LBNL/SLAC Ring and Lattice Issues

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In collaboration with John Byrd, Alex Chao, Yuri Nosochkov, Uli Wienands, and Frank Zimmermann
Luminosity

• Bunch luminosity

\[ L_b = f_{rev} \frac{N_b^2}{4\pi\sigma_x\sigma_y} R_h \]

where \( R_h \) is a geometrical reduction from the hourglass effect and is written as

\[ R_h = \sqrt{\frac{2}{\pi}} ae^{a^2} K_0(a^2), a = \frac{\beta^*_y}{\sqrt{2} \sigma_z} \]

• Total luminosity

\[ L = n_b L_b \]
• Given the beam-beam parameter

\[ \xi_y = \frac{r e N_b \beta_{y}^*}{2\pi \gamma \sigma_y (\sigma_x + \sigma_y)} \]

The luminosity can be re-written as

\[ L = \frac{c I \gamma \xi_y}{2r^2 I_A \beta_{y}^*} R_h \]

where \( I_A = 17045 \text{ A} \). Smaller \( \beta_{y}^* \) is absolutely necessary. For example, in our design we have \( I = 7.2 \text{ mA}, E_0 = 120 \text{ GeV}, \xi_y = 0.07, R_h = 0.76, \beta_{y}^* = 1 \text{ mm} \), gives \( 1 \times 10^{34} \text{ cm}^{-2} \text{s}^{-1} \) in luminosity. So what is the beam-beam limit for Higgs factory?
Figure 1: Measured $\xi_y$ at 94.5 GeV versus bunch current. The data is fitted with ("Model fit") and without ("Linear fit") beam-beam limitation.

R. Assmann and K. Cornelis, Proceeding of EPAC 2000, Vienna, Austria
Power Limitation

• Synchrotron radiation

\[ U_0 = \frac{4\pi r e mc^2}{3 \gamma^4} \]

• Beam power given by RF

\[ P_b = U_0 l / e \]

• Limits the total beam current I

For example, \( E_0 = 120 \text{ GeV}, \rho = 2.6 \text{ km}, U_0 = 6.97 \text{ GeV}, \]
\( l = 7.2 \text{ mA}, \) lead to \( P_b = 50 \text{ MW} \) in our design.
Scaling of Luminosity

- If there is a beam-beam limit as suggested by the simulation and beam power is also limited, the luminosity can be re-written as

\[ L = \frac{3c}{8\pi r_e^3} \frac{\xi_y \rho}{\gamma^3 \beta_y^*} \frac{P_b}{P_A} R_h \]

where \( P_A = mc^2 I_A/e = 8.7 \text{ GW} \). This scaling was first given by B. Richter, Nucl. Instr. Meth. 136 (1976) 47-60.
Beamstrahlung Effects

• Beam lifetime due to large single photon emission (for 30 minutes, V.I. Telnov, 2012)

\[ \frac{N_b}{\sigma_x \sigma_z} < 0.1 \eta \frac{\alpha}{3 \gamma r_e^2} \]

• Large RF-buckets and large momentum aperture \( \eta \)
• Large \( \sigma_z \) and \( \sigma_x \). Favors longer and larger horizontal beam size.
• Limits bunch population \( N_b \)

Are there any reasonable solutions?
Analysis of Design Constraints

• To achieve the beam-beam parameter and assuming $\beta_y = \kappa \beta \beta_x$ and $\varepsilon_y = \kappa \varepsilon \varepsilon_x$ we have

$$\frac{N_b}{\varepsilon_x} = \frac{2\pi \gamma \xi_y}{r_e} \sqrt{\frac{\kappa}{\kappa_x}}$$

• To have adequate beam lifetime (due to beamstrahlung)

$$\frac{N_b}{\sqrt{\varepsilon_x}} < 0.1 \eta \frac{\alpha \sigma_z}{3 \gamma r_e^2} \sqrt{\frac{\beta^*_y}{\kappa \beta}}$$

• Clearly, smaller coupling $\kappa_e$ is better and larger momentum acceptance $\eta$ is better but they have their own limits.
Solution of the Constraints

• Given a momentum acceptance $\eta$, we solve

$$\varepsilon_x < \left( \frac{0.1\eta\alpha\sigma}{6\pi\gamma^2\xi_y r_e} \right)^2 \frac{\beta_y^*}{\kappa_e}$$

Note that it does not depend on $\kappa_\beta$. We can use $\kappa_\beta$ to adjust the bunch population $N_b$ or the number of bunches $n_b$. Clearly, there are many possible solutions. But is there any self-consistent one?

• The requirement of accommodating beamstrahlung is translated to design a low emittance lattice.

• Normally, the emittance scales as $\gamma^2$. This relation requires a scaling of $\gamma^4$, indicating a difficulty to design a machine with much higher energy than 120 GeV.
# LBNL/SLAC Design Parameters

<table>
<thead>
<tr>
<th></th>
<th>LEP2</th>
<th>LBNL/SLAC Design</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Beam Energy [GeV]</strong></td>
<td>104.5</td>
<td>120</td>
</tr>
<tr>
<td><strong>Circumference [km]</strong></td>
<td>26.7</td>
<td>26.7</td>
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<tr>
<td><strong>Beam current [mA]</strong></td>
<td>4</td>
<td>7.2</td>
</tr>
<tr>
<td><strong>Number of bunches</strong></td>
<td>4</td>
<td>50</td>
</tr>
<tr>
<td><strong>Bunch population [10^{10}]</strong></td>
<td>57.5</td>
<td>8.0</td>
</tr>
<tr>
<td><strong>Horizontal emittance [nm]</strong></td>
<td>48</td>
<td>4.3</td>
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<tr>
<td><strong>Vertical emittance [nm]</strong></td>
<td>0.25</td>
<td>0.0215</td>
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<tr>
<td><strong>Momentum compaction factor</strong></td>
<td>18.5x10^{-5}</td>
<td>2.4x10^{-5}</td>
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<tr>
<td><em><em>β_x</em> [mm]</em>*</td>
<td>1500</td>
<td>50</td>
</tr>
<tr>
<td><em><em>β_y</em> [mm]</em>*</td>
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<tr>
<td><strong>Hourglass factor</strong></td>
<td>0.98</td>
<td>0.76</td>
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<tr>
<td><strong>SR power [MW]</strong></td>
<td>11</td>
<td>50</td>
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<tr>
<td><strong>Bunch length [mm]</strong></td>
<td>16.1</td>
<td>1.5</td>
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<tr>
<td><strong>Beam-beam parameter</strong></td>
<td>0.07</td>
<td>0.07</td>
</tr>
<tr>
<td><strong>Luminosity [10^{34} cm^{-2}s^{-1}]</strong></td>
<td>0.0125</td>
<td>1.01</td>
</tr>
</tbody>
</table>
12 cells make an achromat (unit transformation).
Quasi (4\textsuperscript{th} Order) Achromat

\textbf{3\textsuperscript{rd} order driving terms}

\textbf{4\textsuperscript{th} order driving terms}

The cancellation occurs in every 12 cells in arcs. Only non-vanishing resonance is \(4\nu_x\).
Arc Design

90°/60° FODO Lattice

Dynamic/Momentum Aperture

Emittance:

\[ \varepsilon_x = F_c \frac{C_q \gamma^2}{J_x} \theta^3 \]

where \( \theta \) is the bending angle in a cell.

Natural emittance: \( \varepsilon_x = 4.3 \) nm and cell length is 28.375 m

Yunhai Cai, HF2012 Workshop

11/15/2012
Lattice of Collider Ring

at IP:
- $\beta_x = 50 \text{ mm}$
- $\beta_y = 1 \text{ mm}$
- $L^* = 4 \text{ m}$
Final Focus System

at IP:
\( \beta_x = 50 \text{ mm} \)
\( \beta_y = 1 \text{ mm} \)
\( L^* = 4 \text{ m} \)

\( \delta_{\phi\rho\xi} = 0. \)

Table name = TWISS
Local Chromatic Correction

Use two pairs of sextupole reaching residual of 0.05%.
Chromatic Beating in Collider Ring
Four families of sextupole used in the optimization, achieving ±2% momentum bandwidth.
Beta Functions at IP vs. Momentum

Good region of ±0.4% in $\Delta p/p$ is necessary for the core in beam distribution.
Beam Lifetime due to Beamstrahlung

Estimate for LBNL/SLAC ring based on Telnov’s paper.
Benchmark of Beam-Beam Code

3D PIC Simulation (BBI)

![Graph showing 3D PIC Simulation](image)

94.3 GeV

Measurement

![Graph showing Measurement](image)

Figure 1: Measured $\xi_y$ at 94.5 GeV versus bunch current. The data is fitted with ("Model fit") and without ("Linear fit") beam-beam limitation.

Parameters provided by
Frank Zimmermann
Helmut Burkhardt

http://hbu.web.cern.ch/hbu/BeamDyn/LEP.html

$\varepsilon_y = 0.19$ nm, without beam-beam perturbation

R. Assmann and K. Cornelis,
Proceeding of EPAC 2000, Vienna, Austria
Beam-beam limit can be as low as 0.05 if synchrotron tune is too high, even with very strong radiation damping.
Longitudinal Dynamics

• Synchrotron tune:

\[ \nu_s = \sqrt{\frac{\hbar \alpha_p}{2\pi}} \frac{eV_{RF}}{E_0} \cos \phi_s \]

to lower \( \nu_s \) one wants lower RF frequency and momentum compaction factor. This also makes RF bucket height

\[ \delta_{RF,max} = \sqrt{\frac{F(q)U_0}{\pi \hbar \alpha_p E_0}} \]

larger and allows a larger momentum acceptance.
Beam-Beam Simulation
LBNL/SLAC Design \( (f_{RF} = 700 \text{ MHz and } \alpha_p = 2.4 \times 10^{-5}) \)

Beam-beam parameter

Bunch luminosity

There is no surprise. It simply confirms calculations.
Lattice Issues

- Ultra-low beta (1mm) IR design with large momentum bandwidth (3%)
- Low emittance lattice and ultra-low beta IR with adequate dynamic aperture (10 $\sigma$)
- Large synchrotron radiation, saw-tooth (1-2%) in arcs of two beams in single ring
- Machine tolerances, especially alignment tolerance and orbit stability
Risks & Mitigations & RD Items

• Ultra-low beta* with large energy bandwidth in ring
  \( \beta_y^* = 1 \text{ mm and } \eta > 3\% \) (lower emittance)

• RF parameters: frequency, voltage, gradient?
  What’s length is necessary for RF system
  \( (f_{RF}=700 \text{ MHz and } V_{RF}=12 \text{ GV}) \) ?
  is HOM a problem?

• What is the shortest bunch we can make?
  Coherent synchrotron radiation, heating

• What is energy reach of ring? How large is can be?
  How about 80 km?
Summary

• Impact on design due to beamstrahlung is analyzed. We found a formula of minimum natural emittance that is necessary for beam lifetime.

• A systematic design procedure is outlined. There are many possible solutions. The final choice should be made with other considerations, including the interaction region design.

• We have achieved 2% momentum bandwidth in a lattice with an ultra-low beta interaction region.