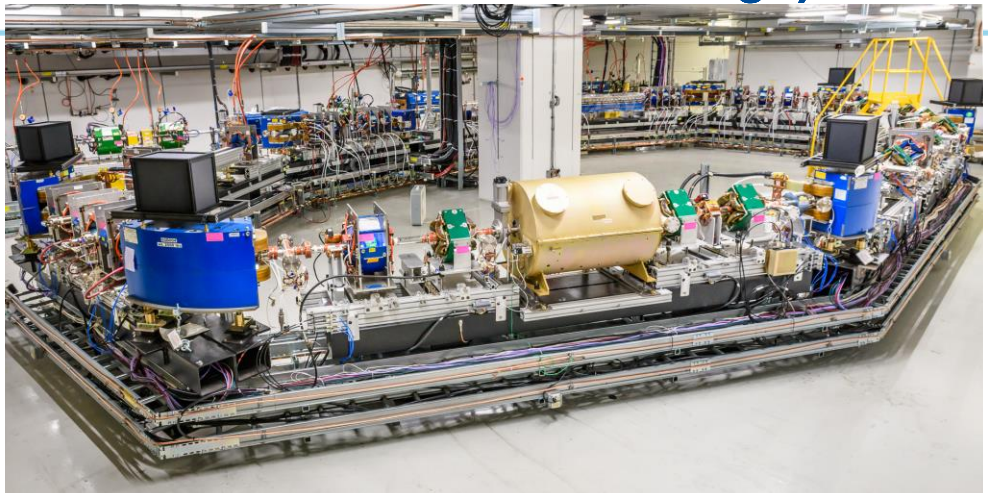
Managed by Fermi Research Alliance, LLC for the U.S. Department of Energy Office of Science

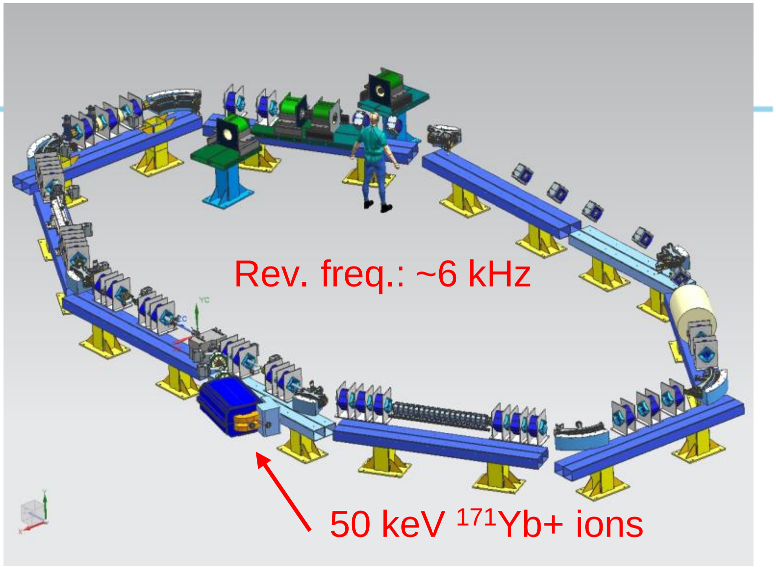
Ion scattering on residual gas

Sergei Nagaitsev

Dec 1, 2022

Fermilab IOTA ring (40-m circumference, 50 – 150 MeV/c momentum range)





- Present status: operating with relativistic electrons (~100-150 MeV)

- **This proposal:** add a 50-keV (120-MeV/c momentum) ^{171}Yb+ ion source, install counter-propagating lasers for Doppler laser cooling and extra ion diagnostics.

Ion parameters

Ions: 171 Yb +

$A := 171 \quad M := 931.5 \quad T := 300 \quad k := 1.38 \cdot 10^{-23}$

$c := 3 \cdot 10^{10} \quad L := 4000$ m -- IOTA circumference

$K := 50 \cdot 10^{-3}$ MeV -- kinetic energy

$$\gamma := \frac{K + A \cdot M}{A \cdot M} \qquad \beta := \sqrt{1 - \frac{1}{\gamma^2}}$$

$\beta \cdot c = 2.377 \times 10^7$ cm/s -- ion velocity

$\frac{\beta \cdot c}{L} = 5.943 \times 10^3$ Hz

$\beta \cdot A \cdot M = 126.209$ MeV/c

$\frac{K \cdot 10^6}{A} = 292.398$ eV/amu -- kinetic energy per nucleon

Charge-exchange cross section (example)

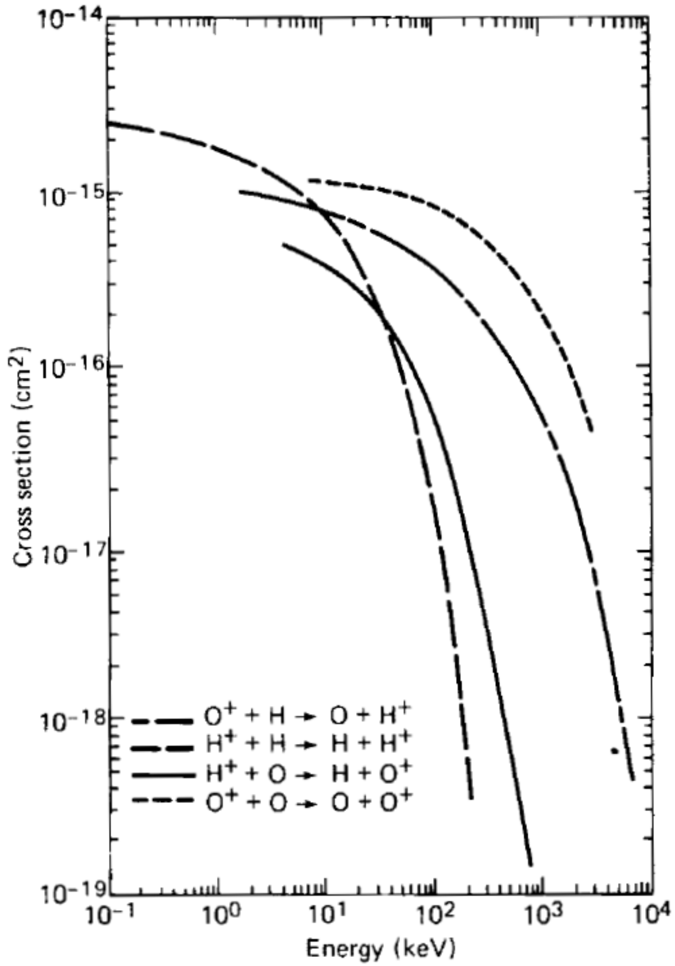


Fig. 1: Charge-exchange cross sections of energetic H^+ and O^+ ions as a function of total ion energy for electron pickup from cold neutral hydrogen and oxygen (figure is taken from a compilation by McEntire and Mitchell, 1989).

- From: Detection of Energetic Neutral Atoms, Peter Wurz

http://wurz.space.unibe.ch/paper_bad_honnef.pdf

Lifetime estimates

- Assume that the main loss mechanism is the charge exchange with residual (neutral) atoms/molecules:
  - $A^+ + B \rightarrow A + B^+$
  - In our case the kinetic energy is about 300 eV/nucleon
  - Cross-section is estimated at 1e-16 cm^{-2}

Pressure: $p := 1 \cdot 10^{-10}$ torr

$$n := \frac{p \cdot 133.3}{k \cdot T \cdot 10^6} \qquad n = 3.22 \times 10^6 \quad \text{1/cm\^{}3}$$

$\sigma := 1 \cdot 10^{-16}$ cross-section, cm^-2

$\tau := (n \cdot \sigma \cdot \beta \cdot c)^{-1} \qquad \tau = 130.659$ seconds

Residual gas parameters

- Typical composition: N_2/CO and H_2
- T = 300 K
  - Ave. velocity: 5e4 cm/s (N2) and 2e5 cm/s (H2)
  - Thus, the thermal velocities are much smaller than Yb+ ion velocity (2.4e7 cm/s) at 50 keV

Coulomb scattering between Yb+ ions and neutral molecules

- Consider the minimum approach distance between nuclei Yb (Z = 70, A1 = 171) and N2 (z=7, A2=28), H2 (z=1, A2=2)
  - In the CM frame, Yb is almost stationary.
  - Nitrogen:

$$Z := 70 \qquad z := 7$$

$$A2 := 28 \qquad r_p := 2.8\cdot 10^{-13}\cdot\frac{0.511}{938}$$

$$a_{min} := \frac{2\cdot Z\cdot z\cdot r_p}{A2\cdot\beta^2} \qquad a_{min} = 8.504\times 10^{-9} \text{ cm}$$

  - Hydrogen:

$$Z := 70 \qquad z := 1$$

$$A2 := 2 \qquad r_p := 2.8\cdot 10^{-13}\cdot\frac{0.511}{938}$$

$$a_{min} := \frac{2\cdot Z\cdot z\cdot r_p}{A2\cdot\beta^2} \qquad a_{min} = 1.701\times 10^{-8} \text{ cm}$$

  - It looks like the minimum approach distance is about equal to the atomic size, thus the Coulomb interaction is highly screened by atomic electrons.

Small-angle Coulomb scattering in accelerators

- Emittance growth due to multiple scattering (standard formula)

$$\frac{d}{dt}\begin{bmatrix}\varepsilon_x \\ \varepsilon_y\end{bmatrix} = \frac{2\pi c r_e^2}{\gamma^2\beta^3}\sum_k Z_k\left(Z_k+1\right)\ln\left(\frac{\theta_k^{\max}}{\theta_k^{\min}}\right)\left\langle\begin{bmatrix}\beta_x(s) \\ \beta_y(s)\end{bmatrix} n_k(s)\right\rangle_s$$

where $\theta_k^{\min} = \frac{\hbar}{p a_{atom}} \approx \frac{\sqrt[3]{Z_k}\, m_e c}{192\, p}$ is set by atom size

and

$$\theta_k^{\max} = \frac{\hbar}{p a_{nucl}} \to \approx \min\left(\frac{274 m_e c}{\sqrt[3]{A_k}\, p}, \sqrt{\frac{\varepsilon_{mx,my}}{\beta_{x,y}}}\right)$$

is set by nuclear size or the ring acceptance

$p = Mc\beta\gamma$ — and M is the mass of accelerated particle

This formulas may not be applicable in our case since Yb+ ion velocity is small: $\beta < Z\alpha$

https://www.sr-niel.org/index.php/sr-niel-long-write-up/protons-and-ions-scattering-on-screened-coulomb

Scattering by polarized neutral atoms

This might be an important mechanism in our case (50 keV Yb+ ions)

PHYSICAL REVIEW VOLUME 95, NUMBER 5 SEPTEMBER 1, 1954

Scattering of Ions by Polarization Forces

ERICH VOGT* AND GREGORY H. WANNIER
Bell Telephone Laboratories, Murray Hill, New Jersey

(Received May 19, 1954)

The interaction potential of a charge carrier and a gaseous atom is attractive at large distances, varying as r^{-4}. This potential has simple classical properties since it has a cross section which varies inversely as the speed. For most ions the mechanism which removes the singularity of the potential is irrelevant classically —we find that this is also the case in quantum theory: the well-known indeterminacy of the wave function for singular potentials can be removed in an obvious way and a cross section of the capture type can be computed. This cross section oscillates sinusoidally about its classical value but has apparently no average deviation over one cycle, even when the de Broglie wavelength is long. In the limit of low velocities, the quantum-mechanical cross section has twice the classical value. These two facts combine to make the classical law of variation of the cross section approximately valid even in the quantum range.

I. INTRODUCTION

WHEN gaseous ions or electrons move through a gas whose molecules are not too large, then the two interact according to the law

$$V = -\tfrac{1}{2}e^2\alpha/r^4, \qquad (1)$$

where e is the ionic charge, α the molecular polarizability, and r the distance between the ion and the molecule. The classical theory of the motion under this force is simple because the cross sections derived from this force are proportional to $1/v$, where v is the relative velocity. This feature of the classical theory can be derived from a dimensional argument. The cross sections must be constructed from the quantities $e^2\alpha$, v, and m, where m is the reduced mass. This construction can be made only in a single way, namely,

$$\sigma = \text{const}(e^2\alpha/mv^2)^{\frac{1}{2}}, \qquad (2)$$

where the constant is a pure number. However, this simple result of classical mechanics will be modified if

is, a case in which the negative energy states cannot be quantized. Such states have been considered in the study of Case.[1] Case points out that the attractive potential is always terminated in reality by a repulsive wall and that this wall will determine the choice of the phase of the rapidly oscillating wave function. While this observation is undoubtedly true, in a great number of physical situations the choice of phase at the repulsive wall is very complicated. Further, this manner of pointing out that the singular potentials are "really" not singular is actually side-stepping the issue; there are many classical situations, generally in the positive energy spectrum, where the presence of a repulsive term in addition to (1) has no importance; furthermore, when it is important the simple classical properties just described are destroyed. Extremely complicated treatments are then required as, for instance, those of Langevin,[2] Hassé and Cook,[3] and others. Hence, to follow up the suggestion of Case of introducing the repulsive wall explicitly into the theory would simply