DESIGN STUDY OF RACE- TRACK FFAG RING FOR NUSTORM

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MUON DECAY RING FOR NUSTORM

- nuSTORM
  - Physics : sterile neutrinos, precise νe/νe-bar cross-section.
  - Technology : demo. of neutrino factory, detector studies.
- Muon decay ring
  - Store low energy (~3.8GeV/c) muons in storage ring with relatively long straight section: straight section ~60-70% of total ring circumference.
  - Small beam divergence of neutrino beam : x', y', deflection $<<1/\gamma$

DESIGN PARAMETERS SUMMARY

- Beam energy
  - Optimization of ν detector efficiency — 3.8GeV/c
- Long straight section
  - Large ν beam intensity — (60-)70% of circumference
- Small ν beam divergence
  - Small size ν detector — x', y' <<1/γ ~ 25mrad
- Large acceptance
  - Large μ beam intensity — Δp/p>+-16%: amap
    εx, y(normalized)>10^{-2}m.rad

MUON DECAY RING WITH ZERO-CHROMATIC FFAG

- Zero-chromatic FFAG(fixed-field alternating gradient) has some advantages in particle storage ring.
- Advantages
  - Beam acceptance is large, beam momentum acceptance in particluar.

  > **Zero chromaticity**

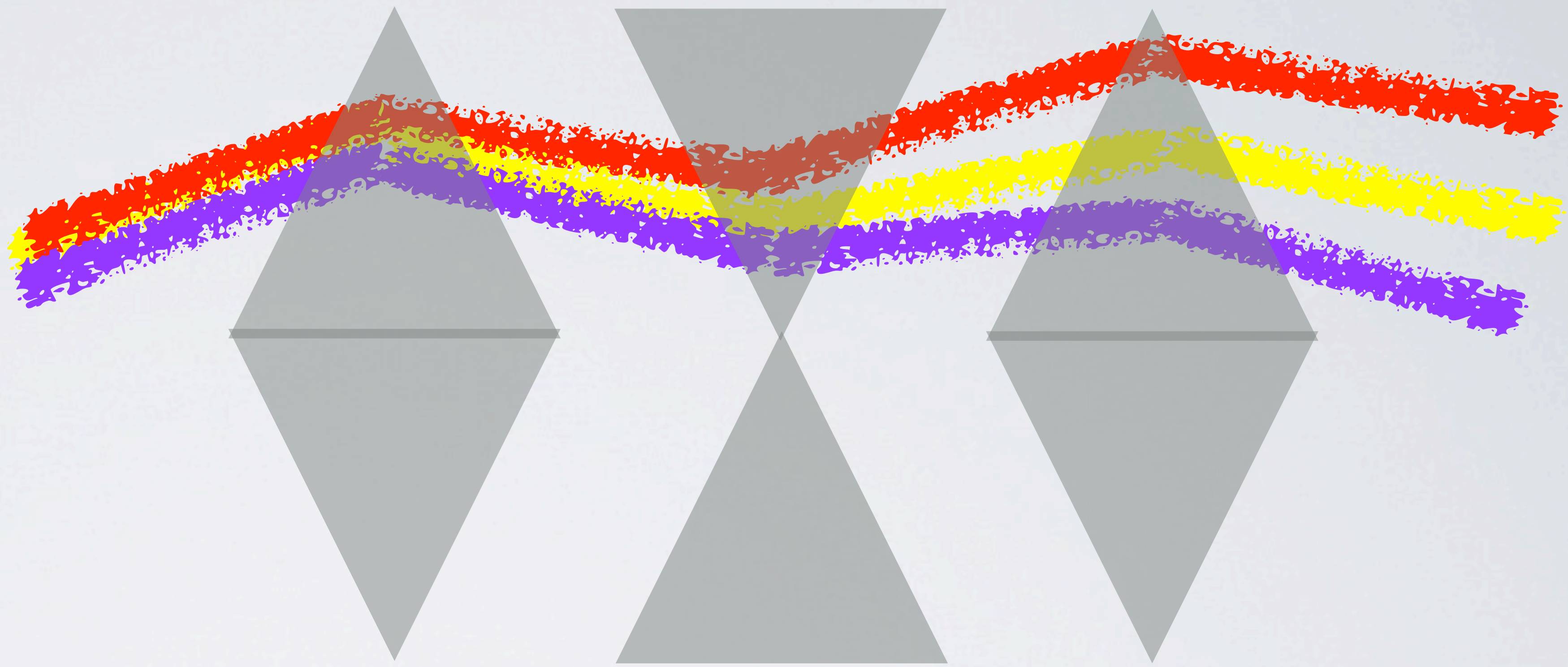
    - Constant betatron tunes for largely momentum spread beam: No Resonance Crossing for large momentum range (not for non-scaling FFAG)
    - Large acceptance in longitudinal & transverse directions : Ax,z >10mm.rad, $\Delta p/p$> 10%
- Issue : How to make a race-track ring?
  - Conventional zero-chromatic FFAG is circular symmetric and zero-chromaticity for straight line has not been found.
  - Need to conceive zero-chromatic FFAG optics for straight line.

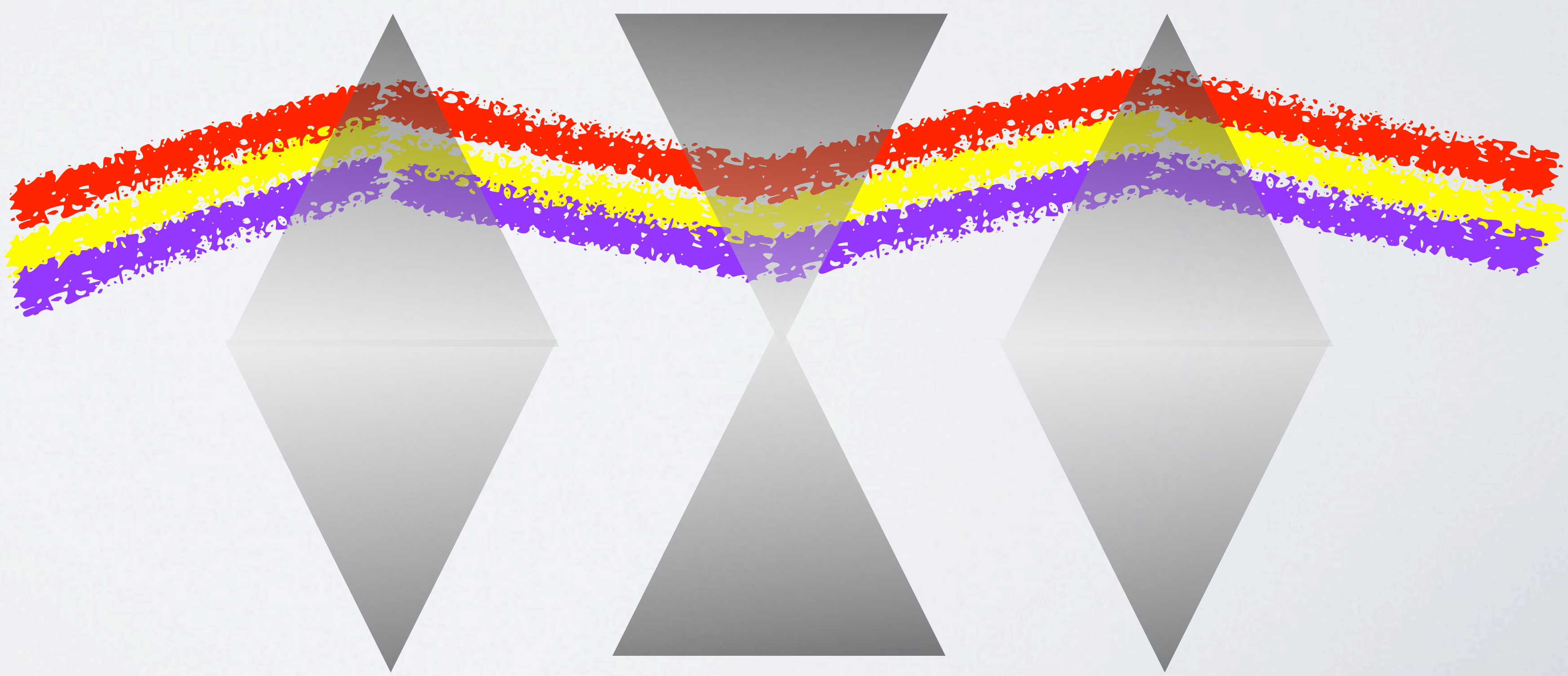
ZERO CHROMATICITY

- Chromatic effect of beam optics
  - Focusing force for different energy beam is changed.
  - Betatron tunes are varied..
- Zero chromaticity
  - Focusing force for different energy beam is not changed, always constant.
  - Betatron tunes are constant.

Optics conditions for zero-chromaticity

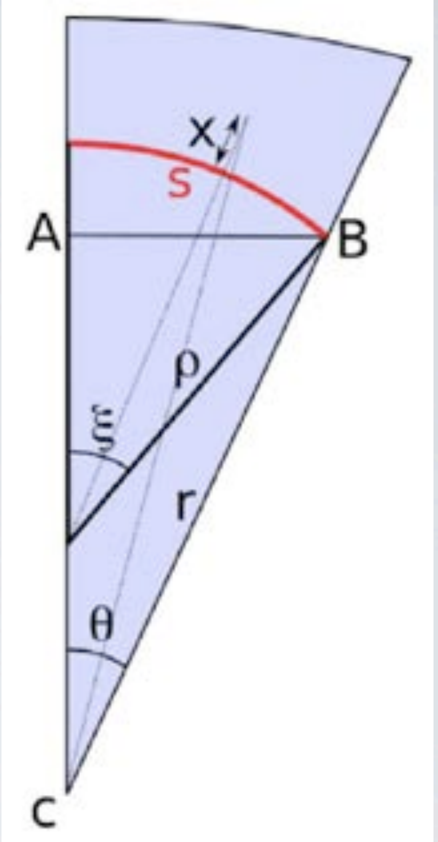
- **Orbit similarity**
- **Constant field index**





ZERO CHROMATICITY ARC SECTION

-CONVENTIONAL FFAG(1950s)-



- Betatron oscillation (cylindrical coordinate)

$$\frac{d^2x}{d\theta^2}+\frac{r^2}{\rho^2}\left(1-K\rho^2\right)x=0$$

$$\frac{d^2z}{d\theta^2}+\frac{r^2}{\rho^2}\left(K\rho^2\right)z=0$$

$$K=-\frac{1}{B\rho}\frac{\partial B}{\partial r}$$

- Zero chromaticity : constant betatron tunes during acceleration

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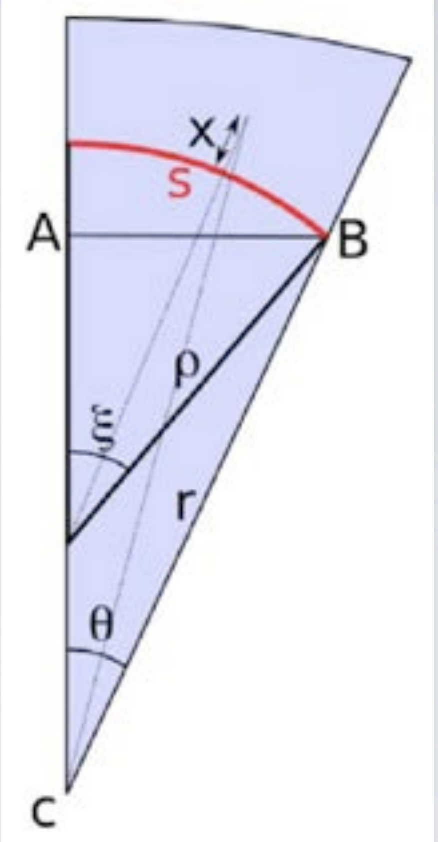

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- Zero chromaticity : constant betatron tunes during acceleration

$$\frac{d\left(r^2/\rho^2\right)}{dp} = 0$$

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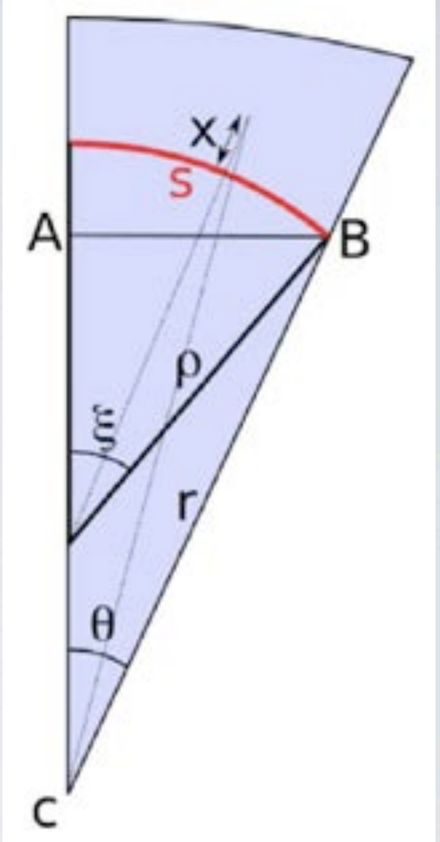


- Zero chromaticity : constant betatron tunes during acceleration

$$\frac{d\left(r^2/\rho^2\right)}{dp} = 0$$

$$\frac{d\left(K\rho^2\right)}{dp} = 0$$

$$\longrightarrow \begin{cases} r \propto \rho \\ \dfrac{r}{B}\left[\dfrac{\partial B_z}{\partial r}\right]_{z=0} = k \end{cases}$$

$$\boxed{B_z = B_0\left(\frac{r}{r_0}\right)^k f(\theta)}$$

AG FOCUSING LATTICE OF SCALING FFAG RING

$$B_z = B_0\left(\frac{r}{r_0}\right)^k f(\theta)$$

- AG focusing : FODO lattice
- Radial sector
  - F: positive bend
  - D:negative bend

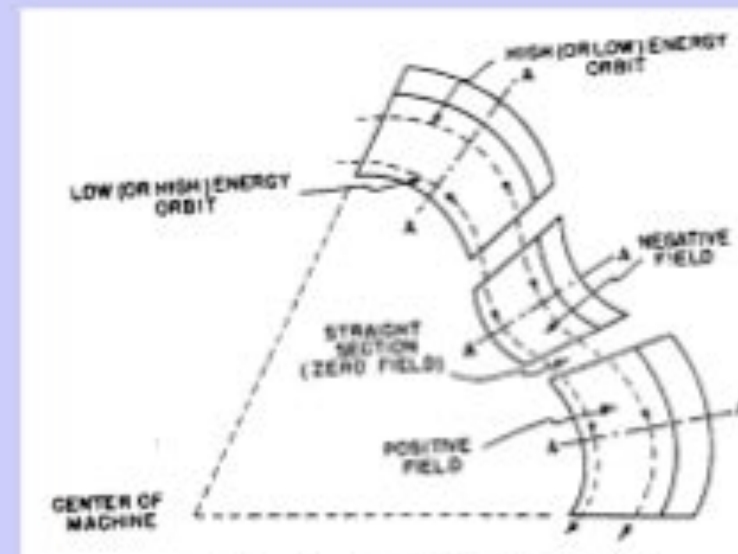


FIG. 2. Plan view of radial-sector magnets.

- Spiral sector
  - F: positive bending
  - D: edge focusing

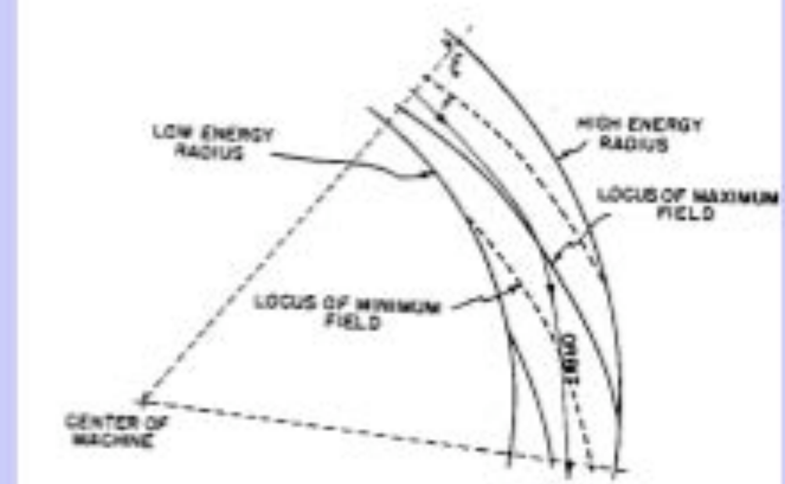
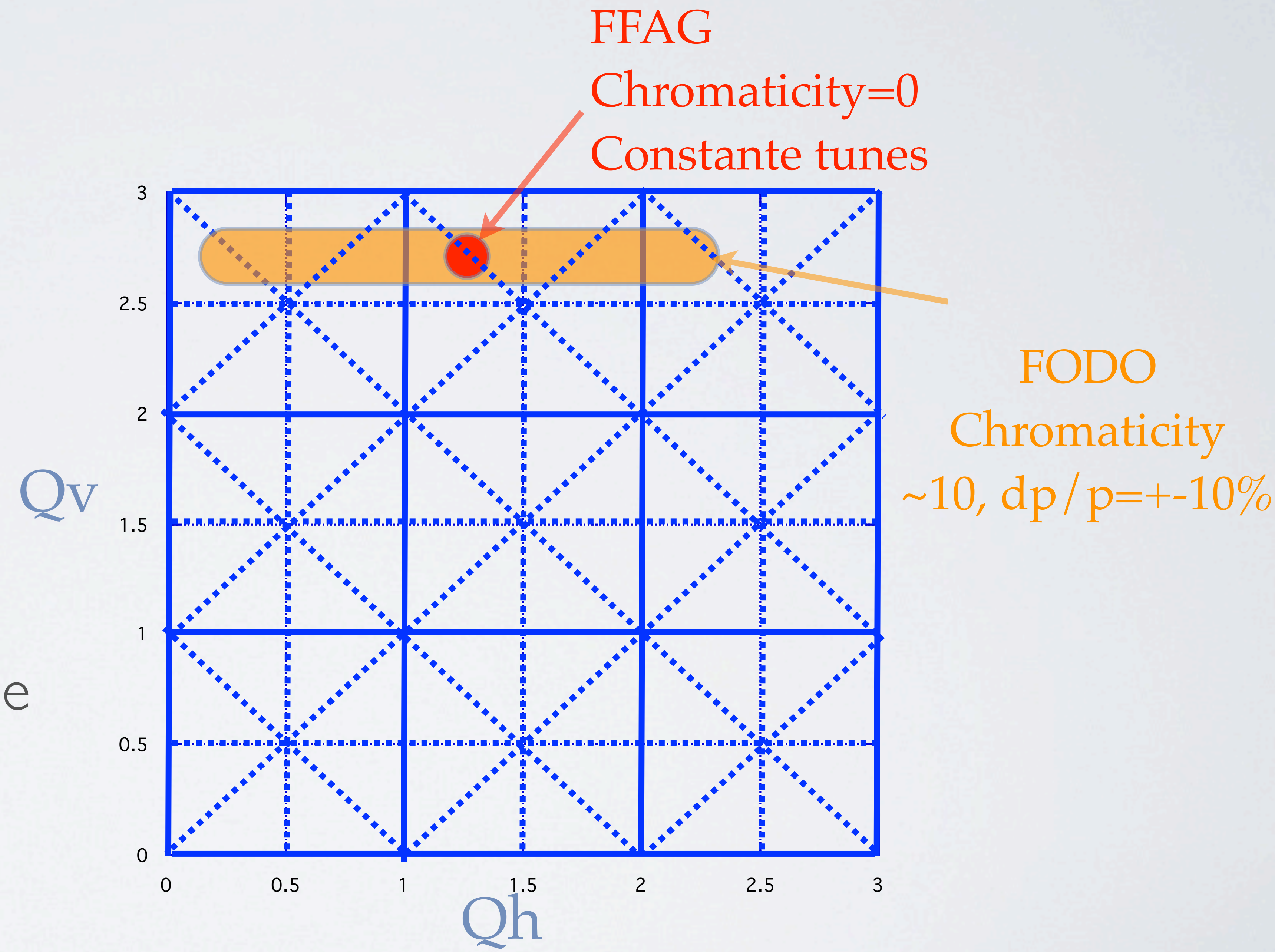


FIG. 3. Spiral-sector configuration.

FFAG CIRCULAR LATTICE COMPARED WITH FODO LATTICE

- Tune diagram
  - FFAG lattice
    - constat tunes : no-resonance crossing
  - FODO lattice
    - Non-zero natural chromaticiy: resonance crossing,



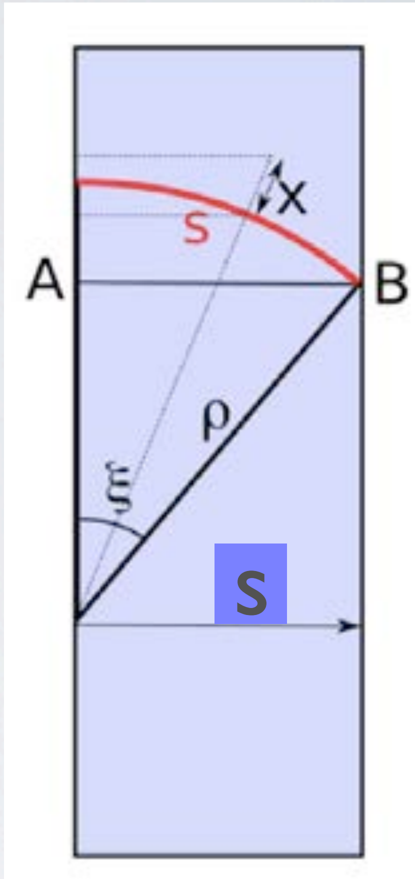
ZERO CHROMATICITY STRAIGHT LINE:NEW FINDINGS

-ADVANCEMENTS OF FFAG (2010s)-

- Betatron oscillation (linear coordinate)

$$\frac{d^2x}{ds^2}+\frac{1}{\rho^2}\left(1-K\rho^2\right)x=0$$

$$\frac{d^2z}{ds^2}+\frac{1}{\rho^2}\left(K\rho^2\right)z=0 \qquad K=-\frac{1}{B\rho}\frac{\partial B}{\partial x}$$



- Zero chromaticity : constant betatron tunes for various beam momentum

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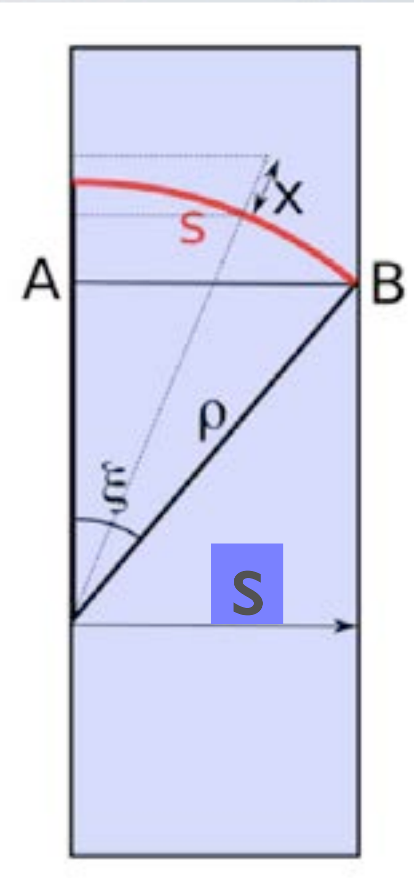

- Zero chromaticity : constant betatron tunes for various beam momentum

$$\frac{d\left(1/\rho^2\right)}{dp}=0$$

$$\frac{d\left(K\rho^2\right)}{dp}=0$$

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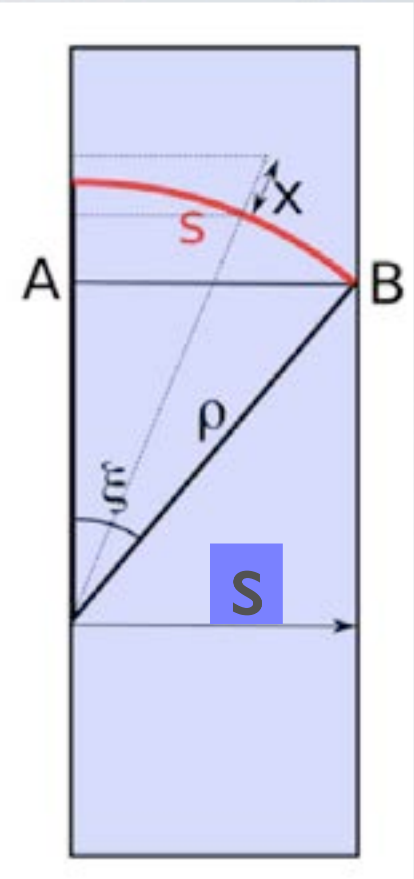


- Zero chromaticity : constant betatron tunes for various beam momentum

$$\frac{d\left(1/\rho^2\right)}{dp} = 0$$

$$\frac{d\left(K\rho^2\right)}{dp} = 0$$

$$\longrightarrow \begin{cases} \rho = const. \\ \frac{1}{B}\left[\frac{\partial B_z}{\partial x}\right]_{z=0} = \frac{n}{\rho} \end{cases}$$

$$B_z = B_0 \exp\left[\frac{n}{\rho}x\right]$$

OPTICS

- Scaling(zero-chromatic) FFAG straight line (JB. Lagrange, Y.Mori)
  - Example
    - Proton beam
    - Energy range : E=80~200MeV

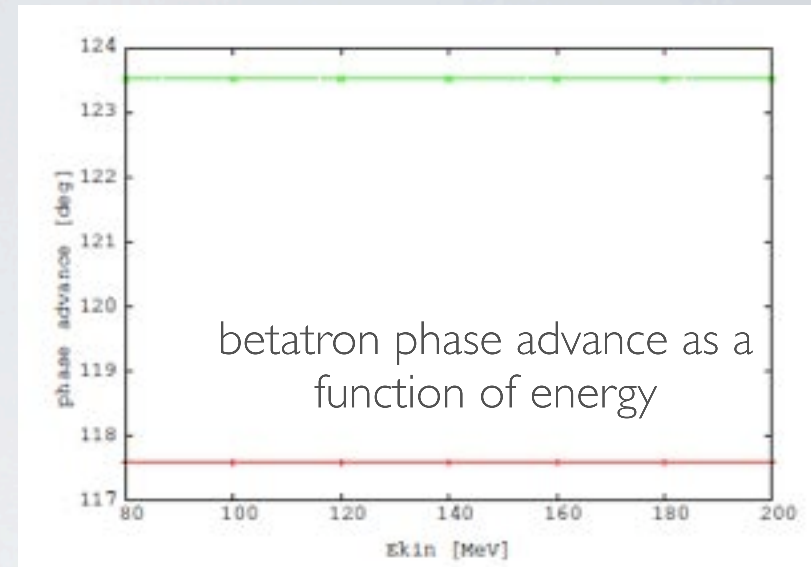
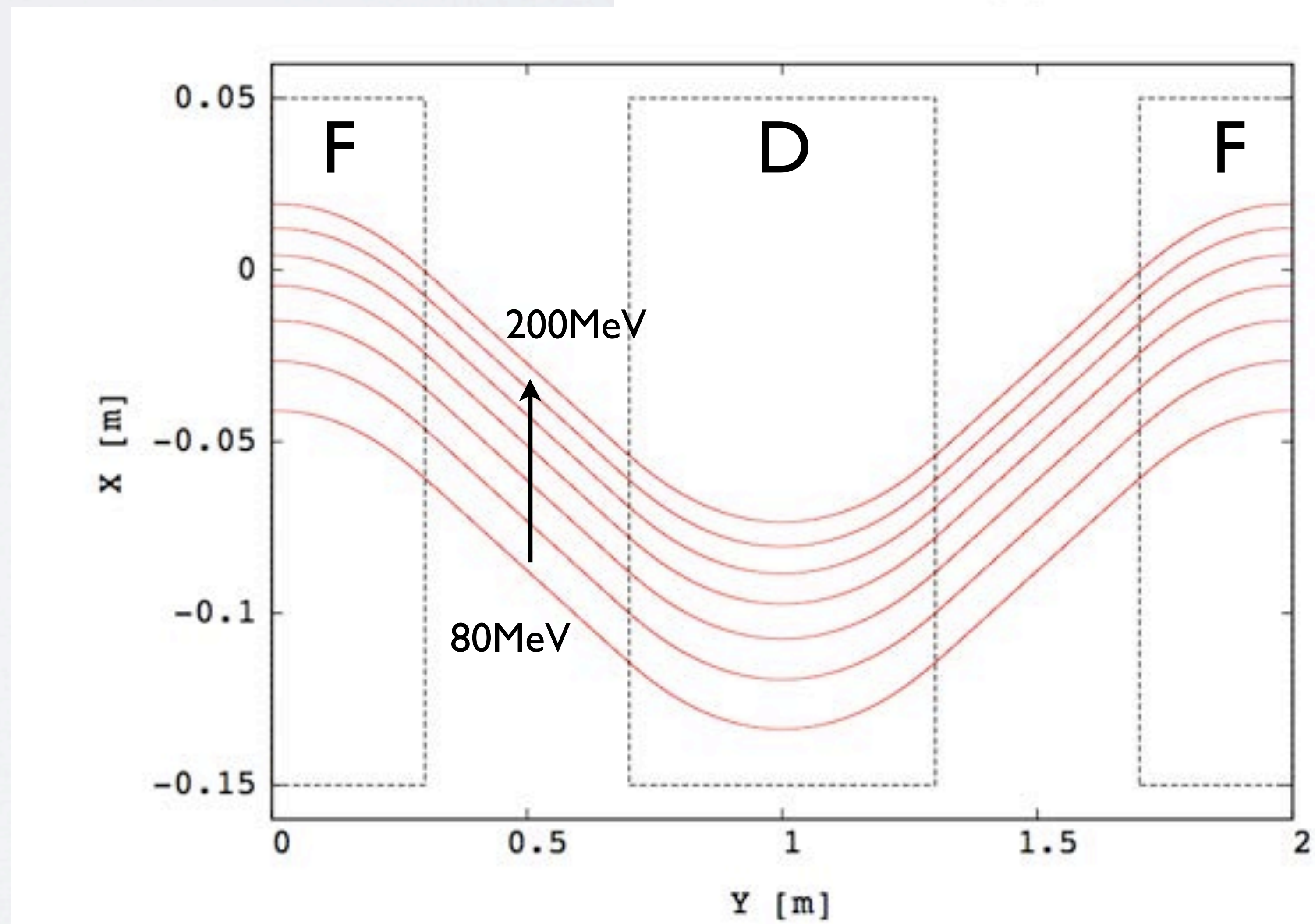


Table 1: Tracking parameters

Length of the magnets	60 cm
Drift	40 cm
Kinetic energy range	80 to 200 MeV
Field index	17
Local curvature radius	2.1 m
Step size	1 mm
Phase advances:	
horizontal μ_x	104.8 deg.
vertical μ_z	112.5 deg.



EXPERIMENT OF SCALING FFAG STRAIGHT LINE

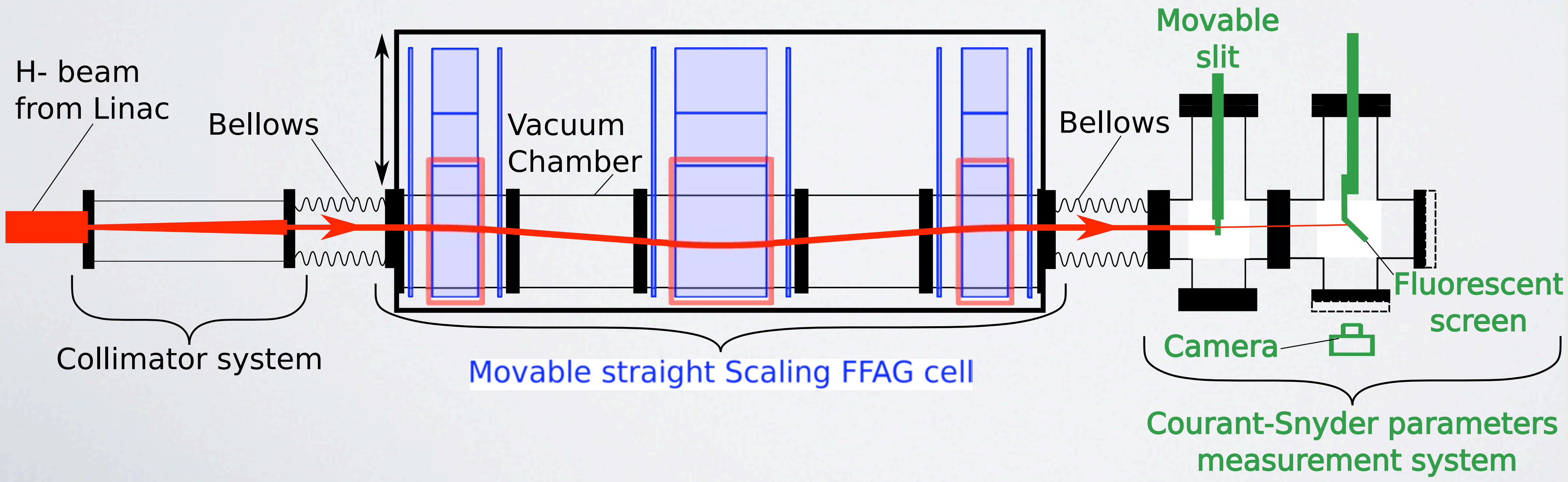
- Clarify the FFAG straight line experimentally with π-section
  - Dispersion suppressor
  - Insertion matching

$$B_z = B_0 \exp\left[\frac{n}{\rho} x\right]$$

Lagrange, Mori
IPAC'12

- Momentum range
  - 0.0811 - 0.1441 GeV/c
  - H- ion beam

	$\bar{x_1}$ (mm)	$\bar{x'_1}$ (mrad)	$\bar{\beta_1}$ (m)	$\bar{\alpha_1}$	$\psi_{exp.}$ (deg)	$\psi_{track.}$ (deg)
11 MeV	**2.0**	**-2.4**	**17.7**	**-1.5**	**87.5** ± 3.3	**87.5**
7 MeV	**1.8**	**-2.1**	**11.7**	**-1.0**	**86.1** ± 9.6	**87.6**



RACE-TRACK SCALING FFAG

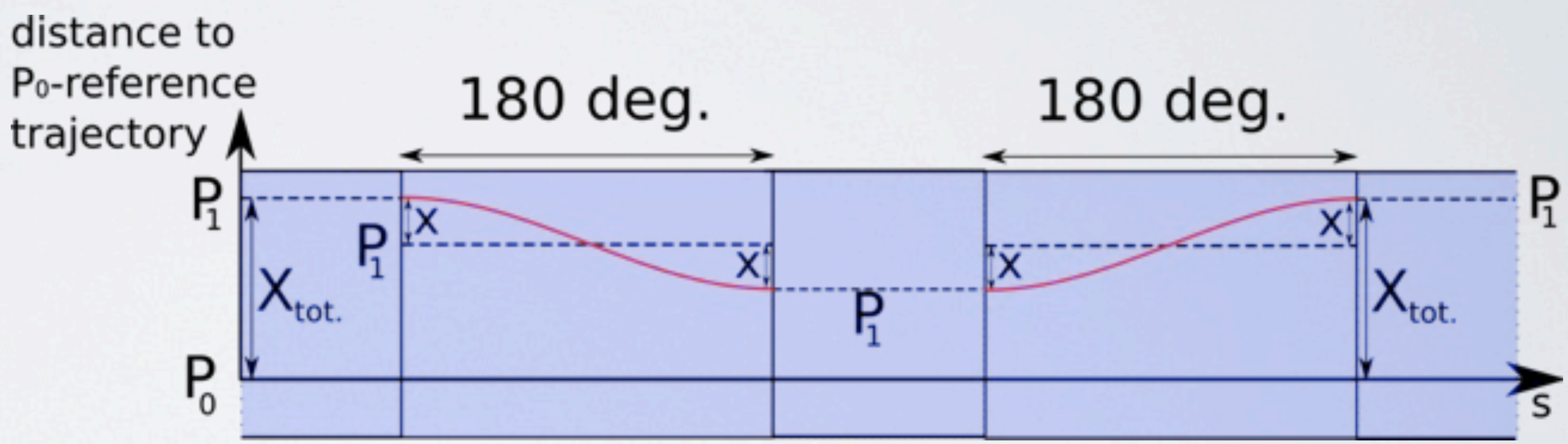
- Optics issues:
  - Suppressing dispersion in straight line:
  - Matching between arc section and straight line.
    - Twiss parameters
    - Dispersion function

- How?

DISPERSION SUPPRESSOR/ INSERTION MATCHING

- Dispersion suppressor
  - Successive π-cells in the horizontal plane can suppress the dispersion.
  - Help to reduce the size of apertures of the magnet and rf cavity.

$$x = \ln\left(\frac{P_1}{P_0}\right)\left(\frac{\rho_0}{n_0} - \frac{\rho_1}{n_1}\right)$$



- Insertion matching
  - Matching condition for closed orbit between ring and straight line

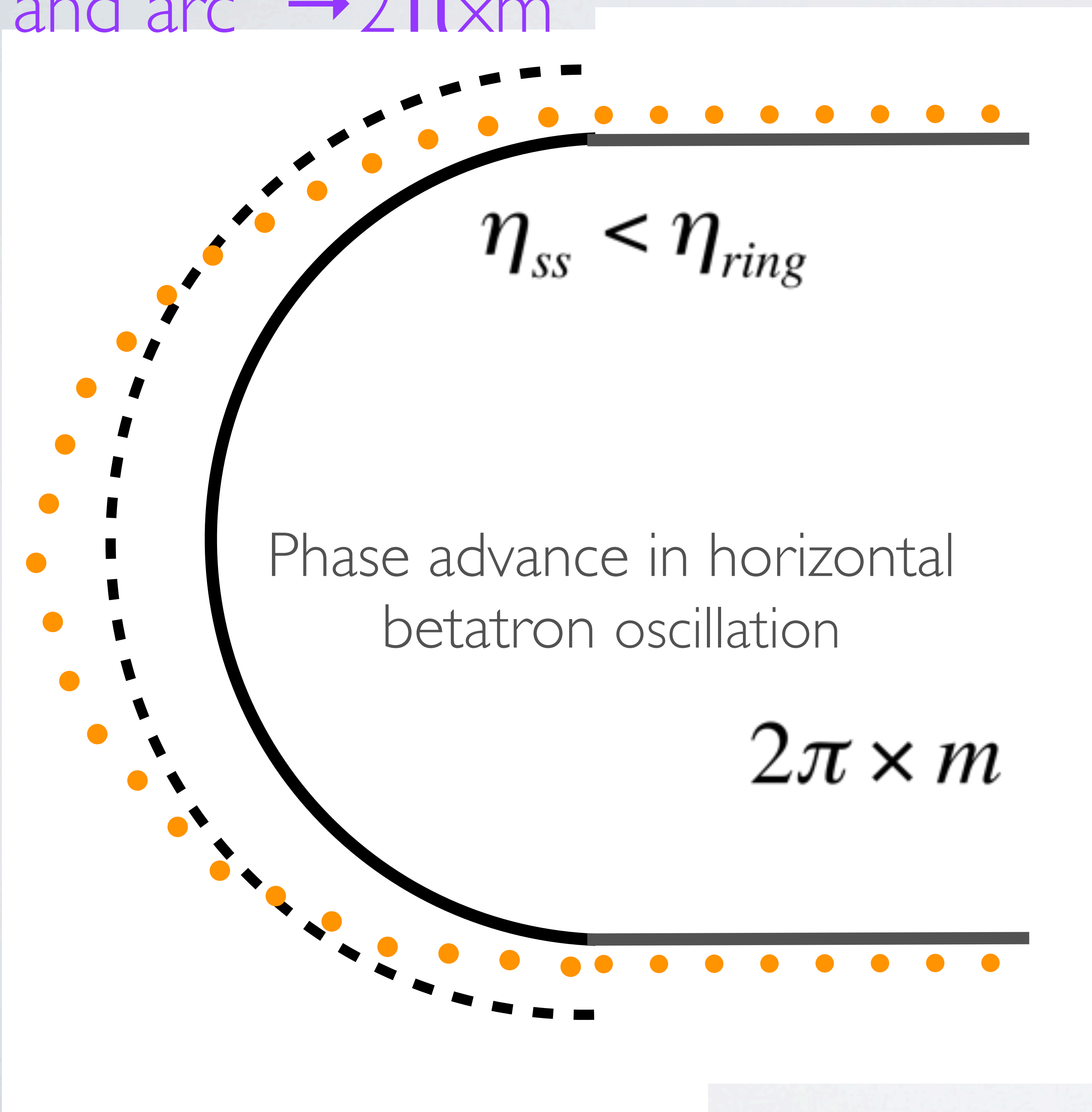
ring linear line

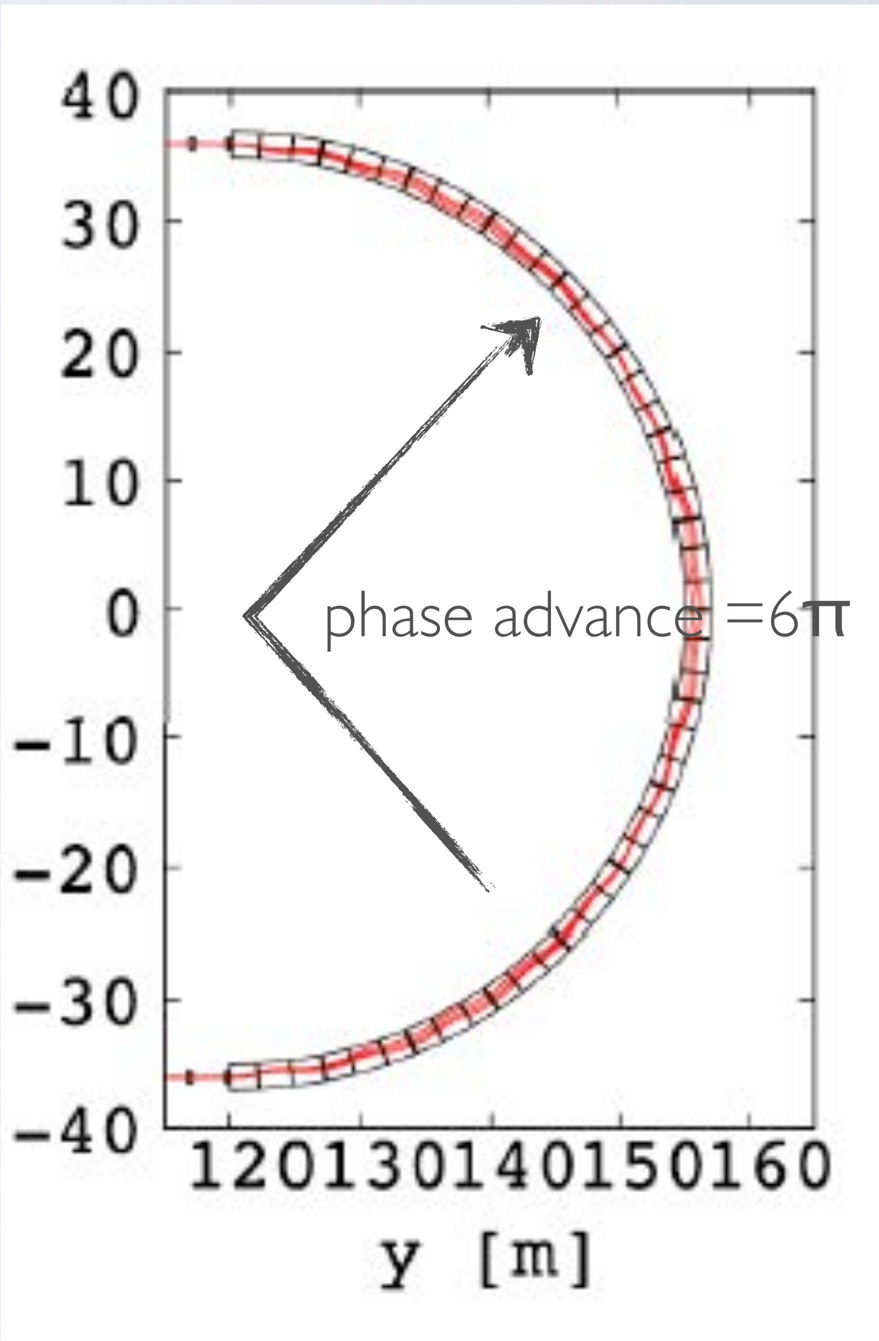
$$\left(1 + \frac{x}{r_m}\right)^{k+1} = \exp\left(\frac{n}{\rho}x\right) \Rightarrow \boxed{\frac{k+1}{r_m} = \frac{n}{\rho}}$$

← 1st order

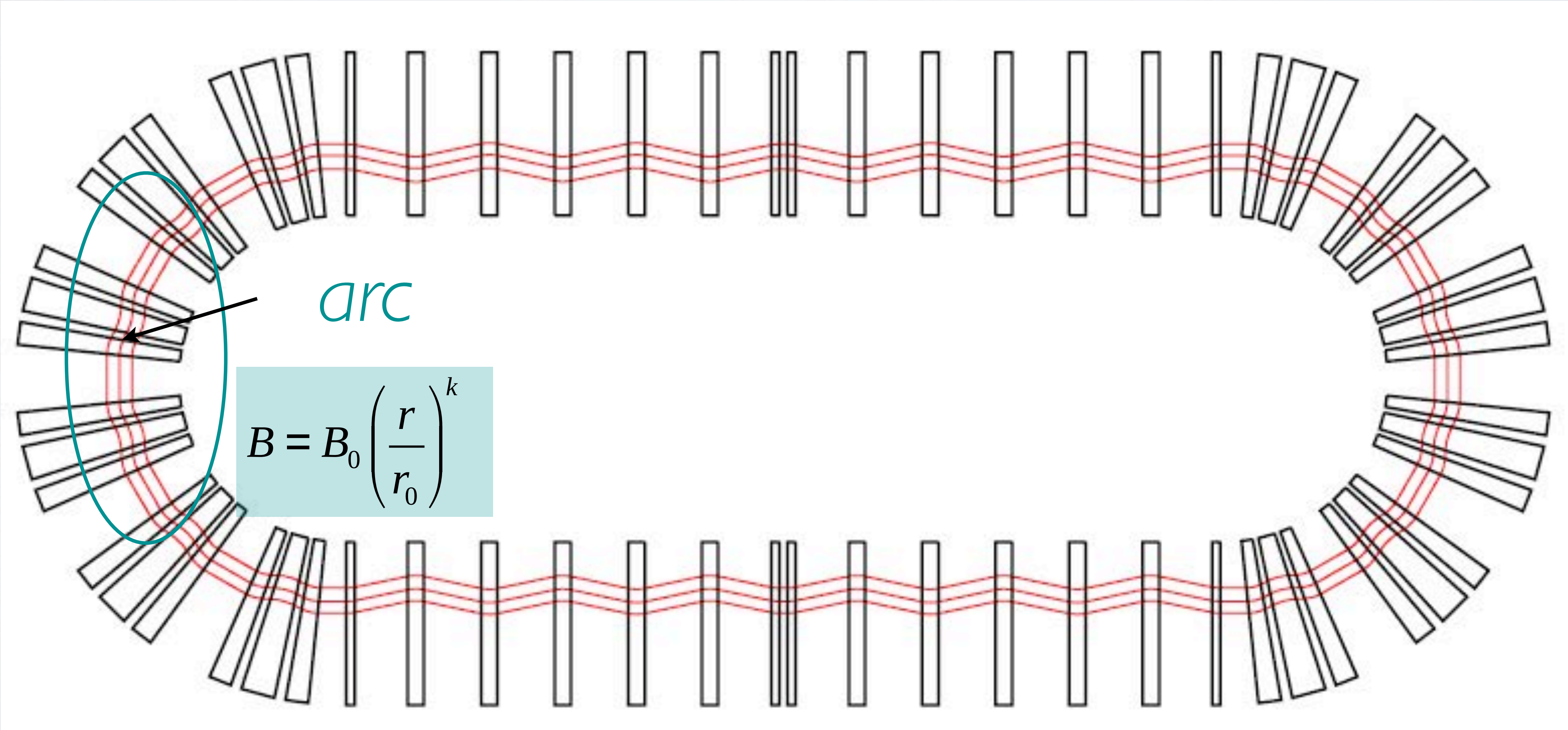
DISPERSION MATCHING BETWEEN ARC AND STRAIGHT

- Phase advance in horizontal betatron oscillation/btw. straight and arc →2π×m





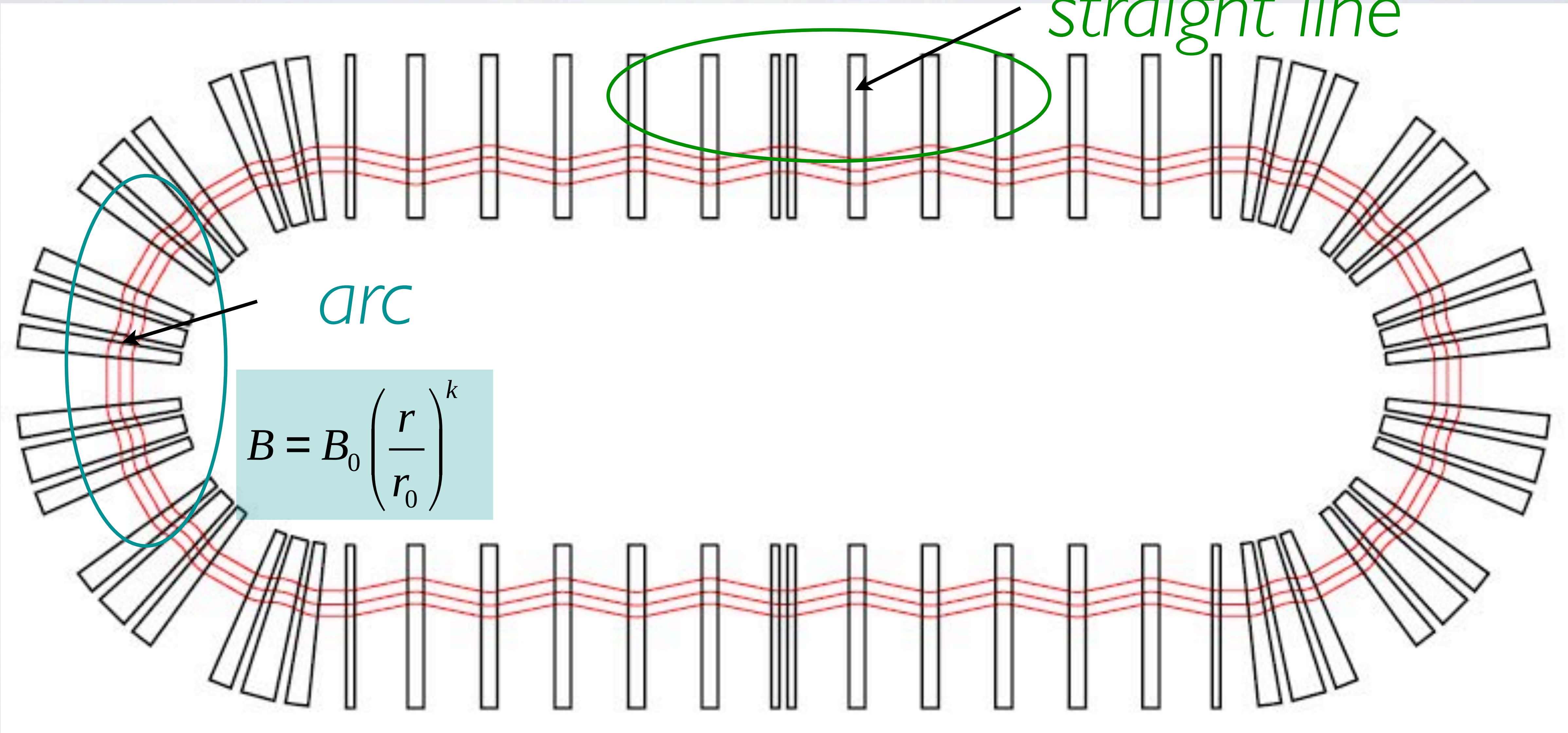
RACE TRACK FFAG RING



$$B = B_0 \left(\frac{r}{r_0} \right)^k$$

RACE TRACK FFAG RING

$$B_z = B_0 \exp\left[\frac{n}{\rho} x\right]$$



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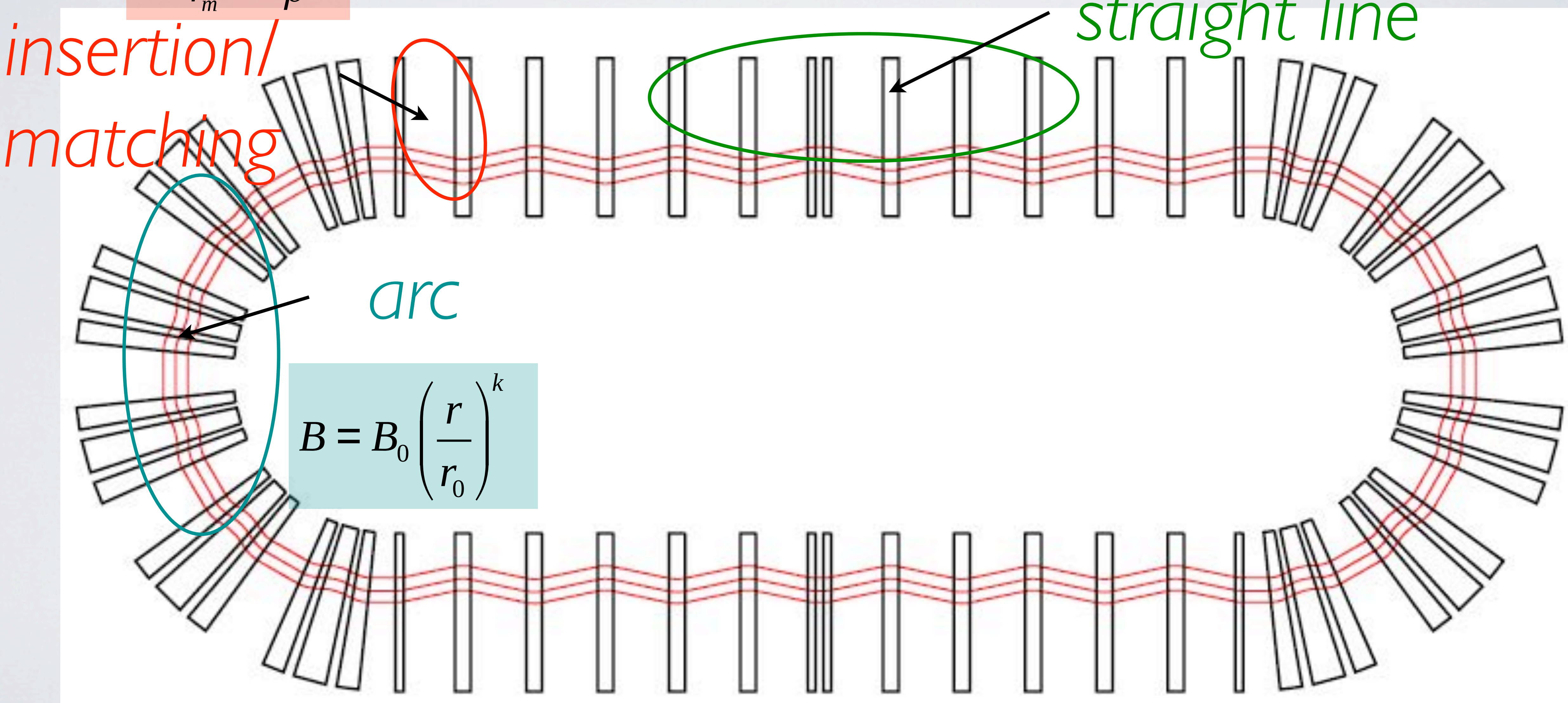

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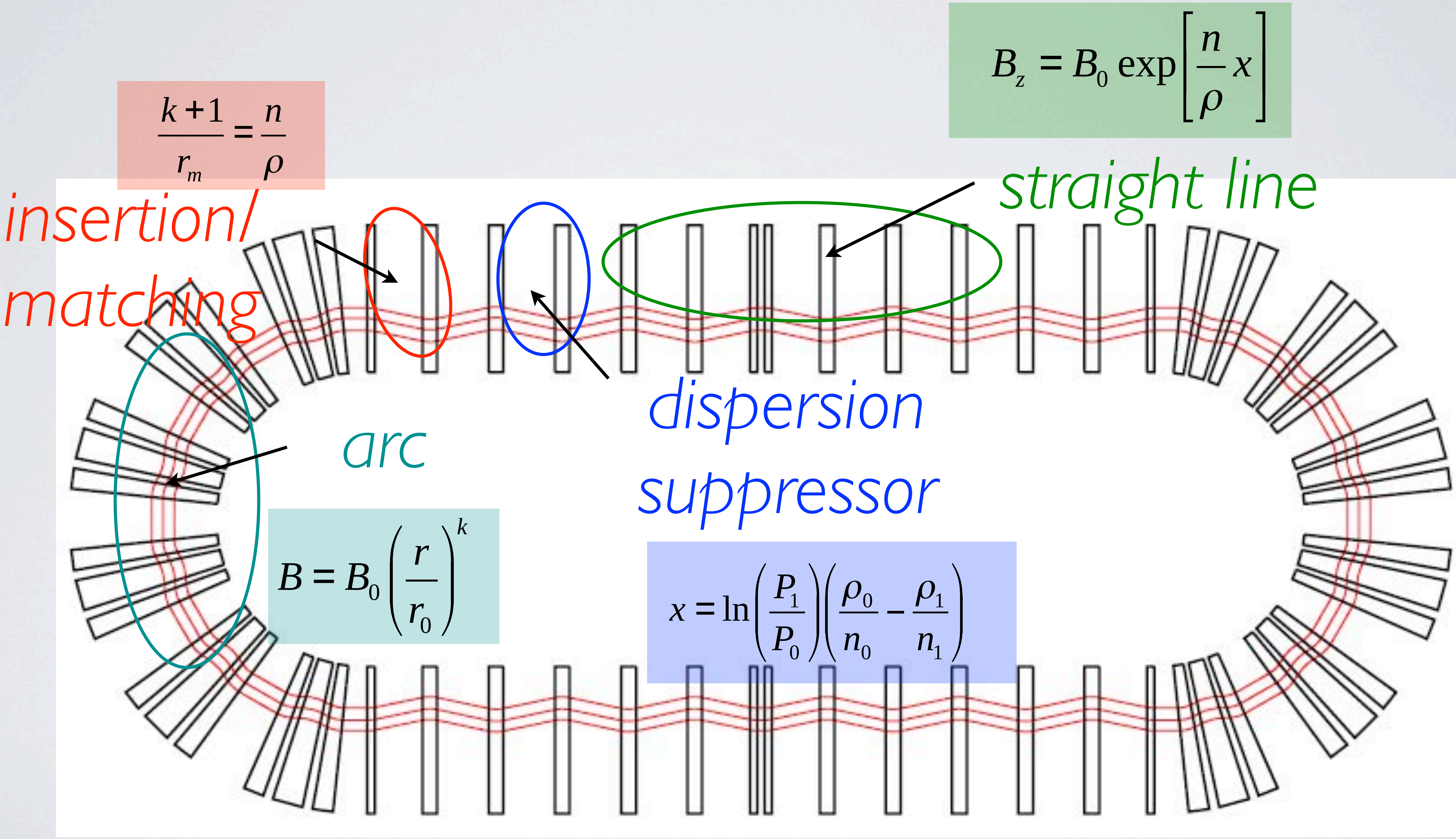
RACE TRACK FFAG RING

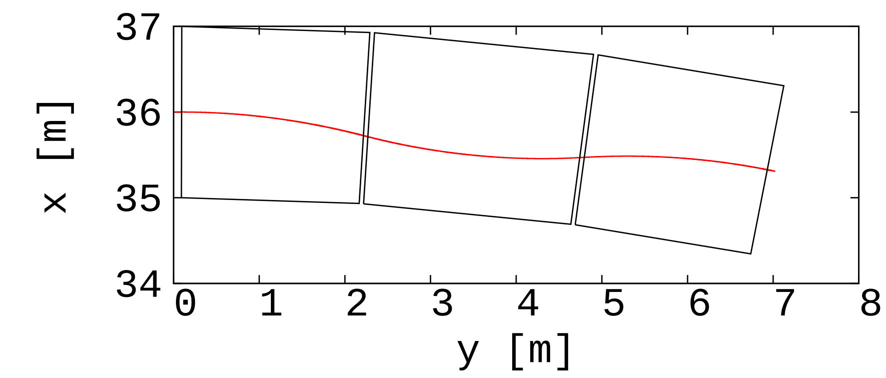


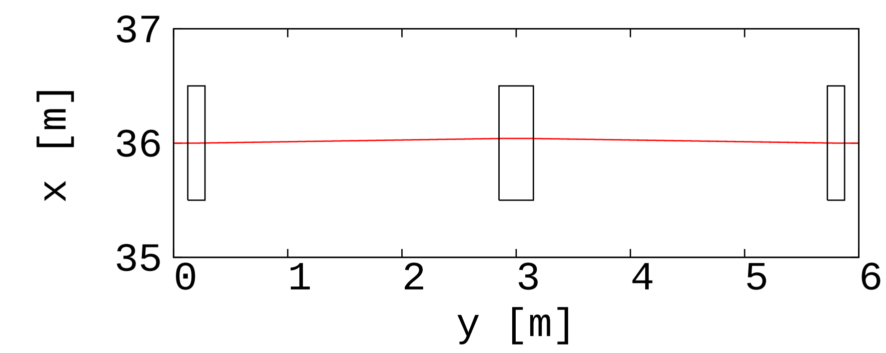
PRELIMINARY DESIGN OF MUON DECAY FFAG RING FOR LENF

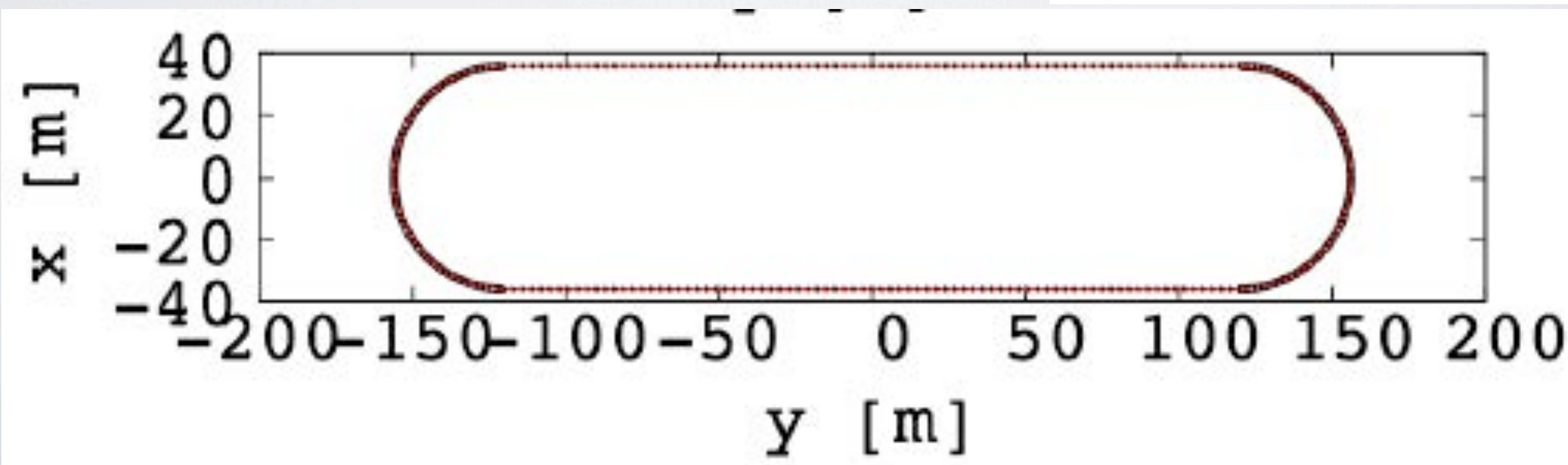
- Specifications to be achieved.
  - energy 3.8GeV (2GeV)
  - SS/circumference 70%
  - bending angle in SS <10 mrad
  - max. magnetic field <1.8 T
  - Acceptance
  - Transverse $\varepsilon_{h,v}^{*}$ > 15,000mm.mrad(normalized: a half of that for NF)
  - Longitudinal dp/p > 20%
- Design strategy
  - Optics matching between arc and straight
    - Dispersion matching: Closed orbits between arc and straight has to be matched for small dispersion in the straight section.
    - Optical transparency: Arc should be transparent to straight.
- Total phase advance/half arc should be mx2π. (m:integer)

3.8GEV/C RACE-TRACK FFAG

	Circular Section	Straight Section
Type	FDF triplet	DFD triplet
k-value or m-value	10.85	2.65m–1
Opening angle or Cell length	11.25 deg	6m
Packing factor	0.96	0.1
Periodic cell dispersion	1.39 m	0.38 m
Horizontal phase advance	67.5 deg.	13.1 deg.
Vertical phase advance	11.25 deg.	16.7 deg.
Number of cells in the ring	32	80
Total circumference	706 m	
Ring Tune	(8.9,4.75)	
Useful part/SS	68%	



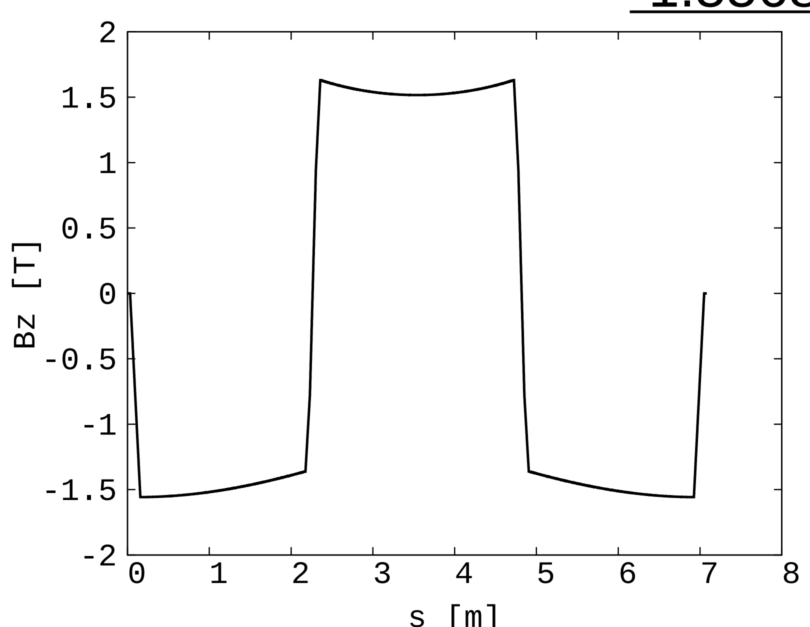


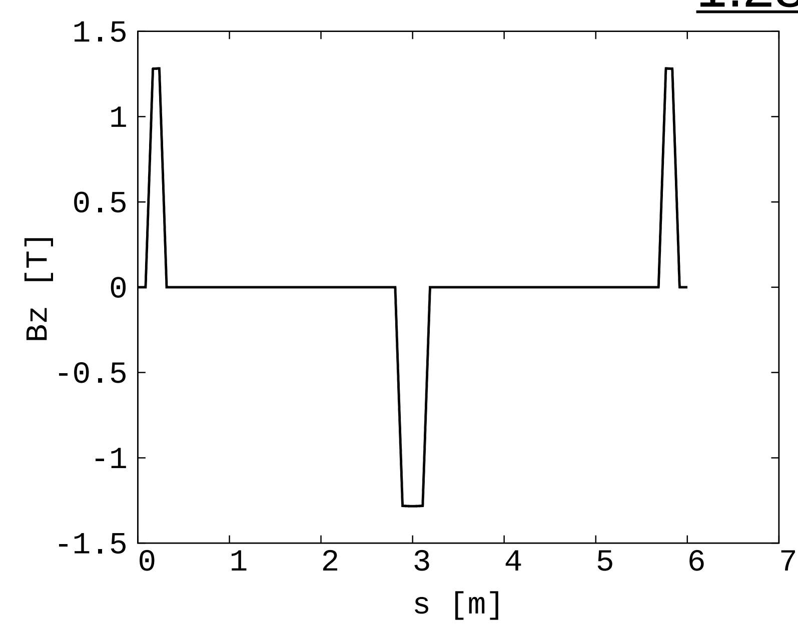


MAGNET PARAMETERS

ARC SECTION	
F1 magnet parameters	
Magnet center	1.85 deg
Magnet length	3.4 deg
	-1.55684 T
D magnet parameters	
Magnet center	5.625 deg
Magnet length	4.0 deg
	1.91025 T
F2 magnet parameters	
Magnet center	9.4 deg
Magnet length	3.4 deg
	-1.55684 T

STRAIGHT SECTION	
D1 magnet parameters	
Magnet center	0.2 m
Magnet length	0.15 m
	1.28067 T
F magnet parameters	
Magnet center	3 m
Magnet length	0.3 m
	1.1503 - 7 T
D2 magnet parameters	
Magnet center	5.8 m
Magnet length	0.15 m
	1.28067 T

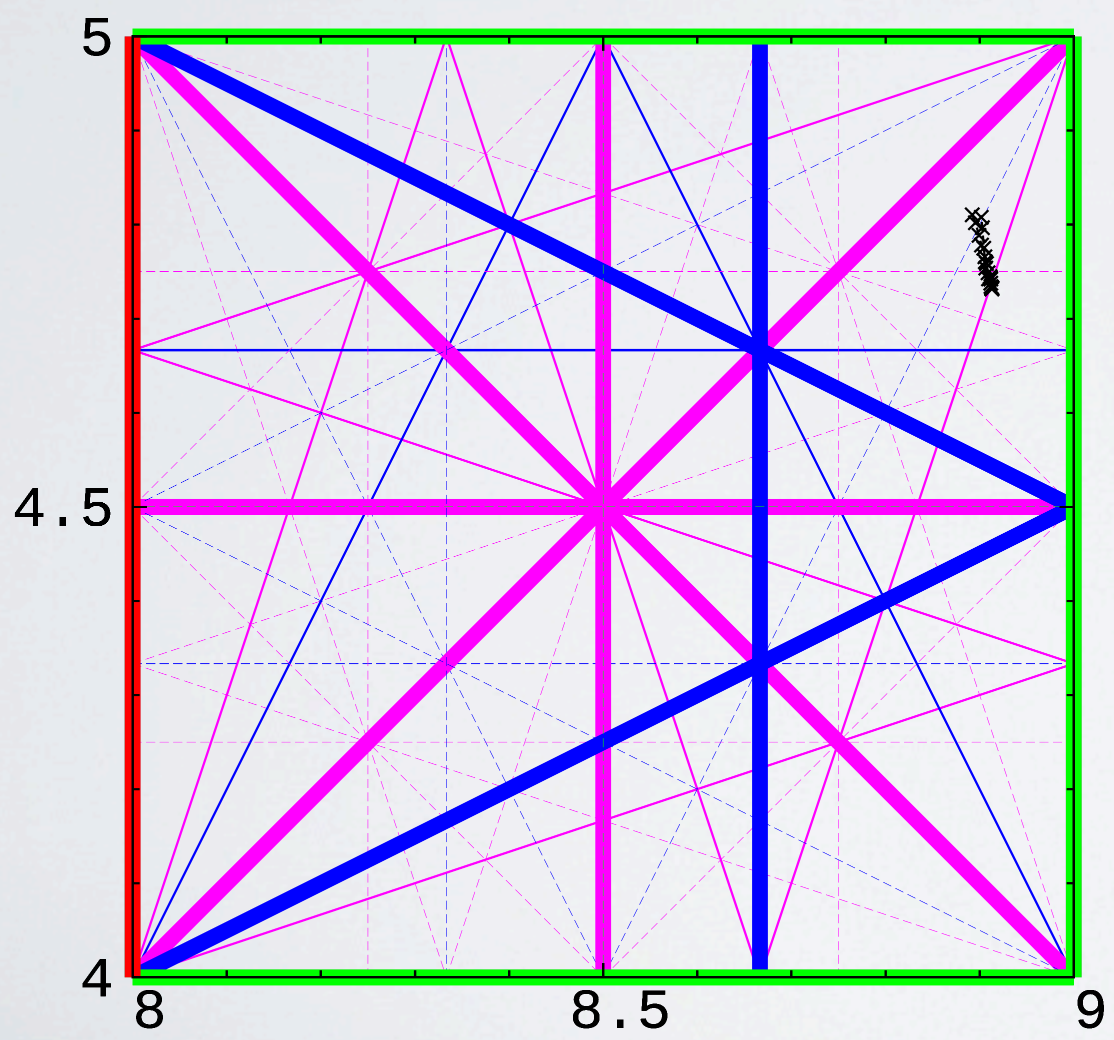


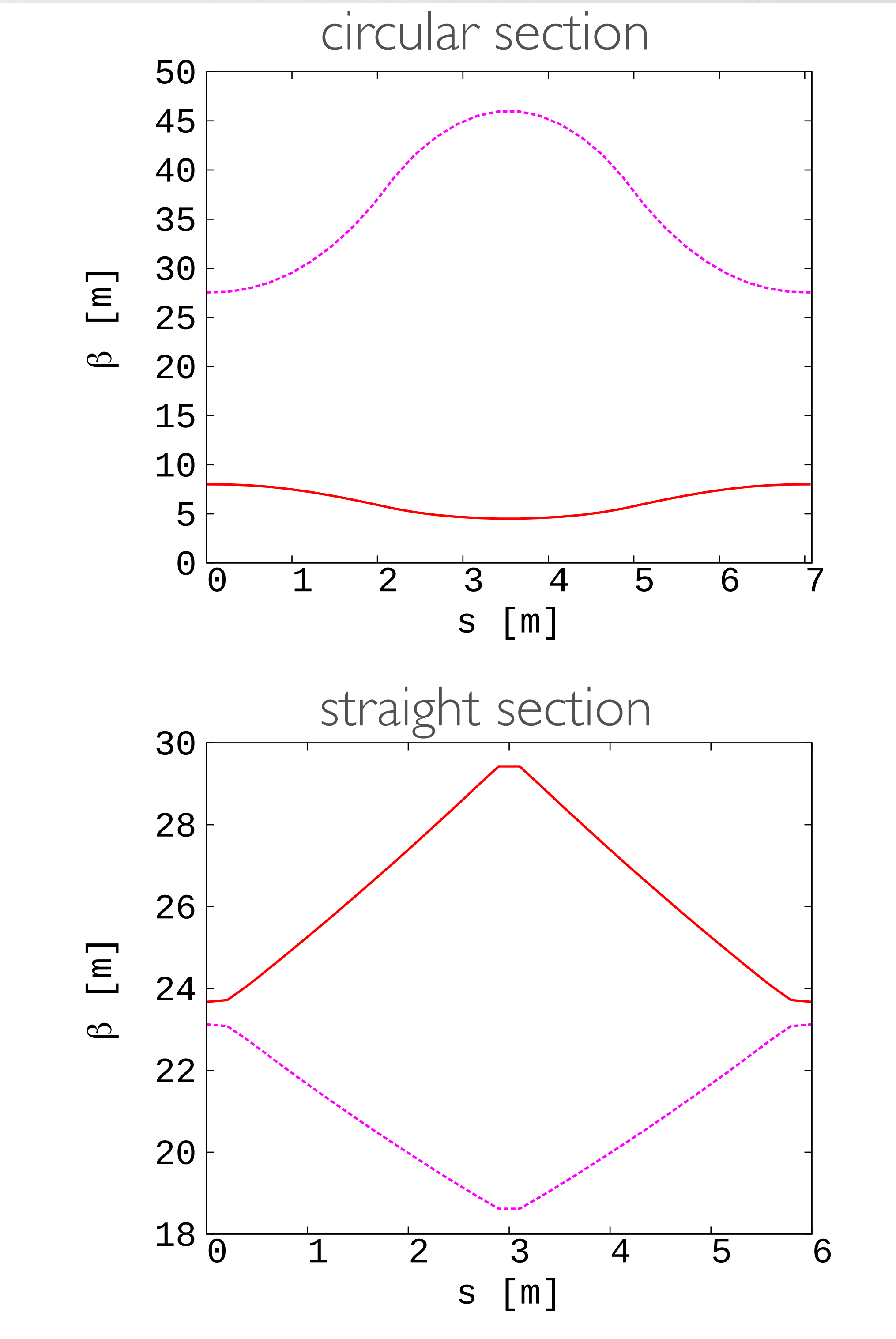


- All magnets can be normal conducting.
  - Super-ferric magnets are favorable for low power consumption and low operational cost.

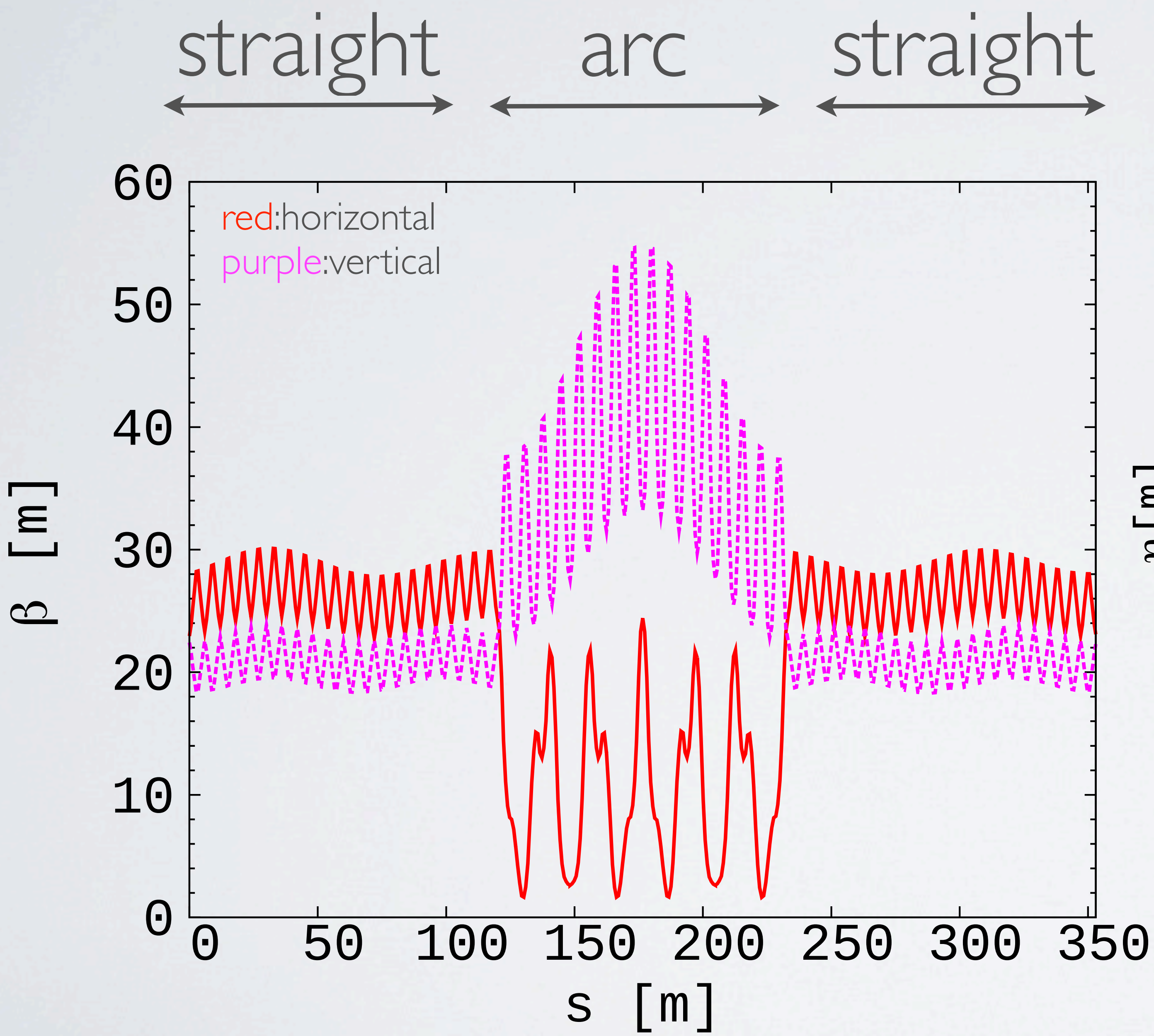
TUNE DIAGRAM

Tune shift ΔQh=0.02, ΔQv=0.1
for Δp/p=+-16%

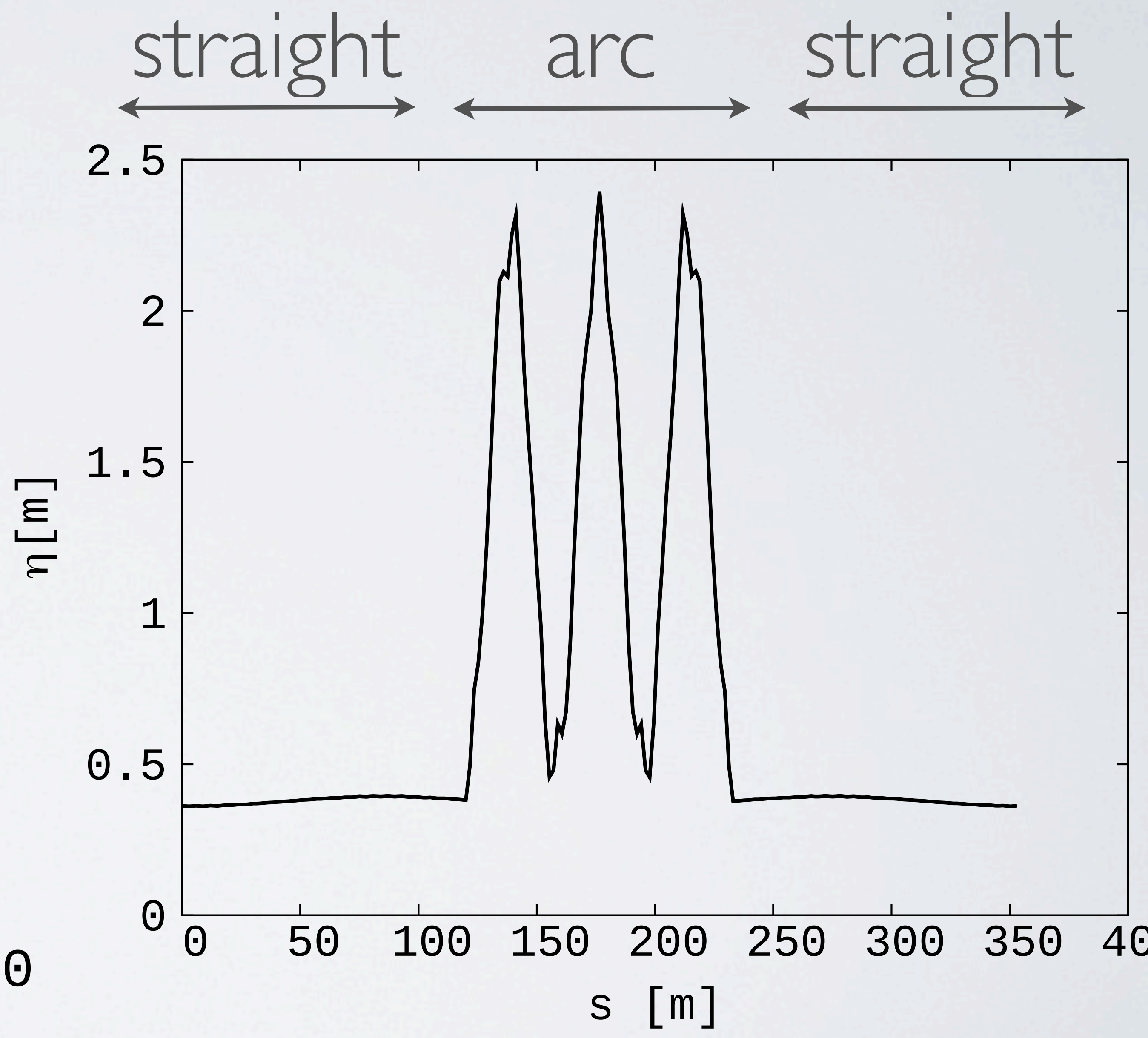




BEAM PARAMETERS



beta function

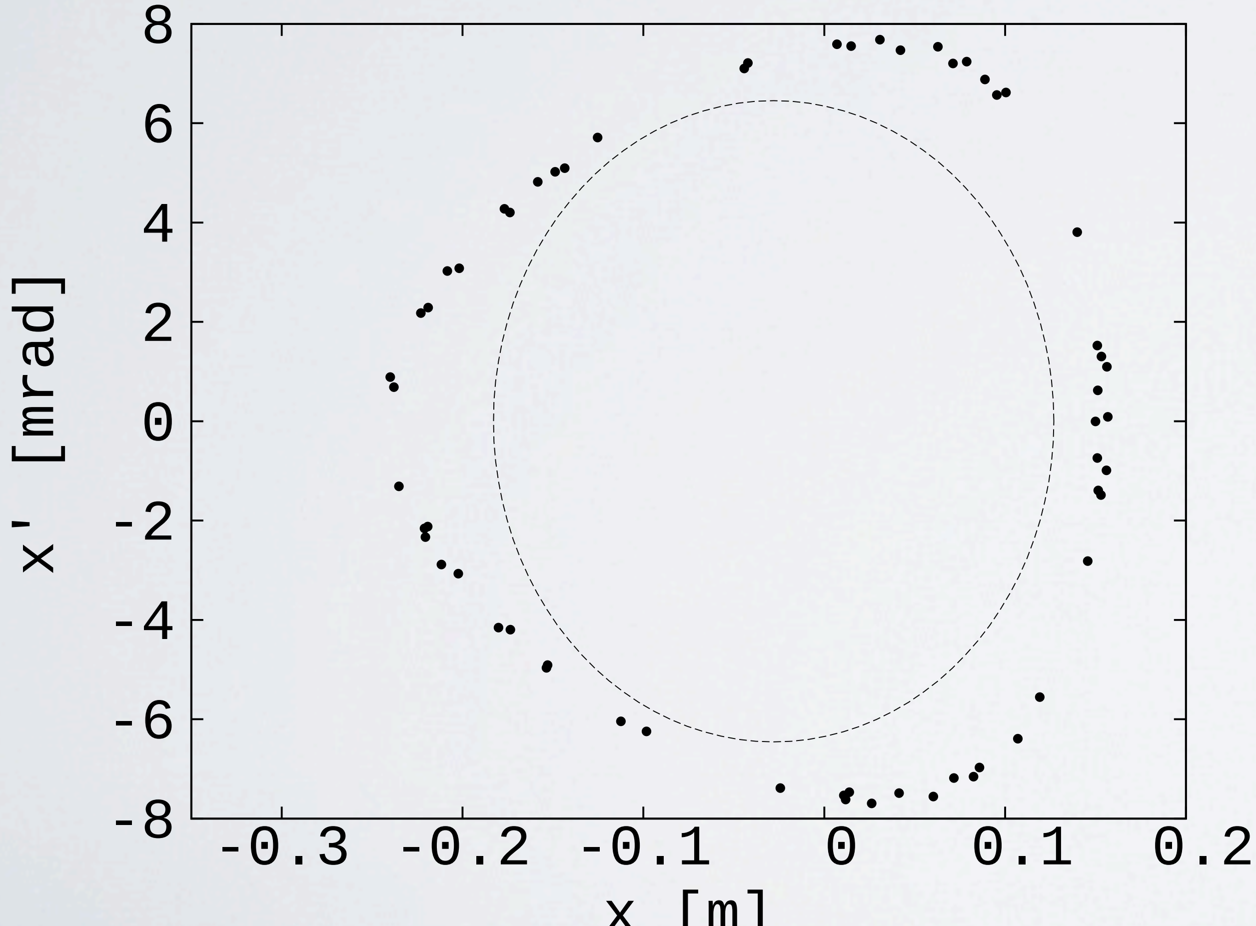


dispersion

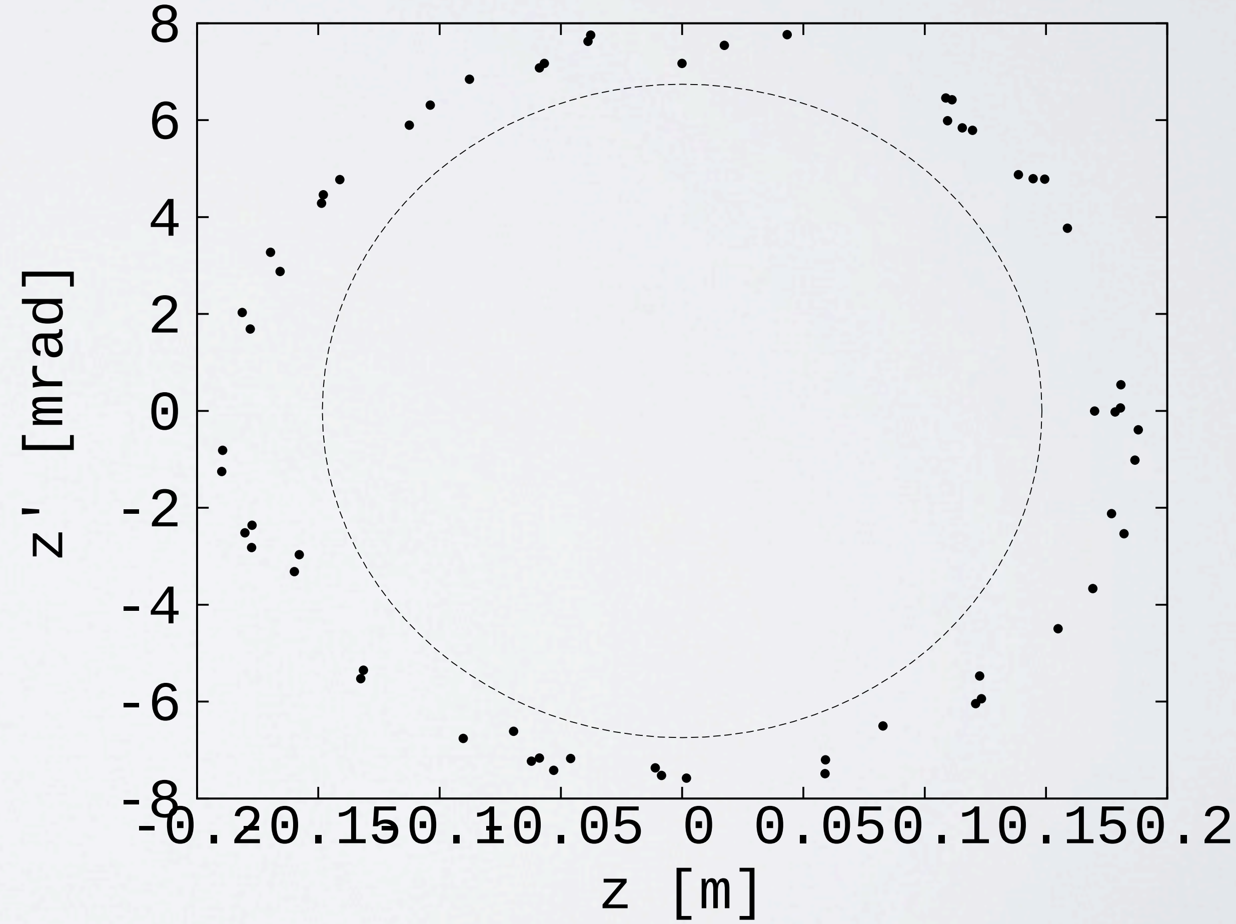
ACCEPTANCE:TRANSVERSE

- Acceptance estimated by single-particle tracking

$\varepsilon_h \sim 20{,}000\text{mm.mrad(normalized)}$



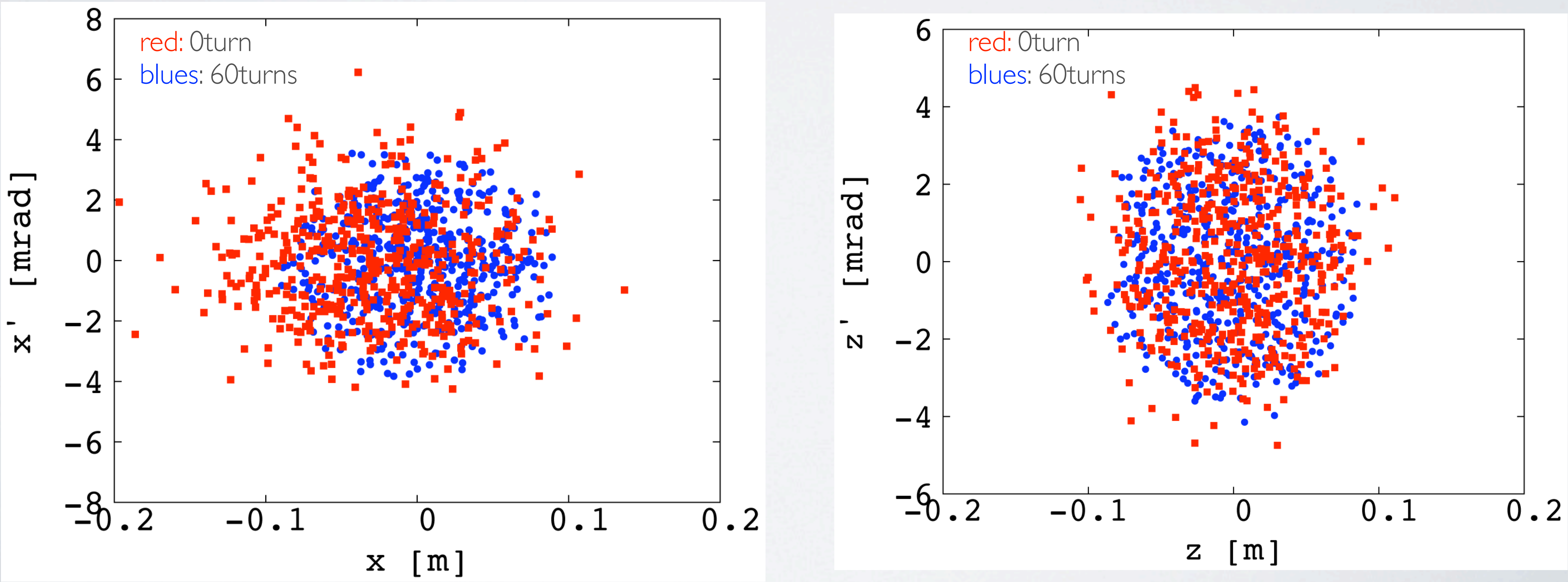
$\varepsilon_v \sim 20{,}000\text{mm.mrad(normalized)}$



cf. $\varepsilon_{h,v}^* = 30{,}000\text{mm.mrad(normalized) for NF}$

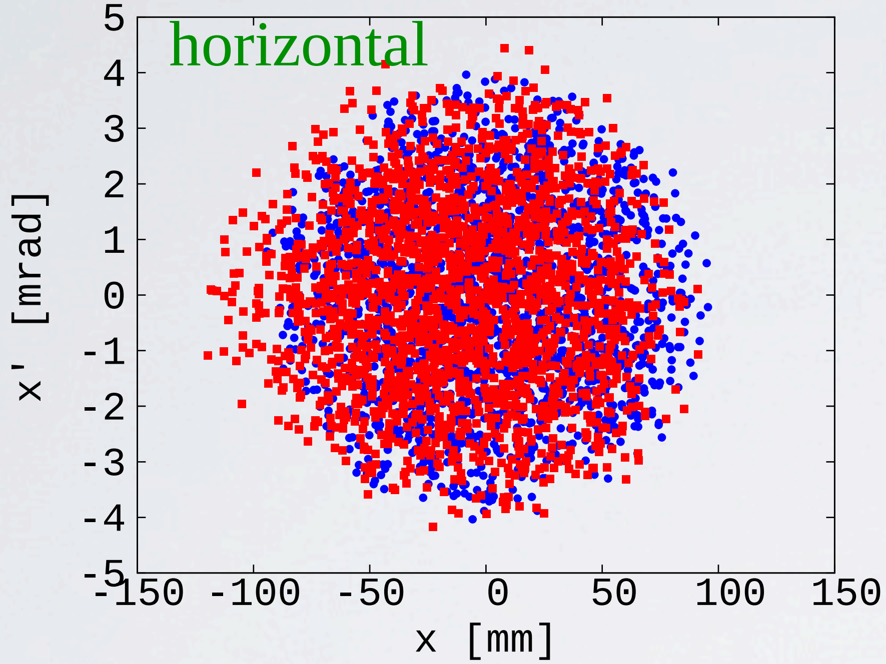
6-D TRACKING

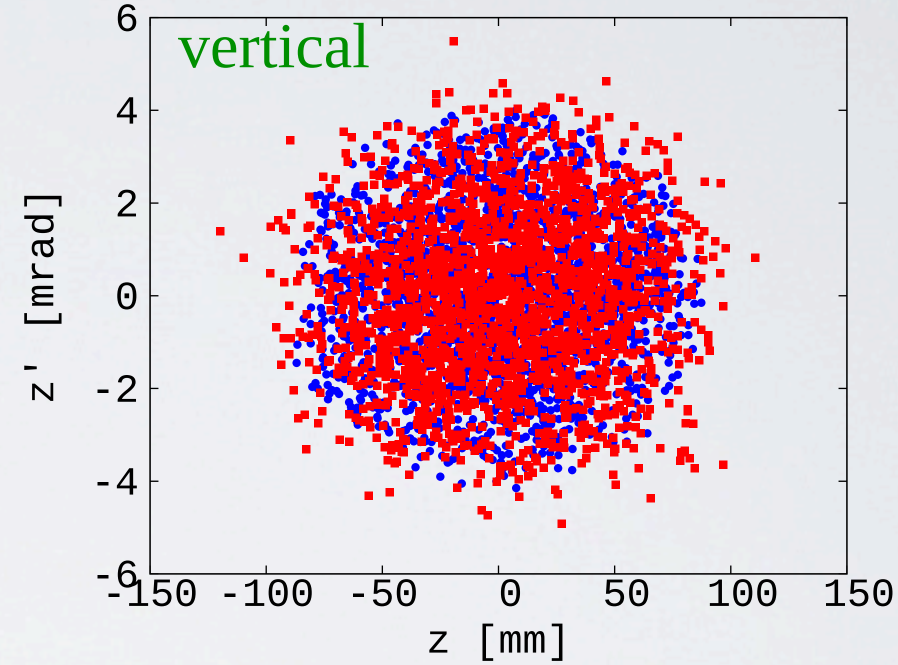
- Before & after 60 turns, 2100 particles: **Δp/p=+-16%**
  - Emittance (transverse): 4D ellipsoid (Water bag;homogeneous in 4D)
    - Horizontal : 16,000mm.mrad(normalized)
    - Vertical : 16,000mm.mrad(normalized)
  - Momentum : Δp/p=+-16%, homogeneous→ dispersion matched injection
- **No loss after 60 turns**



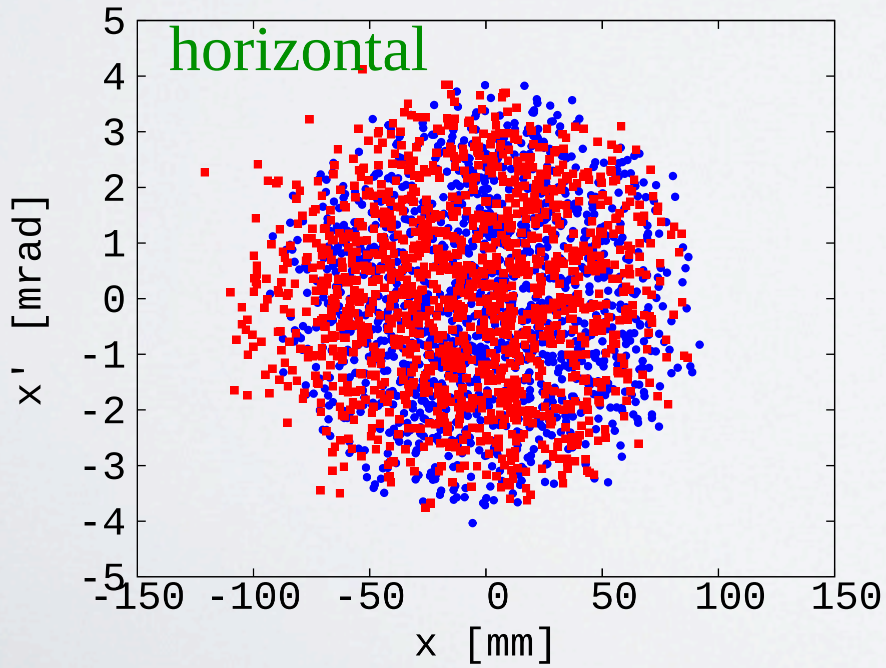
6-D TRACKING

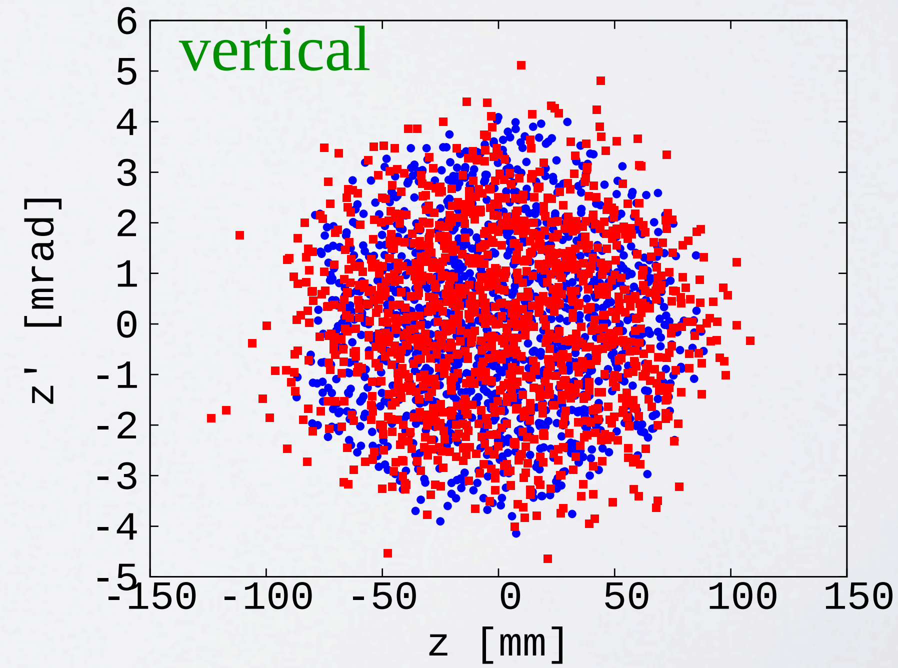
- Δp/p=+-20%; No particle loss after 60turns





- Δp/p=+-26%; 0.7% particle loss after 60turns





SUMMARY

- Zero-chromatic race-track FFAG muon decay ring for nuSTORM has been studied.
  - Zero chromaticity is essential to have large acceptance especially for largely momentum spread beam.
  - Zero-chromatic FFAG straight conceived recently makes the race-track FFAG in reality.
- A preliminary design for 3.8GeV/c race-track FFAG ring has been done.
  - Main parameters
    - total circumference ~ 706m
    - energy 3.8GeV/c
    - SS/circumference 70%
    - bending angle in SS <10 mrad
    - maximum B field ~ 1.9T max. at orbit
    - A_h^*~20,000mm.mrad, A_v^*~20,000mm.mrad, $\Delta p/p$=+-16%(+-20,+-26%)
- **Large acceptance.**
- **No particle loss in 6D simulation. cf. G4beamline (Sato-san's talk)**
- **All magnets can be normal conducting. Super conducting (super-ferric) magnets, in particular for arc sections, are also favorable in their operational power and cost.**

SUMMARY

- Issues to be studied.
  - Better optics matching between arc and straight.
    - Present design; ΔQh=0.02, ΔQv=0.1 for Δp/p=+-16%. Mismatch exists.
    - Smaller tune shifts by better matching increase momentum acceptance.
  - Smaller circumference
    - Normal conducting magnets in arc section → Super conducting magnets
    - Optical performance of the ring does not change.
    - Straight line magnets are still normal conducting.
      - cf. 1.9T(normal) → 2.7T(SC, may be Super-ferric) : total circumference ~ 500m.
    - Large bore combined function type of SC magnet is needed.
      - cf. Bore size: 100cm(hor.) x 22cm(ver.)