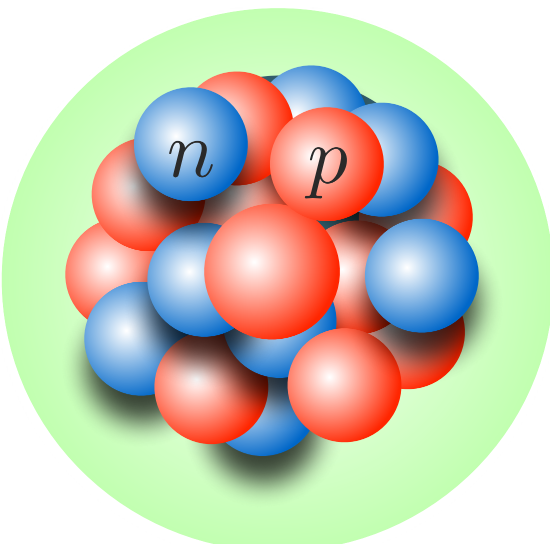
Theory Systematics

Noemi Rocco

Workshop on Neutrino Event Generator
March 15-17, 2023

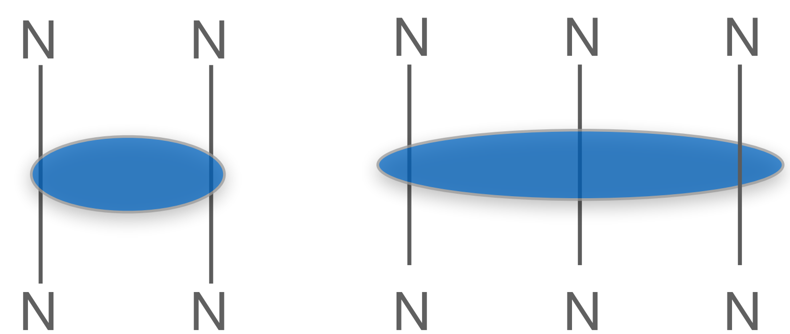
Inputs for the nuclear model

At low energy, the effective degrees of freedom are pions and nucleons:



$$H = \sum_i \frac{\mathbf{p}_i^2}{2m} + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \dots$$

1-body **2-body** **3-body**

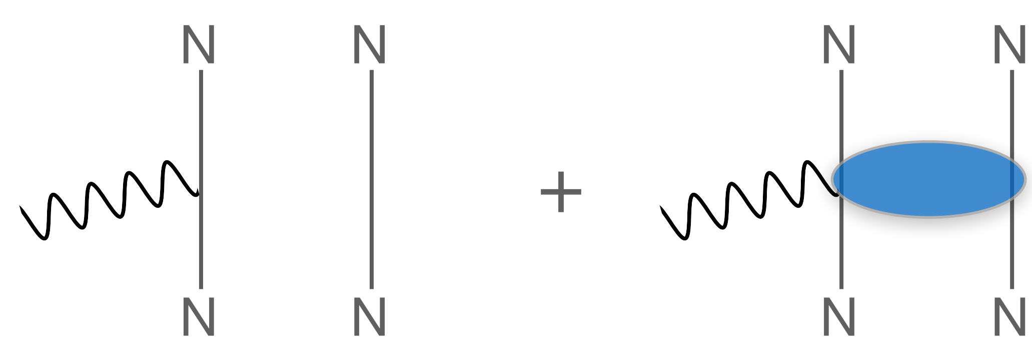


The electromagnetic current is constrained by the Hamiltonian through the **continuity equation**

$$\nabla \cdot \mathbf{J}_{\rm EM} + i[H, J^0_{\rm EM}] = 0 \qquad [v_{ij}, j_i^0] \neq 0$$

The above equation implies that the current operator includes one and two-body contributions

$$J^\mu(q) = \sum_i j_i^\mu + \sum_{i<j} j_{ij}^\mu + \dots$$

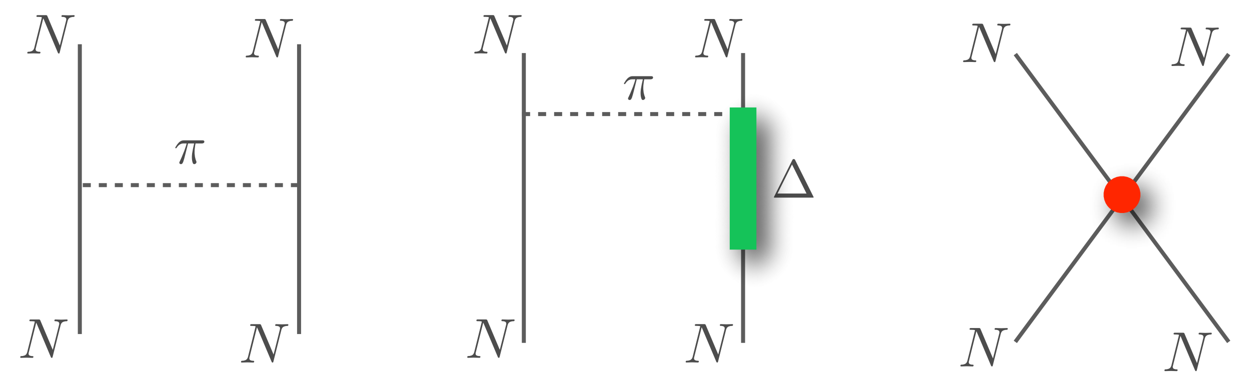


Phenomenological potential: av18 + IL7

Phenomenological potentials explicitly include the **long-range one-pion exchange interaction** and a set of **intermediate- and short-range phenomenological terms**

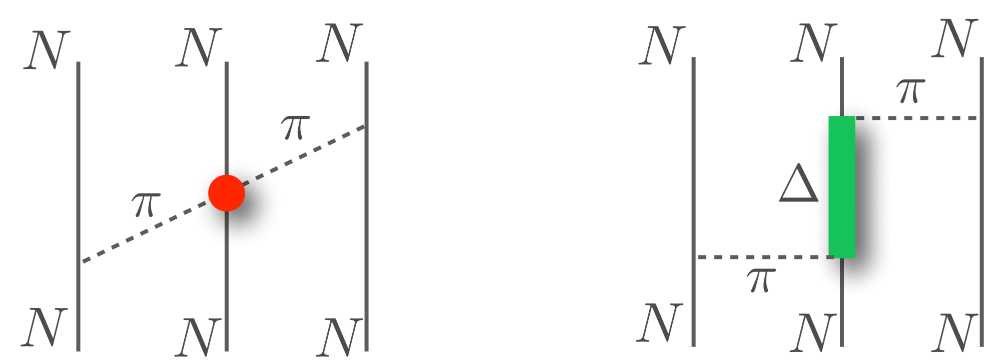
- **Argonne v18** is a finite, local, configuration-space potential controlled by ~4300 np and pp scattering data below 350 MeV of the Nijmegen database

$$v_{ij} = \sum_{p=1}^{18} v^p(r_{ij}) O^p_{ij} \longleftrightarrow$$



- Phenomenological three-nucleon interactions, like the **Illinois 7**, effectively include the lowest nucleon excitation, the Δ(1232) resonance, end other nuclear effects

$$V^{3\mathrm{N}}_{ijk} = A^{PW}_{2\pi} O^{2\pi,PW}_{ijk} + A^{SW}_{2\pi} O^{2\pi,SW}_{ijk} + A^{\Delta R}_{3\pi} O^{3\pi,\Delta R}_{ijk} + A^{R} O^{R}_{ijk} \longleftrightarrow$$



The parameters of the AV18 + IL7 are fit to properties of **exactly solvable light nuclear systems**.

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Chiral effective field theory

The EFT program consists of the following steps:

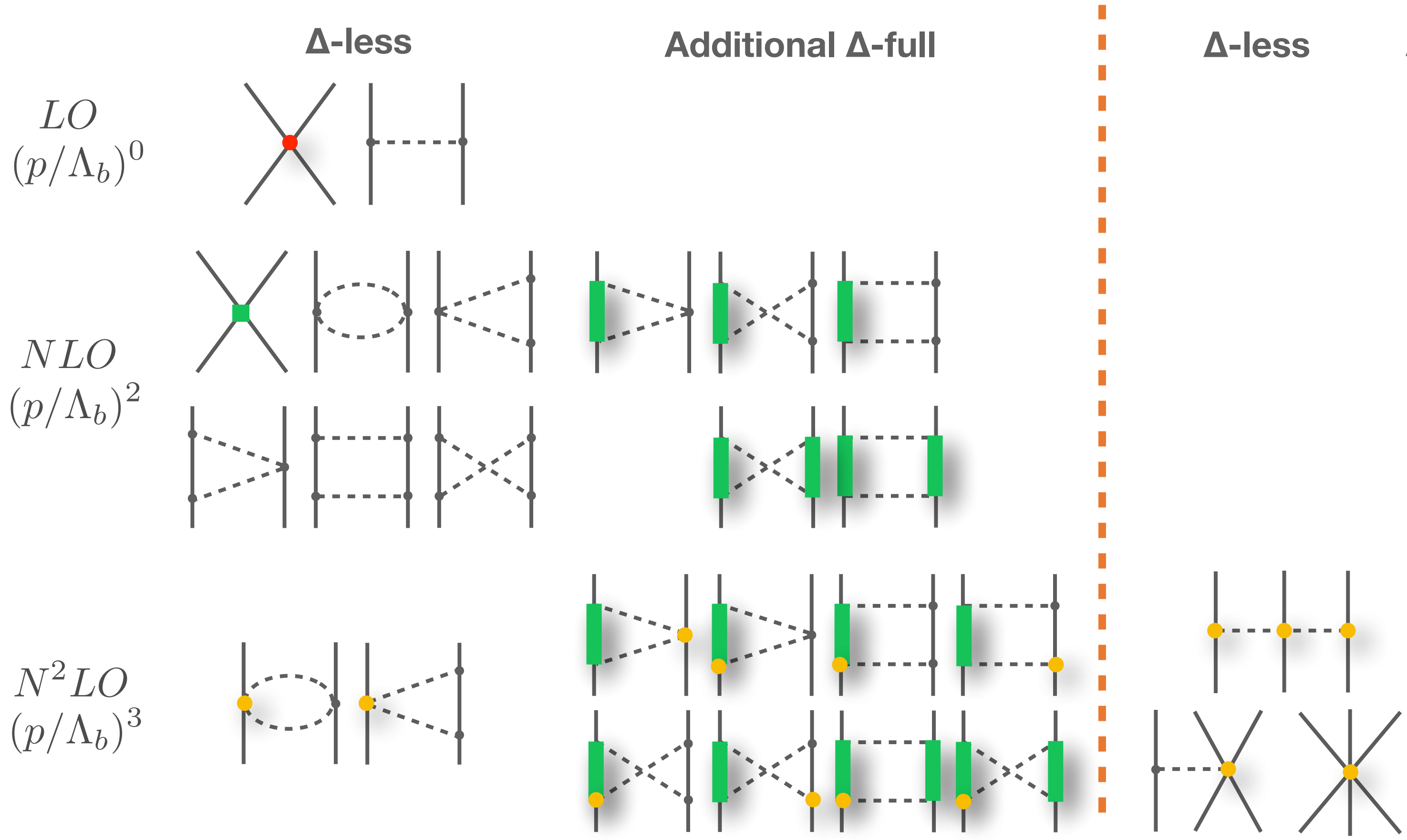
Identify the soft and hard scale of the problem $\mathcal{L}^{(n)} \sim \left(\frac{q}{\Lambda_b}\right)^n$ ~ 100 MeV soft scale, ~ 1 GeV hard scale

Exploits the (approximate) broken chiral symmetry of QCD to construct interactions

Construct the most general Lagrangian consistent with these symmetries

Design an organizational scheme that can distinguish between more and less important terms:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}^{(0)} + \mathcal{L}^{(1)} + \mathcal{L}^{(2)}$$



Contact interactions lead to LEC; determined fitting properties of finite nuclei

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Efforts to provide UQ for interactions

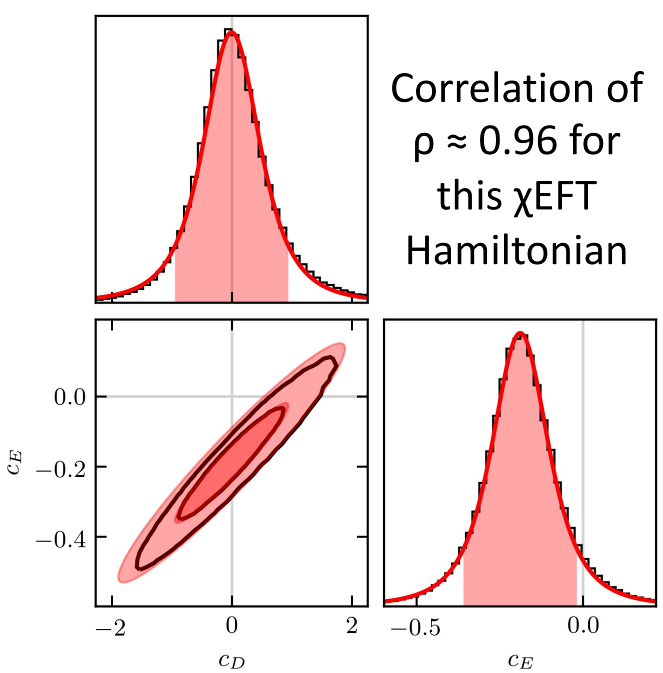
Full Bayesian approach to constrain parameters (LECs)

Propagate errors throughout the calculation (pdf for cD and cE to fit observables)

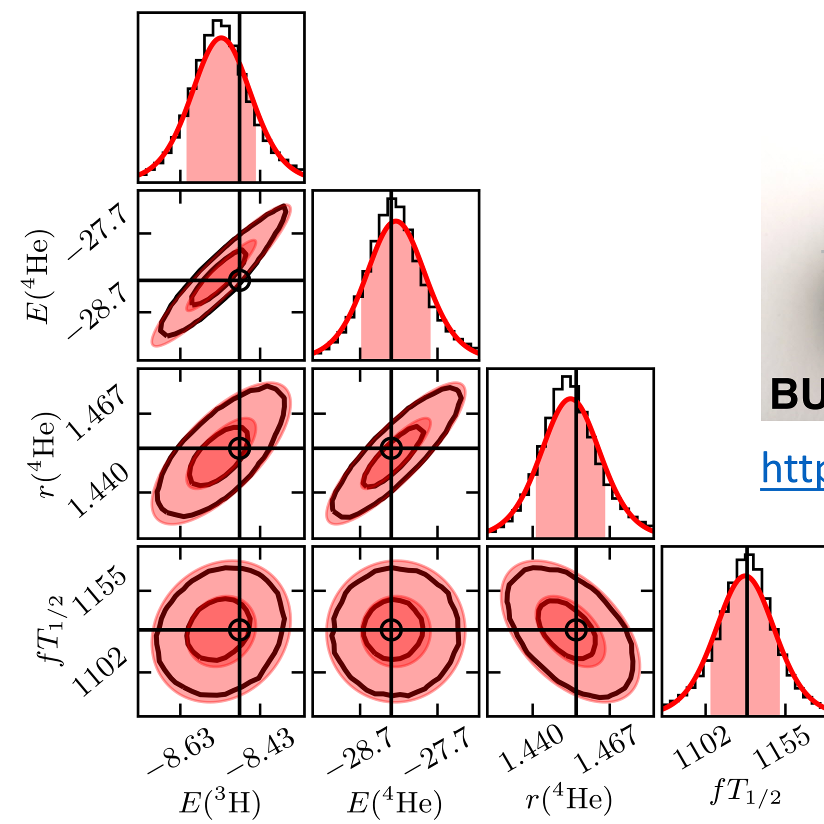
Formulate statistical models for uncertainties: Bayesian estimates of EFT truncation errors

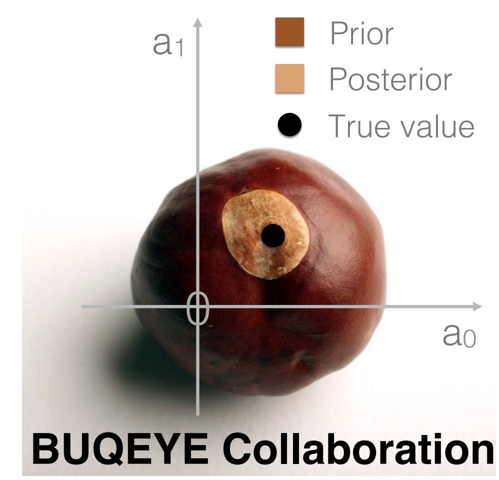
WashU group is using MCMC to optimize determination of LEC and provide UQ for the Delta-full chiral potentials used in QMC calculations

Posterior for c_D and c_E



Correlation of ρ ≈ 0.96 for this χEFT Hamiltonian





https://buqeye.github.io/

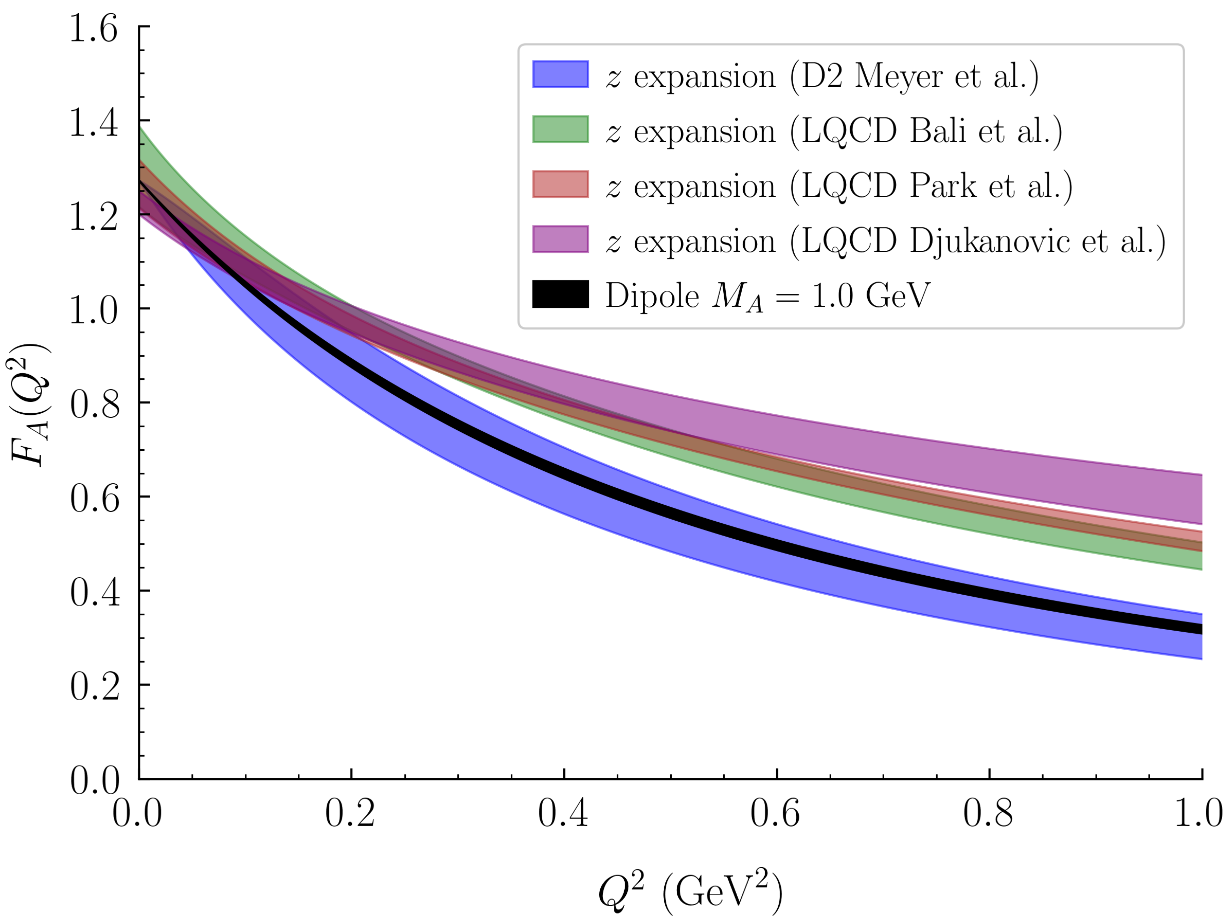
S. Wesolowski, et al, PRC 104, 064001 (2021)

Elementary Input: Form Factors

- Axial one-body contribution: $$J_A^\mu = -\gamma^\mu \gamma_5 \mathcal{F}_A - q^\mu \gamma_5 \frac{\mathcal{F}_P}{M}$$

Standard parametrization of the axial form factor:

Dipole: $$F_A(Q^2) = \frac{g_A}{(1+Q^2/M_A^2)^2},$$ with $$g_A = 1.2723(23)$$ $$M_A = 1.014 \pm 0.014 \text{ GeV}$$



Different determinations of nucleon axial form factor using the z-expansion

D2 Meyer et al: fits to neutrino-deuteron scattering data

LQCD result: general agreement between the different calculations

LQCD results are 2-3σ larger than D2 Meyer ones for $Q^2 > 0.3$ GeV^2

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Many-Body method: GFMC

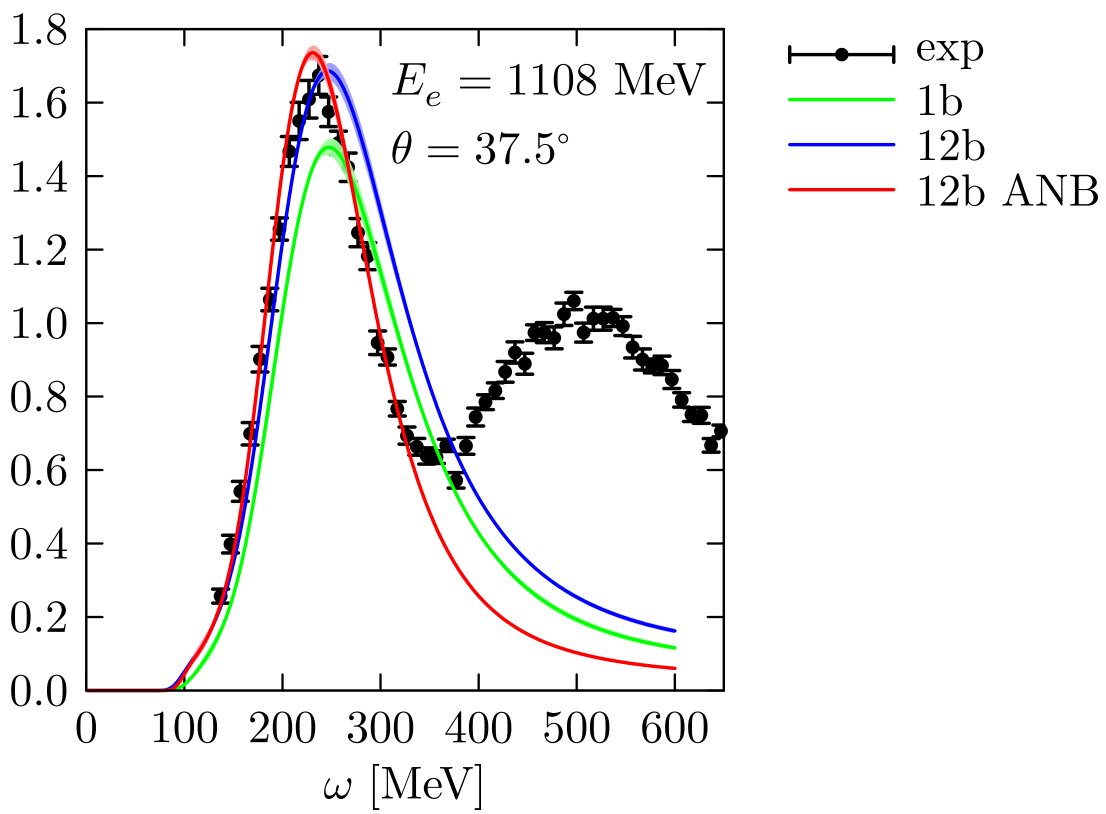
QMC techniques **projects out the exact lowest-energy state:** $e^{-(H-E_0)\tau}|\Psi_T\rangle \rightarrow |\Psi_0\rangle$

Nuclear response function involves evaluating a number of transition amplitudes.
Valuable information can be obtained from the **integral transform of the response function**

$$E_{\alpha\beta}(\sigma,\mathbf{q}) = \int d\omega K(\sigma,\omega)R_{\alpha\beta}(\omega,\mathbf{q}) = \langle\psi_0|J^\dagger_\alpha(\mathbf{q})K(\sigma,H-E_0)J_\beta(\mathbf{q})|\psi_0\rangle$$

Inverting the Laplace transform is a complicated problem

A. Lovato et al, PRL117 (2016), 082501, PRC97 (2018), 022502



Inclusive results which are virtually correct in the QE

Different Hamiltonians can be used in the time-evolution operator

Relies on non-relativistic treatment of the kinematics

Can not handle explicit pion degrees of freedom

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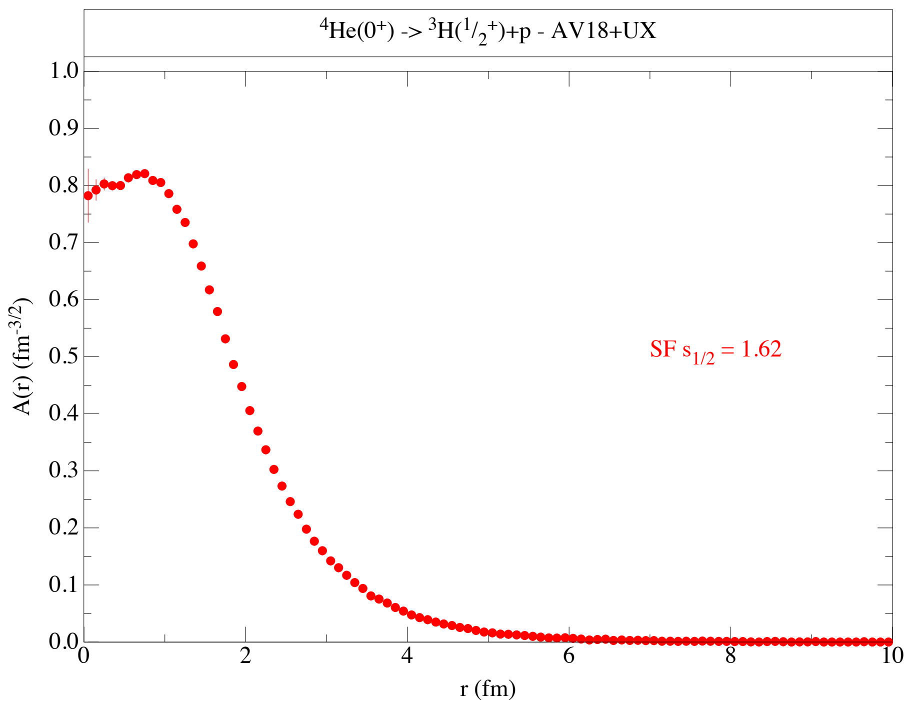
Many-Body method: QMC Spectral Function

- Single-nucleon spectral function:

$$P_{p,n}(\mathbf{k},E)=\sum_n \Big|\langle\Psi_0^A|[|k\rangle\otimes|\Psi_n^{A-1}\rangle]\Big|^2\delta(E+E_0^A-E_n^{A-1})=P^{MF}(\mathbf{k},E)+P^{\rm corr}(\mathbf{k},E)$$

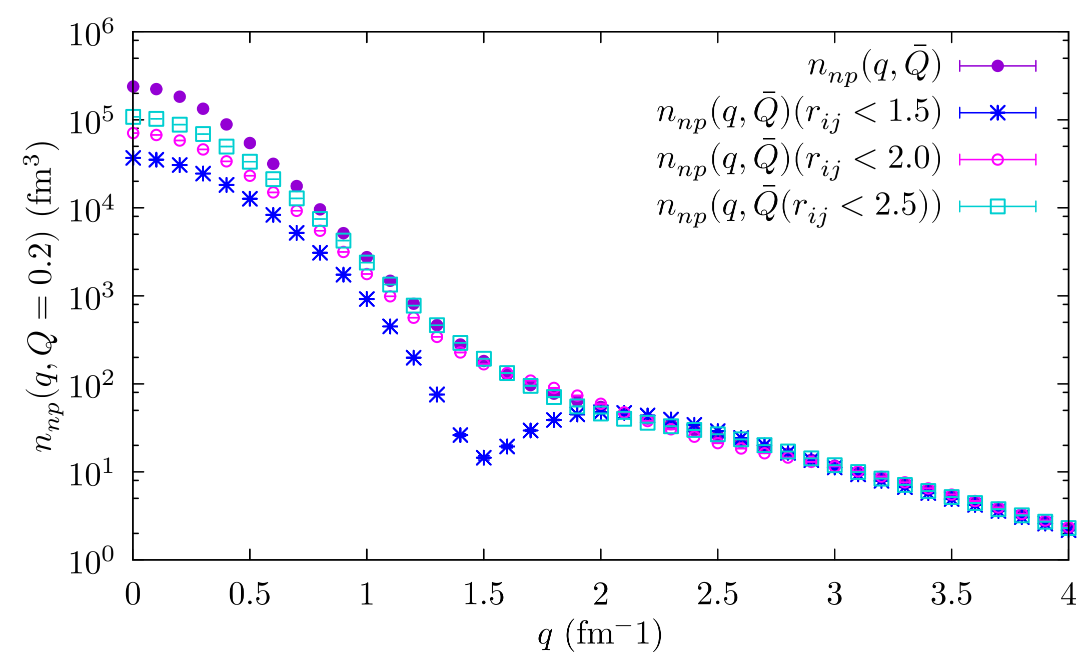


$$P^{MF}(\mathbf{k},E)=\Big|\langle\Psi_0^A|[|k\rangle\otimes|\Psi_n^{A-1}\rangle]\Big|^2$$
$$\times\,\delta\Big(E-B_A+B_{A-1}-\frac{\mathbf{k}^2}{2m_{A-1}}\Big)$$

- The single-nucleon overlap has been computed within VMC (center of mass motion fully accounted for)

$$P^{\rm corr}(\mathbf{k},E)=\int d^3k'\Big|\langle\Psi_0^A|[|k\rangle|k'\rangle\otimes|\Psi_n^{A-2}\rangle]\Big|^2$$
$$\times\,\delta\Big(E-B_A-e(\mathbf{k}')+B_{A-2}-\frac{(\mathbf{k}+\mathbf{k}')^2}{2m_{A-2}}\Big)$$

- Written in terms of two-body momentum distribution



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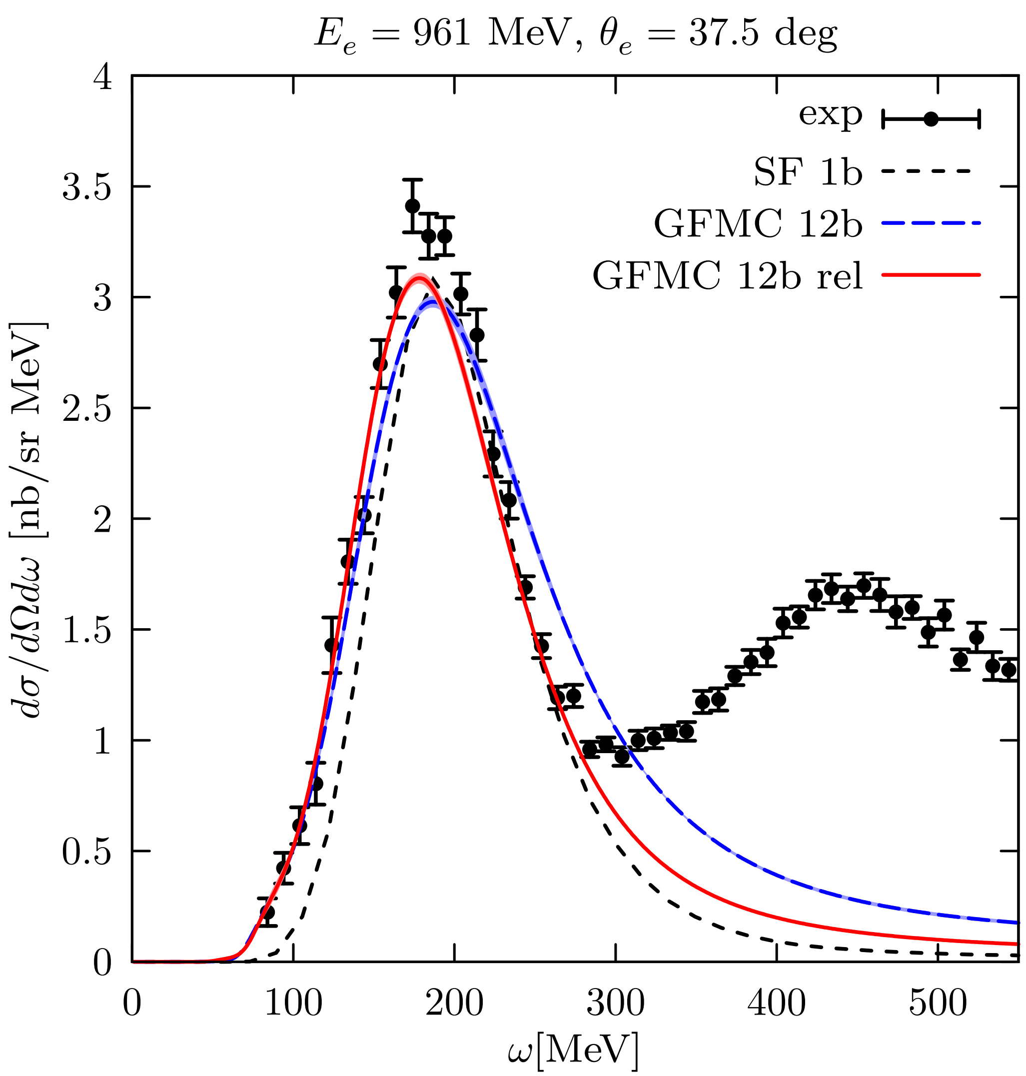
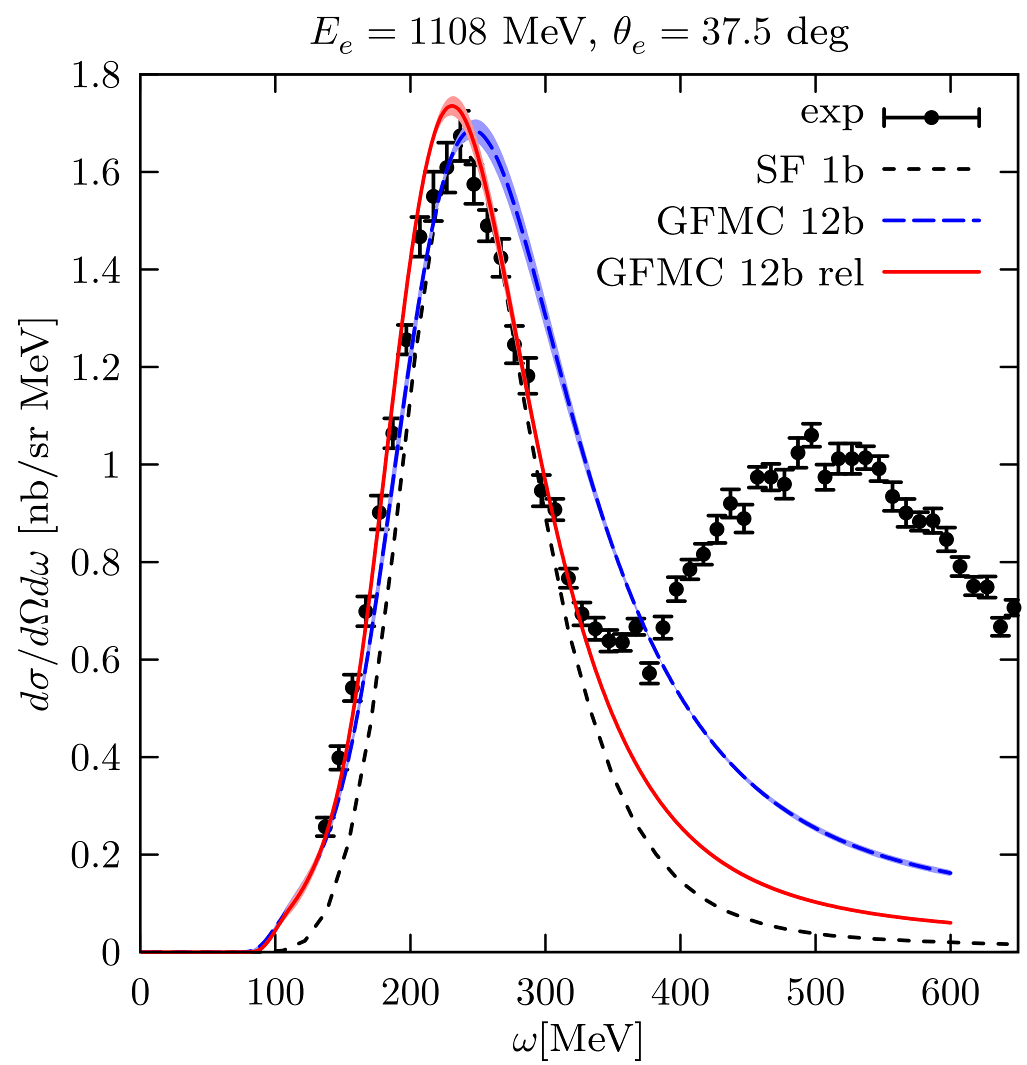
Comparing different many-body methods

- <u>e -⁴He:</u> inclusive cross section





- Comparisons among GFMC, and SF approach: first step to precisely **quantify the uncertainties i**nherent to the factorization of the final state.

- Gauge the role of **relativistic effects** in the energy region relevant for neutrino experiments.

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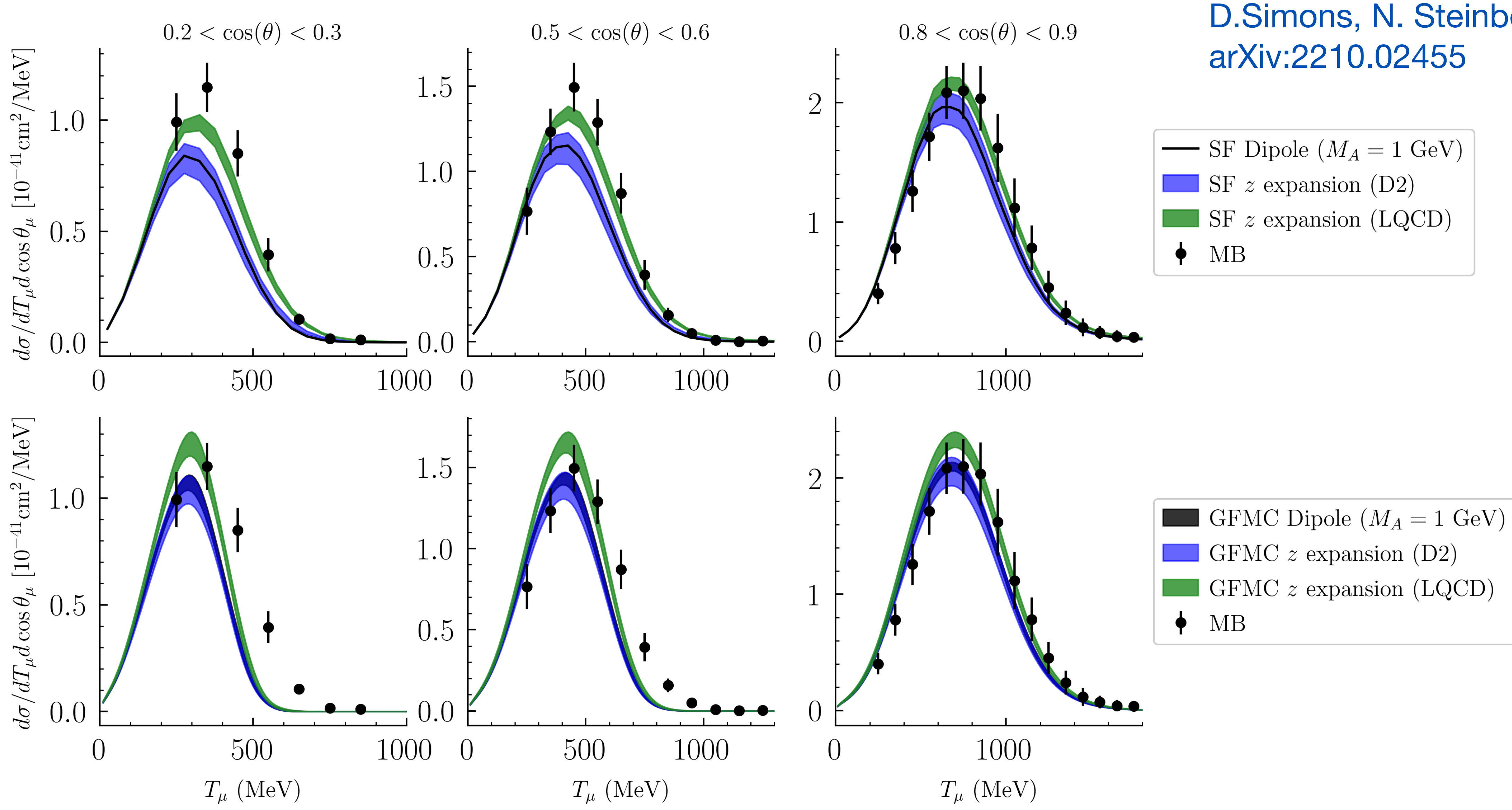
Study of model dependence in neutrino predictions

MiniBooNE results; breakdown into one- and two-body contributions for the SF and GFMC

D.Simons, N. Steinberg et al
arXiv:2210.02455



MiniBooNE	$0.2 < \cos\theta_\mu < 0.3$	$0.5 < \cos\theta_\mu < 0.6$	$0.8 < \cos\theta_\mu < 0.9$
SF Difference in $d\sigma_{\text{peak}}$ (%)	16.3	17.1	9.3
GFMC Difference in $d\sigma_{\text{peak}}$ (%)	18.6	17.1	12.2

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Study of model dependence in neutrino predictions

T2K results; breakdown into one- and two-body contributions for the SF and GFMC

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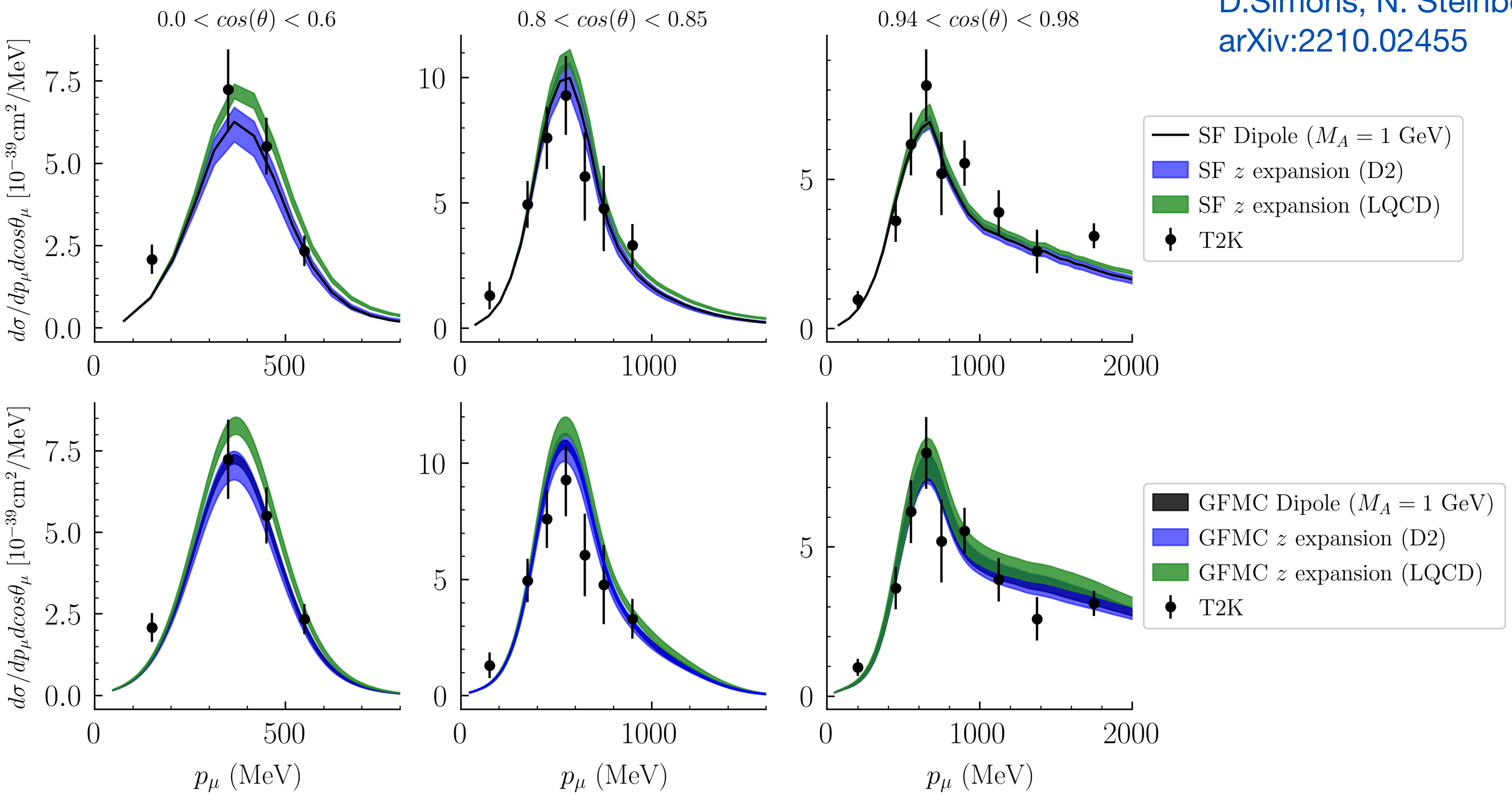

T2K	$0.0 < \cos\theta_\mu < 0.6$	$0.80 < \cos\theta_\mu < 0.85$	$0.94 < \cos\theta_\mu < 0.98$
SF difference in $d\sigma_{\mathrm{peak}}$ (%)	15.3	8.2	3.3
GFMC difference in $d\sigma_{\mathrm{peak}}$ (%)	15.8	8.0	4.6

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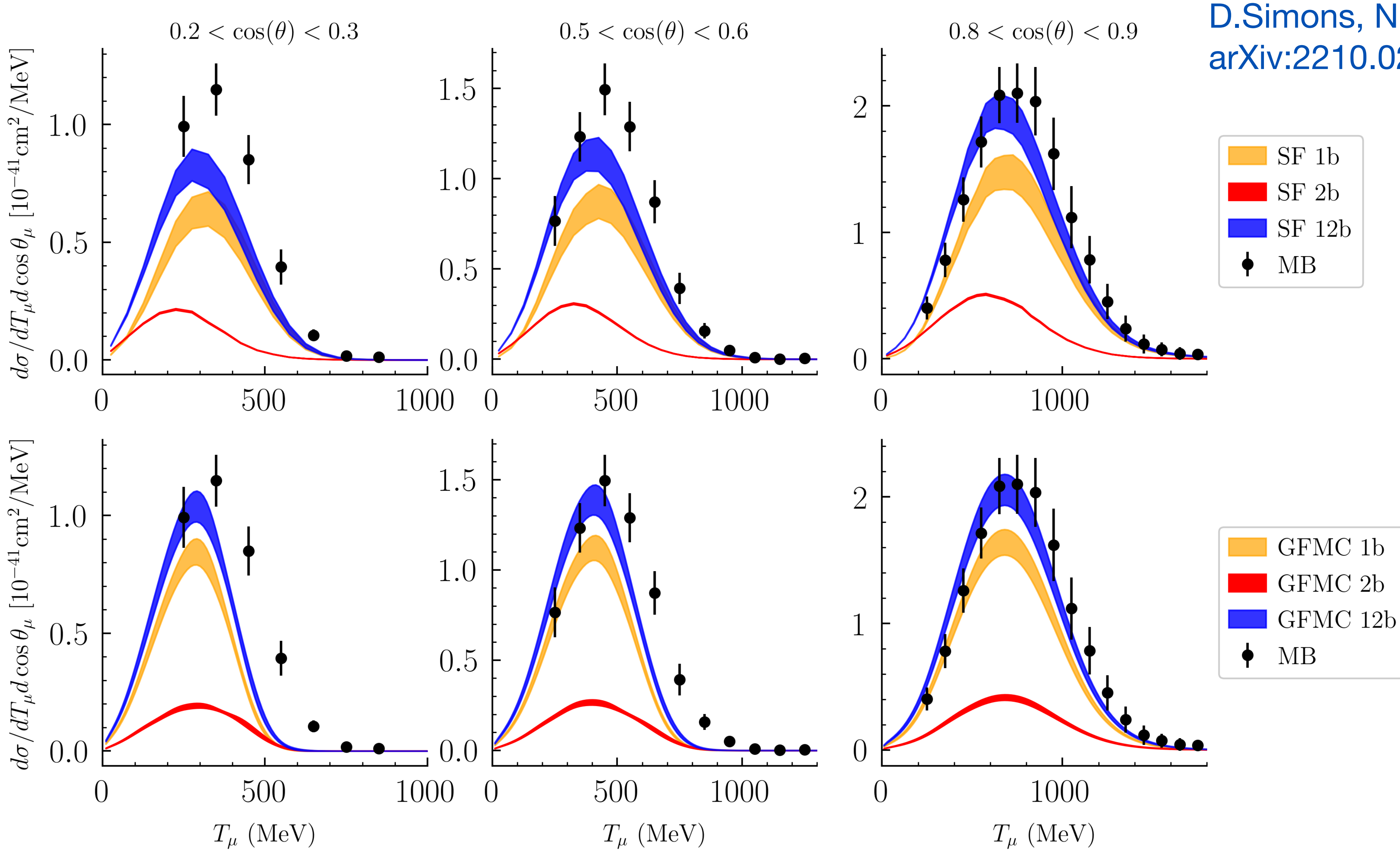

MiniBooNE	$0.2 < \cos\theta_\mu < 0.3$	$0.5 < \cos\theta_\mu < 0.6$	$0.8 < \cos\theta_\mu < 0.9$
GFMC/SF difference in $d\sigma_{\text{peak}}$ (%)	22.8	20.3	5.6

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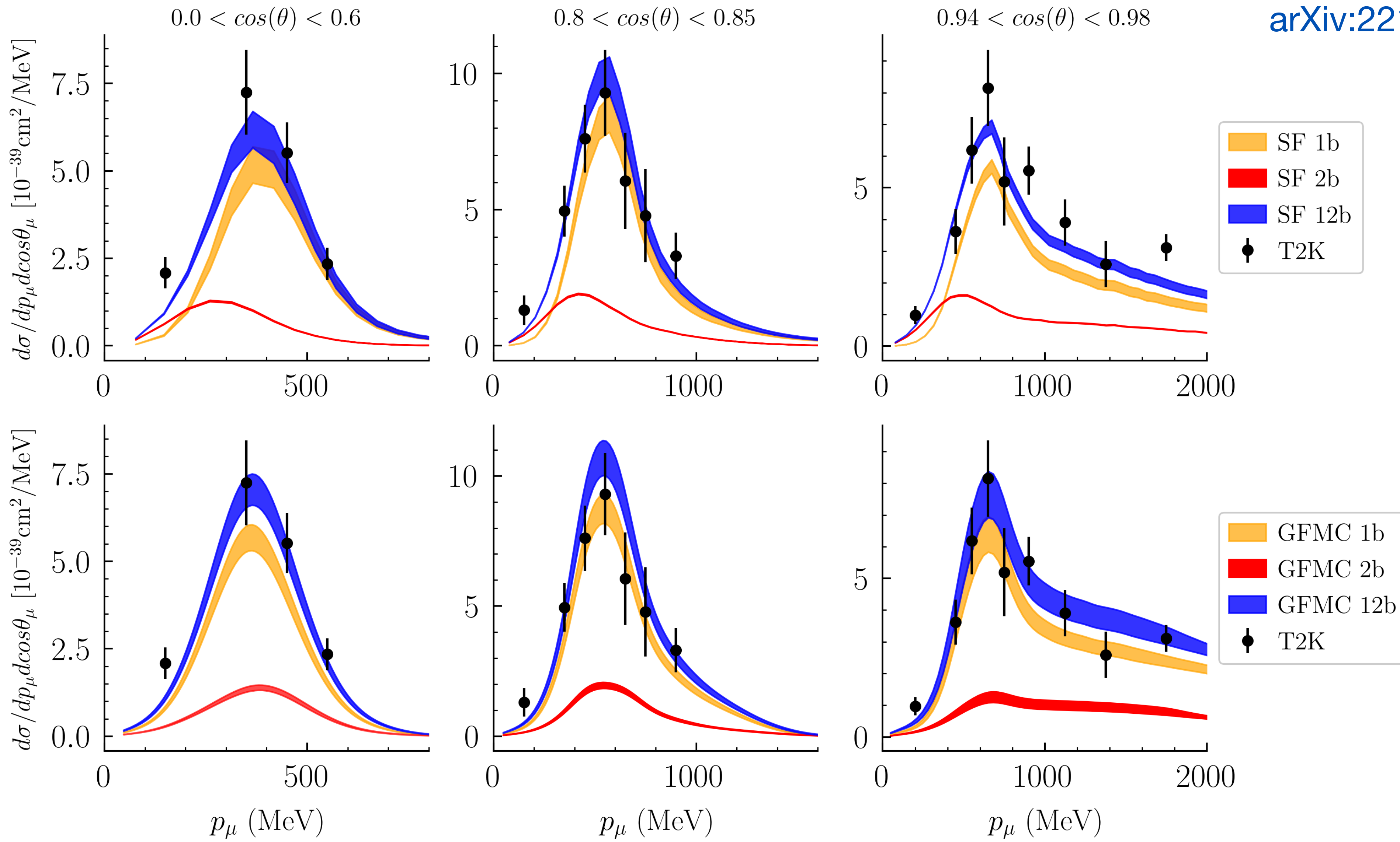

T2K	$0.0 < \cos\theta_\mu < 0.6$	$0.80 < \cos\theta_\mu < 0.85$	$0.94 < \cos\theta_\mu < 0.98$
GFMC/SF difference in $d\sigma_{\text{peak}}$ (%)	13.4	7.3	10.0

Conclusions

Different sources of uncertainties can be considered:

Nuclear Hamiltonians: different efforts in place to provide UQ in chiral EFT

Form factors: one- and two-body currents.

Error of factorizing the hard interaction vertex / using a non relativistic approach

These errors need to be consistently propagated / combined through the intra-nuclear cascade

At the level of event generators: reweighting procedures only allow one to propagate a subset of model uncertainties. We want to simulate the entire process using different inputs without resorting to event-reweighting techniques. This requires highly optimized codes

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