Nuclear Medium Effects in Neutrino-Nucleus Deep Inelastic Scattering

H. Haider

Aligarh Muslim University

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Generator Workshop

General process for the deep inelastic scattering is

$$l(k) + N(p) \longrightarrow l'(k') + X(p'), \quad l, l' = e^{\pm}, \mu^{\pm}, \nu_l, \bar{\nu}_l, \ N = n, p$$

- $Q^2 = -q^2 = -(k - k')^2 = 4EE' \sin^2 \frac{\theta}{2}$
- $M_N^2 = p^2$
- $\nu = p.q = M_N(E - E')$
- $x = \frac{Q^2}{2M_N\nu} = \frac{Q^2}{2p.q} = \frac{Q^2}{2M_N E y}$
- $y = \frac{p.q}{p.k} = 1 - \frac{E'}{E}$
- $W^2 = M_N^2 + 2p.q - Q^2$

Formalism: $\nu_l/\bar{\nu}_l − N$ scattering

The basic reaction for the (anti)neutrino induced charged current deep inelastic scattering process on a free nucleon target is given by

$$\nu_l(k)/\bar{\nu}_l(k) + N(p) \quad \rightarrow \quad l^-(k')/l^+(k') + X(p'), \qquad (l = e, \nu, \tau)$$

The general expression for the double differential scattering cross section (DCX):

$$\frac{d^2\sigma}{dxdy} = \frac{yM_N}{\pi} \frac{E}{E'} \frac{|\mathbf{k}'|}{|\mathbf{k}|} \frac{G_F^2}{2} \left(\frac{M_W^2}{Q^2 + M_W^2} \right)^2 L_{\mu\nu} W_N^{\mu\nu} ,$$

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Leptonic tensor: $\qquad\qquad L_{\mu\nu} = 8(k_\mu k'_\nu + k_\nu k'_\mu - k.k' g_{\mu\nu} \pm i\varepsilon_{\mu\nu\rho\sigma}k^\rho k'^\sigma)$

Hadronic tensor:

$$\begin{aligned} W_N^{\mu\nu} &= -g^{\mu\nu}\, W_{1N}(\nu, Q^2) + W_{2N}(\nu, Q^2)\frac{p^\mu p^\nu}{M_N^2} - \frac{i}{M_N^2}\,\varepsilon^{\mu\nu\rho\sigma}\,p_\rho q_\sigma W_{3N}(\nu, Q^2) + \frac{W_{4N}(\nu, Q^2)}{M_N^2}\,q^\mu q^\nu \\ &\quad + \frac{W_{5N}(\nu, Q^2)}{M_N^2}\,(p^\mu q^\nu + q^\mu p^\nu) + \frac{i}{M_N^2}\,(p^\mu q^\nu - q^\mu p^\nu)W_{6N}(\nu, Q^2)\,. \end{aligned}$$

$W_{iN}(\nu, Q^2)$ $(i = 1 - 6)$ are the weak nucleon structure functions

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$$+ \frac{W_{5N}(\nu,Q^2)}{M_N^2} (p^\mu q^\nu + q^\mu p^\nu) + \frac{i}{M_N^2} (p^\mu q^\nu - q^\mu p^\nu) W_{6N}(\nu,Q^2).$$

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In the limit of $Q^2 \rightarrow \infty$, $\nu \rightarrow \infty$, $x \rightarrow$ finite and $W_{iN}(\nu,Q^2)$ $(i = 1-5)$ are written in terms of the dimensionless nucleon structure functions as:

$$F_{1N}(x) = W_{1N}(\nu,Q^2) \quad F_{2N}(x) = \frac{Q^2}{2xM_N^2} W_{2N}(\nu,Q^2) \quad F_{3N}(x) = \frac{Q^2}{xM_N^2} W_{3N}(\nu,Q^2)$$

$$F_{4N}(x) = \frac{Q^2}{2M_N^2} W_{4N}(\nu,Q^2) \quad F_{5N}(x) = \frac{Q^2}{2xM_N^2} W_{5N}(\nu,Q^2)$$

Formalism: $\nu_l/\bar{\nu}_l - N$ scattering

The differential scattering cross section is given by

$$
\frac{d^2\sigma}{dxdy} = \frac{G_F^2 M_N E_\nu}{\pi(1+\frac{Q^2}{M_W^2})^2} \left\{ \left[y^2 x + \frac{m_l^2 y}{2E_\nu M_N} \right] F_{1N}(x,Q^2) + \left[\left(1 - \frac{m_l^2}{4E_\nu^2}\right) - \left(1 + \frac{M_N x}{2E_\nu}\right)y \right] F_{2N}(x,Q^2) \right.
$$

$$
\pm \left[xy\left(1 - \frac{y}{2}\right) - \frac{m_l^2 y}{4E_\nu M_N} \right] F_{3N}(x,Q^2) + \frac{m_l^2(m_l^2 + Q^2)}{4E_\nu^2 M_N^2 x} F_{4N}(x,Q^2) - \frac{m_l^2}{E_\nu M_N} F_{5N}(x,Q^2) \left. \right\}.
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For $\nu(\bar{\nu})$-proton scattering	
$F_{2p}^\nu(x)$	$= 2x[d(x)+s(x)+\bar{u}(x)+\bar{c}(x)]$
$F_{2p}^{\bar{\nu}}(x)$	$= 2x[u(x)+c(x)+\bar{d}(x)+\bar{s}(x)]$
$xF_{3p}^\nu(x)$	$= 2x[d(x)+s(x)-\bar{u}(x)-\bar{c}(x)]$
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For $\nu(\bar{\nu})$-neutron scattering	
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$xF_{3n}^{\bar{\nu}}(x)$	$= 2x[d(x)+c(x)-\bar{u}(x)-\bar{s}(x)].$

At the leading order

Callan-Gross relation:

$$F_2(x)=2xF_1(x)$$

Albright-Jarlskog relations:

$$F_4(x) = 0 \qquad F_2(x) = 2xF_5(x)$$

In this work MMHT PDFs parameterization (Harland-Lang *et al.*, Eur. Phys. J. C **75**, no. 5, 204 (2015)) has been used.

Charm quark is considered to be a massive object and in four flavor scheme we consider:

$$F_{iN}(x,Q^2) = F_{iN}^{n_f=4}(x,Q^2) = \underbrace{F_{iN}^{n_f=3}(x,Q^2)}_{\text{for massless}(u,d,s)\text{ quarks}} + \underbrace{F_{iN}^{n_f=1}(x,Q^2)}_{\text{for massive charm quark}}$$

Details in ref. Ansari *et al.*, Phys. Rev. D **102**, 113007 (2020)

Perturbation and nonperturbative effects at nucleon level

In the kinematic region of low and moderate Q^2, both the higher order perturbative and the nonperturbative ($\propto \frac{1}{Q^2}$) QCD effects come into play.

- Perturbative effects like the QCD corrections at the next-to-next-to-leading order (NNLO) in the strong coupling constant α_s.

- In the present work we have evaluated the structure functions at NLO, following the works of Kretzer and Reno (Phys. Rev. D **66**, 113007 (2002); ibid **69**, 034002 (2004)) and Jeong and Reno (Phys. Rev. D **82**, 033010 (2010).)

- The nonperturbative effect like:
  - The target mass correction (TMC) is associated with the finite mass of the target nucleon, and is incorporated using Ref. Kretzer and Reno, Phys. Rev.D **69**, 034002 (2004).
  - HT originates due to the interactions of struck quarks with the other quarks via gluon exchange and is incorporated following the works of Dasgupta *et al.*, Phys. Lett. B **382**, 273 (1996).

- The effects of tau lepton mass, charm quark mass and threshold effect have been taken into account

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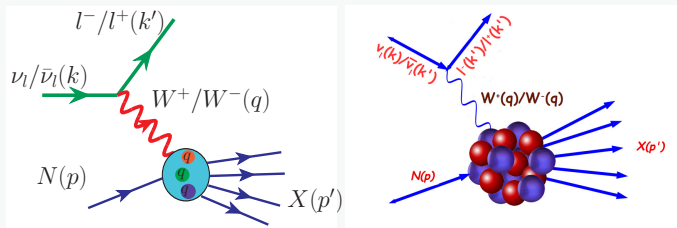
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Free nucleon structure functions: $F_{iN}^{WI}(x, Q^2)$ vs x (i = 1 − 5)
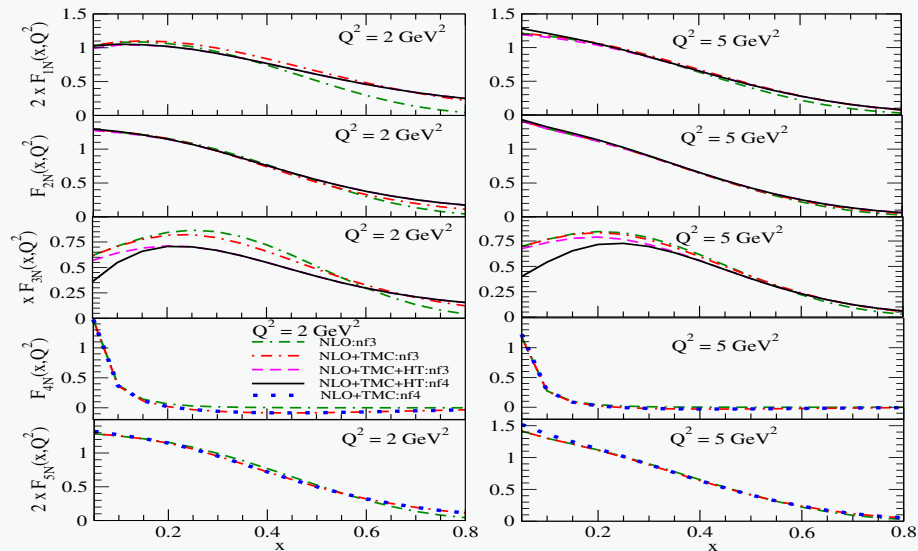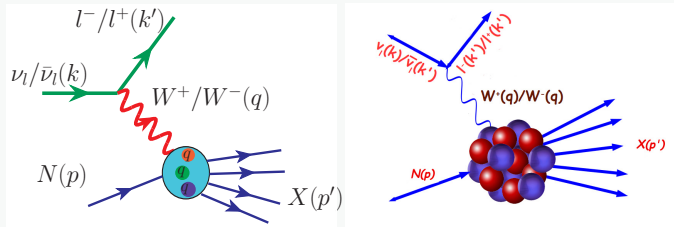

Ansari *et al.*, Phys. Rev. D **102**, 113007 (2020)
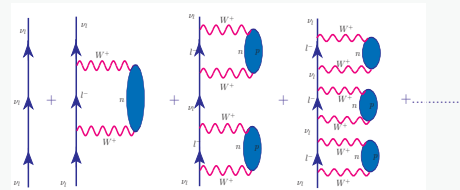
$\nu_l/\bar{\nu}_l - A$ scattering

- Fermi motion, binding energy and nucleon correlations through spectral function and is calculated using Lehmann's representation for the relativistic nucleon propagator.

- Nuclear many body theory is used to calculate it for an interacting Fermi sea in nuclear matter. A local density approximation is then applied to translate these results to finite nuclei.

- There are virtual mesons associated with the nucleon bound inside the nucleus. These meson clouds get strengthened by the strong attractive nature of nucleon-nucleon interactions.

- This leads to an increase in the interaction probability of virtual mediating quanta with the meson cloud. The effect of meson cloud is more pronounced in heavier nuclear targets and dominate in the intermediate region of $x(0.2<x<0.6)$.

- The shadowing suppression at small x occurs due to coherent multiple scattering of quark-anti quark pair coming from the virtual boson with destructive interference of the amplitudes and is incorporated following the works of Kulagin and Petti. Phys. Rev. D **76**, 094033(2007).

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In the nuclear matter the dressed nucleon propagator is written as:

$$G(p) = \frac{M_N}{E(\mathbf{p})} \sum_r u_r(\mathbf{p}) \bar{u}_r(\mathbf{p}) \left[\int_{-\infty}^{\mu} d\omega \frac{S_h(\omega, \mathbf{p})}{p^0 - \omega - i\eta} + \int_{\mu}^{\infty} d\omega \frac{S_p(\omega, \mathbf{p})}{p^0 - \omega + i\eta} \right],$$

The spectral function $F_{iA,N}(x_A, Q^2)$ $(i = 1 - 5))$ are obtained as:

$$F_{iA,N}(x_A, Q^2) = 4 \int d^3r \int \frac{d^3p}{(2\pi)^3} \frac{M_N}{E_N(\mathbf{p})} \int_{-\infty}^{\mu} dp^0 \, S_h(p^0, \mathbf{p}, \rho(r)) \times f_{iN}(x, Q^2),$$

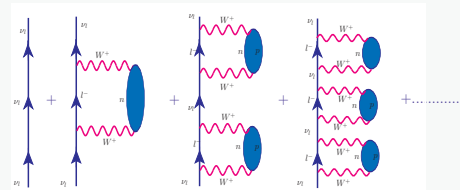In our model, the nuclear structure functions is given by:

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Zaidi *et al.* Phys. Rev. D **101** (2020), 033001, Phys. Rev. D **99** (2019), 093011

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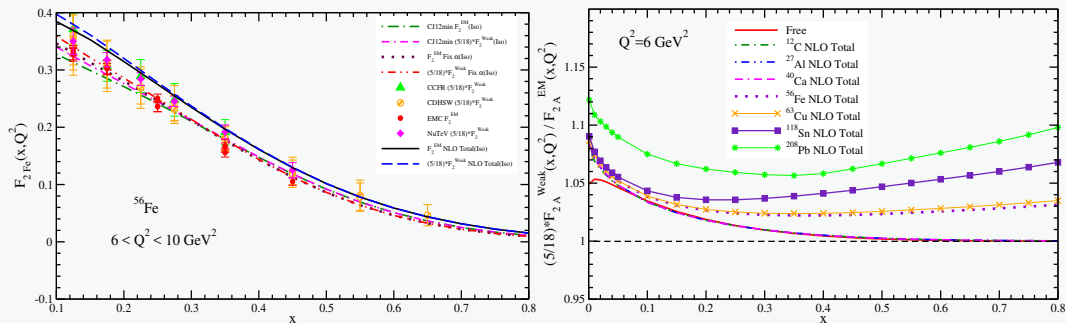
EM Nuclear Structure Functions
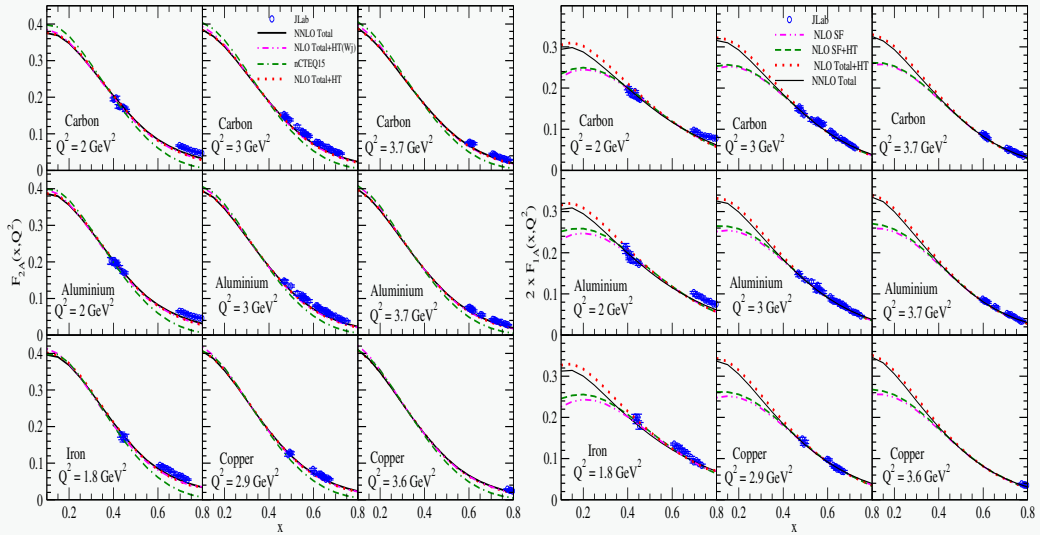
Zaidi *et al.*, Phys. Rev. D **99**, 093011 (2019).

Nuclear structure functions: $2xF_{1A}(x, Q^2)$, $F_{2A}(x, Q^2)$ and $xF_{3A}(x, Q^2)$ for ^{40}Ar
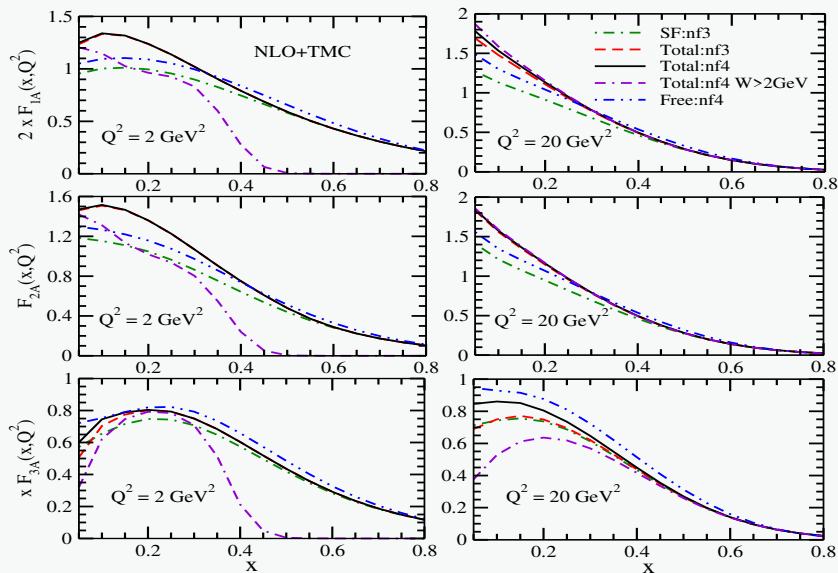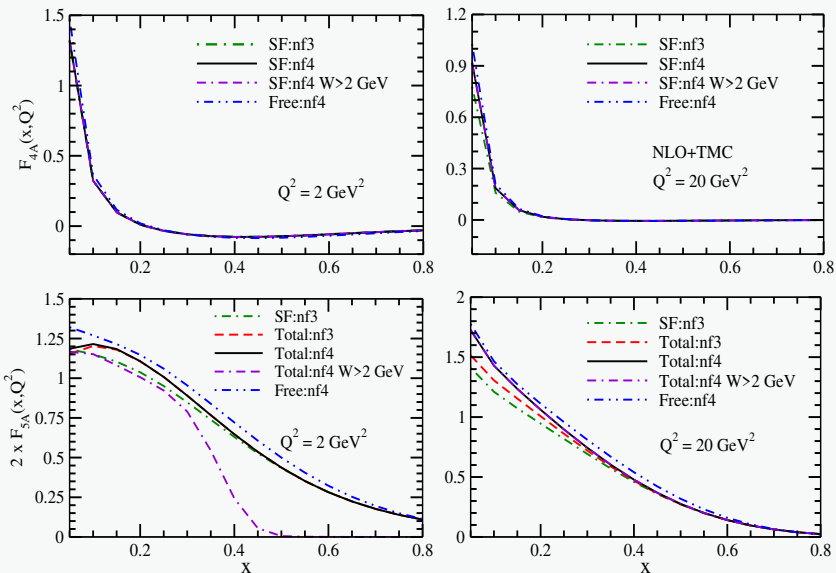
Main take away for Nuclear Medium Effects, Left: are different in EM and Weak interactions. Right: show A dependence
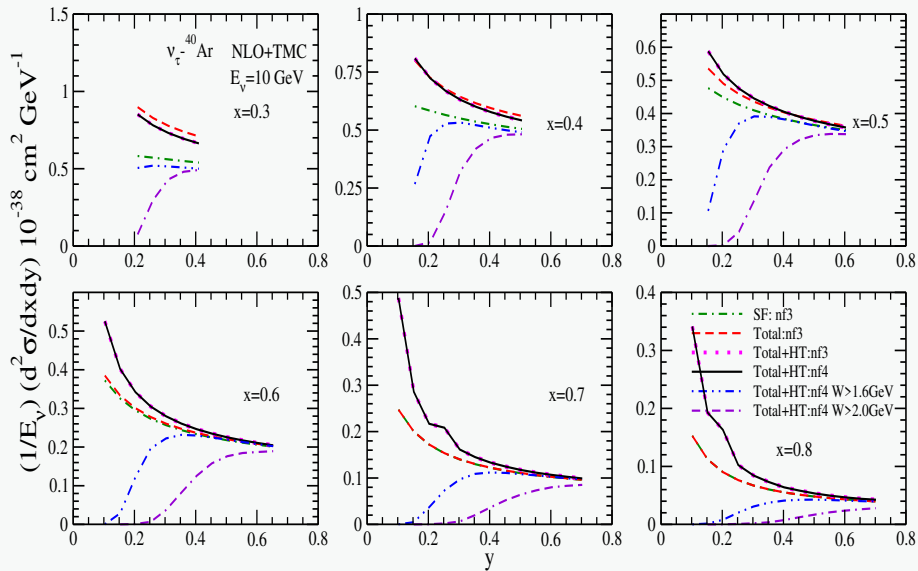

Haider *et al.*, Nuc Phys A, 955, 2016, 58

Nuclear structure functions: $F_{4A}(x, Q^2)$ and $2xF_{5A}(x, Q^2)$ for ^{40}Ar

$\frac{1}{E_\nu}\frac{d^2\sigma}{dxdy}$ *vs y at* $E_\nu = 10 GeV$

Conclusions

- Perturbative and nonperturbative effects are quite important in the evaluation of nucleon structure functions as well as the differential cross section. These effects are important in the different regions of x and Q^2.

- We have studied nuclear medium effects in electromagnetic and weak nuclear structure functions.

- The theoretical results presented here show that the difference between $F_2^{EM}(x, Q^2)$ and $F_2^{Weak}(x, Q^2)$ is quite small at large $x (x > 0.3)$. Nuclear Medium Effects are A dependent.

- The additional structure functions $F_4(x, Q^2)$ and $F_5(x, Q^2)$ leads to a significant reduction in the $v_\tau / \bar{v}_\tau - N(A)$ cross section as compared to muon and electron neutrino cross sections.

- These theoretical results would be helpful for MINERvA, MicroBooNE, NOvA, and upcomig DUNE experiment.

- These results would also be helpful in atmospheric neutrino analysis and other experiments which are planning to observe the v_τ events.

*GENIE DIS model Development update*

- Validation of Code for Free Nucleon Case(Fortran to C++ ready).
- Under Validation of Code for Nucleus Case(Fortran to C++ ready)
- Steve already placed the AMUValDIS model in my GENIE area. Need to validate with the code which is outside the GENIE
- Remaining validations and modifications will be done in collaboraton with New Mexico State University: Professor Stephen Pate-Morales and his Research group
- Timeline:May be 4Months

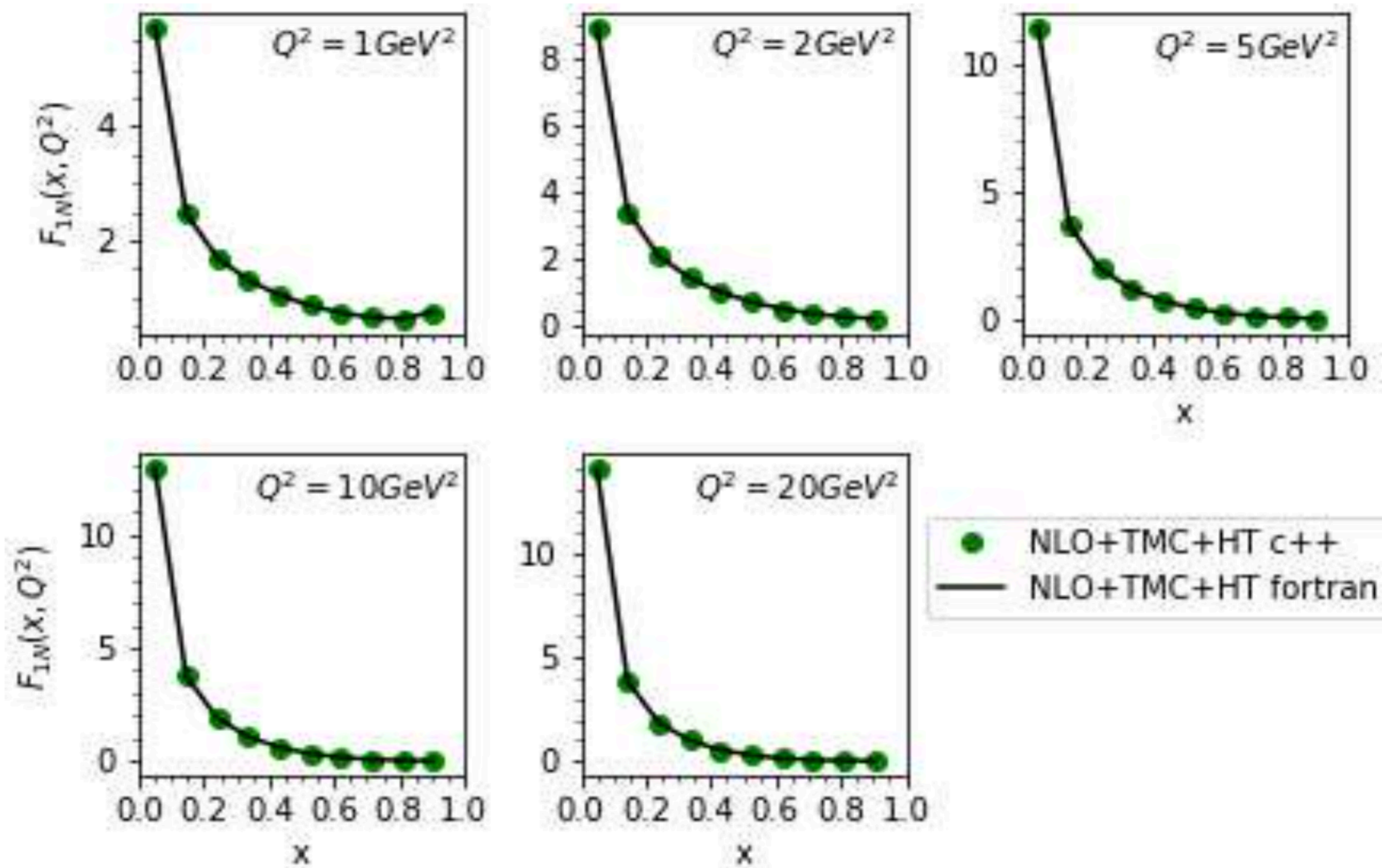$F_1N(x, Q^2)$, $F_{2N}(x, Q^2)$ and $F_{3N}(x, Q^2)$ comparisions in C++ vs Fortran

Validation done by Vaniya Ansari(AMU) and Deeksha (Ph.D student of Mary Hall Reno Iowa University)

$Ratio = F_{2N}^{Fortran}(x, Q^2)/F_{2N}^{c++}(x, Q^2)$ **at LO**

NLO+TMC+HT Fortran
NLO+TMC+HT c++

$$Ratio = F_{2N}^{Fortran}(x, Q^2)/F_{2N}^{c++}(x, Q^2) \text{ at NLO+TMC+HT}$$
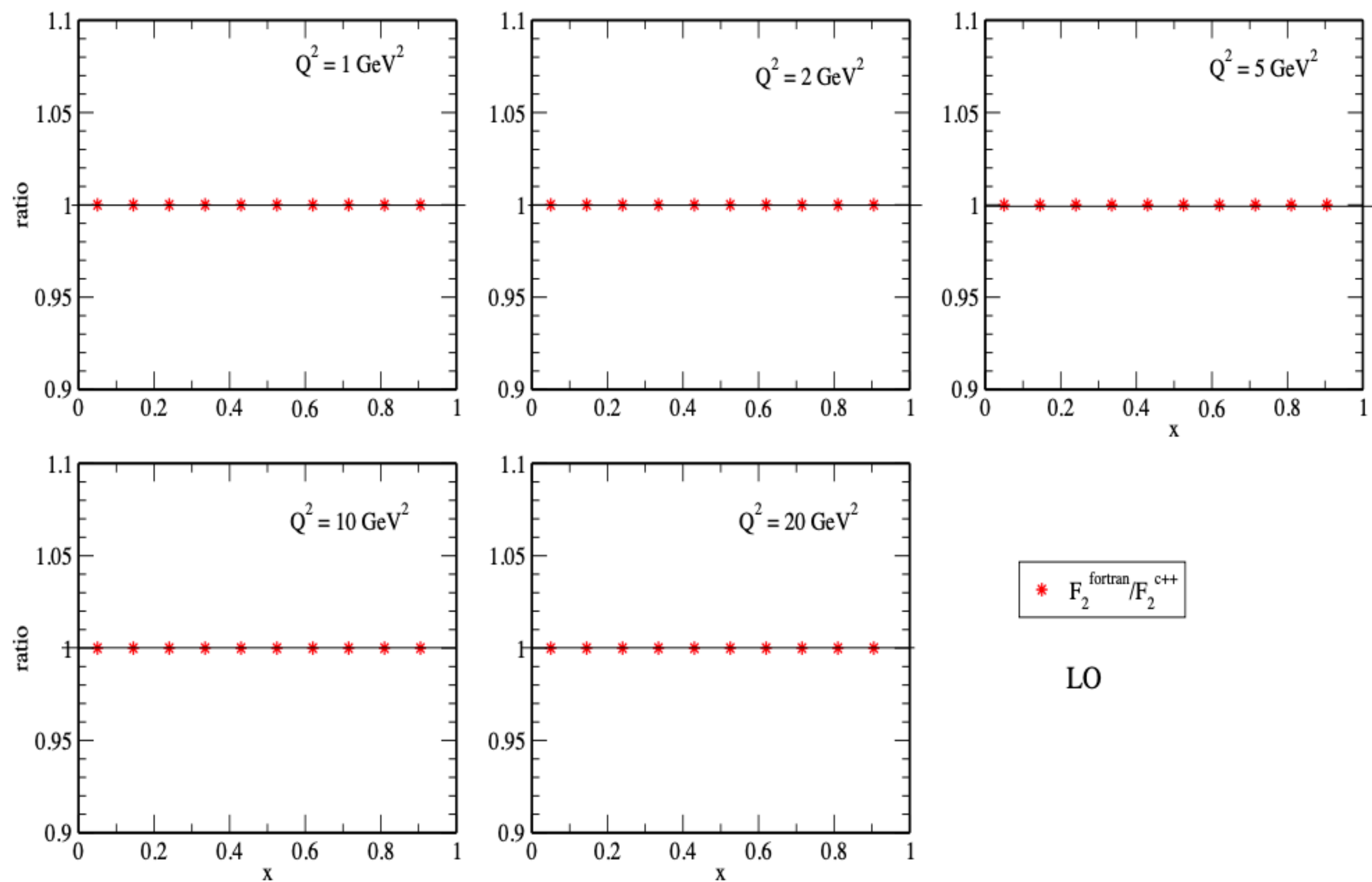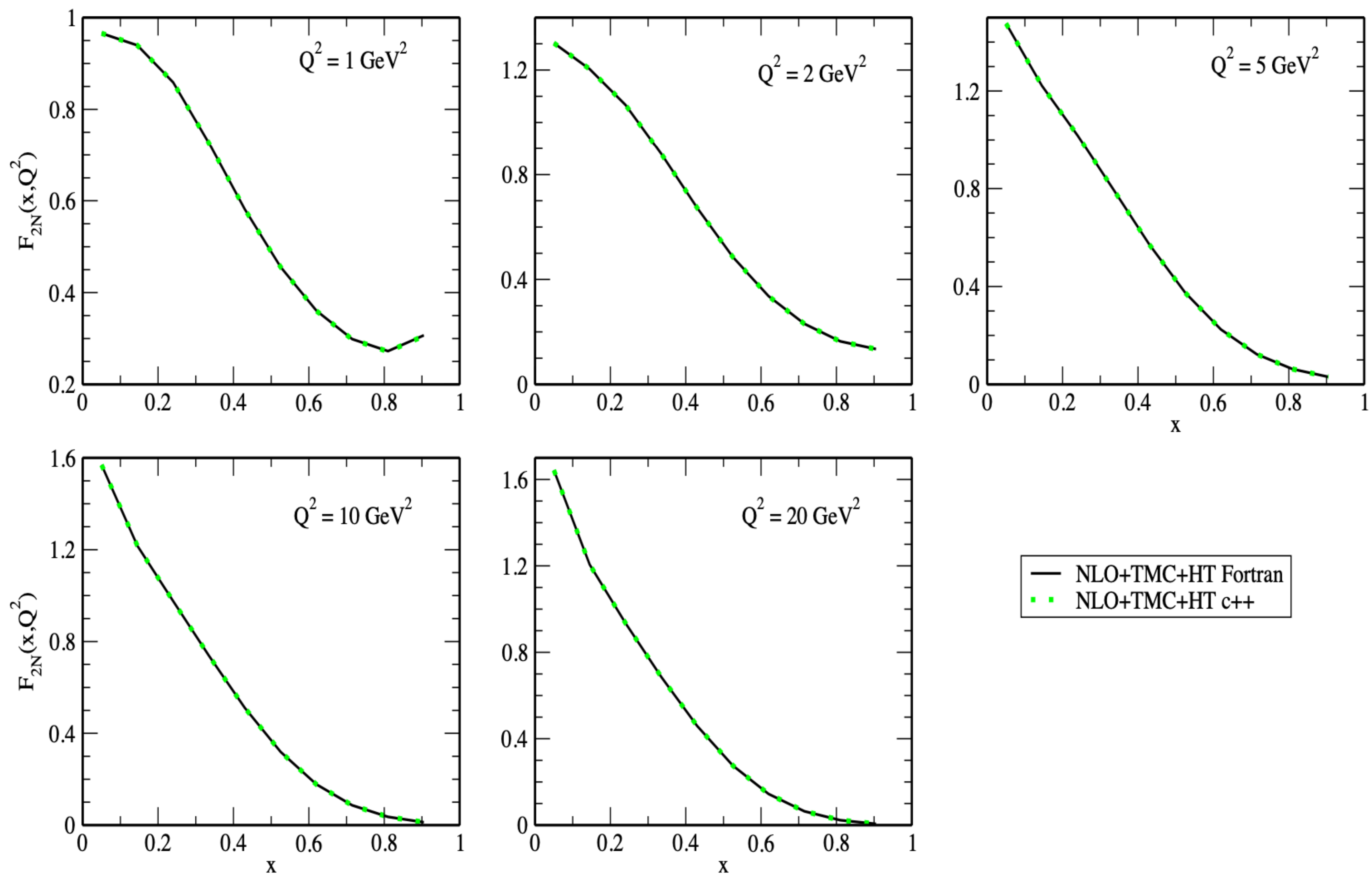
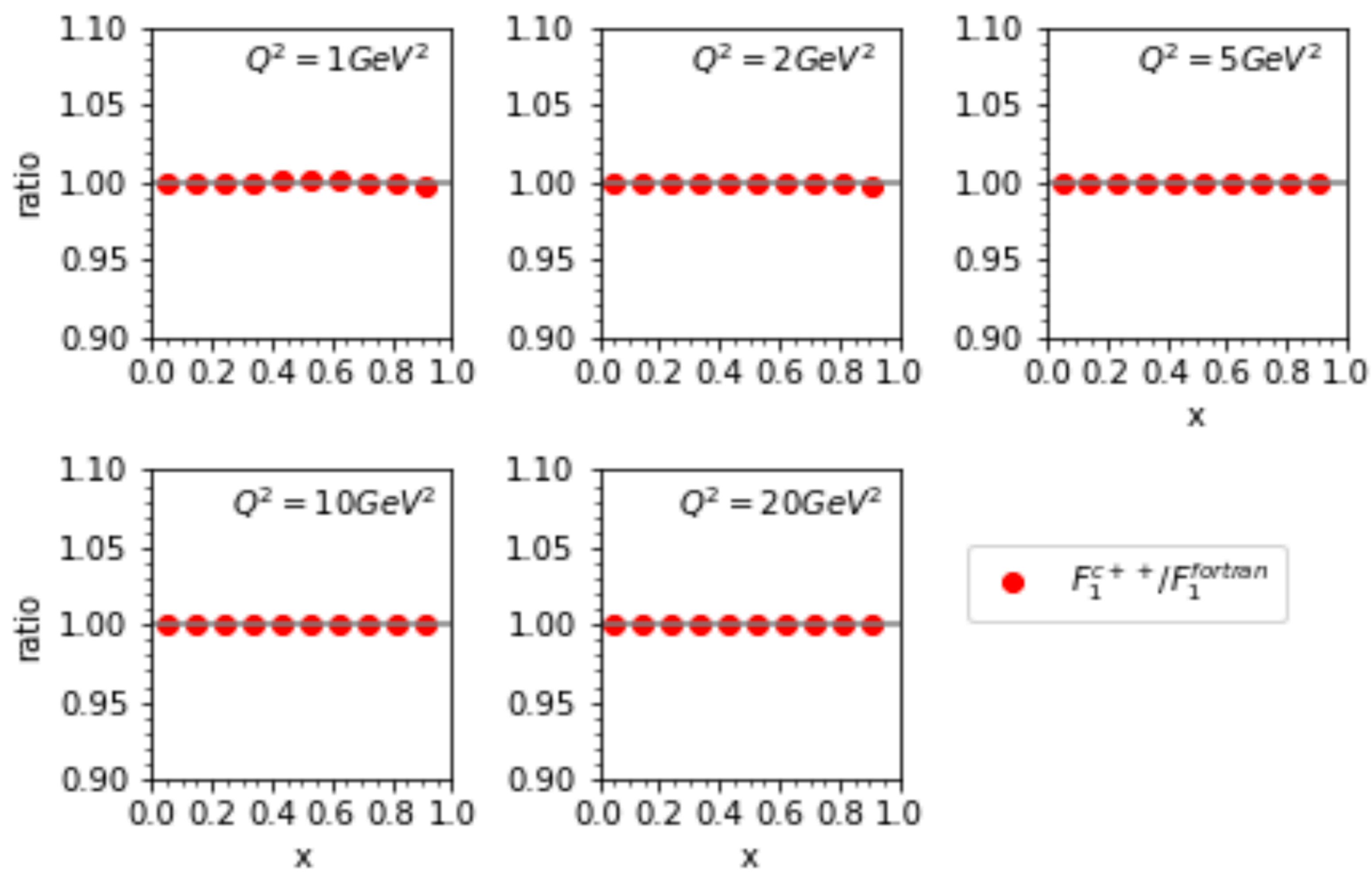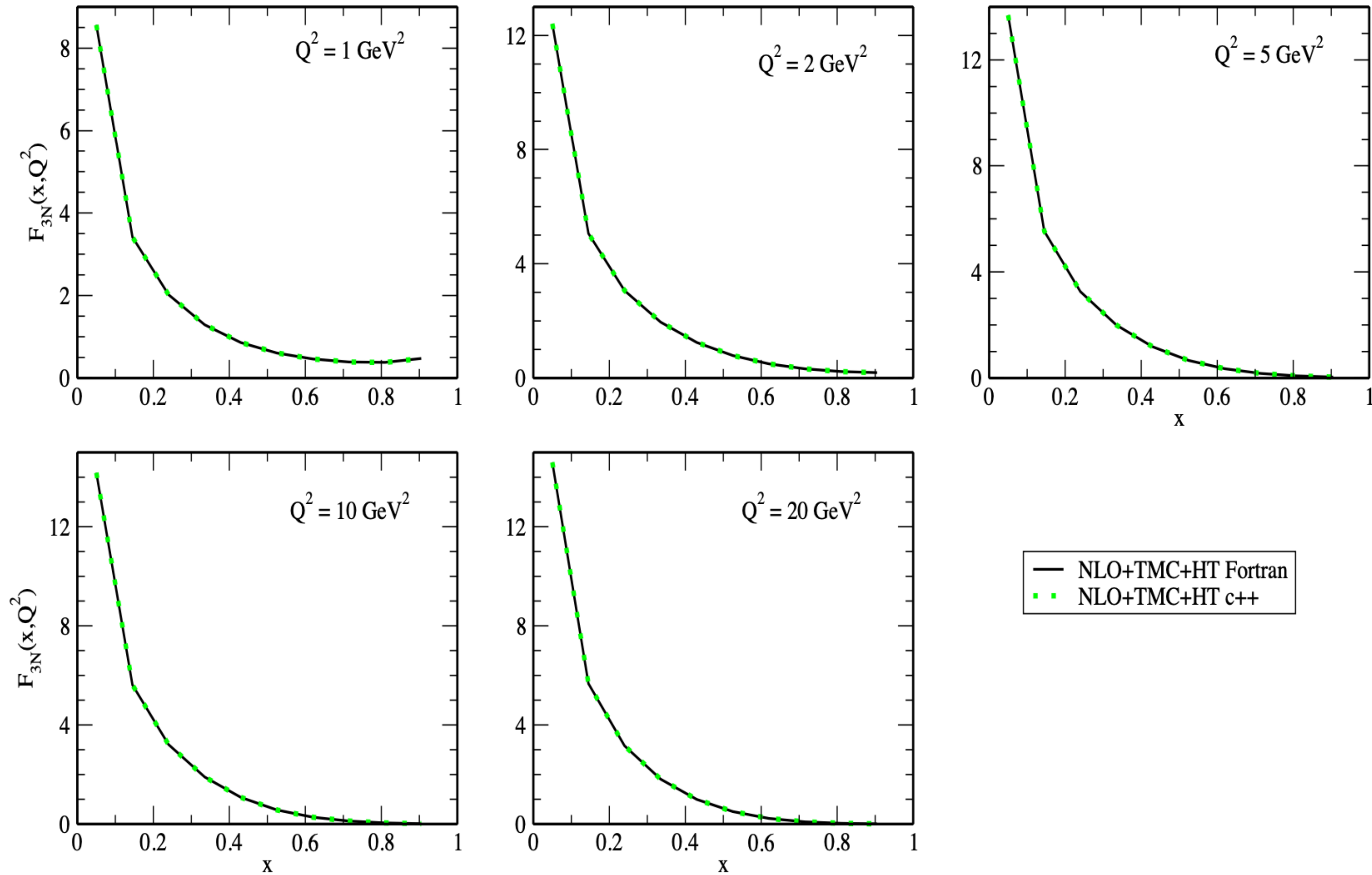# $F_{1N}(x, Q^2)$ at NLO+TMC+HT

Ratio $= F_{1N}^{c++}(x, Q^2)/F_{1N}^{Fortran}(x, Q^2)$ at NLO+TMC+HT

$F_{3N}(x, Q^2)$ at NLO+TMC+HT

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