Limitation on the luminosity of e+e- storage rings due to beamstrahlung

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Introduction

- Everybody knows that at storage rings energy losses due to synchrotron radiation in bending magnets $\Delta E \propto E^4 / R$ per turn, therefore for energies higher than LEP-2 one should use linear colliders: ILC or CLIC on the energy $2E_0$=200-500 GeV (up to 1000 and 3000 GeV).
- After LEP-2 there was a proposal of e+e- ring with 2E=90-400 GeV in VLHC tunnel (T.Sen, J. Norem, 2002), (see T.S. talk at this workshop)
- More recently A.Blondel and F.Zimmermann (arXiv:1112.2518) have proposed LEP-3 in the LHC ring on the energy 2E=240 GeV for study of the Higgs boson, (see F.Z. talk at this workshop)
- Inspired by the above suggestion K.Oide has proposed (13 Feb. 2012) SuperTRISTAN on the energy 2E=240-500 GeV (with $2\pi R$=40-60 km) with and without crab-waist collisions. The expected L with c-w were higher than at the ILC.
- If all correct, such ring collider would be easier, cheaper than LC and can provide a higher luminosity. That means the end of ILC!

Beamstrahlung

Comparing rings and linear colliders K.Oide very correctly noted in his transparences “Beamstrahlung-free, Ring better than Linear? Needs detailed calculation.”

I learned about Oide’s report at KEK from BINP colleagues and immediately checked this issue, first by my simulation code for LC-PLC, then analytically. It became clear that beamstruhlung is very important for considered collider parameters. My conclusions I reported at BINP at the end of February and then published arXiv:1203.6563 (March 29), to be published in PRL.

Two weeks later I found a 34 years old conference paper (w-gr. report) J.E.Augustin, N.Dikansky Y.Derbenev, J.Rees, B.Richter, A.Skrinsky, M.Tigner and H.Wiedemann, Limitations on Performance of e+ e- Storage rings and Linear Colliding Beam Systems at High Energy, 1st Workshop on Possibilities and Limitations of Accelerators and Detectors 15-21 Oct 1978. eConf C781015, (1978) 009.

This never cited paper was devoted exactly to consideration of beamstrahlung at high energy storage rings, they introduced the term “beamstrahlung”.

In fact, Augustin et al. have estimated, an additional beam energy spread due to beamstrahlung.

Being not aware about their report I also calculated the beam energy spread, but found that even more important is the emission of single photons in the tail of the beamstrahlung spectra. Namely this effect determines the beam lifetime (while the increase of the energy spread is still acceptable).

(Initial) parameters of rings under study (Blondel-Zimmermann, Oide)

TABLE I. Parameters of LEP and several recently proposed storage-ring colliders [6, 7]. "STR" refers to "SuperTRISTAN" [7]. Use of the crab-waist collision scheme [11, 12] is denoted by "cr-w". The luminosities and the numbers of bunches for all projects are normalized to the total synchrotron-radiation power of 100 MW. Beamstrahlung-related quantities derived in this paper are listed below the double horizontal line.

	LEP	LEP3	DLEP	STR1	STR2	STR3 cr-w	STR4 cr-w	STR5 cr-w	STR6 cr-w
$2E_0$, GeV	209	240	240	240	240	240	400	400	500
Circumference, km	27	27	53	40	60	40	40	60	80
Beam current, mA	4	7.2	14.4	14.5	23	14.7	1.5	2.7	1.55
Bunches/beam	4	3	60	20	49	15	1	1.4	2.2
N, 10^{11}	5.8	13.5	2.6	6	6	8.3	12.5	25.	11.7
σ_z, mm	16	3	1.5	3	3	1.9	1.3	1.4	1.9
ε_x, nm	48	20	5	23.3	24.6	3	2	3.2	3.4
ε_y, nm	0.25	0.15	0.05	0.09	0.09	0.011	0.011	0.017	0.013
β_x, mm	1500	150	200	80	80	26	20	30	34
β_y, mm	50	1.2	2	2.5	2.5	0.25	0.2	0.32	0.26
σ_x, μm	270	54	32	43	44	8.8	6.3	9.8	10.7
σ_y, μm	3.5	0.42	0.32	0.47	0.47	0.05	0.047	0.074	0.06
SR power, MW	22	100	100	100	100	100	100	100	100
Energy loss/turn, GeV	3.4	7	3.47	3.42	2.15	3.42	33.9	18.5	32.45
$\mathcal{L}$, 10^{34} cm^{-2}s^{-1}	0.013	1.3	1.6	1.7	2.7	17.6	4	7	2.2
$E_{c,\max}/E_0$, 10^{-3}	0.09	6.3	4.2	3.5	3.4	38	194	232	91
n_γ/electron	0.09	1.1	0.37	0.61	0.6	4.2	8.7	11.3	4.8
lifetime(SR@IP), s	$\sim\infty$	0.02	0.3	0.2	0.4	0.005	0.001	0.0005	0.005
$\mathcal{L}_{\rm corr}$, 10^{34} cm^{-2}s^{-1}	0.013	0.2	0.4	0.5	0.8	0.46	0.02	0.03	0.024

The critical energy of beamstrahlung is too high. If the electron loses more than about 1% of its energy (1.2% at LEP2), it will leave the beam. The beam lifetimes at these parameters are very short. One can decrease N and simultaneously increase the number of bunches, then L will be much lower (Lcorr, the last line).

Beam lifetime due to beamstrahlung

Electron loses the beam after emission of beamstrahlung photon with an energy greater than the threshould energy $E_{th}=\eta E_0$, where η~0.01. These photons have energies larger than the critical energy

$$E_c = \hbar\omega_c = \hbar\frac{3\gamma^3 c}{2\rho};$$

The spectrum per unit length at $u = E_\gamma / E_c >> 1$

$$\frac{dn}{dx} = \sqrt{\frac{3\pi}{2}}\frac{\alpha\gamma}{2\pi\rho}\frac{e^{-u}}{\sqrt{u}}du,$$

The number of photons on collision length *l* with $E_\gamma > \eta E_0$

$$n_\gamma(E_\gamma \geq \eta E_0) \approx \frac{\alpha^2\eta l}{\sqrt{6\pi}r_e\gamma u^{3/2}}e^{-u}; \quad u = \frac{\eta E_0}{E_c},$$

$l \approx \sigma_z / 2$ for head-on and $l \approx \beta_y / 2$ for crab-waist collisions

The beam lifetime depends exponentially on the critical energy (prop. to the beam field). And inversely, the critical energy (acceptable beam parameters, luminosity) depend logarithmically on the beam lifetime, therefore we can make useful approximations in calculation of the lifetime.

Let us assume that the electron crosses the region with strongest field with 10% probability. The average number of collisions n_{col} before an electron leaves the beam is found from $0.1 n_{col} n_{\gamma} = 1$, that gives

$$n_{\rm col} \approx 10 \frac{\sqrt{6\pi} r_e \gamma u^{3/2}}{\alpha^2 \eta l} e^u; \qquad \tau = n_{\rm col} \frac{2\pi R}{c}.$$

Assuming (in front of the exponent) E_0=150 GeV, l=1 mm, η=0.01, 2πR=50 km and the beam lifetime 30 min we get

$$u = \eta E_0 / E_{\rm c} \approx 8.5; \qquad E_{\rm c} \approx 0.12 \eta E_0 \sim 0.1 \eta E_0.$$ *

Note, the accuracy of this expression is quite good for any ring collider, because it depends logarithmically on collider parameters.

The maximum (effective, E+B) field for Gaussian beams $B \approx 2eN/\sigma_x \sigma_z$, then the (max) critical energy

$$\frac{E_{\rm c}}{E_0} = \frac{3\gamma {r_e}^2 N}{\alpha \sigma_x \sigma_z}.$$

Eq. * imposes the following restriction on the beam parameters:

$$\frac{N}{\sigma_x \sigma_z} < 0.1\eta \frac{\alpha}{3\gamma {r_e}^2} \qquad (**)$$

This formula is the basis for the following discussions.

In collision of Gaussian beams the average number of beamstrahlung photons $\langle n_\gamma \rangle = 2.12 \; N\alpha r_e / \sigma_x$, their average energy $\langle E_\gamma \rangle \approx 0.31 \langle E_c \rangle$ and $\langle E_c \rangle \approx 0.42 E_{c,\max}$, that gives $\langle E_\gamma \rangle \approx 0.13 E_{c,\max}$

Hence, the beam lifetime is determined by photons with energy 8.5/0.13 = 65 times greater than $\langle E_\gamma \rangle$.

With account of (**)

$$\langle n_\gamma \rangle = \frac{0.07\eta\alpha^2\sigma_z}{r_e\gamma} \approx \frac{0.067(\sigma_z/\mathrm{mm})}{(E_0/100\,\mathrm{GeV})}\left(\frac{\eta}{0.01}\right)$$

$$\langle E_\gamma \rangle \approx 0.13 \times 0.1\eta E_0 \approx 1.3 \times 10^{-4} E_0 \left(\frac{\eta}{0.01}\right)$$

The beam energy spread

The energy spread due to beamstrahlung

$$\frac{\sigma_E^2}{E_0^2} = \frac{\tau_s}{4E_0^2}\dot{n}_\gamma \langle E_\gamma^2 \rangle$$

where the damping time (due to radiation in bending magnets)

$$\tau_s \approx T_{rev}E_0/\Delta E_{rev}, \quad \dot{n}_\gamma = \langle n_\gamma \rangle / T_{rev}; \qquad \langle E_\gamma^2 \rangle \approx 4.3 \langle E_\gamma \rangle^2$$

which gives with account of (**)

$$\frac{\sigma_E^2}{E_0^2} \approx \frac{\langle n_\gamma \rangle \langle E_\gamma \rangle^2}{E_0 \Delta E_{rev}} \approx \frac{1.15 \cdot 10^{-9} (\sigma_z / \mathrm{mm})}{(\mathrm{E}_0 / 100\ \mathrm{GeV})(\Delta \mathrm{E}_{rev} / E_0)} \left(\frac{\eta}{0.01} \right)^3$$

For E_0=120 GeV, σ_z=5 mm, $\Delta E_{rev}/E_0$=0.05

$$\left(\frac{\sigma_E}{E} \right)_{BS} \approx 3 \cdot 10^{-4} \left(\frac{\eta}{0.01} \right)^{3/2}$$

The energy spread due to radiation in bending magnets

$$\left(\frac{\sigma_E^2}{E_0^2}\right)_{\mathrm{SR}} = \frac{55\sqrt{3}}{128\pi\alpha J_{\mathrm{s}}}\,\frac{mc^2}{E_0}\,\frac{\Delta E_{\mathrm{rev}}}{E_0} = \frac{0.016}{J_{\mathrm{s}} E_0(\,\mathrm{GeV})}\,\frac{\Delta E_{\mathrm{rev}}}{E_0}$$

For E_0=120 GeV, $\Delta E_{rev}/E_0$=0.05, J_s=1.5

$$\left(\frac{\sigma_E}{E_0}\right)_{SR} \approx 2\cdot 10^{-3}$$

For the given example, the energy spread due to BS dominates at η>0.035 (if the lifetime due to single beamstrahlung is kept about 30 min).

The lifetime due to the beam energy spread

In order to have acceptable lifetime due to the energy spread one needs $\eta > 6(\sigma_E / E)_{BS.}$

If the lifetime due to single beamstrahlung is kept large (30 min) (condition **), then the BS energy spread contributes to the lifetime only when (follows from previous formulas)

$$\eta > \frac{2.5}{\sigma_z(\mathrm{mm})}\left(\frac{\Delta E_{\mathrm{rev}}}{10\,\mathrm{GeV}}\right)$$

For typical cases this value is very large ($\eta > 0.3$) compared with a reasonable $\eta \sim 0.01\text{-}0.03$. Therefore the beam lifetime due to beamstrahlungis is always determined by emission of single photons.

Head-on and "crab-waist" collision schemes

Below we consider two collision schemes: head-on and crab-waist.
In the crab-waist scheme the beams collide at an angle $\theta >> \sigma_x / \sigma_z$
This scheme allows a higher luminosity, if it is determined by the tune shift (beam-beam strength parameter).
For head-on collisions the tune shift ($\xi_y \le 0.1 - 0.15$) and the luminosity

(1) $$\xi_y = \frac{N r_e \beta_y}{2\pi\gamma\sigma_x\sigma_y} \approx \frac{N r_e \sigma_z}{2\pi\gamma\sigma_x\sigma_y} \quad \text{for} \quad \beta_y \approx \sigma_z \qquad \mathcal{L} \approx \frac{N^2 f}{4\pi\sigma_x\sigma_y} \approx \frac{N f \gamma \xi_y}{2 r_e \sigma_z}$$

For the crab-waist scheme

(2) $$\xi_y = \frac{N r_e \beta_y^2}{\pi\gamma\sigma_x\sigma_y\sigma_z} \quad \text{for} \quad \beta_y \approx \sigma_x/\theta \qquad \mathcal{L} \approx \frac{N^2 f}{2\pi\sigma_y\sigma_z\theta} \approx \frac{N^2 \beta_y f}{2\pi\sigma_x\sigma_y\sigma_z} \approx \frac{N f \gamma \xi_y}{2 r_e \beta_y}$$

In the crab-waist scheme one can make $\beta_y << \sigma_z$, therefore the luminosity is higher. Nf is determined by SR power. The only free parameters in L are σ_z (for head-on) and β_y (crab-waist), they are constrained by beamstrahlung condition

(3) $$\frac{N}{\sigma_x\sigma_z} < 0.1\eta \frac{\alpha}{3\gamma r_e{}^2}$$

Comparing (1),(2),(3) one can find the minimum beam energy when beamstrahlung becomes important.

For head-on collisions

$$\gamma_{\min} = \left(\frac{0.1\eta\alpha\sigma_z^2}{6\pi r_e \xi_y \sigma_y}\right)^{1/2} \propto \frac{\sigma_z^{3/4}}{\xi_y^{1/2}\varepsilon_y^{1/4}}$$

For "crab-waist" collisions

$$\gamma_{\min} = \left(\frac{0.1\eta\alpha\beta_y^2}{3\pi r_e \xi_y \sigma_y}\right)^{1/2} \propto \frac{2^{1/2}\beta_y^{3/4}}{\xi_y^{1/2}\varepsilon_y^{1/4}}$$

In the crab-waist scheme the beamstrahlung becomes important at much low energies because $\beta_y << \sigma_z$. For typical values of parameters in Table 1 E_{min}>70 GeV for head-on collisions and E_{min}>20 GeV for "crab-waist".

For considered colliders with $2E_0$>240 GeV beamstrahlung is important in both schemes.

Luminosities with account of beamstrahlung

For head-on collisions

$$\mathcal{L} \approx \frac{(Nf)N}{4\pi\sigma_x\sigma_y}, \; \xi_y \approx \frac{Nr_e\sigma_z}{2\pi\gamma\sigma_x\sigma_y}, \; \frac{N}{\sigma_x\sigma_z} \equiv k \approx 0.1\eta\frac{\alpha}{3\gamma {r_e}^2} \qquad \sigma_y \approx \sqrt{\varepsilon_y\sigma_z}$$

This can be rewritten as

$$\mathcal{L} \approx \frac{(Nf)k\sigma_z}{4\pi\sigma_y}, \qquad \xi_y \approx \frac{kr_e\sigma_z^2}{2\pi\gamma\sigma_y}, \qquad \sigma_y \approx \sqrt{\varepsilon_y\sigma_z}.$$

One can see that in beamstrahlung dominated regime the luminosity is proportional to the bunch length and its maximum value is determined by the tune shift. Together these equations give

$$\mathcal{L} \approx \frac{Nf}{4\pi}\left(\frac{0.1\eta\alpha}{3}\right)^{2/3}\left(\frac{2\pi\xi_y}{\gamma r_e^5\varepsilon_y}\right)^{1/3}$$

$$\sigma_{z,\mathrm{opt}} = \varepsilon_y^{1/3}\left(\frac{6\pi\gamma^2 r_e\xi_y}{0.1\eta\alpha}\right)^{2/3}$$

Luminosities with account of beamstrahlung

Similarly for the crab-waist collisions

$$\mathcal{L} \approx \frac{(Nf)N\beta_y}{2\pi\sigma_x\sigma_y\sigma_z}, \ \xi_y \approx \frac{N r_e \beta_y^2}{\pi\gamma\sigma_x\sigma_y\sigma_z}, \ \frac{N}{\sigma_x\sigma_z} \equiv k \approx 0.1\eta\frac{\alpha}{3\gamma r_e{}^2} \qquad \sigma_y \approx \sqrt{\varepsilon_y\beta_y}$$

Substituting, we obtain

$$\mathcal{L} \approx \frac{(Nf)k\beta_y}{2\pi\sigma_y}, \qquad \frac{k r_e \beta_y^2}{\pi\gamma\sigma_y} \approx \xi_y, \qquad \sigma_y \approx \sqrt{\varepsilon_y\beta_y}$$

These relations are similar to those for head on collisions if to replace σ_z by β_y and k by 2k. The corresponding solutions are

$$\mathcal{L} \approx \frac{Nf}{4\pi}\left(\frac{0.2\eta\alpha}{3}\right)^{2/3}\left(\frac{2\pi\xi_y}{\gamma r_e^5\varepsilon_y}\right)^{1/3}$$

$$\beta_{y,\mathrm{opt}} = \varepsilon_y^{1/3}\left(\frac{3\pi\gamma^2 r_e\xi_y}{0.1\eta\alpha}\right)^{2/3}$$

In the beamstrahlung dominated regime the luminosities in crab-waist and head-on collisions are practically the same (difference $2^{2/3}$) !

As soon as the crab-waist gives no profit at high energies, further we will consider only the head-on scheme.

The optimum bunch length in practical unites

$$\frac{\sigma_{z,\mathrm{opt}}}{\mathrm{mm}} \approx \frac{2\xi_y^{2/3}}{\eta^{2/3}} \left(\frac{\varepsilon_y}{\mathrm{nm}}\right)^{1/3} \left(\frac{E_0}{100\,\mathrm{GeV}}\right)^{4/3};$$

typically from several mm to cm

The maximum luminosity with account of beamstranlung

$$\mathcal{L} \approx h\frac{N^2 f}{4\pi\sigma_x\sigma_y} = h\frac{Nf}{4\pi}\left(\frac{0.1\eta\alpha}{3}\right)^{2/3}\left(\frac{2\pi\xi_y}{\gamma r_e^5\varepsilon_y}\right)^{1/3}$$

where h is the hourglass loss factor, $f=n_b c/2\pi R$. SR power in rings

$$P = 2\delta E\frac{cNn_{\mathrm{b}}}{2\pi R} = \frac{4e^2\gamma^4 cNn_{\mathrm{b}}}{3RR_{\mathrm{b}}}$$

Finally, the luminosity

$$\mathcal{L} \approx h\frac{(0.1\eta\alpha)^{2/3}PR}{32\pi^2\gamma^{13/3}r_e^3}\left(\frac{R_{\mathrm{b}}}{R}\right)\left(\frac{6\pi\xi_y r_e}{\varepsilon_y}\right)^{1/3}$$

In practical units

$$\frac{\mathcal{L}}{10^{34}\,\mathrm{cm}^{-2}\mathrm{s}^{-1}} \approx \frac{100h\eta^{2/3}\xi_y^{1/3}}{(E_0/100\,\mathrm{GeV})^{13/3}(\varepsilon_y/\,\mathrm{nm})^{\frac{1}{3}}}\left(\frac{P}{100\,\mathrm{MW}}\right)\left(\frac{2\pi R}{100\,\mathrm{km}}\right)\frac{R_{\mathrm{b}}}{R}$$

The beamstrahlung suppresses the luminosity by a factor $\sigma_z/\sigma_{opt}=(E_{min}/E_0)^{4/3}$ for the energies above E_{min}, which is about 70 GeV for head-on and 20 GeV for crab-waist schemes.

Beamstrahlung and the tune-shift determine σ_z and the combination N/σ_x. Assuming P=100 MW, h=0.8, ξ_y =0.15, η=0.01 and other parameters from Table 1 we obtain optimized parameters for projects from Table 1:

TABLE II. Realistically achievable luminosities and other beam parameters for the projects listed in Table I at synchrotron-radiation power $P = 100$ MW. Only the parameters that differ from those in Table I are shown.

	LEP	LEP3	DLEP	STR1	STR2	STR3 cr-w	STR4 cr-w	STR5 cr-w	STR6 cr-w
$2E_0$, GeV	209	240	240	240	240	240	400	400	500
Circumference, km	27	27	53	40	60	40	40	60	80
Bunches/beam	~ 2	~ 7	70	24	53	240	36	45	31
N, 10^{11}	33	5.9	2.35	3.9	4.	0.4	0.34	0.6	0.65
σ_z, mm	8.1	8.1	5.7	6.9	6.9	3.4	6.7	7.8	9.6
σ_y, μm	1.4	1.1	0.53	0.78	0.78	0.19	0.27	0.36	0.35
$\mathcal{L}$, 10^{34} cm^{-2}s^{-1}	0.47	0.31	0.89	0.55	0.83	1.1	0.12	0.16	0.087

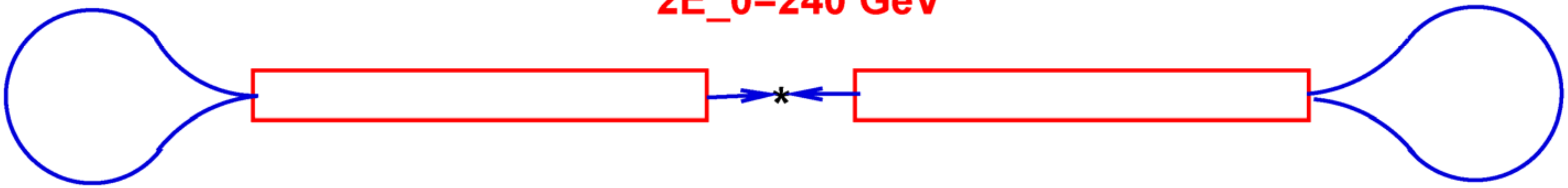
From $$L \sim \frac{\eta^{2/3}\xi_y^{1/3}}{\varepsilon^{1/3}} R$$ one can see the ways to increase L.

Thus, the luminosity of linear colliders is limited by wall-plug power, they are not energy-effective because each bunch is used only once.

The luminosity of high energy storage rings is also determined by wall-plug power due to severe synchrotron radiation.

Is there any solution of the problem?

CW Linear collider with a recuperation?



If η is the energy acceptance of the ring, the maximum energy of beamstrahlung photons should be ηE (not ηE_0). This reduce L by a factor of $(E/E_0)^{2/3}$~0.25. However, due to much lower SR losses (E^4/R) one can increase Nf by a very large factor and thus to increase the luminosity by 1-2 orders of magnitude ($>10^{35}$).

Unfortunately, there are many stoppers which kill this scheme:

1. Refrigeration power is about 150-200 MW (accel. grad. ~15 MeV/m, $Q=2\cdot10^{10}$)
2. Parasitic collision of beams inside the linac. One can separate beams (pretzel scheme), but the beam attraction leads to the beam instability.
3. The transverse wake field problem for beams shifted from the axis.
4. The energy difference between the head and tail becomes unacceptable after deceleration (beam loading helps during acceleration, but makes worse during deceleration).

That is a good idea, but technically impossible. LC schemes with recuperation were considered in 1970's and were also rejected.

Conclusion

❖ Luminosities of high energy storage rings are limited by single beamstrahlung. The beam lifetime is large enough when the critical energy $E_{c,\,max} < 0.1\eta E_0$ (η is the ring energy acceptance).

❖ Luminosities for head-on and crab-waist schemes are similar.

❖ Attainable luminosities at “the Higgs energy” $2E_0$=240 GeV at e+e- storage rings and linear colliders are comparable: about 10^{34}.

❖ For $2E_0$=400-500 GeV the storage ring luminosities would be a factor of 15-25 smaller than desired (may be sufficient for very large rings).

❖SC LC with recuperation could have (in ideal) a higher luminosity, but technically unfeasible.