The effect of RF on polarization in a muon storage ring

Rajendran Raja

Fermi National Accelerator Laboratory

P.O. Box 500

Batavia, IL 60510

Abstract

We investigate the preservation of polarization in an idealized muon storage rings with muon energy 30 GeV and 50 GeV for beams with momentum acceptance $\delta p/p$=2%. The beams depolarize rapdily due to the differential rates of precession as a function of muon momentum. When an rf voltage is applied, inducing synchrotron oscillations, the polarization is seen to be preserved for synchrotron tunes of the order of 0.2. We investigate the dependence of depolarization as a function of synchrotron tune and rf voltage for 30 GeV and 50 GeV muon storage rings.

I. INTRODUCTION

The preservation of polarization of muons in a muon storage ring is investigated. As the muons circulate around the ring, the spin vector of a muon precesses faster than the momentum vector by an amount $2\pi\gamma(g-2)/2$, where g is the gyromagnetic ratio of the muon and γ is its Lorentz gamma factor. See ref [1] for a discussion of this in detail and references therein. Since there is a spread of momenta in the ring, there will be differential precession of the polarization leading to a loss of the average polarization of the muons as the muons circulate the ring. If the momentum of a muon remains the same from turn to turn, the amount of spin precession will be the same relative to the average spin precession and the depolarization will be cumulative. This effect has been known for a long time [2] and its remedy is to induce synchrotron oscillations causing the individual muon spin precession to vary from turn to turn. We will estimate the magnitude of the effect and its remedy using simple simulations and then investigate further using more realistic simulations that take into account synchrotron oscillations properly.

II. SIMPLE SIMULATION

We construct an idealized storage ring of 50 GeV muon energy with the parameters as shown in table *I*. We inject 10,000 muons into the ring with $\delta p/p$ of 2% and allowed to circulate for 100 turns. The muons are assumed to be fully longitudinally polarized and the polarization is computed at the end of each turn for those muons whose decay electrons end up in a calorimeter of radial extent 5cm-100cm placed around the beam pipe. The polarization is assumed to precess entirely in the horizontal plane and the magnitude of the average polarization vector is plotted from turn to turn. The muons are decayed with a vertex that is uniformly distributed in the decay beam pipe. Figure 1 shows the loss of polarization as a function of turn number for the 50 GeV case in the absence of any *rf* (Synch=0). The same muons are used for the next turn with the same values of momenta and the decay is preformed again. The number of muons circulating turn by turn is decreased by the actual number that would have decayed. This is similar to the technique used in [1] to determine the energy scale of the muon collider.

A. Generating synchrotron oscillations

At the end of each turn, the muons are subjected to an *rf* voltage and their momenta will be re-arranged depending on the phase of their arrival at the *rf*. Each of the muons will undergo a synchrotron oscillation so that their energies E as a function of turn number t will be given by the expression

$$E = Acos(\phi_0 + 2\pi Q_s t) \tag{2.1}$$

where Q_s is the synchrotron tune (fractional) and A is the amplitude of oscillation and ϕ_0 is an arbitrary phase. It is impossible from knowing the momentum of each muon alone to generate synchrotron oscillations, since one needs to compute both A and ϕ_0. The trick is to generate the oscillations in such a way that the mean energy and its standard deviation are preserved from turn to turn. This is accomplished by the following neat construction. For each muon, generate two random numbers r_1 and r_2 such that both of these are Gaussian distributed with standard deviation δp. The two-dimensional distribution of these two Gaussian numbers is cylindrically symmetrical and is exponentially distributed in $r_1^2 + r_2^2$, where r_1 and r_2 are pairs of Gaussian random numbers. The projection of this distribution along any radial direction is Gaussian. Synchrotron oscillations are obtained by rotating this distribution by the angle $2\pi Q_s t$ and taking the x- component of the vectors. This will have the effect of introducing oscillations while at the same time preserving the mean and standard deviation of the distribution from turn to turn.

B. $\delta p/p$=2% case

We investigate the case for a beam energy spread of 2% at the 1σ level. Figure 1 shows the polarization as a function of turn number for synchrotron tunes 0.05 0.1,0.15,0.2 and 0.25. For a synchrotron tune of 0.05, the polarization is stable for 100 turns, but contains an oscillation term. These oscillations persist with increased frequency but much reduced amplitude for synchrotron tune values 0.1-0.25. Figure 2 shows the corresponding distributions for the 30 GeV case. The loss of polarization for the no *rf* case is slower in the 30 GeV case as expected, since spins precess slower.

C. $\delta p/p$=0.7% case

It has since been pointed out that the current Fermilab design for a muon storage ring envisages rms $\delta p/p = 0.7\%$. Figures 3 and 4 show the depolarization as a function of turn number for synchrotron tunes of 0.01-0.05. It can be seen that in this case a synchrotron tune of 0.01 is sufficient to maintain polarization up to 100 turns.

III. MORE REALISTIC SIMULATION

The above simulation assumes that all the particles are in the *rf* bucket and undergo synchrotron oscillations with the same tune. In practice, this is not the case and we will now attempt to simulate realistic synchrotron oscillations using the synchrtorn oscillation difference equations [3].

$$\Delta E_{n+1} = \Delta E_n + eV(sin\phi_n - sin\phi_s) \tag{3.1}$$

$$\phi_{n+1} = \phi_n + \frac{2\pi h\eta \Delta E_{n+1}}{\beta^2 E_s} \tag{3.2}$$

where ΔE_{n+1} is the difference between the energy gained by a particle at the end of turn n as it traverses the *rf* with phase ϕ_n, and a synchronous particle as it traverses the *rf* with phase ϕ_s. The "slip factor" η is defined as $1/\gamma_t^2 - 1/\gamma^2$, where γ_t is the Lorentz gamma factor of particles at the transition energy and γ is the Lorentz factor of the synchronous particle. eV is the *rf* Voltage times the charge of the particle, β is the velocity of the particle expressed as a function of the velocity of light and h is the "harmonic number", which is the integral number of *rf* oscillations during the time it takes the synchronous particle to traverse the ring. E_s is the energy of the synchronous particle. One can show [3] that this results in synchrotron oscillations with the synchrotron tune Q_s being given by

$$Q_s = \sqrt{\frac{-h\eta eV cos\phi_s}{2\pi\beta^2 E_s}} \tag{3.3}$$

where Q_s is defined as the fractional number of synchrotron oscillations during one turn around the ring. The *rf* voltage needed can be written as

$$eV = -\frac{2\pi\beta^2 E_s}{h\eta cos\phi_s} Q_S^2 \tag{3.4}$$

We want to operate the *rf* such that there is no net acceleration. This implies a synchronous phase of π radians, since we are above transition, and cosϕ_s= -1. This is known in the jargon as a stationary bucket. The fractional momentum spread $\delta p_{3\sigma}/p$ at the 3σ level supported by the stationary bucket, can be written as

$$\frac{\delta p_{3\sigma}}{p} = \frac{2Q_s}{h|\eta|} \tag{3.5}$$

We would like $\approx 3\sigma$ of a Gaussian energy spread to be contained in the bucket, hence the notation. Given the synchrotron tune and $\delta p_{3\sigma}/p$, one can write

$$eV = \pi Q_s \beta^2 E_s \frac{\delta p_{3\sigma}}{p} \tag{3.6}$$

for the stationary bucket. We take the transition energy in the ring to be 2 GeV for muons[1] and a total circumference of 2 kilometers for the ring. Note that we are now leaving the idealized parameters of the previous simulation and inching closer to a realistic design. This leads to the separatrix curves shown in figure 5, which depict the turn by turn motion of particles with various inital conditions. The plot shows two *rf* buckets with synchronous phases π and $-\pi$. Inside the separatrix, which is a curve that starts with zero initial phase, the particles are contained. Outside the separatrix, the particles are "unbounded" and propagate around the ring with energies that do not oscillate around the synchronous energy. It is the oscillations that take the particles on either side of the synchronous energy that will enable the polarization to be maintained, since these oscillations cause a particle to experience different spin precessions from turn to turn, which do not cumulatively add up to values different from the mean spin precession of the synchronous particle.

[1]This leads to a value of 0.00278 for the η function for the 30 GeV ring and 0.00279 for the 50 GeV ring.

Figure 6 shows the oscillations of the particles in various contours shown in figure 5, starting with the contour closest to the synchronous particle and moving further away, as a function of turn number. The particle on the contour closest to the synchronous phase shows oscillations consistent with the synchronous tune, whereas the oscillations slow down as one gets further away from the center of the bucket. On the separatrix, the oscillations take an infinite amount of time.

Figure 7 shows the initial distribution of the beam as it is injected. The beam has a momentum spread $\delta p/p$ of 0.7% at the 1σ level. The beam energy chosen is 30 GeV. Figure 8 shows the evolution of the beam at the end of 100 turns. Notice that the beam shape in the bucket is maintained whereas the particles outside the bucket drift in phase.

A. 30 GeV Results

We now compute the polarization for 1000 particles turn by turn assuming a longitudinal polarization of 1 at injection and precessing each particle by the amount appropriate to it given its energy for that turn. Figure 9 shows the variation of polarization as a function of turn number for Q_s=0.01 for *rf* voltages ranging from 5-25 MV. It can be seen that 5 MV is too small and does not change the polarization appreciably from the no rf case. At 15 MV, the oscillations are more or less optimal to maintain the polarization for 100 turns. At 25 MV, one gets too much *rf* and too much energy gain and a greater amplitude in the polarization oscillation.Figures 10-12 show the corresponding curves for Q_s=0.015,0.020 and 0.025 respectively. It can be seen that the optimum polarization behavior is for Q_s=0.015 and *rf* voltage in the vicinity of 20MV-25MV.Figure 13 shows the curve for Q_s=0.005. This value of synchrotron tune is clearly too low, since increasing the *rf* voltage causes depolarization due to the particles energy spread increasing as a result of not traversing their trajectories fast enough in 100 turns. Table II summarizes the results for all the spin tunes for 30 GeV muon storage ring energy. For a 30 GeV storage ring, 34.3% of the muons survive after 100 turns. P is the average polarization over 100 turns and P_W is the intensity weighted polarization over 100 turns. When no *rf* is applied, one gets P=0.72 and P_W=0.79. It can again be seen that for Q_s=0.015 and *rf*=25MV, one gets a bucket that is capable of supporting $\approx$ 2.1% at the 3σ level and one obtains P=0.95 and P_W=0.96.

B. 50 GeV Results

We repeat the above set of calculations for the muon storage ring energy of 50 GeV and $\delta p/p$)=.7% at the 1σ level. The values of P and P_W for the no *rf* case are 0.49 and 0.55 respectively. This is due to faster spin precession. The *spin tune* for 50 GeV is 0.551 whereas for the 30 GeV ring it is 0.331. Figures 14-17 show the variation of polarization as a function of turn number for *rf* voltages ranging from 10MV -50MV. Figure 18 again shows that a Q_s=0.005 is too small and the presence of *rf* actually makes matters worse. For a 50 GeV storage ring, 52.6% of the muons survive after 100 turns. Table III summarizes the results for the 50 GeV case. It appears as though an *rf* voltage of 50 MV and Q_s=0.02 yields the best results of P=0.91 and P_W=0.92.

IV. SCALING RELATIONS

Using equations 3.6 and 3.5 and the results presented here, one can arrive at the following expression for the *rf* needed and its frequency.

$$eV = \left(\frac{Q_s}{0.015}\right)\left(\frac{\delta p/p}{0.007}\right) 25MV \tag{4.1}$$

$$rf\ frequency \approx \left(\frac{Q_S}{0.015}\right)\left(\frac{0.007}{\delta p/p}\right) 92MHz \tag{4.2}$$

V. HIGHER OREDER EFFECTS

We have so far considered an ideal storage ring which supports a large momentum acceptance.In practice there will be sextupole elements present, which could introduce spin

Parameter	Value	Parameter	Value
Muon Energy	50 GeV	γ	473.22
spin precession in one turn	3.4667 radians	Magnetic field	4.0 Tesla
radius of ring	41.66666 meters	beam circulation time	0.87327E-06 sec
dilated muon life time	0.10397E-02 sec	turn by turn decay constant	0.8399E-03

TABLE I. Parameters of an idealized muon storage ring

tune dependence on the particle position as well the momentum. This will introduce faster depolarizations than simulated here. Further work needs to be done to investigate the effect and size of *rf* needed for such realistic rings.

VI. ACKNOWLEDGEMENTS

The author wishes to acknowledge useful conversations with Chuck Ankenbrandt, Norbert Holtkamp, Carol Johnstone, David Neuffer and Alvin Tollestrup.

VII. CONCLUSIONS

We have shown that loss of polarization due to differential spin precession can be stemmed by the introduction of an *rf* voltage and explored the dependence on spin tune. Once it can be shown that preservation of polarization in the ring extends significantly the physics reach of the machine (work in progress), the case for the relatively modest amounts of *rf* required to preserve the polarization will seem more compelling.

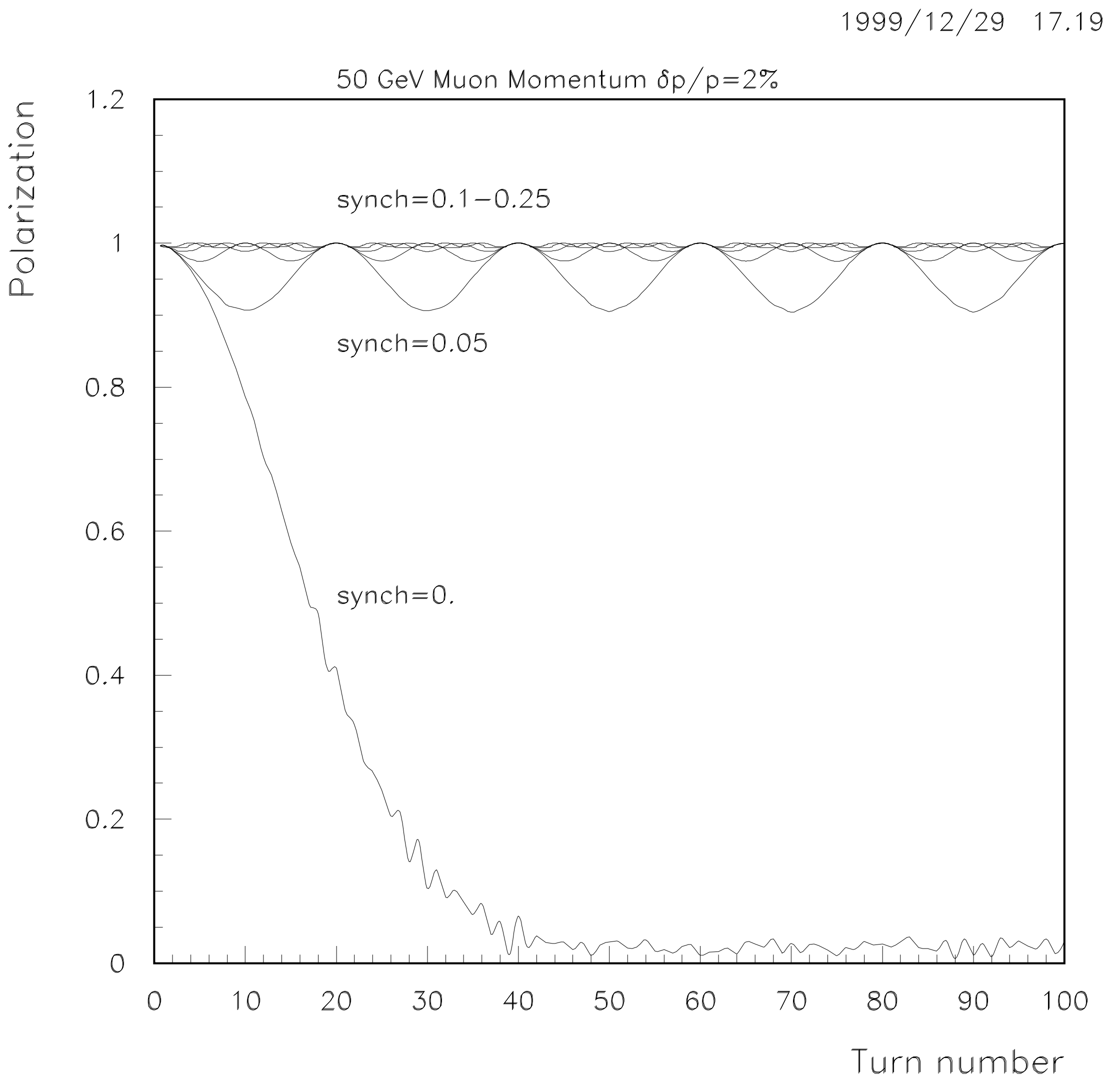

FIG. 1. Figure shows the rate of change of polarization with turn number for a muon storage ring of 50 GeV momentum for various values of the synchroton oscillation tune.

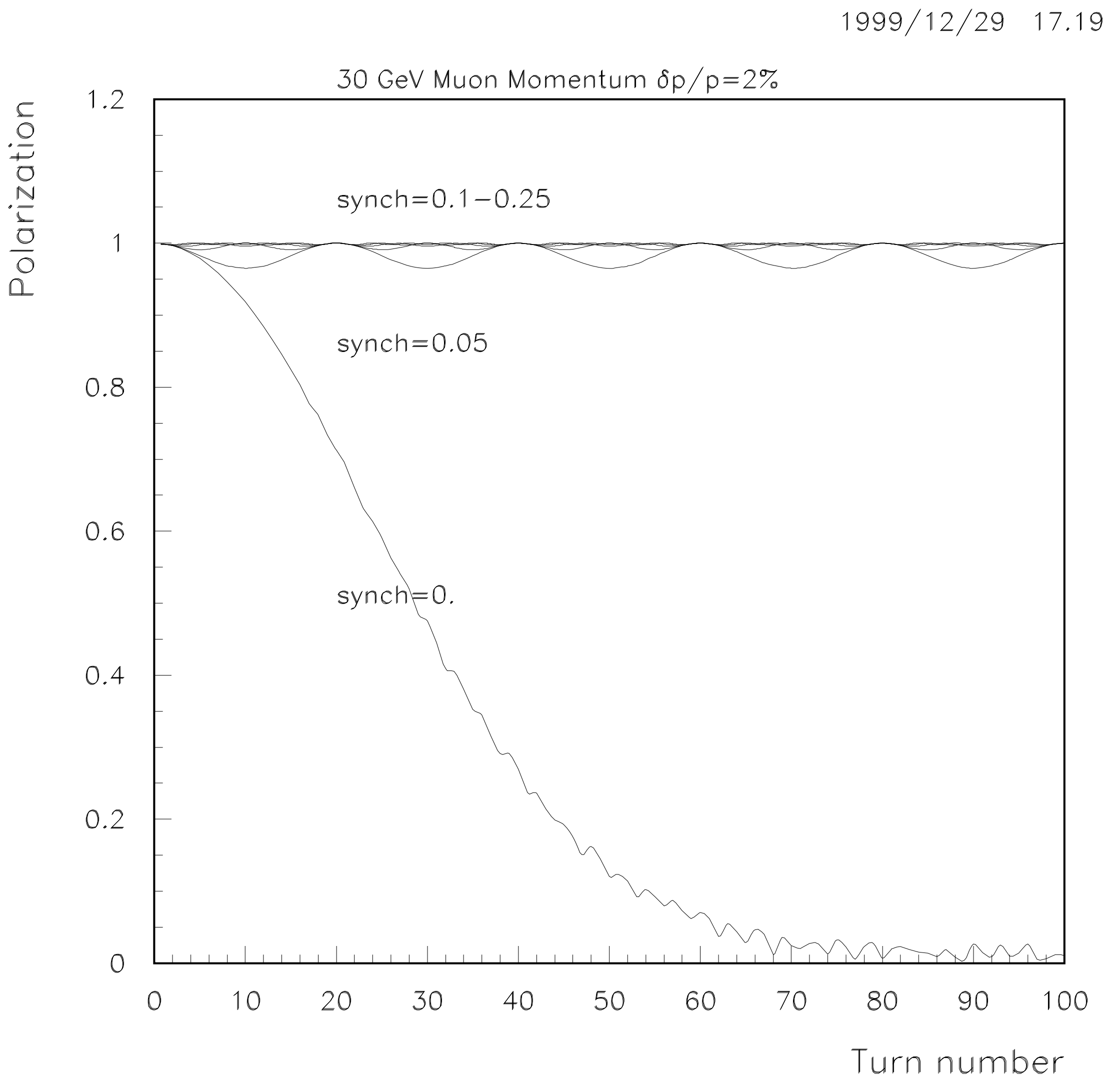

FIG. 2. Figure shows the rate of change of polarization with turn number for a muon storage ring of 30 GeV momentum for various values of the synchroton oscillation tune.

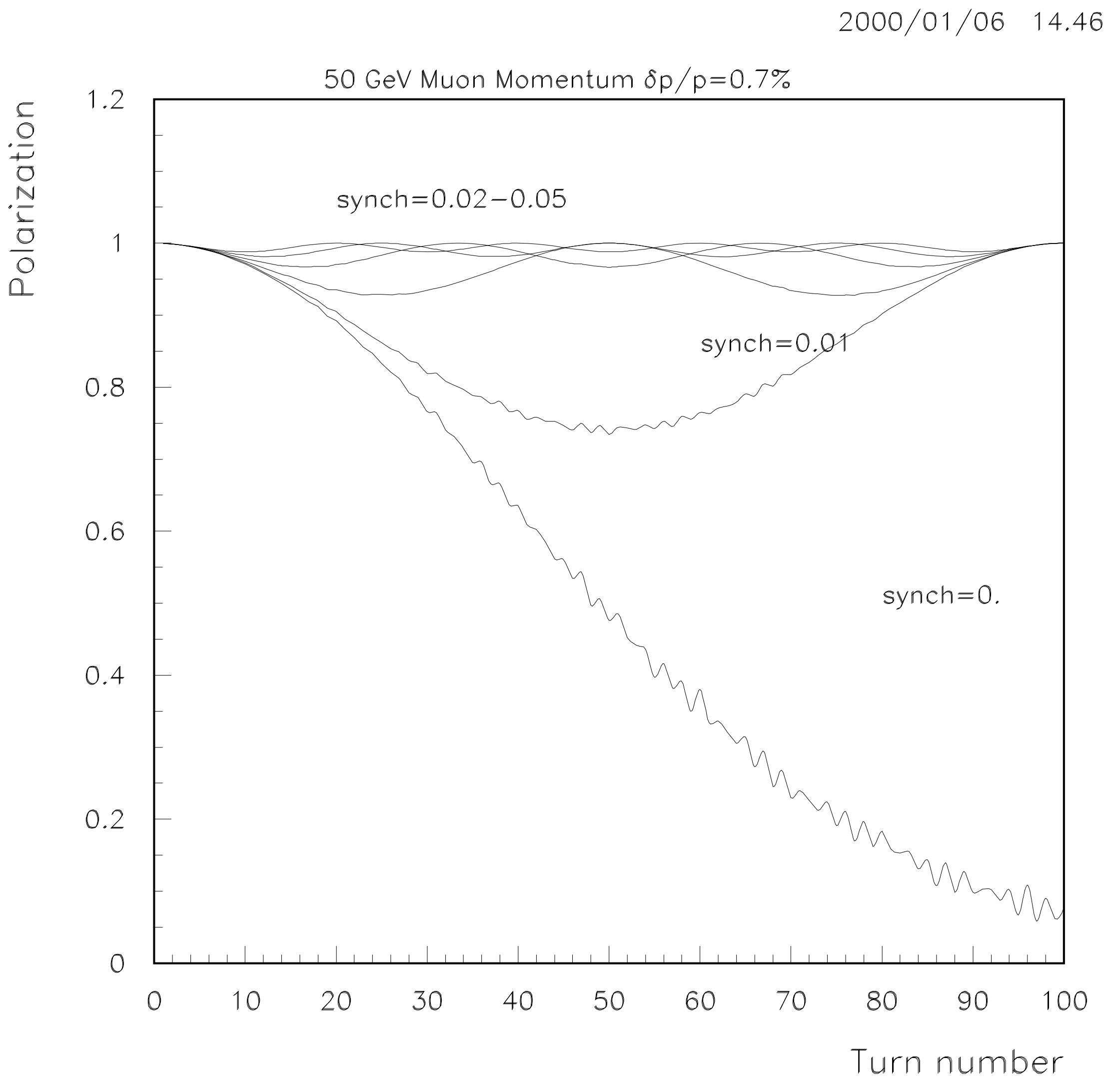

FIG. 3. Figure shows the rate of change of polarization with turn number for a muon storage ring of 50 GeV momentum for various values of the synchroton oscillation tune.

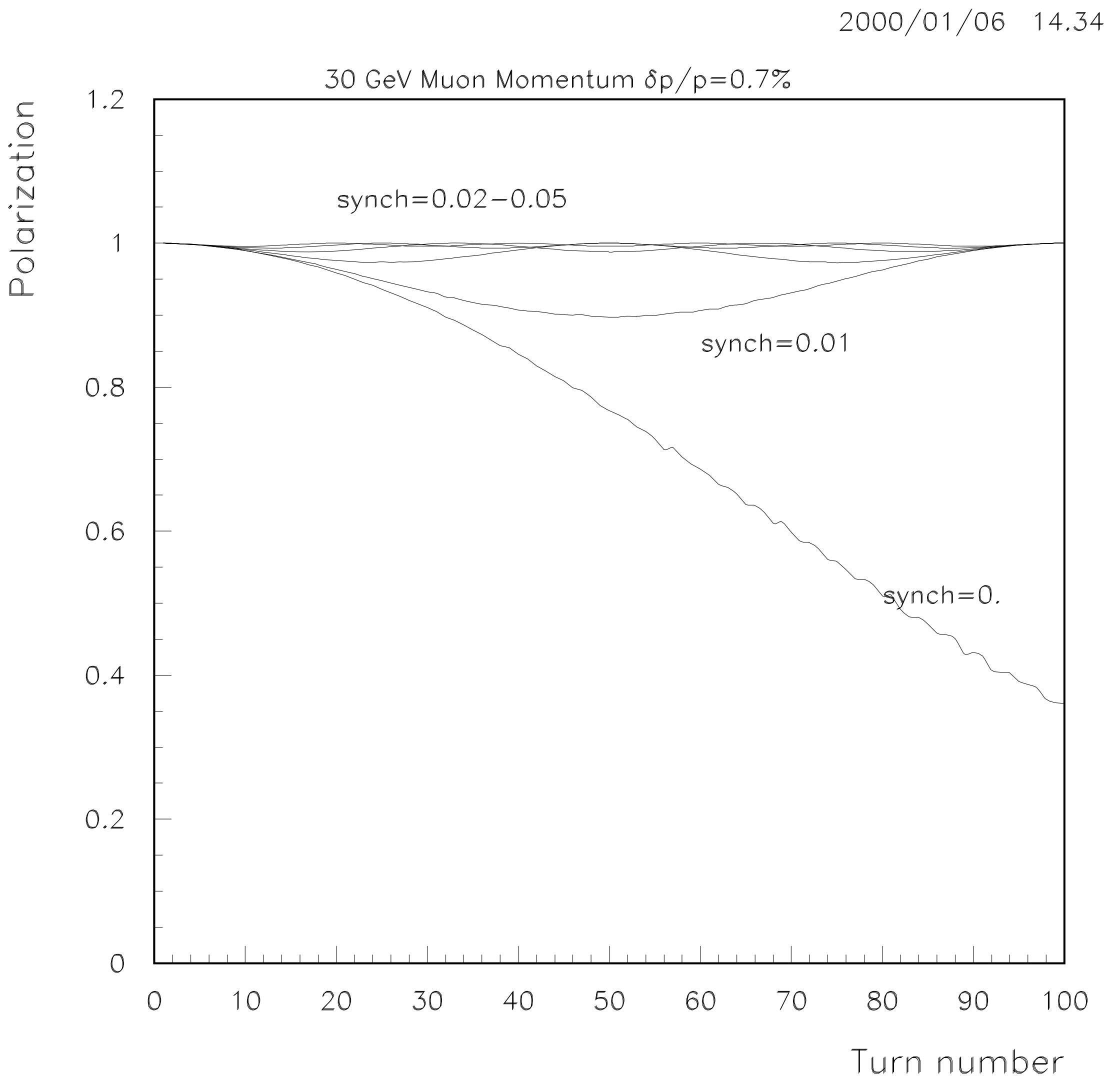

FIG. 4. Figure shows the rate of change of polarization with turn number for a muon storage ring of 30 GeV momentum for various values of the synchroton oscillation tune.

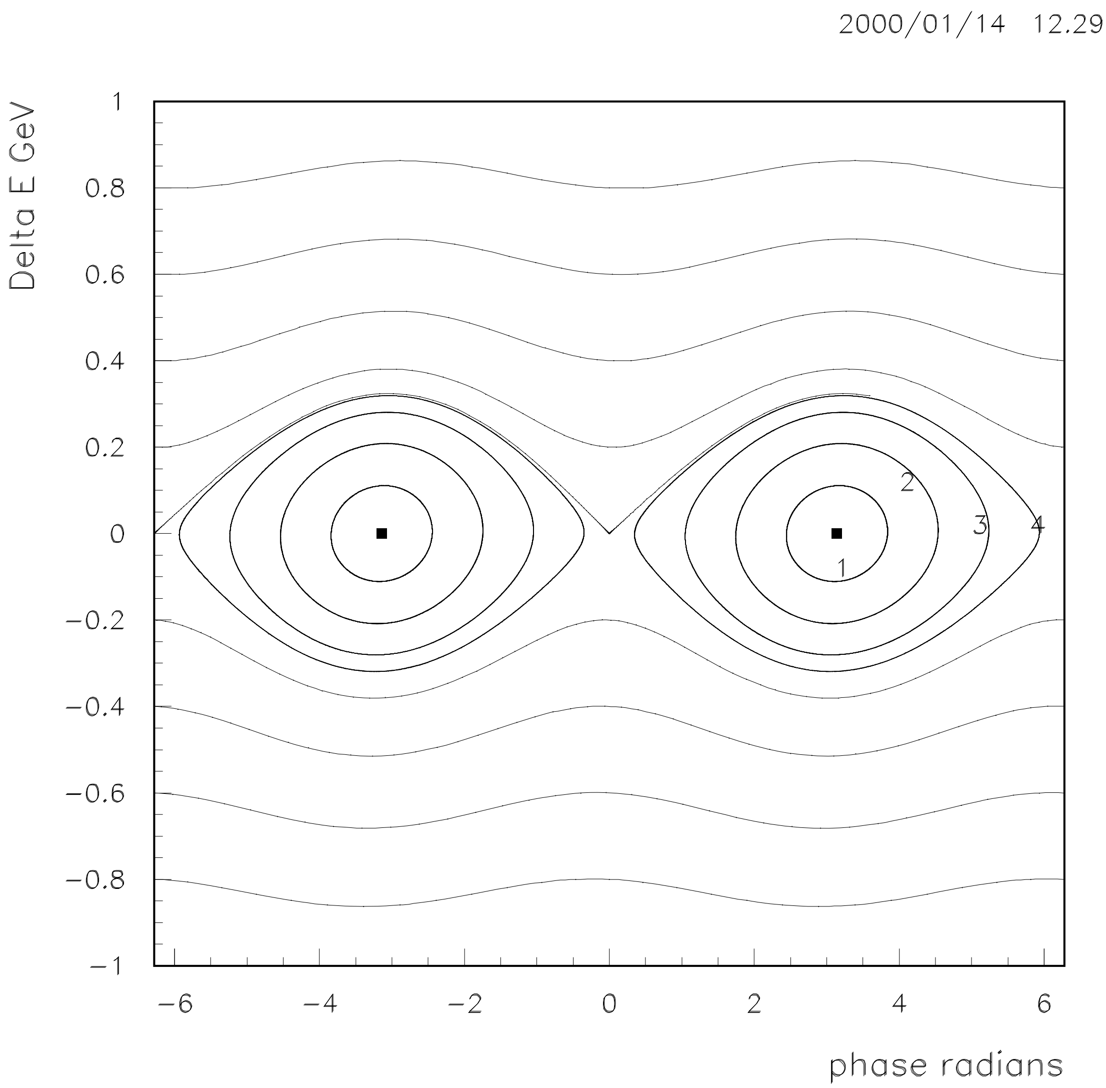

FIG. 5. Figure shows the separatrix for an *rf* voltage 15 MV and a synchrotron tune of 0.015. The two stationary buckets shown have synchronous phases of $\pm\pi$ respectively. The contours show the path of particles from turn to turn. The particles circulate in a clockwise fashion along the closed contours, from right to left in the unbounded contours below the separatrix and from left to right in the unbounded contours above the separatrix. The oscillation of energy along the numbered contours are shown in figure 6.

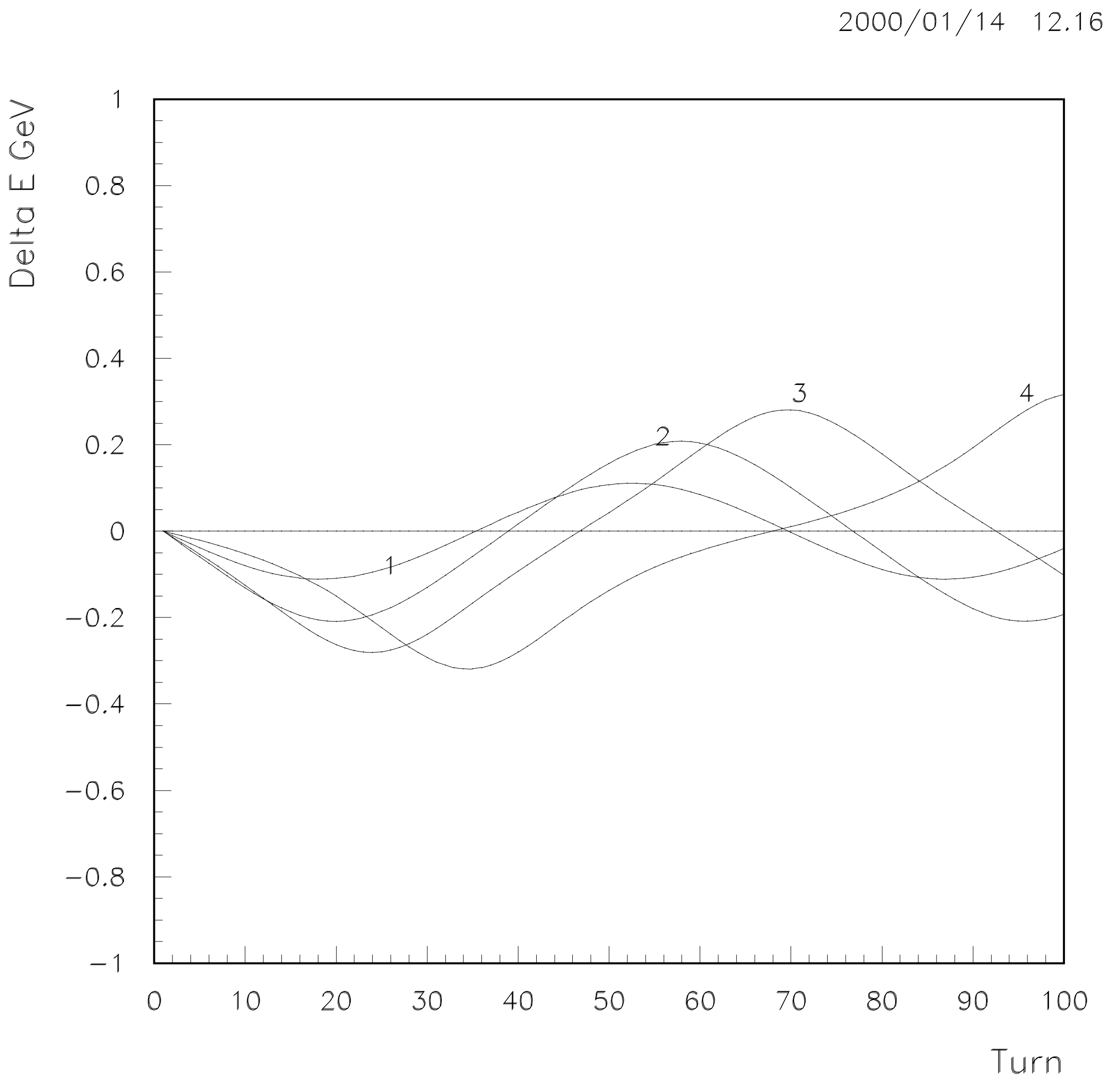

FIG. 6. Figure shows the oscillations in energy of particles on contours numbered 1-4 of figure 5. The further one is from the synchronous particle, the slower the oscillations.

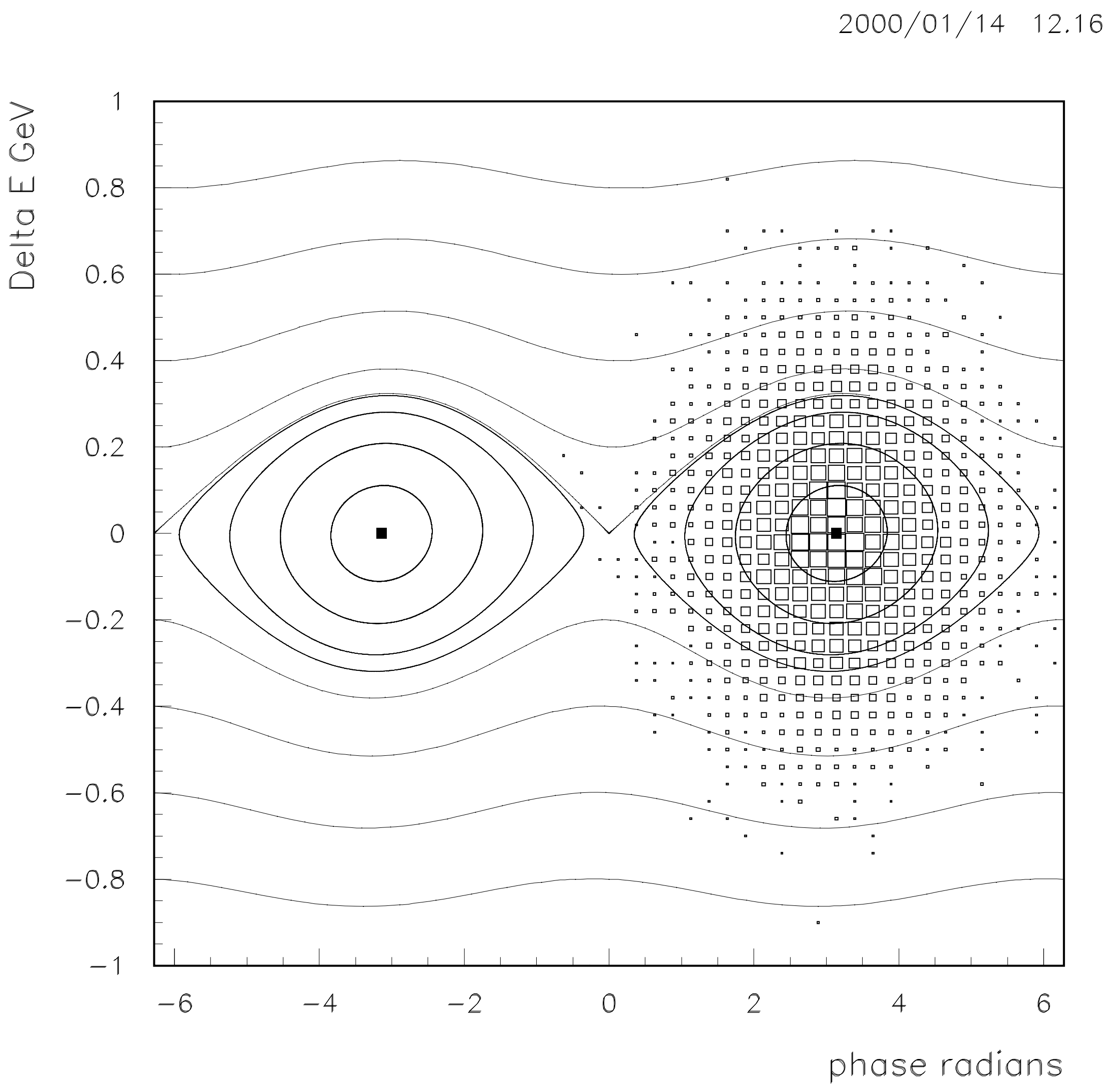

FIG. 7. The beam profile at injection superimposed on the separatrix of figure 5

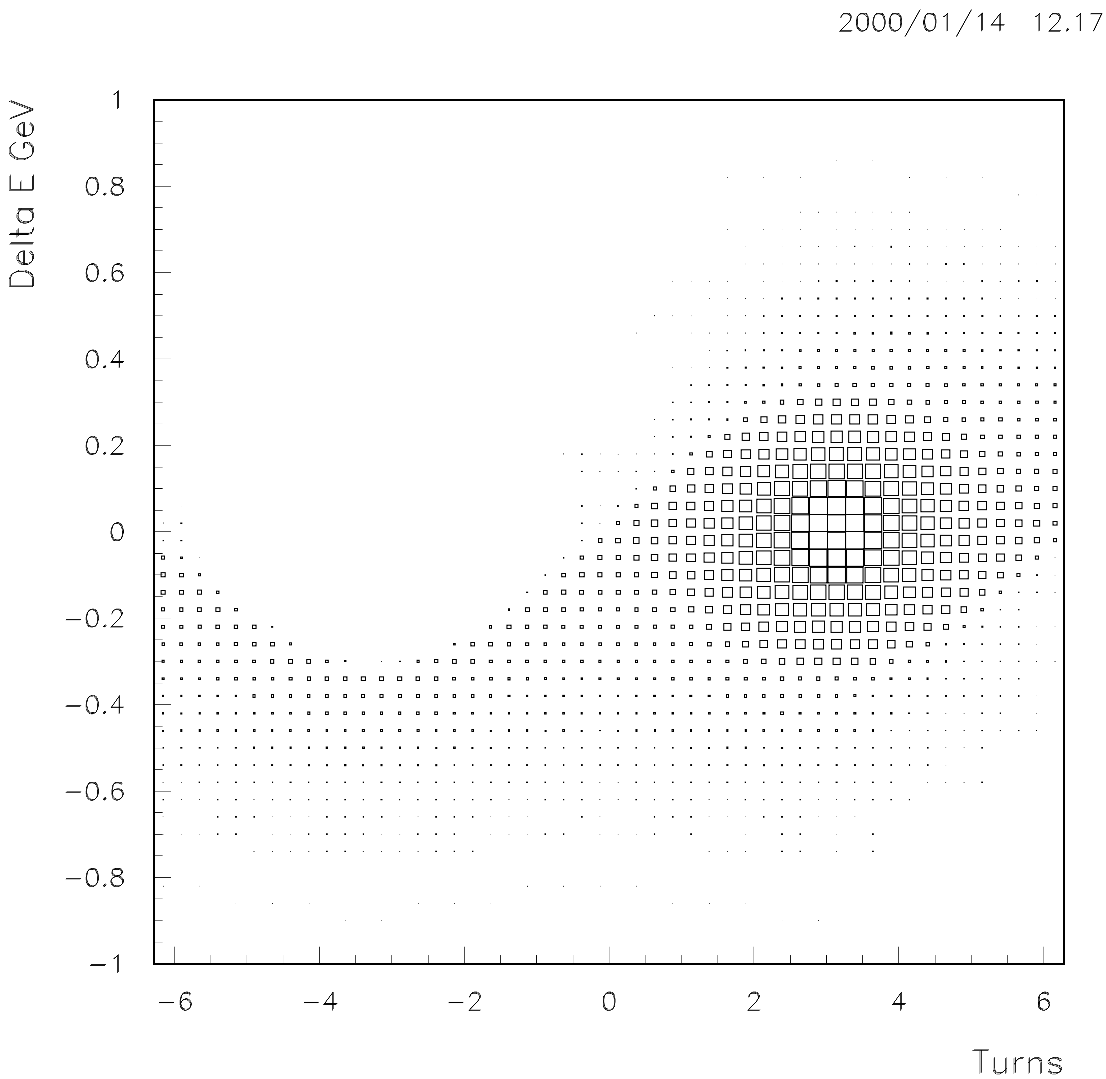

FIG. 8. The shape of the beam at the end of 100 turns. The beam inside the separatrix remains as is, whereas the particles outside drift in phase, with the ones with positive ΔE arriving at later times (larger phases).

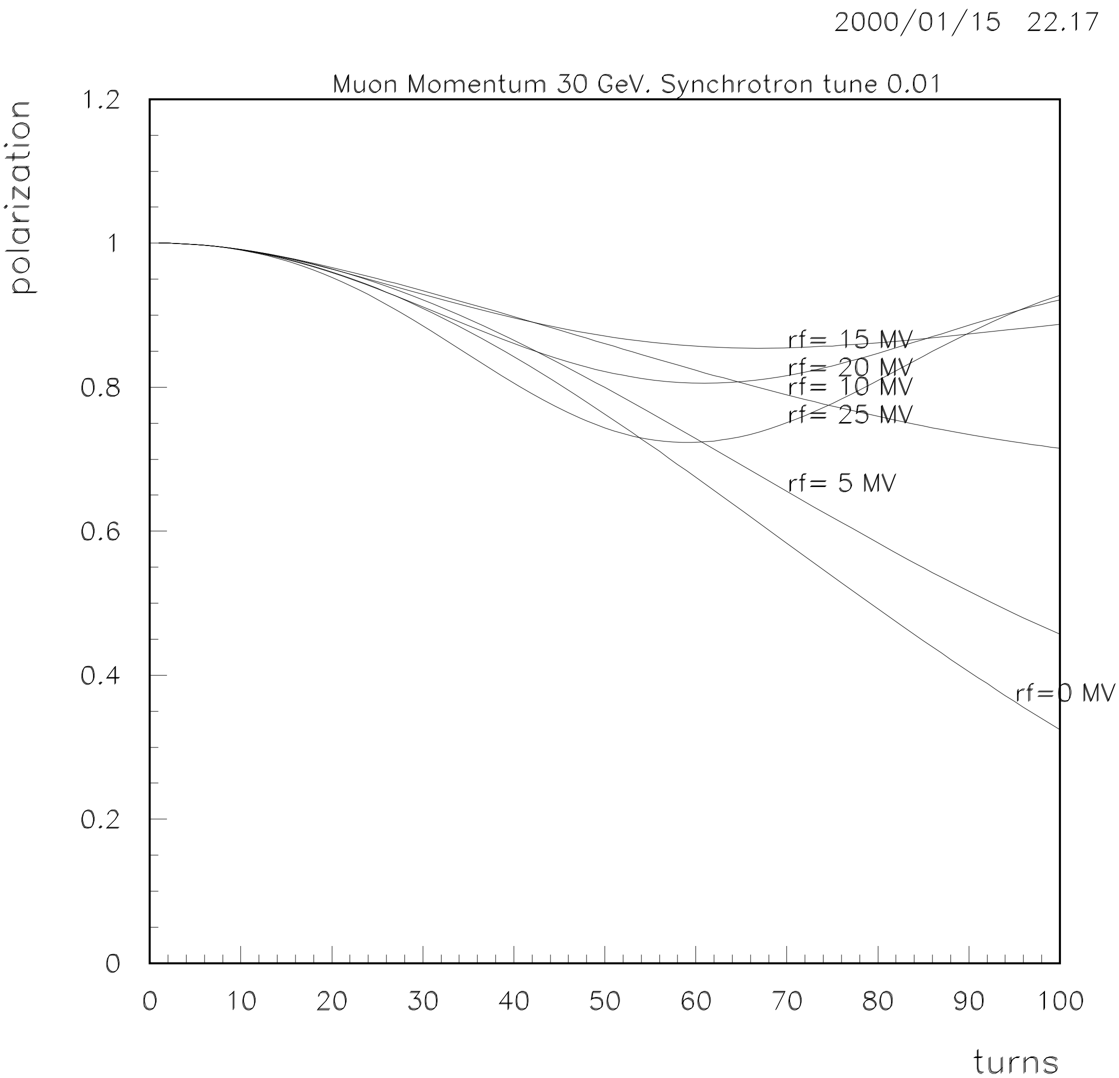

FIG. 9. Variation of polarization as a function of turn for various rf voltages ranging from 5MV to 25 MV. Beam Energy 30 GeV, synchrotron tune = 0.01.

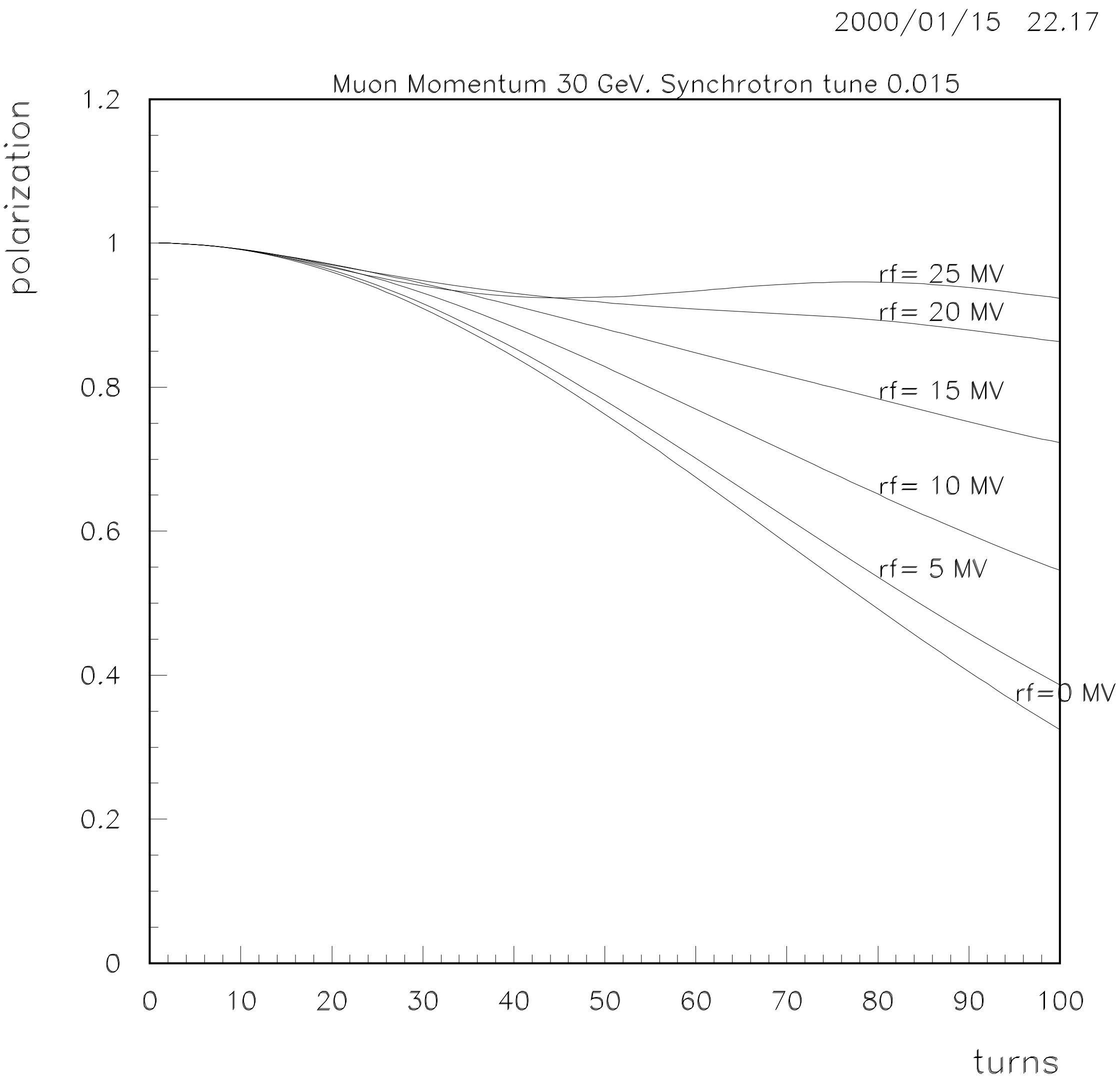

FIG. 10. Variation of polarization as a function of turn for various rf voltages ranging from 5MV to 25 MV. Beam Energy 30 GeV, synchrotron tune = 0.015.

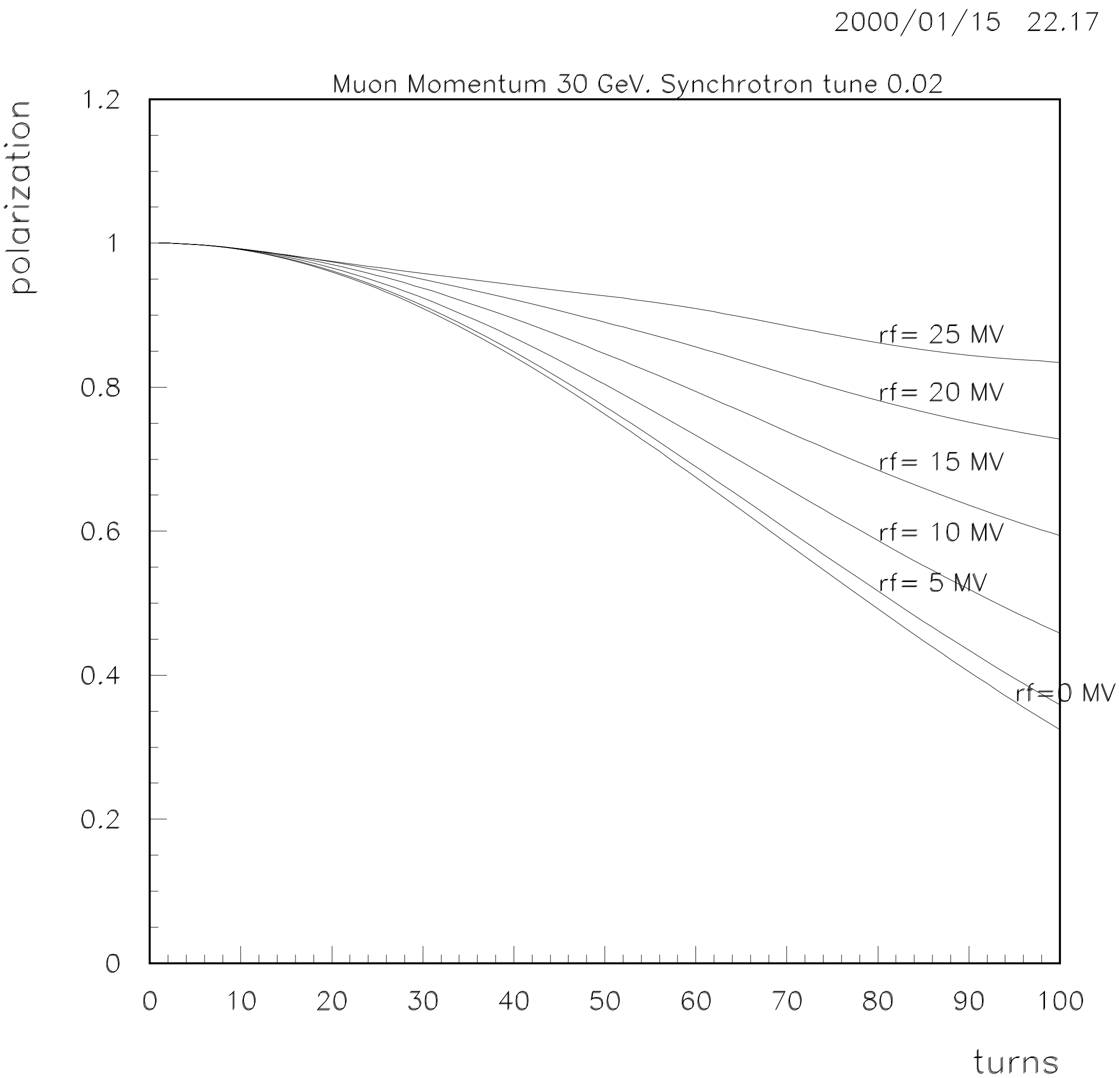

FIG. 11. Variation of polarization as a function of turn for various rf voltages ranging from 5MV to 25 MV. Beam Energy 30 GeV, synchrotron tune = 0.02.

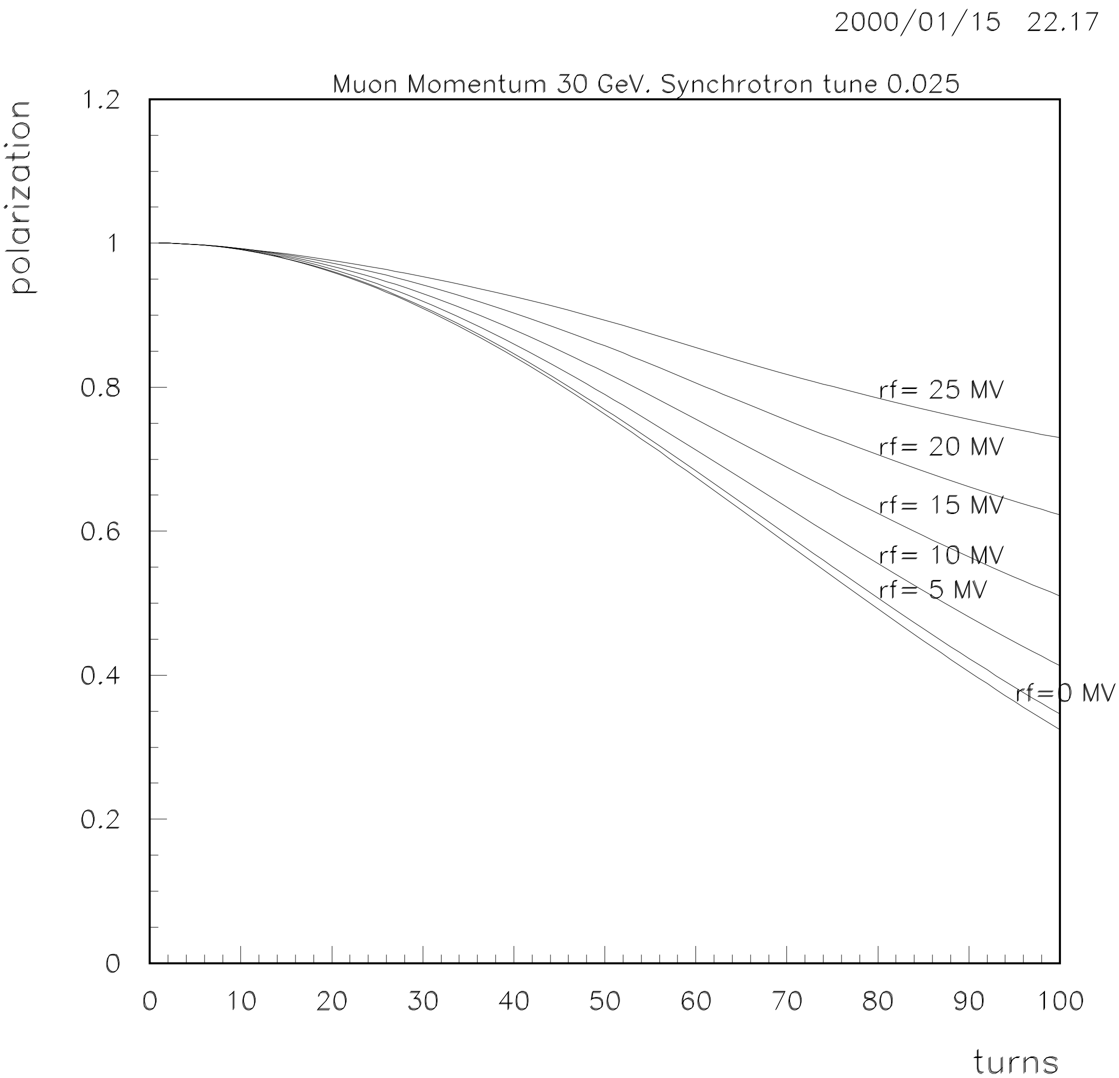

FIG. 12. Variation of polarization as a function of turn for various rf voltages ranging from 5MV to 25 MV. Beam Energy 30 GeV, synchrotron tune = 0.025.

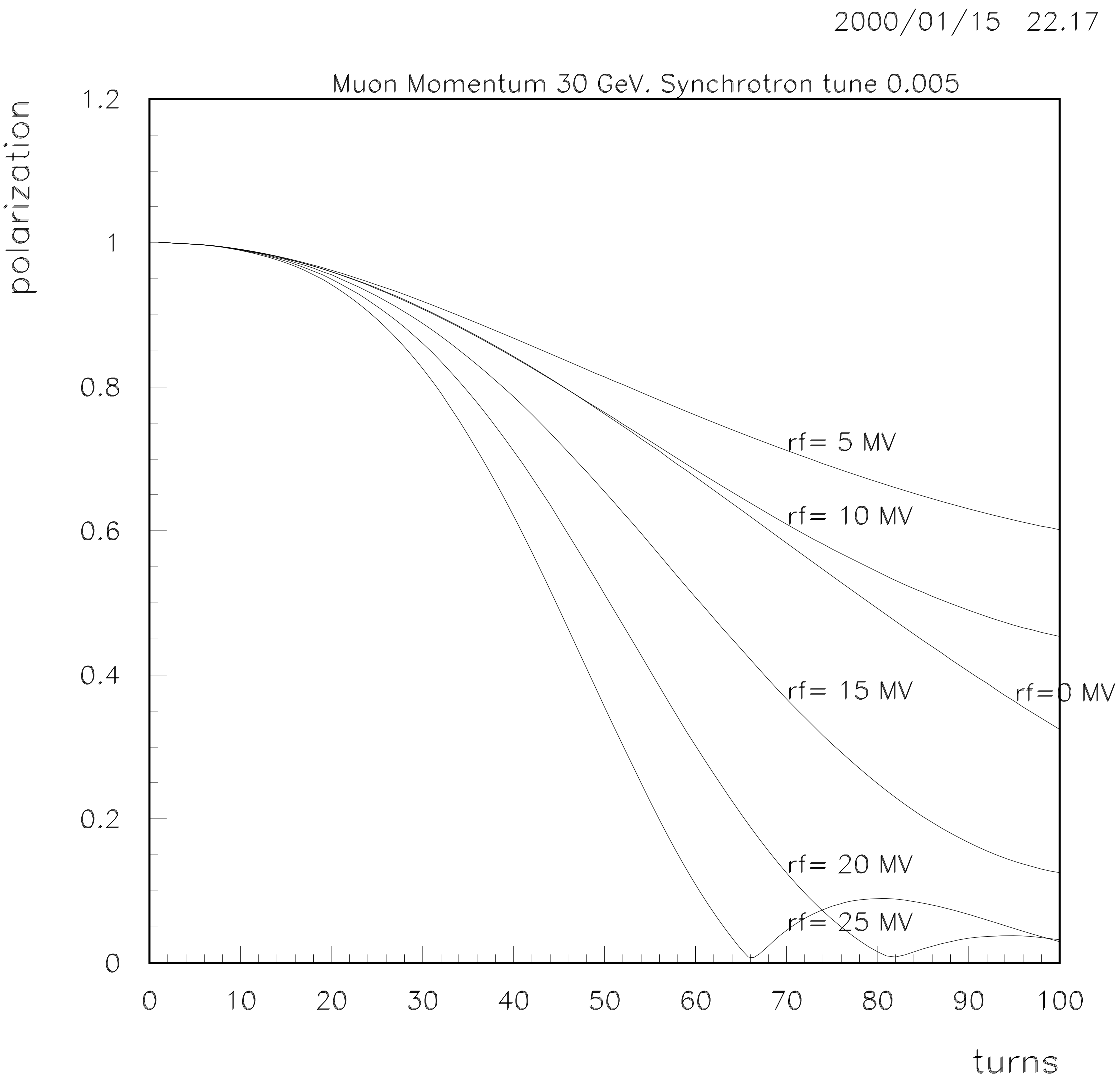

FIG. 13. Variation of polarization as a function of turn for various rf voltages ranging from 5MV to 25 MV. Beam Energy 30 GeV, synchrotron tune = 0.005.

REFERENCES

[1] "Calibrating the energy of a 50×50 GeV muon collider using spin precession", Rajendran Raja and Alvin Tollestrup, Phys.Rev.D 58(1998)013005.

[2] R.Rossmanith, Talk given at the 1997 Fermilab Workshop on Physics at the First Muon Collider and Front End of a muon Collider.
A. Blondel, Proceedings of the NUFACT99 Workshop, Lyon, France. (1990).

[3] An introduction to the physics of high energy accelerators, D.J.Edwards, M.J.Syphers,John Wiley and Sons, Page 34.

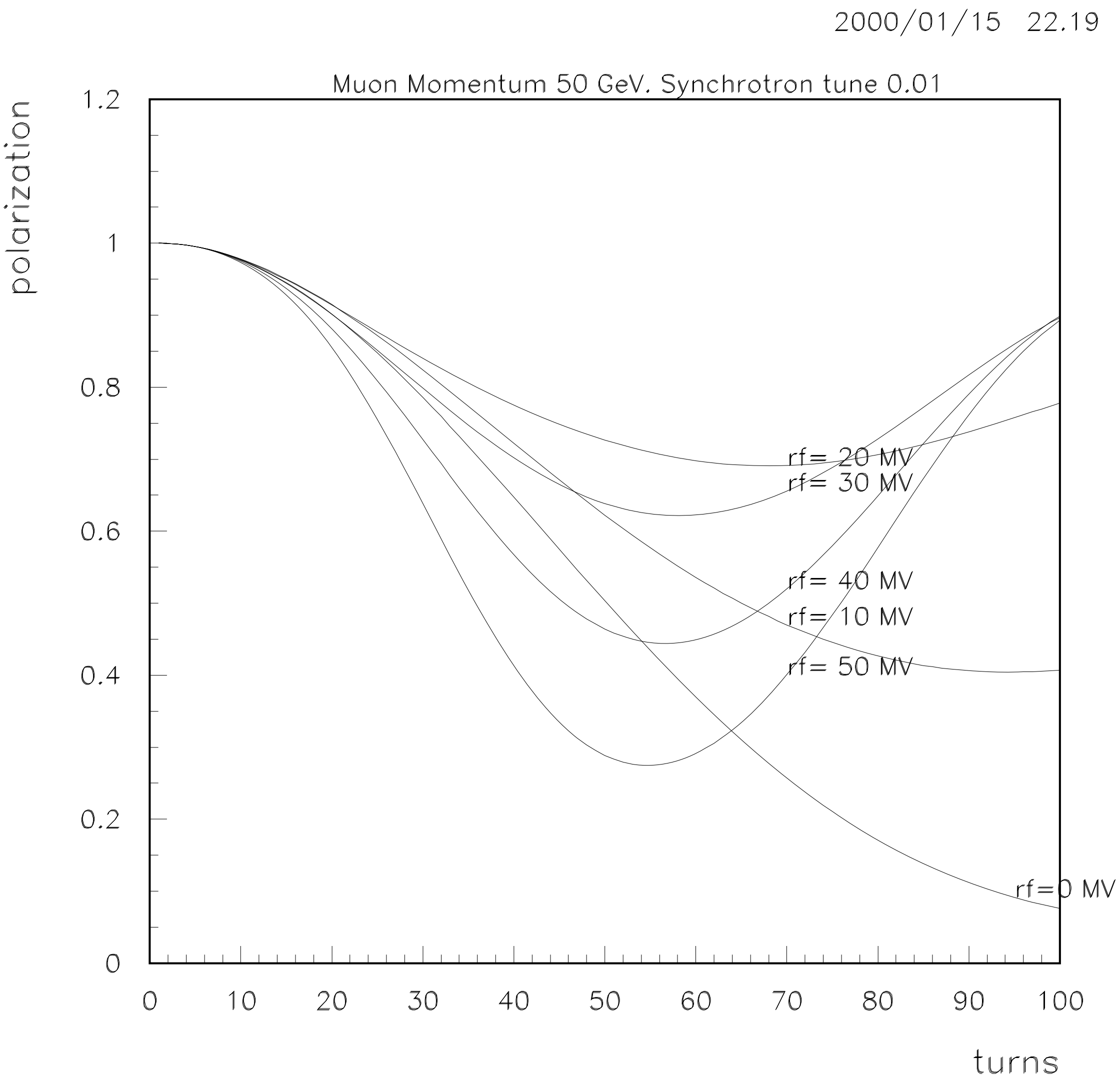

FIG. 14. Variation of polarization as a function of turn for various rf voltages ranging from 5MV to 25 MV. Beam Energy 50 GeV, synchrotron tune = 0.01.

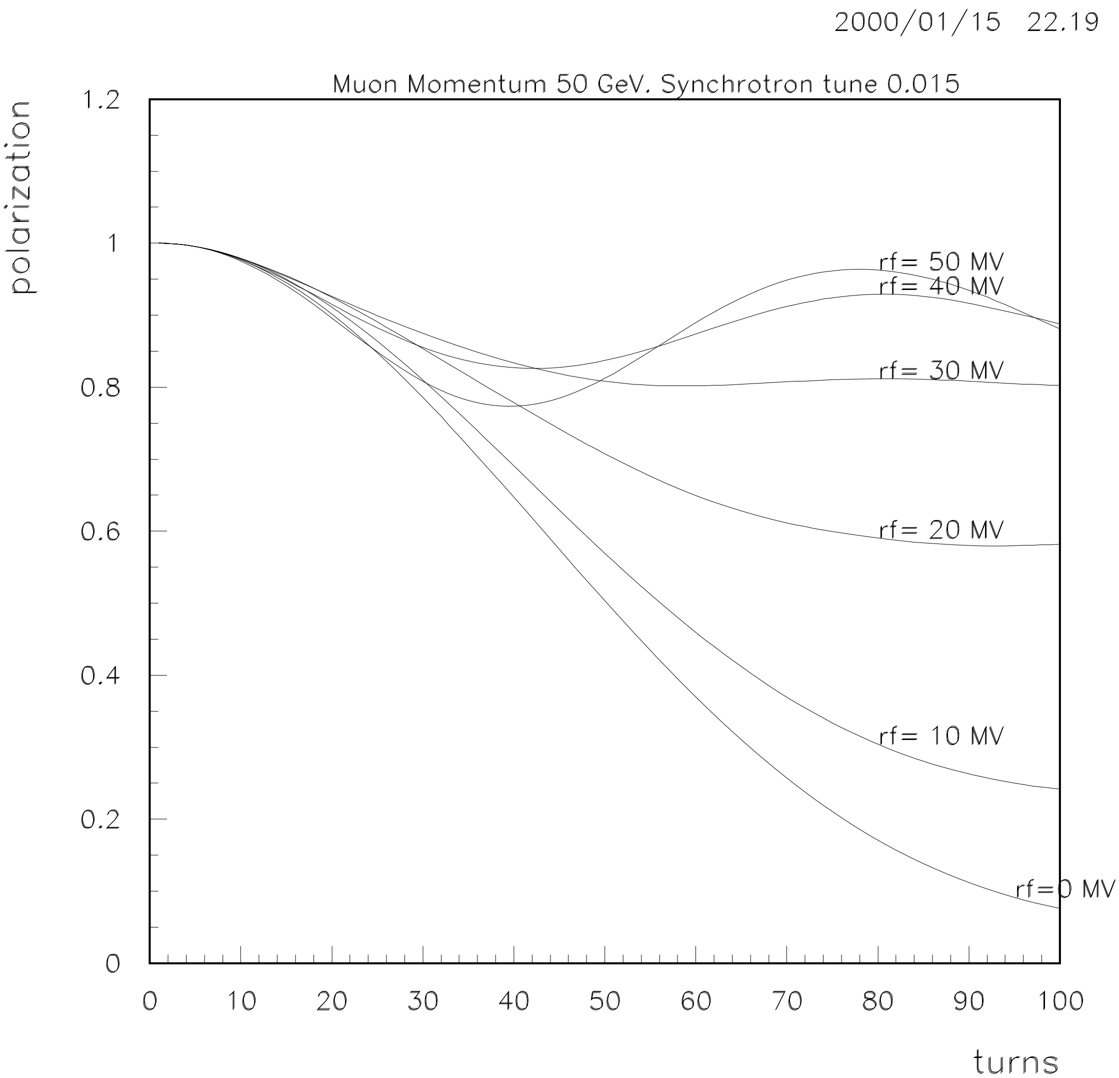

FIG. 15. Variation of polarization as a function of turn for various rf voltages ranging from 5MV to 25 MV. Beam Energy 50 GeV, synchrotron tune = 0.015.

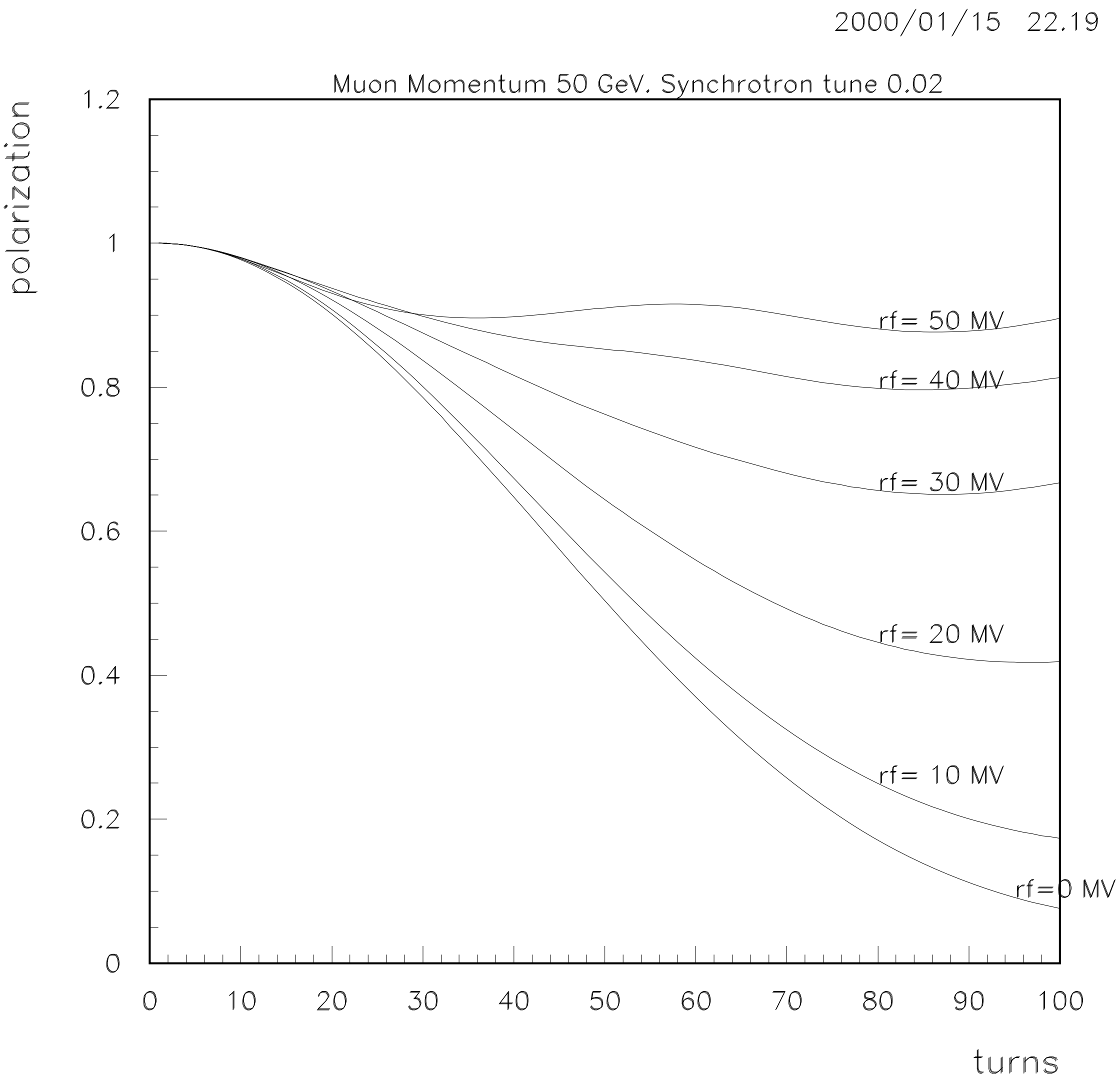

FIG. 16. Variation of polarization as a function of turn for various rf voltages ranging from 5MV to 25 MV. Beam Energy 50 GeV, synchrotron tune = 0.02.

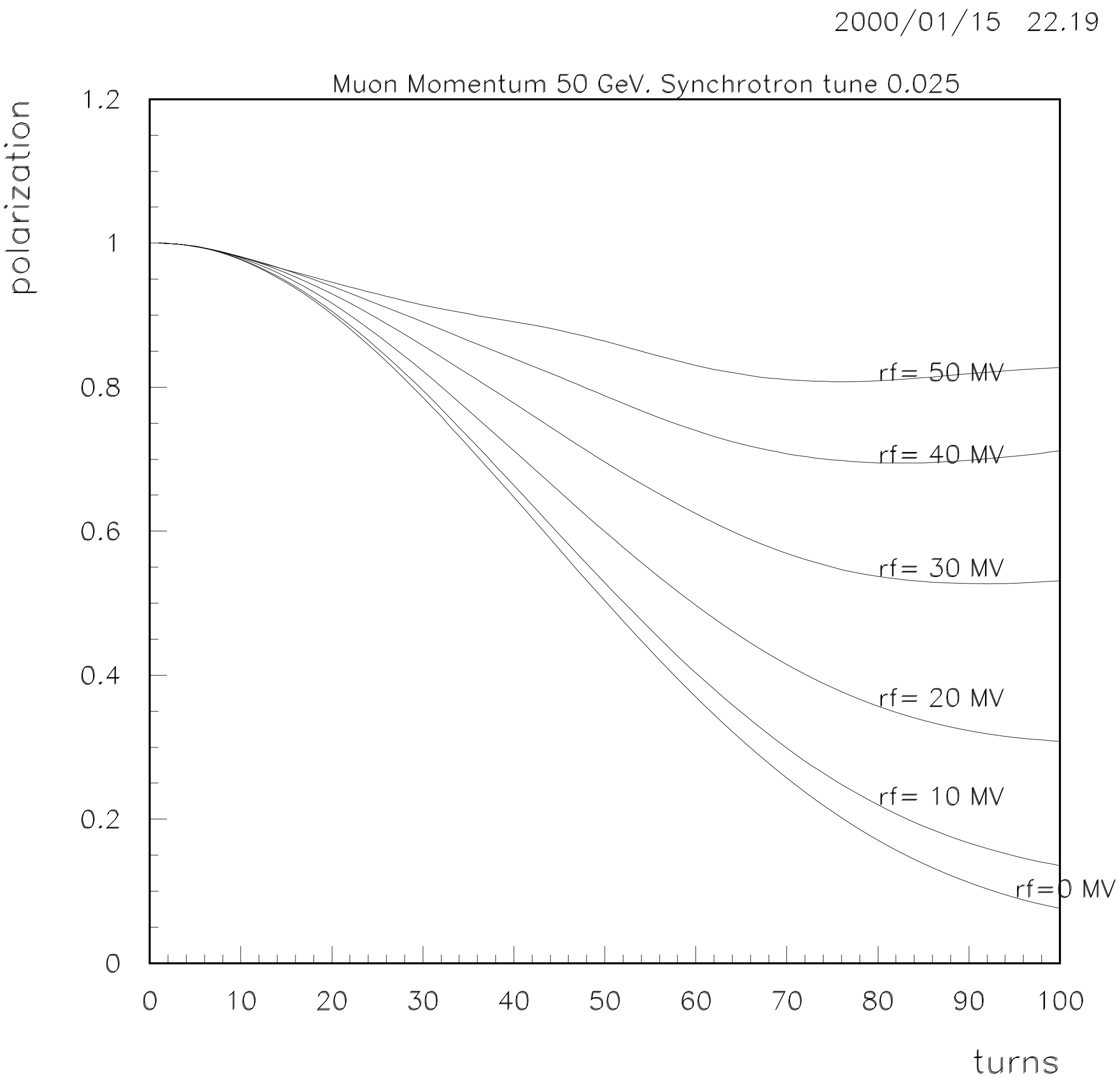

FIG. 17. Variation of polarization as a function of turn for various rf voltages ranging from 5MV to 25 MV. Beam Energy 50 GeV, synchrotron tune = 0.025.

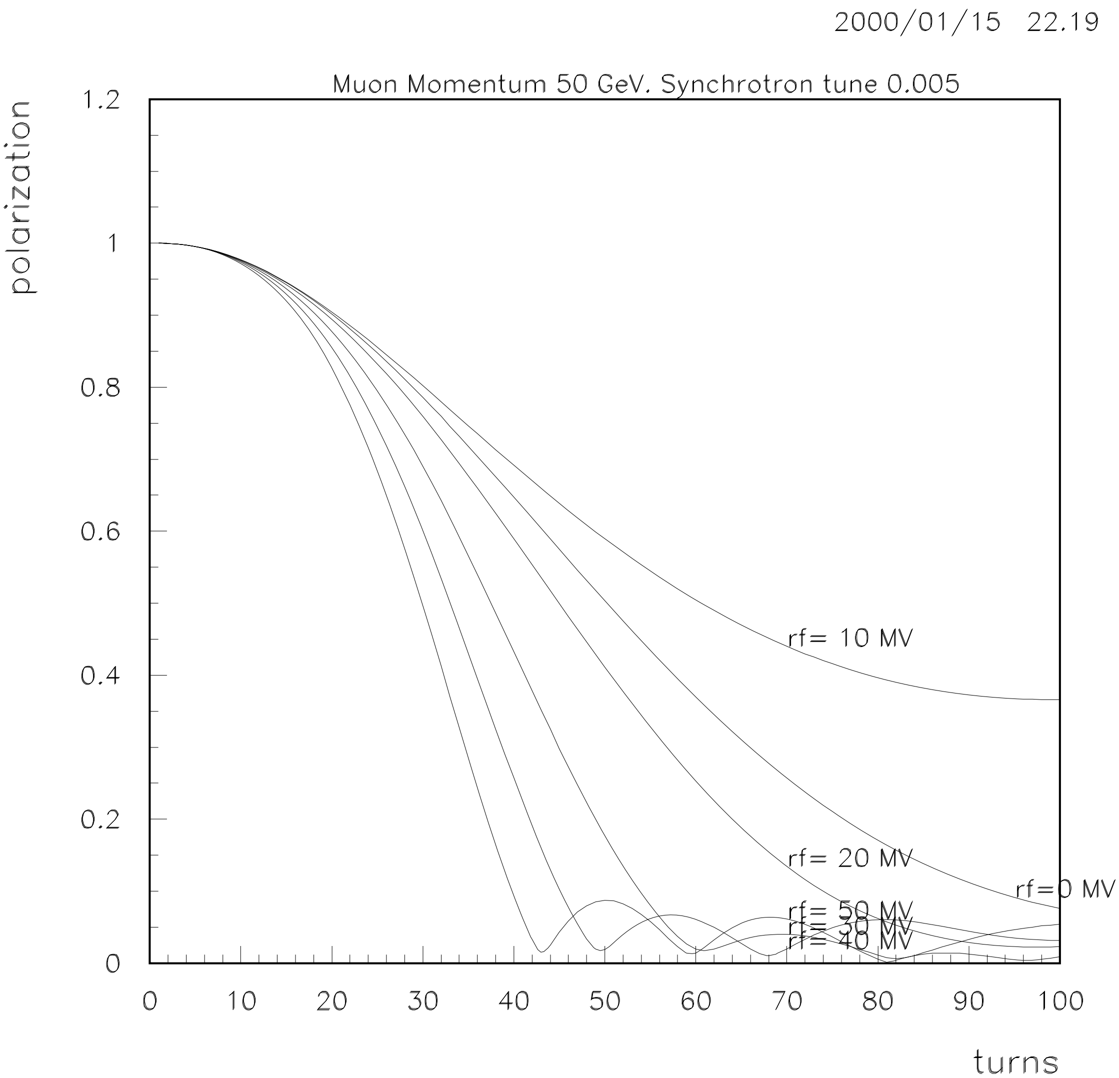

FIG. 18. Variation of polarization as a function of turn for various rf voltages ranging from 5MV to 25 MV. Beam Energy 50 GeV, synchrotron tune = 0.005.

Q_S	rf MV	rf freq. MHz	$\delta p_{3\sigma}/p$	P	P_W
0.005	5.0	50.8	0.011	0.811	0.851
0.005	10.0	25.4	0.021	0.749	0.805
0.005	15.0	16.9	0.032	0.609	0.700
0.005	20.0	12.7	0.042	0.497	0.611
0.005	25.0	10.2	0.053	0.450	0.564
0.010	5.0	203.4	0.005	0.773	0.825
0.010	10.0	101.7	0.011	0.860	0.888
0.010	15.0	67.8	0.016	0.904	0.917
0.010	20.0	50.8	0.021	0.885	0.899
0.010	25.0	40.7	0.027	0.849	0.867
0.015	5.0	457.6	0.004	0.749	0.807
0.015	10.0	228.8	0.007	0.808	0.852
0.015	15.0	152.5	0.011	0.875	0.901
0.015	20.0	114.4	0.014	0.927	0.939
0.015	25.0	91.5	0.018	0.947	0.952
0.020	5.0	813.5	0.003	0.739	0.799
0.020	10.0	406.8	0.005	0.776	0.828
0.020	15.0	271.2	0.008	0.828	0.866
0.020	20.0	203.4	0.011	0.879	0.905
0.020	25.0	162.7	0.013	0.920	0.936
0.025	5.0	1271.1	0.002	0.734	0.795
0.025	10.0	635.6	0.004	0.759	0.815
0.025	15.0	423.7	0.006	0.796	0.843
0.025	20.0	317.8	0.008	0.839	0.875
0.025	25.0	254.2	0.011	0.881	0.907

TABLE II. Summary of results for 30 GeV muon storage ring. P is the average poalrization for 100 turns and P_W is the intensity weighted polarization for 100 turns.

Q_S	rf MV	rf freq. MHz	$\delta p_{3\sigma}/p$	P	P_W
0.005	10.0	42.2	0.013	0.633	0.674
0.005	20.0	21.1	0.025	0.457	0.523
0.005	30.0	14.1	0.038	0.374	0.441
0.005	40.0	10.6	0.051	0.337	0.401
0.005	50.0	8.4	0.064	0.303	0.366
0.010	10.0	169.0	0.006	0.657	0.696
0.010	20.0	84.5	0.013	0.794	0.811
0.010	30.0	56.3	0.019	0.778	0.790
0.010	40.0	42.2	0.025	0.696	0.712
0.010	50.0	33.8	0.032	0.613	0.632
0.015	10.0	380.2	0.004	0.594	0.643
0.015	20.0	190.1	0.008	0.745	0.772
0.015	30.0	126.7	0.013	0.854	0.865
0.015	40.0	95.1	0.017	0.898	0.901
0.015	50.0	76.0	0.021	0.894	0.894
0.020	10.0	676.0	0.003	0.565	0.618
0.020	20.0	338.0	0.006	0.672	0.710
0.020	30.0	225.3	0.010	0.789	0.811
0.020	40.0	169.0	0.013	0.868	0.880
0.020	50.0	135.2	0.016	0.913	0.918
0.025	10.0	1056.2	0.003	0.549	0.604
0.025	20.0	528.1	0.005	0.624	0.669
0.025	30.0	352.1	0.008	0.724	0.755
0.025	40.0	264.1	0.010	0.812	0.832
0.025	50.0	211.2	0.013	0.877	0.888

TABLE III. Summary of results for 50 GeV muon storage ring. P is the average poalrization for 100 turns and P_W is the intensity weighted polarization for 100 turns.