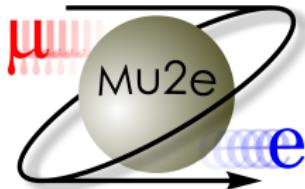


# Z-A dependence of muon to electron conversion

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Caltech

# Goal

## Goal

Improve the calculation of the muon conversion rate by adding muonic x-ray experimental dataset and taking into account nuclear deformation

- ① Previous calculation primarily use electron scattering data  
(Kitano, de Vries)
- ② The conversion rate currently ignores nuclear deformation
- ③ Differentiate between proton and neutron distribution in the nucleus

R. Kitano et al., Phys. Rev. D 66 (2002)

H. De Vries et al., Atom. Data and Nucl. Data Tabl. 36 (1987)

# Equations

Most general Lepton Flavor Violation interaction Lagrangian (Kitano et al.):

$$\begin{aligned}\mathcal{L} = & -\frac{4G_F}{\sqrt{2}} (m_\mu A_R \bar{\mu} \sigma^{\mu\nu} P_L e F_{\mu\nu} + m_\mu A_L \bar{\mu} \sigma^{\mu\nu} P_R e F_{\mu\nu} + h.c.) \\ & - \frac{G_F}{\sqrt{2}} \sum_{q=u,d,s} \left[ \left( g_{LS(q)} \bar{e} P_R \mu + g_{RS(q)} \bar{e} P_L \mu \right) \bar{q} q \right. \\ & + \left( g_{LP(q)} \bar{e} P_R \mu + g_{RP(q)} \bar{e} P_L \mu \right) \bar{q} \gamma_5 q \\ & + \left( g_{LV(q)} \bar{e} \gamma^\mu P_L \mu + g_{RV(q)} \bar{e} \gamma^\mu P_R \mu \right) \bar{q} \gamma_\mu q \\ & + \left( g_{LA(q)} \bar{e} \gamma^\mu P_L \mu + g_{RA(q)} \bar{e} \gamma^\mu P_R \mu \right) \bar{q} \gamma_\mu \gamma_5 q \\ & \left. + \frac{1}{2} \left( g_{LT(q)} \bar{e} \sigma^{\mu\nu} P_R \mu + g_{RT(q)} \bar{e} \sigma^{\mu\nu} P_L \mu \right) \bar{q} \sigma_{\mu\nu} q + h.c. \right]\end{aligned}$$

# Equations

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# Equations

Radial solution:

$$\psi_\kappa^\mu = \begin{pmatrix} g(r) \chi_\kappa^\mu(\theta, \phi) \\ i f(r) \chi_\kappa^\mu(\theta, \phi) \end{pmatrix}$$

Quantum numbers  $\mu$  and  $\kappa$ :

$$(\sigma \cdot I + 1) \chi_\kappa^\mu = -\kappa \chi_\kappa^\mu, \quad j_z \chi_\kappa^\mu = \mu \chi_\kappa^\mu$$

Dirac equation of the radial component (where  $u_1(r) = r g(r)$  and  $u_2(r) = r f(r)$ ):

$$\frac{d}{dr} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} -\kappa/r & W - V + m_i \\ -(W - V - m_i) & \kappa/r \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

# Equations

conversion rate:

$$\omega_{conv} = 2G_f^2 \left| A_R^* D + \tilde{g}_{LS}^{(p)} S^{(p)} + \tilde{g}_{LS}^{(n)} S^{(n)} + \tilde{g}_{LV}^{(p)} V^{(p)} + \tilde{g}_{LV}^{(n)} S^{(n)} \right|^2 \\ + 2G_f^2 \left| A_L^* D + \tilde{g}_{RS}^{(p)} S^{(p)} + \tilde{g}_{RS}^{(n)} S^{(n)} + \tilde{g}_{RV}^{(p)} V^{(p)} + \tilde{g}_{RV}^{(n)} S^{(n)} \right|^2$$

Overlap integrals:

$$D = \frac{4}{\sqrt{2}} m_\mu \int_0^\infty (-E(r)) (g_e^- f_\mu^- + f_e^- g_\mu^-) r^2 dr$$

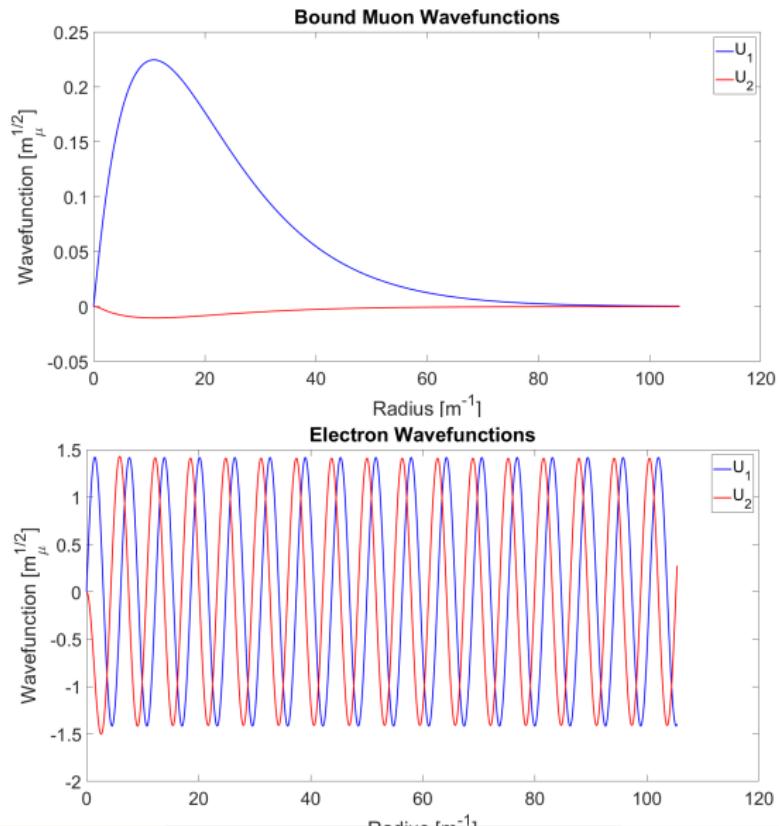
$$S^{(p)} = \frac{1}{2\sqrt{2}} \int_0^\infty Z \rho^{(p)}(r) (g_e^- g_\mu^- - f_e^- f_\mu^-) r^2 dr$$

$$S^{(n)} = \frac{1}{2\sqrt{2}} \int_0^\infty (A - Z) \rho^{(n)}(r) (g_e^- g_\mu^- - f_e^- f_\mu^-) r^2 dr$$

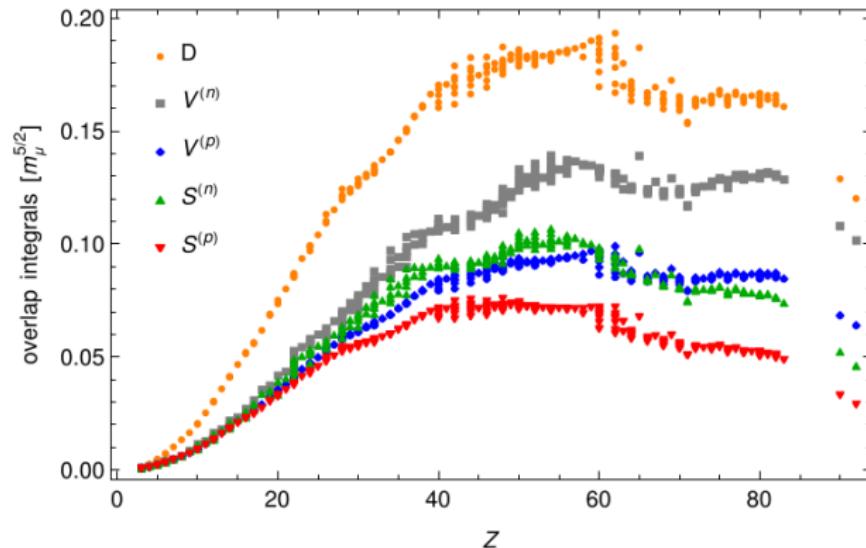
$$V^{(p)} = \frac{1}{2\sqrt{2}} \int_0^\infty Z \rho^{(p)}(r) (g_e^- g_\mu^- + f_e^- f_\mu^-) r^2 dr$$

$$V^{(n)} = \frac{1}{2\sqrt{2}} \int_0^\infty (A - Z) \rho^{(n)}(r) (g_e^- g_\mu^- + f_e^- f_\mu^-) r^2 dr$$

# Wavefunctions (for Al nucleus)



# Previous work



J. Heeck et al., Nucl. Phys.  
B 980 (2022)

- Neutron distribution is equal to the proton distribution scaled by  $N/Z$
- The nucleon distribution is computed from the rms radius only for a number of isotopes

# Including nuclear deformation

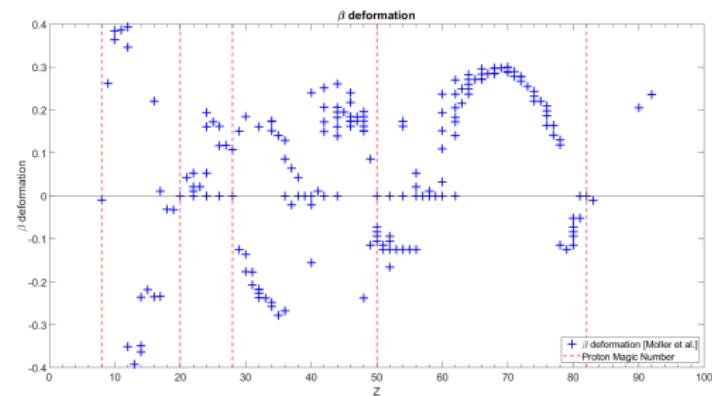
- Start with 2pF (2-parameter Fermi distribution)  $\rho(r) = \frac{\rho_0}{1+\exp\left[\frac{r-c}{t}\right]}$  (or Fourier-Bessel distribution from literature)
- include quadrupole deformation parameter  $\beta$  into the deformed 2pF (non-spherically symmetric)  $\rho(r, \theta) = \frac{\rho_0}{1+\exp\left[\frac{r-c(1+\beta Y_{20}(\theta))}{t}\right]}$
- adjust the t parameter in the (spherically symmetric) 2pF to have the same **rms radius** as the deformed 2pF:  $\langle r^2 \rangle = \frac{4\pi}{Z} \int_0^\infty \rho(r) r^4 dr$
- solve (spherically symmetric) Dirac equation with the (spherically symmetric) equivalent 2pF

# Using Barrett Moment

- Start with 2pF (2-parameter Fermi distribution)  $\rho(r) = \frac{\rho_0}{1+\exp\left[\frac{r-c}{t}\right]}$
- include quadrupole deformation parameter  $\beta$  into the deformed 2pF (non-spherically symmetric)  $\rho(r, \theta) = \frac{\rho_0}{1+\exp\left[\frac{r-c(1+\beta Y_{20}(\theta))}{t}\right]}$
- adjust the t parameter in the (spherically symmetric) 2pF to have the same **Barrett Moment** as the deformed 2pF:  
$$\langle e^{-\alpha r} r^k \rangle = \frac{4\pi}{Z} \int_0^\infty \rho(r) e^{-\alpha r} r^{k+2} dr$$
- solve (spherically symmetric) Dirac equation with the (spherically symmetric) equivalent 2pF

R.C. Barrett, Rep. Prog. Phys. 37 (1974)

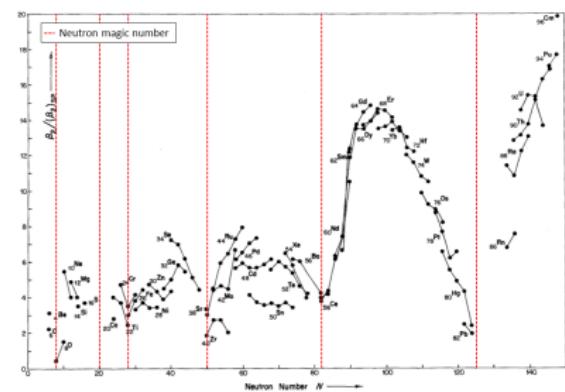
# Adding neutron distribution



P. Moller et al., Atom. Data Nucl. Data Tabl. 109  
(2016)

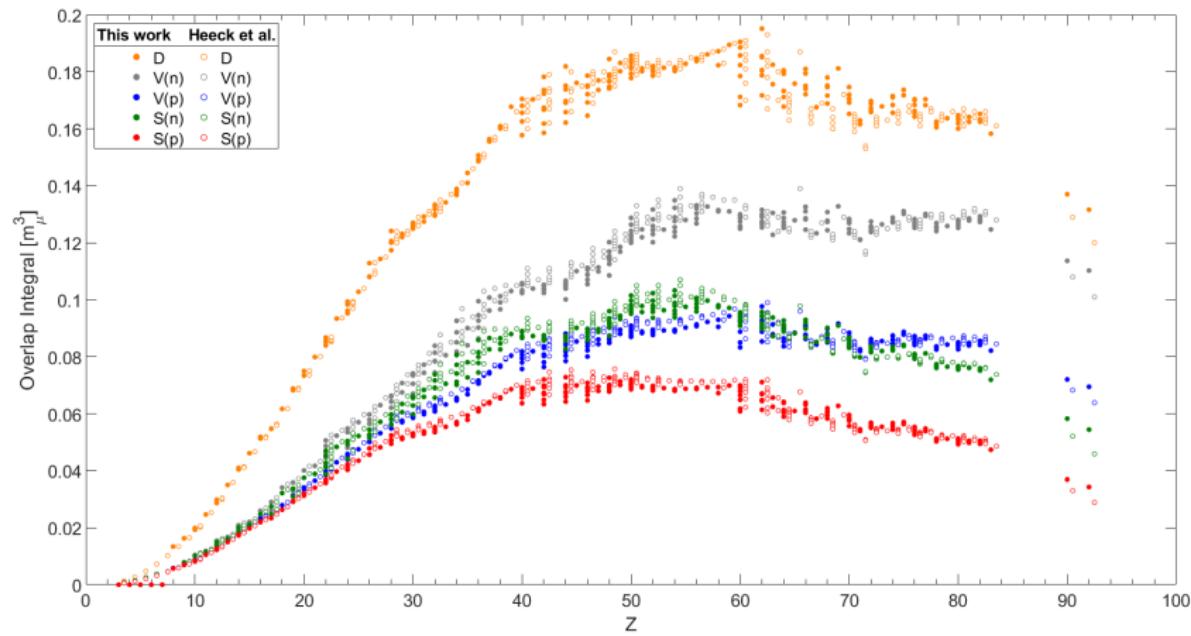
Calculation based on relativistic Hartree-Bogoliubov model for  
even-even nuclei

K. Zhang et al., Atom. Data Nucl. Data Tabl. 144 (2022)

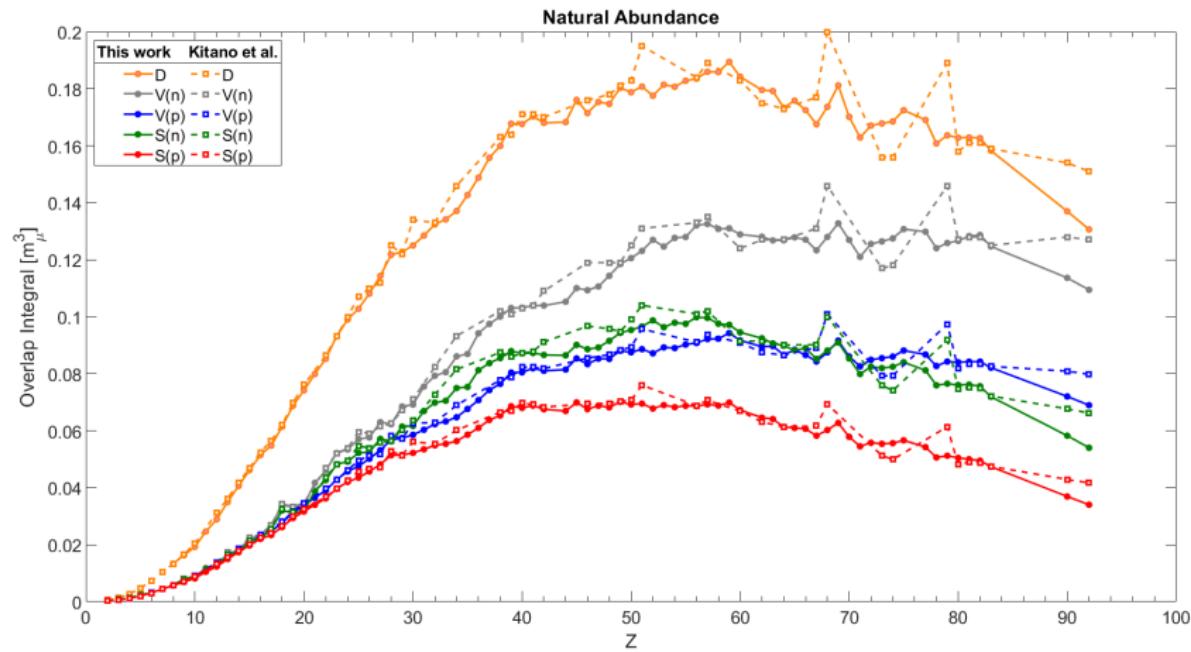


P. Stelson and L. Grodzins, Nucl.  
Data Sheets 1 (1965)

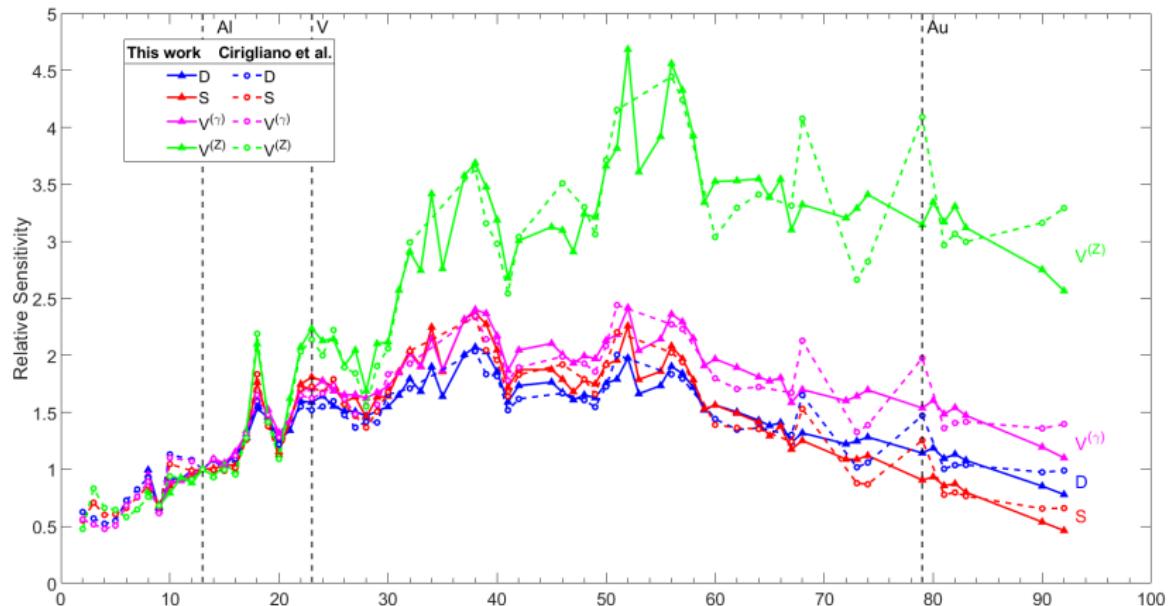
# Results: all isotopes



# Results: natural abundance



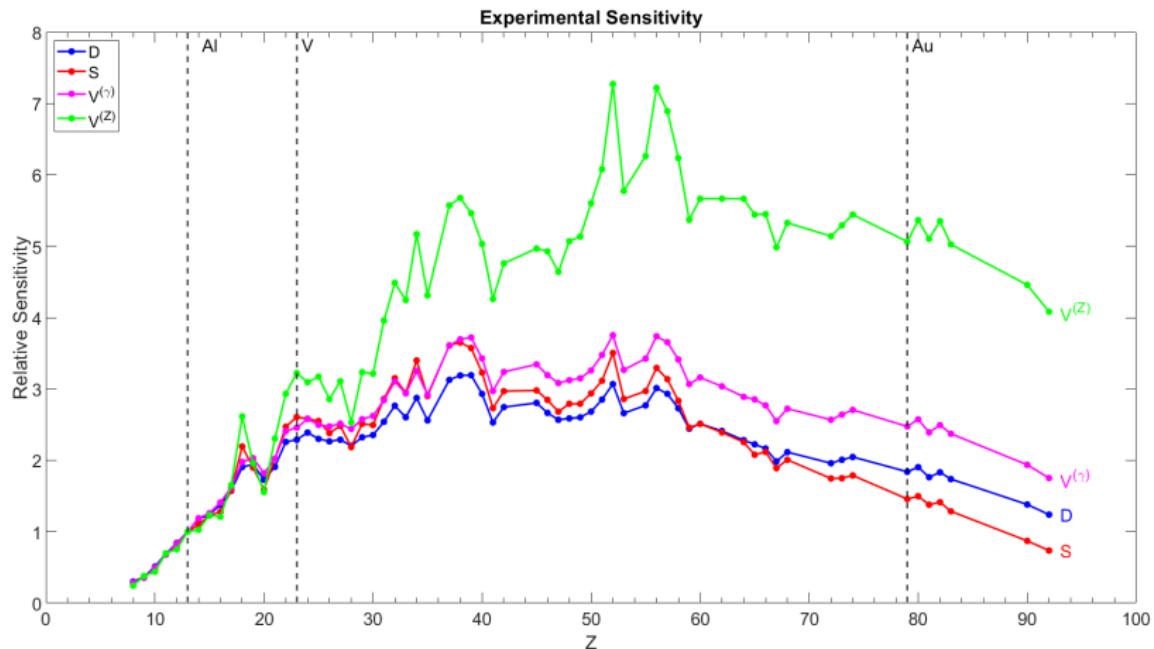
# Experimental sensitivity plot (muon capture rate)



$$R_{\mu e}(A, Z) = \frac{\Gamma(\mu^- + N(A, Z) \rightarrow e^- + N(A, Z))}{\Gamma(\mu^- + N(A, Z) \rightarrow \nu_\mu + N'(A, Z-1))}$$

V. Cirigliano et al., Phys. Rev. D 80 (2009)

# Experimental sensitivity plot (muon lifetime)



$$R_{\mu e}(A, Z) = \Gamma \left( \mu^- + N(A, Z) \rightarrow e^- + N(A, Z) \right) \times \tau_\mu(Z, A)$$

# Conclusion and future work

Improved calculation of muon conversion rate:

- include experimental muonic x-ray dataset
- include quadrupole deformation
- differentiate proton and neutron distribution
- New normalization for the experimental sensitivity (see next talk by David Hitlin for more details)
- paper coming soon

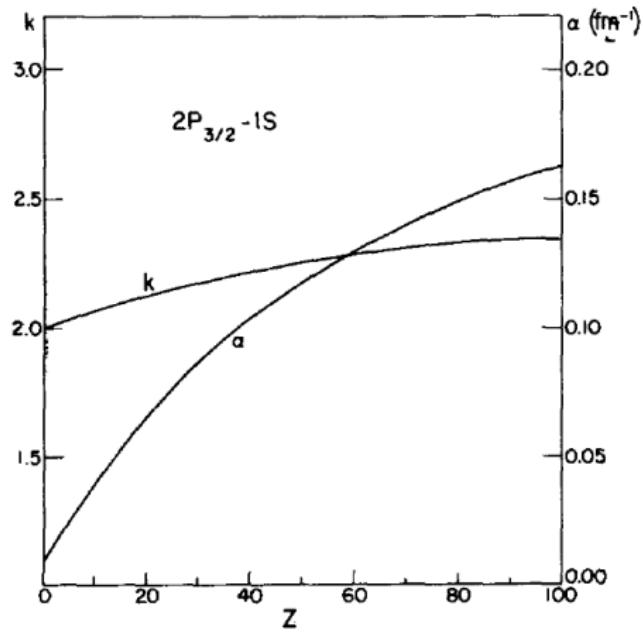
# Equations

Dirac equation in the central force system:

$$W\psi = \left[ -i\gamma_5\sigma_r \left( \frac{\partial}{\partial r} + \frac{1}{r} - \frac{\beta}{r} K \right) + V(r) + m_i\beta \right] \psi$$

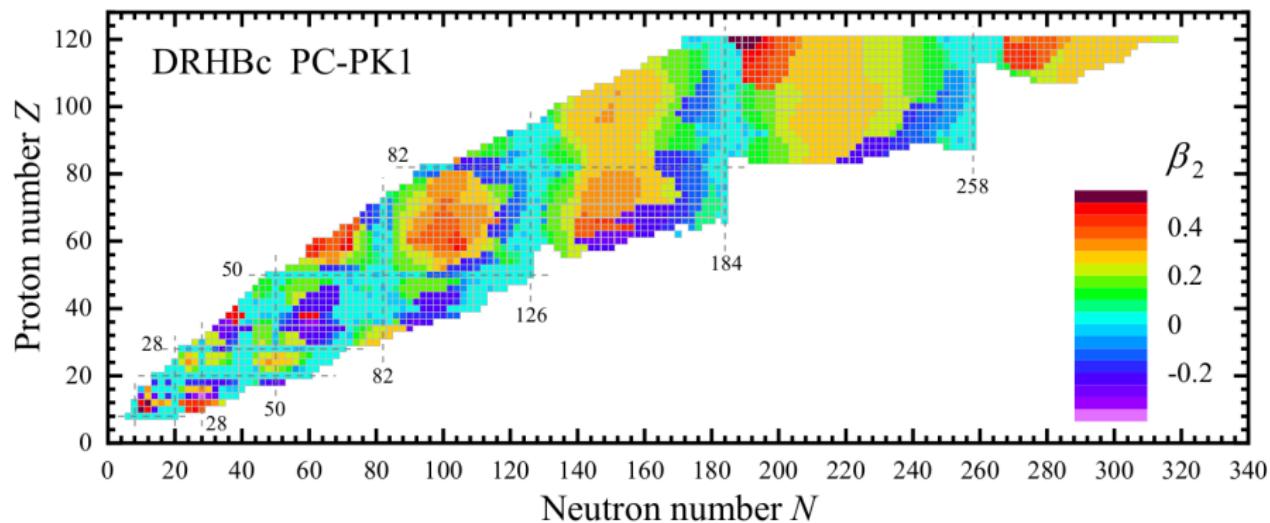
$$\gamma_5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \sigma_r = \begin{pmatrix} \sigma \cdot r & 0 \\ 0 & \sigma \cdot r \end{pmatrix},$$
$$K = \begin{pmatrix} \sigma \cdot l + 1 & 0 \\ 0 & -(\sigma \cdot l + 1) \end{pmatrix}$$

# Barrett Moment parameter



R.C. Barrett, Rep. Prog. Phys. 37 (1974)

# Proton-neutron quadrupole deformation



K. Zhang et al., Atom. Data Nucl. Data Tabl. 144 (2022)