

# theory thoughts on $\mu \rightarrow e$ at one event in $10^{20}$

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1. intro  $\left\{ \begin{array}{l} \text{why LFV?} \\ \text{why } \mu \leftrightarrow e? \\ \text{is } 10^{-20} \text{ special?} \end{array} \right.$
2. are  $\mu \rightarrow e\gamma$ ,  $\mu \rightarrow e\bar{e}e$ ,  $\mu A \rightarrow eA$  sufficient for discovery?
3. if we see some pattern of  $\mu \rightarrow e\gamma$ ,  $\mu \rightarrow e\bar{e}e$ ,  $\mu A \rightarrow eA$ —what can we learn?
  - complementarity + reach of three processes
  - reject models?
  - anything about  $\tau \leftrightarrow l$ ?
4. some theory about  $\mu A \rightarrow eA$

## why $\mu \rightarrow e$ LFV? (we are sure $\mu \leftrightarrow e$ is interesting—why?)

- expt {DM,Baryon Asym., DM,  $[m_\nu]$ } and theory problems {hierarchy, CP, flavour} say NP is somewhere... ...and we like discovering :)  
in particular,  $[m_\nu] \Rightarrow$  LFV is an NP signature that exists !
- most reassuring way to discover New Particles is to produce at colliders...but LFV (probably) crucial to identify NP flavour structure  
by analogy with SM, where CKM+CP from quark flavour physics + masses at colliders
- independent info on LFV from colliders ( $h \rightarrow e^\pm \mu^\mp, \dots$ ), (quark flavour-changing) meson decays ( $K \rightarrow \bar{e}\mu, \dots$ ) and lepton decays ( $\mu \rightarrow e\gamma, \dots, \tau \rightarrow \mu\rho \dots$ )  
**exptal reach in low-E  $\mu \rightarrow e$  makes it promising for discovery!**  
(models may predict bigger rates for  $\tau \leftrightarrow l \dots$  'cuze exptal  $\mu \leftrightarrow e$  bds are stronger :) )  
more channels in  $\tau \leftrightarrow l$  for distinguishing models

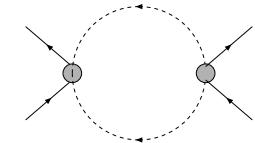


# theoretical milestones —is $\mu \leftrightarrow e$ at $10^{-20}$ “special”?

theoretical guesstimates for lower bd on BRs:

1. calculate loops with EW bosons and  $m_\nu$  (SM for Dirac  $m_\nu$ , in EFT for Majorana):

$$BR_{LFV} \gtrsim \left| \frac{m_\nu^2}{16\pi^2 v^2} \log \frac{\Lambda_{NP}}{v} \right|^2 \gtrsim 10^{-55}$$

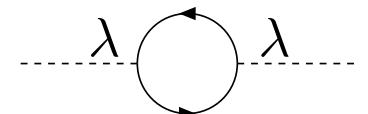


ugh....but  $m_\nu$ , LFV different dependance on NP scale  $\Lambda_{NP}$ (and cplgs  $\lambda$ ?):

$$m_\nu \sim \frac{\lambda^2 v^2}{\Lambda_{NP}} \quad , \quad \sqrt{BR_{LFV}} \sim \frac{\lambda^2 v^2}{\Lambda_{NP}^2}$$

2. input other relation for  $\lambda$  and  $\Lambda_{NP}$ :  $\Delta m_H^2 \Big|_{NP} \sim \frac{\lambda^2 \Lambda_{NP}^2}{16\pi^2} < v^2$

$$\Rightarrow BR_{LFV} \gtrsim \left| \frac{1}{16\pi^2} \left[ \frac{m_\nu}{\pi v} \right]^{4/3} \right|^2 \sim 10^{-38}$$



still too small... but used same coupling  $\lambda$  for LNV and LFV...

3. many models separate LFV from LNV (SUSY seesaw, type II and inverse seesaws, NP for flavour anomalies...) and predict  $BR_{LFV} \lesssim \text{expt}$

## which processes: $\mu \rightarrow e$ , $\Delta F_Q = 0$ + heavy NP

process	current bd on BR	future sensitivity	dreams
$\mu \rightarrow e\gamma$	$< 4.2 \times 10^{-13}$ (MEG)	$6 \times 10^{-14}$ (MEGII)	$10^{-(14 \rightarrow 20)}$
$\mu \rightarrow e\bar{e}e$	$< 1.0 \times 10^{-12}$ (SINDRUM)	$10^{-16}$ (202x, Mu3e)	$10^{-20}$
$\mu Au \rightarrow eAu$	$< 7 \times 10^{-13}$ , (SINDRUMII)		$10^{-20}$
$\mu Ti \rightarrow eTi$	$< 6.1 \times 10^{-13}$ , (SINDRUMII)	$10^{-(16 \rightarrow ?)}$ (Mu2e, COMET)	$10^{-(18 \rightarrow 20)}$
?( $\mu Li \rightarrow eLi$ )			
$(\mu \rightarrow e\gamma\gamma)$	$< 7.2 \times 10^{-11}$ (CrystalBox)	?	??

NP heavy  $\Rightarrow$  no  $\mu \rightarrow ea$ , neglect  $\mu \rightarrow e\gamma\gamma$  (dim8,  $\mu A \rightarrow eA$  + sensitive), neglect SD  $\mu A \rightarrow eA \ll SI$  (2007.09612)

- $\mu \rightarrow e_L$  processes described at exptal scale by 6 operators:

$$\begin{aligned} \delta \mathcal{L} = & \frac{1}{\Lambda_{NP}^2} \left[ C_D (m_\mu \bar{e} \sigma^{\alpha\beta} P_R \mu) F_{\alpha\beta} + C_S (\bar{e} P_R \mu) (\bar{e} P_R e) + C_{VR} (\bar{e} \gamma^\alpha P_L \mu) (\bar{e} \gamma_\alpha P_R e) \right. \\ & \left. + C_{VL} (\bar{e} \gamma^\alpha P_L \mu) (\bar{e} \gamma_\alpha P_R e) + C_{Alight} \mathcal{O}_{Alight} + C_{Aheavy\perp} \mathcal{O}_{Aheavy\perp} \right] \end{aligned}$$

$\{C\}$  are  $\mathcal{O}(1)$  dimless numbers that can be exptally measured

$\mathcal{O}_{Alight}$  = combo of 4fermion operators probed by light targets (Al, Ti)

$\mathcal{O}_{Aheavy\perp}$  = indep. combo of 4fermion ops probed by heavy targets (Au)

# Are $\mu \rightarrow e\gamma$ , $\mu \rightarrow e\bar{e}e$ , $\mu A \rightarrow eA$ sufficient for discovery?

3 processes, probe a few ops— if  $\Delta F_Q=0$ ,  $\mu \rightarrow e$  occurs, will it contribute to

$\mu \rightarrow e\gamma$ ,  $\mu \rightarrow e\bar{e}e$  or  $\mu A \rightarrow eA$ ?

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**Probably yes:** SM loops ensure almost every  $\Delta QF = 0$ ,  $\mu \rightarrow e$  interaction with  $\leq 4$  legs, contributes  $\gtrsim \mathcal{O}(10^{-3})$  to amplitudes  $\mu \rightarrow e\gamma$ ,  $\mu \rightarrow e\bar{e}e$  and/or  $\mu A \rightarrow eA$  (not  $\bar{e}\mu G\tilde{G}$ ,  $\bar{e}\mu F\tilde{F}$ ,  $\bar{e}\gamma\mu F\partial F\dots$ ) (but reach in  $\Lambda_{NP}$  reduced by factor  $\mathcal{O}(30)$ , for operators contributing via loops)

coefficient	$\mu \rightarrow e\gamma$	$\mu \rightarrow e\bar{e}e$	$\mu A \rightarrow eA$
$ C_{D,X} $	$1.12 \times 10^{-8}$	$4.30 \times 10^{-7}$	$2.35 \times 10^{-7}$
$ C_{V,XX}^{ee} $	$1.10 \times 10^{-4}$	$7.80 \times 10^{-7}$	$1.86 \times 10^{-5}$
$ C_{V,XY}^{ee} $	$2.55 \times 10^{-4}$	$9.34 \times 10^{-7}$	$3.77 \times 10^{-5}$
$ C_{S,XX}^{ee} $	$1.73 \times 10^{-4}$	$2.8 \times 10^{-6}$	$(3.64 \times 10^{-3})$
$ C_{V,XX}^{\mu\mu} $	$1.10 \times 10^{-4}$	$5.60 \times 10^{-5}$	$1.85 \times 10^{-5}$
$ C_{V,XY}^{\mu\mu} $	$2.56 \times 10^{-4}$	$1.12 \times 10^{-4}$	$3.77 \times 10^{-5}$
$ C_{S,XX}^{\mu\mu} $	$8.24 \times 10^{-7}$	$(1.58 \times 10^{-5})$	$(1.73 \times 10^{-5})$
$ C_{V,XX}^{\tau\tau} $	$3.80 \times 10^{-4}$	$1.95 \times 10^{-4}$	$1.24 \times 10^{-5}$
$ C_{V,XY}^{\tau\tau} $	$4.40 \times 10^{-4}$	$1.91 \times 10^{-4}$	$1.25 \times 10^{-5}$
$ C_{S,XX}^{\tau\tau} $	$5.33 \times 10^{-6}$	$1.02 \times 10^{-4}$	$1.12 \times 10^{-4}$
$ C_{S,XY}^{\tau\tau} $	—	—	—
$ C_{T,XX}^{\tau\tau} $	$1.10 \times 10^{-8}$	$(4.20 \times 10^{-7})$	$(2.30 \times 10^{-7})$

**sensitivities/1-at-a-time bds** for  $\delta\mathcal{L} = 2\sqrt{2}G_F C_i \mathcal{O}_i$ ; if model gives smaller coefficients, it is consistent with data. If it generates larger coefficients, need to arrange a cancellation...

## what can we learn if see $\mu \rightarrow e\gamma$ , $\mu \rightarrow e\bar{e}e$ , or $\mu A \rightarrow eA$ ?

EFT gives recipe to address this question.

(we want to find *the* model, so can't study this question in a model...)

- to do EFT, need:

- 1) an operator basis: all the operators that expt can be sensitive to
- 2) a recipe to change scale ( $\supset$  include loops): 1-loop RGEs(+some 2loop bits)

- focus on 1); want basis appropriate for our question... (no physics in basis choice)

“Usual” basis theory-motivated (gauge invar., remove derivatives, ...),

below  $m_W$  contains  $\sim 100$   $\mu \rightarrow e$ ,  $\Delta q_F=0$  operators with  $\leq 4$  legs

$\mu \rightarrow e_L \gamma$ ,  $\mu \rightarrow e_L \bar{e}e$ , and  $\mu A \rightarrow e_L A$  sensitive to *only six* operators at exptal scale

$\Rightarrow$  stay in 6-d exptally-motivated subspace, its what expt can probe!

concretely: define, eg  $\vec{v}_{DL}(m_\mu, \Lambda_{NP})$  in coeff space, such that

$$BR(\mu \rightarrow e_L \gamma) = 384\pi^2 |\vec{C}(\Lambda_{NP}) \cdot \vec{v}_{DL}(m_\mu, \Lambda_{NP})|^2$$

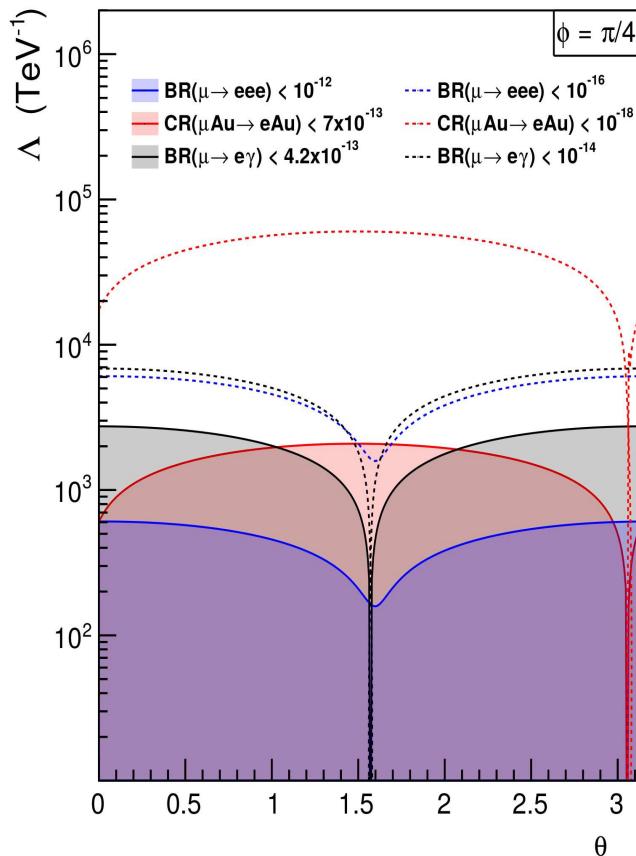
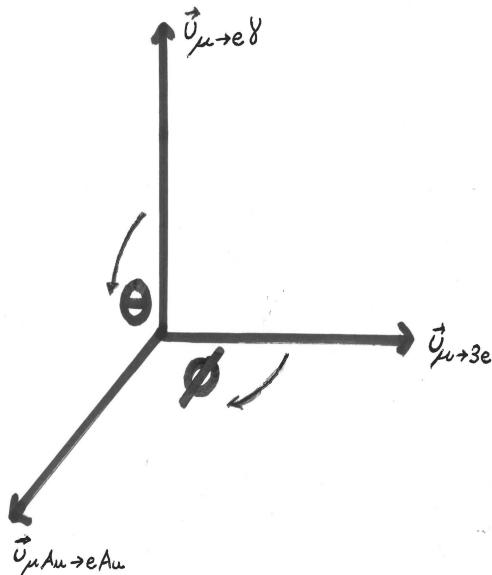
where coefficients predicted by models at  $\Lambda_{NP}$  are lined up in  $\vec{C}$ .

- Can now : 2) match to models and explore if observations can reject models!
  - 1) check that processes give complementary info about NP: $\kappa$ -plots

# Are the observables complementary? make a plot!

Restrict to 3-d space of coefficients of  $\vec{v}_{\mu \rightarrow e_L \gamma}$ ,  $\vec{v}_{\mu \rightarrow 3e_L}$ ,  $\vec{v}_{\mu Al \rightarrow e_L Al}$  ( $= z, x, y$ ). Model predicts a vector  $\vec{C}/\Lambda_{NP}^2$ ; can fix  $|\vec{C}| = 1$  and constrain  $\Lambda_{NP}(\theta, \phi)$ :

$$\vec{C} \cdot \vec{v}_{\mu \rightarrow e_L \gamma} \equiv \frac{v^2 \cos \theta}{\Lambda_{NP}^2}$$



see 2204.00564

# Plot complementarity+reach of $\mu \rightarrow e\gamma$ , $\mu \rightarrow e\bar{e}e$ and $\mu A \rightarrow eA$

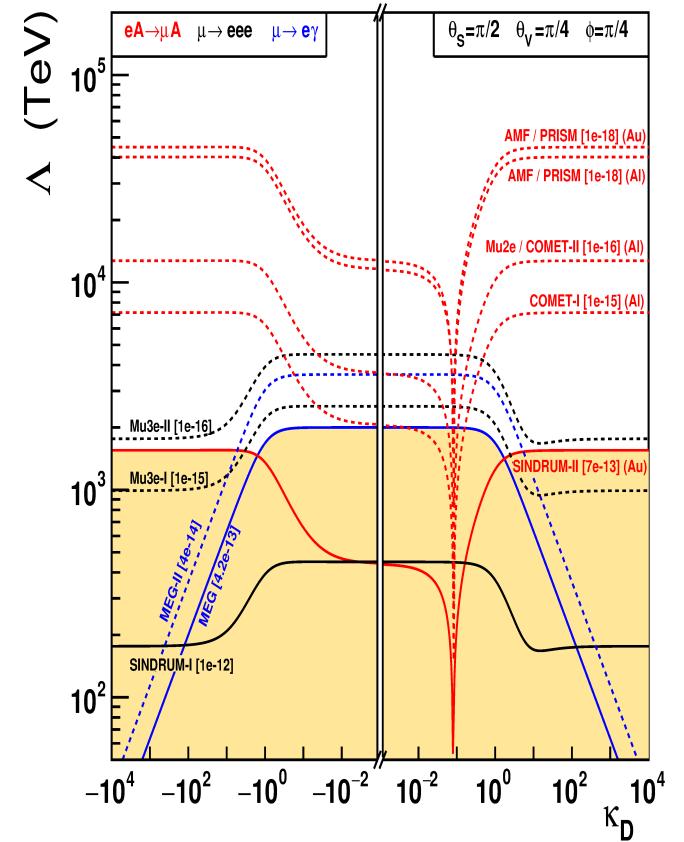
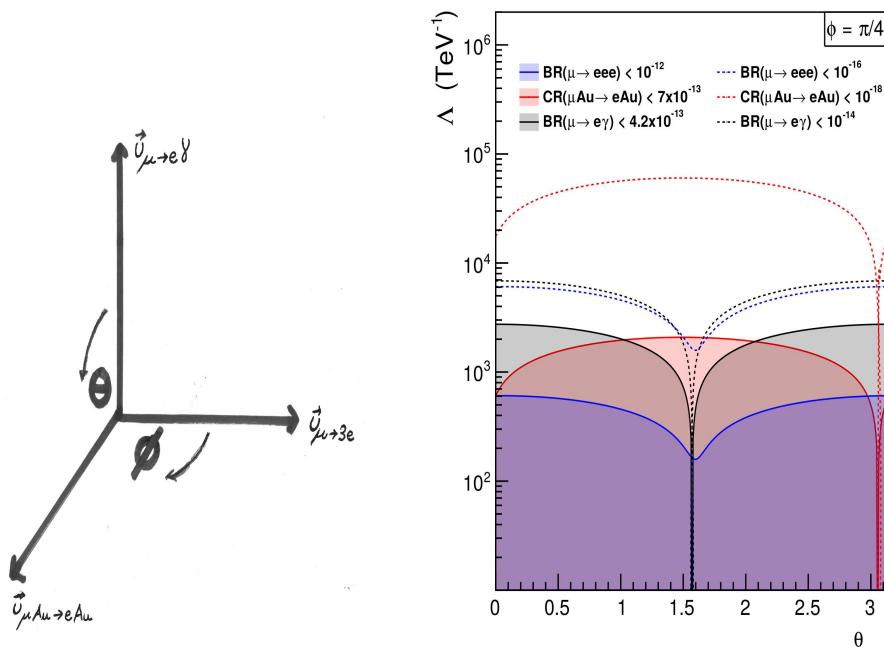
(in theoretically self-consistent EFT, including LO loops, cancellations...)

Restrict to 3-d space of coefficients of  $\vec{v}_{\mu \rightarrow e_L \gamma}$ ,  $\vec{v}_{\mu \rightarrow 3e_L}$ ,  $\vec{v}_{\mu A u \rightarrow e_L A u}$  ( $= z, x, y$ ).

Impose  $|\vec{C}|=1$  and use spher. coord.:

$$\vec{C} \cdot \vec{v}_{\mu \rightarrow e_L \gamma} \equiv \frac{v^2 \cos \theta}{\Lambda_{\text{NP}}^2}$$

Define  $\kappa_D = \cot g(\theta_D - \pi/2)$



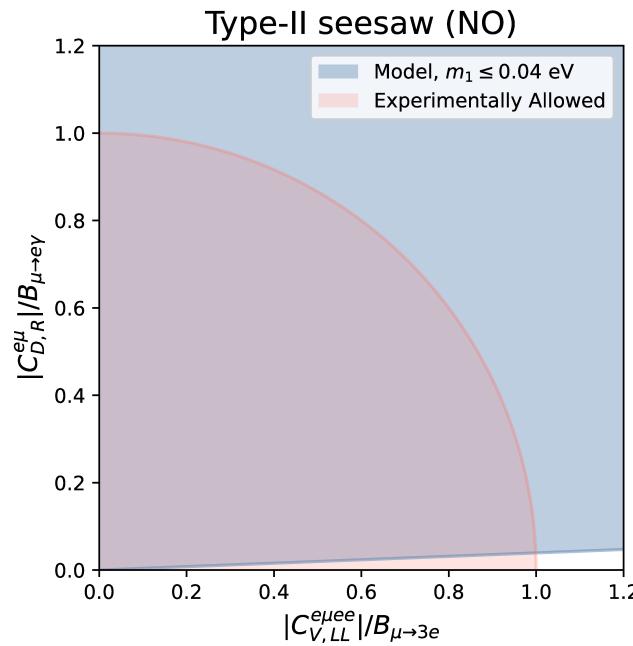
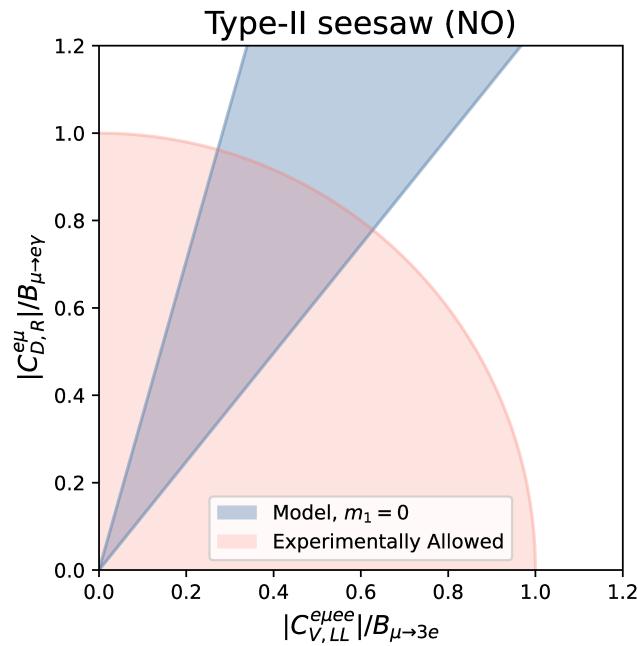
see 2204.00564

## match to models, and explore what we can learn

**Ex:** Type II seesaw = add triplet scalar  $\vec{T}$ ,  $[m_\nu] \propto [Y]\lambda_H$

$$\mathcal{L} \supset \left( [Y]_{\alpha\beta} \bar{\ell}_\alpha^c \varepsilon \vec{\tau} \cdot \vec{T} \ell_\beta + M_T \lambda_H H \varepsilon \vec{\tau} \cdot \vec{T}^* H + \text{h.c.} \right) + \dots$$

Do the model predictions fill the whole experimentally accessible region?



vertical axis  $\sim C_{DL}(m_\mu) \leftrightarrow \mu \rightarrow e\gamma$ , horizontal axis  $C_{VL}(m_\mu) \leftrightarrow \mu \rightarrow e\bar{e}e$ .  
Normalised to current bd.

LEFT:  $m_1 = 0$

RIGHT:  $m_1 < 0.4$  eV; ( $4l$  coeff  $\propto$  mass scale, can vanish like  $m_{ee}$ )  
(NB  $2q2l \propto \alpha_e/4\pi$ , so  $\mu A \rightarrow eA$  mediated by dipole)

# can $\mu \leftrightarrow e$ tell us anything about $\tau \leftrightarrow l$ ?

(resurrect dim8:  $[\mu \rightarrow \tau] \times [\tau \rightarrow e] = [\mu \rightarrow e]$ )

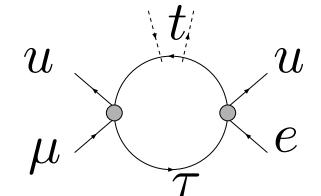
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recall exptal reach:  $\text{BR}(\mu \rightarrow e) \rightarrow 10^{-(18 \rightarrow 20)} \sim [\text{BR}(\tau \rightarrow l) \rightarrow 10^{-9}]^2$

1. if model has  $(\mu \rightarrow \tau), (\tau \rightarrow e)$ , then no conserved flavour, so “expect”  $\mu \rightarrow e$

2. calculate something model-independent: In SMEFT,  $(\text{dim6})^2 \rightarrow \text{dim8}$ ,  
eg  $\bar{\ell}_\tau \mu \varepsilon \bar{q}_3 u \times (\bar{\ell}_e \gamma \ell_\tau)(\bar{q}_1 \gamma q_3) \rightarrow \bar{\ell} e \varepsilon \bar{q} u H^\dagger H$

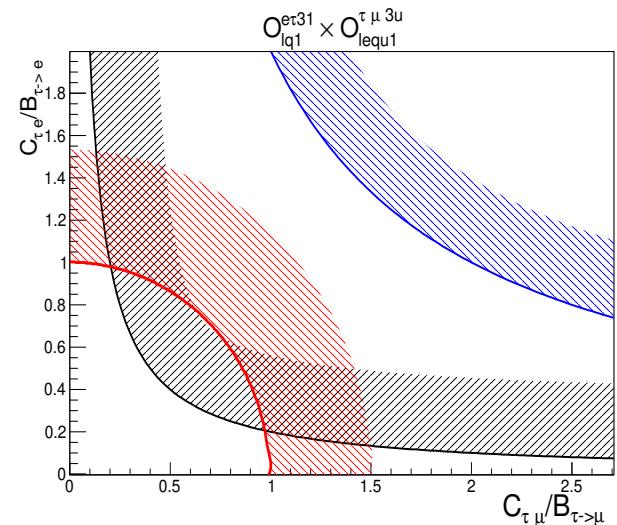
$$\frac{\Delta^{(8)} C^{e\mu uu}}{\Lambda_{\text{NP}}^4} \simeq \frac{\{y_t^2, g^2\}}{16\pi^2} \frac{C_{LQ}^{e\tau ut}}{\Lambda_{\text{NP}}^2} \frac{C_{LEQU}^{\tau\mu tu}}{\Lambda_{\text{NP}}^2}$$



so effective low-energy 4-fermion interaction  $2\sqrt{2}G_F C_S$

$$\Delta^{(6)} C_S^{e\mu uu} \propto \frac{v^4}{16\pi^2 \Lambda_{\text{NP}}^4} C^{e\tau ut} C^{\tau\mu tu}$$

3. eg  $\mu A \rightarrow e A$  sensitivity,  $(\text{BR} \leq 7 * 10^{-13} \leq 10^{-16})$   
complementary to  $B^- \rightarrow \{e, \mu\}\nu$  decays  
for some operators:



## calculating a more accurate $\mu A \rightarrow e A$ rate

Spin Indep Conversion Ratio on target A, from Kitano, Koike, Okada (2002):

$$\frac{32G_F^2 m_\mu^5}{\Gamma_{cap}} \left[ \left| \tilde{C}_{V,R}^{pp} I_{V,A}^{(p)} + \tilde{C}'_{S,L}^{pp} I_{S,A}^{(p)} + \tilde{C}_{V,R}^{nn} I_{V,A}^{(n)} + \tilde{C}'_{S,L}^{nn} I_{S,A}^{(n)} + C_{D,L} \frac{I_{D,A}}{4} \right|^2 + \{L \leftrightarrow R\} \right]$$

$I_{XA}^N$  = overlap of lepton wavefns and S/V density of Ns in target A

improvements in progress (todo list):

- include Spin Dependent (real nuclear phys caln)
- better neutron densities
- more targets
- more operators
- NLO  $\chi$ PT ...

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...lots (yet) to do!

## Summary

$\mu \rightarrow e\gamma$ ,  $\mu \rightarrow e\bar{e}e$  and  $\mu A \rightarrow eA$  have excellent sensitivity ( $\Lambda_{\text{NP}} \gtrsim 10^4 v$  upcoming,  $\Lambda_{\text{NP}} \gtrsim 10^5 \rightarrow 10^6 v$  AMF), to a few operators at low energy

Loop effects described by (leading order) RGEs ensure that almost every  $\mu \rightarrow e$  operator (chiral basis) with  $\leq 4$  legs (in broken EW), contributes to amplitudes for  $\mu \rightarrow e\gamma$ ,  $\mu \rightarrow e\bar{e}e$  and/or  $\mu A \rightarrow eA$ , suppressed by  $\gtrsim \mathcal{O}(10^{-3})$ . Can even have interesting sensitivity to products of some  $(\mu \rightarrow \tau) \times (\tau \rightarrow e)$  interactions!

Prospects for distinguishing between models using  $\mu \rightarrow e$  observations can conveniently be explored in EFT, using operator basis motivated by observables rather than theory.

Some theoretical progress in the calcn of  $\mu A \rightarrow eA$  would be welcome.

# Backup

## What are $\mathcal{O}_{A\text{light}}, \mathcal{O}_{A\text{heavy}\perp}$ ?

$$\begin{aligned}\mathcal{O}_{A\text{light},X} &\sim 0.7(\bar{e}P_X\mu) \left[ (\bar{u}u) + (\bar{d}d) + \dots \right] + 0.13(\bar{e}\gamma^\alpha P_X\mu) \left[ (\bar{u}\gamma_\alpha u) + (\bar{d}\gamma_\alpha d) \right] \\ \mathcal{O}_{A\text{heavy}\perp,X} &\simeq (\bar{e}\gamma^\alpha P_X\mu) \left[ 0.56(\bar{u}\gamma_\alpha u) + 0.8(\bar{d}\gamma_\alpha d) \right] + \dots\end{aligned}$$

obtained by matching nucleons to quarks, then writing

$$\mathcal{O}_{A\text{heavy},X} = \mathcal{O}_{A\text{light},X} + \epsilon \mathcal{O}_{A\text{heavy}\perp,X}$$

where  $\epsilon$  calculable misalignement  $\simeq 5\%$ .

**problem:** scalar density of  $u$  quarks in  $N \in \{n, p\} \simeq$  scalar density of  $d$  quarks  $\Rightarrow$  with current theory uncertainties in  $\mu A \rightarrow eA$ , measuring  $C_S^n$  and  $C_S^p$  only allows to determine  $C_S^u + C_S^d$  (but not  $C_S^u - C_S^d$ ).

## bounds/upcoming reach to $\Delta LF = 1, \Delta QF = 0$

some processes	current constraints on BR	future sensitivities
$\mu \rightarrow e\gamma$	$< 4.2 \times 10^{-13}$	$6 \times 10^{-14}$ (MEG)
$\mu \rightarrow e\bar{e}e$	$< 1.0 \times 10^{-12}$ (SINDRUM)	$10^{-16}$ (202x, Mu3e)
$\mu A \rightarrow eA$	$< 7 \times 10^{-13}$ Au, (SINDRUMII)	$10^{-(16 \rightarrow ?)}$ (Mu2e, COMET) $10^{-(18 \rightarrow ?)}$ (PRISM/PRIME/ENIGMA)
$\tau \rightarrow \{e, \mu\}\gamma$	$< 3.3, 4.4 \times 10^{-8}$	$\text{few} \times 10^{-9}$ (Belle-II)
$\tau \rightarrow e\bar{e}e, \mu\bar{\mu}\mu, e\bar{\mu}\mu\dots$	$< 1.5 - 2.7 \times 10^{-8}$	$\text{few} \times 10^{-9}$ (Belle-II, LHCb?)
$\tau \rightarrow \begin{Bmatrix} e \\ \mu \end{Bmatrix} \{\pi, \rho, \phi, \dots\}$	$\lesssim \text{few} \times 10^{-8}$	$\text{few} \times 10^{-9}$ (Belle-II)
$h \rightarrow \tau^\pm \ell^\mp$	$< 1.5, 2.2 \times 10^{-3}$ (ATLAS/CMS)	$< 2.4 \times 10^{-4}$ (ILC)
$h \rightarrow \mu^\pm e^\mp$	$< 6.1 \times 10^{-5}$ (ATLAS/CMS)	$2.1 \times 10^{-5}$ (ILC)
$Z \rightarrow e^\pm \mu^\mp$	$< 7.5 \times 10^{-7}$ (ATLAS)	
$Z \rightarrow l^\pm \tau^\mp$	$< \times 10^{-7}$ (ATLAS)	

$\mu A \rightarrow eA \equiv \mu$  in  $1s$  state of nucleus  $A$  converts to  $e$

## take observable-motivated basis to $\Lambda_{NP}$ ?

1.  $\mu \rightarrow e\gamma$  measures  $C_{D,R}(m_\mu)$

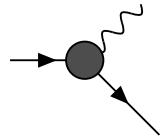
solving RGEs gives  $\vec{C}(m_\mu) = \vec{C}(m_W)\mathbf{G}(m_\mu, m_W)$ ,  $\Rightarrow$  define  $\vec{v}_{\mu \rightarrow e\gamma}(m_\mu, \Lambda)$  such that:

$$\begin{aligned}
 C_{DR}(m_\mu) &= \vec{C}(\Lambda) \cdot \vec{v}_{\mu \rightarrow e\gamma}(m_\mu, \Lambda) \\
 C_{D,X}(m_\mu) &= C_{D,X}(m_W) \left( 1 - 16 \frac{\alpha_e}{4\pi} \ln \frac{m_W}{m_\mu} \right) \\
 &\quad - \frac{\alpha_e}{4\pi e} \ln \frac{m_W}{m_\mu} \left( -8 \frac{m_\tau}{m_\mu} C_{T,XX}^{\tau\tau} + C_{S,XX}^{\mu\mu} + C_{2loop} \right) \\
 &\quad + 16 \frac{\alpha_e^2}{2e(4\pi)^2} \ln^2 \frac{m_W}{m_\mu} \left( \frac{m_\tau}{m_\mu} C_{S,XX}^{\tau\tau} \right) \\
 &\quad - 8\lambda^{a_T} \frac{\alpha_e}{4\pi e} \ln \frac{m_W}{2 \text{ GeV}} \left( -\frac{m_s}{m_\mu} C_{T,XX}^{ss} + 2 \frac{m_c}{m_\mu} C_{T,XX}^{cc} - \frac{m_b}{m_\mu} C_{T,XX}^{bb} \right) f_{TD} \\
 &\quad + 16 \frac{\alpha_e^2}{3e(4\pi)^2} \ln^2 \frac{m_W}{2 \text{ GeV}} \left( \sum_{u,c} 4 \frac{m_q}{m_\mu} C_{S,XX}^{qq} + \sum_{d,s,b} \frac{m_q}{m_\mu} C_{S,XX}^{qq} \right)
 \end{aligned}$$

all coeffs on right side  $C(m_W)$  (basis vectors rotate and change length with scale)  
 $\lambda = \alpha_s(m_W)/\alpha_s(2\text{GeV}) \simeq 0.44$ ,  $f_{TS} \simeq 1.45$ ,  $a_S = 12/23$ ,  $a_T = -4/23$ .

## ...but there are many defns of $\Lambda_{\text{NP}}$

the dipole operator allows on-shell fermion to emit on-shell  $\gamma$   
 induces  $\mu \rightarrow e\gamma$ , edms (and  $g - 2$ )



$$\delta\mathcal{L}_{\mu \rightarrow e\gamma} = \frac{\textcolor{red}{M}}{\Lambda_{\text{NP}}^2} (C_{D,L}\bar{e}_R \sigma^{\alpha\beta} \mu_L + C_{D,R}\bar{e}_L \sigma^{\alpha\beta} \mu_R) F_{\alpha\beta}$$

normalisation: dipole is dim5 at low  $E$ , dim6 in SMEFT... what mass upstairs?  $\textcolor{red}{M}: \{m_\mu, m_e\} \rightarrow v$ ?  
 If NP not chirality-flip, need SM Yukawa (other than external leg might cost loop(s))

Put  $\textcolor{red}{M} = \mu_\mu$  for  $\mu \rightarrow e\gamma$ ,  $\textcolor{red}{M} = m_e$  for  $d_e$ :

$$BR(\mu \rightarrow e\gamma) = 384\pi^2 \left( \frac{v^4 |C_{DR}|^2}{\Lambda^4} + \frac{v^4 |C_{DL}|^2}{\Lambda^4} \right) < 5.7 \times 10^{-13} \Rightarrow \Lambda_{\text{NP}}^{e\mu} \gtrsim \sqrt{C_{DX}} 10^4 v$$

$$d_e = 2\sqrt{2}G_F m_e \left( \frac{v^2}{\Lambda^2} - \frac{v^2}{\Lambda^2} \right) \leq 4.2 \times 10^{-30} \text{ ecm} \Rightarrow \Lambda_{NP}^{ee} \gtrsim \sqrt{\mathcal{I}\{C_{DX}\}} 3 \times 10^4 v$$

Whereas if you put  $\textcolor{red}{M} = v$ ,  $\Lambda_{\text{NP}} \Big|_{d_e} \gg \Lambda_{\text{NP}} \Big|_{\mu \rightarrow e\gamma}$ .

# Counting constraints in space of $\sim 100$ operators

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Count constraints: (write  $\delta\mathcal{L} = C_{Lorentz,XY}^{flavour}/v^n \mathcal{O}_{Lorentz,XY}^{flav}$ ,  $X, Y \in \{L, R\}$ )

$\mu \rightarrow e\gamma$  :  $BR(\mu \rightarrow e\gamma) = 384\pi^2(|C_{D,L}|^2 + |C_{D,R}|^2)$   $\Rightarrow 2$  constraints

$\mu \rightarrow e\bar{e}e$  : ( $e$  relativistic  $\approx$  chiral, neglect interference between  $e_L, e_R$ )

$$BR = \frac{|C_{S,LL}|^2}{8} + 2|C_{V,RR} + 4eC_{D,L}|^2 + (64 \ln \frac{m_\mu}{m_e} - 136)|eC_{D,L}|^2 + |C_{V,RL} + 4eC_{D,L}|^2 + \{L \leftrightarrow R\} \Rightarrow 6$$
 more constraints

$\mu A \rightarrow eA$  : ( $S_A^N, V_A^N$  = integral over nucleus A of N distribution  $\times$  lepton wavefns, different for diff. A)

$$BR_{SI} \sim Z^2 |V_A^p \tilde{C}_{V,L}^p + S_A^p \tilde{C}_{S,R}^p + V_A^n \tilde{C}_{V,L}^n + S_A^b \tilde{C}_{S,R}^n + D_A C_{D,R}|^2 + |L \leftrightarrow R|^2$$

$$BR_{SD} \sim |\tilde{C}_A^N + 2\tilde{C}_T^N|^2$$

SI bds on Au, Ti, (+ SD on ?Ti, Au?)

$\Rightarrow 4 + 2$  more constraints

future: improved theory, 3SI+2SD targets

$\Rightarrow 6 + 4$  constraints

is 12-20 constraints on  $\sim 100$  operators a problem?

## many operators+few constraints=using inconvenient basis

Have 6 (+6) constraints on  $e_L$  ( $e_R$ ) operator coefficients. Focus on  $e_L$ .

**Want to change basis to scale -dependent basis of constrained 6-d subspace.**

1.  $\mu \rightarrow e\gamma$  measures  $C_{D,R}(m_\mu)$

Have RGEs for coefficients (arranged in row vector)

$$\mu \frac{\partial}{\partial \mu} \vec{C}(\mu) = \vec{C}(\mu) \boldsymbol{\Gamma}(\mu, g_s(\mu), \dots) \Rightarrow \vec{C}(m_\mu) = \vec{C}(m_W) \mathbf{G}(m_\mu, m_W)$$

solved as scale-ordered exponential (resummed QCD,  $\alpha \log$ , some  $\alpha^2 \log^2, \alpha^2 \log$ )

$\Rightarrow$  define scale-dep  $\vec{v}_{\mu \rightarrow e\gamma}(\Lambda)$ , column of  $\mathbf{G}$  such that:  $C_{DR}(m_\mu) = \vec{C}(\Lambda) \cdot \vec{v}_{\mu \rightarrow e\gamma}(\Lambda)$   
 **$\vec{v}_{\mu \rightarrow e\gamma}(\Lambda)$  is scale-dep basis vector for constrainable subspace**

2-6. repeat for other independent constraints. So obtain scale-dep basis vectors for the subspace, defined from the observables.

The “flat directions” (experimentally inaccessible) are orthogonal, and therefore irrelevant.

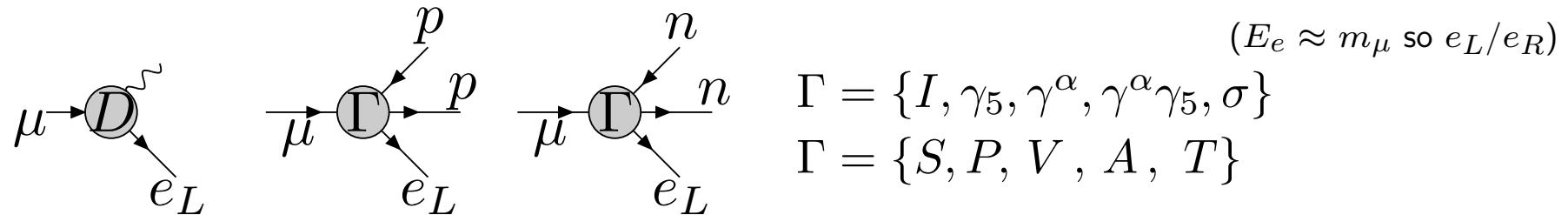
Basis should span the finite-eigenvalue subspace of the correlation matrix.

**what to do with this basis?**

## $\mu A \rightarrow eA$ : most sensitive process, expt + th



- $\mu^-$  captured by *Al* nucleus, tumbles down to  $1s$ . ( $r \sim Z\alpha/m_\mu \gtrsim r_{Al}$ )
- in SM: muon “capture”  $\mu + p \rightarrow \nu + n$ , or decay-in-orbit
- LFV:  $\mu$  interacts with  $\vec{E}$ , nucleons (via  $\tilde{C}_{\Gamma,X}^N(\bar{e}\Gamma P_X N)(\bar{N}\Gamma N)$ ), converts to  $e$



≈ WIMP scattering on nuclei

- 1) “Spin Independent” rate  $\propto A^2$  (amplitude  $\propto \sum_N \propto A$ )

KitanoKoikeOkada

$$BR_{SI} \sim Z^2 |\sum \dots \tilde{C}_{SI}|^2 , \quad \tilde{C}_{SI} \in \{\tilde{C}_V^p, \tilde{C}_S^p, \tilde{C}_V^n, \tilde{C}_S^n, C_D\}$$

- 2) “Spin Dependent” rate  $\sim \Gamma_{SI}/A^2$  (sum over  $N \propto$  spin of only unpaired nucleon)

$$BR_{SD} \sim \dots |\tilde{C}_A^N + 2\tilde{C}_T^N|^2$$

CiriglianoDavidsonKuno  
HoferichterEtal

# Can't we do without RGEs, etc?

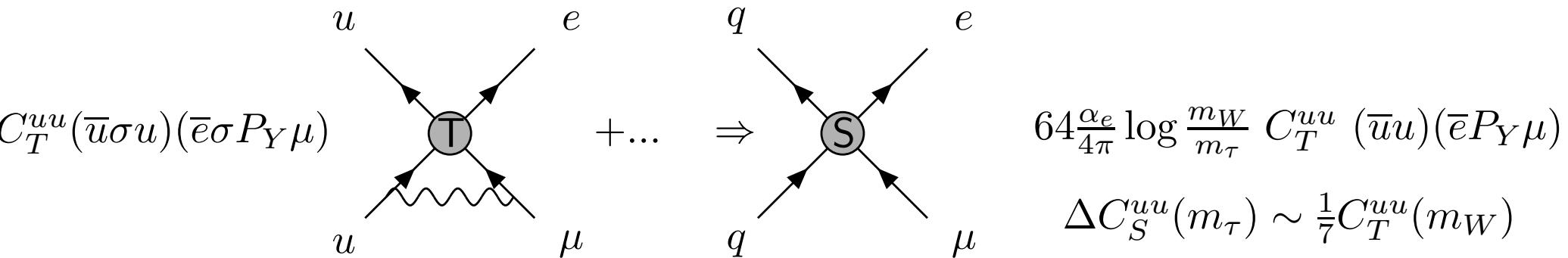
in discovery mode for LFV+electroweak loops are small...include later?

counterex:  $\mu A \rightarrow eA$  in model giving tensor  $2\sqrt{2}G_F C_T^{uu}(\bar{e}\sigma P_R\mu)(\bar{u}\sigma u)$  at weak scale

**1: forget loops** quark tensor matches to nucleon spin  $\bar{N}\gamma\gamma_5 N : (N \in \{n, p\})$

$$\Rightarrow BR(\mu A \rightarrow eA) \approx BR_{SD} \approx \frac{1}{2}|C_T^{uu}|^2 \quad (\text{CiriglianoDKuno Hoferichter et al})$$

**2: include QED loops**  $m_W \rightarrow 2$  GeV:



Then, scalar ops have enhanced nuclear matrix elements, and are SpinIndep:

$$BR(\mu A \rightarrow eA) \approx BR_{SI} \sim Z^2|2C_T^{uu}|^2 \sim 10^3 BR_{SD}$$

**loops can change Lorentz structure/external legs  $\Rightarrow$  different operator whose coefficient better constrained. Important for  $\mu \rightarrow e$ . (?not  $\tau \rightarrow l$ ?)**

# Operator basis $m_\tau \rightarrow m_W : \sim 90$ operators

Add QCD×QED-invar operators, representing all 3,4 point interactions of  $\mu$  with  $e$  and *flavour-diagonal* combination of  $\gamma, g, u, d, s, c, b$ .  $Y \in L, R$ .

$$m_\mu (\bar{e} \sigma^{\alpha\beta} P_Y \mu) F_{\alpha\beta} \quad \text{dim 5}$$

$$\begin{aligned} &(\bar{e} \gamma^\alpha P_Y \mu) (\bar{e} \gamma_\alpha P_Y e) && (\bar{e} \gamma^\alpha P_Y \mu) (\bar{e} \gamma_\alpha P_X e) \\ &(\bar{e} P_Y \mu) (\bar{e} P_Y e) && \text{dim 6} \end{aligned}$$

$$\begin{aligned} &(\bar{e} \gamma^\alpha P_Y \mu) (\bar{\mu} \gamma_\alpha P_X \mu) && (\bar{e} \gamma^\alpha P_Y \mu) (\bar{\mu} \gamma_\alpha P_X \mu) \\ &(\bar{e} P_Y \mu) (\bar{\mu} P_Y \mu) \\ &(\bar{e} \gamma^\alpha P_Y \mu) (\bar{f} \gamma_\alpha P_Y f) && (\bar{e} \gamma^\alpha P_Y \mu) (\bar{f} \gamma_\alpha P_X f) \\ &(\bar{e} P_Y \mu) (\bar{f} P_Y f) && (\bar{e} P_Y \mu) (\bar{f} P_X f) \quad f \in \{u, d, s, c, b, \tau\} \\ &(\bar{e} \sigma P_Y \mu) (\bar{f} \sigma P_Y f) \end{aligned}$$

$$\frac{1}{m_t} (\bar{e} P_Y \mu) G_{\alpha\beta} G^{\alpha\beta} \quad \frac{1}{m_t} (\bar{e} P_Y \mu) G_{\alpha\beta} \tilde{G}^{\alpha\beta} \quad \text{dim 7}$$

$$\frac{1}{m_t} (\bar{e} P_Y \mu) F_{\alpha\beta} F^{\alpha\beta} \quad \frac{1}{m_t} (\bar{e} P_Y \mu) F_{\alpha\beta} \tilde{F}^{\alpha\beta} \quad \text{...zzz...but } \sim 90 \text{ coeffs!}$$

$(P_X, P_Y = (1 \pm \gamma_5)/2)$ , all operators with coeff  $-2\sqrt{2}G_F C$ .

# operators at exptal scale

Kuno Okada

There are dipoles of 2 chiralities

$$D \quad \bar{e}\sigma^{\alpha\beta}P_L\mu F_{\alpha\beta} \quad \bar{e}\sigma^{\alpha\beta}P_R\mu F_{\alpha\beta}$$

which also contribute in  $\mu \rightarrow e\gamma$ ,  $\mu \rightarrow e\bar{e}e$ .

Six 4-fermions for  $\mu \rightarrow e\bar{e}e$ ,  $Y, X \in \{L, R\}$ ,  $Y \neq X$

$V$	$(\bar{e}\gamma^\alpha P_Y \mu)(\bar{e}\gamma_\alpha P_Y e)$	$(\bar{e}\gamma^\alpha P_Y \mu)(\bar{e}\gamma_\alpha P_X e)$
$S$	$(\bar{e}P_Y \mu)(\bar{e}P_Y e)$	

For  $\mu A \rightarrow eA$ , interactions with nucleons  $N \in \{n, p\}$  parametrised by :

$S, V$	$\bar{e}P_X \mu \bar{N} N$	$\bar{e}\gamma^\alpha P_X \mu \bar{N} \gamma_\alpha N$	$X \in \{L, R\}$
$A, T$	$\bar{e}\gamma^\alpha P_X \mu \bar{N} \gamma_\alpha \gamma_5 N$	$\bar{e}\sigma^{\alpha\beta} P_X \mu \bar{N} \sigma_{\alpha\beta} N$	
$P, Der$	$\bar{e}P_X \mu \bar{N} \gamma_5 N$	$\bar{e}\gamma^\alpha P_X \mu (\bar{N} i \overleftrightarrow{\partial}_\alpha \gamma_5 N)$	

Matching in  $\chi$ PT gives Derivative. But absorb in matching chiral basis for the lepton current (relativistic  $e$ ), into  $G_O^{N,q}$  = quark matrix elements in nucleons.  
but not for the non-rel. nucleons.