

AXION COUPLINGS AND CHIRAL ANOMALIES

Axion effective lagrangians

(EMILIAN DUDAS, PRIYANKA LAMBA, SP, KAZUKI SAKURAI)

Dear Marcela and Carlos, my dear, old friends! We know each other for thirty years.

We have aged a bit since then... more me than you :-). We haven't seen each other very often for some time (too bad!) but you are always very close to my heart.

I was just looking at our photos from the conference in Kazimierz in 1998 and 2002.

It is really hard to believe that another twenty years have gone by...

But on the other hand, it was a time of your wonderful careers, which I sincerely congratulate you.

There is still a lot of exciting times ahead of you.

I wish you many great moments for many more years.



Planck conference 1998



Planck conference 2002



Planck conference 2002

GOLDSTONE BOSON COUPLINGS AND CHIRAL ANOMALIES

(A TEXTBOOK TOPIC, RECENTLY AGAIN OF INTEREST)

J. QUEVILLON&CH.SMITH,1903.12559; Q. BONNEFOY et al, 2011.10025

HISTORICALLY: PION DECAY INTO TWO PHOTONS

1949- STEINBERGER DIRECT 1-LOOP CALCULATION IN THE PION- NUCLEON MODEL
GIVES THE GAUGE AND LORENTZ INVARIANT EFFECTIVE LAGRANGIAN

$$\frac{g}{\Lambda} \pi^0 \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$$

$$\frac{g}{\Lambda} = \frac{e^2 G_{\pi N}}{32\pi^2 m_N}$$

THIS RESULT GIVES

$$\Gamma(\pi^0 \rightarrow 2\gamma) = \frac{m_\pi^3}{\pi\Lambda} g^2$$

AND AGREES WITH EXP NUMBER.

1967: VELTMAN AND SUTHERLAND (INDEPENDENTLY) POINT OUT THAT THIS RESULT IS INCONSISTENT WITH THE CONCEPT OF THE PION AS A PSEUDO-NAMBU GOLDSTONE BOSON OF THE SPONTANEOUSLY BROKEN GLOBAL $SU(2)_L \times SU(2)_R$ SYMMETRY OF THE PION-NUCLEON THEORY

π^0 IS THE NGB CORRESPONDING TO THE AXIAL ELEMENT OF THE $U(1) \times U(1)$ ELECTRICALLY NEUTRAL SUBGROUP (LEFT EXPLICITLY UNBROKEN BY ELECTROMAGNETIC INTERACTIONS)

$\Psi \rightarrow e^{i\theta\tau_3\gamma_5} \Psi$ (SYMMETRY IN THE LIMIT OF MASSLESS NUCLEONS)

FOR A GOLDSTONE BOSON, ONLY DERIVATIVE COUPLINGS ARE ALLOWED (CHIRAL AND GAUGE INVARIANT)

$$\partial_\mu \partial^\mu \pi^0 \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma} F_{\mu\nu}$$

SO ADDITIONAL SUPPRESSION BY $\frac{m_\pi^2}{m_N^2}$ (PION IS A PSEUDO-GOLDSTONE)

THEN $\Gamma \sim m_\pi^7$ ($\frac{g}{\Lambda} \partial_\mu \pi^0 \epsilon^{\mu\nu\rho\sigma} A_\nu F_{\rho\sigma}$ is not gauge invariant)

SOLUTION: BELL-JACKIW ANOMALY (AN ANOMALY OF THE AXIAL GLOBAL SYMMETRY CURRENT IN THE PRESENCE OF ELECTROMAGNETIC INTERACTIONS)

A TOY MODEL WITH “UNBROKEN” VECTOR-LIKE U(1) GAUGE SYMMETRY (MIMICING THE PION DECAY INTO PHOTONS AND KSVZ QCD AXION MODEL)

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + \bar{\Psi}_L D_\mu \gamma_\mu \Psi_L + \bar{\Psi}_R D_\mu \gamma_\mu \Psi_R \\ + |\partial_\mu \Phi|^2 - V(|\Phi|^2) - (y\Phi \bar{\Psi}_L \Psi_R + h.c)$$

GAUGE CHARGES

$$D_\mu = \partial_\mu - iqgA_\mu \qquad q_L = q_R = q \qquad q_\Phi = 0$$

$$\Phi = (\sigma + f/\sqrt{2}) \exp(ia(x)/f)$$

CLASSICAL INVARIANCE UNDER **GLOBAL AXIAL SYMMETRY** DEFINED BY THE TRANSFORMATIONS

$$\Psi_L \rightarrow e^{iQ_L\theta} \Psi_L$$

$$\Psi \rightarrow e^{i\theta\gamma_5} \Psi$$

$$\Phi \rightarrow e^{iQ_\Phi\theta}$$

$$\Psi_R \rightarrow e^{iQ_R\theta} \Psi_R$$

AXIAL CHARGES

$$Q_R = -Q_L = Q/2, \quad Q_\Phi = -Q$$

GLOBAL AXIAL SYMMETRY IS SPONTANEOUSLY BROKEN

$$\Phi_0 = \frac{1}{\sqrt{2}} f e^{ia(x)/f}$$

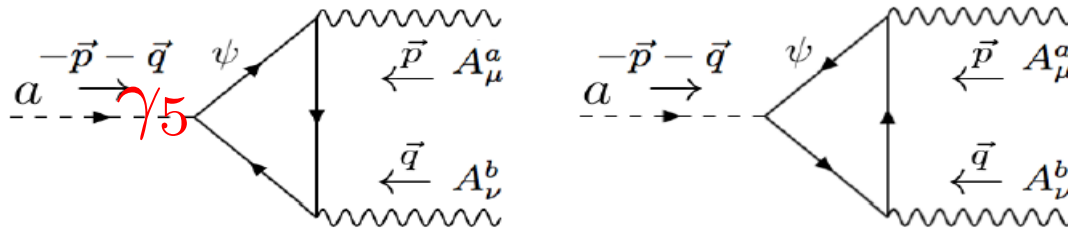
THERE REMAINS SHIFT SYMMETRY ON THE AXION FIELD

$$a(x) \rightarrow a(x) + f\theta$$

AXION DECAY AMPLITUDE DETERMINED BY THE YUKAWA TERM

$$y\Phi\bar{\Psi}_L\Psi_R + h.c. = M\bar{\Psi}\Psi + a(x)\bar{\Psi}\gamma_5\Psi + \dots$$

$$M = yf$$



$$A(p, q) = \sum_f (Q_L^f q_L^f q_L^f - Q_R^f q_R^f q_R^f) \left(\frac{g^2}{4\pi^2 f} \epsilon_{\mu\nu\rho\sigma} p^\rho q^\sigma \right) \epsilon_1^\mu \epsilon_2^\nu$$

THIS RESULT IS GIVEN BY THE EFFECTIVE LAGRANGIAN (AFTER INTEGRATING OUT THE FERMIONS)

$$\mathcal{L} \sim \frac{g^2}{16\pi^2} D_{\gamma\gamma} \frac{a(x)}{f} F \tilde{F} \quad D_{\gamma\gamma} = \text{AXIAL ANOMALY COEFFICIENT}$$

THE REASON:

MASSLESS GAUGE BOSONS AND VECTOR-LIKE WITH CHARGE q :

THE ONLY GAUGE INVARIANT OPERATOR LEADING TO AXION COUPLINGS TO GAUGE BOSONS, UNSUPPRESSED BY THE EFFECTS OF EXPLICIT AXIAL SYMMETRY BREAKING, IS THE SHIFT NON-INVARIANT OPERATOR

$$\frac{a}{f} F \tilde{F} \quad a(x) \rightarrow a(x) + f\theta$$

BUT LET'S CONSIDER NOW A CHIRAL GAUGE THEORY, WITH FERMIONS COUPLED CHIRALLY TO GAUGE BOSONS

Chiral couplings of fermions to the gauge bosons (still U(1) gauge theory)

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + \bar{\Psi}_L D_\mu^L \gamma_\mu \Psi_L + \bar{\Psi}_R D_\mu^R \gamma_\mu \Psi_R \\ + |D_\mu^{\phi_1} \phi_1|^2 + |D_\mu^{\phi_2} \phi_2|^2 - V(|\phi_1|^2, |\phi_2|^2) - (y\phi_1 \bar{\Psi}_L \Psi_R + h.c)$$

$$D_\mu^L = \partial_\mu - iq_L g A_\mu \quad D_\mu^R = \partial_\mu - iq_R g A_\mu \quad D_\mu^{\phi_{1,2}} = \partial_\mu - iq_{\phi_{1,2}} g A_\mu$$

GAUGE COUPLINGS

$$q_R \neq q_L, \quad q_{\phi_1} = q_L - q_R, \quad q_{\phi_2} \neq q_{\phi_1}$$

THE SCALAR ϕ_1 HAS TO BE CHARGED UNDER THE GAUGE GROUP AND THE GLOBAL AXIAL AND GAUGE SYMMETRIES ARE SIMULTANEOUSLY BROKEN. AN ADDITIONAL SCALAR IS NEEDED FOR THE BEHMECHANISM + A PHYSICAL AXION

Two symmetries

$$\Psi_L \rightarrow e^{iQ/2\theta} \Psi_L \quad \Psi_R \rightarrow e^{-iQ/2\theta} \Psi_L \quad \phi_1 \rightarrow e^{iQ\theta}$$

$$\phi_2 \rightarrow e^{iQ_2\theta}$$

$$\phi_1 = \frac{1}{\sqrt{2}}(f_1 + \sigma_1)e^{i\frac{a_1(x)}{f_1}} \quad \phi_2 = \frac{1}{\sqrt{2}}(f_2 + \sigma_2)e^{i\frac{a_2(x)}{f_2}}$$

Two Goldstone bosons

$$a_1(x) \quad a_2(x)$$

Their orthogonal linear combinations

$$a(x) \quad \phi(x) \quad f = \sqrt{(q_{\phi_1} f_1)^2 + (q_{\phi_2} f_2)^2}$$

$$a(x) \rightarrow a(x) + f\theta$$

EFFECTIVE LAGRANGIAN CONTAINS NOW TWO GAUGE INVARIANT TERMS

$$\mathcal{L} \sim \frac{g^2}{16\pi^2} D_{gg} \frac{a(x)}{f} F \tilde{F} + \frac{g^2}{16\pi^2} E_{agg} \partial_\mu a(x) \left(\frac{\partial_\nu \phi(x)}{f} - g A_\nu \right) \tilde{F}$$

SHIFT SYMMETRIC “STICKELBERG” TERM

$$D_{gg} = \Sigma_f (Q_L^f q_L^f q_L^f - Q_R^f q_R^f q_R^f)$$

$$E_{agg} = -\frac{2}{3} \Sigma_f ((Q_L^f q_R^f - Q_R^f q_L^f)(q_L^f + q_R^f) - 2(Q_R^f q_L^{f2} - Q_R^f q_R^{f2}))$$

$$E_{agg} = 0 \quad \text{FOR} \quad q_L^f = q_R^f$$

“STICKELBERG” TERM CONTRIBUTES TO THE AXION COUPLING TO THE TWO GAUGE BOSONS EVEN FOR ANOMALY FREE SPECTRUM!

$$\frac{\partial_\mu a(x)}{f} \epsilon^{\mu\nu\rho\sigma} (A_\nu + \frac{\partial_\nu \phi}{f}) F_{\rho\sigma} = \frac{1}{f} \epsilon^{\mu\nu\rho\sigma} \partial_\mu [a(A_\nu + \frac{\partial_\nu \phi}{f}) F_{\rho\sigma}] - \frac{1}{4} a F \tilde{F}$$

AXION EFFECTIVE LAGRANGIAN

$$\mathcal{L} \sim \frac{g^2}{16\pi^2} C_{agg} \frac{a(x)}{f} F \tilde{F}$$

$$C_{agg} = D_{gg} + \frac{1}{2} E_{agg}$$

EASY TO FIND ANOMALY-FREE FERMION SPECTRA WHICH GIVE NON-VANISHING AXION COUPLINGS TO GAUGE BOSONS

MERELY A THEORETICAL CURIOSITY OR SOME PHENOMENOLOGICAL APPLICATIONS?

EXTENSION TO THE SM GAUGE GROUPS?

RELEVANT FOR UV INTERPRETATION OF EVENTUALLY MEASURED

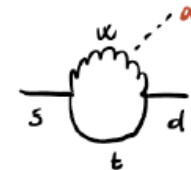
AXION COUPLINGS TO E.G.WW BOSONS?

Flavor changing neutral current

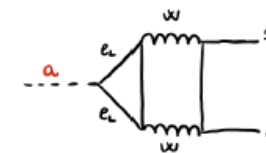
They arise in models with

- * ALPs mixed with SM neutral pions
(e.g. $K^+ \rightarrow \pi^+ \pi^0 \Rightarrow K^+ \rightarrow \pi^+ a$)

- * ALPs coupling to W or tops



- * ALPs coupling to leptons
(higher loop)



- * Flavor violating ALPs

| | | |
|-------|---------------|-----------|
| K_L | \rightarrow | $\pi^0 a$ |
| K^+ | \rightarrow | $\pi^+ a$ |
| B | \rightarrow | $K a$ |



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Understanding the Early Universe:
interplay of theory and collider experiments

Joint research project between the University of Warsaw & University of Bergen