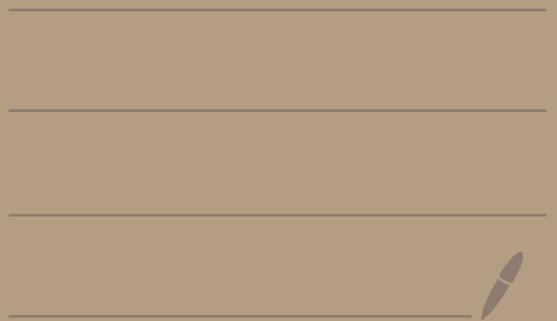


Neutrinoless double beta

and

(ν, N) system



(J, N) system

$$M_{J,N} = \begin{pmatrix} 0 & \mu_D \\ \mu_D & \mu_N \end{pmatrix}$$

$$\Downarrow$$
$$\mu_{1,2} = \frac{\mu_N \pm \sqrt{\mu_N^2 + 4\mu_D^2}}{2}$$

$$\frac{1}{2} \tan 2\theta = \frac{\mu_D}{\mu_N}$$

$$O = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$(1) \quad m_N \gg m_D$$

$$\Rightarrow \quad \theta \approx m_D/m_N, \quad m_1 \approx -m_D^2/m_N$$

$$D \approx \begin{pmatrix} 1 & \theta \\ -\theta & 1 \end{pmatrix} \quad m_2 \approx m_N$$

$$\Rightarrow \quad \boxed{\nu_{A0\nu 2\nu} \propto m_1 = m_\nu}$$

as expected

$$(1.5) \quad m_D \gg m_N$$



$$\theta = 45^\circ + \varepsilon$$

$$\cot 2\theta = \frac{\mu_N}{2\mu_D}$$

$$\begin{aligned}\cos \theta &= \cos 45 \cos \varepsilon - \sin 45 \sin \varepsilon \\ &= \frac{1}{\sqrt{2}} (\cos \varepsilon - \sin \varepsilon)\end{aligned}$$

$$\begin{aligned}\sin \theta &= \sin 45 \cos \varepsilon + \cos 45 \sin \varepsilon \\ &= \frac{1}{\sqrt{2}} (\cos \varepsilon + \sin \varepsilon)\end{aligned}$$

$$\begin{aligned}\Rightarrow \cos 2\theta &= \frac{1}{2} \left[(\cos \varepsilon - \sin \varepsilon)^2 - \right. \\ &\quad \left. - (\cos \varepsilon + \sin \varepsilon)^2 \right] \\ &= \frac{1}{2} (-4 \cos \varepsilon \sin \varepsilon) \approx -2\varepsilon\end{aligned}$$

$$\cos 2\theta \approx -2\epsilon$$

$$\sin 2\theta = 2 \frac{1}{2} (\cos \epsilon - \sin \epsilon) / (\cos \epsilon + \sin \epsilon)$$

$$= \cos^2 \epsilon - \sin^2 \epsilon \approx 1$$



$$\cos 2\theta \approx -2\epsilon = \frac{\mu_N}{2m_D}$$

$$\epsilon \approx -\frac{1}{4} \frac{\mu_N}{m_D}$$

$$0 \approx \frac{1}{\sqrt{2}} \begin{pmatrix} 1-\epsilon & 1+\epsilon \\ -(1+\epsilon) & 1-\epsilon \end{pmatrix}$$

$$\psi = \frac{1}{\sqrt{2}} \left[(1-\epsilon)^2 \psi_1 + (1+\epsilon)^2 \psi_2 \right]$$

⇓

$$m_1 = \frac{m_N}{2} + \frac{1}{2} \sqrt{4m_D^2 + m_N^2}$$

$$m_2 = \frac{m_N}{2} - \frac{1}{2} \sqrt{4m_D^2 + m_N^2}$$

⇓

$$m_1 \approx m_N/2 + \textcircled{m_D} + O\left(\frac{m_D^2}{m_N}\right)$$

$$u_2 = u_N/2 - u_D - O\left(\frac{u_N^2}{u_D}\right)$$



$$A_{\text{avg}} = \frac{1}{2} \left[(1-\epsilon)^2 u_1 + (1+\epsilon)^2 u_2 \right]$$

$$= \frac{1}{2} \left[(1-2\epsilon) u_1 + (1+2\epsilon) u_2 \right]$$

$$= \frac{1}{2} \left[(1-2\epsilon) \left(\frac{u_N}{2} + u_D \right) + \right.$$

$$\left. + (1+2\epsilon) \left(\frac{u_N}{2} - u_D \right) \right]$$

$$= \frac{1}{2} \left[\frac{u_N}{2} + \frac{u_N}{2} - 4\epsilon u_D \right]$$

$$= \frac{1}{2} \left[u_N - (-) u_N \right] = u_N$$

as expected

