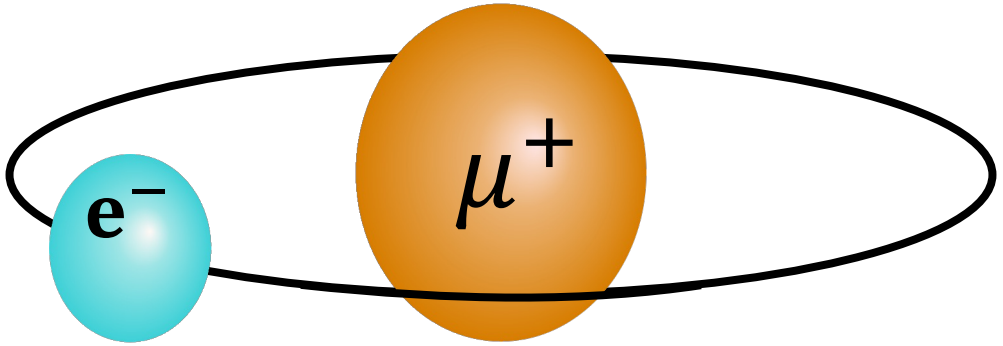
Muonium Spectroscopy

Dave Kawall, University of Massachusetts Amherst



Acknowledgements:
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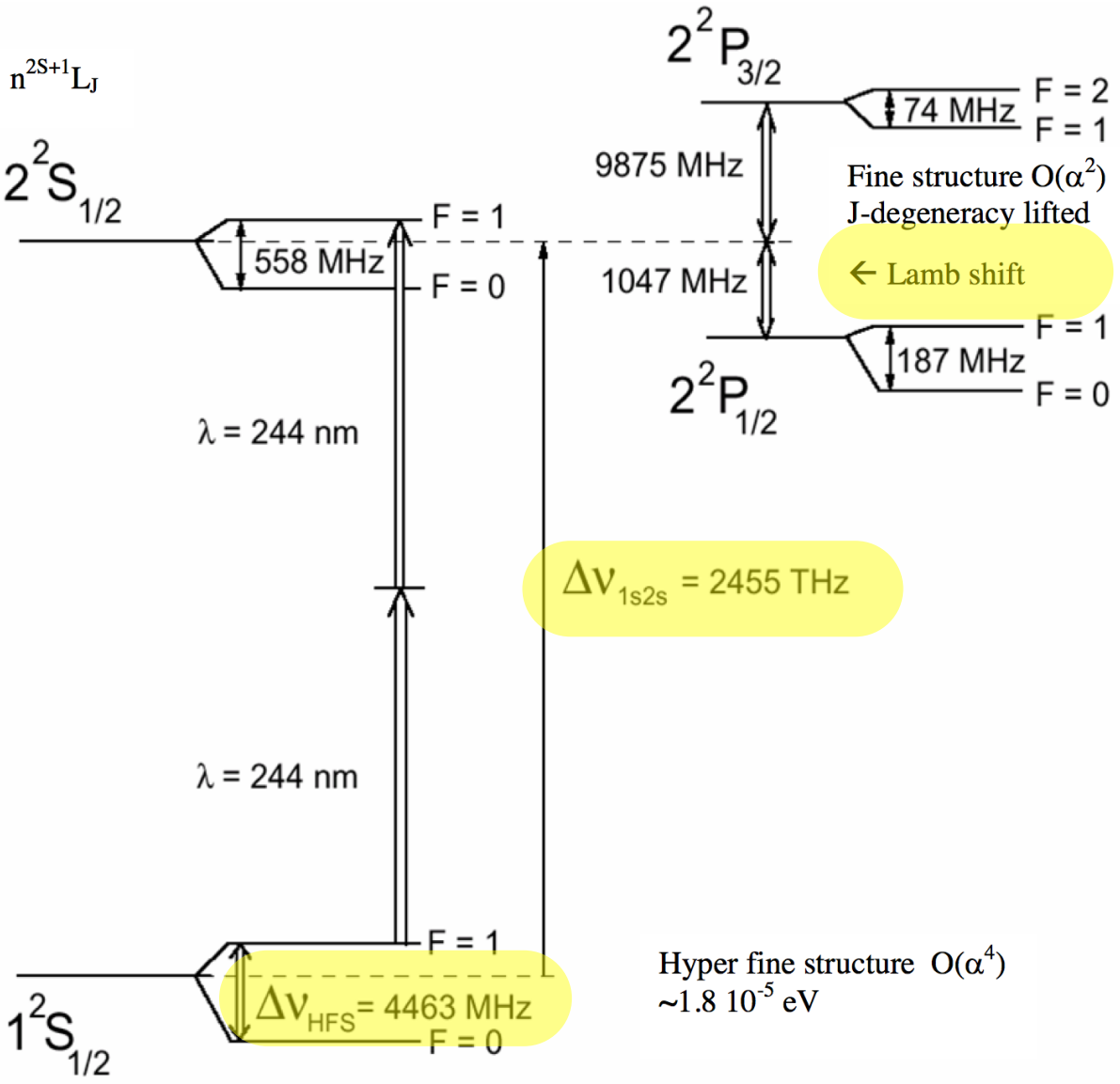
MuSEUM material: Tsutomu Mibe, Shoichiro Nishimura, Koichiro Shimomura, Sohtara Kanda
Mu-MASS material: Paolo Crivelli

Muonium Energy Levels



- Hydrogen-like, but purely leptonic, free of nuclear size effects
- Can produce nearly $10^8/\mathrm{s}$
- Live 2.2 μs, linewidth 145 kHz
- Amenable and interesting for precision spectroscopy
- Extract important constants
- Test bound state QED, search for new physics

$$a_\mu = \frac{\omega_a}{\tilde{\omega}'_p(T_r)} \frac{\mu'_p(T_r)}{\mu_e(H)} \frac{\mu_e(H)}{\mu_e} \frac{m_\mu}{m_e} \frac{g_e}{2}$$

Muonium Ground State Hyperfine Interval Theory Prediction

$$\Delta\nu(\text{theory}) \approx \frac{8\pi}{3} g'_e \mu_B g'_\mu \mu_B^\mu \frac{1}{\pi a_\mu^3}$$

$$= \underbrace{\frac{16}{3}\,\alpha^2\, c\, R_\infty\, \frac{m_e}{m_\mu}\left[1+\frac{m_e}{m_\mu}\right]^{-3}}_{\mathrm{E_{Fermi}}}\underbrace{(1+\Delta)}_{\text{Higher Order}}$$

$$= 4463.302\,868\,(515)\ \text{MHz},\ (120\ \text{ppb})$$

$$= 4463.302\,720\,(511,\ m_\mu/m_e)\,(70,\ \text{QED})\,(2,\ \alpha)\ \text{MHz (QED Contribution)} +$$
$$-65\ \text{Hz (Weak Contribution,}\ 15\ \text{ppb)} +$$
$$232\,(1)\ \text{Hz (Hadronic Vacuum Polarization Contribution,}\ 56\ \text{ppb)} +$$
$$5\,(2)\ \text{Hz (Higher order Hadronic Vacuum Polarization Contribution)}$$

- Largest error from uncertainty in m_μ/m_e (120 ppb)
- Error from higher order terms $\approx$ 70 Hz (16 ppb)
- Effort to reduce QED uncertainty to 10 Hz (M. Eides and colleagues)

Phys. Rev. A **86**, 024501 (2012), PRL **112**, 173004 (2014), Phys. Rev. D **89**, 014034 (2014)

Breit-Rabi Hamiltonian for Ground State Muonium Hyperfine Levels in a Magnetic Field

$$H = h\Delta\nu \mathbf{I}_\mu \cdot \mathbf{J} - g'_\mu \mu_B^\mu \mathbf{I}_\mu \cdot \mathbf{H} + g_J \mu_B^e \mathbf{J} \cdot \mathbf{H}$$

- Energy of $e \cdot \mu$ spin-spin interaction characterized by hyperfine interval $\Delta\nu \approx 4463$ MHz
- Binding corrections to gyromagnetic ratios :

$$g'_\mu = g_\mu \left(1 - \frac{\alpha^2}{3} + \frac{\alpha^2}{2}\frac{m_e}{m_\mu}\right)$$
$$g_J = g_e \left(1 - \frac{\alpha^2}{3} + \frac{\alpha^2}{2}\frac{m_e}{m_\mu} + \frac{\alpha^3}{4\pi}\right)$$

- Two spin 1/2s in a magnetic field $\Rightarrow$ 4 substates
- Good quantum numbers : $\mathbf{F} = \mathbf{I}_\mu + \mathbf{J}$, $M_\mathbf{F}$

$$E_{\mathbf{F}=\frac{1}{2}\pm\frac{1}{2}, M_\mathbf{F}} = -\frac{1}{4}h\Delta\nu - g'_\mu \mu_B^\mu M_\mathbf{F} H \pm \frac{1}{2}h\Delta\nu\sqrt{1 + 2M_\mathbf{F}x + x^2}$$
$$\text{where } x = \left(g_J \mu_B^e + g'_\mu \mu_B^\mu\right) H/h\Delta\nu \approx H(kG)/1.58$$

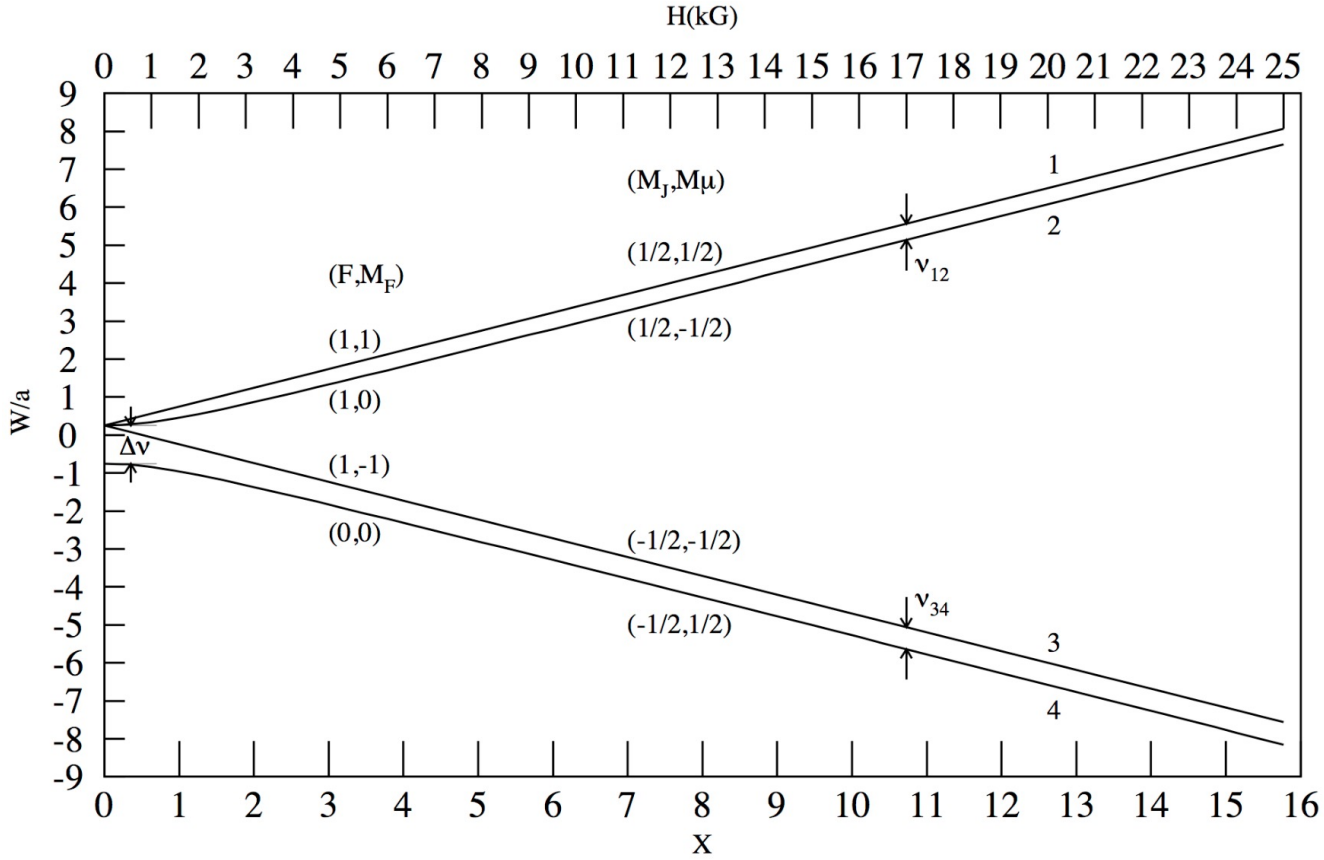
- Out of six possible transitions, consider two :

$$(\mathbf{F}, M_\mathbf{F}) \Longleftrightarrow (\mathbf{F}', M'_\mathbf{F})$$
$$\nu_{12} = (\ 1,\ \ 1) \Longleftrightarrow (\ 1,\ 0\ )$$
$$\nu_{34} = (\ 1, -1) \Longleftrightarrow (\ 0,\ 0\ )$$

$$\text{In a strong field } x \gg 1: \qquad (M_J, M_\mu) \Longleftrightarrow (M'_J, M'_\mu)$$

$$\nu_{12} \approx \left(\frac{1}{2}, \frac{1}{2}\right) \Longleftrightarrow \left(\frac{1}{2}, -\frac{1}{2}\right)$$

$$\nu_{34} \approx \left(-\frac{1}{2}, -\frac{1}{2}\right) \Longleftrightarrow \left(-\frac{1}{2}, \frac{1}{2}\right)$$



- The two microwave transitions measured roughly correspond to muon spin flip

x: measure of Zeeman, Larmor frequency compared to ground state hyperfine interval frequency

From ν_{12} and ν_{34} to $\Delta\nu$, μ_μ/μ_p, and m_μ/m_e

- Use the Larmor relation to express the magnetic field, H, in terms of the proton NMR frequency, ν_p :

$$h\nu_p = 2\mu_p H$$

- Then the transition frequencies are given by :

$$\begin{aligned}
\nu_{12} &= -\nu_p \frac{g'_\mu}{g_\mu}\frac{\mu_\mu}{\mu_p} + \frac{\Delta\nu}{2}\left[(1+x) - \sqrt{1+x^2}\right] \\
\nu_{34} &= +\nu_p \frac{g'_\mu}{g_\mu}\frac{\mu_\mu}{\mu_p} + \frac{\Delta\nu}{2}\left[(1-x) + \sqrt{1+x^2}\right]
\end{aligned}$$

- If we measure ν_{12} and ν_{34} in the same magnetic field :

$$\begin{aligned}
\nu_{12} + \nu_{34} &= \Delta\nu \\
\nu_{34} - \nu_{12} &= 2\nu_p \frac{g'_\mu}{g_\mu}\frac{\mu_\mu}{\mu_p} + \Delta\nu\left[\sqrt{1+x^2} - x\right]
\end{aligned}$$

small

- Knowing ν_{12}, ν_{34}, and ν_p yields $\Delta\nu$ and μ_μ/μ_p.

From ν_{12} and ν_{34} to $\Delta\nu$, μ_μ/μ_p, and m_μ/m_e

- We can derive m_μ/m_e from direct measurement of μ_μ/μ_p through the relation :

$$\frac{m_\mu}{m_e} = \frac{g_\mu}{2} \; \frac{\mu_p}{\mu_\mu} \; \frac{\mu_B^e}{\mu_p}$$

- Alternatively, theoretical prediction for $\Delta\nu$ is a function of m_μ/m_e :

$$\Delta\nu(\text{theory}) = \frac{16}{3}\,\alpha^2\,c\,R_\infty\,\frac{m_e}{m_\mu}\left[1+\frac{m_e}{m_\mu}\right]^{-3}(1+\Delta)$$

- Can measure $\Delta\nu$, solve for m_μ/m_e.
- Gives more precise result, but result depends on the theory

- Free particle tests of QED ($a_e,\ a_\mu$) performed at 5 loops in QED
- Free particle QED is the most successful theory in all of science
- Bound-state QED investigated by hydrogen & muonium HFS, 1s-2s, ... calculations performed at 2-3 loops
- Muonium tests bound-state QED *methods* without complications of proton structure, electron-electron interactions, or strong coupling (binding $(Z\alpha)^2 m_\mu c^2$ small)
- We extract m_μ/m_e using bound-state QED and measurement of muonium ground-state hyperfine interval *but:*

"These two QED theories, the free QED and the bound state QED, are very different in their approaches, problems and applications and it is worth to consider their tests separately... the bound state QED is not a well-established theory and there are no published common prescriptions for the relativistic quantum bound problem. It involves different effective approaches to solve the two-body bound problem."

S. Karshenboim, Physics Reports **422**, 1-63 (2005).

- We have to be careful: success of free particle QED doesn't necessarily apply to bound state QED: saying we know m_μ/m_e to 22 ppb relies on:
  - Bound state QED methods applied to Mu HFS are correct at the 22 ppb level (need expert opinion for careful, conservative error estimation)
  - New physics doesn't perturb the hyperfine interval at the 22 ppb level
- We can extract m_μ/m_e essentially directly from $measurements$ of μ_μ/μ_p from muonium which relies only on correctness of Breit-Rabi Hamiltonian, but still have some bound-state corrections
- Theory-independent ratio μ_μ/μ_p known to about 120 ppb: MuSEUM should improve this direct measurement by a factor 5
- To test compatibility of model of new physics with a_μ, just need to include its influence consistently when extracting m_μ/m_e from muonium

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High Precision Measurements of the Ground State Hyperfine Structure Interval of Muonium and of the Muon Magnetic Moment

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(Received 21 August 1998)

High precision measurements of two Zeeman hyperfine transitions in the ground state of muonium in a strong magnetic field have been made at LAMPF using microwave magnetic resonance spectroscopy and a resonance line narrowing technique. These determine the most precise values of the ground state hyperfine structure interval of muonium $\Delta\nu = 4\,463\,302\,765(53)$ Hz (12 ppb), and of the ratio of magnetic moments $\mu_\mu/\mu_p = 3.183\,345\,13(39)$ (120 ppb), representing a factor of 3 improvement. Values of the mass ratio m_μ/m_e and the fine structure constant α are derived from these results.

$\Delta\nu = 4\,463\,302\,765\,(53)$ Hz (12 ppb) (50 Hz (stat) and 21 Hz (syst))

$m_\mu/m_e = 206.768\,2830\,(46)$ (22 ppb, CODATA 2018)

Present: MuSEUM Experiment at J-PARC

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

New precise spectroscopy of the hyperfine structure in muonium with a high-intensity pulsed muon beam

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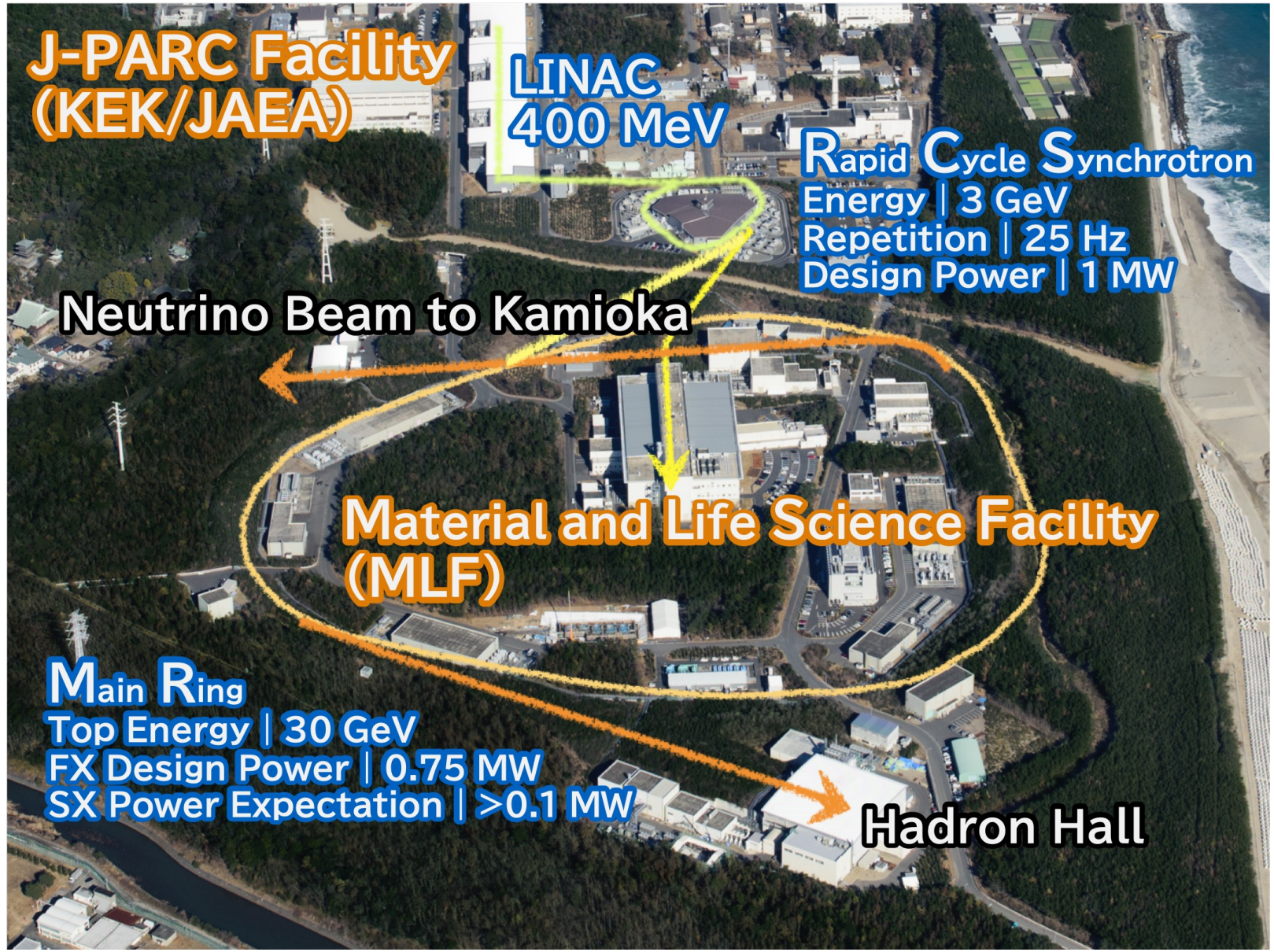
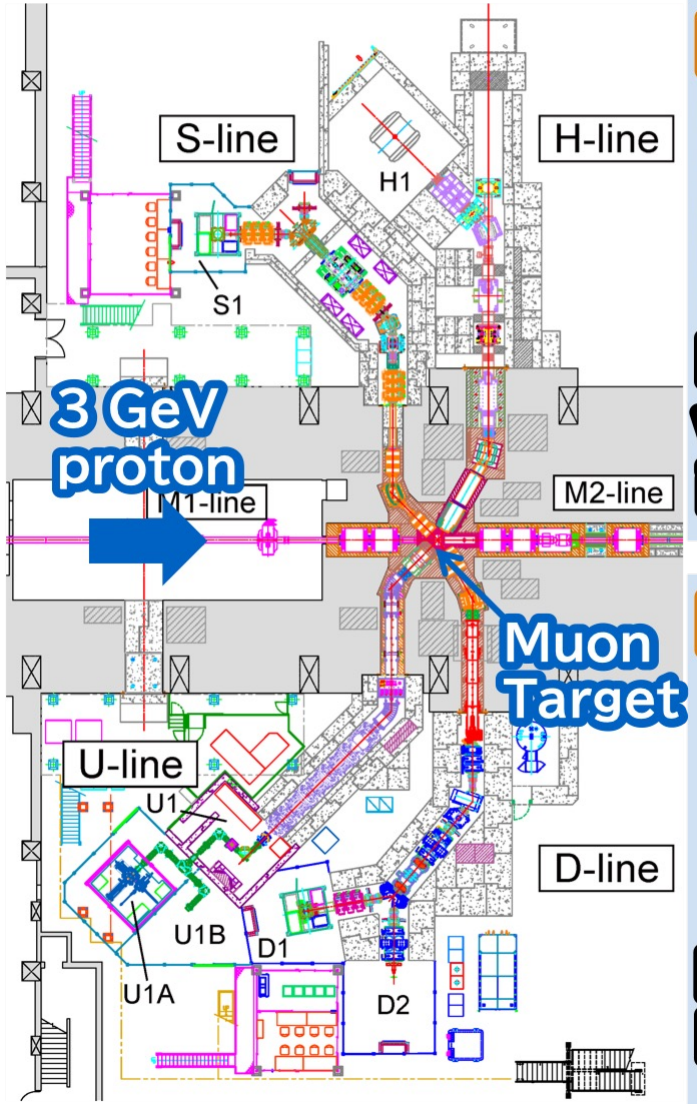
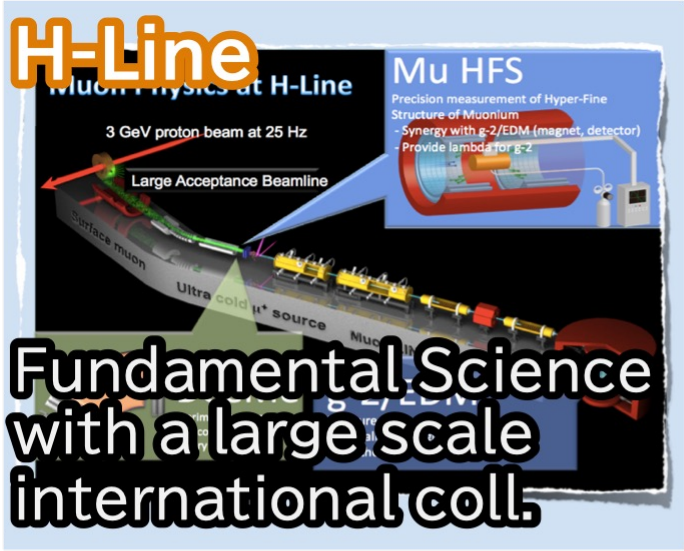
Goal: Stat. and Syst. uncertainties on $\Delta\nu \approx 5$ Hz (1.2 ppb) each, 40 days running
Uncertainty on $m_\mu/m_e \approx 12$ ppb directly
If theory uncertainty $\Rightarrow$ 10 Hz, $\delta(m_\mu/m_e) \approx 3$ ppb using bound state QED theory

Present: MuSEUM Experiment at J-PARC




S-Line
Accommodate many
μSR experiments

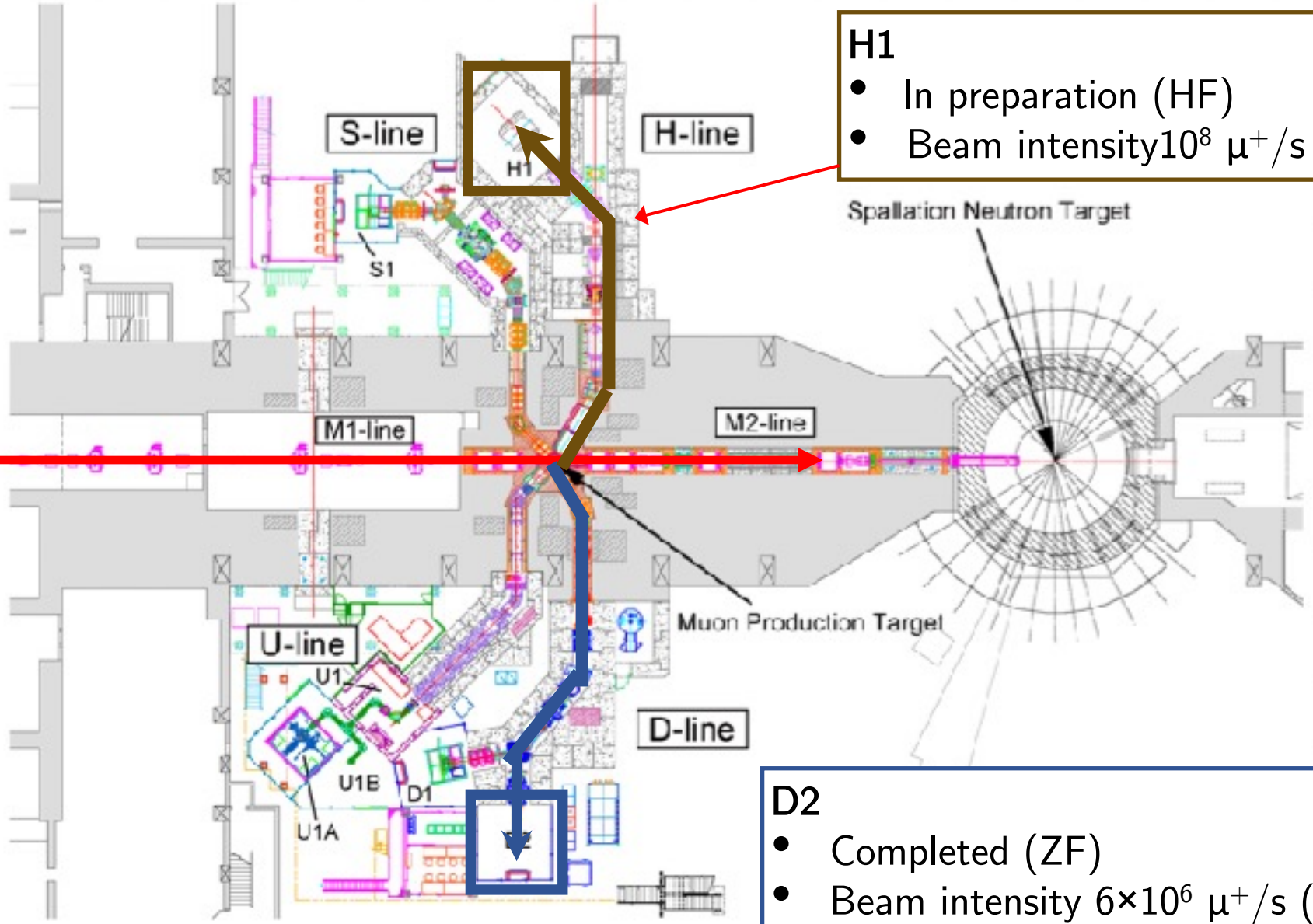

S-line
H-line
H1
S1
3 GeV
proton
M1-line
M2-line
Muon
Target
U-line
U1
U1B
U1A
D1
D2
D-line


H-Line
Mu HFS
Precision measurement of Hyper-Fine Structure of Muonium
- Synergy with g-2/EDM (magnet, detector)
- Provide lambda for g-2
3 GeV proton beam at 25 Hz
Large Acceptance Beamline
Fundamental Science
with a large scale
international coll.


U-Line
Very unique Ultra-
Slow Muon Beam


D-Line
Mid-intensity beam line
For general use

Beam Lines used for Muonium Spectroscopy at J-PARC MLF MUSE



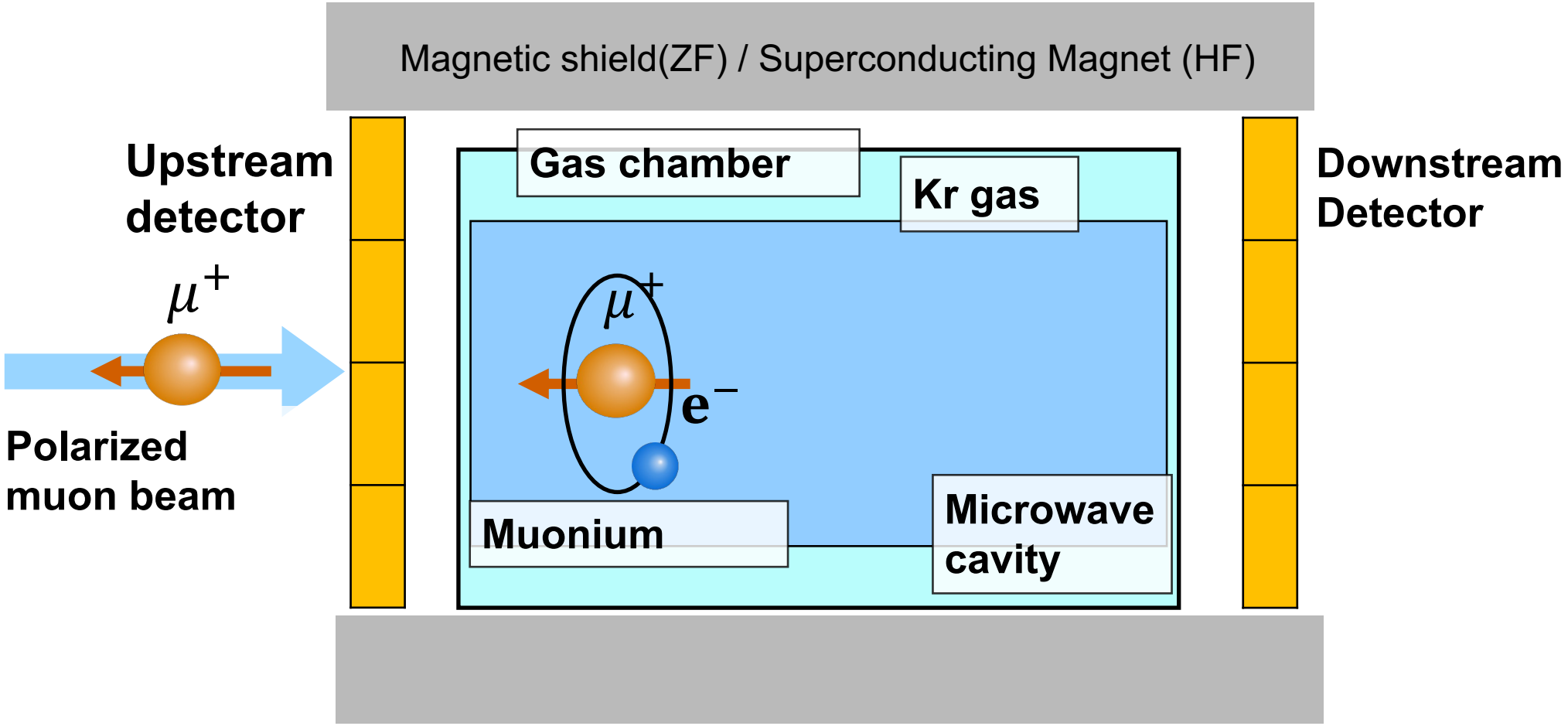
3 GeV proton

25 Hz pulsed

H1
- In preparation (HF)
- Beam intensity 10^8 μ^+/s

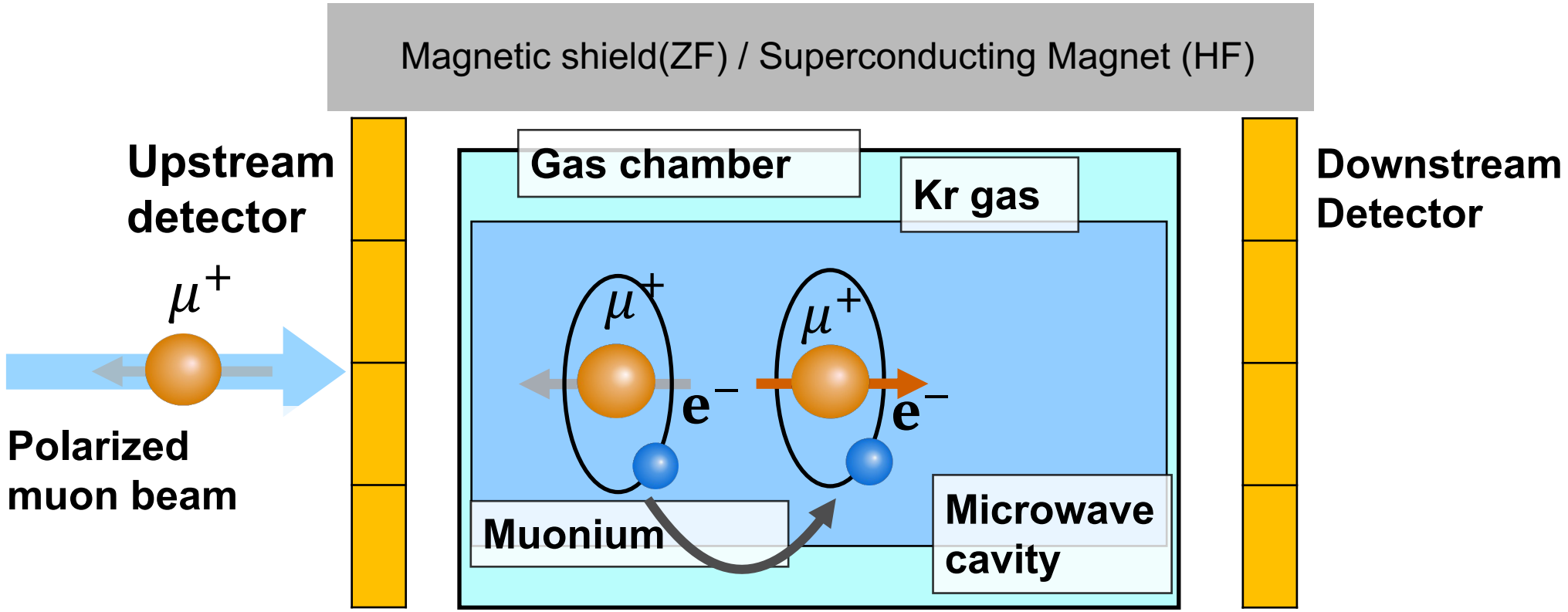
D2
- Completed (ZF)
- Beam intensity 6×10^6 μ^+/s (0.6 MW)

Experimental Technique



- Pion decays at rest yield 28 MeV/c muons, spin polarized opposite to momentum
- Transport μ^+ to Kr gas target (0.3 atm or 1.0 atm) in 1.7 T field along beam axis
- Muons thermalize, picks up e^- from Kr, forms polarized n=1 muonium in $|m_J, m_\mu\rangle = |+1/2, -1/2\rangle$ and $|-1/2, -1/2\rangle$

Experimental Technique



- If we do nothing, muon decays: $\mu^+ \to e^+ \bar{\nu}_\mu \nu_e$

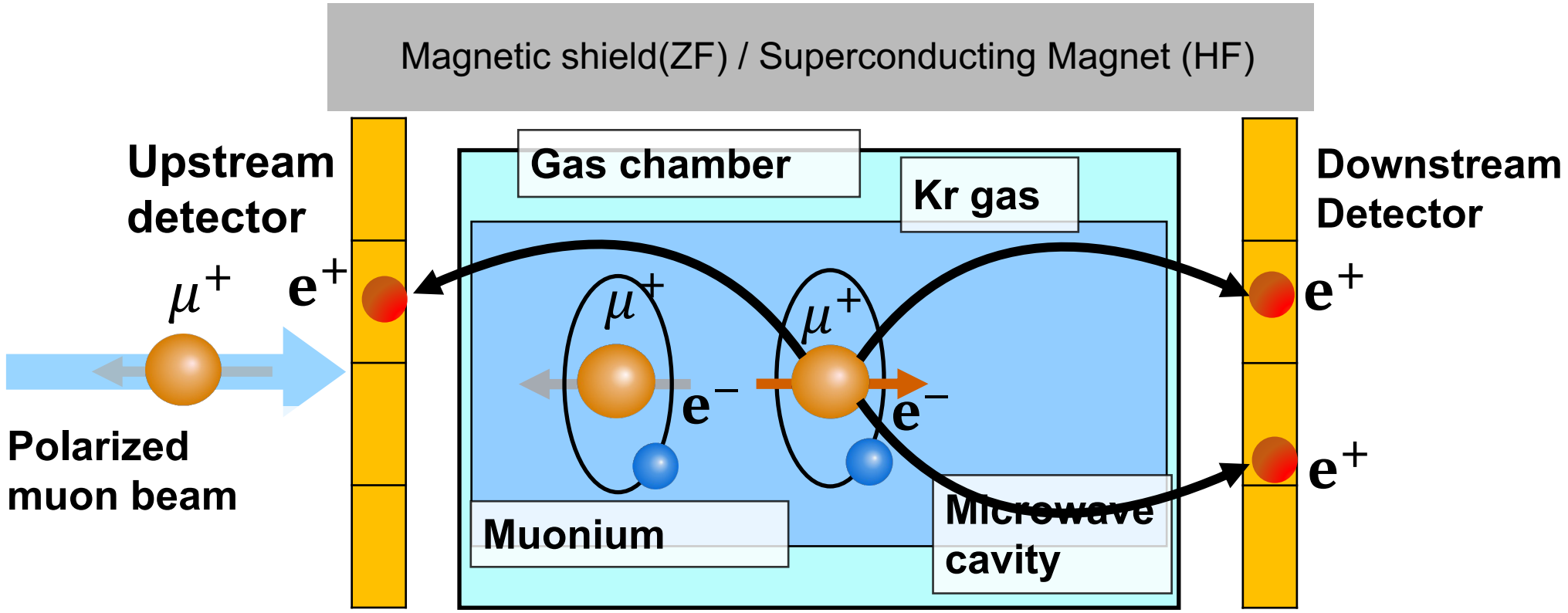
  $\Rightarrow$ Due to parity violation, highest energy e^+ emitted preferentially in μ^+ spin direction (upstream)

- NMR: apply a perpendicular microwave magnetic field at the ν_{12} or ν_{34} transition frequency:

  $\Rightarrow$ Zeeman transition flips the muon spin direction - highest energy e^+ emitted downstream

  $\Rightarrow$ $\Delta\nu$ determined from microwave frequency dependence in counts downstream/upstream

Experimental Technique



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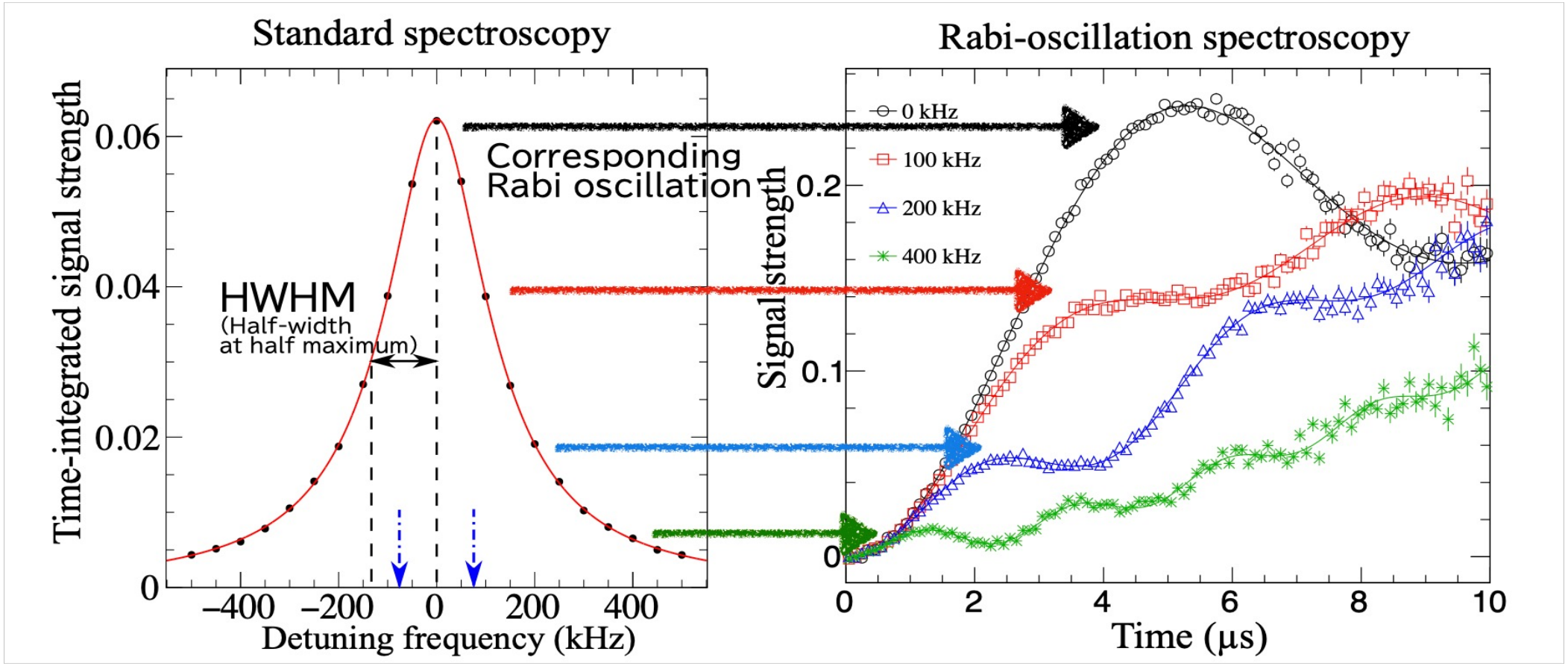
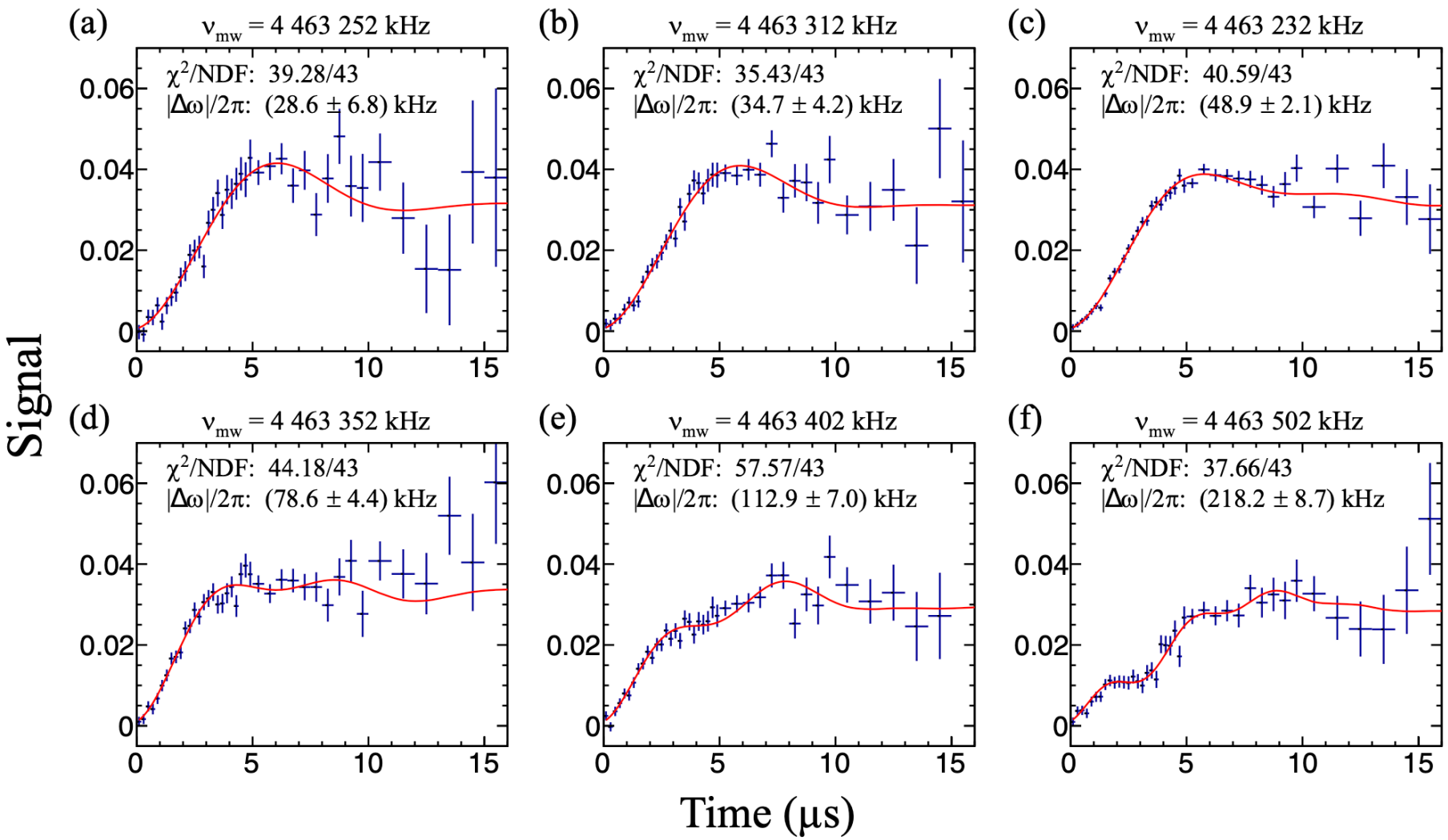
New MuSEUM Experimental Technique: Rabi-oscillation spectroscopy



- Signal depends on microwave detuning, microwave power, magnetic field, detector acceptance
- Los Alamos: looming systematic of asymmetry in microwave power across line
- MuSEUM: Rabi-oscillation: fit signal time dependence to get detuning, microwave power

New MuSEUM Experimental Technique: Rabi-oscillation spectroscopy

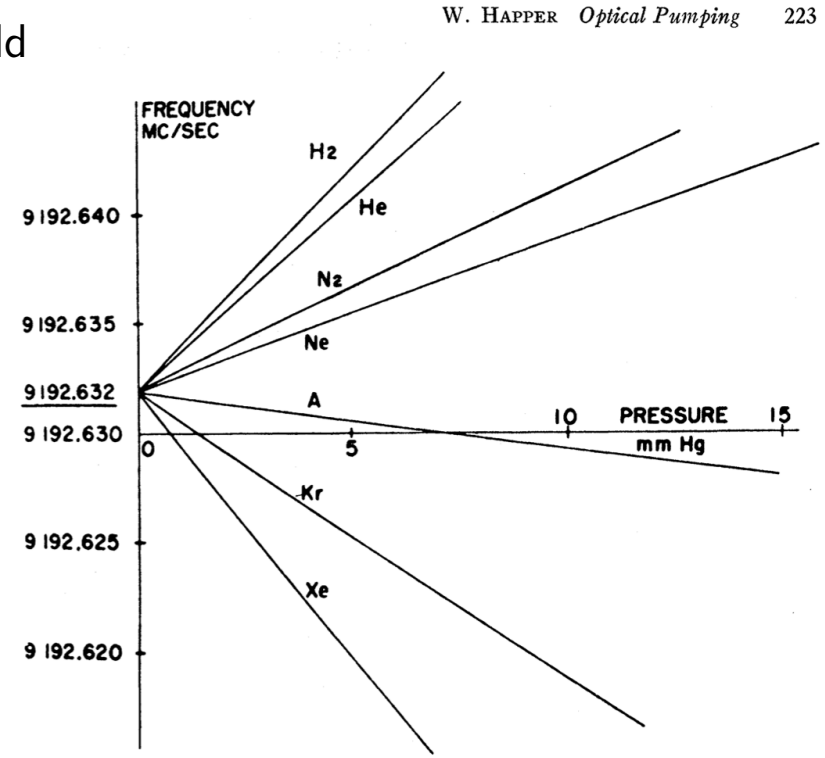


MuSEUM Rabi-oscillation spectroscopy results at zero B field: PRA **104,** L020801 (2021)

MuSEUM Improvements, Comments

- Muon flux roughly 10^8/s, roughly 10× Los Alamos, expect roughly 200 times more statistics
- Reduced MW-related uncertainties, improved MW power monitoring, Rabi-oscillation technique
- Longer cavity (more stopped muons), lower gas pressure (smaller uncertainty from pressure shift)
- Improved muon beam and stopping distribution measurements (higher resolution detector)
- High-rate capable segmented positron counters (reduced pileup systematics)
- Improved, higher resolution NMR system, reduced detector impact on magnetic field
- Improved, redundant pressure monitoring systems

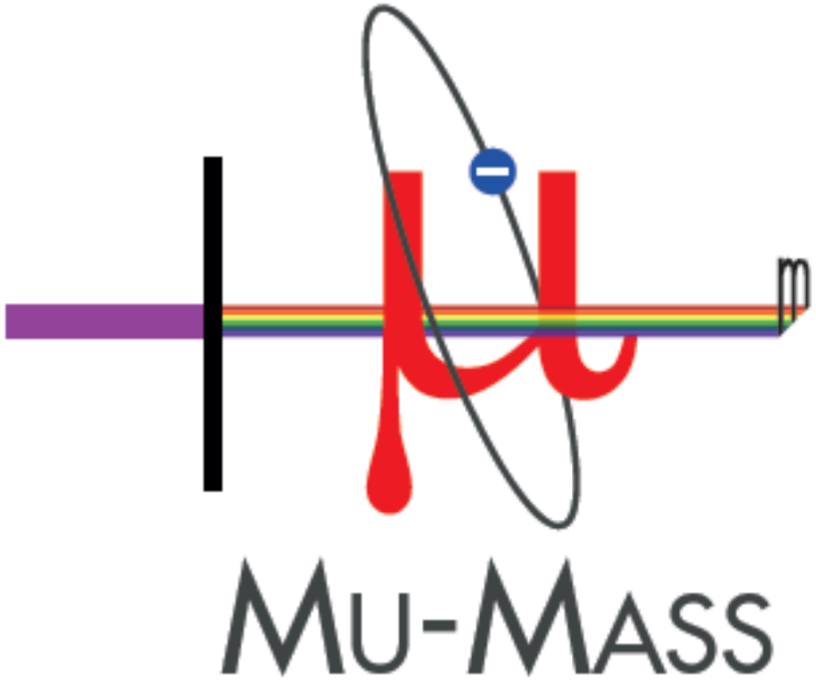
- Pressures shifts $\frac{d\nu_{12}}{dP} \approx -16.4\,\text{kHz/atm}$, $\frac{d\nu_{34}}{dP} \approx -19.6\,\text{kHz/atm}$
- HF pressure shifts in Cs (W. Happer, Rev. Mod Phys. **44**, 169 (1972).)
- Combine Kr with He or Ne to produce target gas with near-zero presssure shift; might also make more compact stopping distribution, reducing many systematics
- Telegdi: run at "magic" B=1.134 T where $d\Delta\nu_{ij}/dB = 0$, insensitive to B field



Mu-MASS: Measure 1S-2S interval to 10 kHz, determine m_μ/m_e to 1 ppb

ETH zürich

IPA



Mu-MASS (MuoniuM lAser SpectroScopy)

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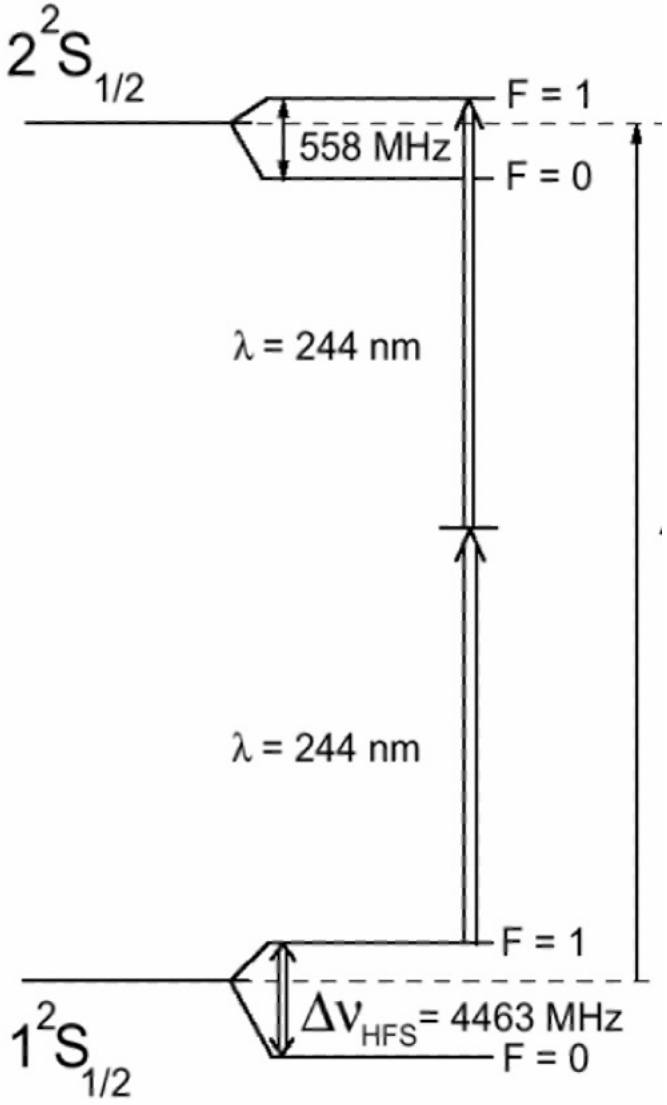
https://www.psi.ch/en/ltp/mu-mass

Mu-MASS material: Paolo Crivelli

Mu-MASS: Measure 1S-2S interval to 10 kHz, determine m_μ/m_e to 1 ppb



1S-2S

$$\Delta\nu_{1S2S}(\text{expt.}) = 2455528941.0(9.8)\,\text{MHz}$$

Meyer et al. PRL84, 1136 (2000), limited by frequency chirp of pulsed laser

$$\Delta\nu_{1S2S}(\text{theory}) = 2455528935.4(1.4)\,\text{MHz}$$

I. Cortinovis et al. (manuscript in preparation)

$$E_n = -\frac{R_\infty c}{n^2(1+m_e/m_\mu)}(1+\mathcal{F})$$

$$\nu_{1S-2S} \approx \frac{3}{4}\frac{R_\infty c}{1+m_e/m_\mu}$$

-Prediction limited by knowledge of muon mass
-QED calculation uncertainty at 6 kHz

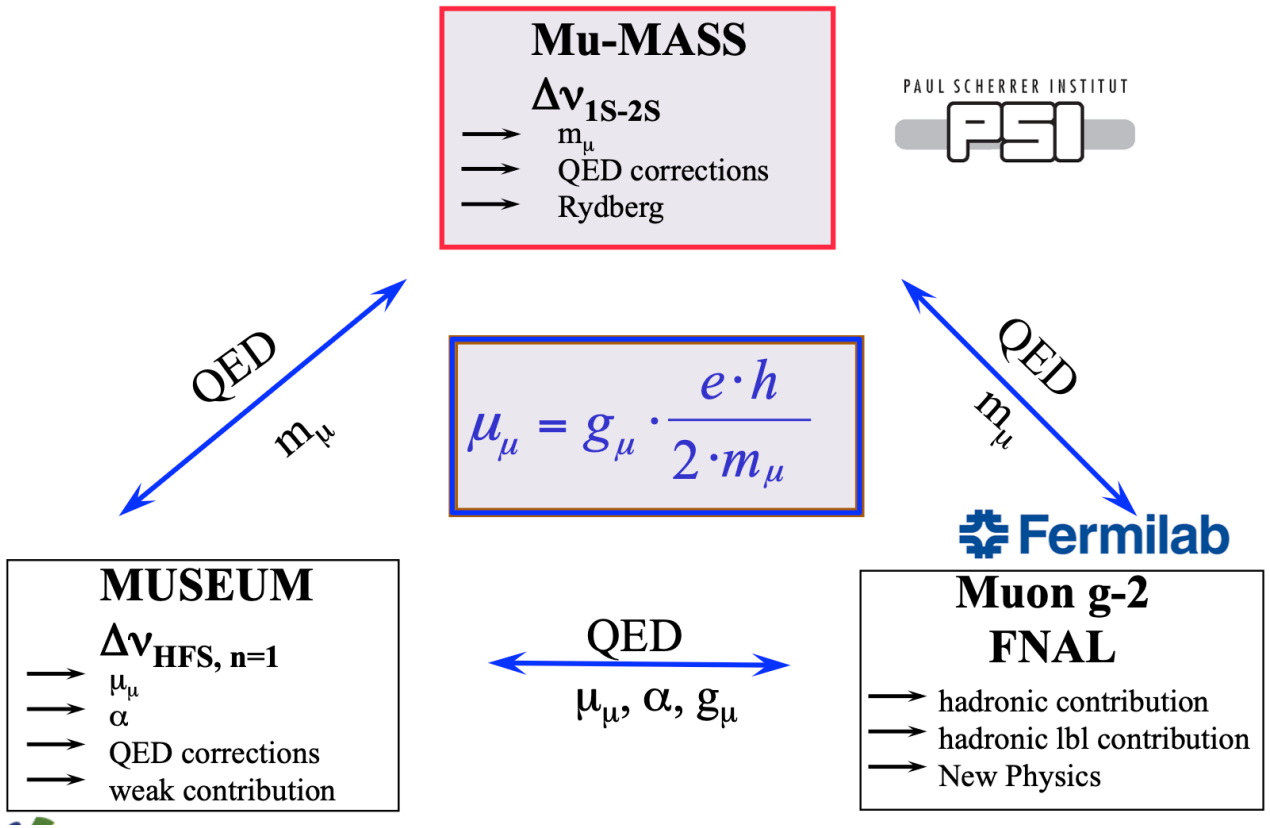
Irene Cortinovis et al., arXiv:2302.22883v3

Mu-MASS: Measure 1S-2S interval to 10 kHz, determine m_μ/m_e to 1 ppb

Mu-MASS: Measure **1S-2S transition** with Doppler free laser spectroscopy
GOAL: improve by 3 orders of magnitude (10 kHz, 4 ppt)

OUTPUT

- → **Muon mass @ 1 ppb**
- → Ratio of q_e/q_μ @ 1 ppt
- → Search for New Physics
- → **Test of bound state QED (1×10^{-9})**
- → Input to muon g-2 theory
- → **Rydberg constant @ ppt level**
- → New determination of α @ 1 ppb



Mu-MASS: Experimental Technique (Paulo Crivelli)

3. 1S-2S laser excitation of Muonium: High power UV CW laser at 244 nm, cavity-enhanced to >20 W of stable intracavity power
4. Photoionization of 2S state with pulsed laser

Opt. Express 29, 27450 (2021)

2. Muonium formation in SiO2 target

PRL 108, 143401 (2012)

In parallel: M formation diagnostics system

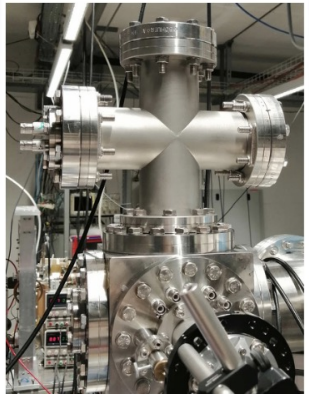
1. Low energy μ^+ beam (LEM beamline at PSI, ~10k μ^+/s)

JINST 10, P10025 (2015)

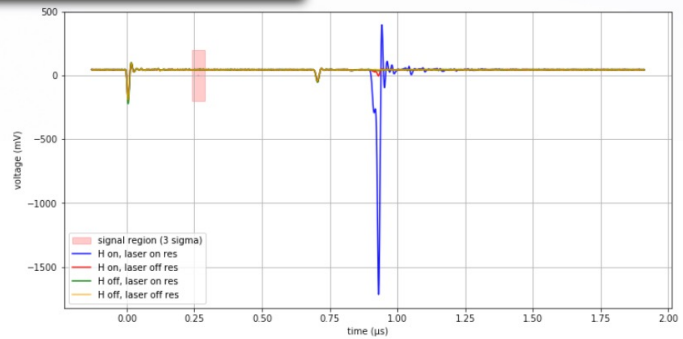
7. Coincidence with positron from decay, in scintillators

6. Muon detection in MCP

5. Guiding ionized muon further away in a "protected" area



Commissioning with residual hydrogen in vacuum chamber+ pulsed UV laser



Mu-MASS Current Status

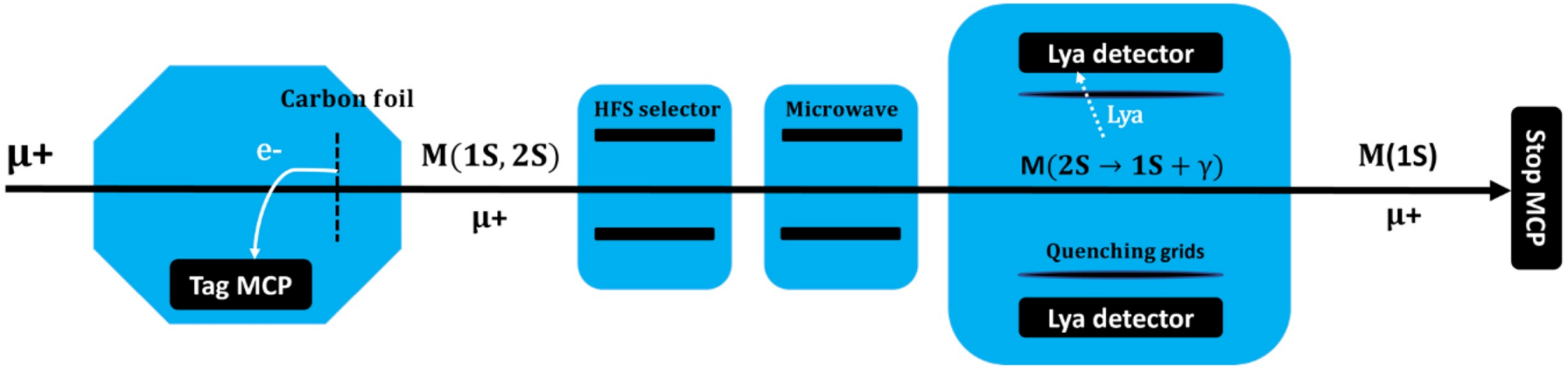
The 1S-2S signal rate is proportional to

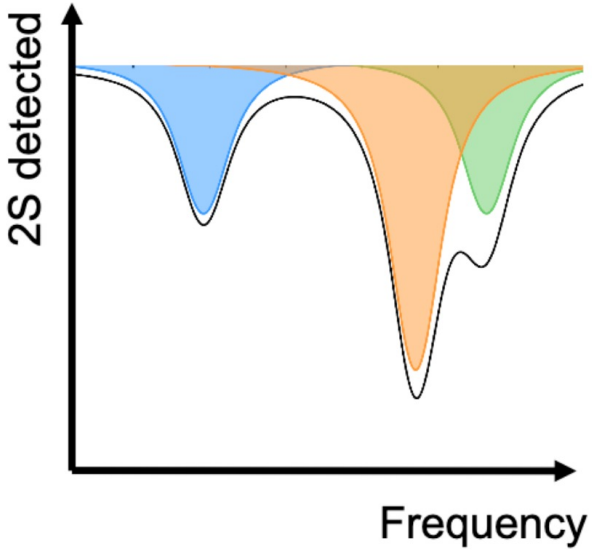
$$R \sim N_{\mathrm{Mu}} \cdot I^2 \cdot t^2 \quad \text{where} \quad \begin{cases} N_{\mathrm{Mu}}: & \text{Muonium production rate} \\ I: & \text{Laser intensity} \\ t \sim v^{-1}: & \text{Interaction time} \end{cases}$$

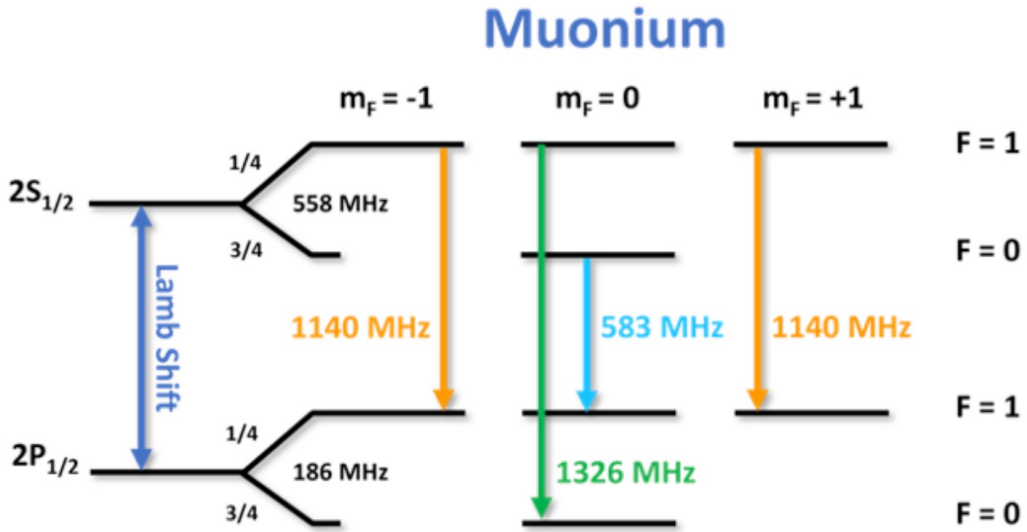
Need a Mu source with high yield and low energy

- Extremely challenging, few events per days, low background required (and demonstrated)
- Have demonstrated >25 W circulating laser power in QCW mode
- Triggerable 355 nm pulse laser for ionization ready
- Took data Dec 2023, physics runs in 2023-2024

Lamb shift measurements by Mu-MASS at LEM at PSI: Paulo Crivelli

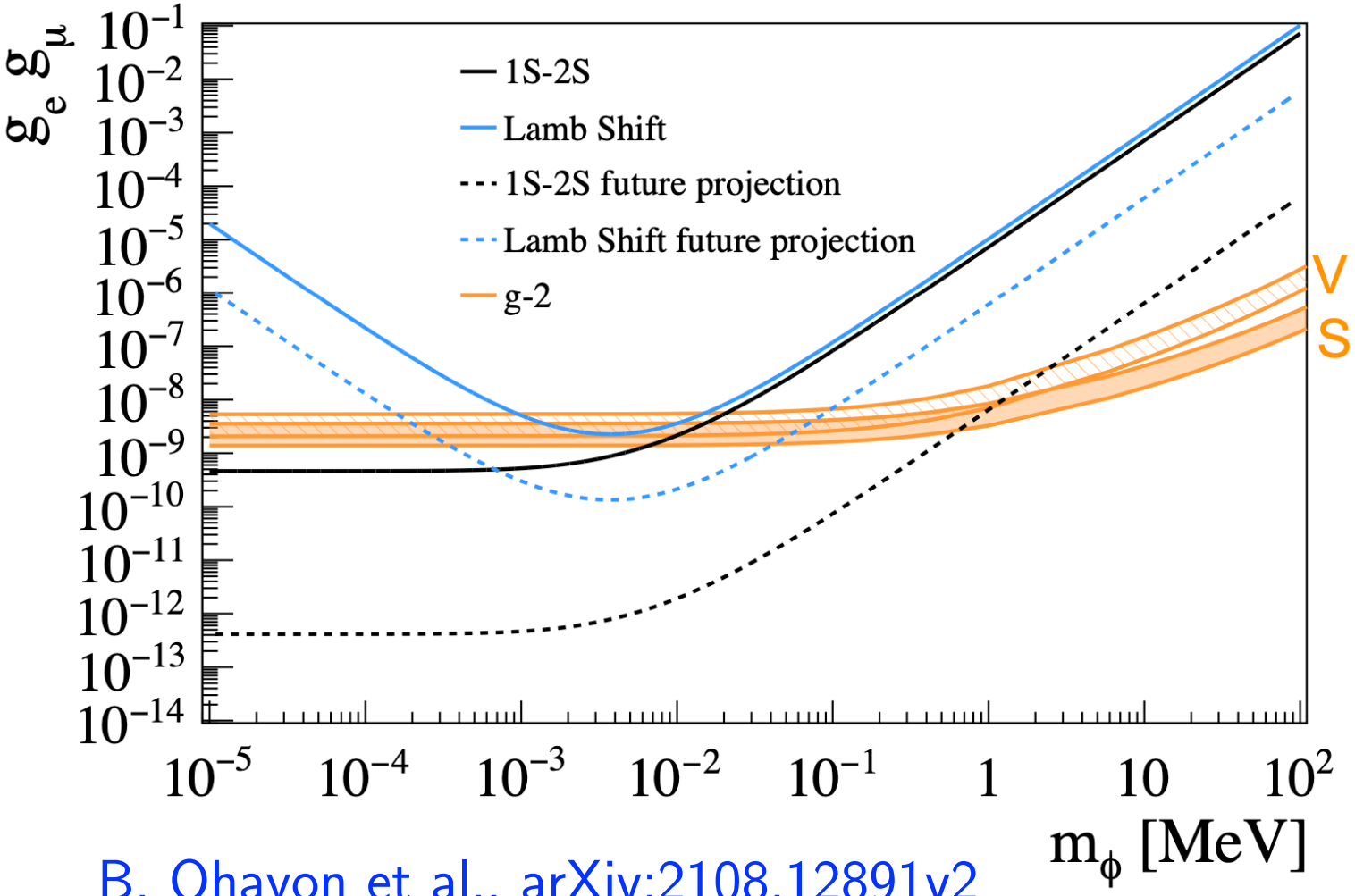






- Measured n=2 Lamb shift: $1047.2\,(2.3)_{\text{stat}}(1.1)_{\text{syst}}$ MHz, factor 10 improvement
- Precision measurement of the Lamb shift in Muonium, B. Ohayon *et al.*,, arXiv:2108.12891v2

Lamb shift measurements by Mu-MASS at LEM at PSI: Paulo Crivelli



$$V_{ss}(\vec{r}) = -g_e^s g_\mu^s \frac{e^{-m_s r}}{4\pi r}$$

Can search for new physics

B. Ohayon et al., arXiv:2108.12891v2

Determine Muon Magnetic Moment Anomaly from Muonium Spectroscopy

PHYSICAL REVIEW LETTERS **127,** 251801 (2021)

Towards an Independent Determination of Muon $g-2$ from Muonium Spectroscopy

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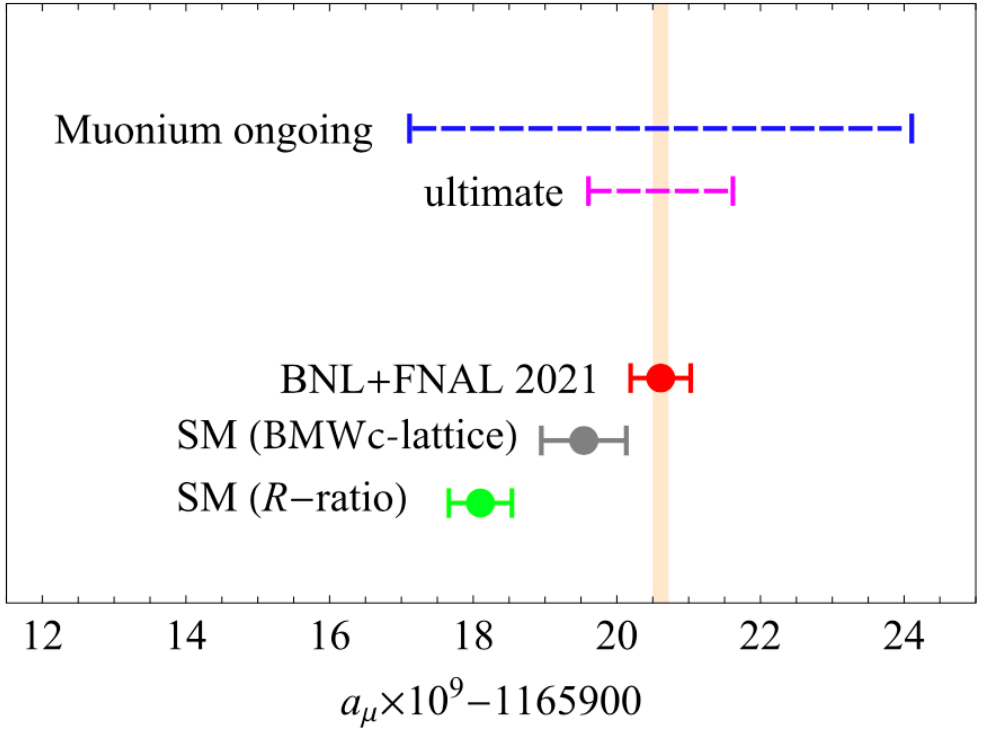
We show that muonium spectroscopy in the coming years can reach a precision high enough to determine the anomalous magnetic moment of the muon below one part per million (ppm). Such an independent determination of muon $g-2$ would certainly shed light on the ~2 ppm difference currently observed between spin-precession measurements and (R-ratio based) standard model predictions. The magnetic dipole interaction between electrons and (anti)muons bound in muonium gives rise to a hyperfine splitting (HFS) of the ground state which is sensitive to the muon anomalous magnetic moment. A direct comparison of the muonium frequency measurements of the HFS at J-PARC and the 1S-2S transition at PSI with theory predictions will allow us to extract muon $g-2$ with high precision. Improving the accuracy of QED calculations of these transitions by about 1 order of magnitude is also required. Moreover, the good agreement between theory and experiment for the electron $g-2$ indicates that new physics interactions are unlikely to affect muonium spectroscopy down to the envisaged precision.

DOI: 10.1103/PhysRevLett.127.251801



$$\vec{\mu}_\ell = g_\ell \left(\frac{q_\ell}{2m_\ell}\right)\vec{S}_\ell,$$

$$\Delta\nu \approx \frac{8\pi}{3} g'_e \mu_B g'_\mu \mu_B^\mu \frac{1}{\pi a_\mu^3}$$

Measure magnetic moment at ppb level, get a_μ at or below 1 ppm level

Muonium Spectroscopy: Summary

- Some extremely interesting, challenging, and consequential experiments underway
- Improvements by factor 10-20 in muon mass possible
- Long term possibility of sensitivity to new physics
- New, more intense source of low energy muons, surface muons would help