Low emittance rings: Colliders

mainly based on experiences of KEKB and SuperKEKB design*)

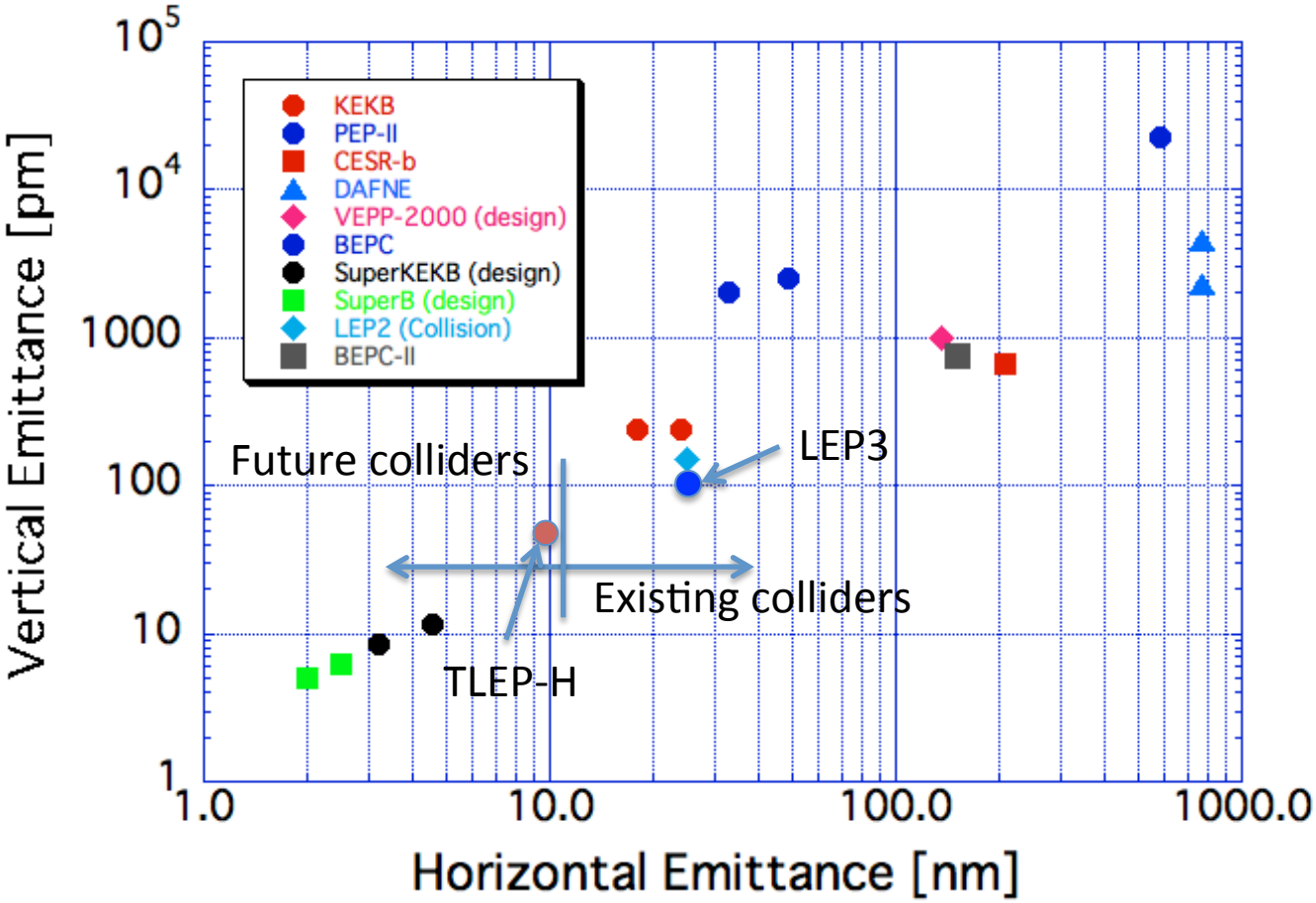
Y. Funakoshi

KEK

*) Based on the works done by KEK/SuperKEKB optics group
members: K. Oide, H. Koiso, Y. Ohnishi, A. Morita, H. Sugimoto

Comparison of emittances of colliders

Comparison of emittances of colliders



From Beam Dynamics Newsletter No. 31
Courtesy of F. Zimmermann, H. Burkhardt and Q. Qin

LEP3/TLEP parameters -1

F. Zimmmerman

	LEP2	LHeC	LEP3	TLEP-Z	TLEP-H	TLEP-t
beam energy E_b [GeV]	104.5	60	120	45.5	120	175
circumference [km]	26.7	26.7	26.7	80	80	80
beam current [mA]	4	100	7.2	1180	24.3	5.4
#bunches/beam	4	2808	4	2625	80	12
#e-/beam [10^{12}]	2.3	56	4.0	2000	40.5	9.0
horizontal emittance [nm]	48	5	25	30.8	9.4	20
vertical emittance [nm]	0.25	2.5	0.10	0.15	0.05	0.1
bending radius [km]	3.1	2.6	2.6	9.0	9.0	9.0
partition number J_ε	1.1	1.5	1.5	1.0	1.0	1.0
momentum comp. α_c [10^{-5}]	18.5	8.1	8.1	9.0	1.0	1.0
SR power/beam [MW]	11	44	50	50	50	50
β^*_x [m]	1.5	0.18	0.2	0.2	0.2	0.2
β^*_y [cm]	5	10	0.1	0.1	0.1	0.1
σ^*_x [μm]	270	30	71	78	43	63
σ^*_y [μm]	3.5	16	0.32	0.39	0.22	0.32
hourglass F_{hg}	0.98	0.99	0.59	0.71	0.75	0.65
ΔE^{SR}_{loss}/turn [GeV]	3.41	0.44	6.99	0.04	2.1	9.3

EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

CERN – SL DIVISION

CERN-SL-2000-078-OP

Emittance optimization with dispersion free steering at LEP

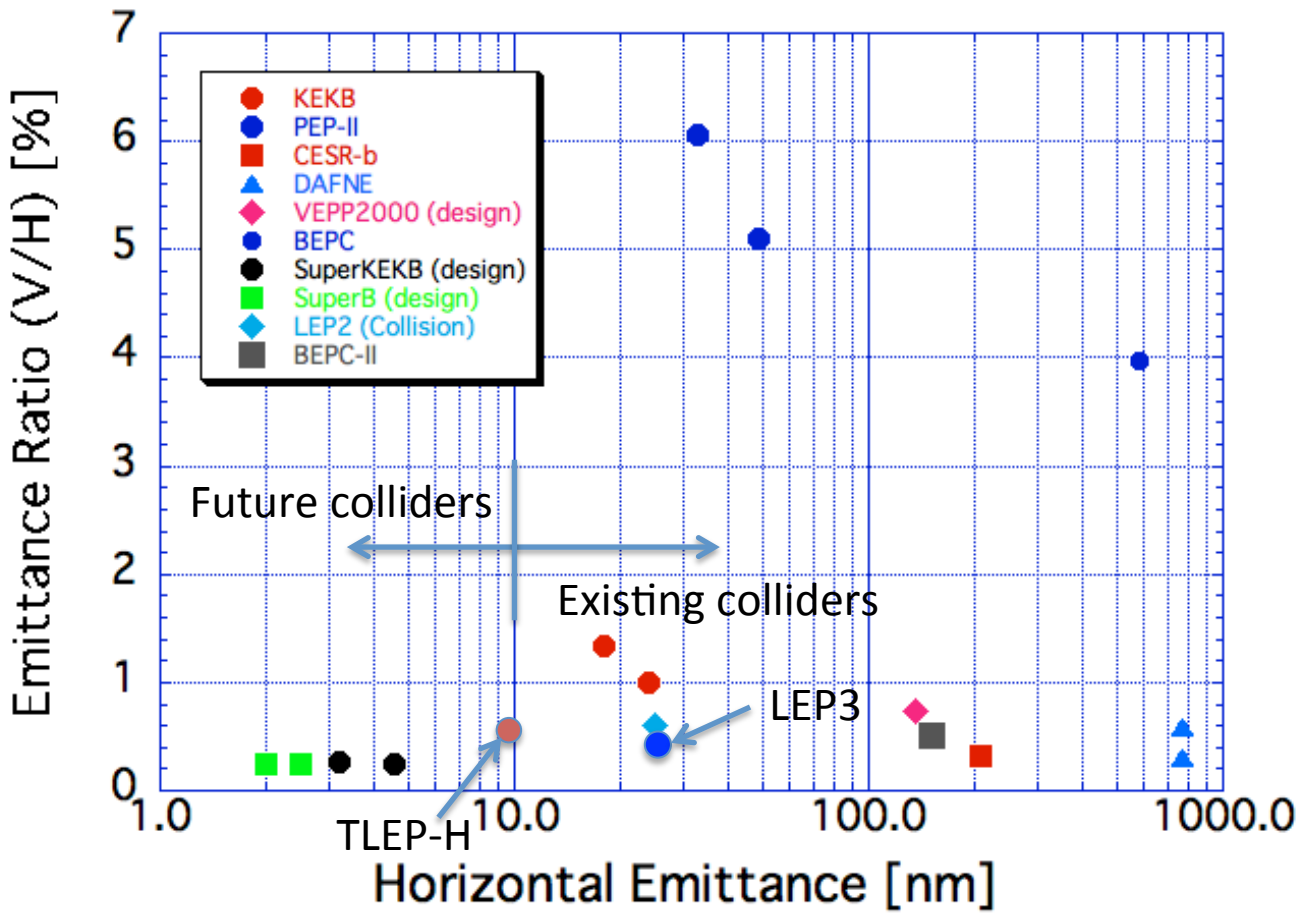
R. Assmann, P. Raimondi *, G. Roy, J. Wenninger

Abstract

Residual vertical dispersion can be a significant performance limitation for the LEP collider because the associated vertical emittance increase reduces the luminosity of the machine. To make the search for orbits yielding small vertical emittances fast and deterministic, a simultaneous correction of the closed orbit and the residual dispersion was implemented at LEP. The principle of the correction and the resulting performance gains are discussed.

Published in Phys. Rev. ST Accel. Beams 3, 121001 (2000)

Comparison of emittances of colliders [cont'd]

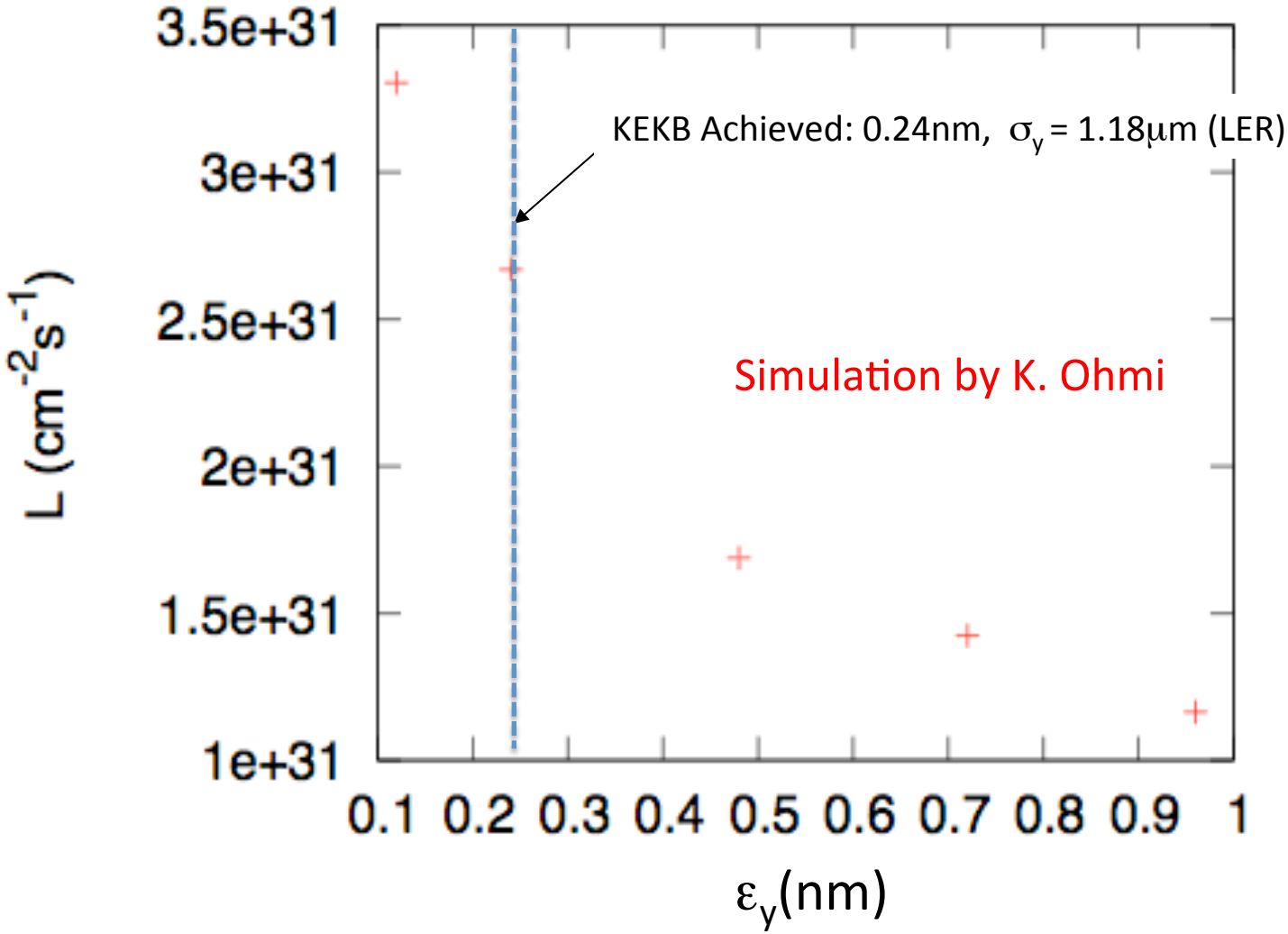


Necessity of low emittance in colliers

Importance of vertical emittance

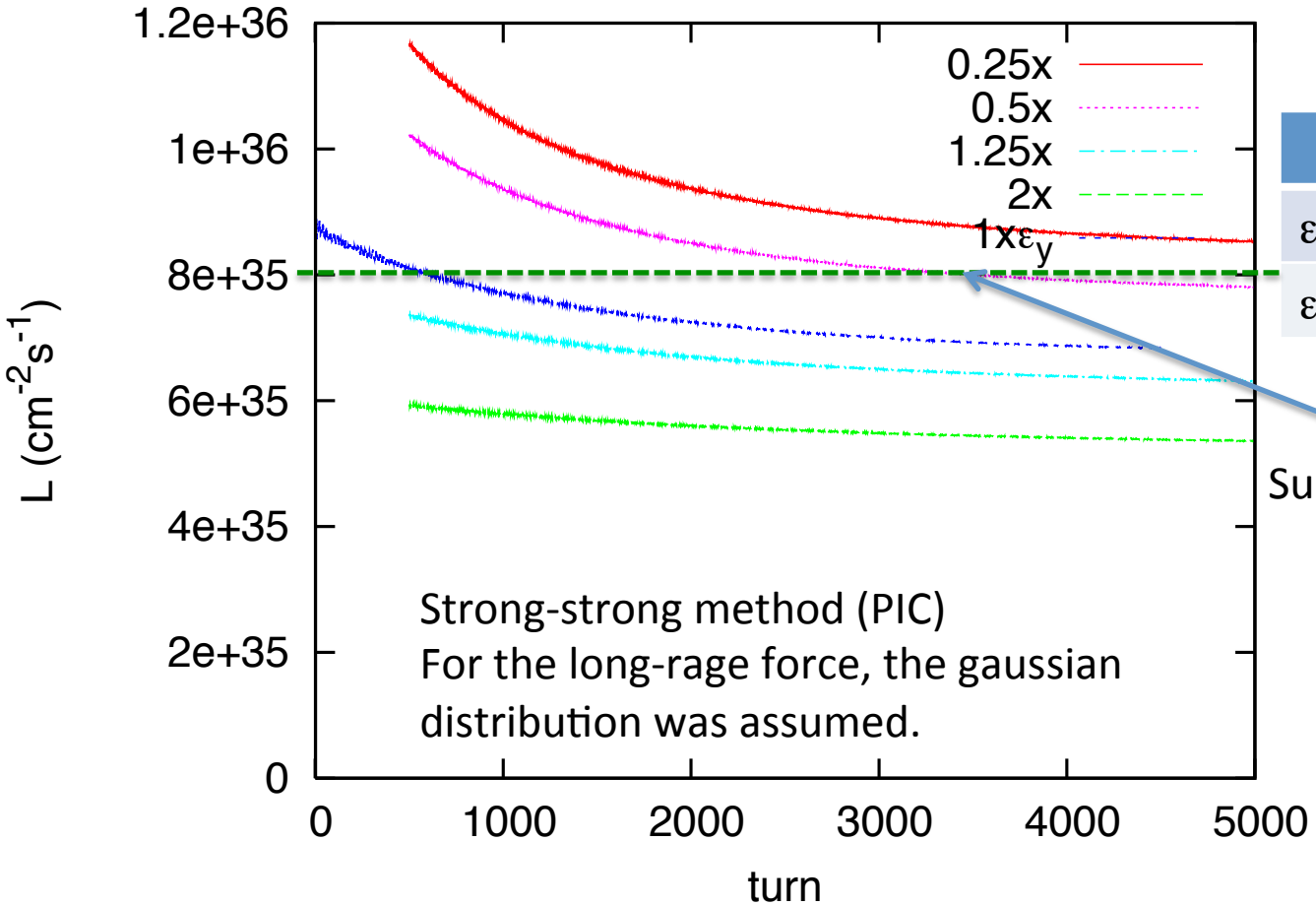
- In many high luminosity colliders such as KEKB and PEP-II, the machines were operated well above the beam-beam limit. Usually the vertical beam-beam limit is lower than the horizontal one.
- Even above the beam-beam limit, the luminosity increases linearly as function of the beam currents.
- In this situation, it is commonly considered that the vertical emittance in single beam is not important for the luminosity, since the vertical emittance is determined by the beam-beam blowup.
- However, the beam-beam simulation predicts that the smaller vertical emittance in single beam is important for a higher luminosity.
- In some machines such as SuperKEKB, SuperB and TLEP, a very low vertical emittance is necessary to achieve the design beam-beam parameter.

Simulation: KEKB Luminosity vs. Vertical emittance (single beam)



- Smaller vertical emittance always gives better performance.

SuperKEKB beam-beam simulation



SuperKEKB design

	LER	HER
ε_x [nm]	3.2	4.6
ε_y [pm]	8.64	11.5

SuperKEKB design luminosity

Smaller vertical emittance gives higher luminosity.
Much lower vertical emittance than the design will be needed to achieve the design luminosity.

K. Ohmi

Methods for low emittances

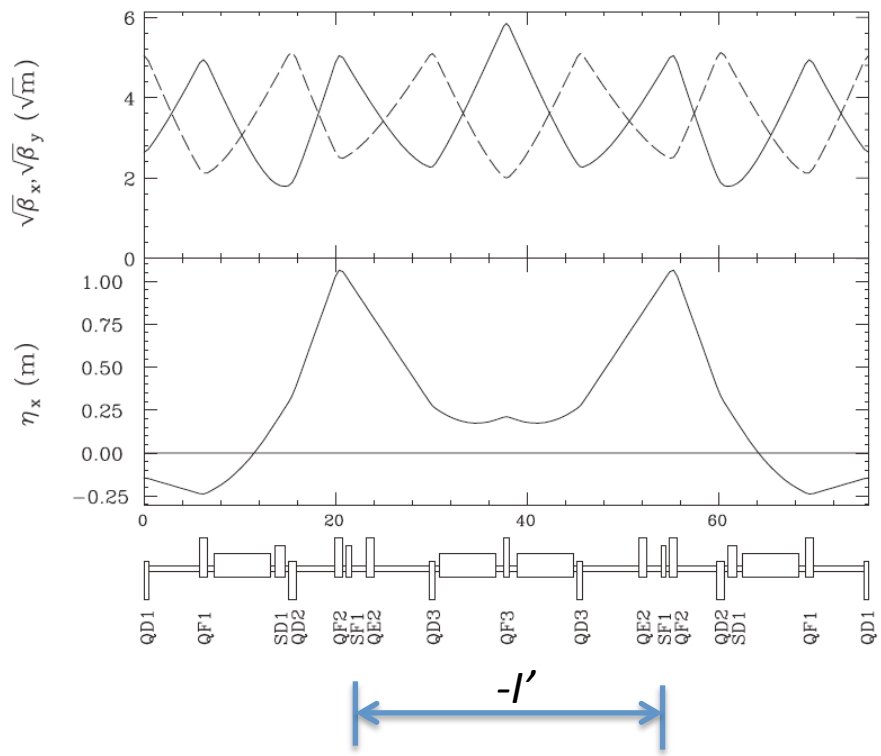
Horizontal emittance

- Suppress radiation excitation
  - Photon spectrum (longer bending magnets)
  - Reduce the integral of *H*
    - Low emittance lattice
      - phase advance of FODO cell, special lattice
- Enhance radiation damping
  - Damping wiggler
  - Damping partition number
    - Ex. TRISTAN(Δf_{RF} = +3kHz -> εx = 165nm -> 80nm)

$$\varepsilon_x = \frac{C_\gamma}{J_x}\gamma^2 \oint \frac{H}{|\rho^3|} ds \Big/ \oint \frac{ds}{\rho^2} \qquad H = \gamma_x \eta_x^2 + 2\alpha_x \eta_x \eta_x' + \beta_x \eta_x'^2$$

2.5 π cell lattice (KEKB, SuperKEKB)

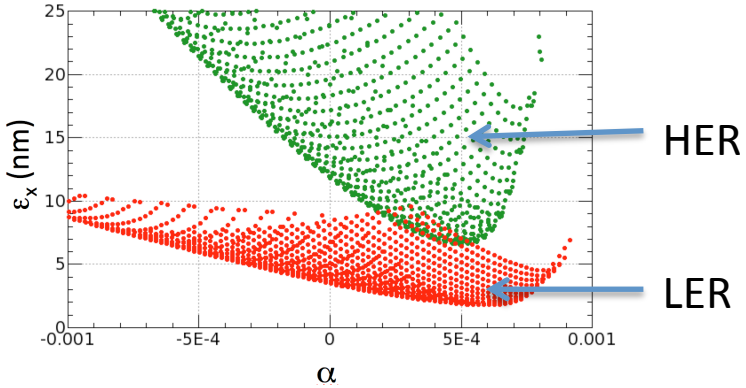
- Five $\pi/2(90°)$ cells are merged. Ten dipole magnets are reduced to four.
- 13 quadruple magnets/cell.
- Emittance and α (momentum compaction) are tunable by changing the pattern of dispersion in wide range.
- Non-interleaved sextupole scheme was adopted. A sextuple pair is connected by $-I'$ transformer.



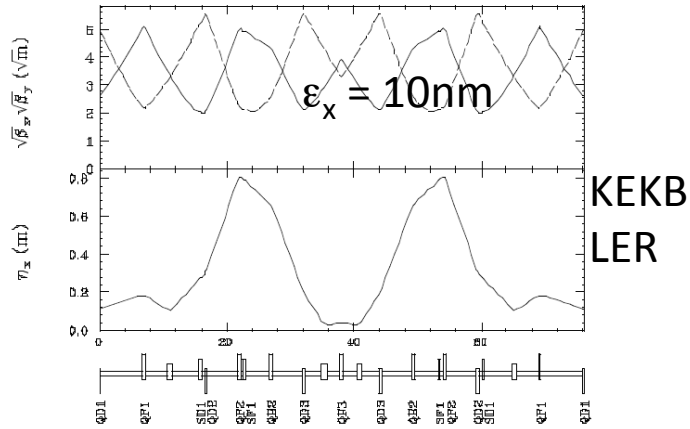
$$-I' = \begin{pmatrix} -1 & 0 & 0 & 0 \\ m_{21} & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & m_{42} & -1 \end{pmatrix}$$

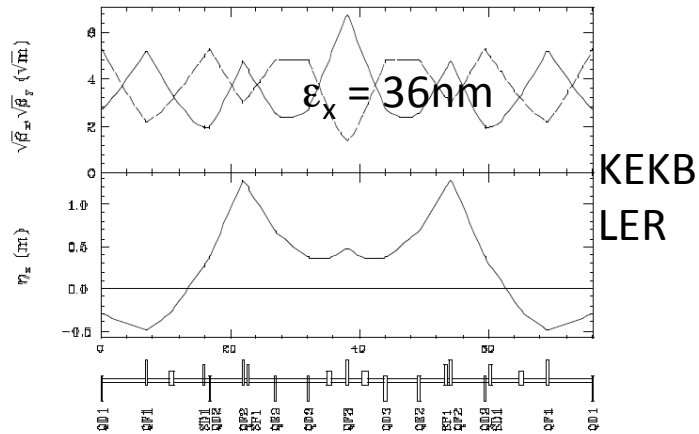
Tunability of 2.5 π cell lattice

- Tunability of emittance and α



$\varepsilon_x$and α: tunability (SuperKEKB)





●In SuperKEKB, emittance is minimized by making use of this tunability.

●α can be negative

$$\alpha = \frac{1}{C}\oint \frac{\eta}{\rho} ds$$

Vertical emittance

- Vertical emittance (single beam, zero current)

$$\varepsilon_y = \kappa\varepsilon_x + A\left(\eta_y^{rms}\right)^2 + \varepsilon_y^{OA}$$

coupling dispersion opening angle

  - x-y coupling
    - Machine errors (such as mis-alignment of Q or SX mganets.)
      - The coupling correction can reduce residual coupling value.
  - Vertical dispersion
    - Machine errors (such as mis-alignment of Q or SX mganets.)
      - The dispersion correction can reduce residual dispersions.
    - Vertical dispersion in design
      - Fringe field of detector solenoid, vertical bending magnets
  - Opening angle of radiation
    - Usually negligible (or ultimate limit of vertical dispersion)
      - ε_y ~ 1 fm (LEP3)

Issues

Issues of low emittance colliders

- Vertical emittance in design
- Machine errors
  - Optics correction (Low emittance tuning)
  - Optics correction with large energy saw-tooth -> skip
- Beam-beam blowup -> skip
- Intra-beam scattering -> skip
- Instabilities -> skip
  - Blowup due to electron clouds
- Touschek lifetime -> skip
- Maintenance of beam collision -> skip

Issue 1
Vertical emittance in design

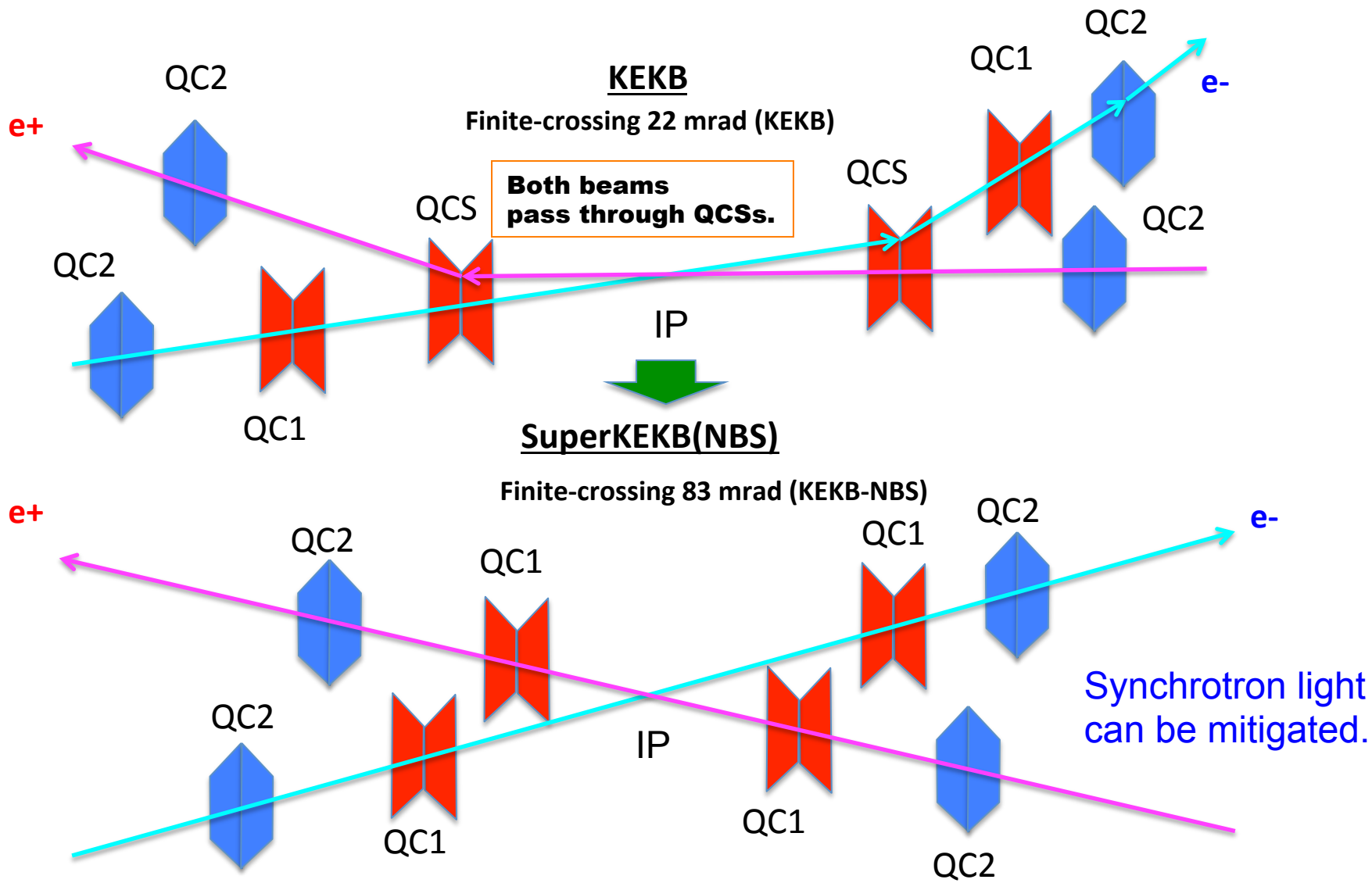
Vertical Emittance originated from Solenoid Fringe (SuperKEKB)

- The vertical kick by fringe field of detector solenoid could create a large vertical emittance.

$$\varepsilon_y \propto \int \frac{H}{\rho^3} ds \qquad \frac{1}{\rho} \propto B_x \cong -\frac{1}{2} x B_z' \cong -\frac{1}{2} s\phi B_z'$$

- Efforts to decrease vertical emittance from this process
  - Adjustment of solenoid axis -> rotation of Belle detector
  - Effort to avoid steep slope of solenoid by careful design of compensation solenoid magnets
  - Iron shield at the tail part of solenoid field
  - Improvement of modeling of solenoid field
- Latest design values of vertical emittance(zero current)
  - ε_y ~ 0.77pm (LER), ε_y ~1.5pm (HER)

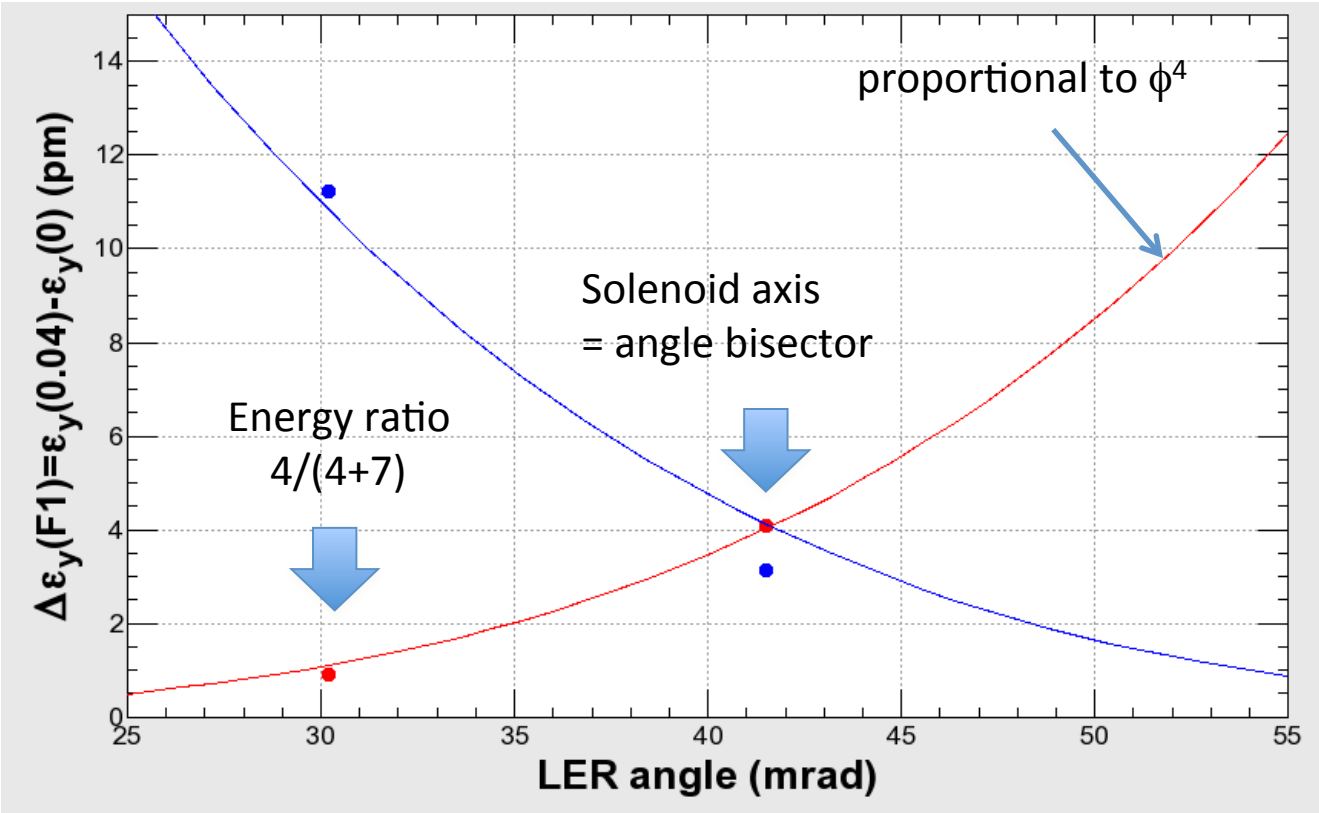
Layout of final focus Quads (KEKB/SuperKEKB)



Each beam passes through centers of quadrupoles.

Vertical Emittance originated from Solenoid Fringe

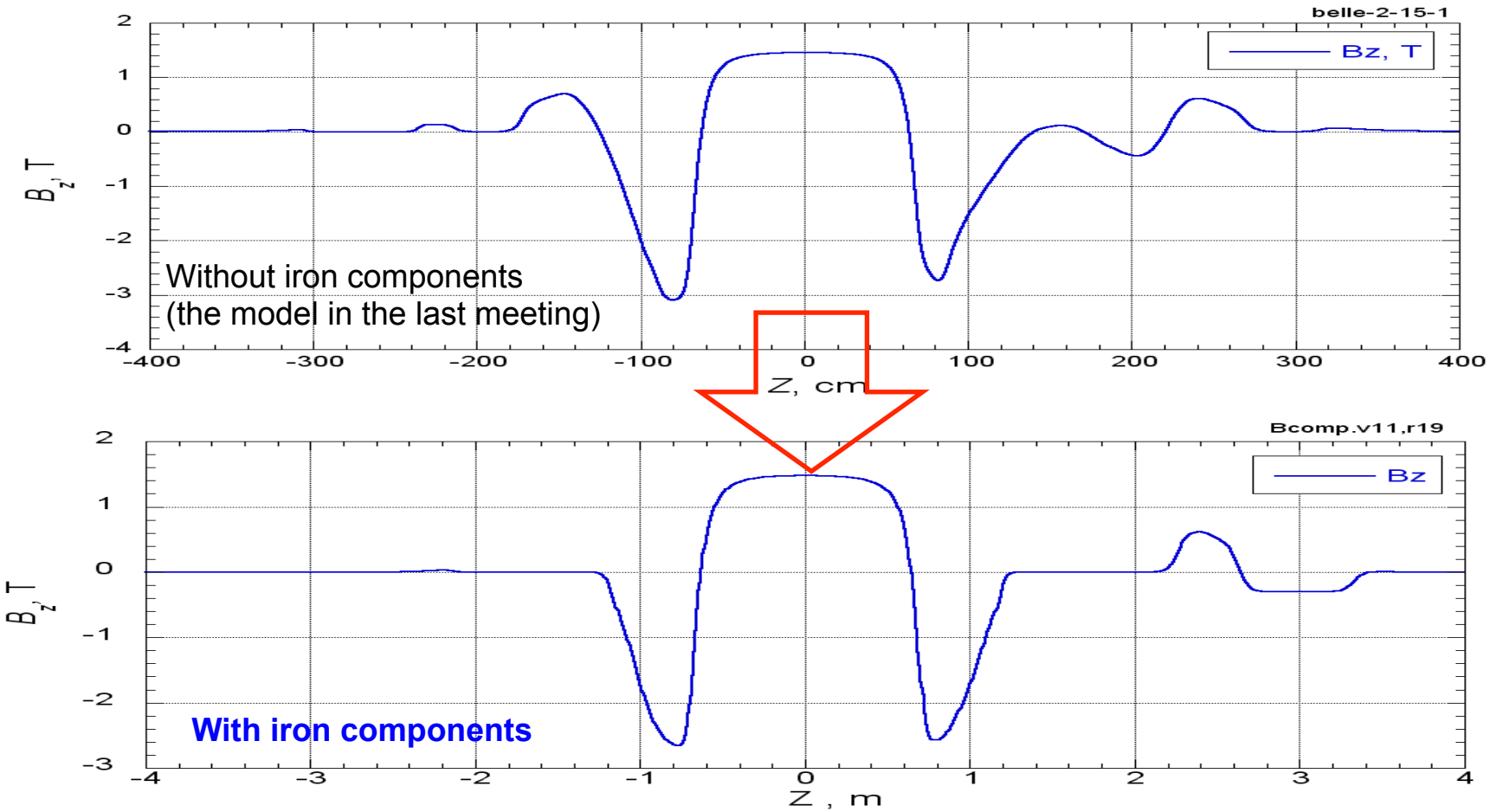
Crossing angle = LER angle + HER angle = 83 mrad



We choose 41.5 mrad and the contribution of solenoid fringe is ~4 pm.

Compensation Solenoids

- Magnetic design of compensation solenoids with iron components
  - By introducing iron components on the beam lines, the solenoid profile was changed.



Issue 2
Optics correction
(Low emittance tuning)

Sextuple mis-alingnment

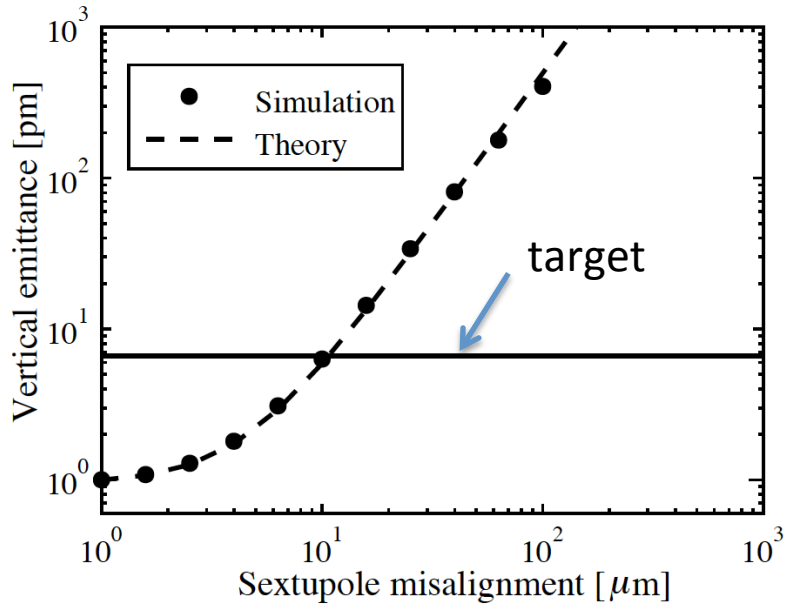
$$\frac{\langle \Delta\varepsilon_y \rangle}{\langle \Delta y_{\text{sext}}^2 \rangle} \cong \frac{J_x}{J_y} \frac{1-\cos 2\pi\nu_x \cos 2\pi\nu_y}{4\left(\cos 2\pi\nu_x - \cos 2\pi\nu_y\right)^2} \varepsilon_x \sum_{\text{sext}} \beta_x \beta_y (k_2 L)^2$$

coupling

$$+ J_z \frac{\sigma_\delta^2}{4\sin^2 \pi\nu_x} \sum_{\text{sext}} \eta_x^2 \beta_y (k_2 L)^2$$

dispersion

ν_x ~0.53
ν_y ~0.57



SuperKEKB HER simulation

Random errors with gaussian distribution
Emittance values are average of 100 random samples.

Typical value of mis-alignment: ~ 100μm
-> We need correction.
-> low emittance tuning

Quadruple mis-alingnment

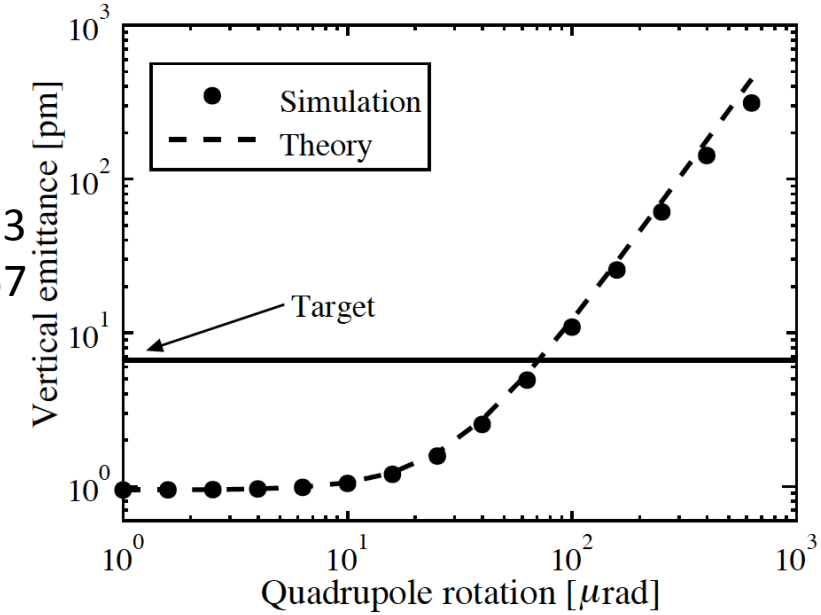
$$\frac{\langle \Delta\varepsilon_y \rangle}{\langle \Delta\theta^2_{\text{quad}} \rangle} \cong \frac{J_x}{J_y} \frac{1-\cos 2\pi\nu_x \cos 2\pi\nu_y}{\left(\cos 2\pi\nu_x - \cos 2\pi\nu_y\right)^2} \varepsilon_x \sum_{\text{quad}} \beta_x \beta_y (k_1 L)^2$$

← coupling

$$+ J_z \frac{\sigma_\delta^2}{\sin^2 \pi\nu_x} \sum_{\text{quad}} \eta_x^2 \beta_y (k_1 L)^2$$

← dispersion

ν_x ~0.53
ν_y ~0.57



SuperKEKB HER simulation

Random errors with gaussian distribution
Emittance values are average of 100 random samples.

Typical value of mis-alignment: ~ 100μrad
-> Quadrupole rotational errors are less serious than the sextuple mis-alignments.

Comparison of coupling term and dispersion term

ν_x ~0.53
ν_y ~0.57

HER(Quad rotation)

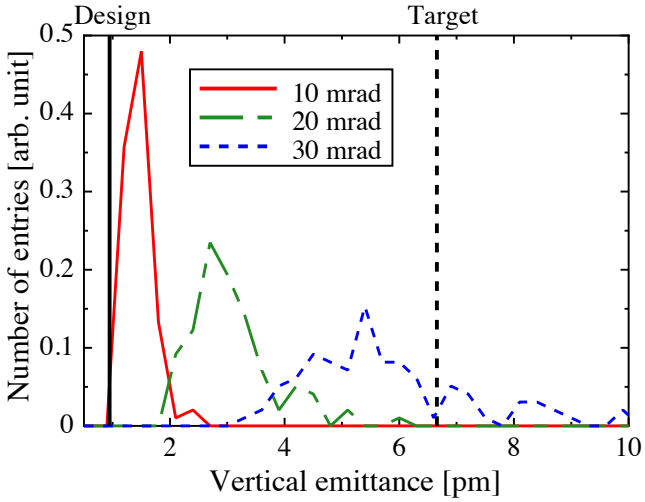
Coupling	.0010816287630558967
Dispersion	3.748599954860665e-05

HER(Sext vertical misalignment)

Coupling	.0449957093193789 76
Dispersion	.0045765442050508375

Dispersion and coupling corrections SuperKEKB simulation

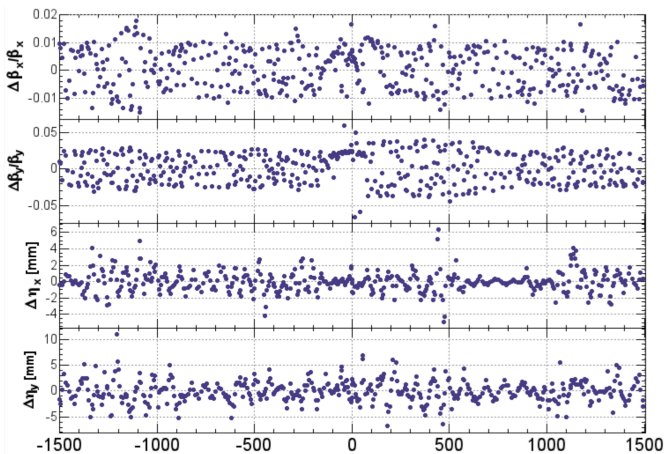
- Machine errors
  - Quadruples: rotational errors, random rms: 100μrad
  - Sextupoles: vertical offset, random rms: 100μm
  - BPM roll: random rms:10mrad, 20mrad, 30mrad
- Method of correction
  - X-y coupling correction
    - Measurement: vertical leaked orbit by kicks with horizontal DC correctors
    - Minimize leaked orbit by using skew-Q windings of sextupole magnets
  - Vertical dispersion
    - Measurement: Orbit change with RF frequency shifts
    - Minimize vertical dispersion by using skew-Q windings of sextupole magnets



Demands for roll errors of BPM
-> less than 20mrad

More realistic simulation SuperKEKB HER

- Corrections: Closed orbit, x-y coupling, dispersion, beta-beat
- Machine errors (random) (except for IR magnets)
  - Quadrupoles: offset:100μm(H,V), rotation: 100μrad, strength: 2.5 x 10^{-4}
  - Sextuples: offset:100μm(H,V)
  - BPM: roll:10mrad, resolution:2μm
- Correctors: DC correctors(H,V), skew-windings of sextuples, Strength of Quadrupoles, horizontal movers of sextupoles

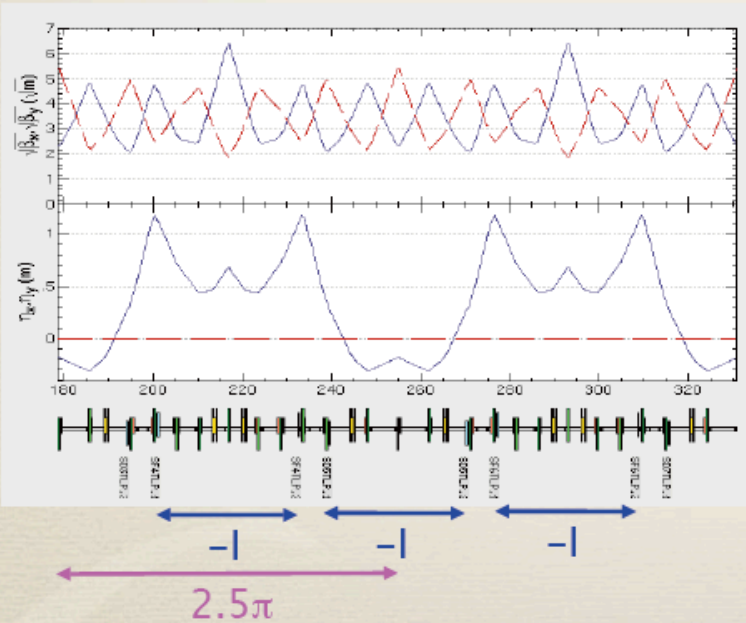


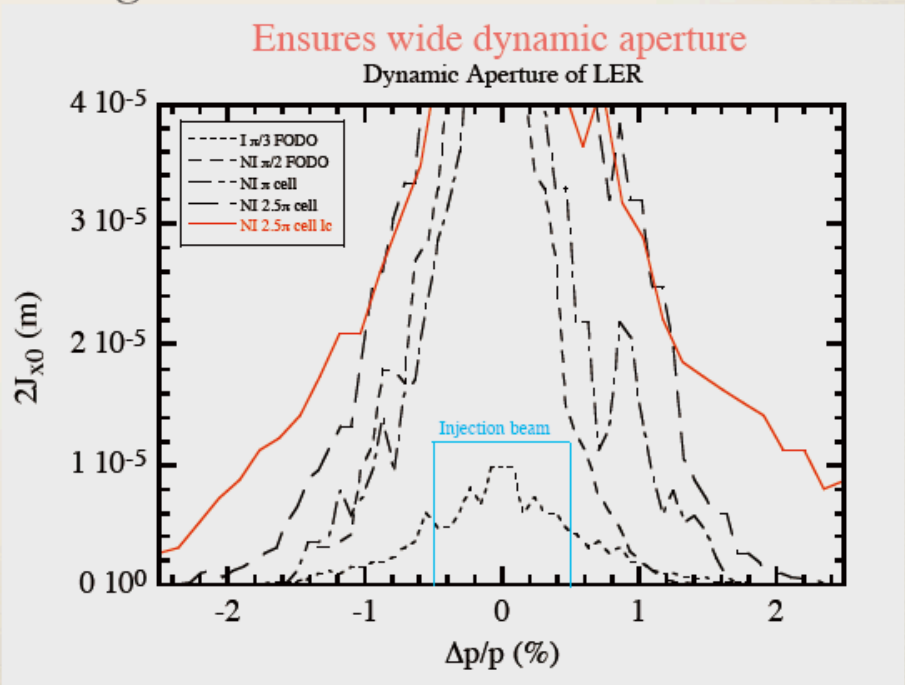
Example of optics corrections (after corrections)

Residuals of optics functions (average of 100 samples)
- horizontal dispersion: rms 1.4mm
- vertical dispersion: rms 2.0mm
- horizontal beta-beat: 1%
- vertical beta-beat: 2%
- vertical emittance 2pm +/- 0.3pm

KEKB Optics

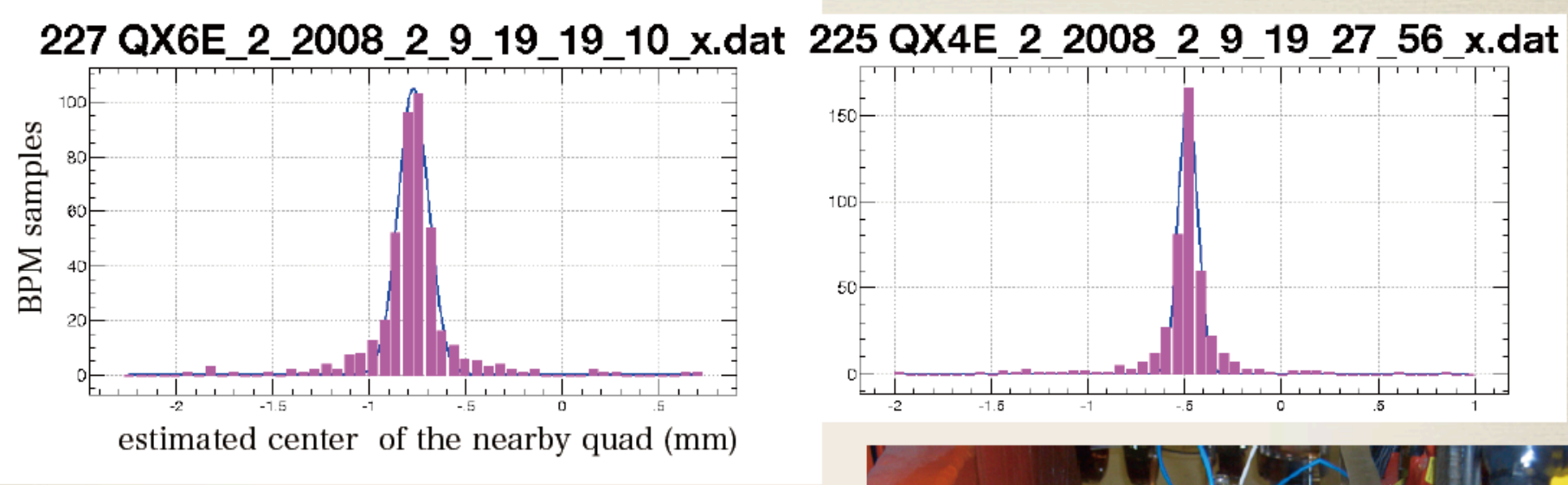
* 3.5 GeV + 8 GeV double ring collider with one collision point, 3016 m circumference.
* About 450 quads, 110 dipoles, 110 sextupoles per ring.
* 2.5 π unit cell, -I transformation between paired sextupoles.
* ν_x close to a half integer: 0.505(LER) & 0.511(HER) at collision.
* 450 BPMs per ring, about 30 per ring are TBT BPMs.



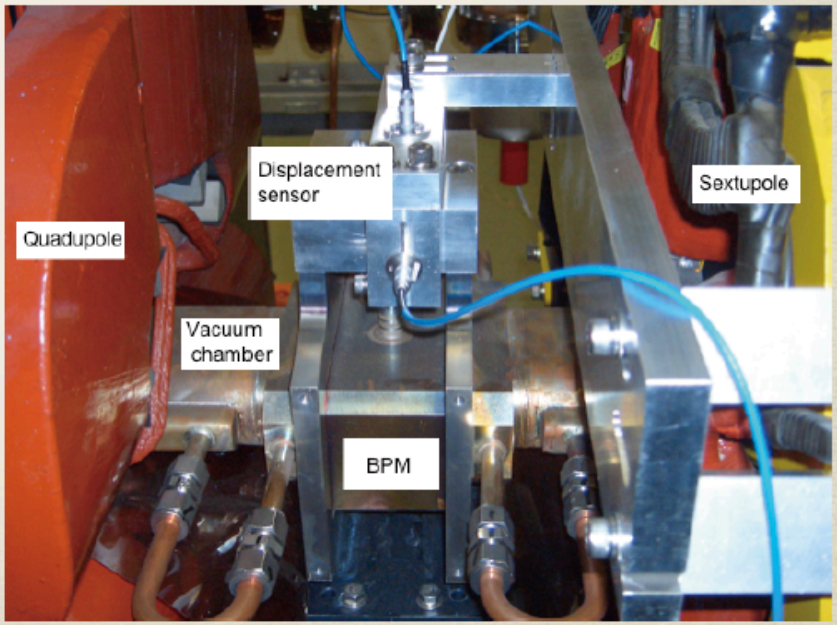


Beam based diagnostics for BPMs (1)

* Beam-based alignment: Quad-BPM



* Once a year, or anything happened to a BPM such as reconnection of cables, realignment, etc.
* BPMs near sextupoles have capacitive sensors to measure relative transverse position of BPMs to sextupoles. →



Beam based diagnostics for BPMs (2)

* Gain mapping of BPM electrodes

CALIBRATION OF KEKB BEAM POSITION MONITORS

PAC97

Kotaro Satoh and Masaki Tejima
KEK, High Energy Accelerator Research Organization
Oho 1-1, Tsukuba, Ibaraki 305 Japan

Abstract

This paper first proposes a practical model for output signals of BPM electrodes. The model is based on a definition of the geometric center of a BPM head, and on the assumption that the character of the head can be specified only by a small number of parameters, the relative gains of electrodes. On the basis of the model, calibration was done to find the relative gains of all KEKB LER BPM heads. The paper reports and discusses the calibration results.

1 INTRODUCTION

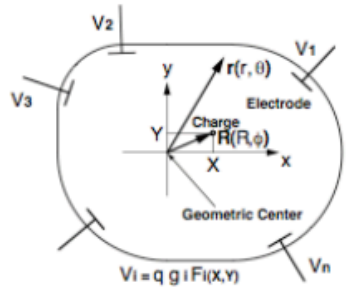
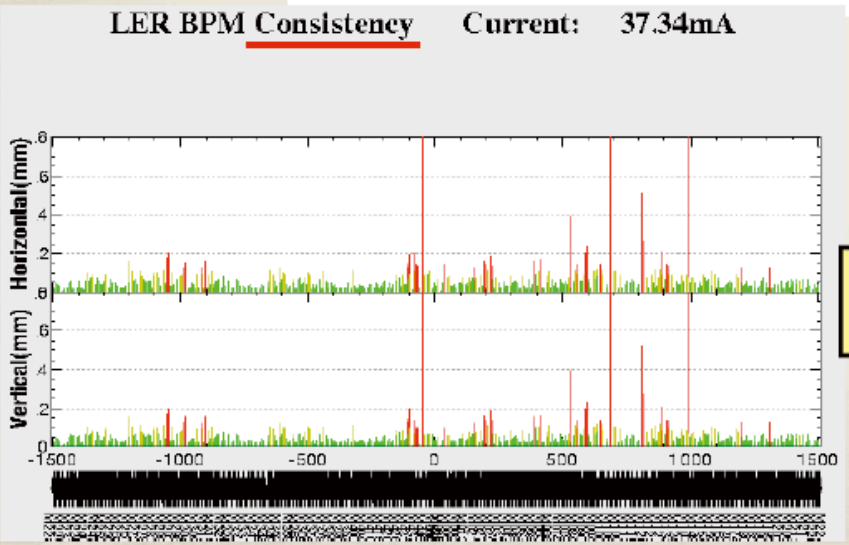
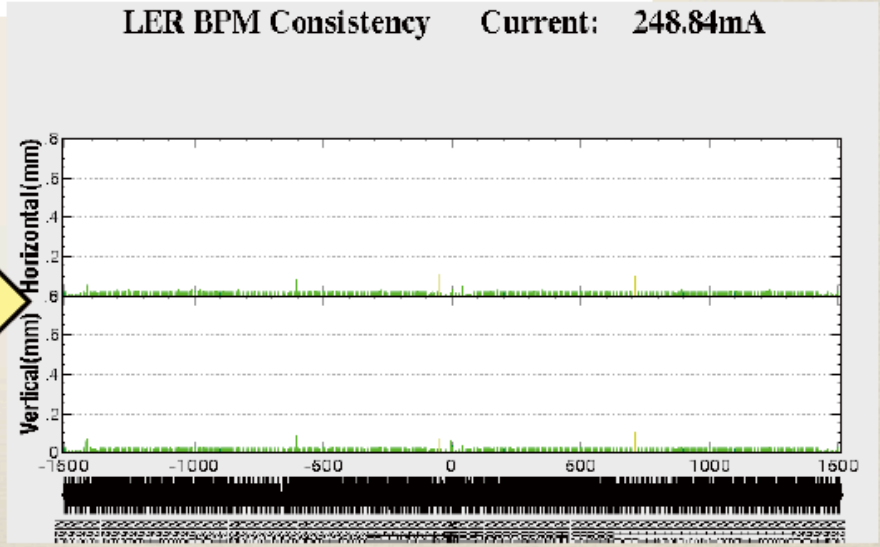


Figure 1: Coordinate system and an image of the model monitor.

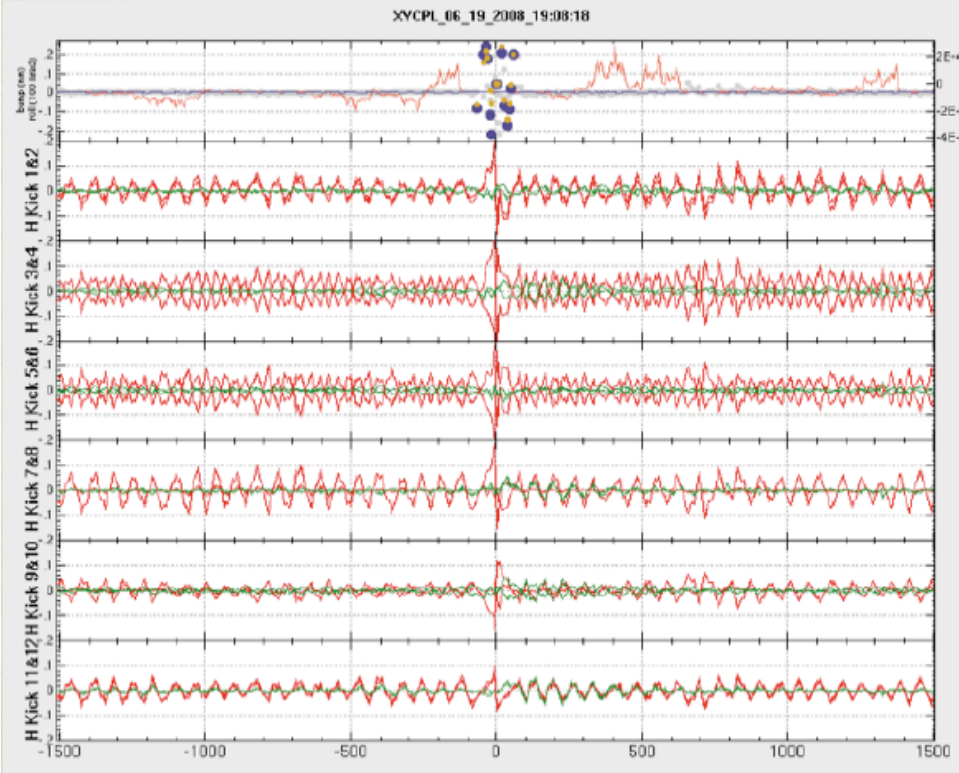


Gain Mapping

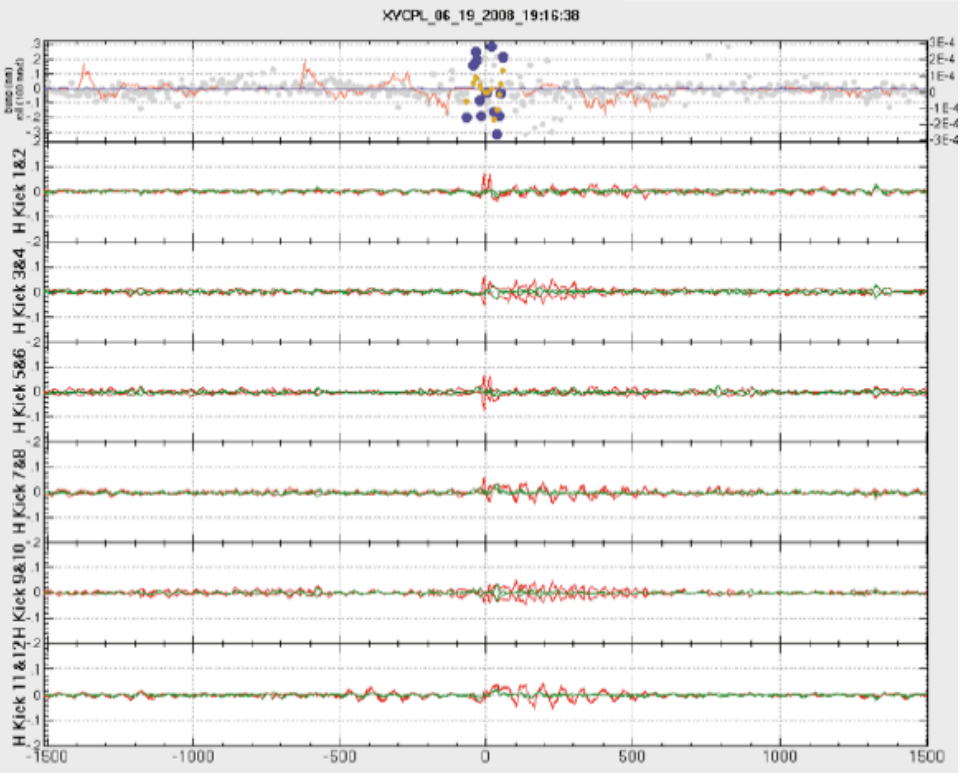


X-y coupling correction

* Kick the beam by horizontal dc correctors at non-coupled, non-dispersive places.
* Measure leaked closed orbit in the vertical plane.
* Correct the leak by **vertical symmetric bumps at sextupole pairs** and skew quads around the IP.
* Only 12 correctors, with equally separated phases, are used.



before



after 1 correction

Dispersion correction

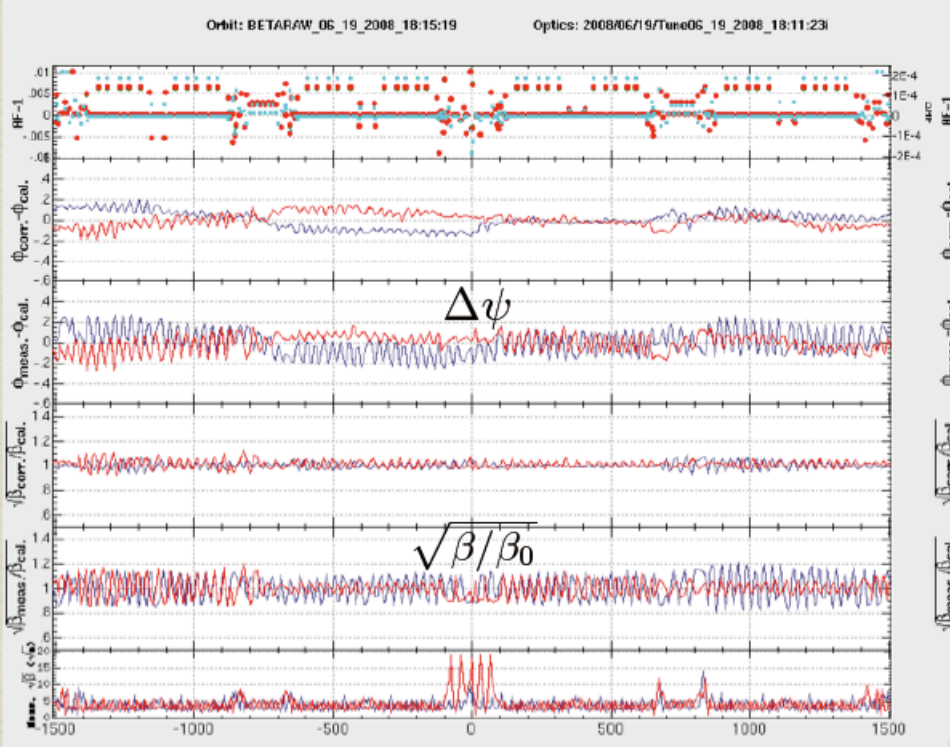
* Change rf frequency by ±100 Hz, measure the orbit change in x and y.
* Correct the difference from the model by horizontal & **vertical antisymmetric bumps at sextupole pairs.**
* Residuals: $\Delta\eta_{x,\mathrm{rms.}} \approx 10$ mm, $\Delta\eta_{y,\mathrm{rms.}} \approx 8$ mm

Before correction

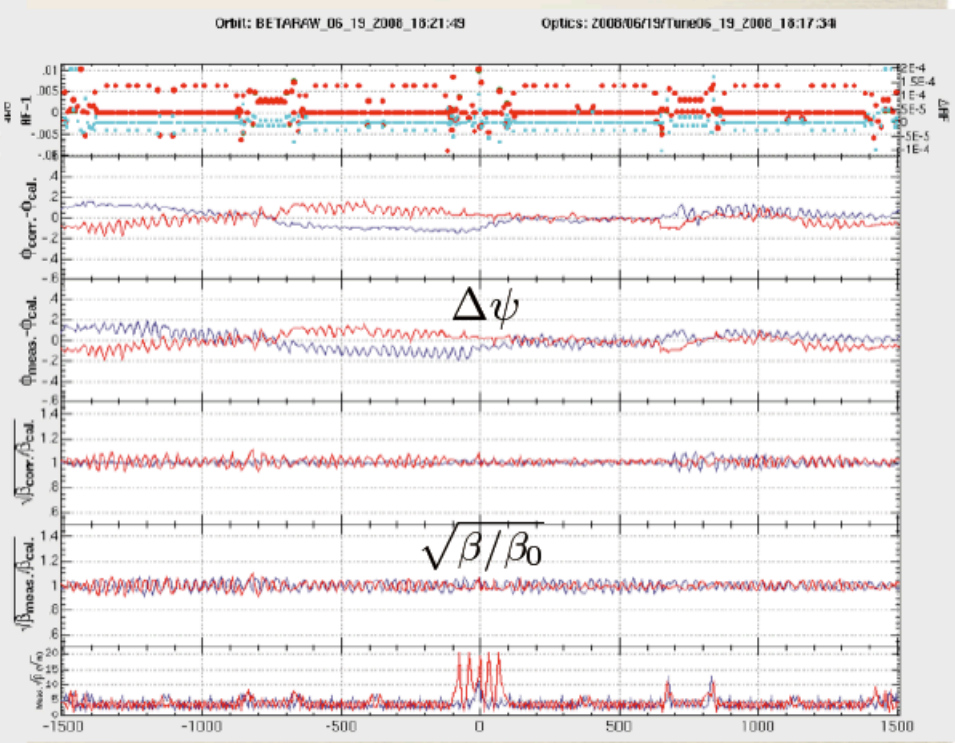
After 1 dispersion and 1 coupling correction applied alternatively.
Note that the vertical scale becomes less than half.

β correction

* Kick the beam by dc correctors in x and y, measure the orbit response in each plane.
* Fit the response with βs and phases at each BPM and the kicked correctors, assuming x-y coupling to have been already corrected. 6 correctors per plane.
* Correct the difference from the model by fudge factors of quads.
* Residuals: $(\Delta\beta/\beta_0)_{x,y\ \mathrm{rms}} \approx 6\%$



Before correction

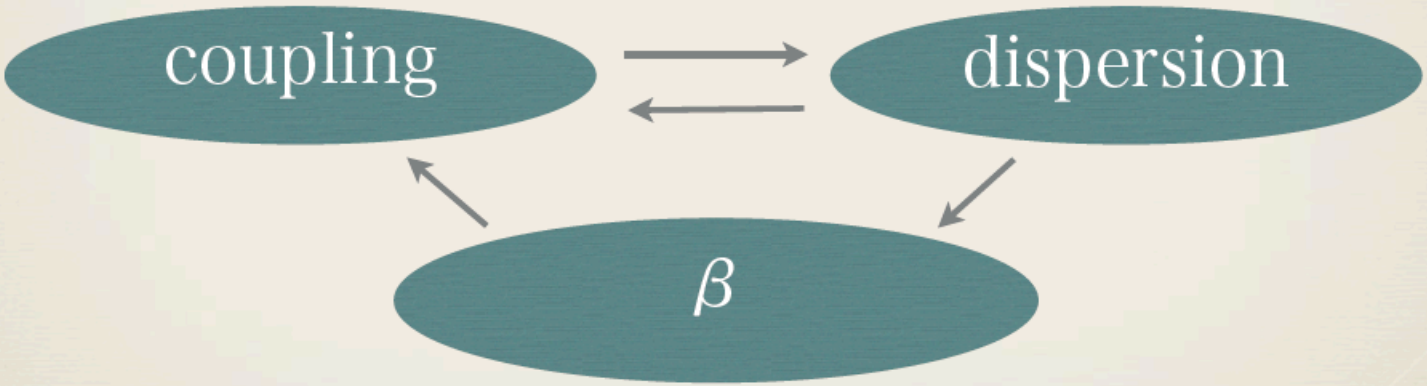


After 1 corrrection.

Iteration

2008_06_19_19_06_29fop	Fill-Length Optimization
2008_06_19_19_06_32luh	Beam Collision Panel
2008_06_19_19_09_12XY_Coupling	MeasOptHER
2008_06_19_19_12_59Dispersion	MeasOptHER
2008_06_19_19_18_27XY_Coupling	MeasOptHER
2008_06_19_19_21_34Dispersion	MeasOptHER
2008_06_19_19_22_29Dispersion	MeasOptHER
2008_06_19_19_23_29Dispersion	MeasOptHER
2008_06_19_19_31_36Global_Beta	MeasOptHER
2008_06_19_19_38_29Global_Beta	MeasOptHER
2008_06_19_20_16_46_amsad8	amsad8 screen capture
2008_06_19_20_34_16_amsad8	amsad8 screen capture

* A loop of coupling, dispersion, β corrections takes **30-60 minutes** per ring to converge. (1 correction takes 3.5 to 7 minutes)



* We do not have to solve the entire problem at once by a single big matrix.

* Although these corrections are not independent, their cross-talks are smaller than the diagonal parts, so the iteration converges quickly.

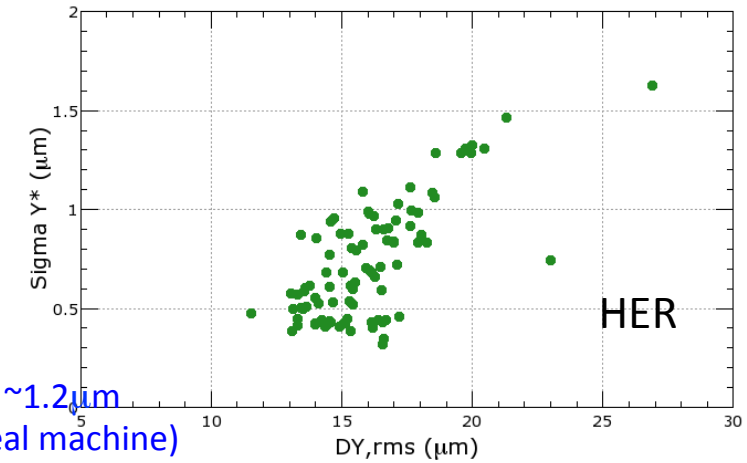
Comparison of simulation and measurement (KEKB)

- Machine errors (random)

	$\sigma_{\Delta x,rms}$ (μm)	$\sigma_{\Delta y,rms}$ (μm)	$\sigma_{\Delta\theta,rms}$ (mrad)	$\sigma_{\Delta K/K,rms}$
Quad	100	meas.*2	meas.*2	1×10^{-4}
Skew Quad	100	meas.*2	meas.*2	1×10^{-4}
Sextupole	100	interpolation +100	0.1	1×10^{-4}
BPM*1	100	interpolation +100	0.1	

*1) BPM jitter error : $\sigma_{\Delta x,\ \Delta y,rms}$ = 2 μm
*2) Measured by MG group

#samples = 100



σ_y^* ~1.2μm (real machine)

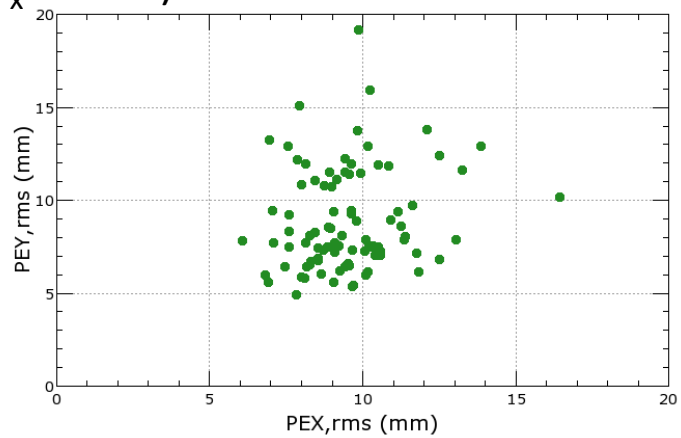
$\Delta y_{,rms}$ ~15-18 μm (real machine)

$\varepsilon_y/\varepsilon_x$ ~ 1.0% (ε_x~24nm)

- Optics corrections
  - Beta-beat
  - X-y coupling
  - dispersions

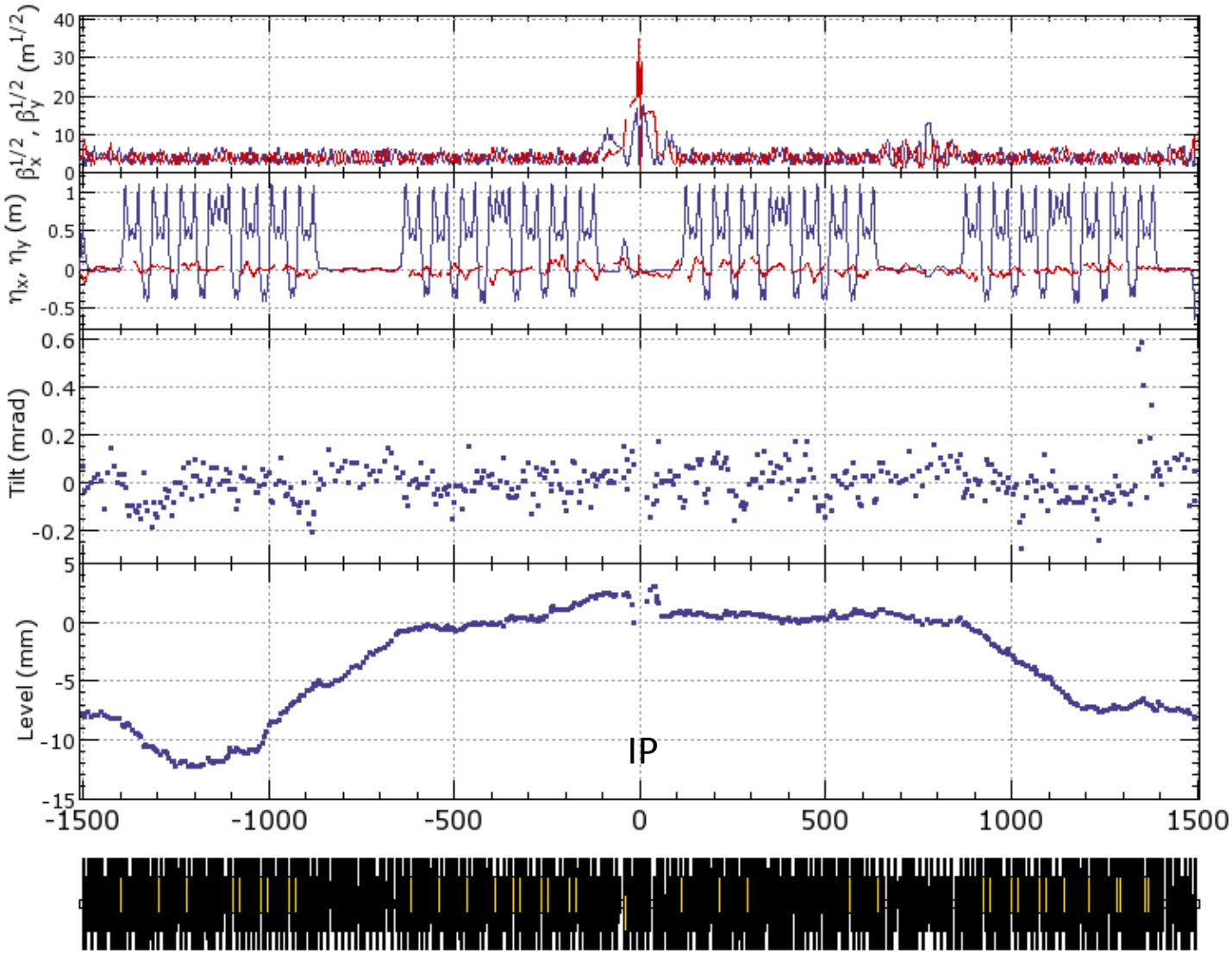
Real machine: $\Delta\eta_{y,rms}$ ~8 mm

ν_x	44.5138
ν_y	41.5900



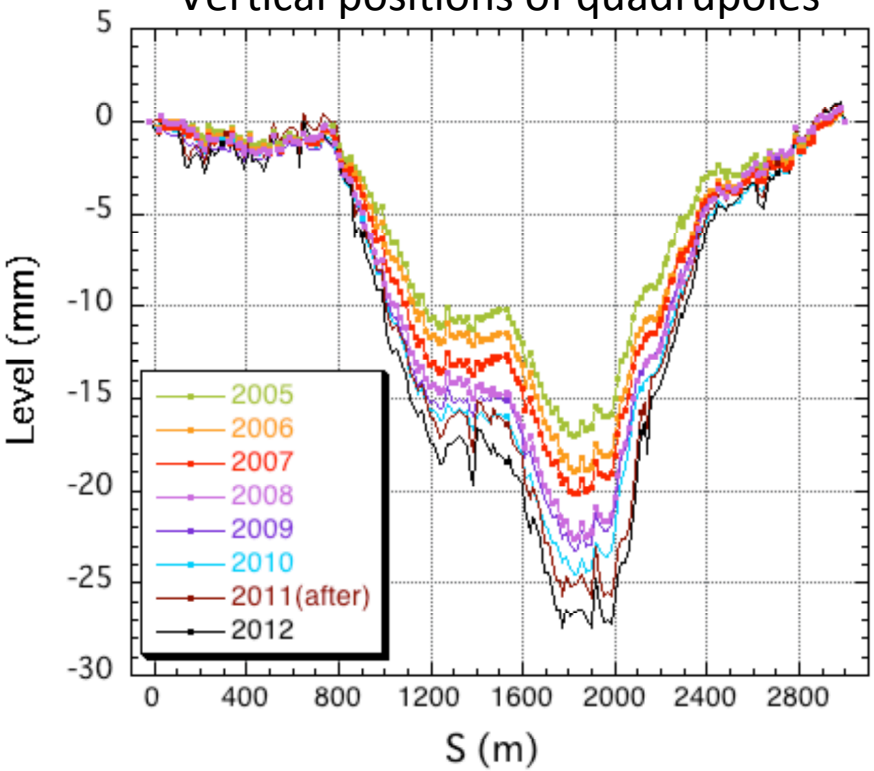
Real machine: $\Delta\eta_{x,rms}$ ~10 mm

KEKB HER Machine Error and Optics Correction



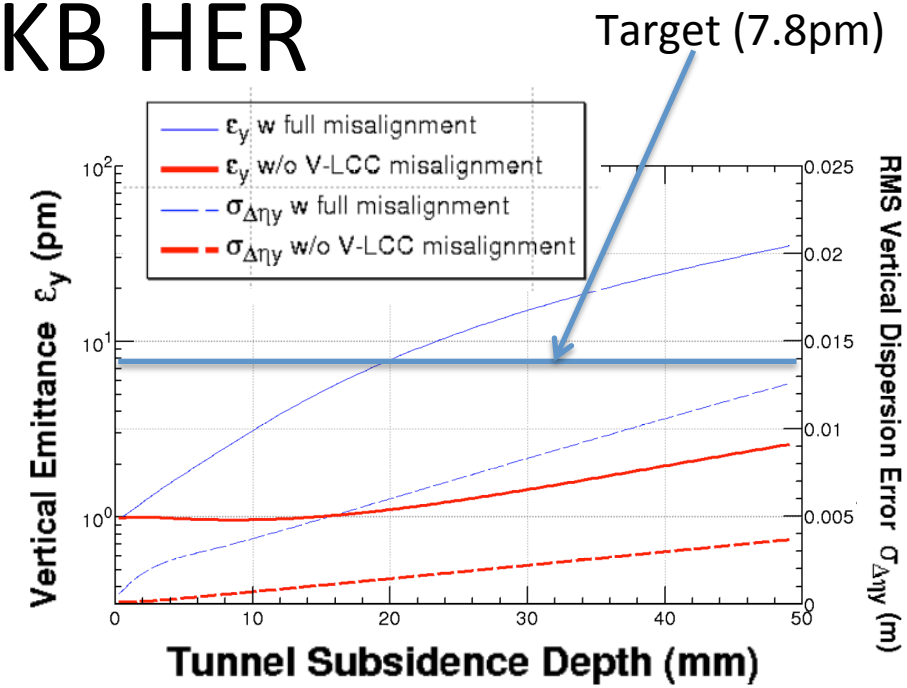
Effects of Tunnel deformation at SuperKEKB HER

Vertical positions of quadrupoles



Tunnel deformation observed at KEKB

- A large subsidence has been observed: ~ 2mm/year and still in progress.
- In the construction period of KEKB (1998), all magnets were aligned on the same plane.



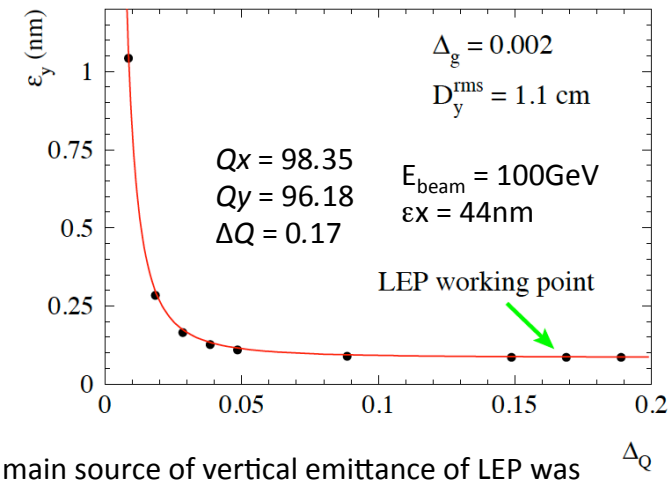
Vertical emittance

-If the alignment error around the V-LCC (vertical local chromaticity correction) area is excluded, the vertical emittance can be preserved well below the target value with optics corrections.

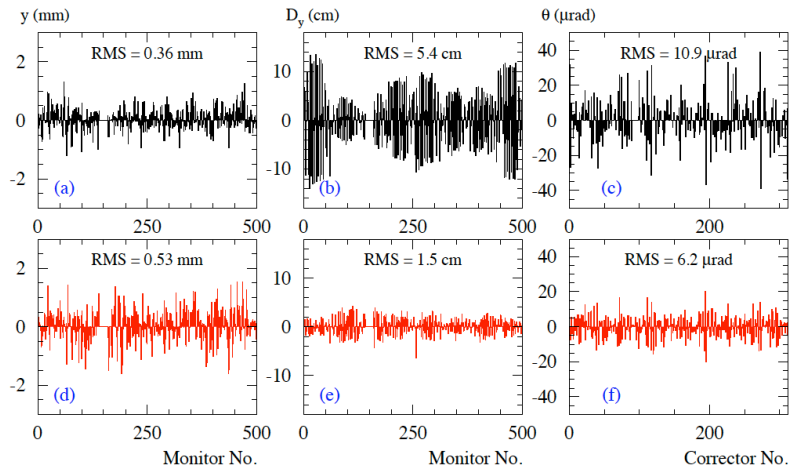
- As for the alignment error of V-LCC, we will need a special care. This is a remaining problem.

Dispersion Free Steering (DFS) method developed at CERN for LEP

- Method
  - simultaneous correction of closed orbit and dispersion
  - SVD inversion of response matrix
  - Correctors: dc corrector magnets

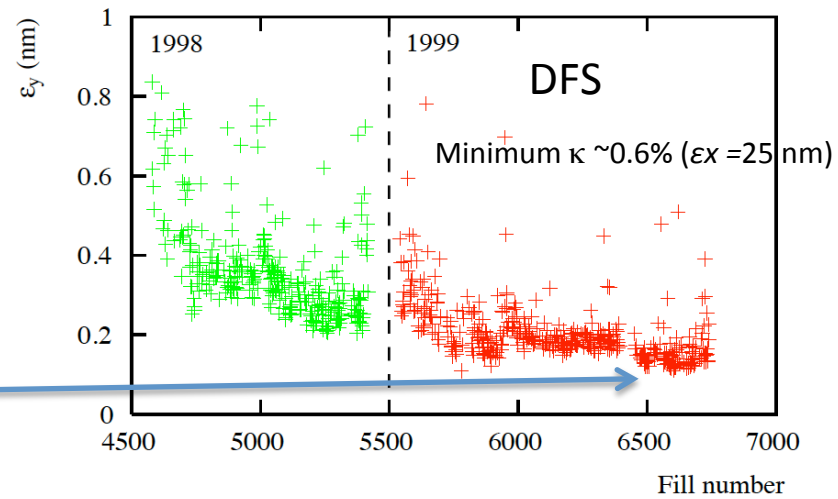


The main source of vertical emittance of LEP was residual vertical dispersion.



MICADO

DFS



Limit of vertical dispersion in LEP came from vertical dispersion generated by separation bumps and from the measurement noise.

Vertical emittance in collision (from luminosity)

Phys. Rev. ST Accel. Beams **3**, *121001 (2000)*

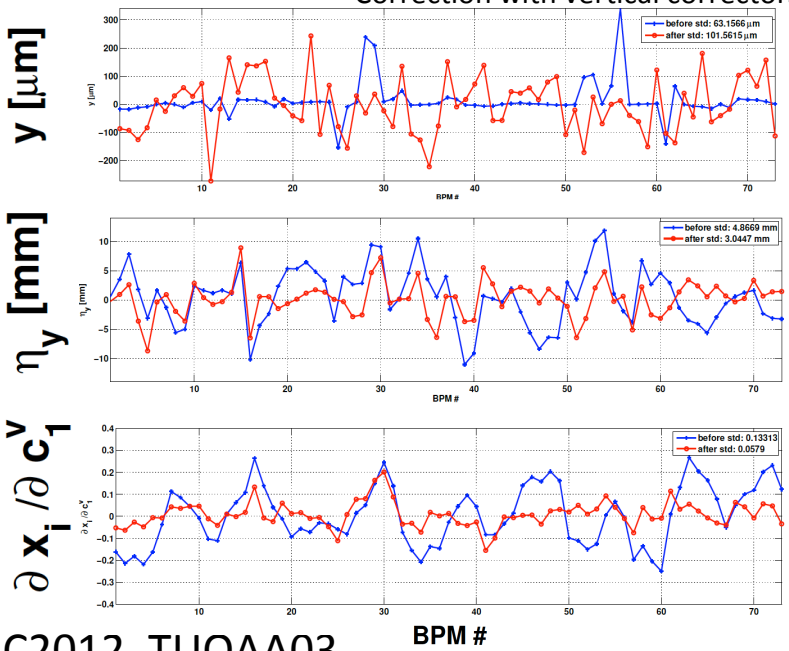
R. Assmann, P. Raimondi, G. Roy, J.Wenninger

Low Emittance Tuning algorism developed for SuperB

- Method
  - simultaneous correction of closed orbit, dispersion and x-y coupling (expansion of DFS)
  - SVD inversion of response matrix
  - Correctors: corrector magnets, skew-Q, BPM roll

Tested at SLS (PSI)

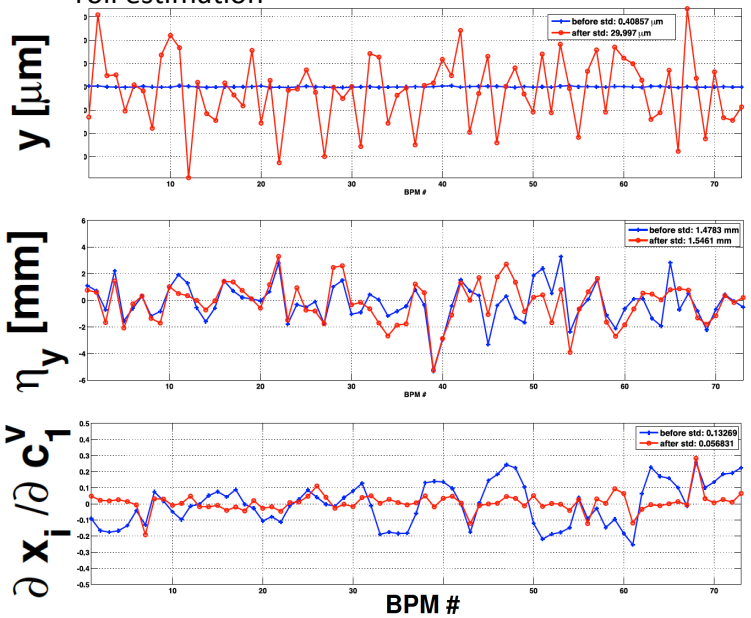
Correction with vertical correctors



IPAC2012, TUOAA03

S.M. Liuzzo, M.E. Biagini, P. Raimondi, INFN/LNF, Frascati (Roma)
M. Aiba, M. Boge, PSI (Villigen)

Correction with skew-Q followed by vertical correction + BPM roll estimation

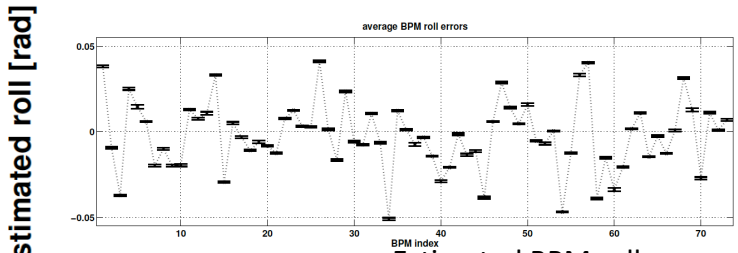


Beam sizes

-σ_y = 16 +/- 0.5 μm (before corrction)

-> 7 +/- 0.5 μm (vertical correctors)

-> 4.4 +/- 0.4(stat) +/- 0.5 (stat) μm → εy ~ 1.3pm



Estimated BPM roll

Difference between SR ring and high energy colliders

- IR
  - Detector solenoid and its compensation
  - Low beta insertion
  - Local chromaticity correction
- Size of rings
  - In larger rings, orbit drift tends to be large.
    - Accuracy of optics measurement with orbit drifts
- Beam-beam interaction
  - Restriction on choice of working point

Summary

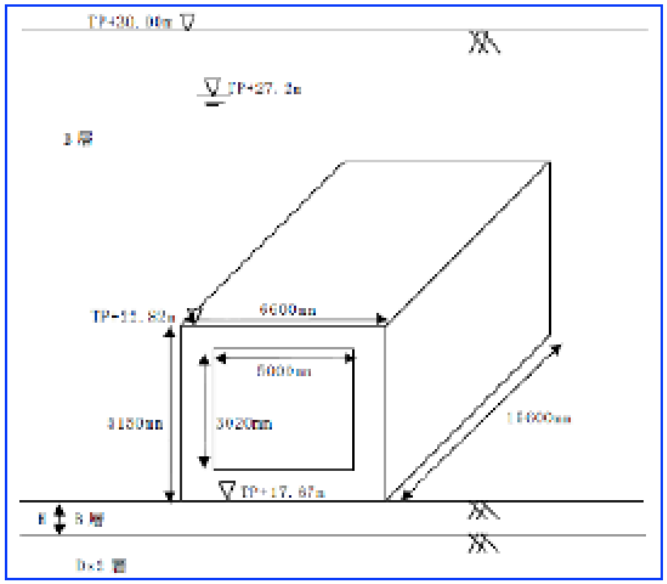
- Emittances of colliders are compared.
- Emittances of proposed ring Higgs factories are somewhat lower than existing colliders.
- Emittance of SuperB and SuperKEKB are much lower in both horizontal and vertical direction.
- In the SuperKEKB design, we found that the fringe field of detector solenoid can be a source of vertical emittance. We succeeded to avoid this problem by design efforts around IP.
- The major sources of the vertical emittance are mis-alignments of the magnets.
- The key issue is to compensate those machine errors by optics correction (low emittance tuning).
- The KEKB/SuperKEKB method for optics correction is shown.
- In the simulations of SupeKEKB, we will be able to achieve the design vertical emittance (except for IR magnet alignment errors). We haven't found any showstopper so far. However, we have to note that there is some discrepancy between simulation and experiment.
- Other methods of correction, DFS developed at CERN and Low Emittance Tuning algorism developed for SuperB, are shown.
- In the future ring colliders, the low emittance will be much more important. To achieve the design vertical emittance of SuperKEKB and SuperB, maybe we need a lot of efforts to compensate machine errors.
- Design vertical emittance of ring Higgs factories seems reachable based on experience of SuperKEKB and SuperB. In addition, experiences in low-emittance SR rings and ATF are also important.

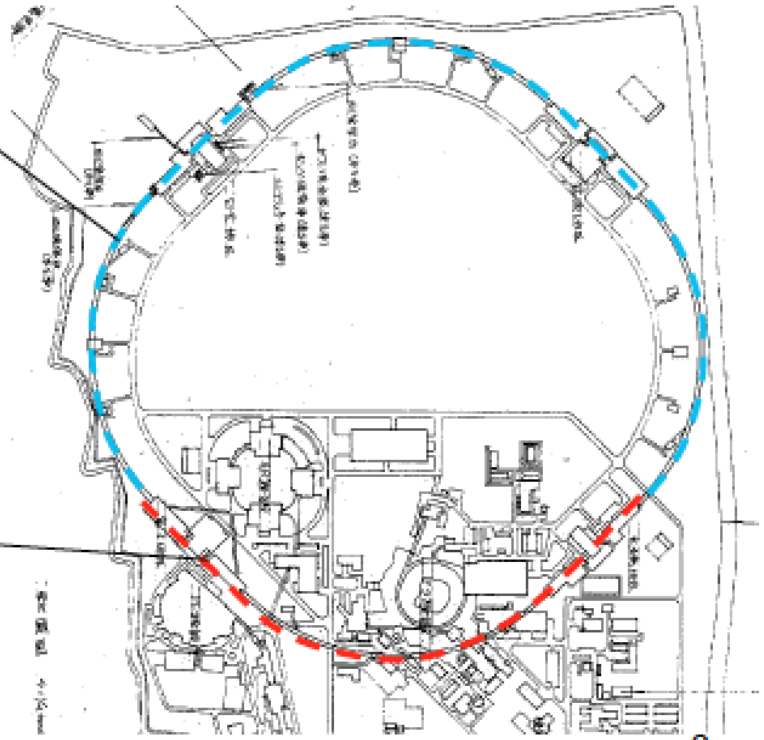
Spare slides

Configuration of the KEKB tunnel

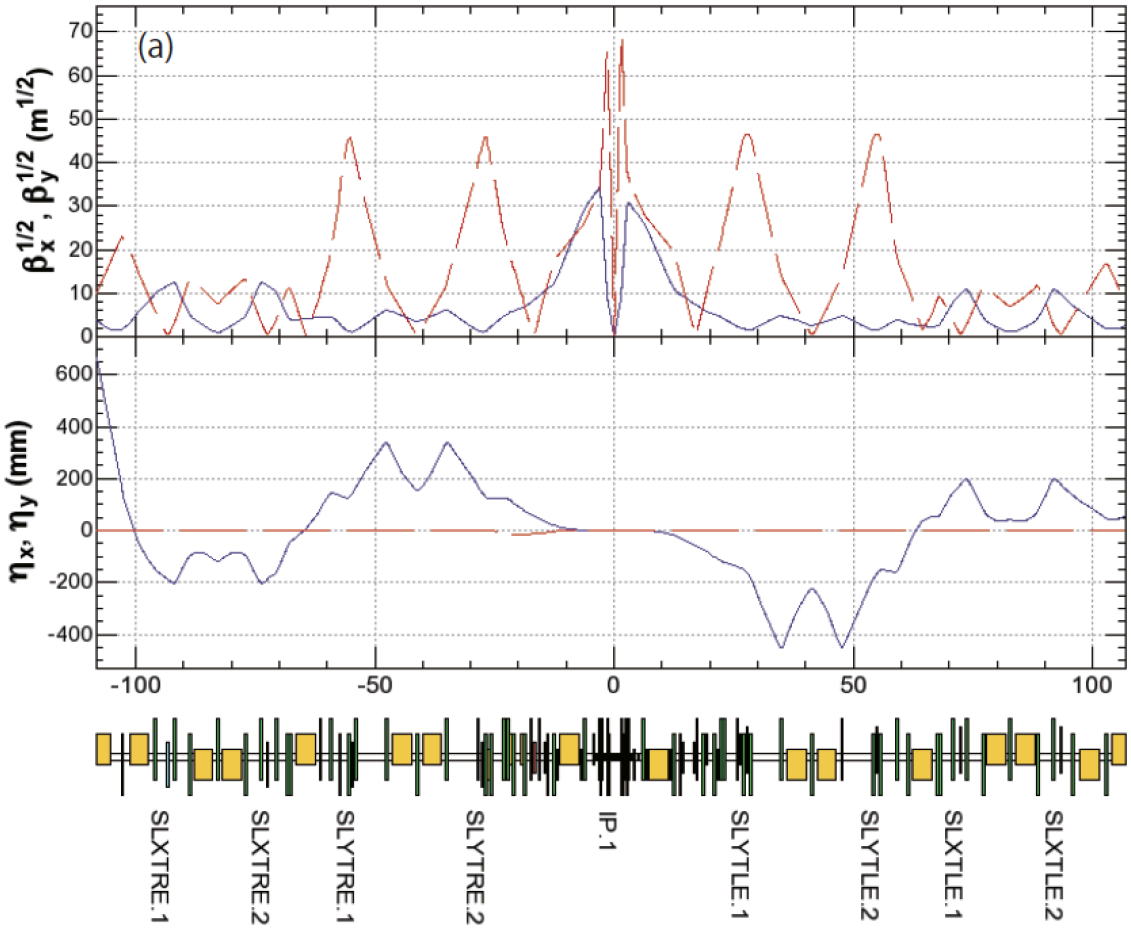
- No piling under the floor at arc-section.
- Refilled soil after complete the tunnel.

Walls to prevent a landslide .





SuperKEKB IR optics



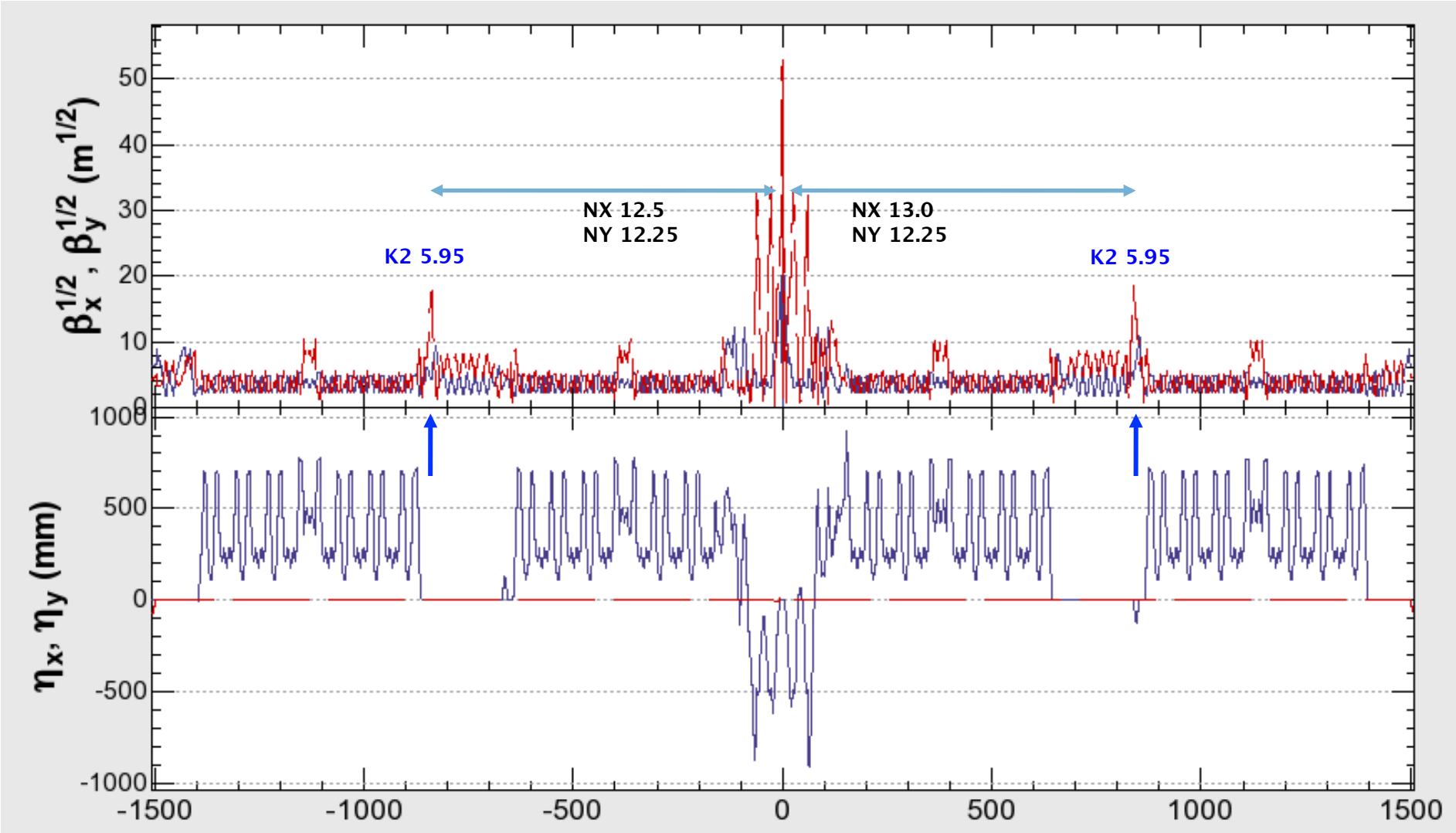
Machine Parameters of the KEKB (June 17 2009)

	LER	HER	
Circumference	3016		m
RF Frequency	508.88		MHz
Horizontal Emittance	18	24	nm
Beam current	1637	1188	mA
Number of bunches	1584 + 1		
Bunch current	1.03	0.750	mA
Bunch spacing	1.84		m
Bunch trains	1		
Total RF volatage Vc	8.0	13.0	MV
Synchrotron tune ν_s	-0.0246	-0.0209	
Betatron tune ν_x / ν_y	45.506/43.561	44.511/41.585	
beta's at IP β_x^* / β_y^*	120/0.59	120/0.59	cm
momentum compaction a	3.31×10^{-4}	3.43×10^{-4}	
Estimated vertical beam size at IP from luminosity σ_y^*	0.94	0.94	μm
beam-beam parameters ξ_x / ξ_y	0.127/0.129	0.102/0.090	
Beam lifetime	133@1637	200@1188	min.@mA
Luminosity (Belle CsI)	21.08		$10^{33}/cm^2/sec$
Luminosity records per day / 7days/ 30days	1.479/8.428/30.208		/fb

Machine Parameters

2011/July/20	LER	HER	unit	
E	4.000	7.007	GeV	
I	3.6	2.6	A	
Number of bunches	2,500			
Bunch Current	1.44	1.04	mA	
Circumference	3,016.315		m	
$\varepsilon_x/\varepsilon_y$	3.2(1.9)/8.64(2.8)	4.6(4.4)/11.5(1.5)	nm/pm	():zero current
Coupling	0.27	0.28		includes beam-beam
β_x^*/β_y^*	32/0.27	25/0.30	mm	
Crossing angle	83		mrad	
α_p	3.25×10^{-4}	4.55×10^{-4}		
σ_δ	$8.08(7.73)\times10^{-4}$	$6.37(6.31)\times10^{-4}$		():zero current
V_c	9.4	15.0	MV	
σ_z	6.0(5.0)	5(4.9)	mm	():zero current
ν_s	-0.0247	-0.0280		
ν_x/ν_y	44.53/44.57	45.53/43.57		
U_0	1.87	2.43	MeV	
$\tau_{x,y}/\tau_s$	43.1/21.6	58.0/29.0	msec	
ξ_x/ξ_y	0.0028/0.0881	0.0012/0.0807		
Luminosity	8×10^{35}		$cm^{-2}s^{-1}$	

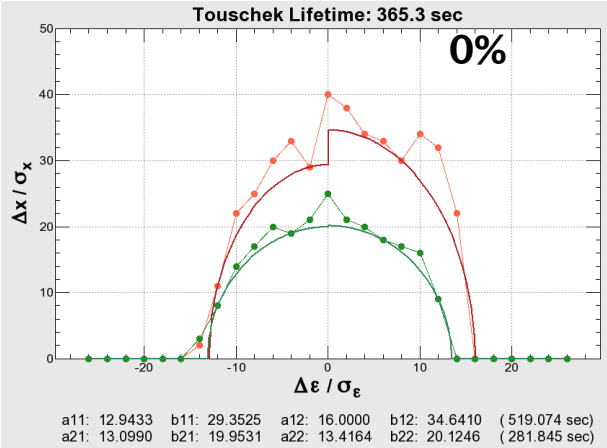
SuperKEKB LER Crab waist

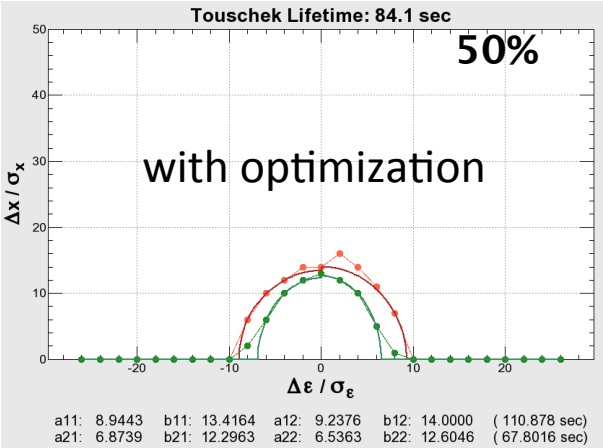


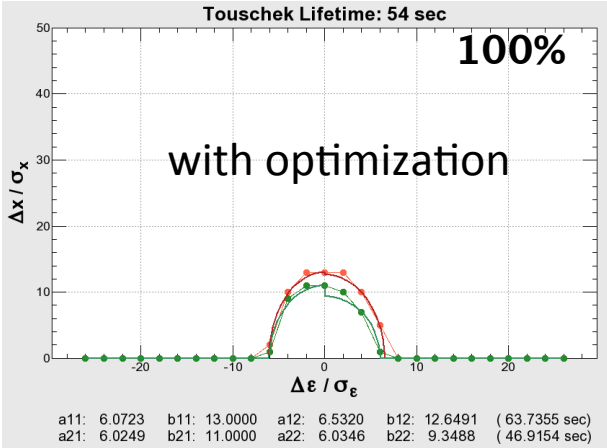
IR: lerfqlc_Oide_1137.sad

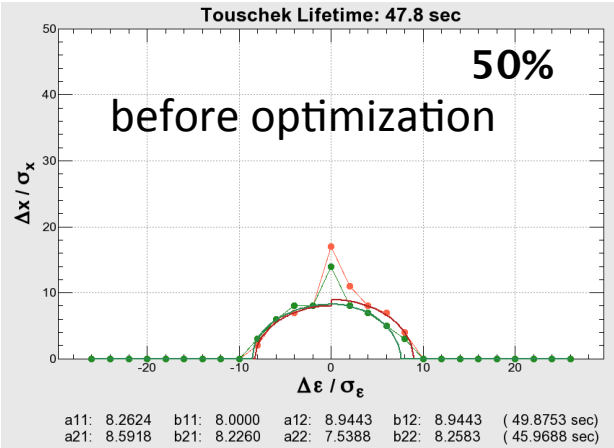
H. Koiso

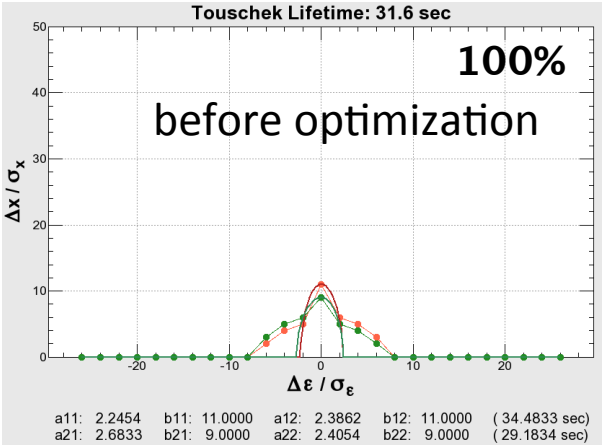
Dynamic aperture with sextupoles for crab waist











Crab Waist schemeは、SuperKEKBではdynamic aperture減少のため
成り立たないと考えられている。

H. Koiso

Importance of low emittance in colliders

- Luminosity formula used for usual colliders does not contain emittances directly.

$$L = \frac{\gamma_{\pm}}{2er_e}\left(1+\frac{\sigma_y^*}{\sigma_x^*}\right)\frac{I_{\pm}\xi_{y\pm}}{\beta_{y\pm}^*}\frac{R_L}{R_{\xi_y}}$$

$$\xi_{y\pm} = \frac{r_e}{2\pi\gamma_{\pm}}\frac{\beta_{y\pm}^* N_{\mp}}{\sigma_{y\mp}^*\left(\sigma_{x\mp}^* + \sigma_{y\mp}^*\right)}R_{\xi\pm}$$

Luminosity Formula

Lorentz factor

Beam current

Beam-Beam parameter

$$L = \frac{\gamma_{\pm}}{2er_e}\left(1+\frac{\sigma_y^*}{\sigma_x^*}\right)\frac{I_{\pm}\xi_{y\pm}}{\beta_{y\pm}^*}\frac{R_L}{R_{\xi_y}}$$

Geometrical reduction factors (crossing angle, hourglass effect)

Beam aspect ratio at IP

Vertical beta function at IP

Minimum value is limited by hourglass effect

Importance of low emittance in colliders [cont'd]

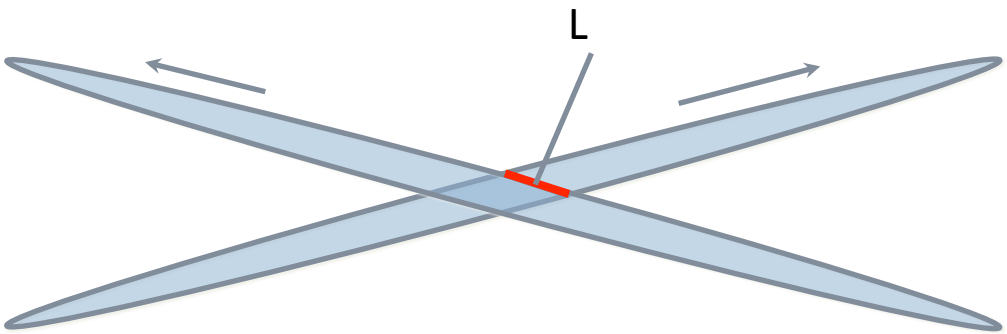
- Low emittances are directly connected to higher luminosity in some situations.
  - The machines which do not reach the beam-beam limit such as TRISTAN or LEP.
  - Ring Higgs factory like LEP3 demands an extremely small vertical emittance.
  - To realize "Nano-beam scheme" in SuperKEKB and SuperB, an extremely low (horizontal) emittance is necessary.
    - Also, an extremely low vertical emittance is demanded to reach the beam-beam limit.
  - Even in conventional ring colliders, a smaller vertical emittance brings a higher luminosity.

NanoBeam scheme

- Head-on collision

hourglass condition: $\beta_y^* >\sim \sigma_s$

- NanoBeam collision



hourglass condition: $\beta_y^* >\sim L$

In the present design, we do not employ the crab waist scheme.

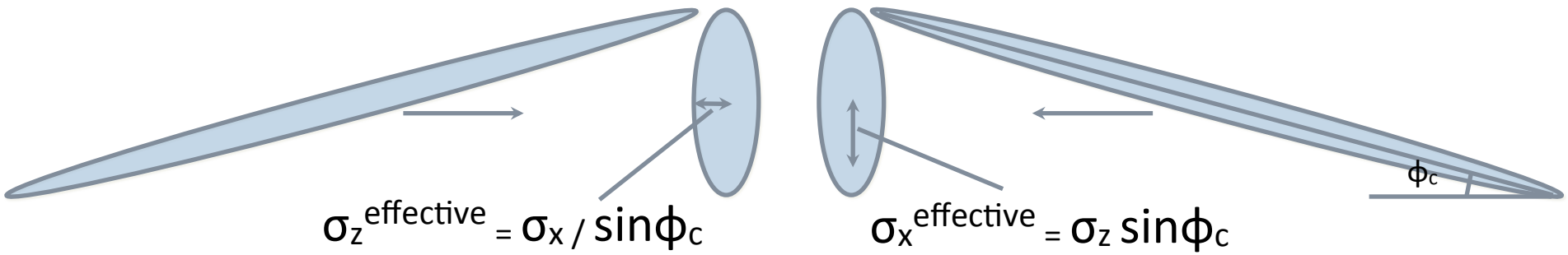
Luminosity and beam-beam parameter with NanoBeam collision

- Luminosity

$$L=\frac{1}{2\sqrt{2}\pi}\frac{N_p N_e}{\sigma_z \phi_c \sqrt{\sigma_{ye}^2+\sigma_{yp}^2}} f_{col} R_L$$

- Beam-beam parameter

$$\xi_{yp}=\frac{r_e}{2\pi\gamma_p}\frac{\beta_{yp}^* N_e}{\sigma_z \phi_c \sigma_{ye}} R_{\xi yp} \quad \xi_{ye}=\frac{r_e}{2\pi\gamma_e}\frac{\beta_{ye}^* N_p}{\sigma_z \phi_c \sigma_{yp}} R_{\xi ye}$$



SuperKEKB Machine Parameters

	KEKB (LER)	SuperKEKB (LER)
Crossing angle	±11mrad	±41.5mrad
β_x^*	1.2m	32mm
β_y^*	5.9mm	0.27mm
ε_x	18nm	3.2nm
ε_y	169pm	8.64pm
$\varepsilon_y / \varepsilon_x$	0.94%	0.27%
σ_{x^*}	147 μm	10.1 μm
σ_{x^*} effective	-	249 μm
σ_z	~7mm	6mm
σ_z effective	-	0.24mm
σ_y	~1μm	48nm

$$L = \frac{N_- N_+}{4\pi\sigma_z \sin\phi_c \sigma_y^*} f R_L$$

$$\xi_{y\pm} = \frac{r_e}{2\pi\gamma_\pm} \frac{\beta_{y\pm}^* N_\mp}{\sigma_{y\mp}^* \left(\sigma_z \sin\phi_c + \sigma_{y\mp}^*\right)} R_{\xi\pm}$$

Very low emittances both in horizontal and vertical directions.

F.Zimmermann

luminosity formulae & constraints

$$L = \frac{f_{rev} n_b N_b^2}{4\pi\sigma_x\sigma_y} = (f_{rev} n_b N_b)\left(\frac{N_b}{\varepsilon_x}\right)\frac{1}{4\pi}\frac{1}{\sqrt{\beta_x\beta_y}}\frac{1}{\sqrt{\varepsilon_y/\varepsilon_x}}$$

$$(f_{rev} n_b N_b) = \frac{P_{SR}\,\rho}{8.8575\times 10^{-5}\,\frac{\mathrm{m}}{\mathrm{GeV}^{-3}}\,E^4}$$ *SR radiation power limit*

$$\frac{N_b}{\varepsilon_x} = \frac{\xi_x 2\pi\gamma(1+\kappa_\sigma)}{r_e}$$ *beam-beam limit*

Higgs Factory
LEP3

$$\frac{N_b}{\sigma_x\sigma_z}\frac{30\,\gamma r_e^2}{\delta_{acc}\,\alpha} < 1$$

>30 min beamstrahlung lifetime (Telnov) → N_b, β_x

→minimize $\kappa_\varepsilon = \varepsilon_y/\varepsilon_x$, $\beta_y \sim \beta_x(\varepsilon_y/\varepsilon_x)$ and respect $\beta_y \geq \sigma_z$

Vertical emittance in LEP3

$$\xi_{x\pm} = \frac{r_e}{2\pi\gamma_\pm} \frac{\beta^*_{x\pm} N_\mp}{\sigma^*_{x\mp}\left(\sigma^*_{x\mp} + \sigma^*_{y\mp}\right)}$$

$$\xi_{y\pm} = \frac{r_e}{2\pi\gamma_\pm} \frac{\beta^*_{y\pm} N_\mp}{\sigma^*_{y\mp}\left(\sigma^*_{x\mp} + \sigma^*_{y\mp}\right)}$$

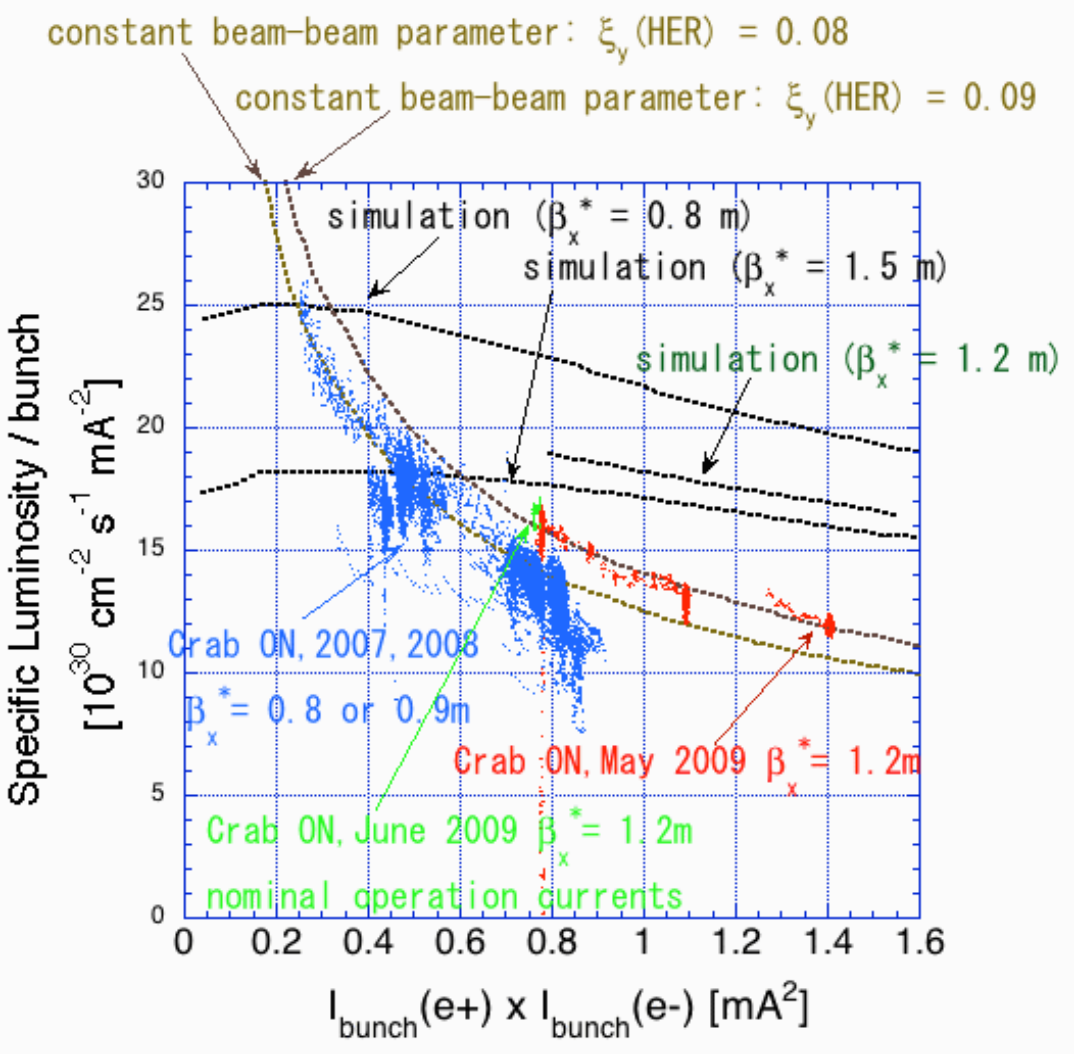
$$\beta^*_y = \beta^*_x \frac{\varepsilon_y}{\varepsilon_x} \longrightarrow \xi_{y\pm} = \xi_{x\pm}$$

Hourglass condition $\beta^*_y \geq \sigma_z$

$$\xi_{y\pm} = \frac{r_e}{2\pi\gamma_\pm} \frac{\beta^*_{y\pm} N_\mp}{\sigma^*_{y\mp}\left(\sigma^*_{x\mp} + \sigma^*_{y\mp}\right)} R_{\xi\pm}$$

A small vertical emittance ($\varepsilon_y/\varepsilon_x$) is needed to achieve the target ξ_y with a small β_y*.

Specific luminosity of KEKB w/ crab cavities



KEKB was operated well above the beam-beam limit.