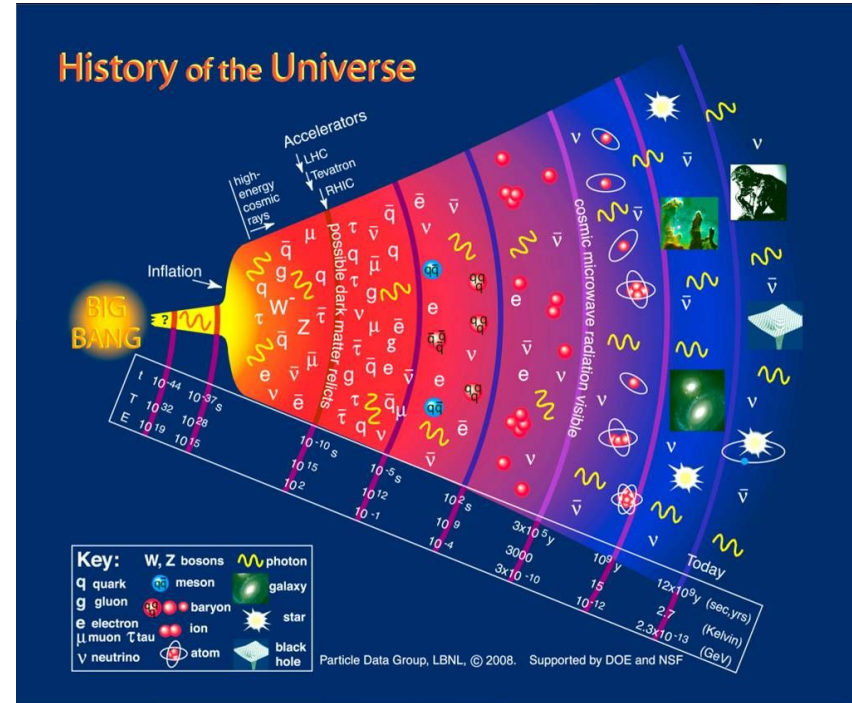


Image Credit: PDG



# Evolution of Plasma Particle Content

- We can define the effective number of relativistic degrees of freedom in equilibrium

$$g_{\star}^{\text{th}}(T) \equiv \sum_{i \in \text{bosons}} g_i + \frac{7}{8} \sum_{j \in \text{fermions}} g_j$$

- Decoupled species contribute with a different temperature

$$g_{\star}^{\text{dec}}(T) \equiv \sum_{i \in \text{bosons}} g_i \left( \frac{T_i}{T} \right)^4 + \frac{7}{8} \sum_{j \in \text{fermions}} g_j \left( \frac{T_j}{T} \right)^4$$

- The total energy density then takes a simple form

$$g_{\star}(T) = g_{\star}^{\text{th}}(T) + g_{\star}^{\text{dec}}(T) \quad \rho(T) = g_{\star}(T) \frac{\pi^2}{30} T^4$$

- We know the particle content of the Standard Model, so we know how  $g_{\star}(T)$  evolves

type		mass	spin	$g$
quarks	$t, \bar{t}$	173 GeV	$\frac{1}{2}$	$2 \cdot 2 \cdot 3 = 12$
	$b, \bar{b}$	4 GeV		
	$c, \bar{c}$	1 GeV		
	$s, \bar{s}$	100 MeV		
	$d, \bar{d}$	5 MeV		
	$u, \bar{u}$	2 MeV		
gluons	$g_i$	0	1	$8 \cdot 2 = 16$
leptons	$\tau^{\pm}$	1777 MeV	$\frac{1}{2}$	$2 \cdot 2 = 4$
	$\mu^{\pm}$	106 MeV		
	$e^{\pm}$	511 keV		
	$\nu_{\tau}, \bar{\nu}_{\tau}$	$< 0.6$ eV	$\frac{1}{2}$	$2 \cdot 1 = 2$
	$\nu_{\mu}, \bar{\nu}_{\mu}$	$< 0.6$ eV		
	$\nu_e, \bar{\nu}_e$	$< 0.6$ eV		
gauge bosons	$W^{+}$	80 GeV	1	3
	$W^{-}$	80 GeV		
	$Z^0$	91 GeV		
	$\gamma$	0		2
Higgs boson	$H^0$	125 GeV	0	1

# Entropy Conservation

- Entropy is conserved, even as the particle content of the plasma changes

$$\frac{d}{dt}(a^3 s(T)) = 0 \qquad s(T) = \frac{\rho(T) + P(T)}{T} = \frac{4\rho(T)}{3T}$$

- We can define an effective number of relativistic species for entropy

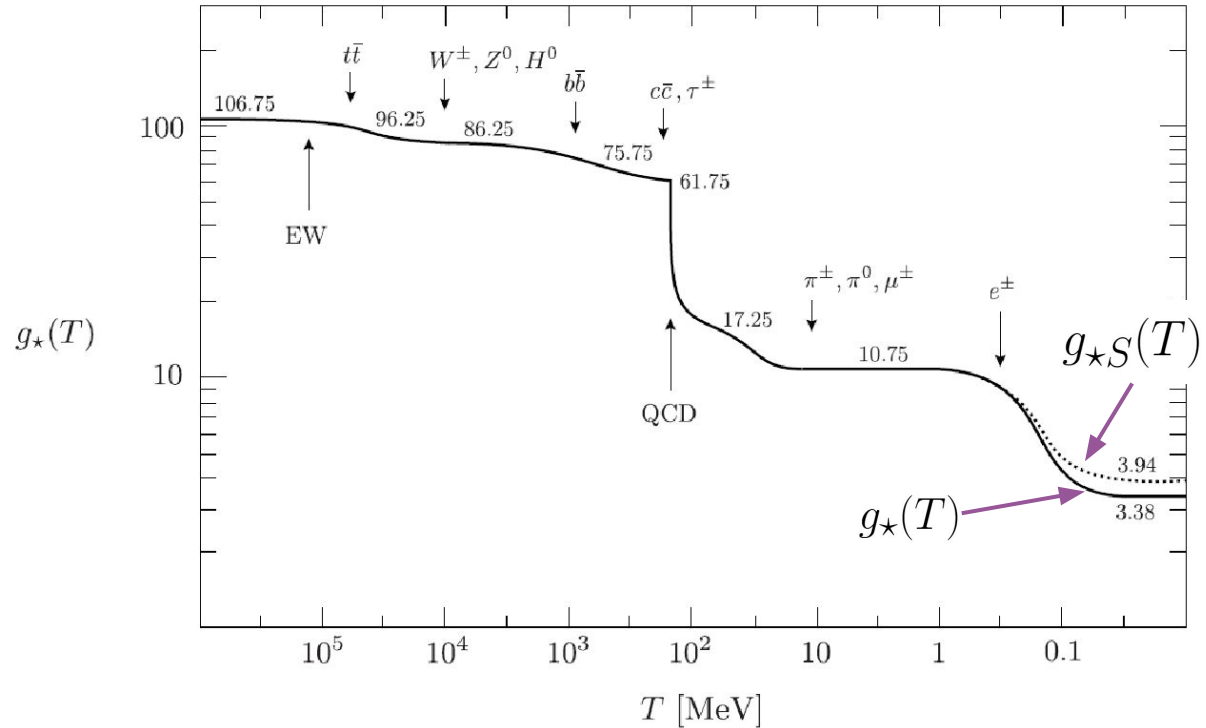
$$s(T) = g_{\star S}(T) \frac{2\pi^2}{45} T^3 \qquad g_{\star S}(T) = g_{\star S}^{\text{th}}(T) + g_{\star S}^{\text{dec}}(T)$$
$$g_{\star S}^{\text{th}}(T) = g_{\star}^{\text{th}}(T) \qquad g_{\star S}^{\text{dec}}(T) \equiv \sum_{i \in \text{bosons}} g_i \left(\frac{T_i}{T}\right)^3 + \frac{7}{8} \sum_{j \in \text{fermions}} g_j \left(\frac{T_j}{T}\right)^3$$

- Conservation of entropy allows us to calculate the temperature evolution

$$T \propto g_{\star S}(T)^{-1/3} a^{-1}$$

# Standard Model Particle Content

- Particle species disappear from equilibrium when the plasma temperature drops below their mass
- The sharp dip in  $g_{\star}(T)$  around 150 MeV results from the QCD phase transition where the relevant degrees of freedom change from quarks and gluons to baryons and hadrons

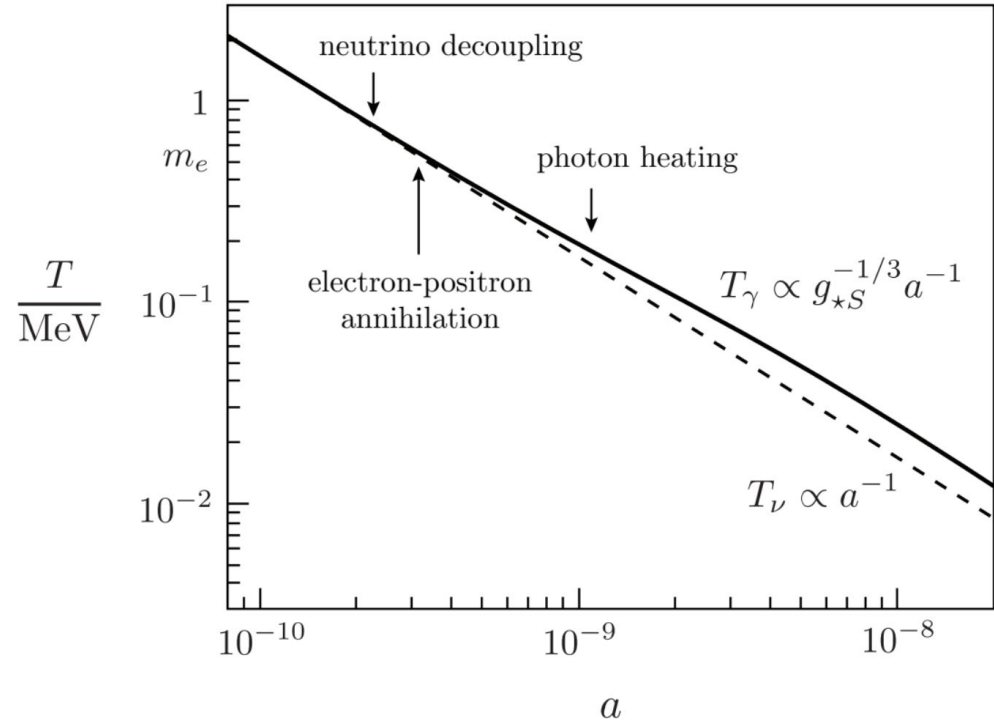


# Cosmic Neutrino Background



# Cosmic Neutrinos as Light Thermal Relics

- Neutrinos were in thermal equilibrium with the plasma until the weak interaction rate became inefficient compared to the Hubble expansion rate around  $T \sim 1$  MeV
- After decoupling, the cosmic neutrino background persisted, undergoing free expansion
- Annihilation of electrons and positrons around  $T \sim 0.5$  MeV heated photons relative to neutrinos



# Cosmic Neutrino Background Temperature

- We can calculate the temperature of cosmic neutrinos relative to photons
- After neutrino decoupling, prior to electron-positron annihilation, we have:

$$g_{\star S}^{\text{th}}(T_+) = 2 + \frac{7}{8} (2 + 2) = \frac{11}{2}$$

$\gamma \qquad e^+ \quad e^-$

- After electron-positron annihilation, we have only photons:

$$g_{\star S}^{\text{th}}(T_-) = 2$$

- Entropy conservation gives the temperature ratio after annihilation:

$$T_\nu = \left( \frac{g_{\star S}^{\text{th}}(T_-)}{g_{\star S}^{\text{th}}(T_+)} \right)^{1/3} T_\gamma = \left( \frac{4}{11} \right)^{1/3}$$

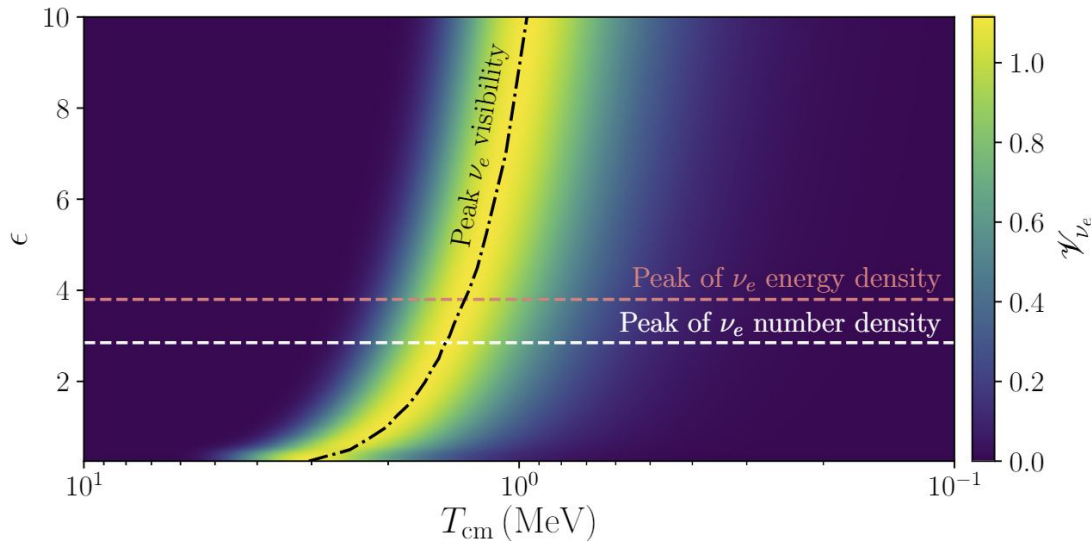
# Effective Number of Neutrino Species

- We can measure the gravitational influence of cosmic neutrinos
- One way this shows up is through the total energy density of neutrinos

$$\rho_r = \rho_\gamma \left( 1 + \frac{7}{8} \left( \frac{4}{11} \right)^{4/3} N_{\text{eff}} \right)$$

- For instantaneous decoupling,  $N_{\text{eff}}$  counts the number of neutrino species
- A complete treatment of neutrino transport gives a slightly larger number in the Standard Model

$$N_{\text{eff}}^{\text{SM}} = 3.044(1)$$

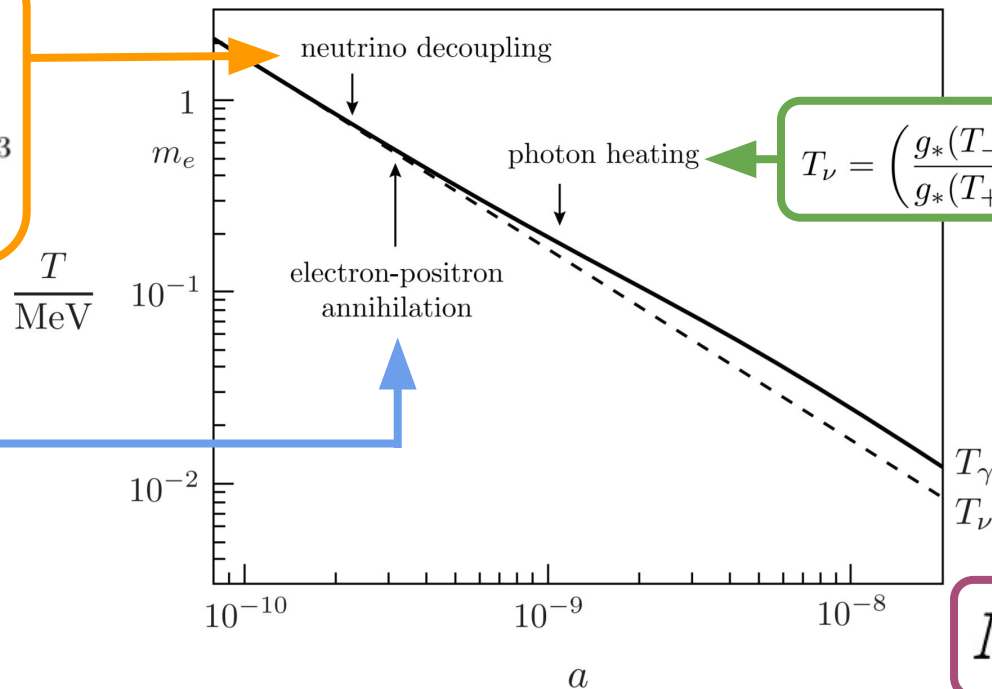
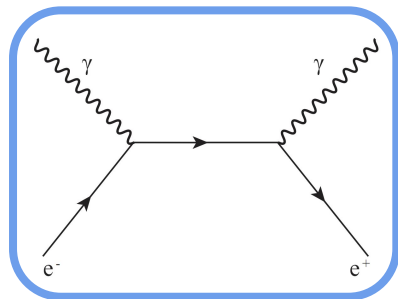


Escudero Abenza (2020); Akita, Yamaguchi (2020);  
Froustey, Pitrou, Volpe (2020); Bennett, et al (2021);  
Bond, Fuller, Grohs, JM, Wilson (In Prep.)

# Summary of Cosmic Neutrino Decoupling

$$\sigma \sim \left| \begin{array}{c} \diagup \diagdown \\ \diagdown \diagup \end{array} \right|^2 \sim G_F^2 T^2$$

$$\frac{\Gamma}{H} \sim \frac{\alpha^2 M_{\text{Pl}} T^3}{M_W^4} \sim \left( \frac{T}{1 \text{ MeV}} \right)^3$$



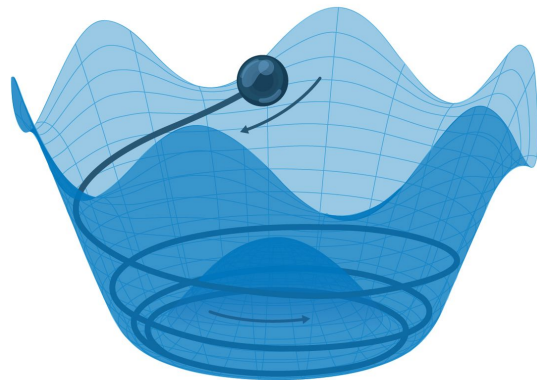
$$T_\nu = \left( \frac{g_*(T_-)}{g_*(T_+)} \right)^{1/3} T_\gamma = \left( \frac{4}{11} \right)^{1/3} T_\gamma$$

$$N_{\text{eff}}^{\text{SM}} = 3.044$$

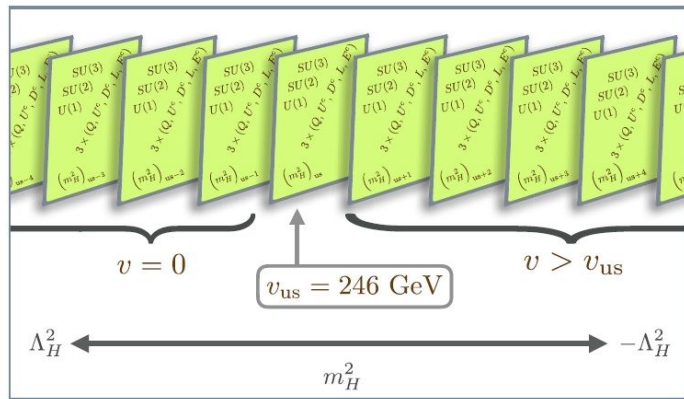


# Light Relics Beyond the Standard Model

# New Light Species are Ubiquitous in Standard Model Extensions



Axions and Axion-Like Particles



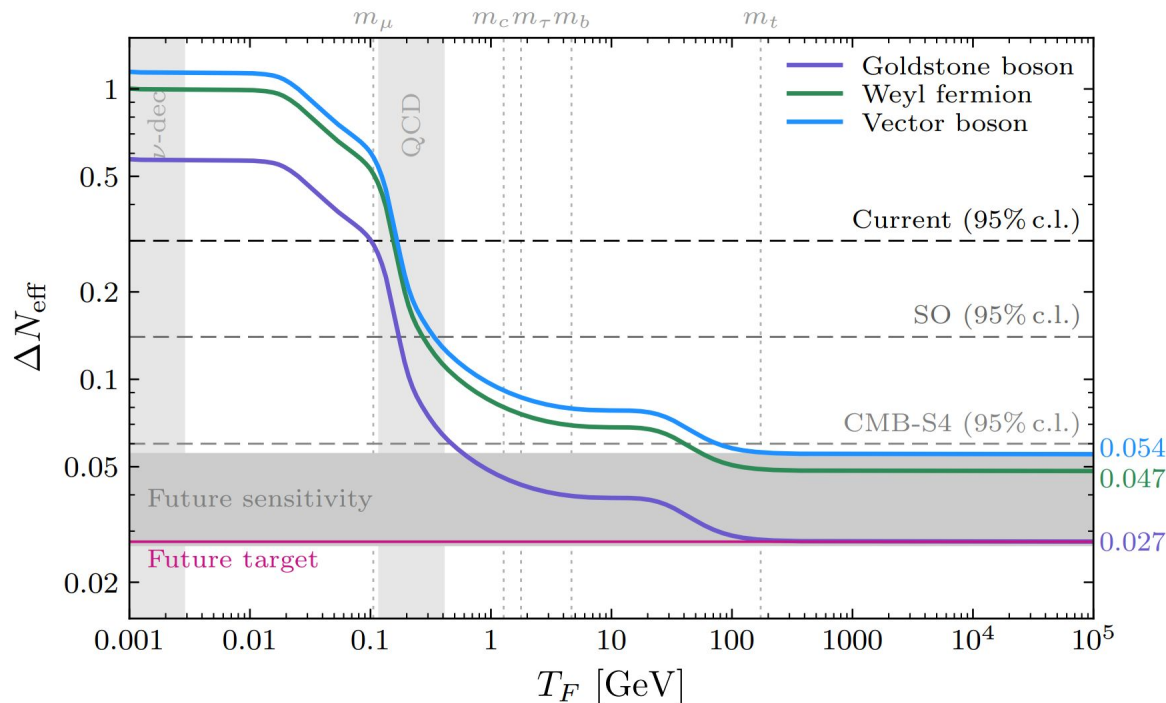
Complex Dark Sectors



Sterile Neutrinos

... and many more

# Light Thermal Relics Set Useful Targets



The relic density of any new light species that was ever in thermal equilibrium with the Standard Model plasma can be computed from its spin and decoupling temperature, setting clear targets for future surveys

$$\Delta N_{\text{eff}} = \frac{4}{7} g_s \left( \frac{43/4}{g_*(T_F)} \right)^{4/3}$$

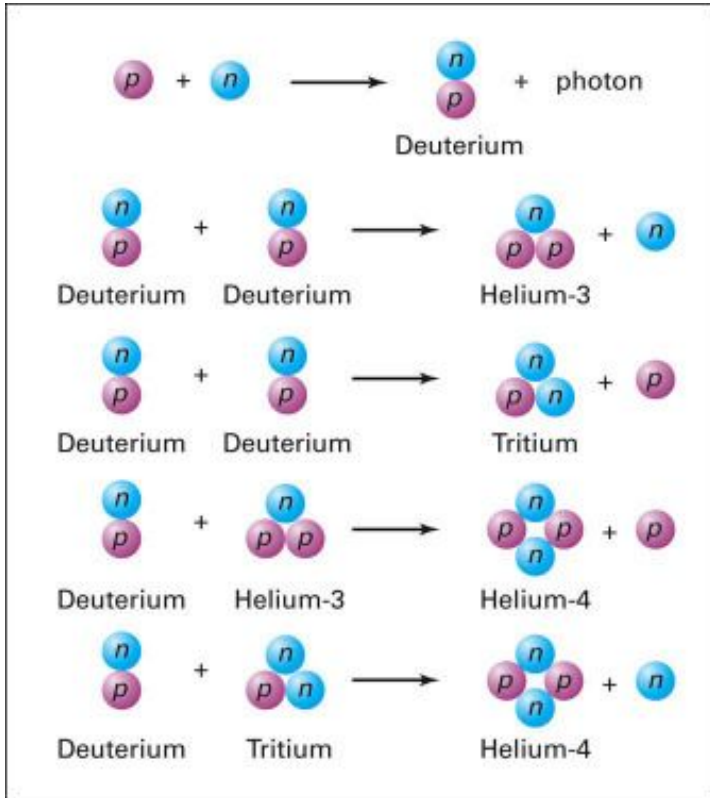
Freeze-out occurs when production rate falls below Hubble rate

$$\Gamma \sim \frac{T^{2n+1}}{\Lambda^n} \quad H \sim \frac{T^2}{M_{\text{pl}}}$$

# Big Bang Nucleosynthesis

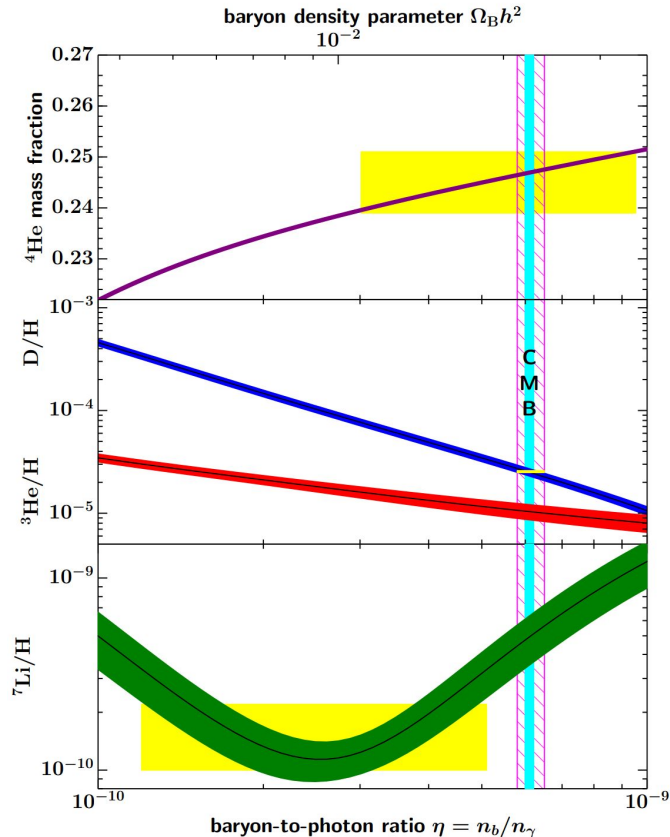


# Standard Big Bang Nucleosynthesis



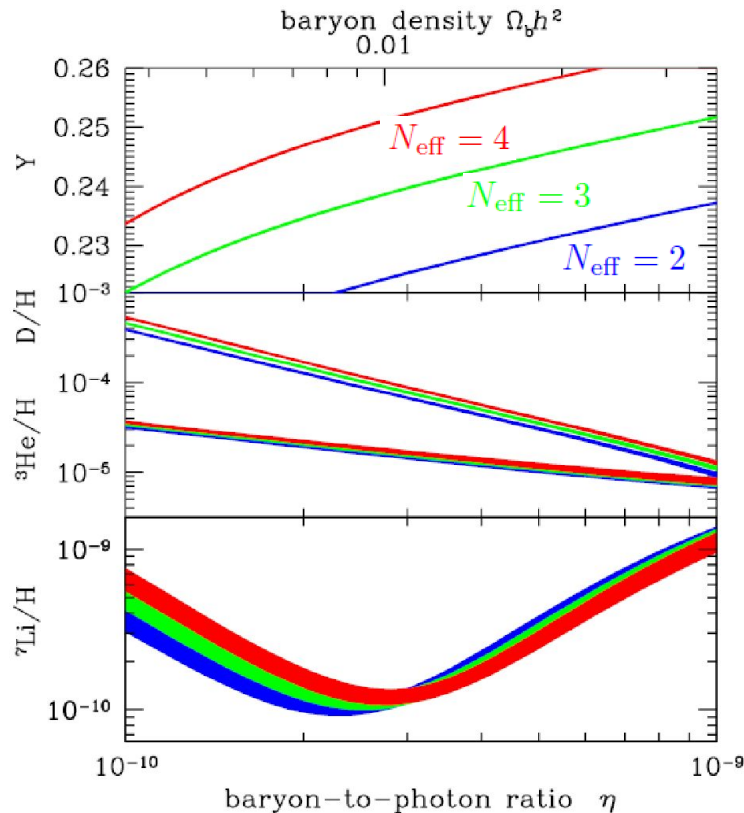
- Big Bang Nucleosynthesis (BBN) is the process by which protons and neutrons became bound into light nuclei in the early universe (around 3 minutes)
- Astronomical measurements of pristine systems allow for inference of the primordial abundances of light elements
  - D: Quasar Absorption Spectra
  - He-4: Emission from Extragalactic HII Regions
  - Li-7: Metal-poor Milky Way Halo Stars (??)

# Standard Big Bang Nucleosynthesis



- Standard BBN depends only on a single parameter, the baryon-to-photon ratio
- Precise measurements of the primordial abundance of Helium-4 and deuterium constrain deviations from the standard cosmic history
- The best estimates of the primordial lithium-7 abundance disagree with the standard theoretical prediction, though the astrophysical inference is suspect

# BBN and Light Relics

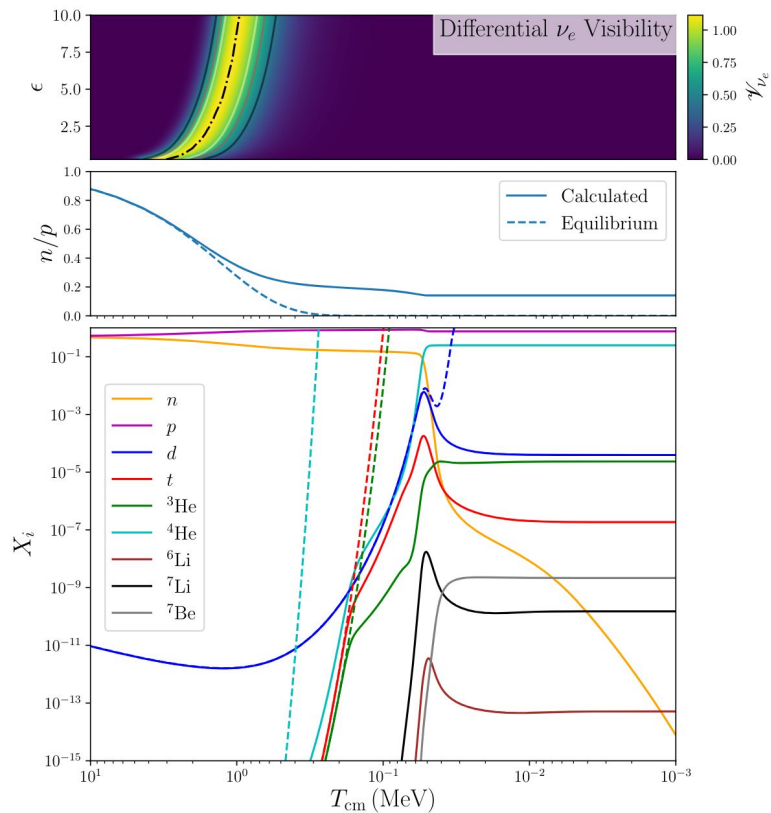


- The light relic density affects the expansion rate during BBN, and thus affects the predicted abundances
- Increasing  $N_{\text{eff}}$  leads to larger expansion rate, leading to higher weak freeze-out temperature, and thus larger neutron-to-proton ratio, ultimately giving a larger helium abundance
- Combined with the CMB constraint on the baryon-to-photon ratio, measurements of the primordial helium-4 and deuterium abundances gives a constraint on  $N_{\text{eff}}$

$$N_{\text{eff}}^{\text{BBN}} = 2.93 \pm 0.23$$

Yeh, Shelton, Olive Fields (2022);  
Figure Credit: Cyburt, et al (2015)

# BBN and New Physics in the Neutrino Sector

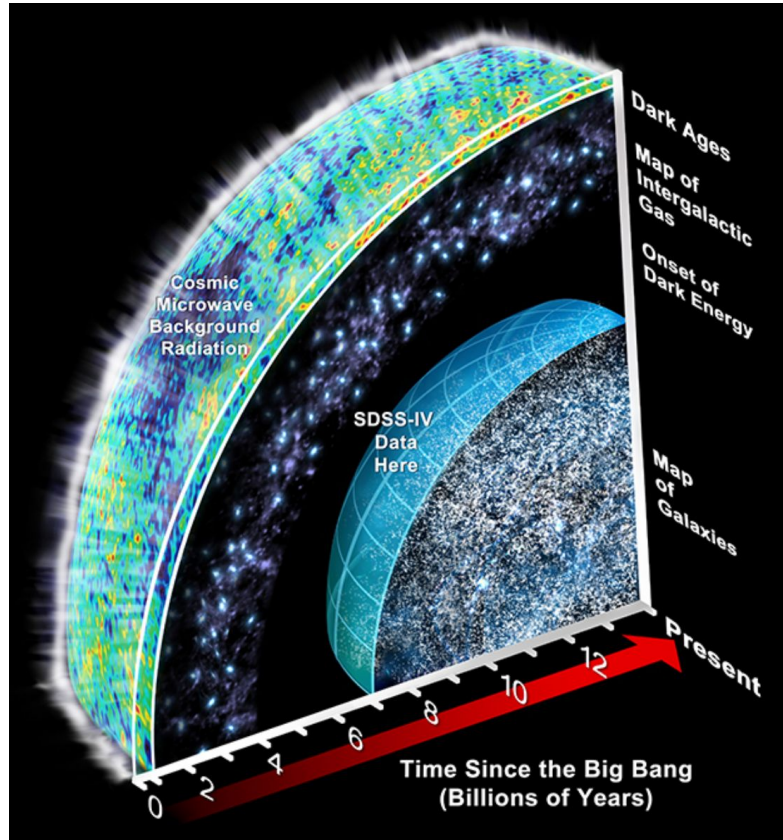


- The precision with which we can measure primordial light element abundances (especially deuterium and Helium-4) allows us to use BBN as a powerful probe of new physics (in particular in the neutrino sector)
- This becomes an even sharper test when combined with CMB constraints



# Cosmic Microwave Background Basics

# The Cosmic Microwave Background



Cosmic Microwave Background (CMB) Spectrum

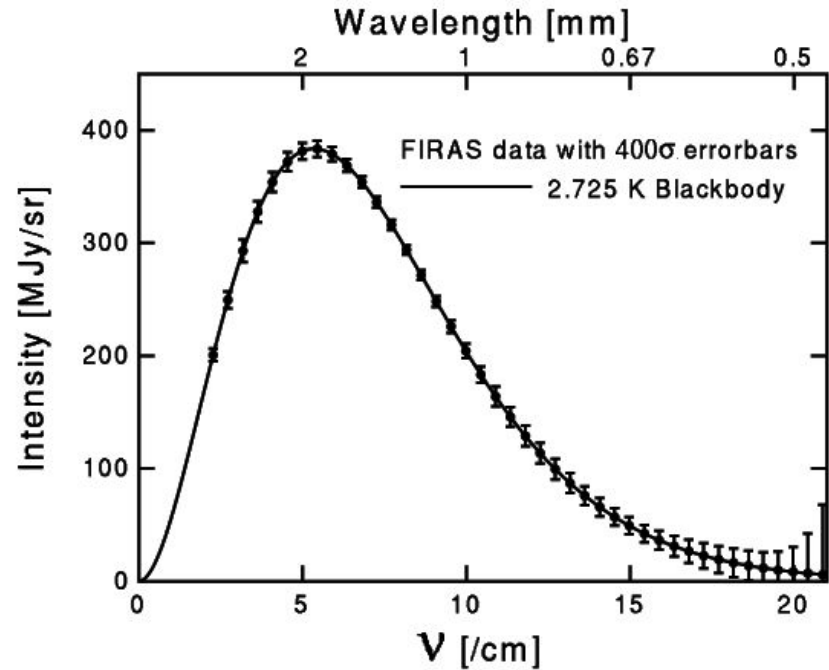


Image Credits: SDSS, COBE FIRAS

# Sound Waves in the Primordial Plasma

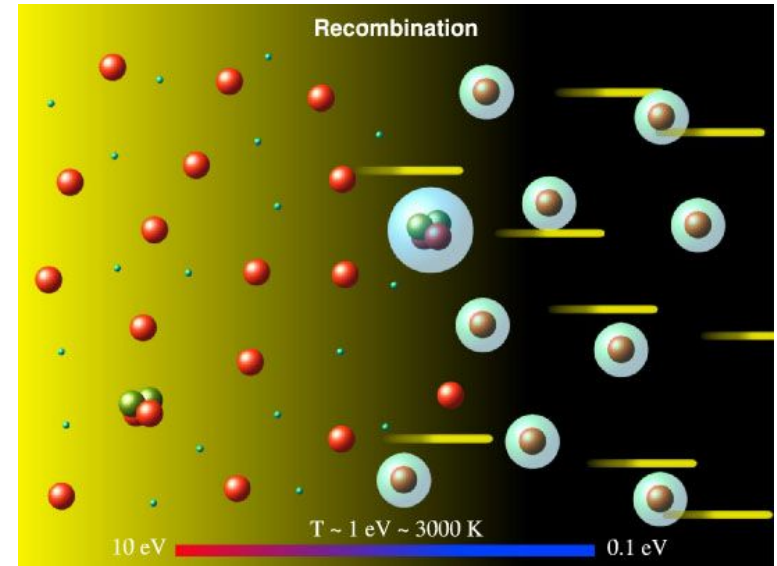
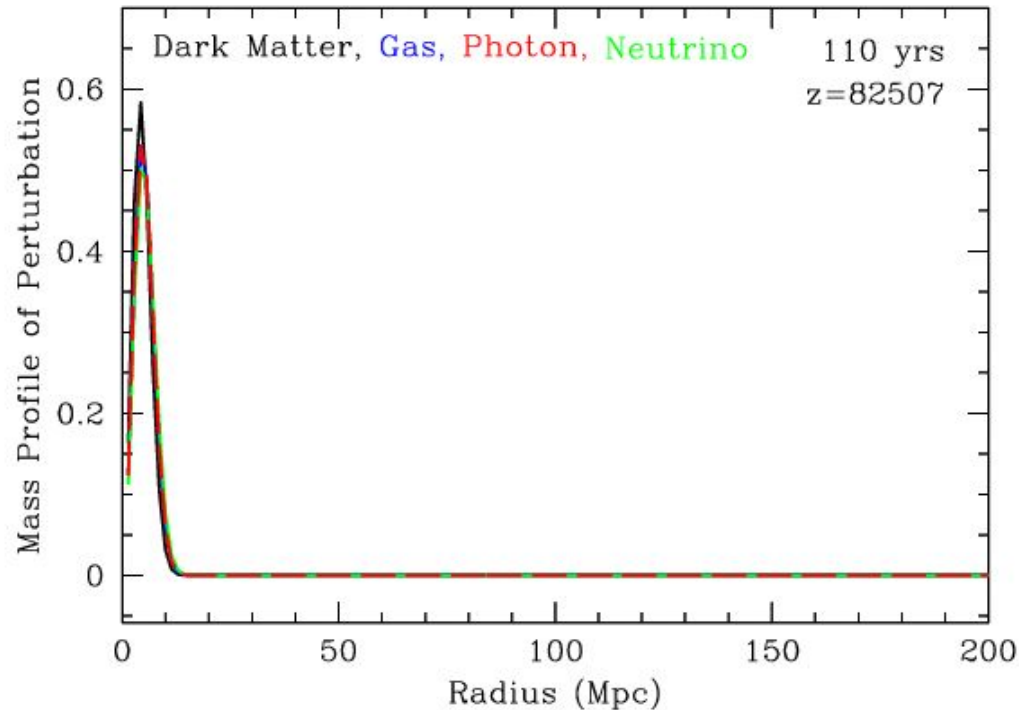
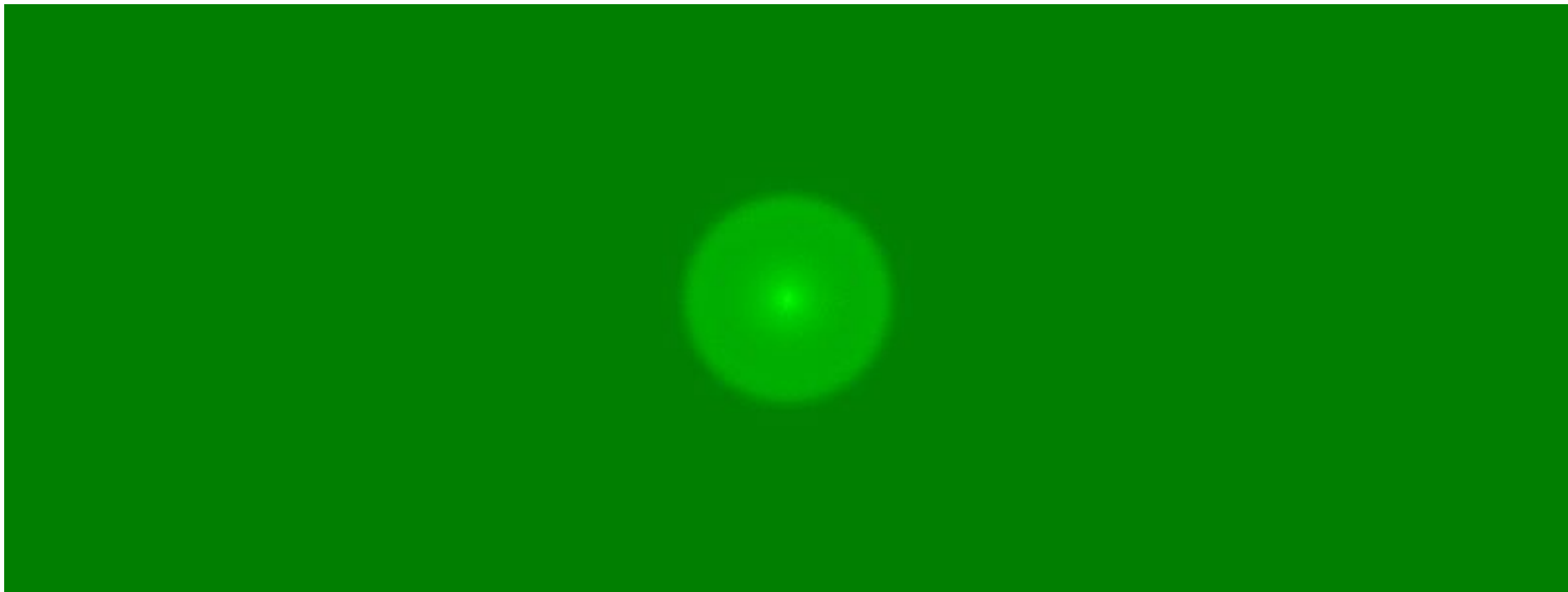
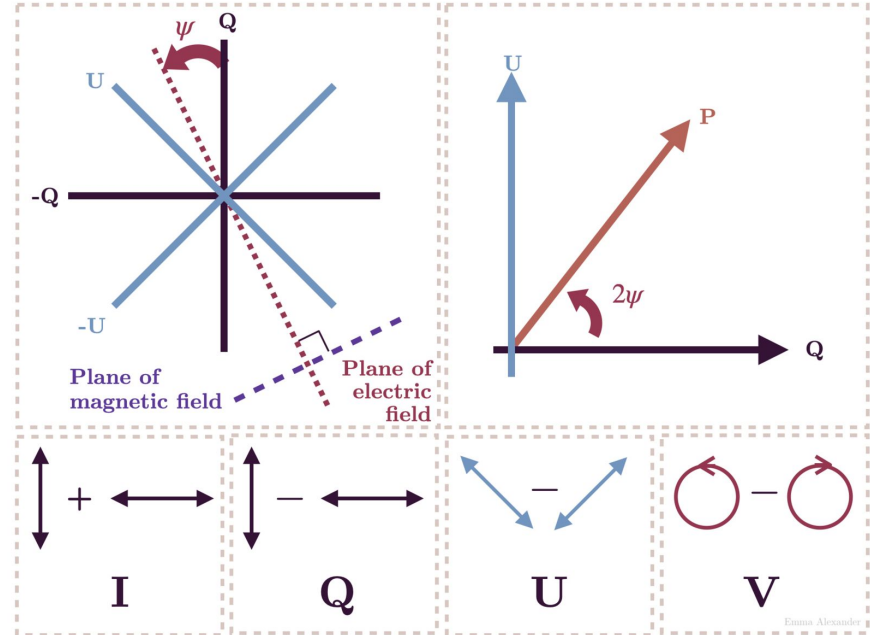
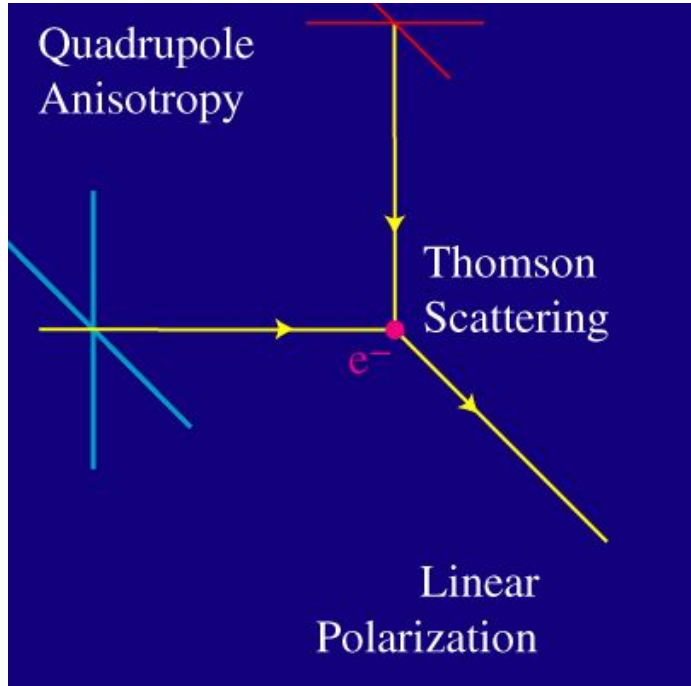


Image Credits: Eisenstein, Kinney

# Superposition of Sound Waves

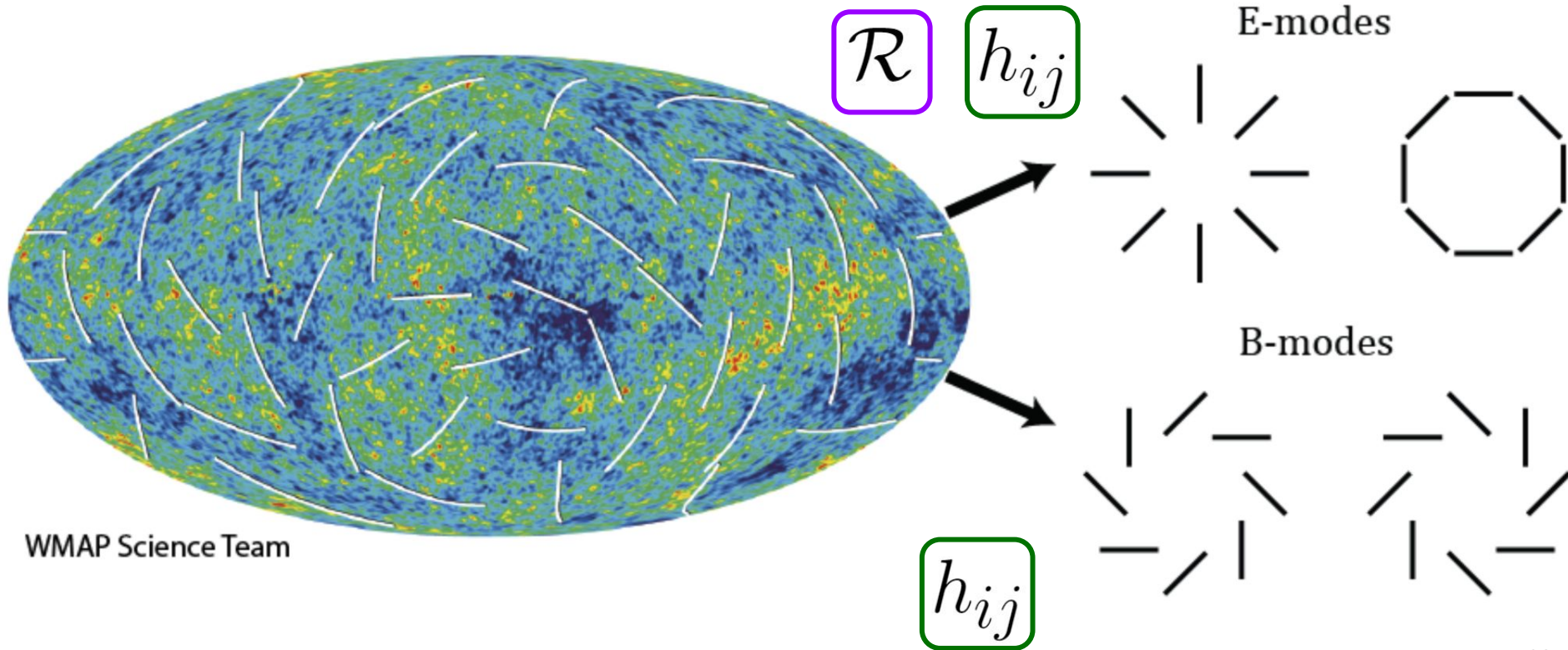


# Linear Polarization of the CMB





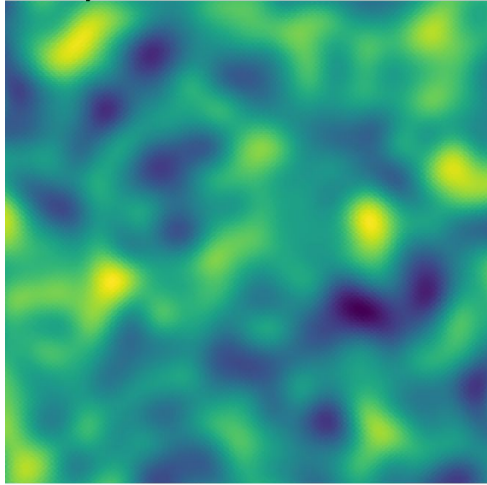
# E and B Modes



# Information In The Cosmic Microwave Background

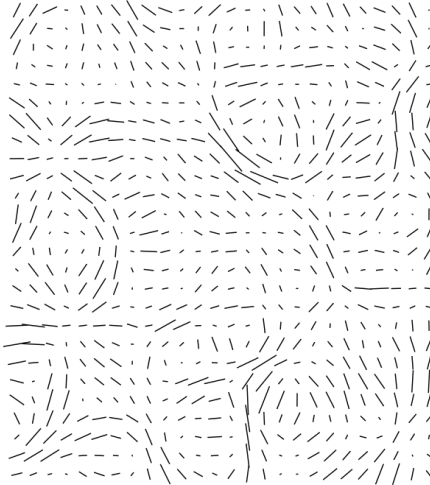
The CMB provides a snapshot of the universe as it existed during recombination

Temperature ~ distribution



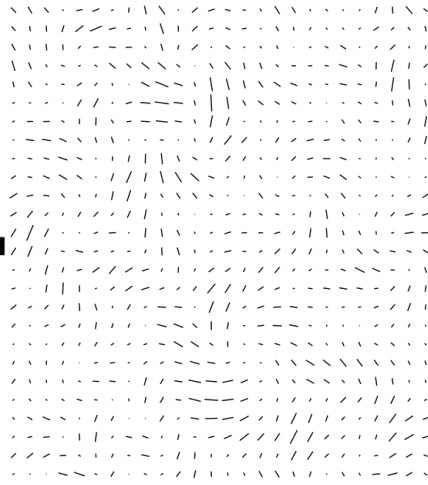
T

Polarization ~ motion



E

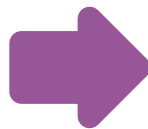
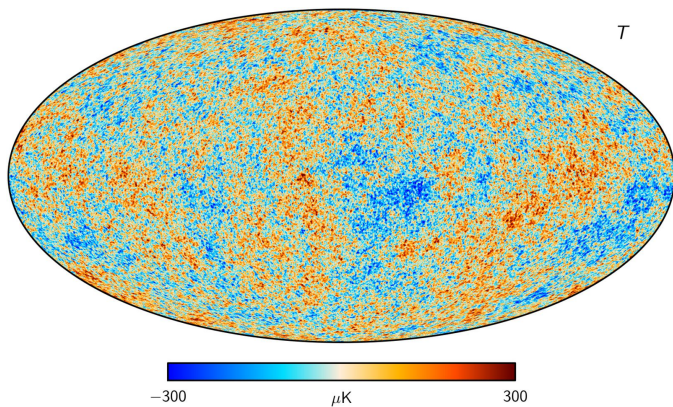
+



B

...plus the imprints of the structure between us and the last scattering surface.

# Statistical Information and Angular Power Spectra



Variance

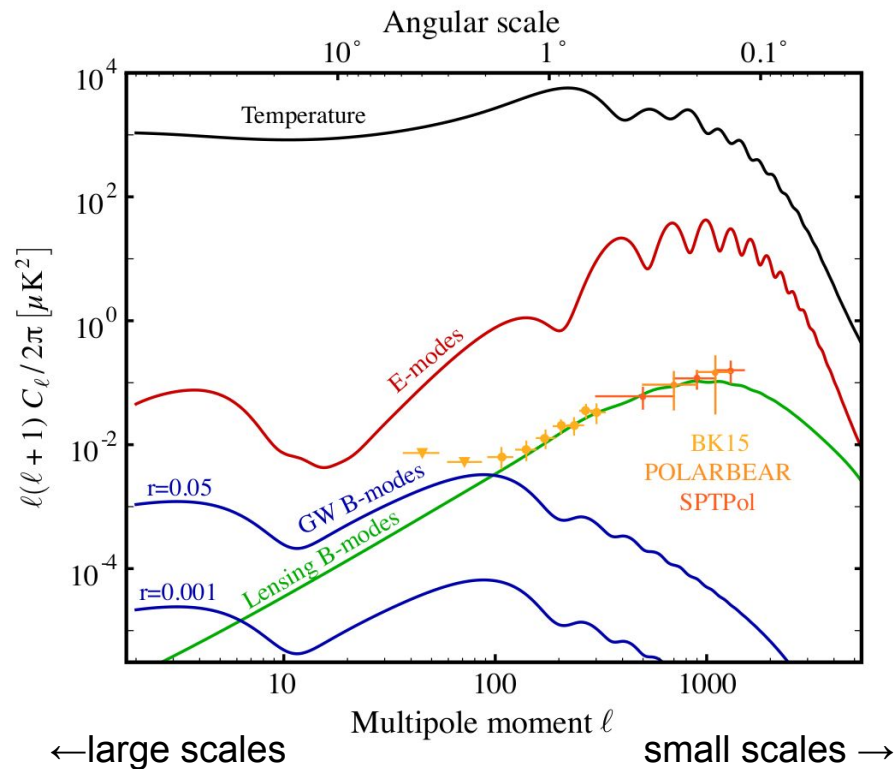
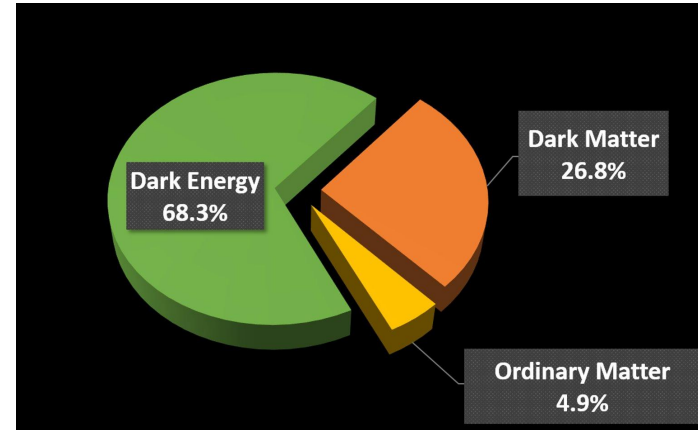
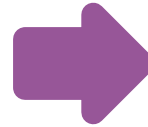
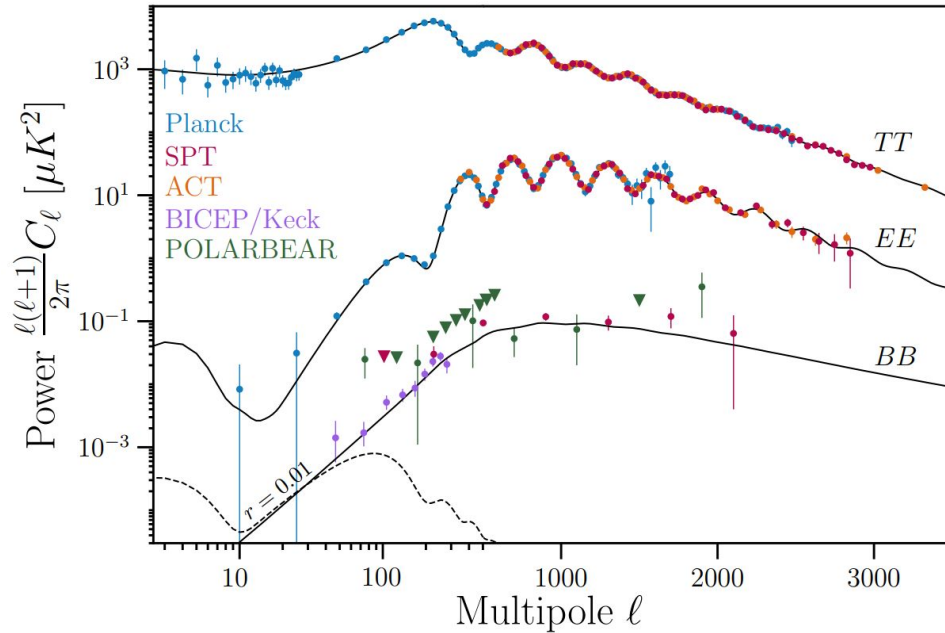


Image Credits: Planck (2018); CMB-S4 (2019)

# CMB Observations and Concordance Flat $\Lambda$ CDM





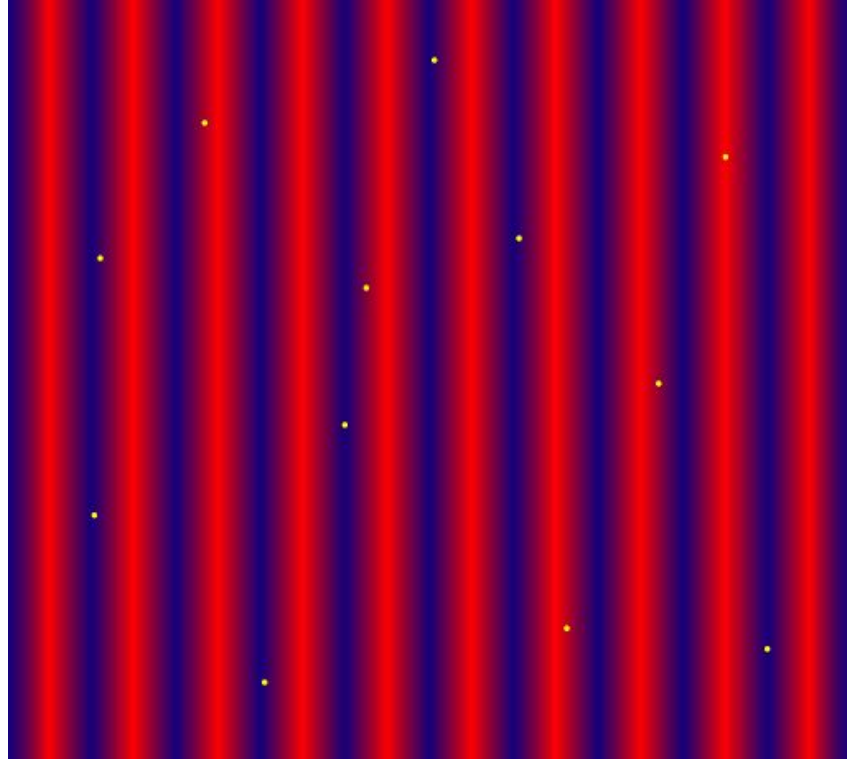
# Measuring Light Relics



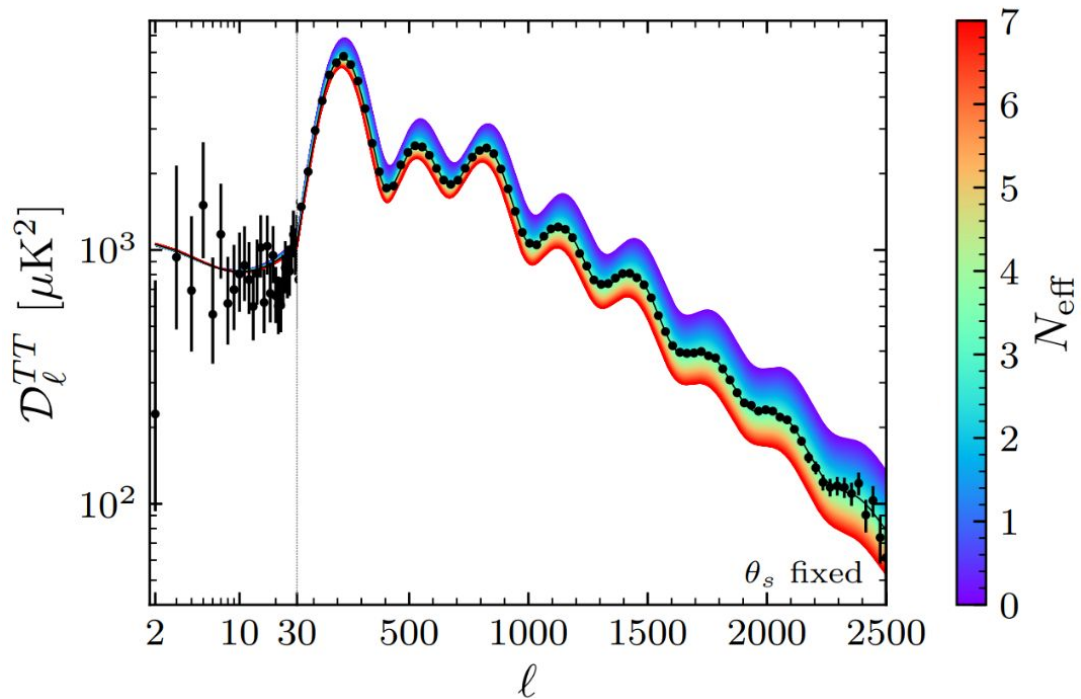
# CMB Diffusion Damping

- Random walk of CMB photons prior to recombination smooths out fluctuations below the free streaming length of photons
- The damping scale of photons is affected by the scattering rate and expansion rate

$$r_d^2 \sim (\sigma_T n_e H)^{-1}$$



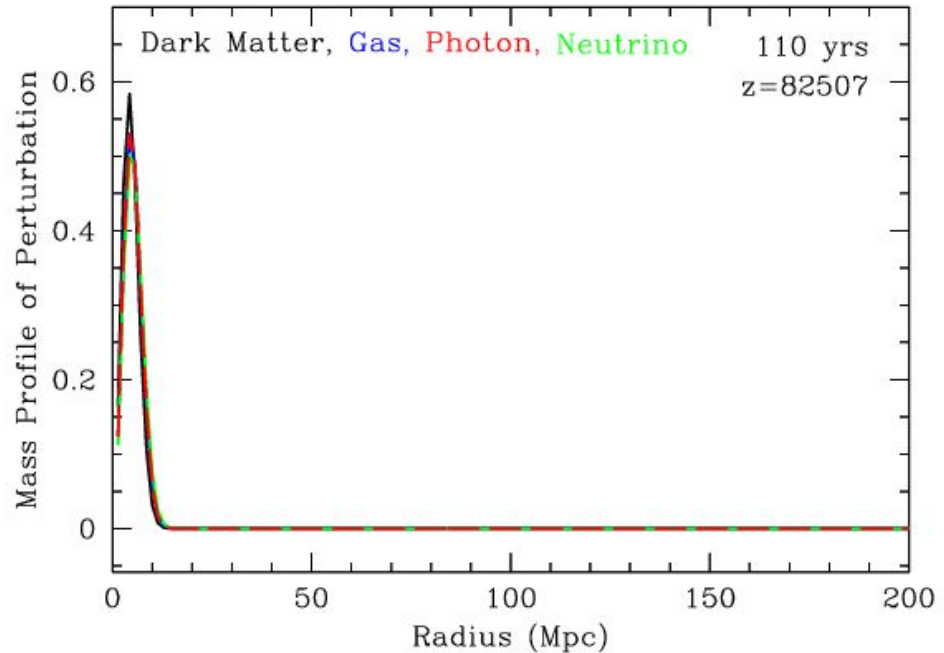
# Light Relics Affect CMB Damping Scale



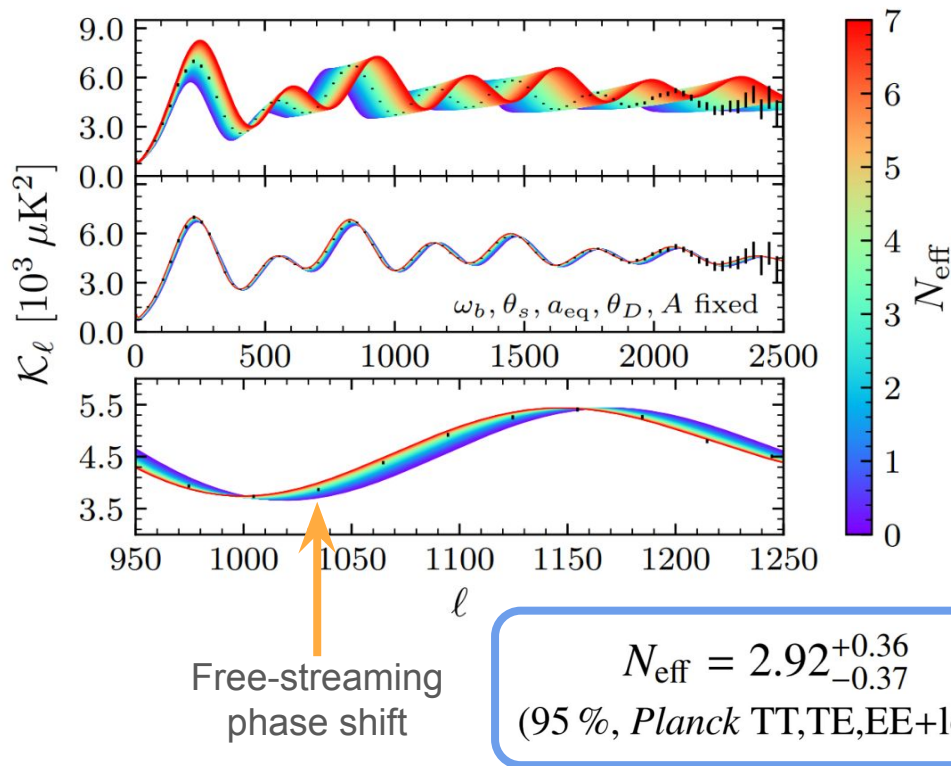
- Increasing  $N_{\text{eff}}$  increases the expansion rate prior to recombination
- With  $\theta_s$  fixed (which is measured very well with current observations), increasing  $N_{\text{eff}}$  leads to increased damping
- The damping scale is also impacted by the free electron density around recombination, which is affected by the primordial helium abundance

# Light Relics Density Perturbations

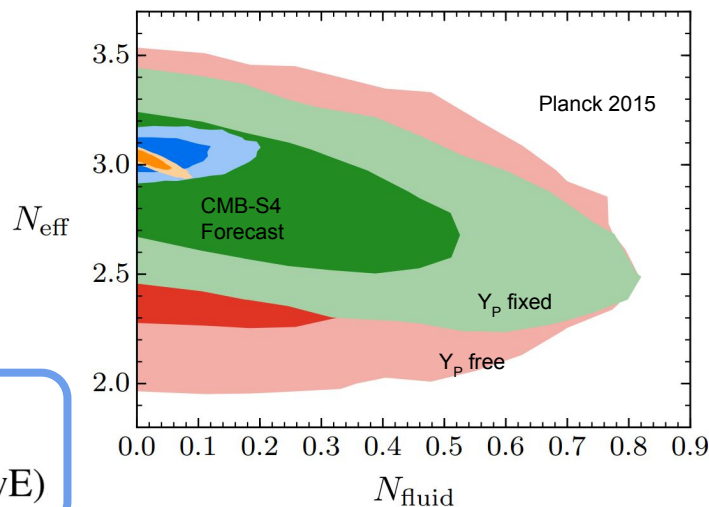
- The density of light relics are perturbed in the same way as the other components (for adiabatic initial conditions)
- The fluctuations of free-streaming light relics propagate at the speed of light, faster than the sound speed of the photon baryon plasma ( $c_s^2 \approx c^2/3$ )
- The gravitational attraction of the light relics pulls the acoustic peak to a larger radius



# Free-Streaming Light Relics and the Phase Shift

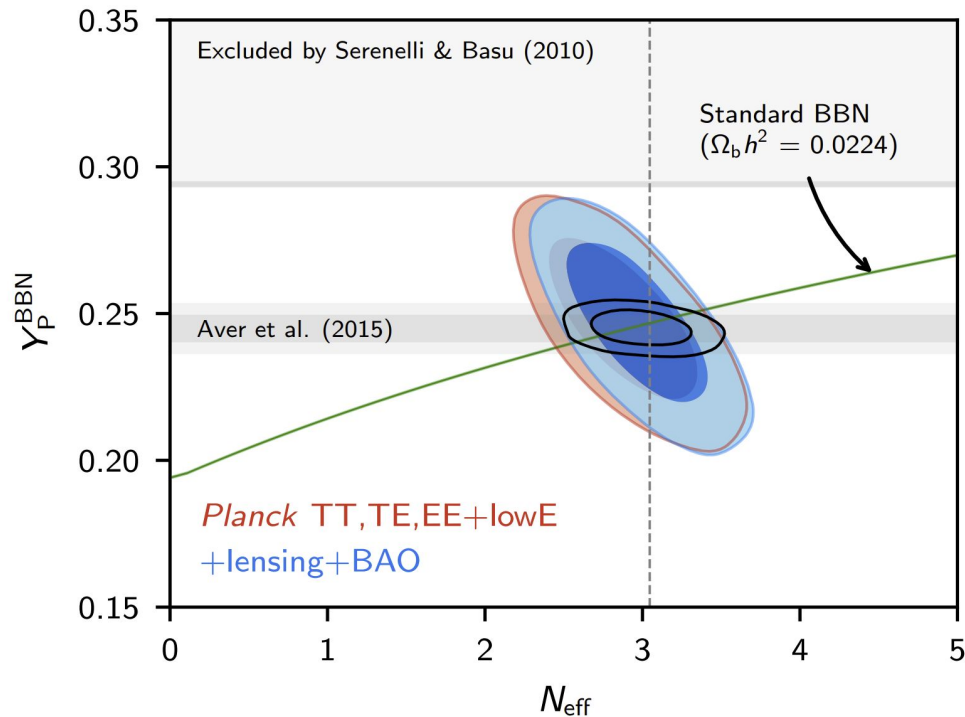


Fluctuations in the density of free-streaming light relics leads to a phase shift of the CMB acoustic peaks, allowing them to be distinguished from fluid-like radiation and changes to primordial helium



# CMB Tests of BBN

- The CMB power spectrum is sensitive to both  $N_{\text{eff}}$  and  $Y_p$  and can therefore be used to test BBN
- Both parameters affect the damping scale, but they are not totally degenerate because  $N_{\text{eff}}$  has other effects (including the phase shift)
- BBN predicts a particular relationship between  $N_{\text{eff}}$  and  $Y_p$
- Current observations are consistent with standard BBN, and place constraints on non-standard scenarios (like time-dependent  $N_{\text{eff}}$ )



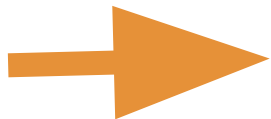


# Constraints on the Light Relic Density

Current  
Constraint

$$N_{\text{eff}} = 2.92^{+0.36}_{-0.37}$$

(95 %, *Planck* TT,TE,EE+lowE)



CMB-S4  
Target

$$\Delta N_{\text{eff}} \leq 0.060 \text{ (95\%)}$$



A night sky with the Milky Way galaxy visible as a bright, colorful band of stars and dust. The colors range from blue and purple to red and orange. At the bottom of the image, the metal framework of a radio telescope is visible, silhouetted against the starry background.

# Thank You!

# Backup Slides

# Timeline of Upcoming CMB Surveys

