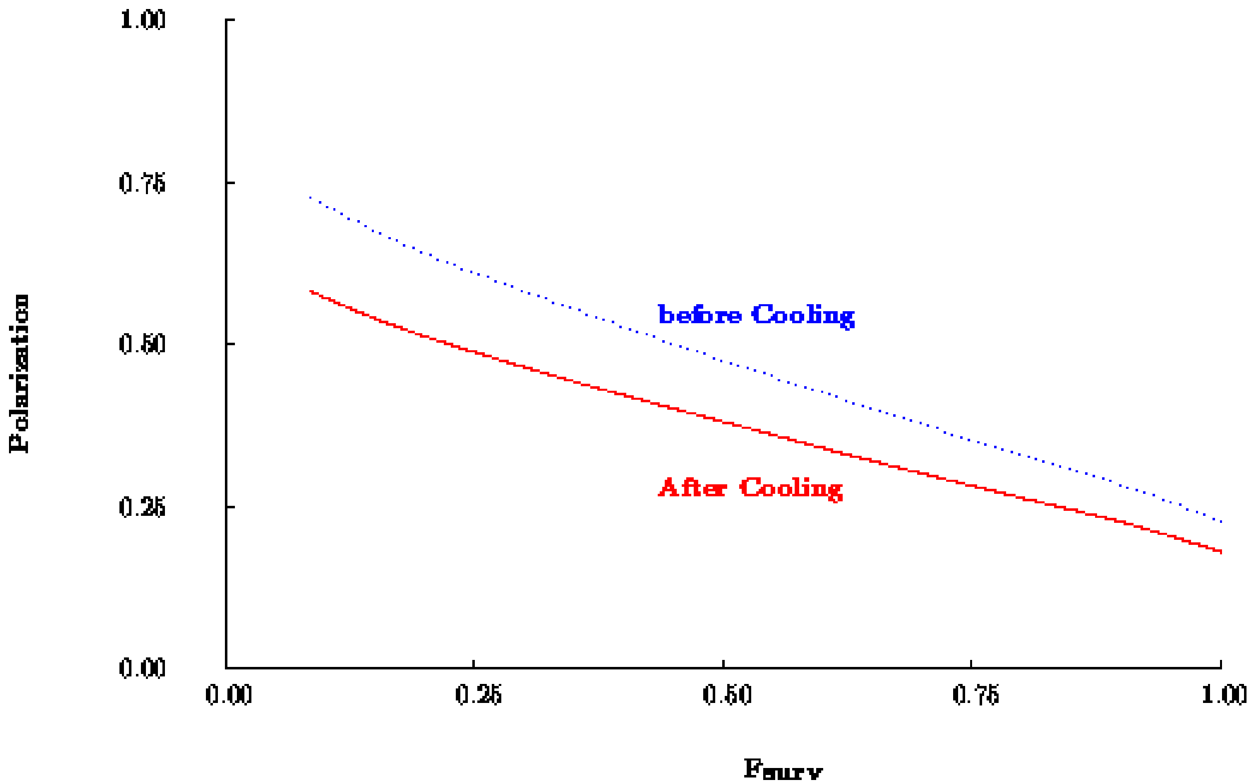
Muon Collider Physics

- Polarization of muons will play a crucial role in many physics areas.
- Both charges polarizable.



Calibrating the energy of the collider to 1E-6

Bargmann-Michel-Telegdi Equation

$$\frac{d\vec{S}}{dt} = \vec{\Omega} \times \vec{S}$$

$$\vec{\Omega} = -\frac{e}{m\gamma}((1+a\gamma)\vec{B}_{\perp} + (1+a)\vec{B}_{\parallel} - (a\gamma + \frac{\gamma}{1+\gamma})\vec{\beta} \times \frac{\vec{E}}{c})$$

$$\vec{\Omega} = \vec{\Omega}_{cyc}(1+a\gamma)$$

$$a = (g-2)/2$$

$B_{\perp}, B_{\parallel}$ are the components of magnetic field perpendicular and parallel particle direction

This equation controls the evolution of the spin vector $\vec{S}$. Polarization is the average of the spin vectors over the muon ensemble. Per revolution spin rotates by $a\gamma 2\pi$ radians more than momentum

Method described in

R.Raja and A. Tollestrup, Phys. Rev. D58(1998)013005

Decay particle energy distribution

In the muon rest frame, E is the energy of the electron. Its fractional energy

expressed in terms of the maximum energy ($m_\mu/2$) is x. N is the number of muon decays. θ is the angle of the electron in the muon center of mass w.r.t muon direction. <E> is the average electron energy and <PL> is the average longitudinal electron momentum in the muon rest frame.

P is the z component of the muon polarization along the muon direction. $\hat{P}$ is charge*P of the muon.

$$x = 2E/m_\mu$$

$$\frac{d^2N}{dx\,d\cos\theta} = N(x^2(3-2x) - \hat{P}x^2(1-2x)\cos\theta)$$

$$<E> = \frac{m_\mu}{2N}\iint x \frac{d^2N}{dx\,d\cos\theta} dx\,d\cos\theta = \frac{7}{10}\frac{m_\mu}{2}$$

$$<P_L> = \frac{m_\mu}{2N}\iint x\cos\theta \frac{d^2N}{dx\,d\cos\theta} dx\,d\cos\theta = \frac{\hat{P}}{10}\frac{m_\mu}{2}$$

Muon neutrinos have identical distribution to electrons. Electron anti-neutrinos have the following distribution.

$$\frac{d^2N}{dx\,d\cos\theta} = 6(x^2(1-x) - \hat{P}\,x^2(1-x)\cos\theta)$$

$$<E> = \frac{6}{20}m_\mu$$

$$<P_L> = -\frac{\hat{P}}{10}m_\mu$$

Electron energy distributions

$$< E_{lab} > = \frac{7}{20} E_{\mu}(1 + \frac{\beta}{7}\hat{P})$$

$$E(t) = Ne^{-\alpha t}(\frac{7}{20} E_{\mu}(1 + \frac{\beta}{7}(\hat{P}\cos\omega t + \phi)))$$

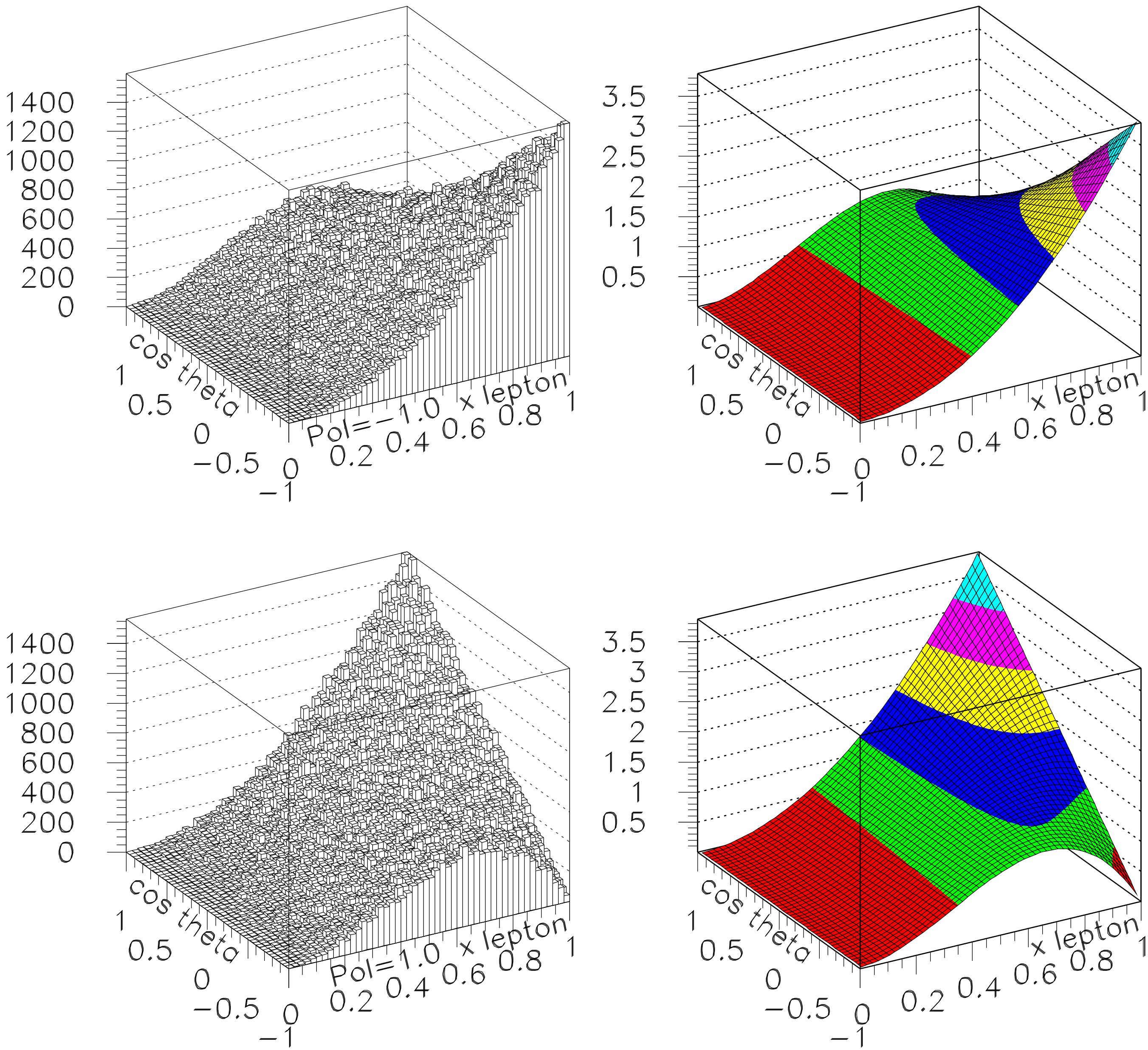
$$\omega = \gamma \frac{g-2}{2} 2\pi$$

$$\alpha = \frac{t_{circ}}{\gamma t_{life}} = \frac{2\pi m_{\mu}}{0.3Bct_{life}}$$

$$f(t) = Ae^{-Bt}(C\cos(D + Et) + F)$$

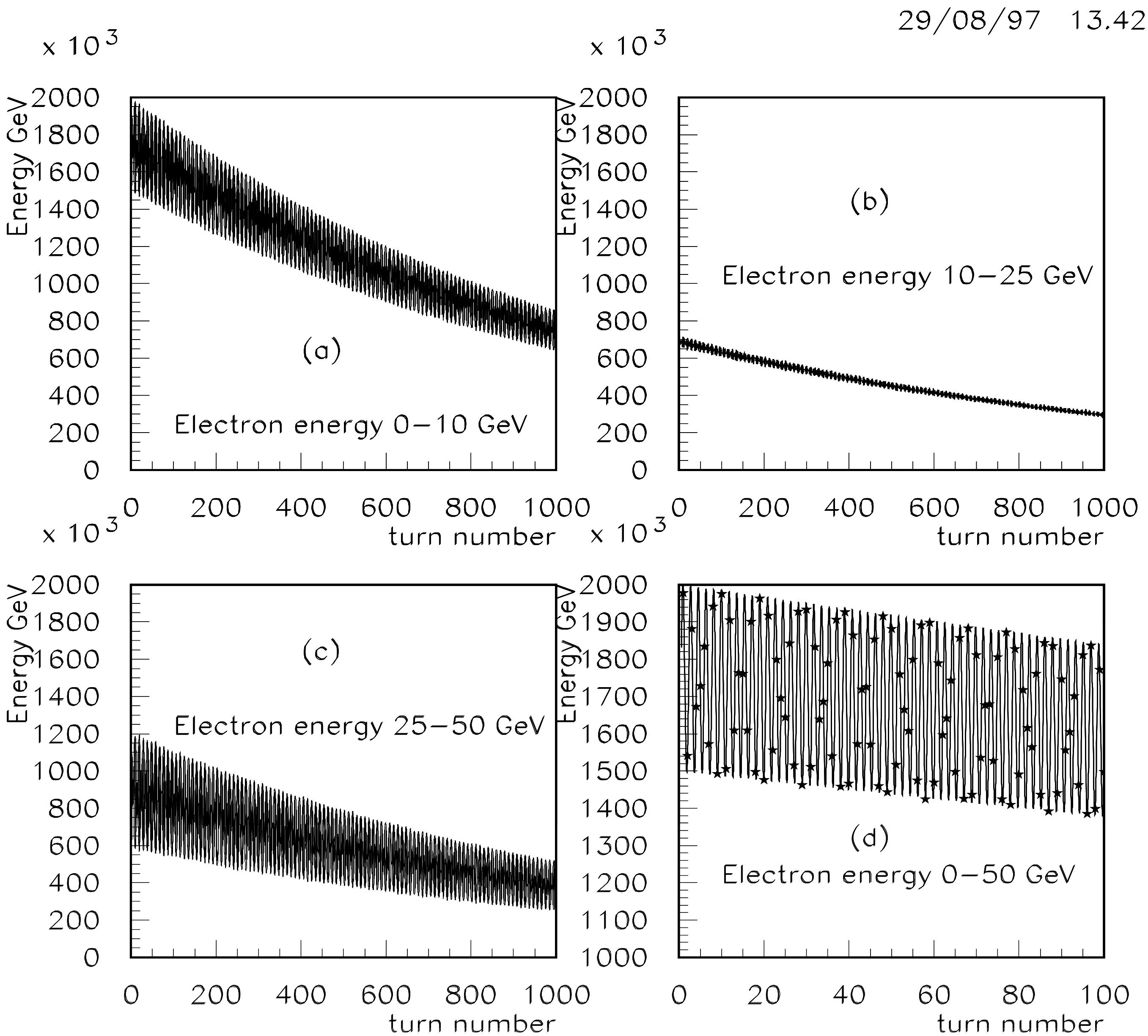
- $<E_{lab}>$ is the average electron energy in lab. E(t) is the total electron energy during turn t. Determine ω to get γ. γ information also present in α.
- f(t) is the fitting function. MINUIT used to fit and extract information.

19/08/97 14.24

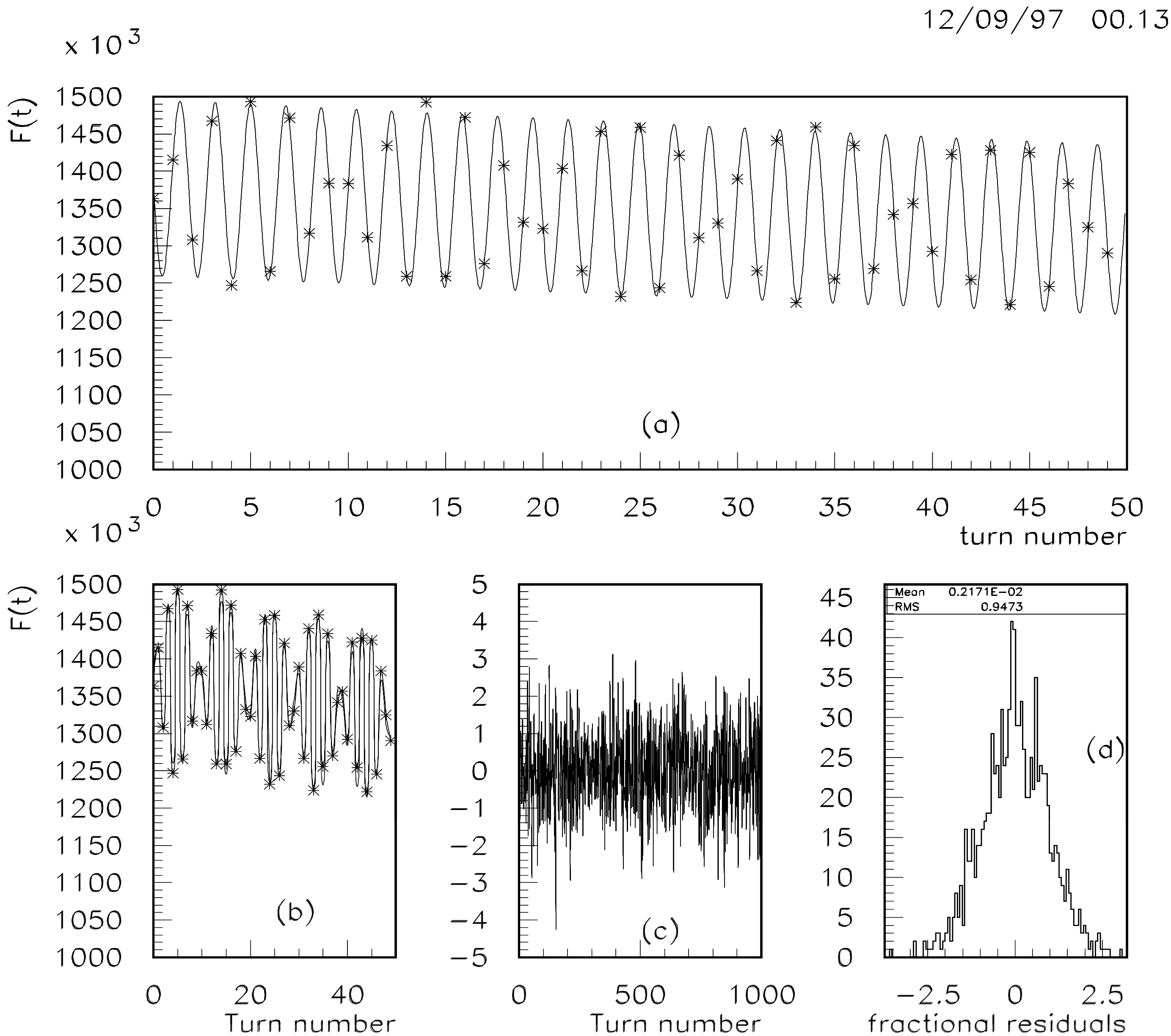


1400
1200
1000
800
600
400
200
0
cos theta
1
0.5
0
−0.5
−1
Pol=−1.0
x lepton
0
0.2
0.4
0.6
0.8
1
3.5
3
2.5
2
1.5
1
0.5
Pol=1.0

Electron lab energy spectrum Pol=1.0, 100K decays



Fit to 50 GeV μ⁻, P=0.26 δp/p=0.03E-2



$\delta\gamma/\gamma$ vs measurement error and Polarization $\delta p/p$=0.03E-2

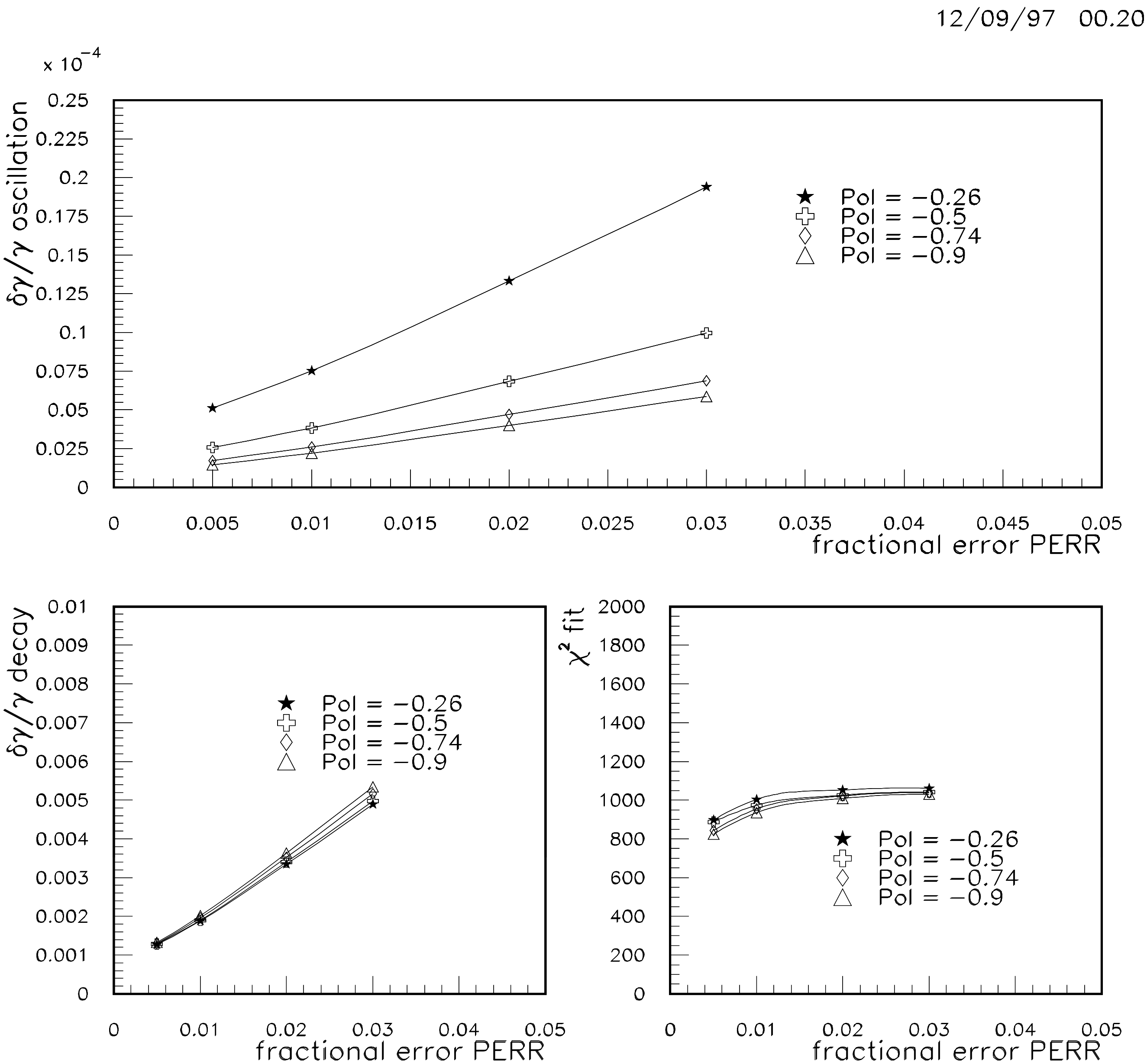
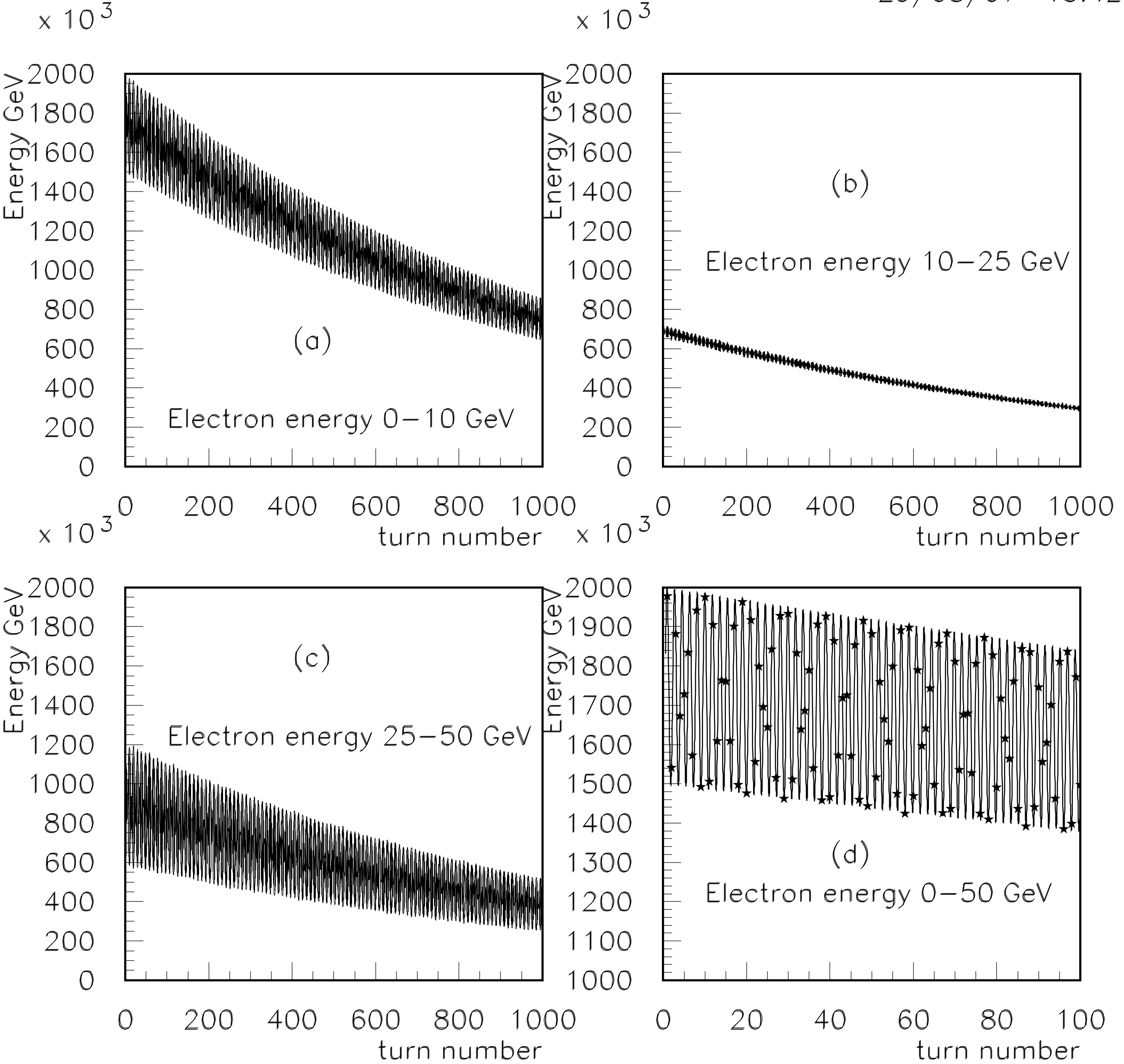


Table of fit parameters

$\hat{P}$	PERR	Number of electrons sampled	$\delta\gamma/\gamma oscillations$	$\delta\gamma/\gamma decay$	χ^2 for NDF=1000
-0.90	0.50E-02	41261	0.14568E-05	0.13227E-02	824.
-0.90	0.10E-01	10315	0.22147E-05	0.20124E-02	936.
-0.90	0.20E-01	2579	0.39999E-05	0.36398E-02	1009.
-0.90	0.30E-01	1146	0.58659E-05	0.53457E-02	1030.
-0.74	0.50E-02	41261	0.17418E-05	0.13019E-02	843.
-0.74	0.10E-01	10315	0.26183E-05	0.19591E-02	954.
-0.74	0.20E-01	2579	0.46981E-05	0.35229E-02	1021.
-0.74	0.30E-01	1146	0.68765E-05	0.51672E-02	1039.
-0.50	0.50E-02	41261	0.25903E-05	0.12813E-02	888.
-0.50	0.10E-01	10315	0.38407E-05	0.19029E-02	973.
-0.50	0.20E-01	2579	0.68338E-05	0.33972E-02	1026.
-0.50	0.30E-01	1146	0.99744E-05	0.49749E-02	1041.
-0.26	0.50E-02	41261	0.51242E-05	0.12688E-02	898.
-0.26	0.10E-01	10315	0.75317E-05	0.18791E-02	1004.
-0.26	0.20E-01	2579	0.13324E-04	0.33447E-02	1053.
-0.26	0.30E-01	1146	0.19380E-04	0.48950E-02	1061.

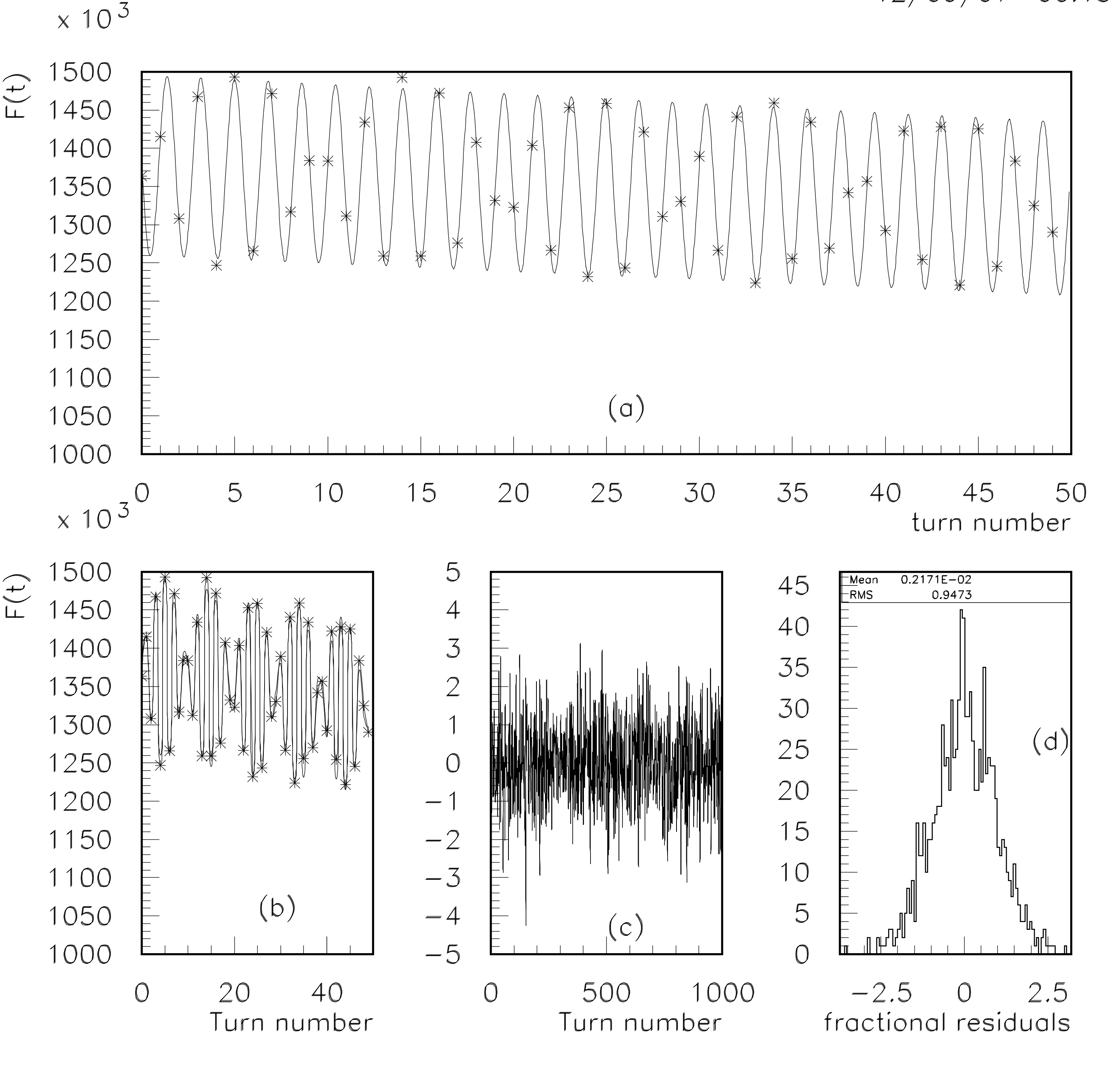
TABLE I. Results of fits for $\delta\gamma/\gamma$ as a function of polarization $\hat{P}$ and noise PERR. Also shown is the χ^2 of the fit for 1000 turns.

29/08/97 13.42

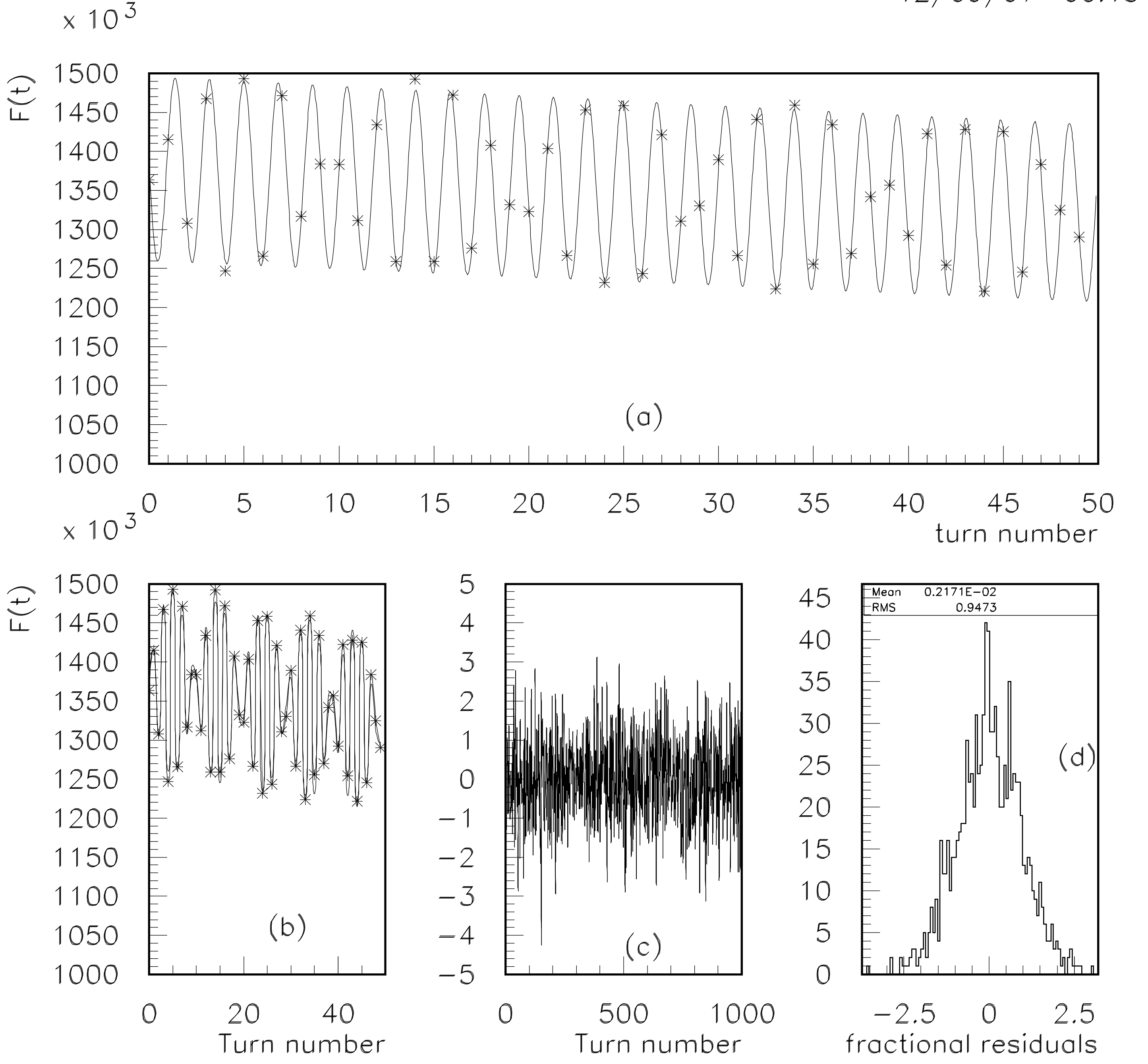


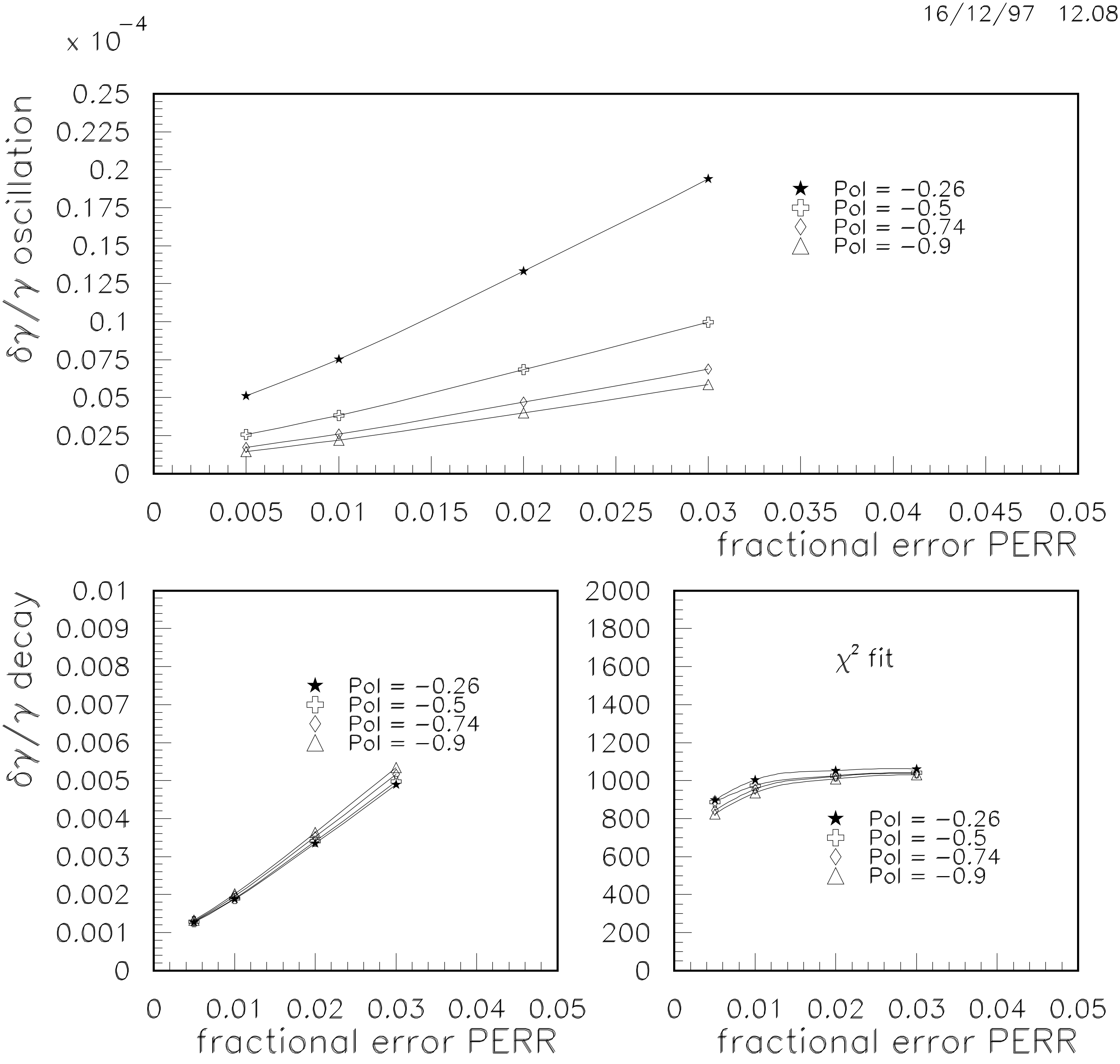
x 10^3
Energy GeV
turn number
(a)
Electron energy 0−10 GeV
(b)
Electron energy 10−25 GeV
(c)
Electron energy 25−50 GeV
(d)
Electron energy 0−50 GeV

12/09/97 00.13

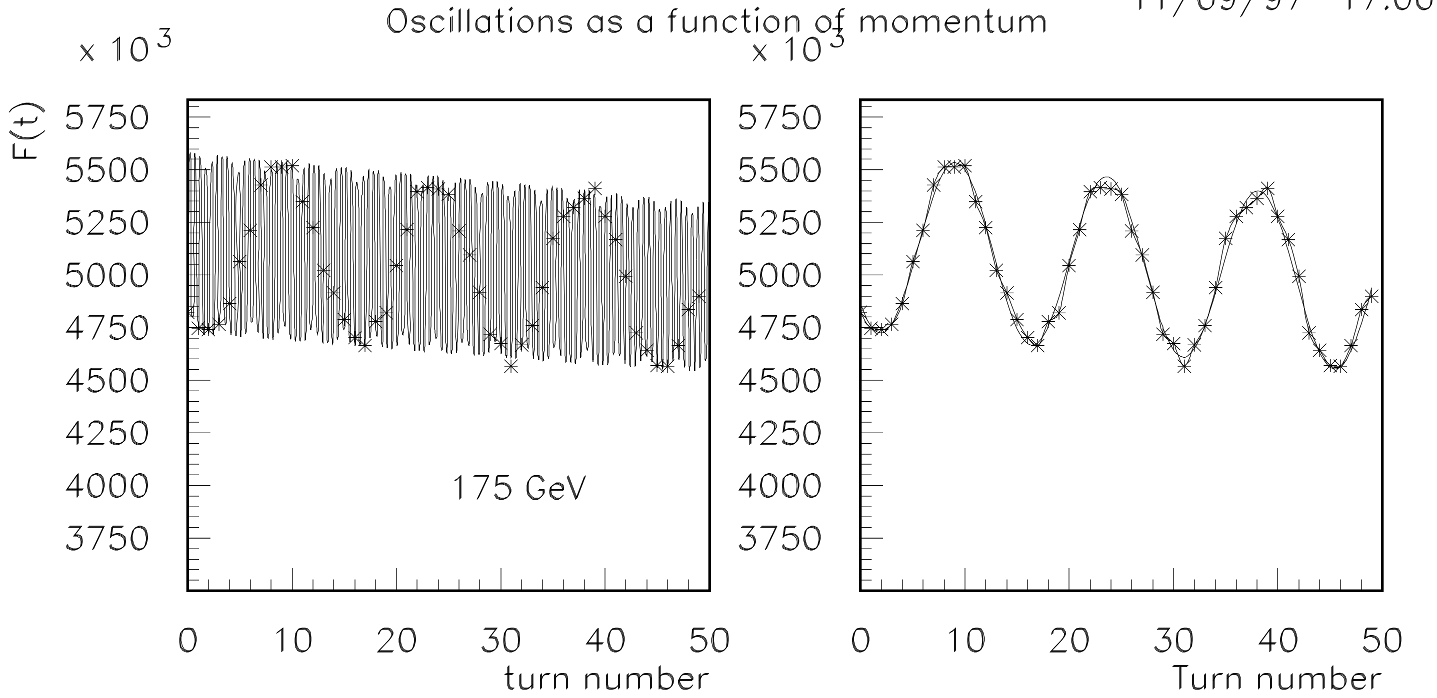

x 10^3
F(t)
(a)
turn number
F(t)
(b)
Turn number
(c)
Turn number
Mean 0.2171E-02
RMS 0.9473
(d)
fractional residuals

12/09/97 00.13

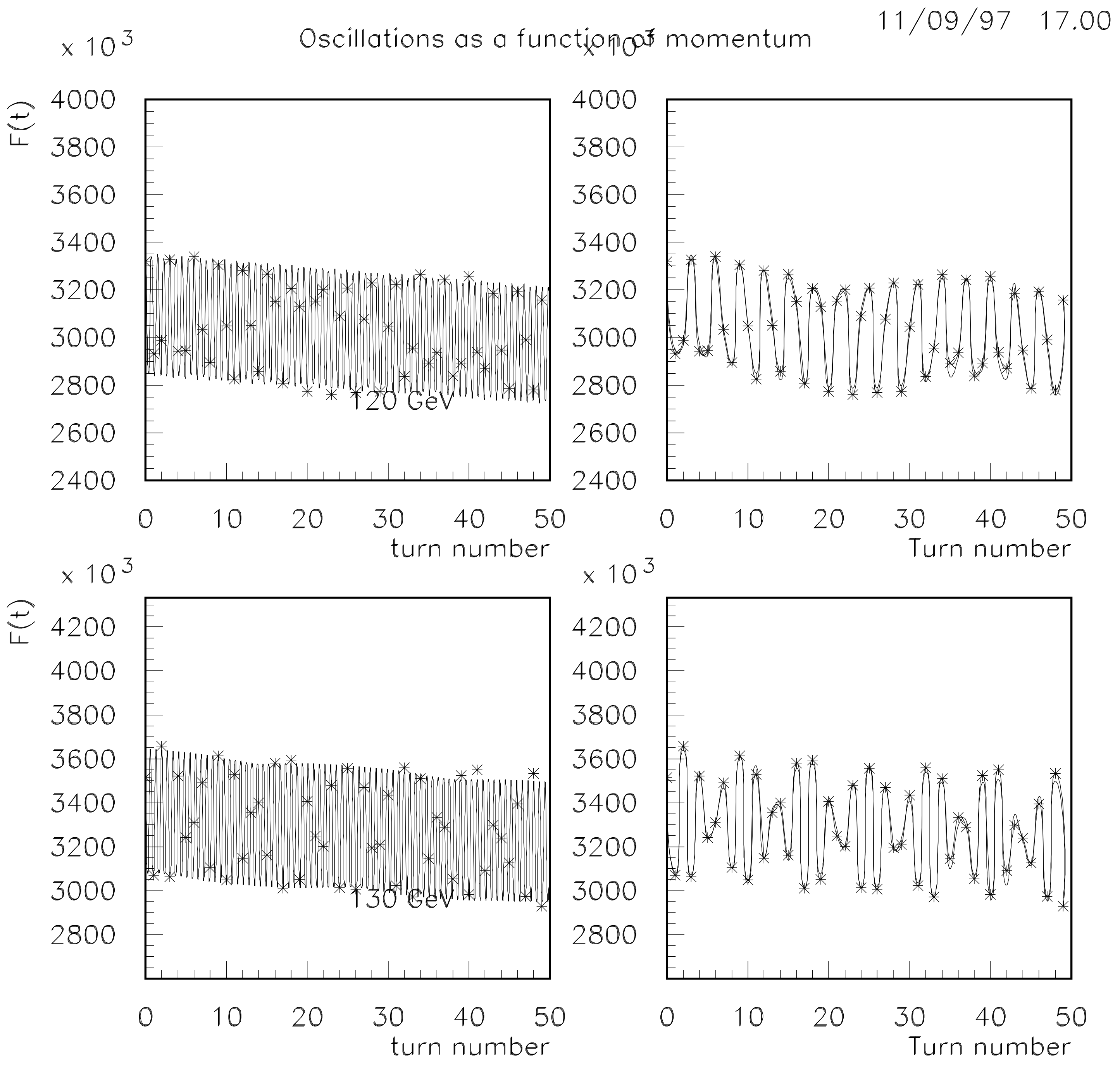

x 10^3
F(t)
(a)
turn number
x 10^3
F(t)
(b)
Turn number
(c)
Turn number
Mean 0.2171E-02
RMS 0.9473
(d)
fractional residuals

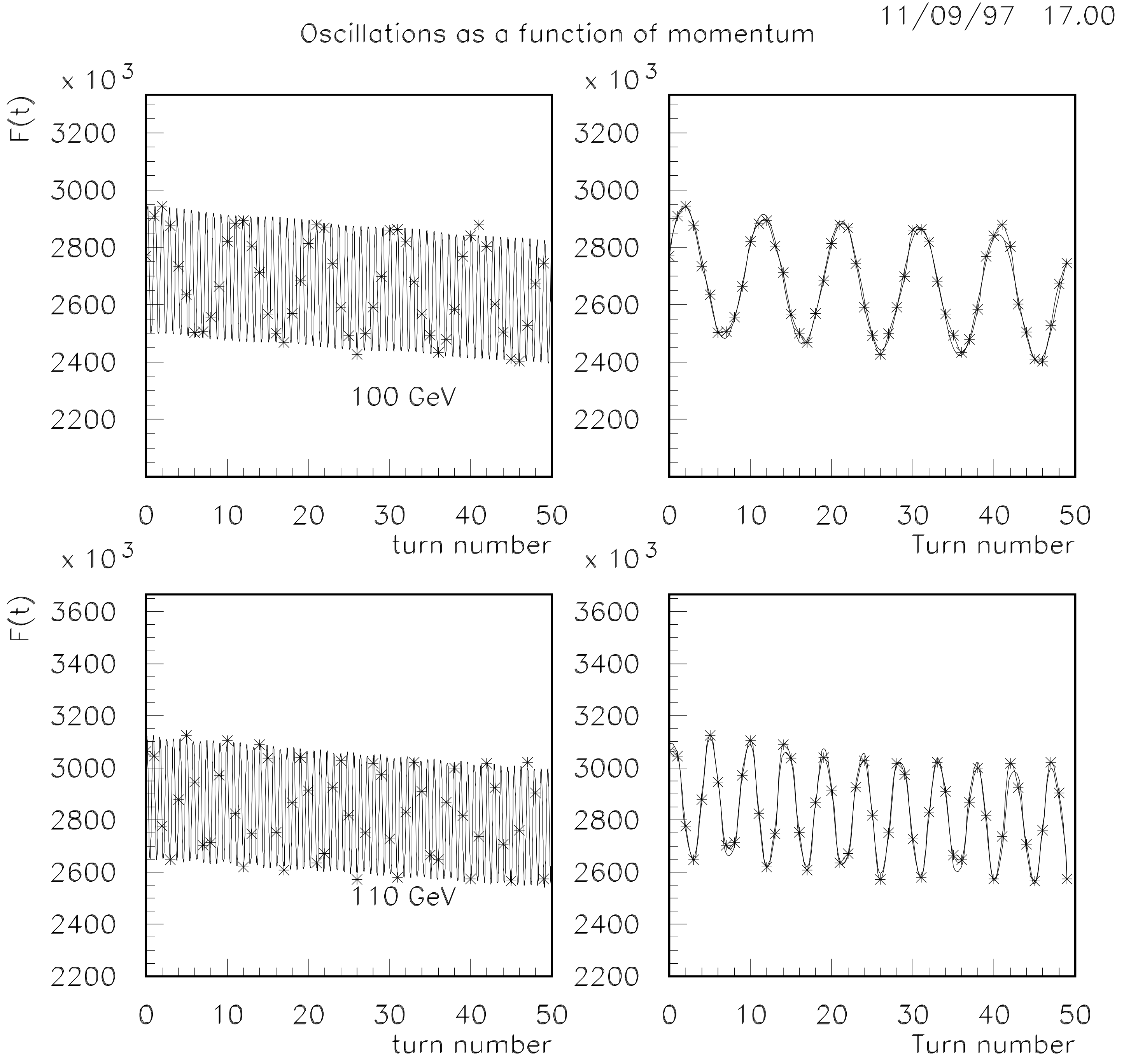

x 10^-4
δγ/γ oscillation
0.25
0.225
0.2
0.175
0.15
0.125
0.1
0.075
0.05
0.025
0
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04
0.045
0.05
fractional error PERR
Pol = −0.26
Pol = −0.5
Pol = −0.74
Pol = −0.9
δγ/γ decay
0.01
0.009
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0
0.01
0.02
0.03
0.04
0.05
fractional error PERR
χ² fit
2000
1800
1600
1400
1200
1000
800
600
400
200
fractional error PERR

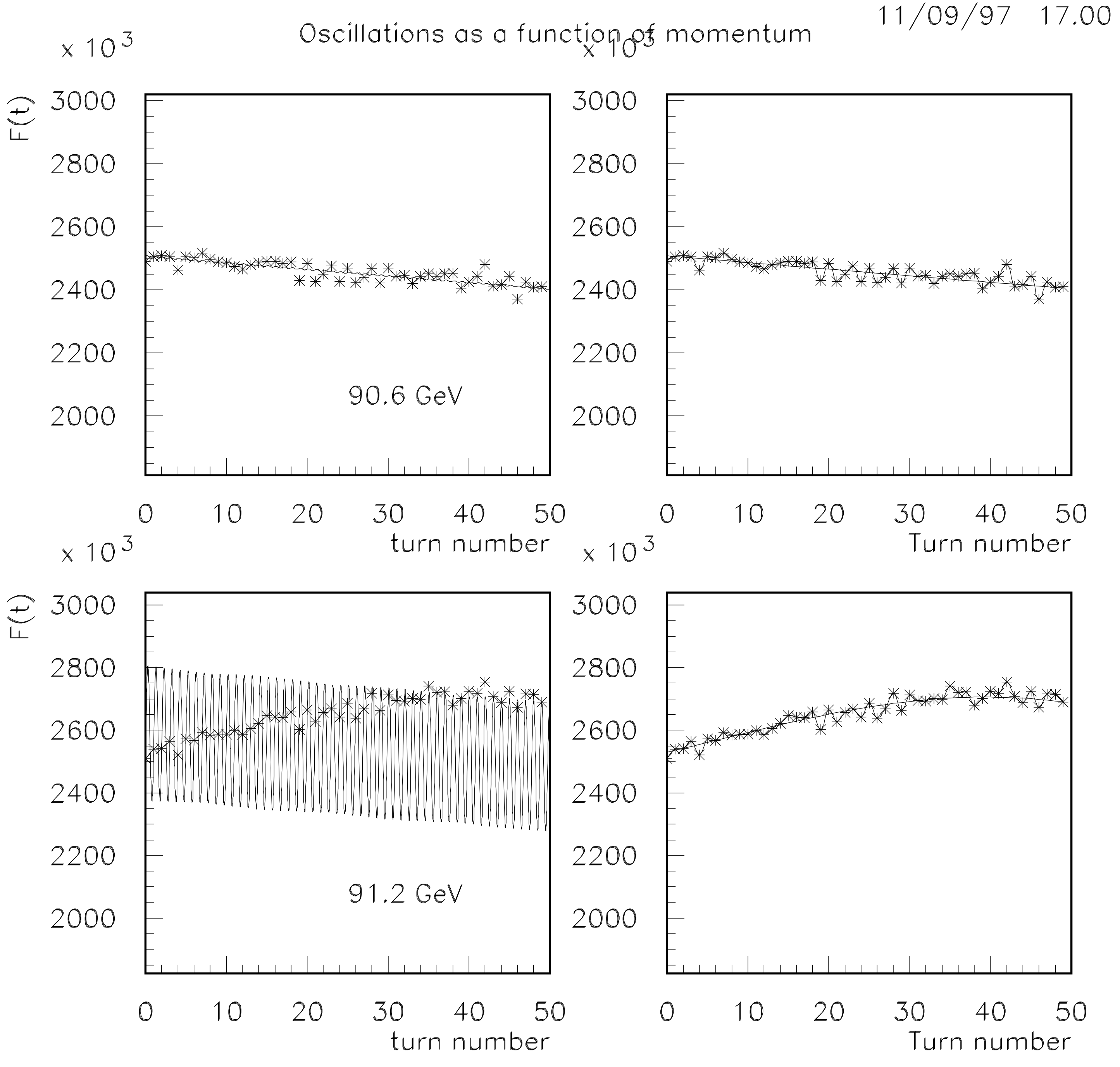
11/09/97 17.00

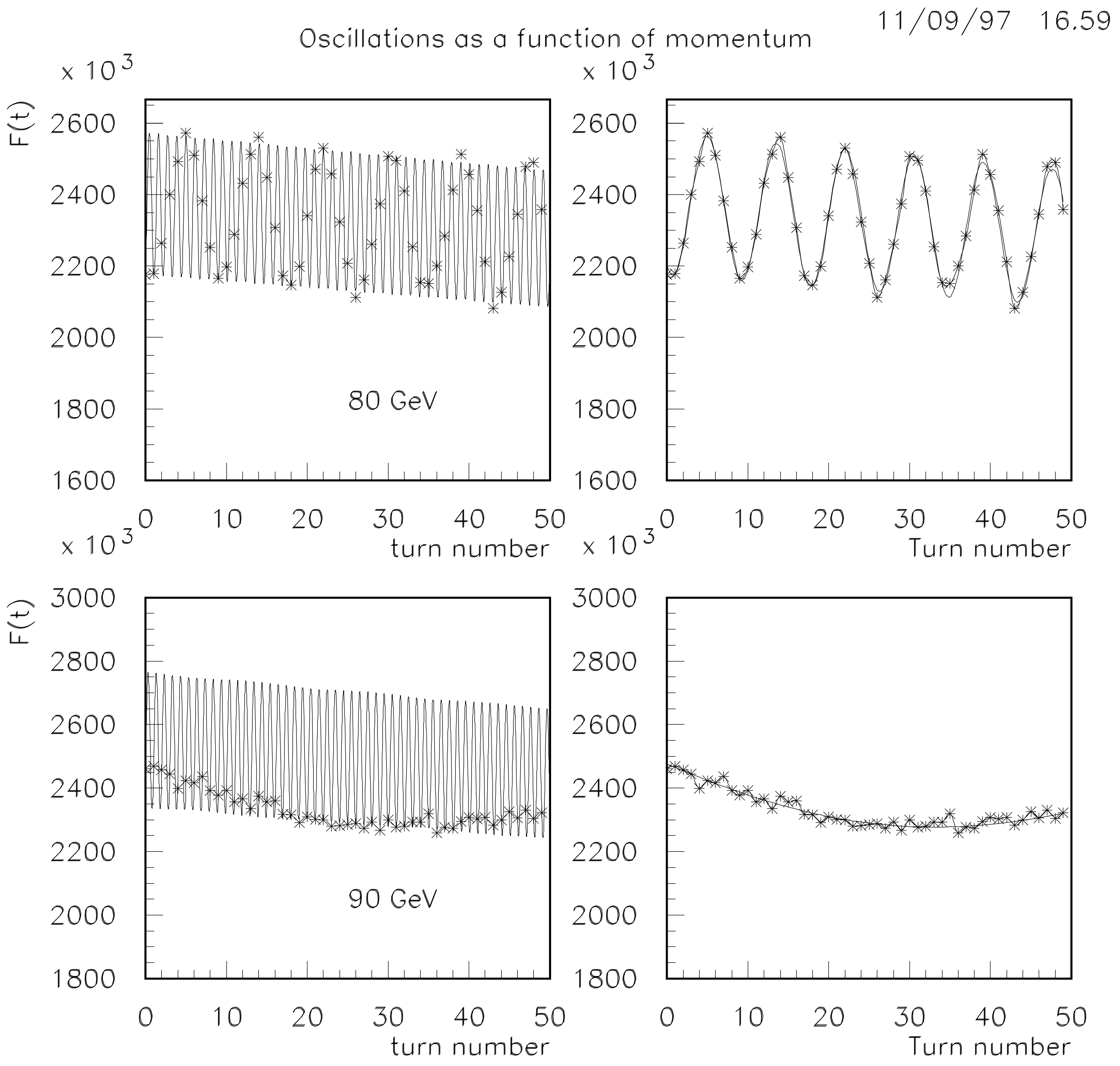


Oscillations as a function of momentum
x 10^3
F(t)
5750
5500
5250
5000
4750
4500
4250
4000
3750
175 GeV
0
10
20
30
40
50
turn number
x 10^3
5750
5500
5250
5000
4750
4500
4250
4000
3750
0
10
20
30
40
50
Turn number


11/09/97 17.00
Oscillations as a function of momentum
x 10^3
F(t)
4000
3800
3600
3400
3200
3000
2800
2600
2400
120 GeV
0
10
20
30
40
50
turn number
Turn number
4200
130 GeV

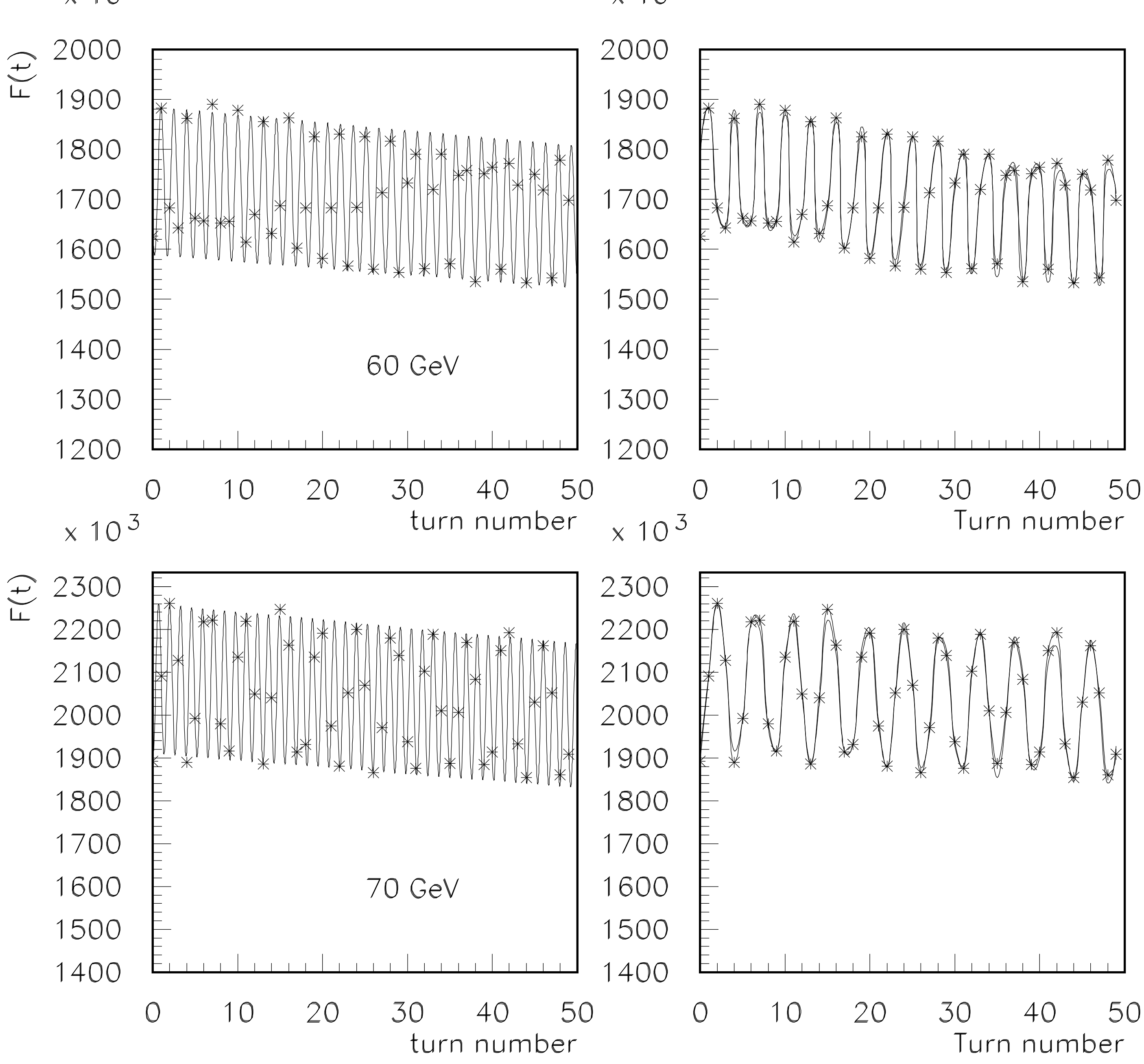

11/09/97 17.00
Oscillations as a function of momentum
x 10^3
F(t)
3200
3000
2800
2600
2400
2200
100 GeV
0
10
20
30
40
50
turn number
Turn number
3600
3400
110 GeV


11/09/97 17.00
Oscillations as a function of momentum
x 10^3
F(t)
3000
2800
2600
2400
2200
2000
90.6 GeV
0 10 20 30 40 50
turn number
Turn number
91.2 GeV

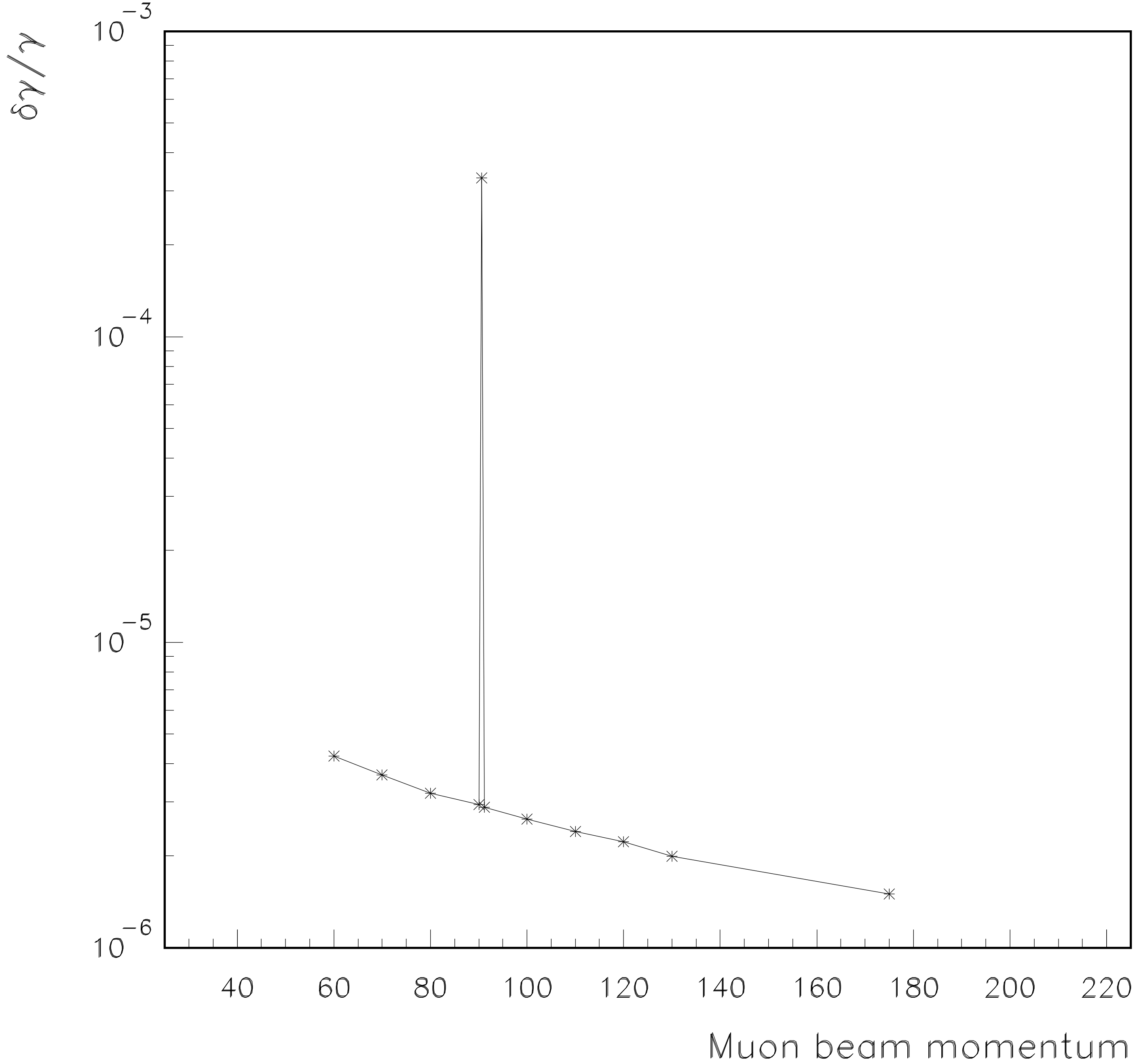

11/09/97 16.59
Oscillations as a function of momentum
x 10^3
F(t)
80 GeV
turn number
Turn number
90 GeV

11/09/97 16.59

Oscillations as a function of momentum



16/12/97 12.17



$\delta\gamma/\gamma$
10^{-3}
10^{-4}
10^{-5}
10^{-6}
40
60
80
100
120
140
160
180
200
220
Muon beam momentum

One can reduce the full width half max of the rf spread by rf cavities. See RR MU-Note.