Black holes and quantum information

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Carlos Wagner and Marcela Karena, Festschrift.
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I first heard from Carlos and Marcela from

Wener Schad

Werner Schad

Werner SCHAD (naskiĝis la 29-an de januaro 1926 en Groß-Gerau, Germanio, mortis en marto 2006 en la montaro apud Bariloche) estis germana esperantisto, sed loĝis ĝis sia forpaso en Bariloche, Argentino, en la suda Patagonio. Li estis tre aktiva movade, kaj senĉese instruis la lingvon kaj klopodis organizi loke kaj regione la laborojn por Esperanto, kun kontakto kaj agado eĉ trans la Andoj, en Ĉilio.

Li devis sperti la duan mondmiliton kiel juna soldato. Post sia liberigo el brita militkaptitejo, li ricevis stipendion por studi en universitato. En 1950 li doktoriĝis pri lingvistiko, specialistiĝo romanidaj lingvoj.

Dum li atendis labori kiel instruisto, li faris kurson pri ĵurnalismo, kaj komencis verki por ĵurnaloj kaj revuoj. *Frankfurter Allgemeine Zeitung*, unu el la plej gravaj ĵurnaloj de Eŭropo, dungis lin, kaj pro lia bona kono de la hispana, sendis lin kiel korespondanton al Venezuelo, kie li kontaktiĝis kun esploristoj, kiujn li akompanis dum multaj vojaĝoj tra indiĝenaj teritorioj en Venezuelo, Brazilo, Peruo kaj aliaj.

Werner Schad	
Persona informo	
Naskiĝo	29-an de januaro 1926 en Groß-Gerau
Morto	1-an de januaro 2006 (79-jara) en Bariloĉe
Lingvoj	Esperanto [#]
Ŝtataneco	Argentino • Germanio [#]
Profesio	
Okupo	ĵurnalisto • esperantisto • verkisto [#]
[#]	**Fonto: Vikidatumoj**
v • d • r	

Kiel ĵurnalisto li verkis multnombrajn artikolojn pri vojaĝoj kaj politiko sud-amerikaj por ĵurnaloj kaj revuoj de Germanio, Svislando, Hispanio, Venezuelo kaj Argentino. Multaj artikoloj de li pri politiko en Sud-Ameriko aperis en la jaroj 1960-1962 en *Frankfurter Allgemeine Zeitung*. Ankaŭ la Esperanto-magazino Monato aperigis plurajn artikolojn liajn ekde 1986 ĝis la lasta en 2003 (Werner Schad: ARGENTINO: Tondrulo ... montoge de la Andoj en Monato (2003/02, p. 18)).

Now to the talk…

We will discuss recent progress on the black hole information problem

References

Review: Almheiri, Hartman, JM, Shaghoulian, Tajdini
(this contains a more general list of references than this talk)

Two important papers in 2019: Penington
Almheiri, Engelhardt, Marolf, Maxfield
...many previous and follow up papers...

Another very interesting
Development that I will not review → Saad, Shenker, Stanford

Quantum information

$$I = S = -Tr[\rho \log \rho]$$

Is quantified by the Von Neuman entropy.

$\rho =$ density matrix of the quantum system.

Quantum information

$$I = S = -Tr[\rho \log \rho]$$

Does not change under unitary time evolution.

A tale of two entropies

Two notions of entropy

- Fine grained entropy or von Neuman entropy. Remains constant under unitary time evolution.
  (sometimes called ``entanglement'' entropy)

$$S = -Tr[\rho \log \rho]$$ ★

- Coarse grained entropy = thermodynamic entropy, or Boltzmann entropy. Obeys 2nd law. Arises from ``sloppiness''

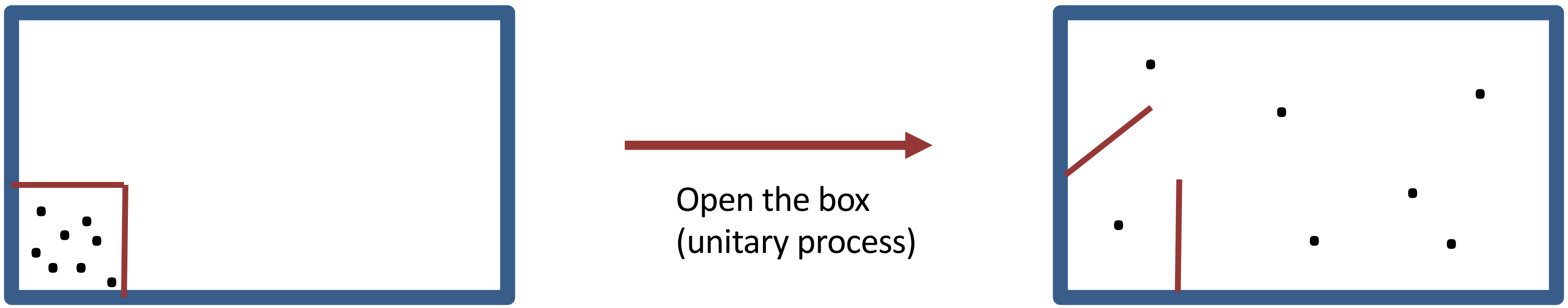
It is subtle to define it precisely. Here we are only mentioning to distinguish it from the star of our show. ★

In thermal equilibrium they are equal.

But we could be out of thermal equilibrium.

The difference in an example

Two kinds of entropy



$$S_{in} = -Tr[\rho_{in} \log \rho_{in}]$$

Von Neumann or fine grained entropy

$$\rho_f = U^{-1}\, \rho_{in} U$$

Fine grained final entropy:

$$S_{fin} = -Tr[\rho_{fin} \log \rho_{fin}] = \mathrm{S}_{\mathrm{in}}$$

Coarse grained, thermodynamic or Boltzmann entropy: $S_{Thermodynamic} > \mathrm{S}_{\mathrm{in}}$

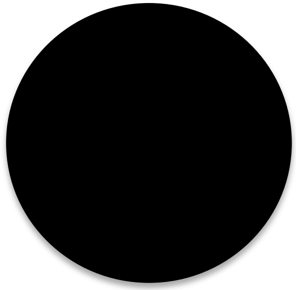
Black holes

Review an influential idea about black holes

Black holes as quantum systems

"Central dogma=central hypothesis"

- A black hole seen from the outside can be described as a quantum system with S degrees of freedom (qubits). $S = \text{Area}/4$ ($l_p = 1$)
- It evolves according to unitary evolution, seen from outside.



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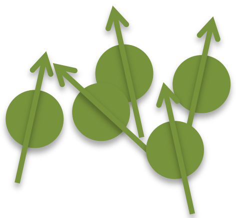
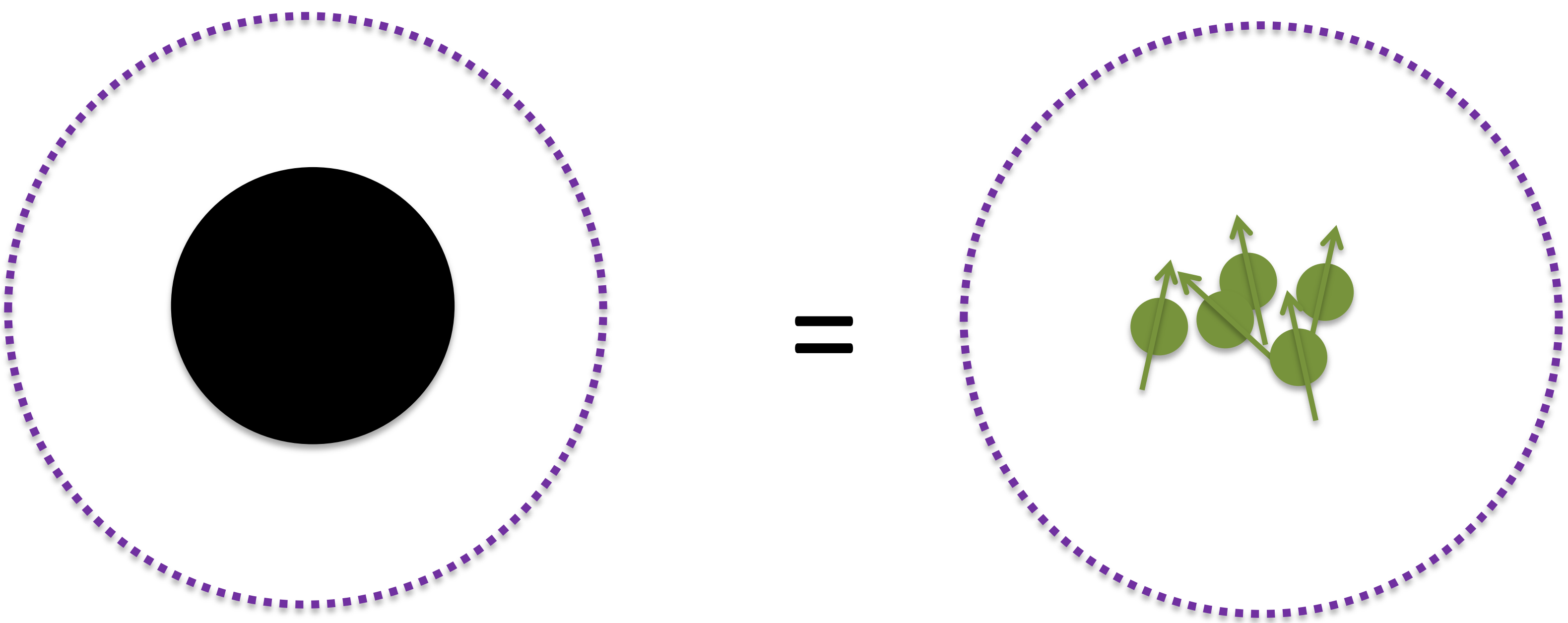


…in other words

- If one includes $A/4G_N$ "mysterious" qubits, then the black hole can be described as an ordinary quantum system.



Is it true ?

Evidence in favor

1) Entropy counting

Special black holes, in special theories (supersymmetric) can be counted precisely using strings/D-branes → reproduce the Area formula. (+ also corrections to this formula)

Strominger Vafa

using results by

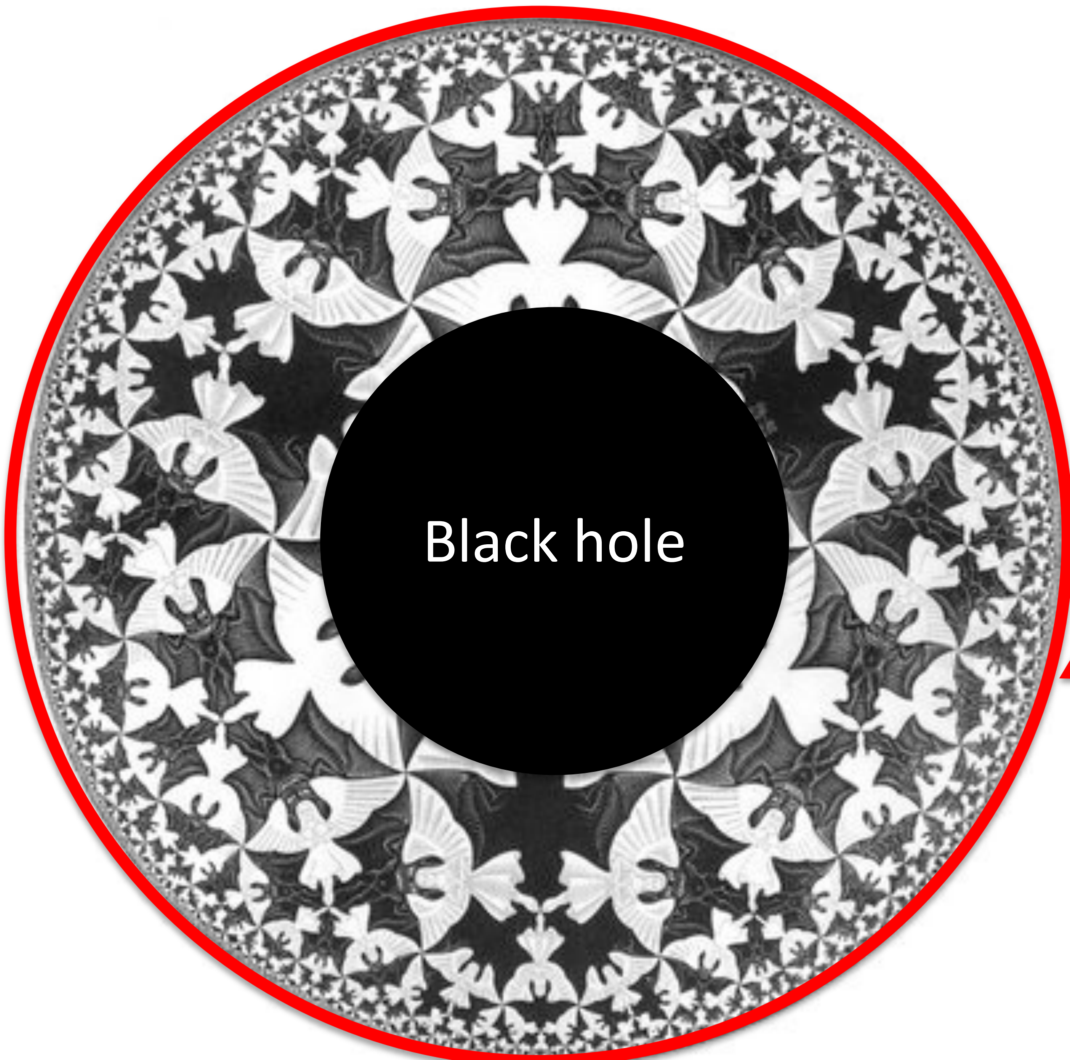
J. Polchinski

…

Sen

…

2) AdS/CFT…



Black hole in a box.
Evolving unitarity.

Hot fluid made out of
<u>very strongly</u>
interacting particles.

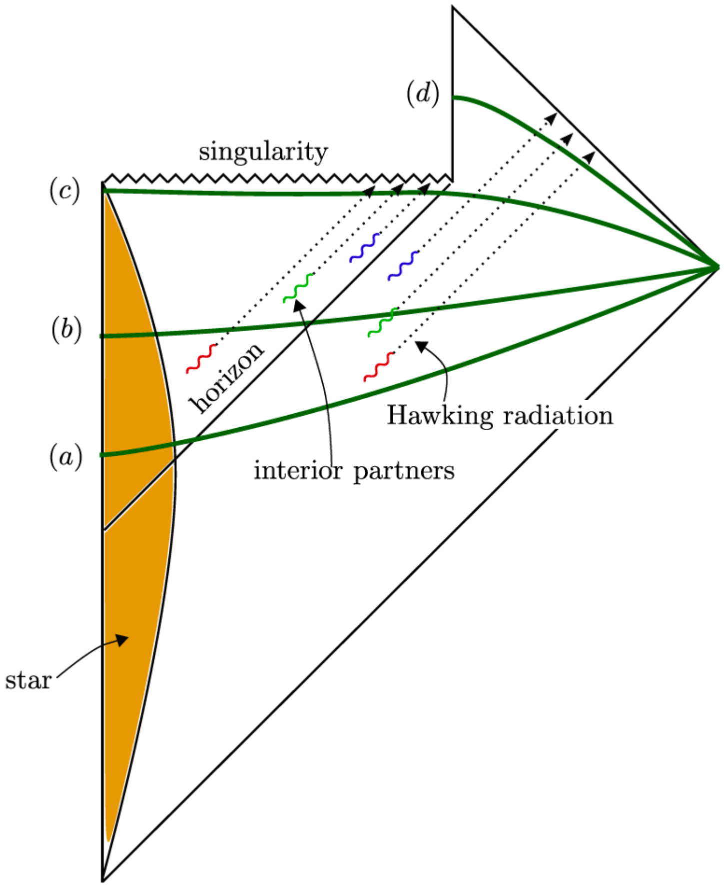
JM
Gubser, Klebanov, Polyakov
Witten

but

Hawking 1976 :

This can’t possibly be true!

Geometry of an evaporating black hole made from collapse

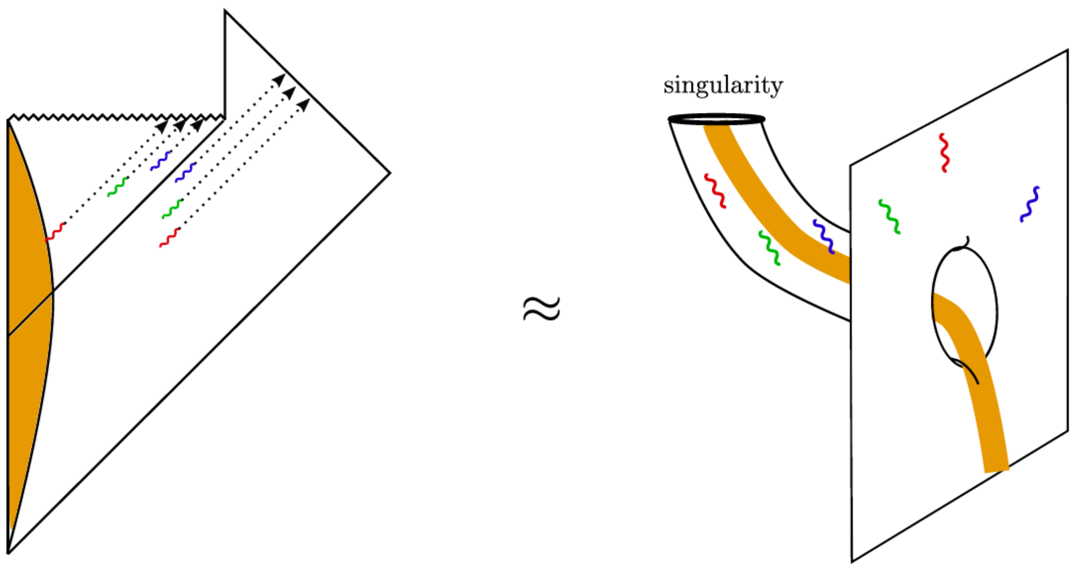


The radiation is entangled with partners of radiation.

Since we do not measure the interior we get a large entropy for the radiation.

A pure state seems to go a mixed state.

The skeptic's view:



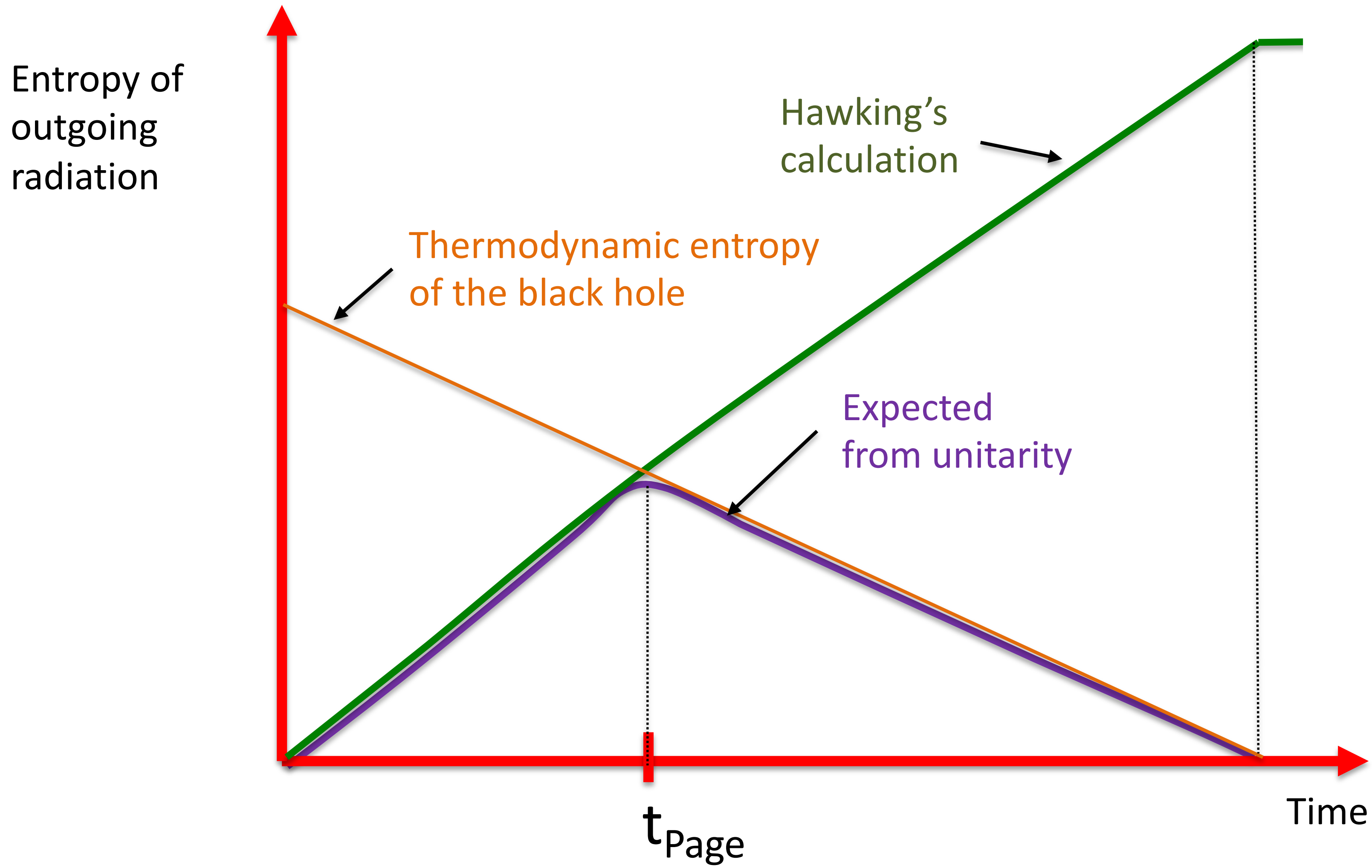
One universe splits of a ``teenage universe''   (big baby universe)

The state is pure if you include both universes, but not if you look only at the original universe.

A better statement of the problem

The Hawking curve vs. the Page curve

Compute the fine grained entropy of the radiation as it comes out of the black hole (formed by a pure state)



This problem involves von Neuman or fine grained entropy

New formulas for computing the entropy of black holes

First review old formulas

Bekenstein Hawking entropy

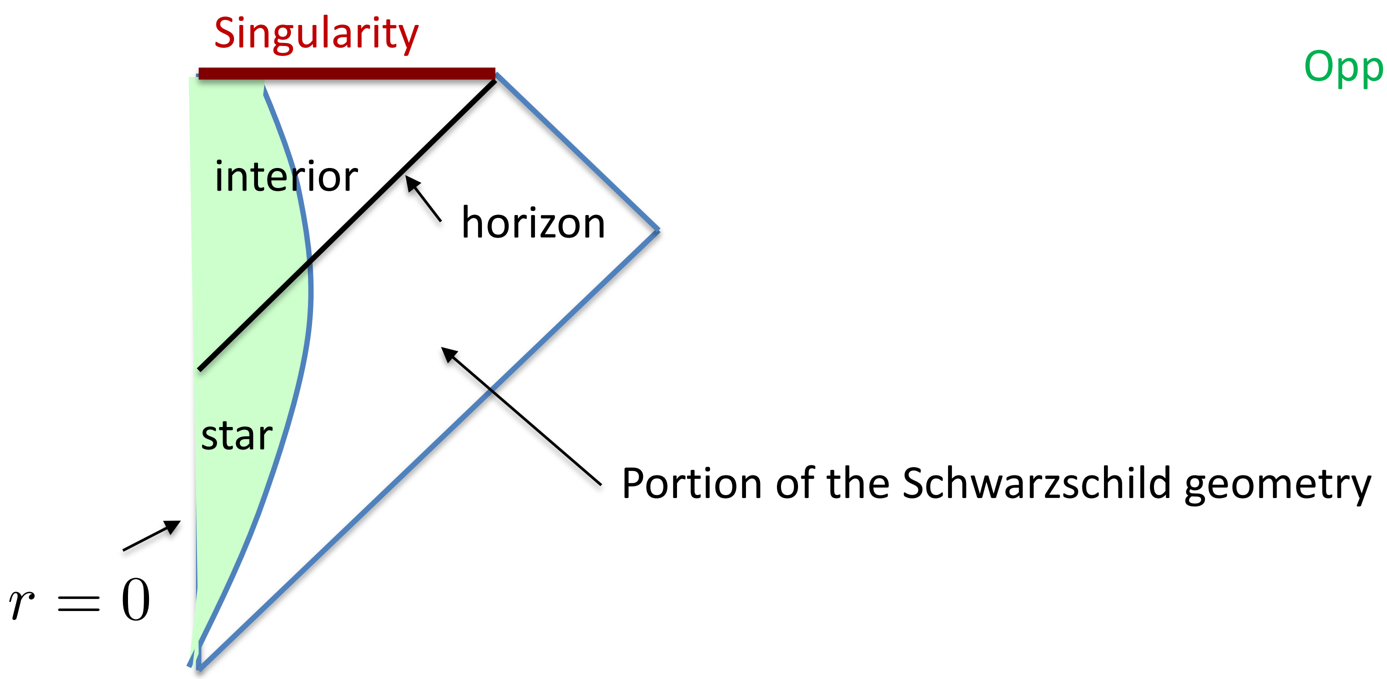
- Use first law:

$$dS = \frac{dE}{T} = \frac{dM}{T} \,, \qquad r_s = G_N M/c^2$$

$$\boxed{S = \frac{\text{Area}}{4G_N} = \frac{\text{Area}}{4l_p^2} = \frac{4\pi r_s^2}{4l_p^2}}$$

Geometry of a Black Hole made from collapse

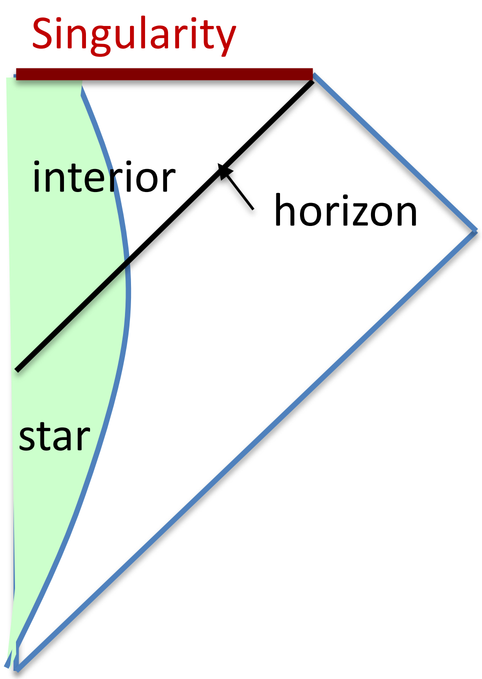
Oppenheimer Snyder 1939



Horizon Area law

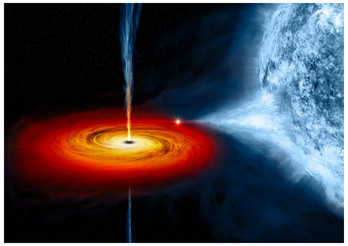
Area law: The area of a black hole horizon always increases. → 2nd law of thermodynamics

Hawking



Starts with small area and it grows to larger area

Generalized entropy



$$S = \frac{\text{Area}_H}{4G_N} + S_{\text{matter}}$$

Bekenstein 70's

Entropy of quantum fields

$$S = \frac{\text{Area}_H}{4G_N} + S_{\text{matter}} = \frac{\text{Area}_H}{4G_N} + S_{\text{QFT}}$$

Includes the entropy of quantum fields

Bombelli, Koul, Lee,Sorkin 1986

Obeys the 2nd Law

Wall 2010

The horizon area computes thermodynamic entropy

Can we compute the fine grained entropy ?

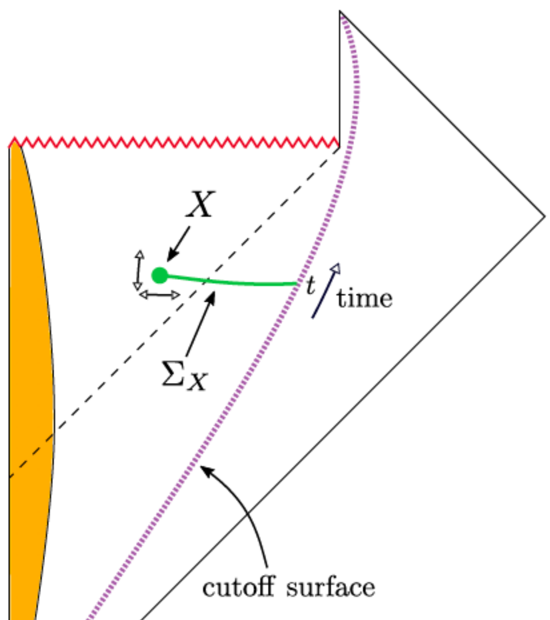
There is another gravitational entropy formula !

Fine grained gravitational entropy

Ryu-Takayanagi 2006
Hubeny, Rangamani, Takayanagi 2007
Faulkner, Lewkowycz, JM 2013
Engelhardt, Wall 2014

$$S = \min_X \left\{ \text{ext}_X \left[\frac{\text{Area(X)}}{4G_N} + S_{\text{semi-cl}}(\Sigma) \right] \right\}$$

The final surface is called minimal quantum extremal surface.



We are allowed to take the surface to the inside. It depends on the geometry of the interior

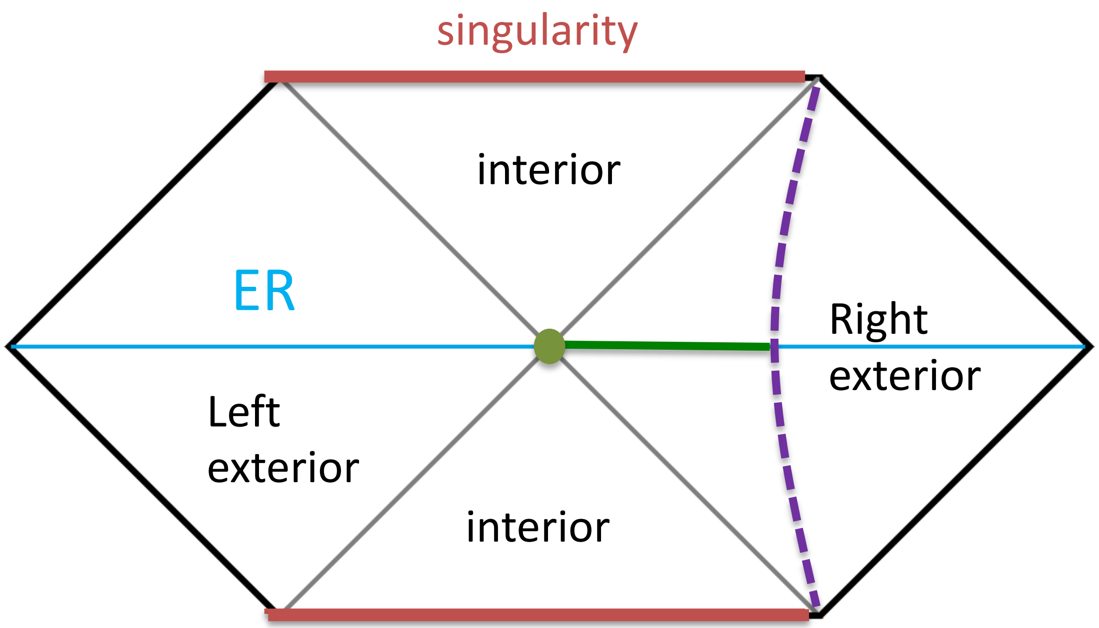
We will discuss its derivation later

For now we will just use it

We should be surprised by the claim that there is a formula for the fine-grained entropy

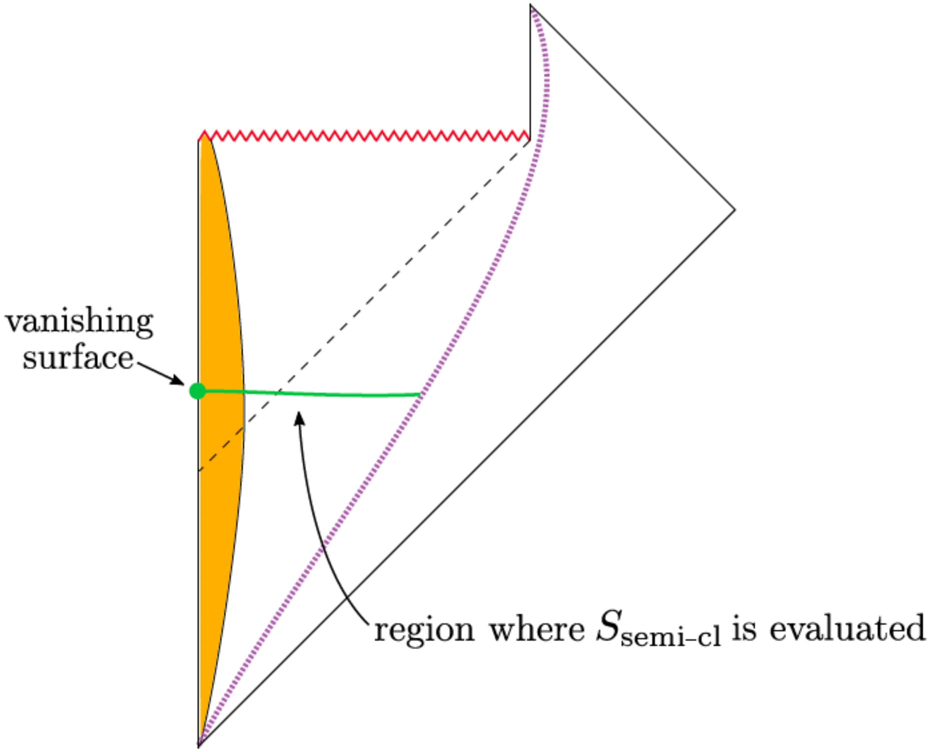
Examples

Full Schwarzschild solution = Two entangled black holes



In this case $S_{vN} = S_{Boltzman}$

Vanishing surface



S = Entropy of star , which is zero if it was in a pure state.

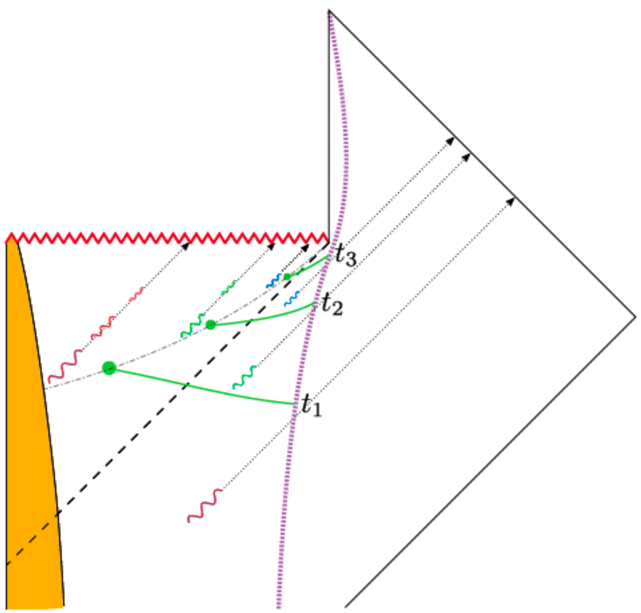
In this case $S_{vN} \ll S_{Boltzman}$

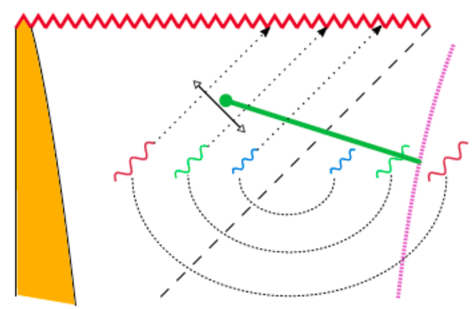
A feature arise when the black hole has been evaporating for a long time

New quantum extremal surface

Penington
Almheiri, Engelhardt, Marolf, Maxfield, 2019

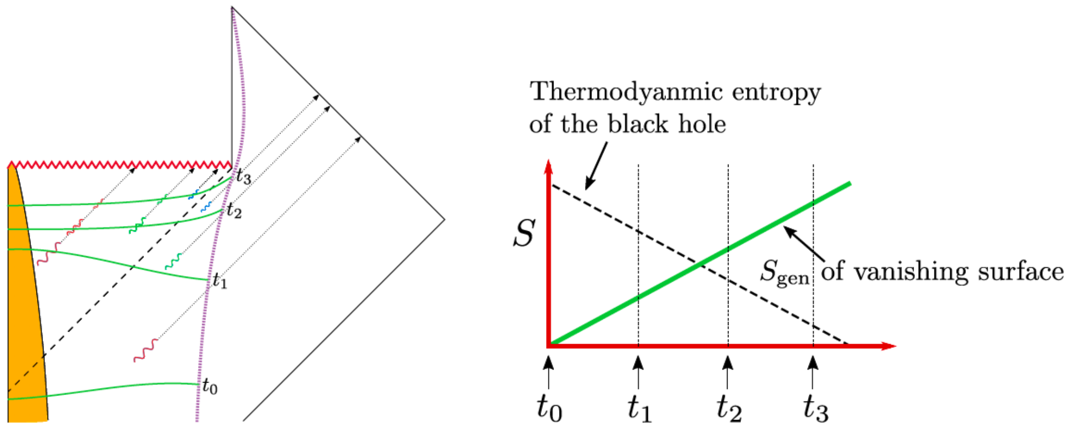
First appears about a scrambling time (r_s log S) after the black hole forms.

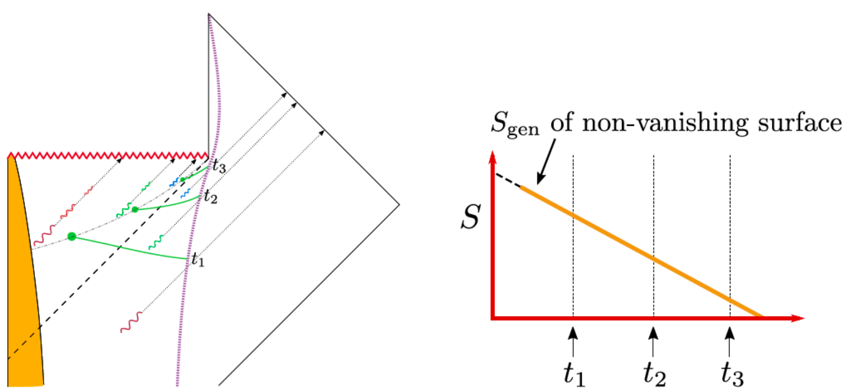




Its entropy is close to the area of the horizon at the time.

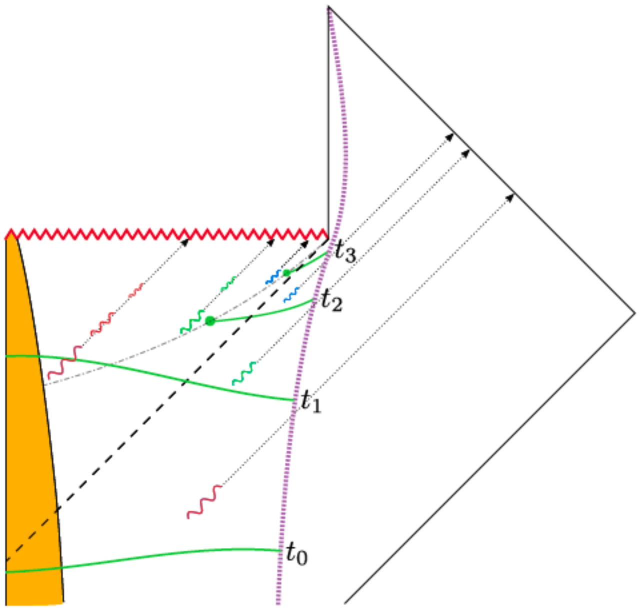
Two quantum extremal surfaces

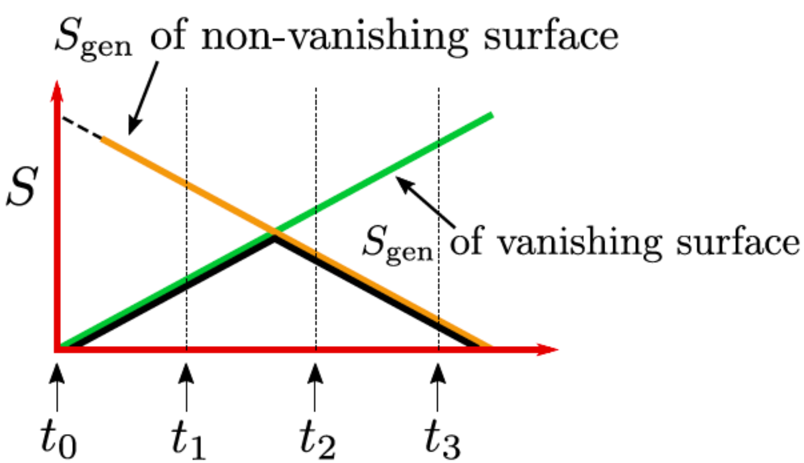




Penington
Almheiri, Engelhardt, Marolf, Maxfield

Choose the minimal one





Penington
Almheiri, Engelhardt, Marolf, Maxfield

…we get the Page curve for the black hole

but

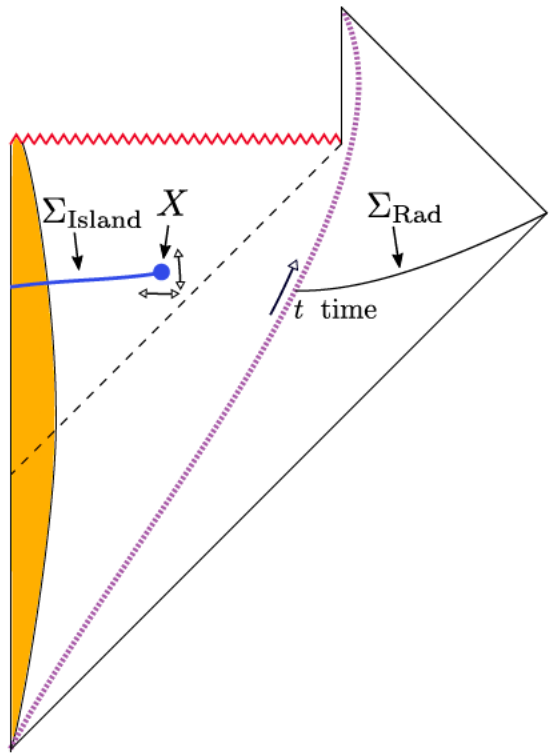
we really wanted the Page curve for the radiation!

- The radiation lives in a region where quantum gravity effects could be very small. (It could be collected into a far away quantum computer).

- Since we obtained the state using gravity → we should apply the gravitational fine-grained entropy formula!

Penington; Almheiri, Mahajan, JM, Zhao

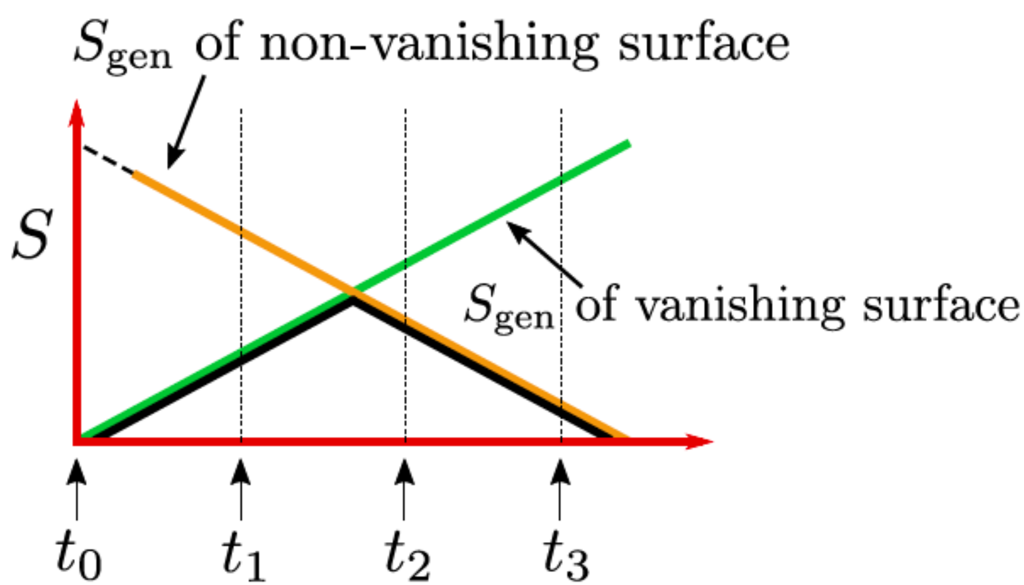
The “Island” formula



$$S_{\rm Rad} = \min_X \left\{ {\rm ext}_X \left[\frac{{\rm Area}(X)}{4G_N} + S_{\text{semi-cl}}[\Sigma_{\rm Rad} \cup \Sigma_{\rm Island}] \right] \right\}$$

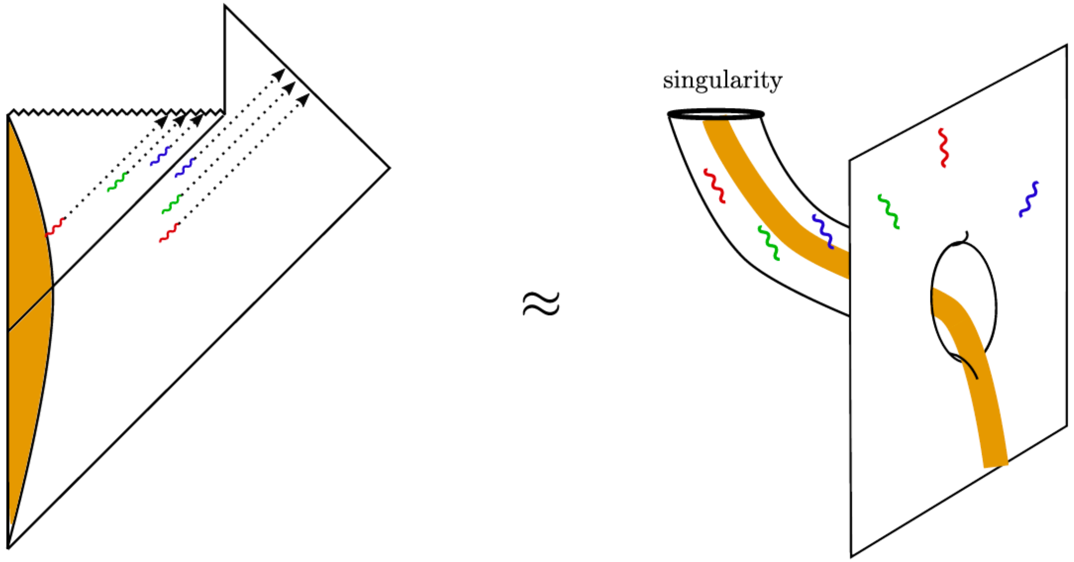
We should view it as just a special case of the general gravitational fine-grained entropy formula

- If the initial matter state is pure, then the quantum extremal surfaces are the same as the ones we discussed before.
- Therefore we get the Page curve for radiation.



The skeptic's complaint

- "This is just an accounting trick!"
- I have always said: "If you include the black hole interior, then the state is pure. The information problem arises because you do not have access to the interior!"



Gravity's accounting ``trick'' $\rightarrow$ ``oracle''

- It can be derived from the gravitational path integral. (Deriving the gravitational fine grained entropy formula)

- Oracle: It gives us the true fine-grained entropy of the <u>exact</u> state, but using only the semiclassical state.

Deriving the fine grained entropy formula

- It is conceptually similar to the derivation of the black hole entropy using the Euclidean black hole.

Lewkowycz, JM ;
Faulkner, Lewkowycz, JM;
Dong, Lewkowycz, Rangamani;
Dong, Lewkowycz

- We can do it without knowing the detailed microstates.

So far, we only talked about entropy.

What is this telling us about the interior?

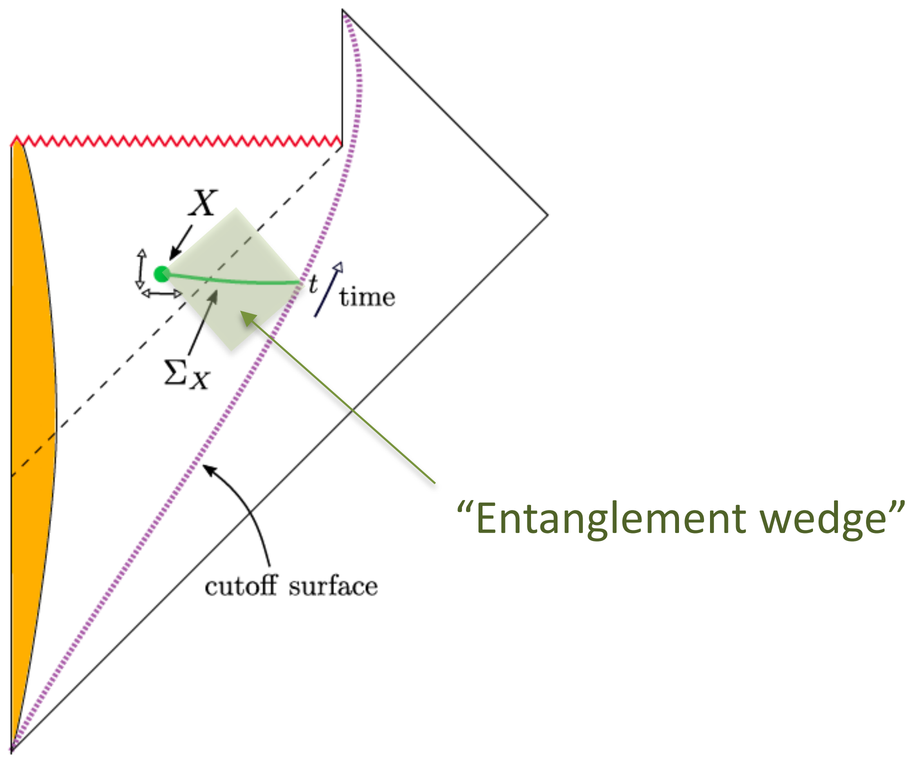
How do we describe the interior?

- The central dogma involves degrees of freedom that describe the black hole from the outside.

- What part of the interior do these degrees of freedom describe?
  - 1) All of the interior?
  - 2) None of the interior?
  - 3) Part of the interior? Which part?

Region described by the black hole degrees of freedom

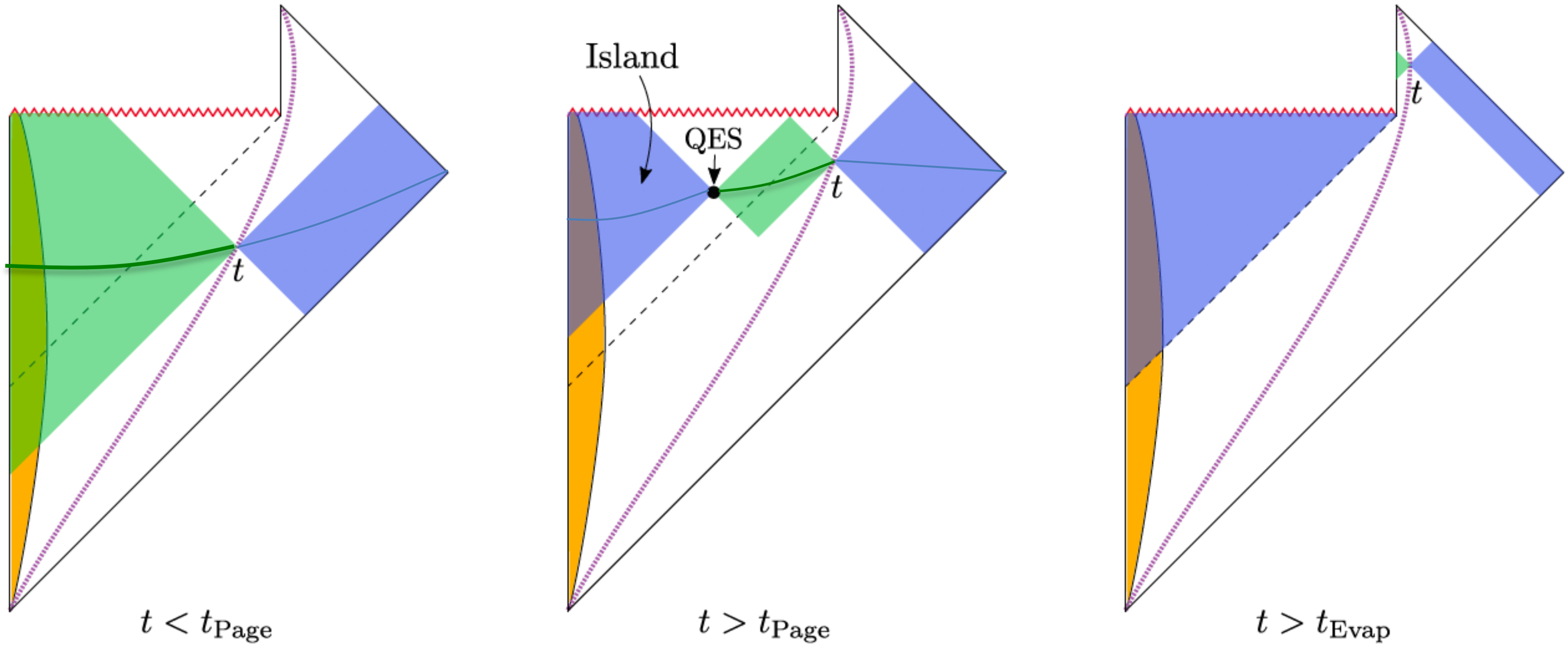
- Region that appears in the computation of the entropy.



Czech, Karczmarek, Nogueira, Van Raamsdonk, Wall, Headrick, Hubeny, Lawrence, Rangamani, Almheiri, Dong, Harlow, Jafferis, Lewkowycz, J.M., Suh, Wall, Faulkner….

Examples

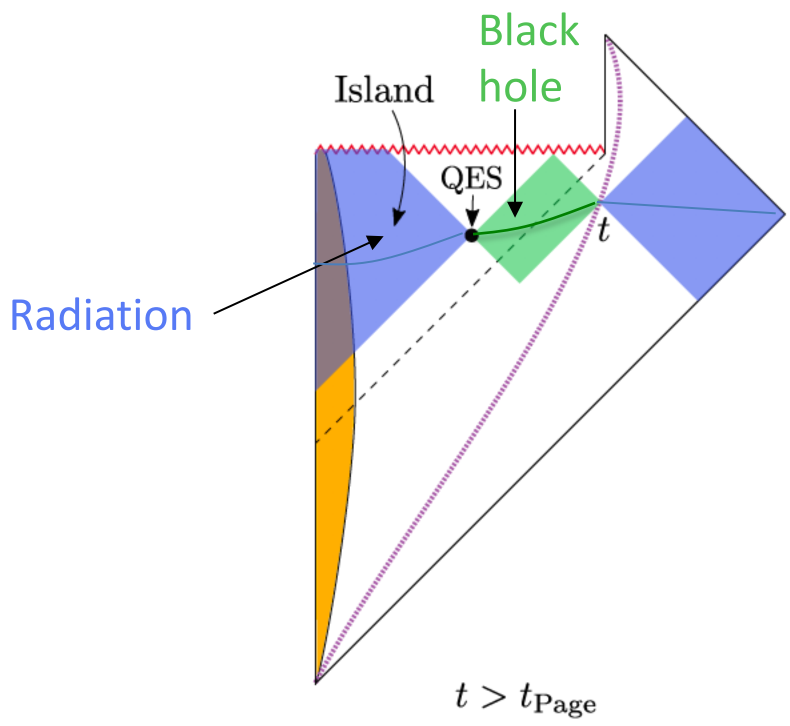
Entanglement wedge of the black hole in green. (black hole = quantum degrees of freedom describe the black hole from the outside)



Entanglement wedge of radiation in blue.
(At late times, it includes part of the black hole interior)

Describing the interior

Part of the interior is described by the black hole degrees of freedom
and another part of the interior to the radiation



- By performing (a very complicated) quantum operation on the radiation → we can extract information from the interior.
- We do not have a clear Lorentzian picture of how that happens.

Gao Jafferis Wall
Petz map: Almheiri, Dong, Harlow; Cotler, Hayden, Penington, Salton, Swingle, Walter; Chen, Penington, Salton; Penington, Stanford, Shenker, Yang

Jafferis, Lewkowycz, JM, Suh; Almheiri, Anous, Lewkowycz;..., Y. Chen

Conclusions

- We reviewed the gravitational fine-grained entropy formula.
- We applied it to the computation of the entropy of radiation and obtained results consistent with unitarity.
- At late times, most of the interior is part of the radiation. It is not part of the ``black hole degrees of freedom''.

What was Hawking's mistake?

- Not to use the fine-grained entropy formula.

(It was not known at the time)

- A lot of what we discussed was derived by thinking about aspects of AdS/CFT, which itself involves string theory.
- But you only need gravity as an effective theory to apply these formulas.

There is an amazingly deep connection between gravity and quantum mechanics!

Is the information puzzle solved?

- One aspect: computing the entropy, <u>yes.</u>

- Another aspect: Understanding what the state is, <u>no</u>.

- We can compute the entropy of the radiation but not its state (at least with the present understanding).

- As with black hole entropy, it is an accounting ``oracle''. The explicit gravitational representation of the states is still mysterious. But the semiclassical solution is representing some aspects of the state.

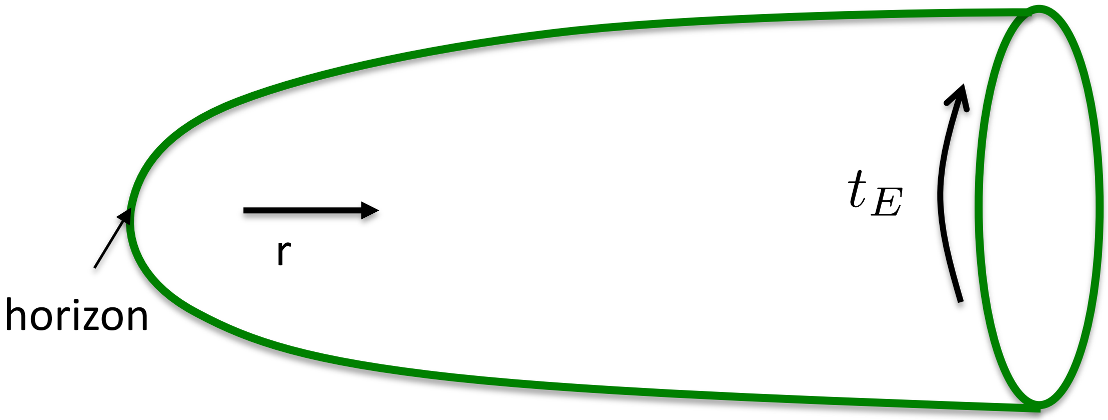
Future

- What further lessons is this teaching us about the interior? The singularity ?
- Implications for cosmology ?

Thank you !

Extra slides

Euclidean black hole



``cigar''

Gibbons
Hawking

$$\mathrm{Tr}[e^{-\beta H}] = Z_{\text{grav}} \sim e^{-I_{grav}} Z_{\text{semi-cl}}$$

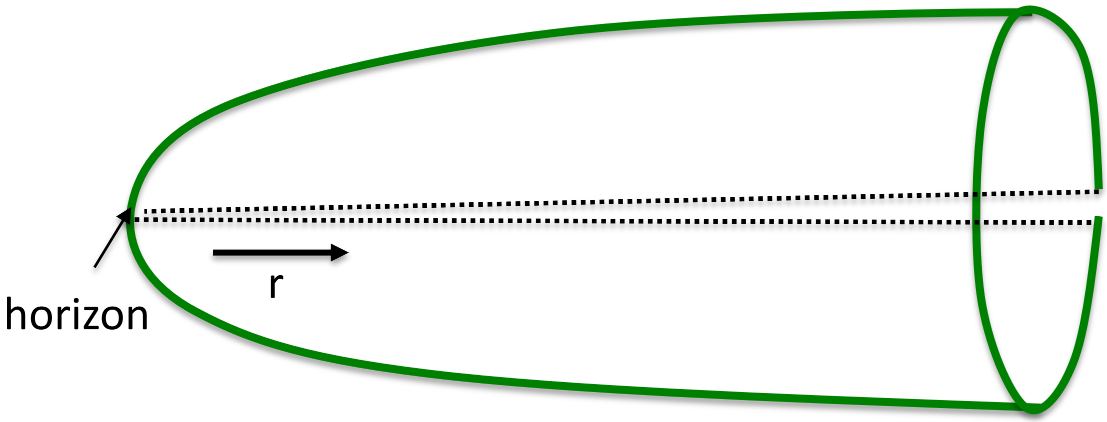
$$I_{grav} \propto -\frac{1}{G_N}\int \sqrt{g}R + \cdots$$

interpretation

$$S = (1-\beta\partial_\beta)\log Z = \frac{\text{Area}}{4G_N} + S_{\text{semi-cl}}$$

Fix the temperature far away, gravity chooses the geometry dynamically.

Euclidean black hole



``cigar''

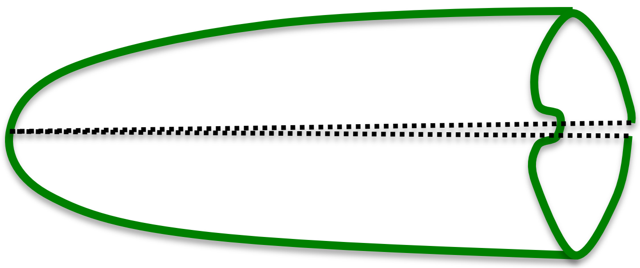
$$\tilde{\rho} \propto \rho$$

Gibbons
Hawking

$$S = (1 - n\partial_n) \log \mathrm{Tr}[\tilde{\rho}^n]|_{n=1} = (1 - n\partial_n) \log Z_n|_{n=1}$$

Computes the entropy if we can only compute the traces, but the actual density matrix itself

Other states prepared by a path integral.



$\tilde{\rho} \propto \rho$

Lewkowycz, JM
Faulkner, Lewkowycz, JM
Dong, Lewkowycz, Rangamani
Dong, Lewkowycz

$\underbrace{\mathrm{Tr}[\tilde{\rho}^3]}_{\text{interpretation}}$

Z_3 symmetry

Z_3 fixed point

$$S = (1 - n\partial_n)\mathrm{Tr}[\tilde{\rho}^n]|_{n=1} = (1 - n\partial_n)\log Z_n|_{n=1}$$

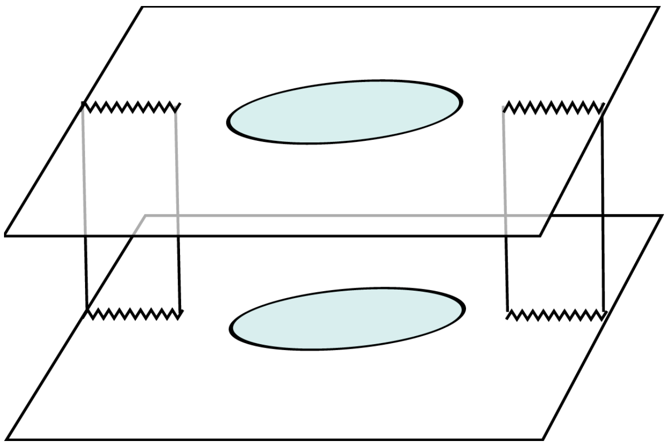
$$S = \min\left\{\mathrm{ext}\left[\frac{\mathrm{Area(X)}}{4G_N} + S_{\mathrm{semi-cl}}(\Sigma)\right]\right\}$$ = fine grained entropy formula

- In the same way that the Euclidean black hole gives us the entropy, this replica trick gives us the gravitational fine-grained entropy formula.

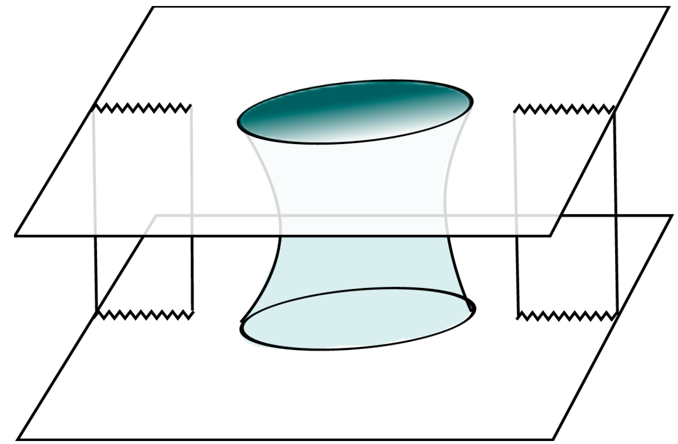
- If the state is prepared by a Euclidean path integral, and it has dynamical gravity only in some regions, then we should allow various topologies in that region.
- Interiors connected by ``replica wormholes'' → island formula.

Penington, Shenker, Stanford, Yang
Almheiri, Hartman, JM, Shaghoulian, Tajdini

Replica wormholes: n=2



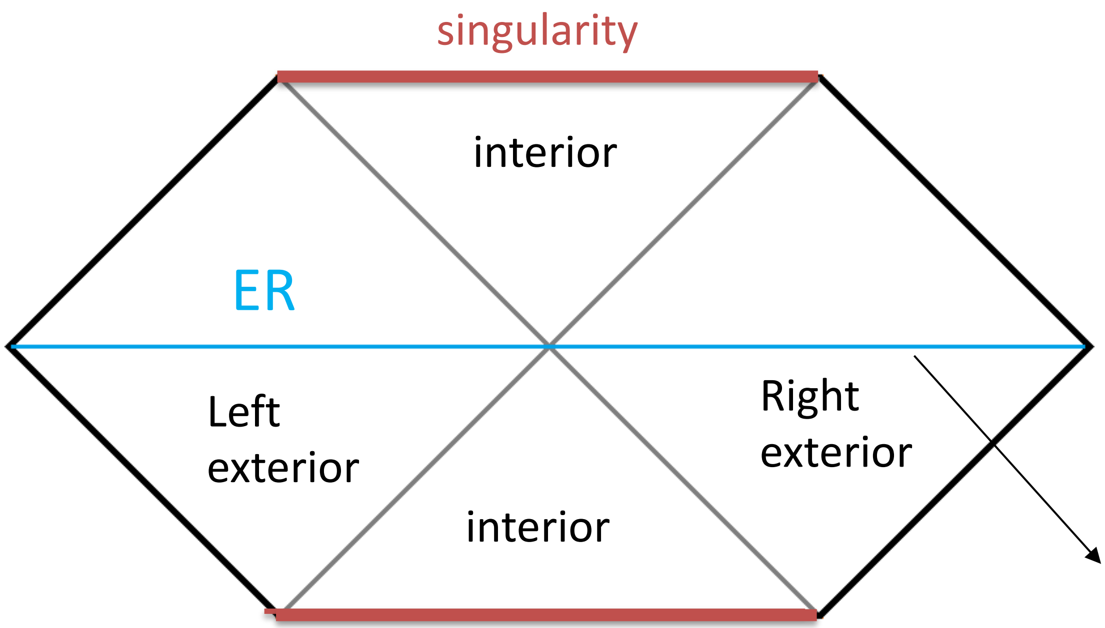
Solution that gives
Hawking's result



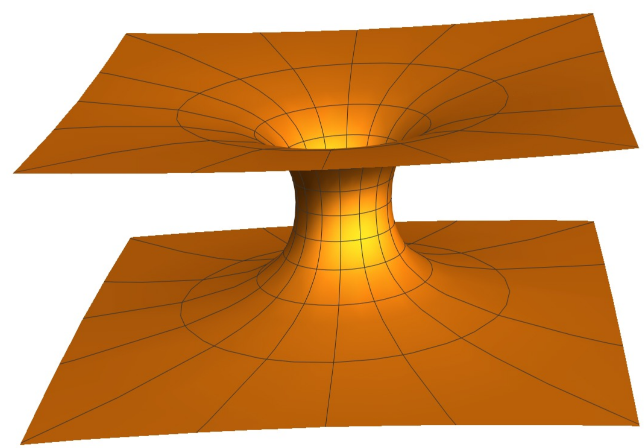
Replica wormhole, giving the
Page answer when it dominates
at late times.

- $Tr[\tilde{\rho}^{\mathrm{n}}]$ is given by these other non-trivial geometries.
- But the von Neuman entropy is given by a computation in the original semiclassical geometry → the fine grained entropy formula that we have seen above

Full Schwarzschild solution



Eddington, Lemaitre, Einstein,
Rosen, Finkelstein,
Kruskal

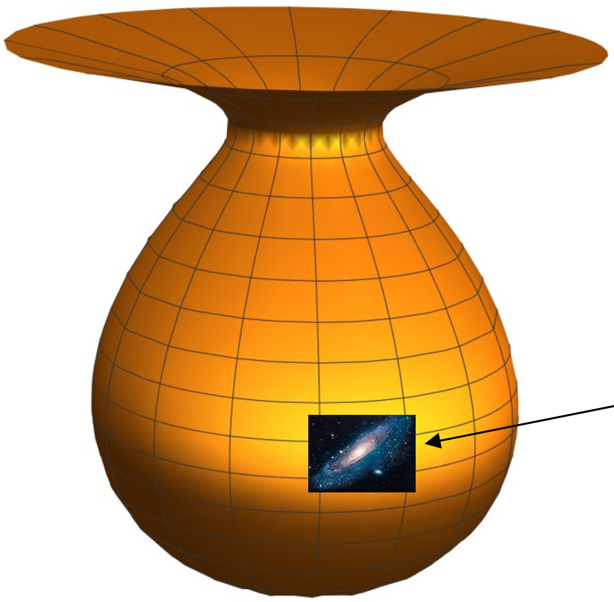


Vacuum solution. No matter.
Two exteriors, sharing the interior.

``Bags of Gold''

Wheeler

Initial slice:



Evolves to a black hole as seen from the outside and a black hole in a closed universe.

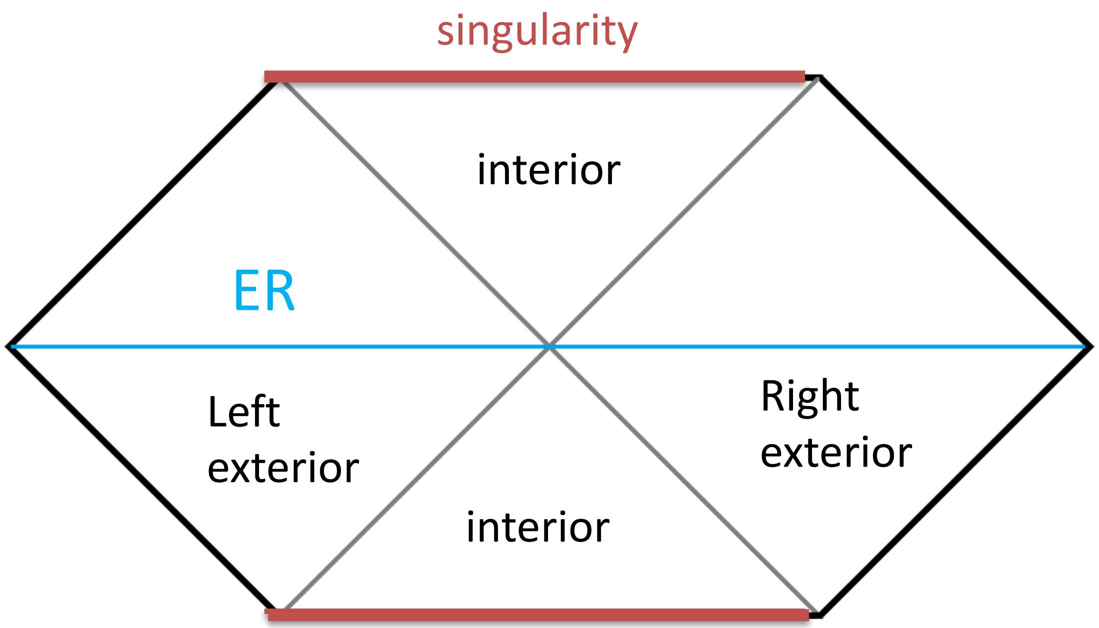
Can have arbitrarily large amount of entropy ``inside''

Counterexample to the statement that the Area entropy counts the entropy "inside" or the entropy of the ``interior''.

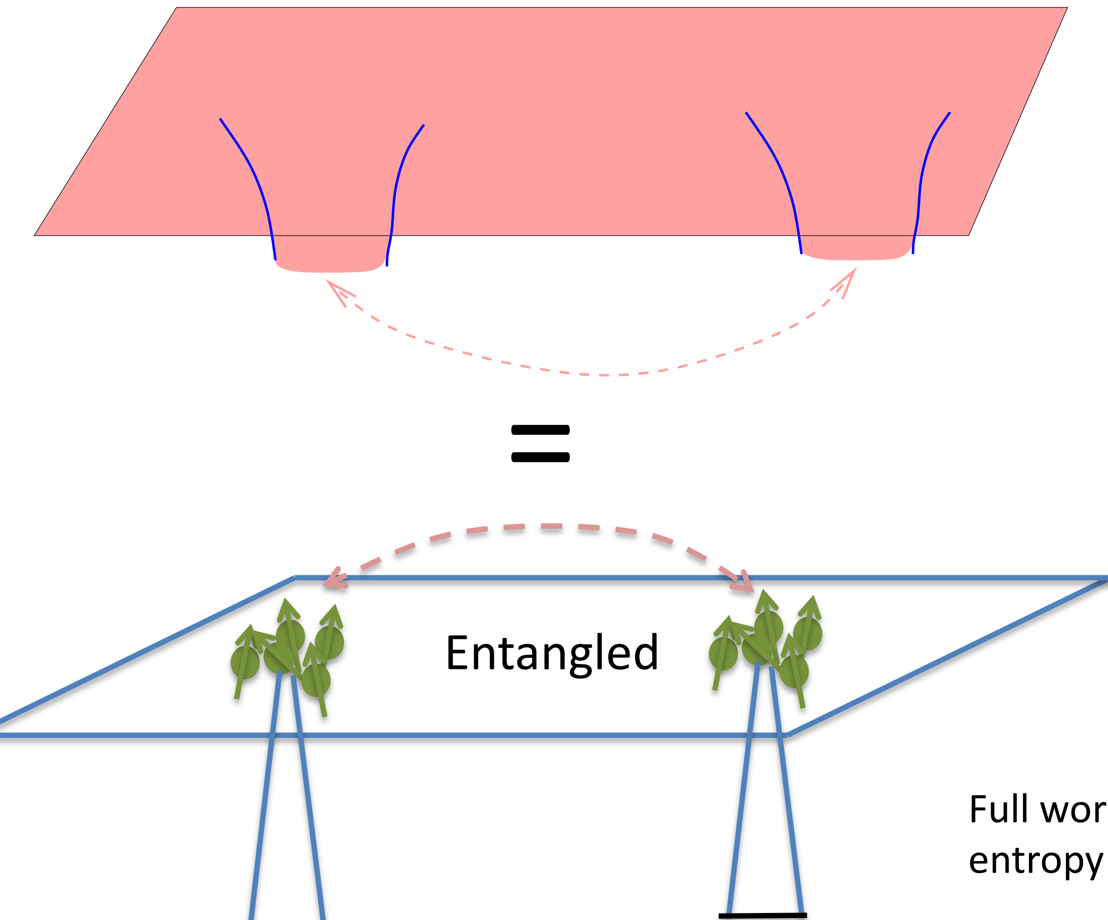
It is only a statement about the black hole as seen from the outside !

No statement has been made about the inside (yet).

Full Schwarzschild solution as a wormhole



Wormholes and entangled states



Connected through the interior

ER = EPR

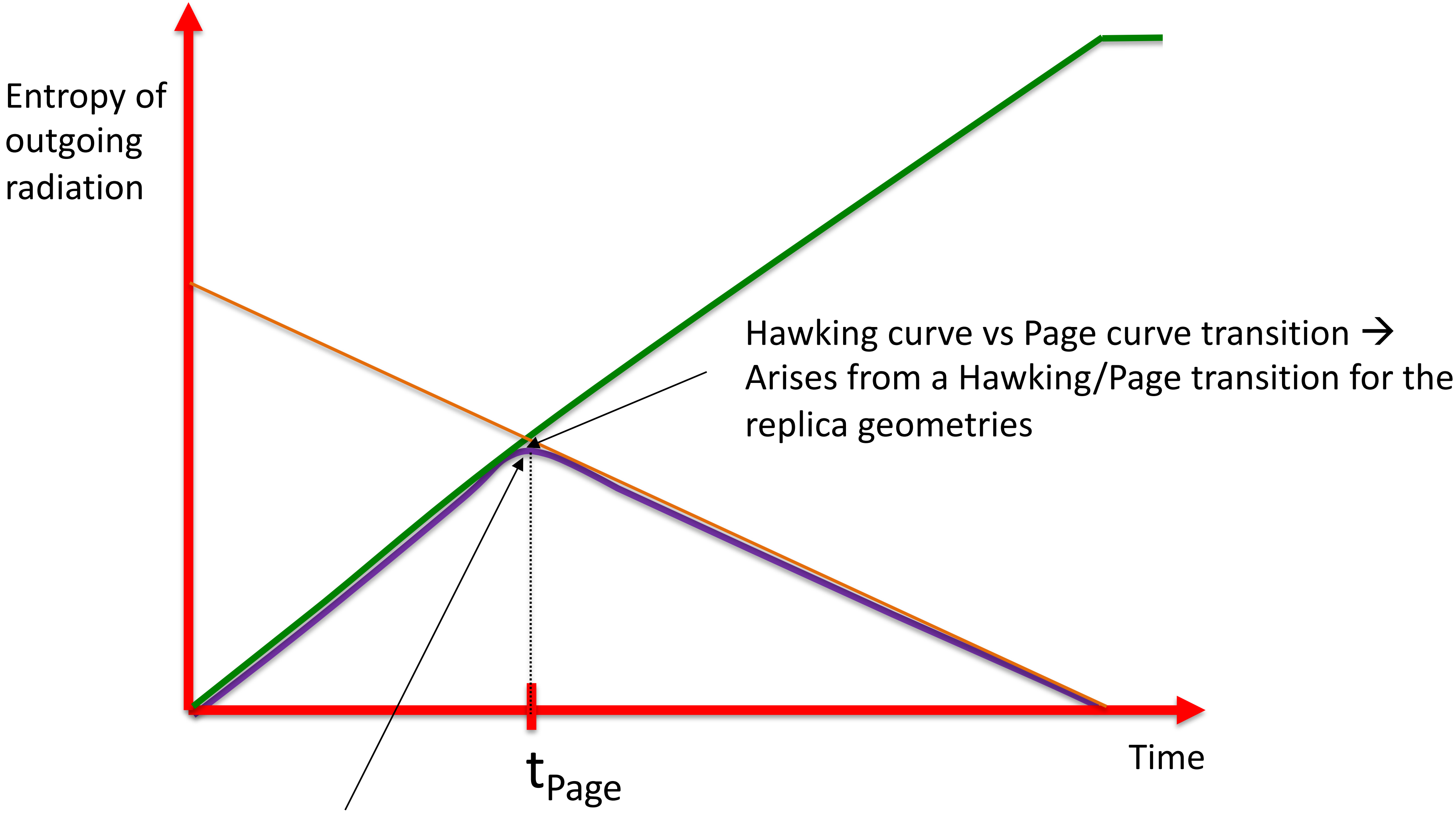
W. Israel
J.M.
JM Suskind

Full wormhole is in a pure state,
entropy of each black hole arises from entanglement

In a particular entangled state

$$|TFD\rangle = \sum_n e^{-\beta E_n/2} |\bar{E}_n\rangle_L |E_n\rangle_R$$

The Hawking curve vs. the Page curve



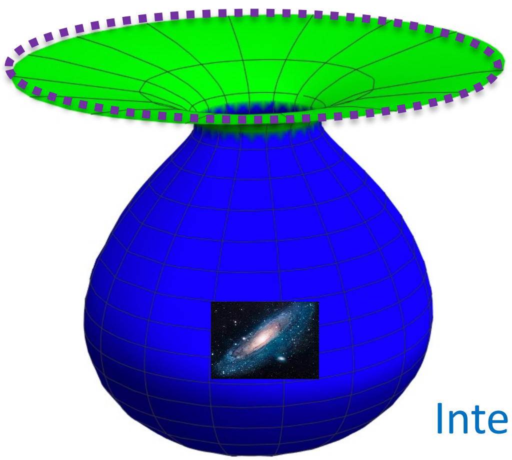
Gets smoothed out by considering the sum over non-replica symmetric geometries

Penington, Shenker, Stanford, Yang; Dong, H. Wang; Marolf, S. Wang, Z. Wang

``Bags of Gold”

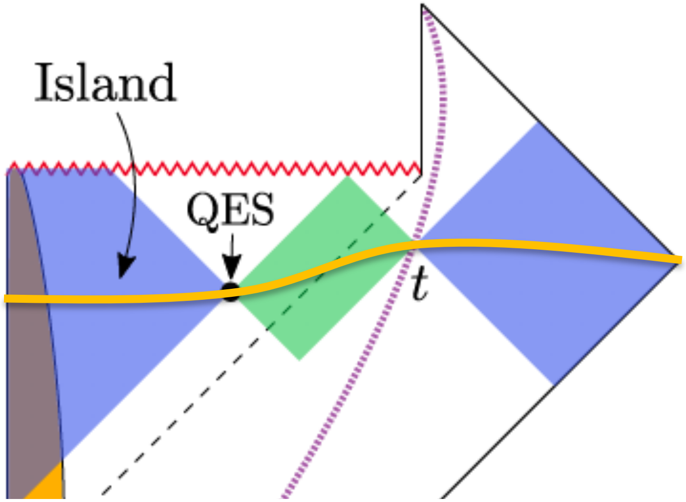
If there is a lot of entropy inside → the entanglement wedge of the black hole ends near the neck

Wall



Interior of the bag belongs to somebody else…

Bag of gold vs old black hole



The geometry and entropy on the orange slice is somewhat similar to the bag of gold.

Now the answer:

Entanglement wedge reconstruction hypothesis

- The quantum system describes everything that is included in its entanglement wedge.

- We can recover the state of a (probe) qubit inside the entanglement wedge.

- Recovery is state dependent (subspace dependent) and similar to quantum error correction.

Czech, Karczmarek, Nogueira, Van Raamsdonk, Wall, Headrick, Hubeny, Lawrence, Rangamani, Almheiri, Dong, Harlow, Jafferis, Lewkowycz, J.M., Suh, Wall, Faulkner….