Muon g-2 in the SM: status and prospects

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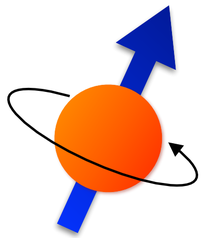
University of Illinois

Captions here

Outline

- Introduction
- Muon g-2 Theory Initiative
- Hadronic corrections in comparison
- HVP
  - puzzles
- HLbL
- Summary and Outlook
- Appendix

- ``The anomalous magnetic moment of the muon in the SM":
  1st White Paper published in 2020 (132 authors, 82 institutions)
  [T. Aoyama et al, arXiv:2006.04822, Phys. Repts. 887 (2020) 1-166.]
- ``Prospects for precise predictions of a_μ in the SM":
  2022 Snowmass Summer Study, arXiv:2203.15810
- Presentations at the Higgs Centre workshop
  (5th Plenary workshop of the Muon g-2 Theory Initiative)



Muon g-2: SM contributions

$$a_\mu = a_\mu(\mathrm{QED}) + a_\mu(\mathrm{EW}) + a_\mu(\mathrm{hadronic})$$

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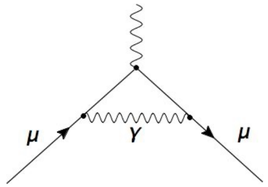
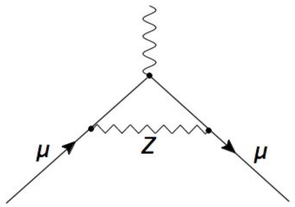
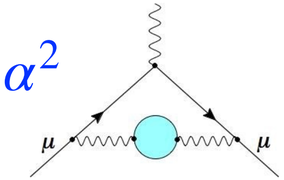
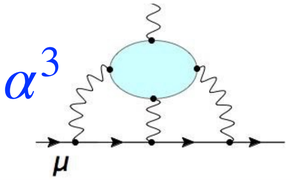
QED		+… (5 loops)	$116\,584\,718.9\,(1) \times 10^{-11}$	0.001 ppm
EW		+… (2 loops)	$153.6\,(1.0) \times 10^{-11}$	0.01 ppm
HVP	α^2 	+… (NNLO)	$6845\,(40) \times 10^{-11}$ [0.6%]	0.34 ppm
HLbL	α^3 	+… (NLO)	$92\,(18) \times 10^{-11}$ [20%]	0.15 ppm

Hadronic corrections

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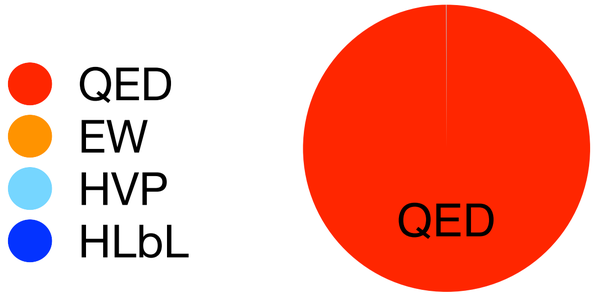
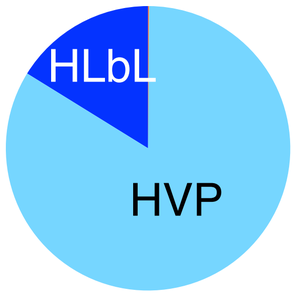
contribution



error^2



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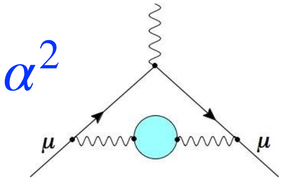
HVP α^2



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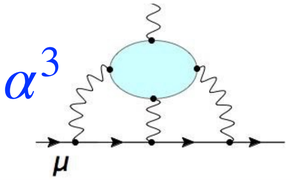

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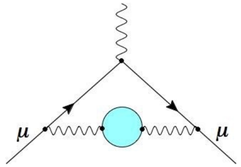

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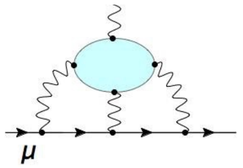
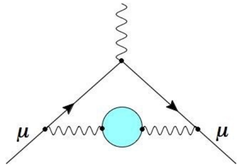


Two different, independent strategies:

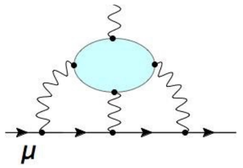
- **For HVP:** use dispersion relations to rewrite integral in terms of hadronic cross section:

Many experiments (over 20+ years) have measured the e^+e^-cross sections for the different channels over the needed energy range with increasing precision.

For HLbL: new dispersive approach



Hadronic Corrections



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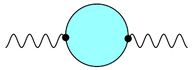
$Im[$  $] \sim |$ $|^2$ ➠ $$a_\mu^{\mathrm{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int dq^2 \omega(q^2)\, \hat{\Pi}(q^2) \;=\; \frac{m_\mu^2}{12\pi^3} \int ds \frac{\hat{K}(s)}{s}\, \sigma_{\mathrm{exp}}(s)$$

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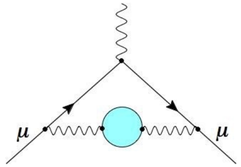
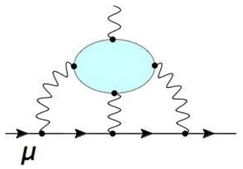


Hadronic Corrections



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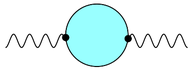
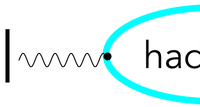
$Im[$  $] \sim |$  hadrons $|^2$ ➠ $$a_\mu^{\mathrm{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int dq^2 \omega(q^2)\,\hat{\Pi}(q^2) = \frac{m_\mu^2}{12\pi^3}\int ds \frac{\hat{K}(s)}{s}\,\sigma_{\mathrm{exp}}(s)$$

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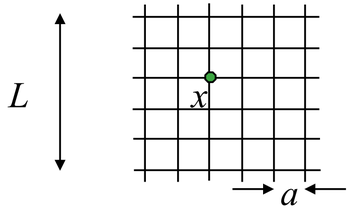
For HLbL: new dispersive approach

- Direct calculation using Euclidean Lattice QCD



Approximations:
discrete space-time (spacing a)
finite spatial volume (L), and time extent (T)
…

Integrals are evaluated numerically using Monte Carlo methods.

- *ab-initio* method to quantify QCD effects
- already used for simple hadronic quantities with high precision
- requires large-scale computational resources
- allows for entirely SM theory based evaluations

Muon g-2 Theory Initiative

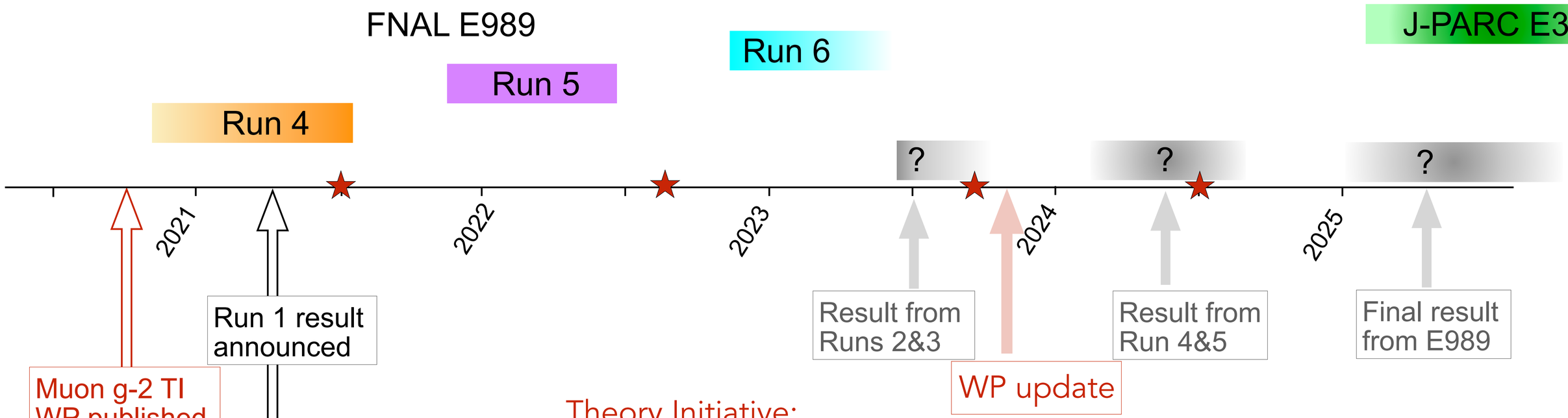
Steering Committee

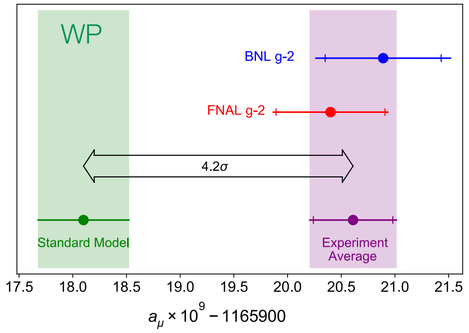
- Gilberto Colangelo (Bern)
- Michel Davier (Orsay) co-chair
- Aida El-Khadra (UIUC & Fermilab) chair
- Martin Hoferichter (Bern)
- Christoph Lehner (Regensburg University) co-chair
- Laurent Lellouch (Marseille)
- Tsutomu Mibe (KEK)
  J-PARC Muon g-2/EDM experiment
- Lee Roberts (Boston)
  Fermilab Muon g-2 experiment
- Thomas Teubner (Liverpool)
- Hartmut Wittig (Mainz)

https://muon-gm2-theory.illinois.edu

- Maximize the impact of the Fermilab and J-PARC experiments
  ➠ quantify and reduce the theoretical uncertainties on the hadronic corrections
- summarize the theory status and assess reliability of uncertainty estimates
- organize workshops to bring the different communities together:
  First plenary workshop @ Fermilab: 3-6 June 2017
  HVP workshop @ KEK: 12-14 February 2018
  HLbL workshop @ U Connecticut: 12-14 March 2018
  Second plenary workshop @ HIM (Mainz): 18-22 June 2018
  Third plenary workshop @ INT (Seattle): 9-13 September 2019
  Lattice HVP at high precision workshop (virtual): 16-20 November 2020
  Fourth plenary workshop @ KEK (virtual): 28 June - 02 July 2021
  Fifth plenary workshop @ Higgs Centre (Edinburgh): 5-9 September 2022
  Sixth plenary workshop @ University of Bern: 4-8 September 2023
  Seventh plenary workshop @ KEK (Japan): June 2024

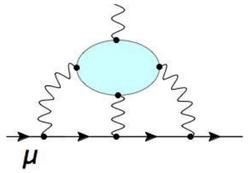
Near-term Timeline



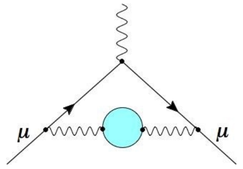


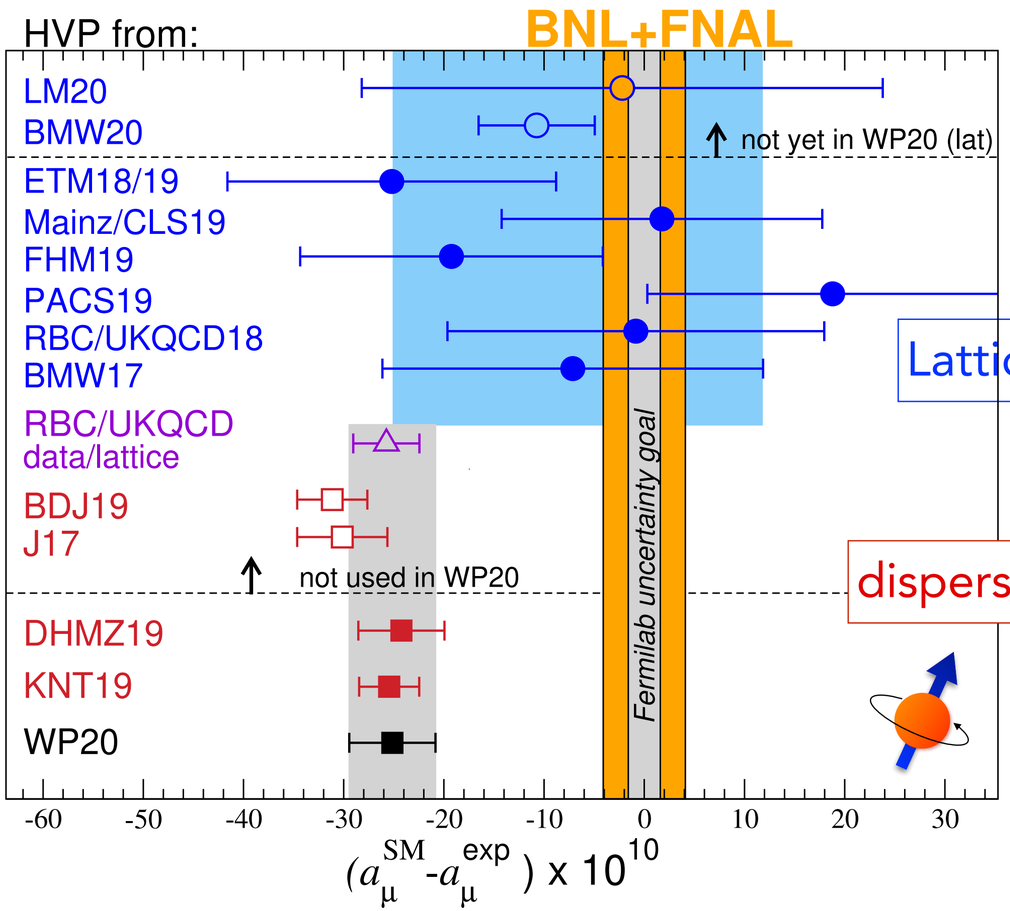
Theory Initiative:

- ☆ ongoing activities: develop method average for Lattice HVP
- ☆ CMD-3 seminar (virtual): **27 March 2023** at 8:00am US CDT
- ☆ WP update with all available results ~ late 2023
  - ★ TI workshops:
    Jun 2021 @ KEK (virtual)
    Sep 2022 @ Higgscentre
    Sep 2023 @ Bern
    Summer 2024 @ KEK



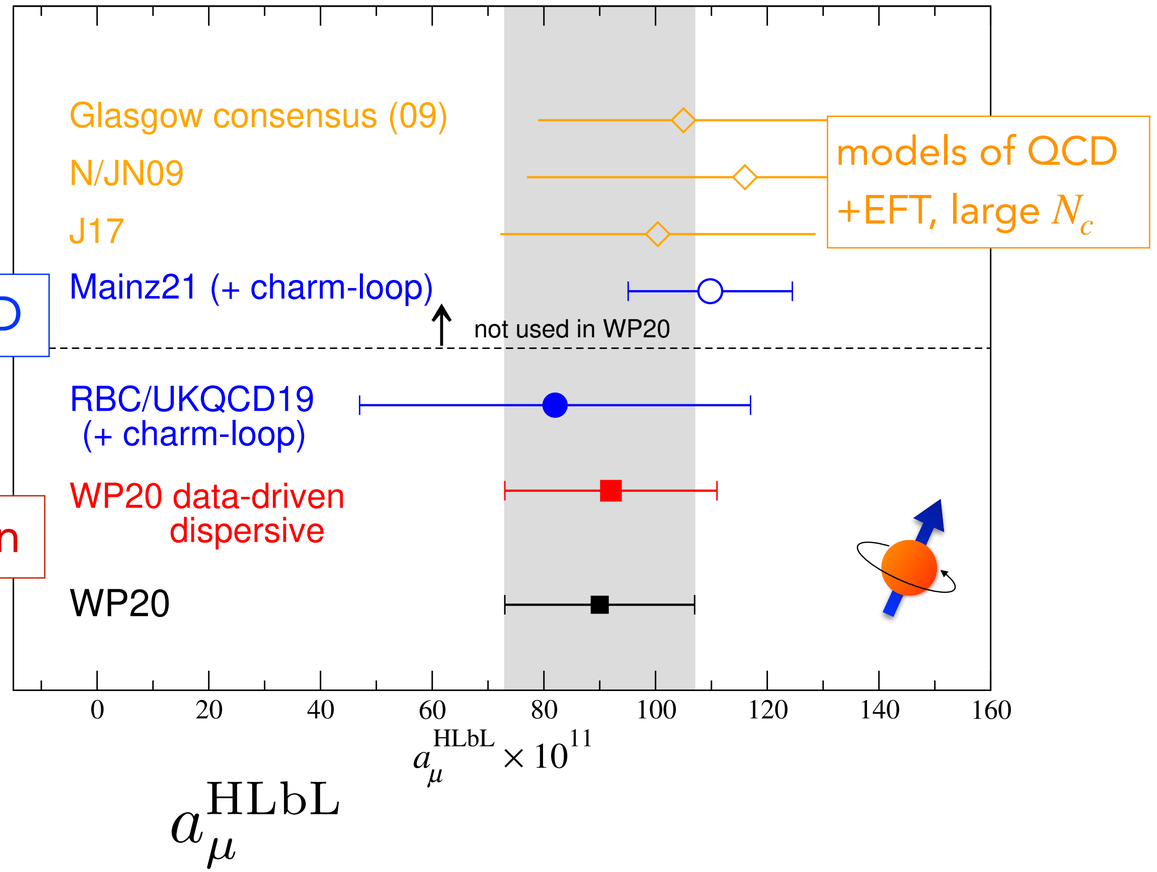
Hadronic Corrections: Comparisons





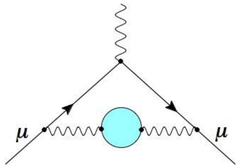
Lattice QCD + QED

dispersive/data driven



a_μ^{HLbL}

$$a_\mu^{\mathrm{SM}} \quad a_\mu^{\mathrm{HVP}} + \left[a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{Weak}} + a_\mu^{\mathrm{HLbL}}\right]$$



HVP: data-driven

In 2020 WP:

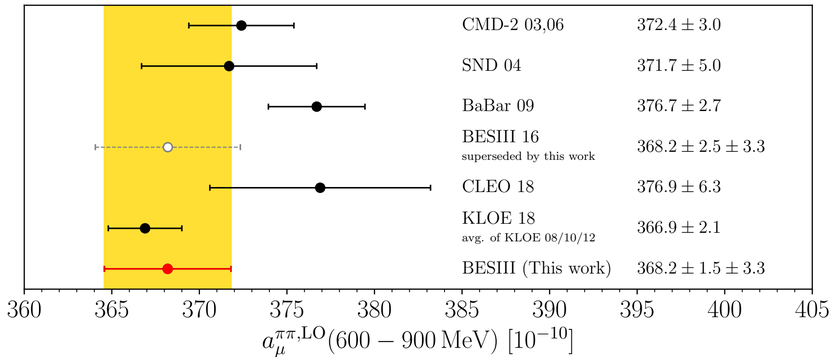
Conservative merging procedure to obtain a realistic assessment of the underlying uncertainties:

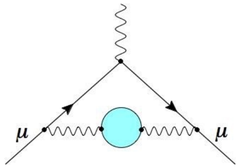
- account for tensions between data sets
- account for differences in methodologies for compilation of experimental inputs
- include correlations between systematic errors
- cross checks from unitarity & analyticity constraints
  [Colangelo et al, 2018; Anantharayan et al, 2018; Davier et al, 2019; Hoferichter et al, 2019]
- Full NLO radiative corrections [Campanario et al, 2019]

$$a_\mu^{\mathrm{HVP,LO}} = 693.1\,(2.8)_{\mathrm{exp}}\,(0.7)_{\mathrm{DV+pQCD}}\,(2.8)_{\mathrm{BaBar-KLOE}} \times 10^{-10}$$

$$= 693.1\,(4.0) \times 10^{-10}$$

[M. Ablikim et al (BES III), arXiv:2009.05011]





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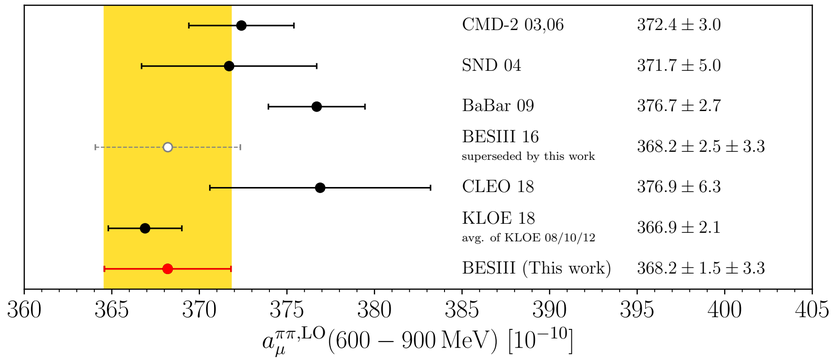
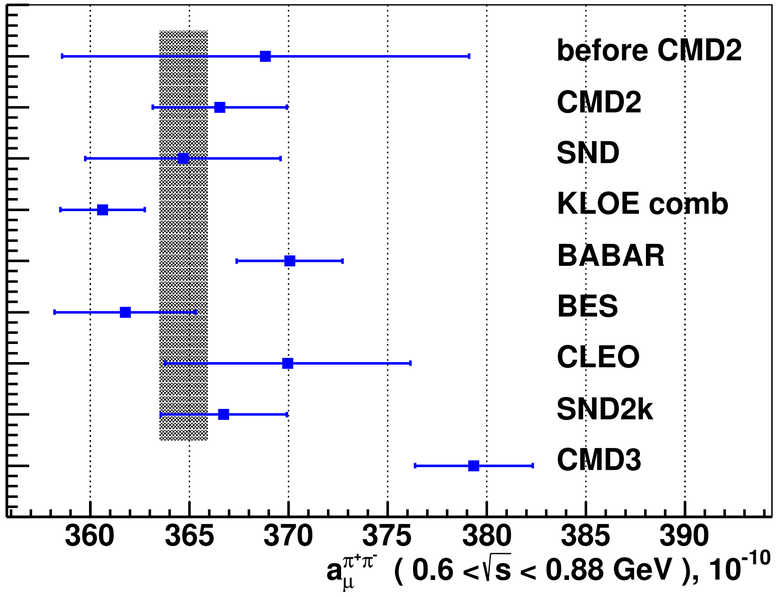
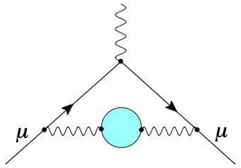


New: from CMD-3 [F. Ignatov et al, arXiv:2302.08834]



A new puzzle!

- discrepancies between experiments now $\gtrsim (3-5)\,\sigma$
  this needs to be understood/resolved
- first step: **March 27 (8:00 –10:00 am US CDT)**
  (virtual) scientific seminar + discussion panel on CMD-3 measurement



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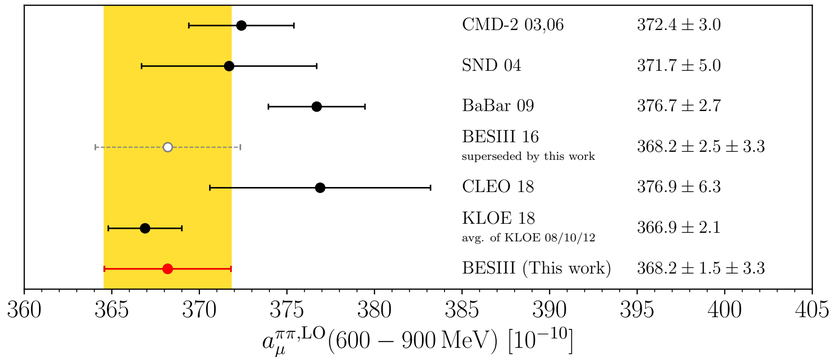
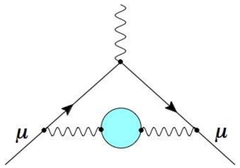


Ongoing work on experimental inputs:

- BaBar: new analysis of large data set in $\pi\pi$ channel, also $\pi\pi\pi$, other channels, other channels
- KLOE: new analysis of large data in $\pi\pi$ channel, other channels
- SND: new results for $\pi\pi$ channel, other channels in progress
- BESIII: new results in 2021 for $\pi\pi$ channel, continued analysis also for $\pi\pi\pi$, other channels
- Belle II: arXiv:2207.06307 (Snowmass WP)
  Better statistics than BaBar or KLOE; similar or better systematics for low-energy cross sections
- STCF: arXiv:2203.06961
- Need blind analyses to resolve the tensions (esp. for $\pi\pi$ channel)



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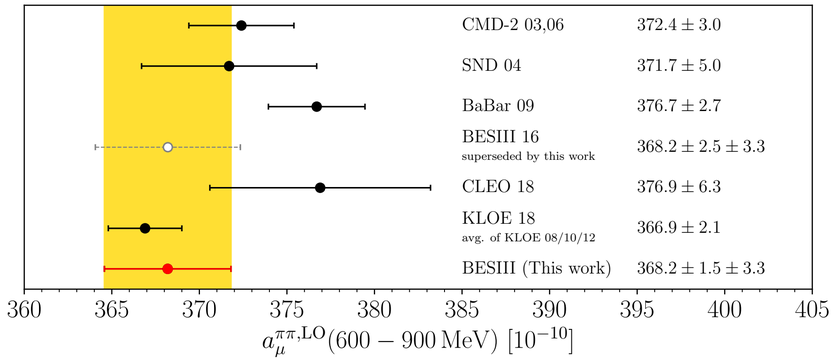
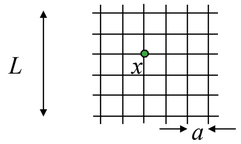


Ongoing work on theoretical aspects:

- Developing NNLO Monte Carlo generators (STRONG 2020 workshop https://agenda.infn.it/event/28089/) [➠ appendix]
- radiative corrections using FsQED (scalar QED + pion form factor)
- charge asymmetry (CMD-3 measurement) vs radiative corrections [Ignatov + Lee, arXiv:2204.12235]
- development of new dispersive treatment of radiative corrections in $\pi\pi$ channel [Colangelo at al, arXiv2207.03495]
- including τ decay data: requires nonperturbative evaluation of IB correction [M. Bruno et al, arXiv:1811.00508]

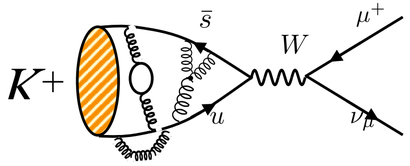
If the differences between experiments are resolved:
data-driven evaluations of HVP with $\sim 0.3\,\%$ feasible by ~2025

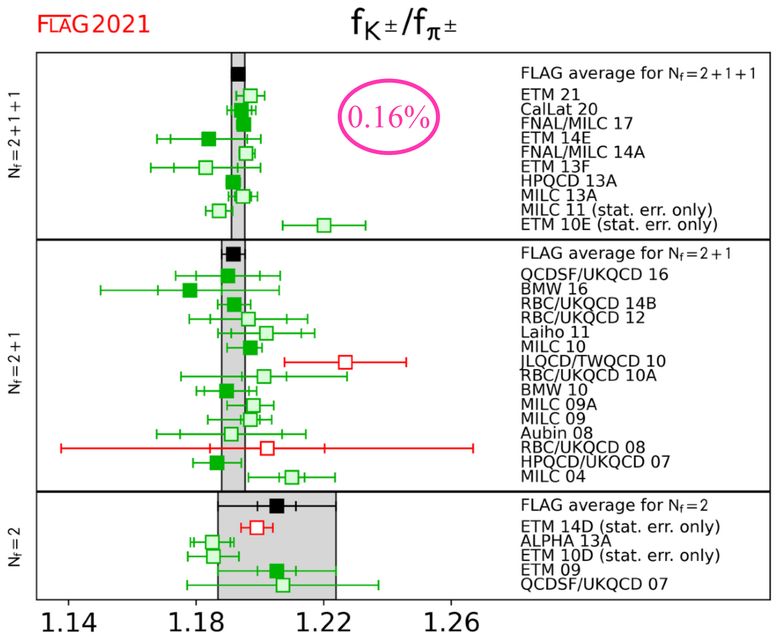
Lattice QCD: the state of the art



Lattice QCD calculations of simple quantities (with at most one stable meson in initial/final state) that **quantitatively account for all systematic effects** (discretization, finite volume, renormalization,…) in some cases with

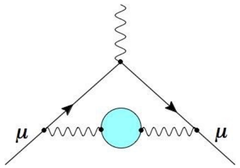
- sub percent precision.
- total errors that are commensurate (or smaller) than corresponding experimental uncertainties.



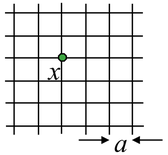


Scope of LQCD calculations is increasing due to continual development of new methods:

- nucleon matrix elements
- nonleptonic decays ($K \to \pi\pi$, …)
- resonances, scattering, long-distance matrix elements, …
- structure: PDFs, GPDs, …
- QED corrections, radiative decay rates …
- 2 - few nucleon systems

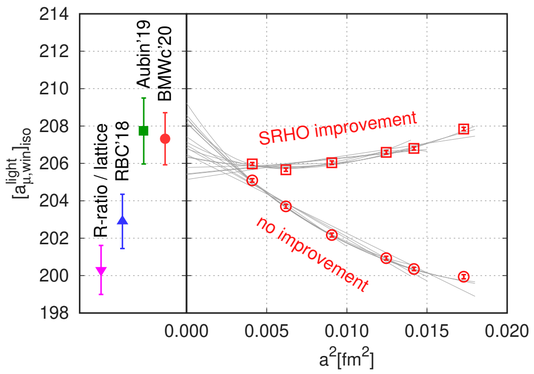


HVP: lattice



In 2020 WP:

- Lattice HVP average at $2.6\,\%$ total uncertainty: $a_\mu^{\mathrm{HVP,LO}} = 711.6\,(18.4) \times 10^{10}$
- BMW 20 (published in 2021)
  first LQCD calculation with sub-percent ($0.8\,\%$) error
  in tension with data-driven HVP (2.1σ)
- Further tensions for intermediate window



-3.7σ tension with data-driven evaluation

-2.2σ tension with RBC/UKQCD18

$$a_\mu^{\mathrm{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt\, \tilde{w}(t)\, C(t)$$

- Use windows in Euclidean time to consider the different time regions separately. [T. Blum et al, arXiv:1801.07224, 2018 PRL]

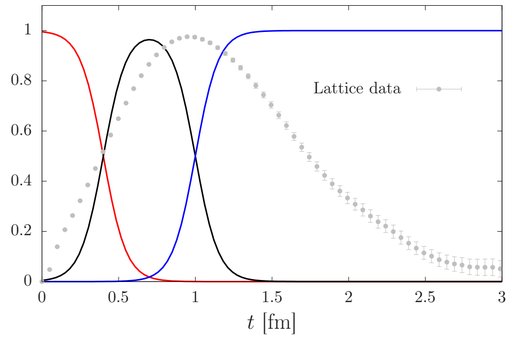
Short Distance (SD) $\quad t : 0 \to t_0$

Intermediate (W) $\quad t : t_0 \to t_1$

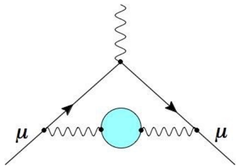
Long Distance (LD) $\quad t : t_1 \to \infty$

$$t_0 = 0.4\,\mathrm{fm}, t_1 = 1.0\,\mathrm{fm}$$

H. Wittig @ Lattice 2021



- disentangle systematics/statistics from long distance/FV and discretization effects
- intermediate window: easy to compute in lattice QCD & using disperse approach:
- Internal cross check: compute each window separately (in continuum, infinite volume limits,…) and combine: $a_\mu = a_\mu^{\mathrm{SD}} + a_\mu^{\mathrm{W}} + a_\mu^{\mathrm{LD}}$



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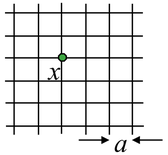

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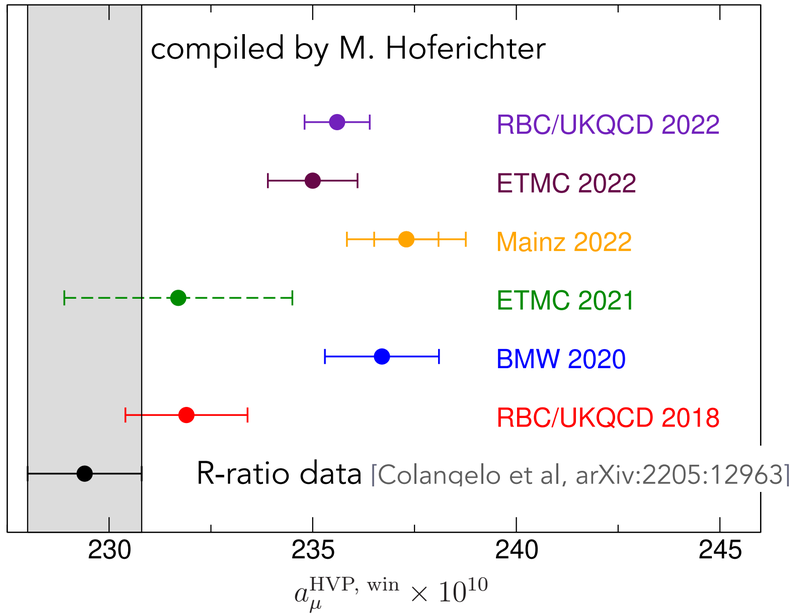
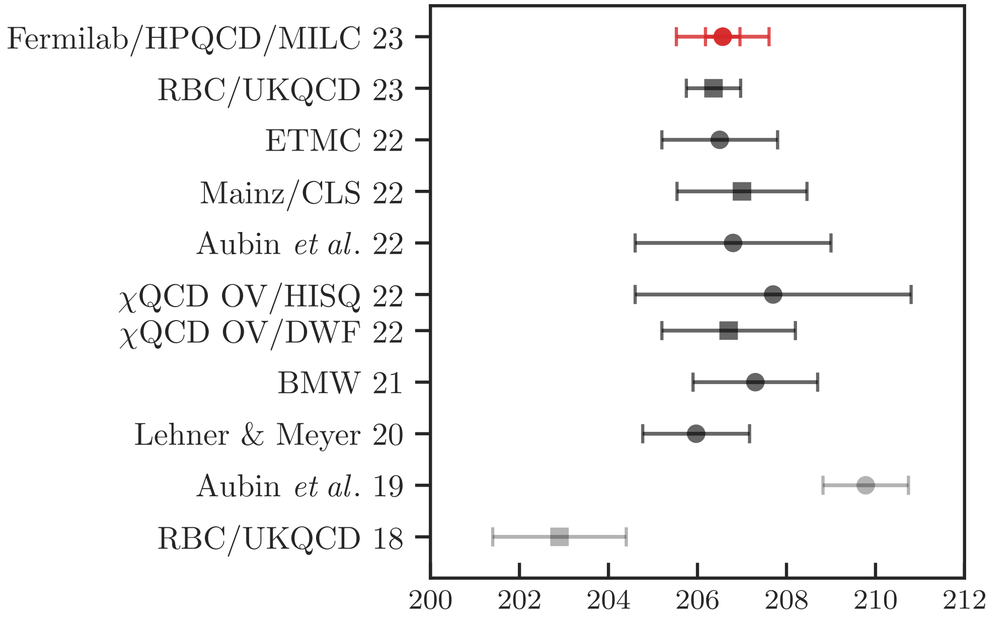
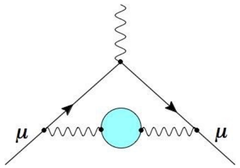


- new results in 2022 for intermediate window, a_μ^W from six different lattice groups.
- Most recently announced unblinded results by RBC/UKQCD and Fermilab/HPQCD/MILC
- lattice-only comparison of light-quark connected contribution to intermediate window:



- Fermilab/HPQCD/MILC: arXiv:2301.08274
- RBC/UKQCD 23: arXiv:2301.08696
- ETM: arXiv:2206.15084
- Mainz: arXiv:2206.06582
- Aubin et al: arXiv:2204.12256
- χQCD: arXiv:2204.01280
- BMW 21: arXiv:2002.12347
- L&M: arXiv:2003.04177
- Aubin et al: arXiv:2204.12256
- RBC/UKQCD 18: arXiv:1801.07224



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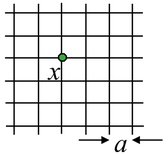

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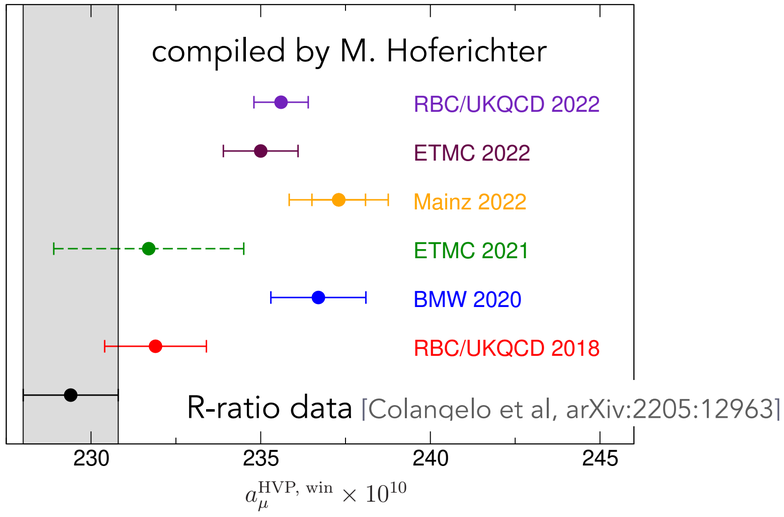


Note: int window ~ 1/3 of $a_\mu^{\mathrm{HVP,LO}}$

Still need independent LQCD calculations of all windows and sub-leading contributions.

Ongoing work:

Evaluations of short-distance windows [ETMC, RBC/UKQCD]

Proposals for computing more windows:

- Use linear combinations of finer windows to locate the tension (if it persists) in $\sqrt{s}$ [Colangelo et al, arXiv:12963]
- Use larger windows, excluding the long-distance region $t \gtrsim 2\,\mathrm{fm}$ to maximize the significance of any tension [Davies at at, arXiv:2207.04765]

For total HVP:

- independent lattice results at sub-percent precision: coming soon!
- Including $\pi\pi$ states for refined long-distance computation (Mainz, RBC/UKQCD, FNAL/MILC)
- include smaller lattice spacings to test continuum extrapolations (needs adequate computational resources)

If results are consistent, Lattice HVP (average) with $\sim 0.5\,\%$ errors feasible by 2025

Summary

- consistent results from independent, precise LQCD calculations for light-quark connected contribution to **intermediate window** $a_\mu^{\rm W}$ (~ 1/3 of $a_\mu^{\rm HVP,LO}$) ➠ $3-4\,\sigma$ tension with data-driven results?
- still need independent LQCD results for long-distance contribution, total HVP: coming soon
  ➠ develop method average for lattice HVP results, **assess tensions (if any)** with data-driven average
- Programs and plans in place to improve by 2025:
  - data-driven HVP: **if differences are resolved/understood,** $\sim 0.3\,\%$
    new measurements from BaBar, KLOE, Belle II, …. will shed light on current discrepancies
    (blind analyses are paramount!)
  - lattice HVP: **if no tensions** between independent lattice results, $\sim 0.5\,\%$
  - dispersive HLbL and lattice HLbL: no puzzles, steady progress, $\sim 10\,\%$
- **IF tensions/differences between data-driven HVP and lattice HVP are resolved,** SM prediction will likely match precision goal of the Fermilab experiment.
- IF NOT, will need detailed comparisons, explore connections between HVP, $\sigma(e^+e^-)$, $\Delta\alpha$, global EW fits.
  - MUonE (space-like measurement of $\Delta\alpha$) will provide important information and cross checks.
    (➠ appendix)

➠ **continued coordination by Theory Initiative: workshops, WPs, …**

Outlook

- Experimental program beyond 2025:
  - J-PARC: Muon g-2/EDM
  - CERN: MUonE
  - Fermilab: future muon campus experiments?
  - Belle II, BESIII, Novosibirsk,…
  - Chiral Belle (?)
- Data-driven/dispersive program beyond 2025:
  - development of NNLO MC generators
  - for HLbL, improved experimental/lattice inputs together with further development of dispersive approach
- MUonE will provide a space-like measurement of HVP
- Lattice QCD beyond 2025:
  - access to future computational resources (coming Exascale) will enable improvements of all errors (statistical and systematic)
  - concurrent development of better methods and algorithms (gauge-field sampling, noise reduction) will accelerate progress
  - beyond g-2: a rich program relevant for all areas of HEP

Outlook

$a_\mu^{\text{HVP LO}}$

$\langle \bar{B}_q^0 | \mathcal{O}_i^{\Delta B=2} | B_q^0 \rangle$ a_μ^{HLbL}

$\langle \bar{D}^0 | \mathcal{O}_i^{\Delta C=2} | D^0 \rangle$

$\hat{B}_K$

$f_{B_{(s)}}$ $\Lambda_b \to p, \Lambda_c, \Lambda$

$f_+^{K\to\pi}$ $f_{+,0}^{B\to D}(q^2), \ldots$

$f_{K^\pm}$ $f_{+,0,T}^{B\to\pi}$ $\ldots$

d_n

PDFs, GPDs, TMDs,..

MEs for light nuclei

$D_s \to \ell\nu\gamma$

$K \to \ell\nu\gamma$

nucleon form factors, ..

g_A, g_T, g_S

$\Delta M_K, \epsilon_K$

$\langle \pi\pi_{(I=0)} | \mathcal{H}^{\Delta S=1} | K^0 \rangle$

$\langle \pi\pi_{(I=2)} | \mathcal{H}^{\Delta S=1} | K^0 \rangle$

$\langle NN | O_i | NN \rangle$

$B \to K^* \ell\ell \to K\pi\,\ell\ell$ $\ldots$

$K^+ \to \ell^+\nu\,(\gamma)$ $\ldots$

$K^+ \to \pi^+\ell^+\ell^-$

$K^+ \to \pi^+\nu\bar{\nu}$ $\ldots$

$B \to X_c \ell\nu,$ other inclusive decay rates, $\ldots$

Complexity

✓

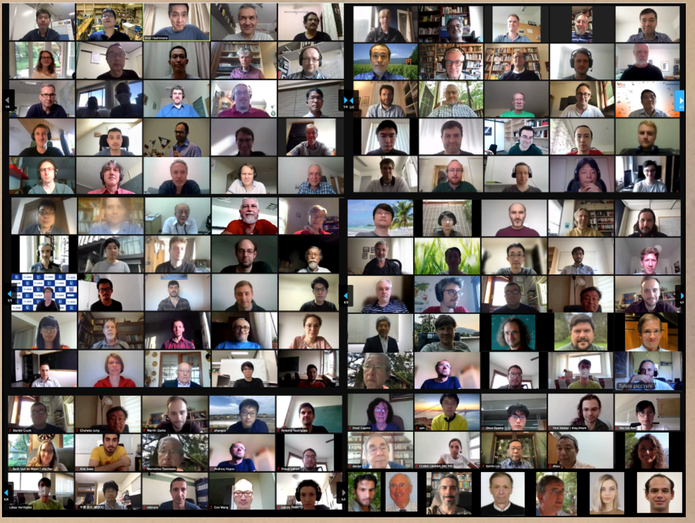
LQCD flagship results

First complete LQCD results, large(ish) errors

First results, physical params, incomplete systematics

new methods, pilot projects, unphysical kinematics

new ideas, first studies



Thank you!

Appendix

Updated WP Summary Table

Contribution	Value $\times 10^{11}$	References
Experimental average (E989+E821)	116592061(41)	Phys.Rev.Lett. 124, 141801
HVP LO (e^+e^-)	6931(40)	Refs. [2–7]
HVP NLO (e^+e^-)	−98.3(7)	Ref. [7]
HVP NNLO (e^+e^-)	12.4(1)	Ref. [8]
HVP LO (lattice, $udsc$)	7116(184)	Refs. [9–17]
HLbL (phenomenology)	92(19)	Refs. [18–30]
HLbL NLO (phenomenology)	2(1)	Ref. [31]
HLbL (lattice, uds)	79(35)	Ref. [32]
HLbL (phenomenology + lattice)	90(17)	Refs. [18–30, 32]
QED	116 584 718.931(104)	Refs. [33, 34]
Electroweak	153.6(1.0)	Refs. [35, 36]
HVP (e^+e^-, LO + NLO + NNLO)	6845(40)	Refs. [2–8]
HLbL (phenomenology + lattice + NLO)	92(18)	Refs. [18–32]
Total SM Value	116 591 810(43)	Refs. [2–8, 18–24, 31–36]
Difference: $\Delta a_\mu := a_\mu^{\rm exp} - a_\mu^{\rm SM}$	251(59)	

website: https://muon-gm2-theory.illinois.edu

Muon g-2: SM contributions

$$a_\mu = a_\mu(\mathrm{QED}) + a_\mu(\mathrm{EW}) + a_\mu(\mathrm{hadronic})$$

Muon g-2: SM contributions

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QED

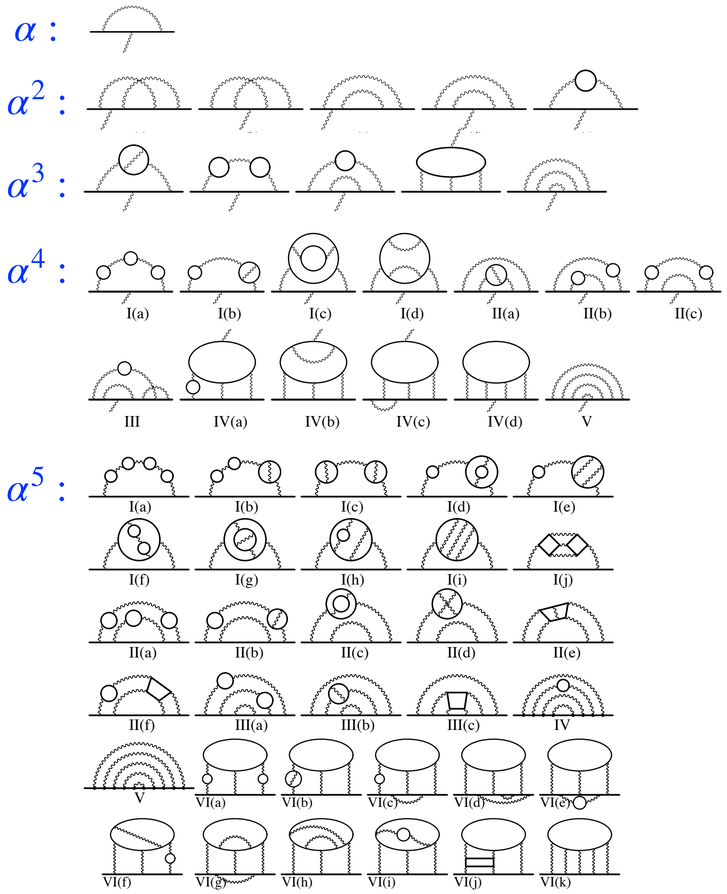
$$a_\mu(\mathrm{QED}) = A_1 \; + \; A_2\left(\frac{m_\mu}{m_e}\right) \; + \; A_2\left(\frac{m_\mu}{m_\tau}\right) \; + \; A_3\left(\frac{m_\mu}{m_e}, \frac{m_\mu}{m_\tau}\right)$$

$$A_i = \sum_{n=0} \left(\frac{\alpha}{\pi}\right)^n A_i^{2n}$$

n	# of diagrams	Contribution x 10^{11}
1	1	116140973.32
2	7	413 217.63
3	71	30141.90
4	891	381.00
5	12672	5.08

$$a_\mu(\mathrm{QED}) = 116\,584\,718.9\,(1) \times 10^{-11}$$

[T. Aoyama et al, arXiv:1205.5370, PRL;
T. Aoyama, T. Kinoshita, M. Nio, Atoms 7 (1) (2019) 28]



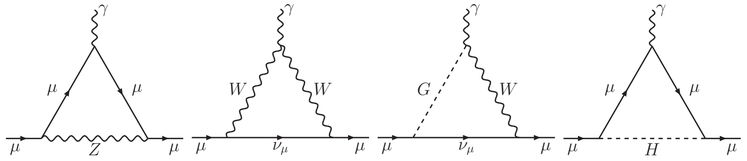
Muon g-2: SM contributions

$$a_\mu = a_\mu(\mathrm{QED}) + a_\mu(\mathrm{EW}) + a_\mu(\mathrm{hadronic})$$

Electroweak

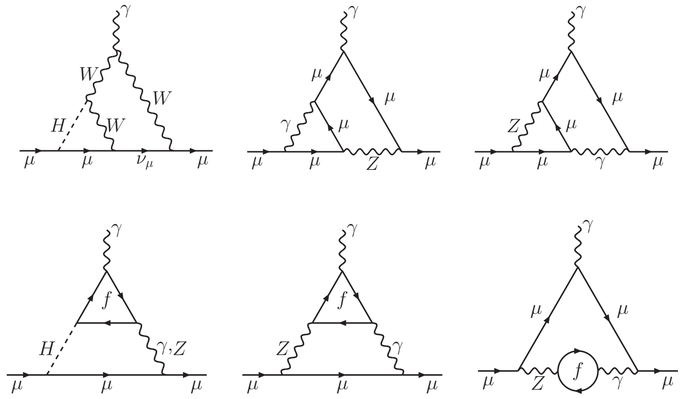
(contributions from W,Z,H bosons)

1-loop



Compared to QED, suppressed by $\sim \frac{m_\mu^2}{M_W^2} \sim 10^{-6}$

2-loop



$$a_\mu(\mathrm{EW}) = 153.6\,(1.0) \times 10^{-11}$$

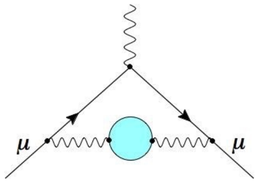
[A. Czarnecki et al, hep-ph/0212229, PRD;
C. Gnendinger et al, arXiv:1306.5546, PRD]

Muon g-2: SM contributions

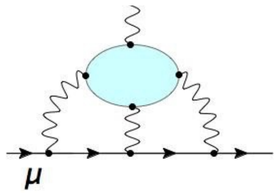
$$a_\mu = a_\mu(\mathrm{QED}) + a_\mu(\mathrm{EW}) + a_\mu(\mathrm{hadronic})$$

leading hadronic

α^2



α^3



✦ The hadronic contributions are written as:

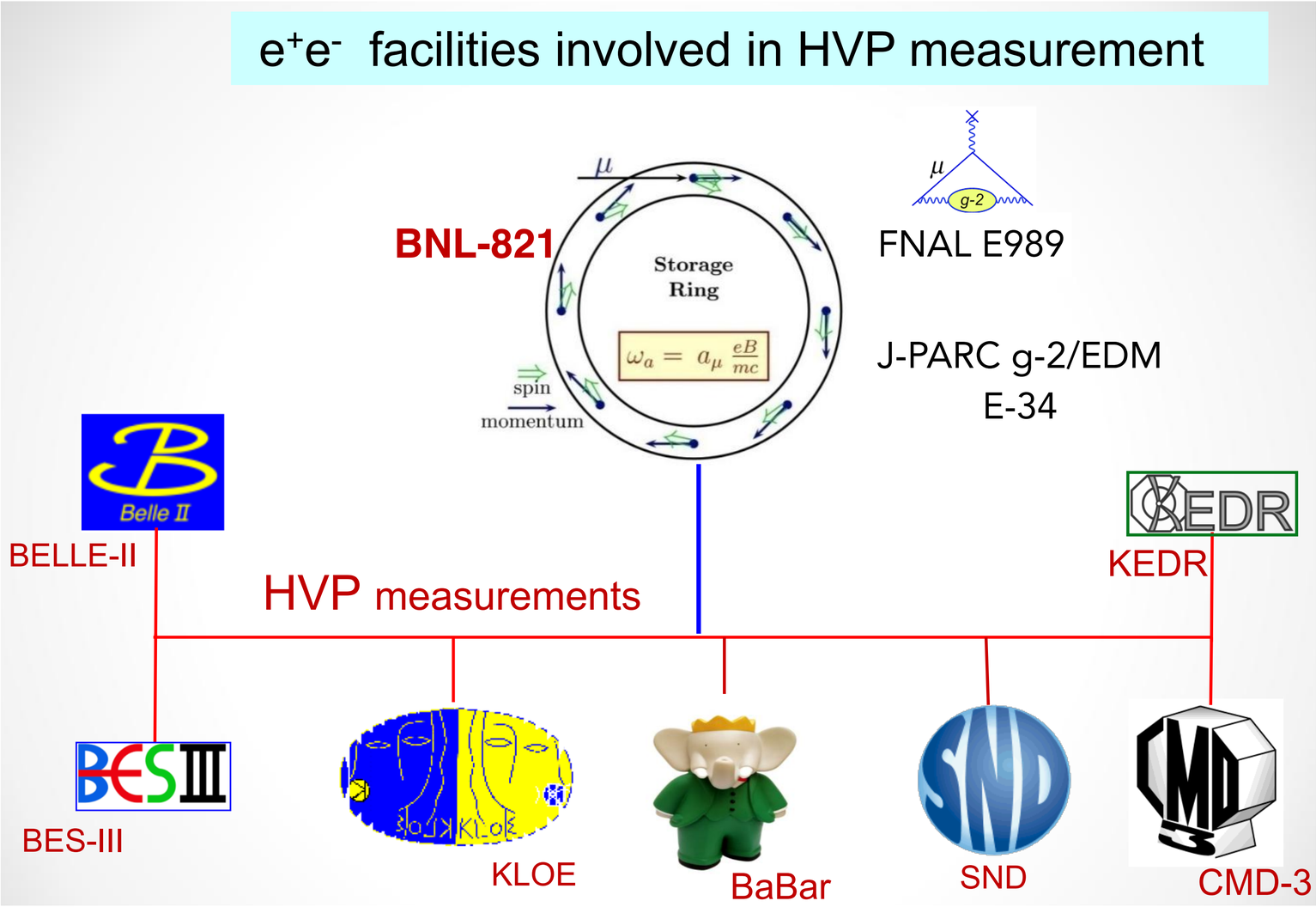
$$a_\ell(\mathrm{hadronic}) = a_\ell^{\mathrm{HVP,\,LO}} + a_\ell^{\mathrm{HVP,\,NLO}} + a_\ell^{\mathrm{HVP,\,NNLO}} + \ldots$$
$$+ a_\ell^{\mathrm{HLbL}} + a_\ell^{\mathrm{HLbL,\,NLO}} + \ldots$$

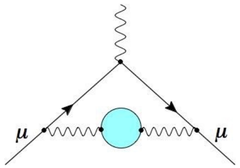
α^2 α^3 α^4

$\sim 10^{-7}$

Experimental Inputs to HVP

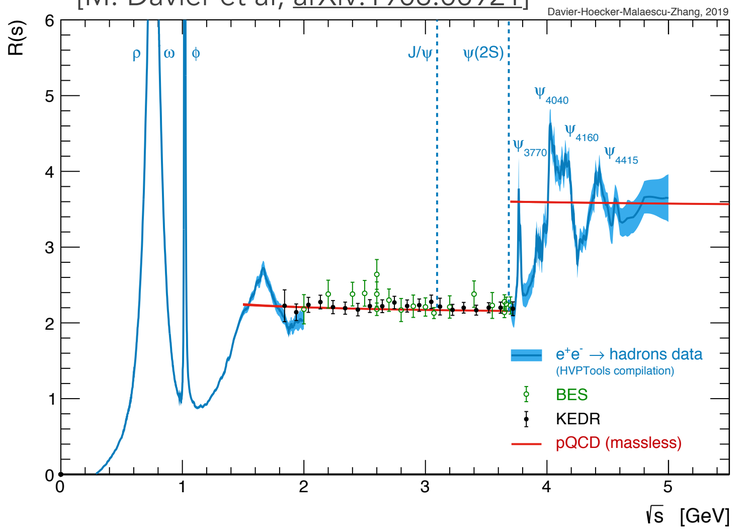
S. Serednyakov (for SND) @ HVP KEK workshop

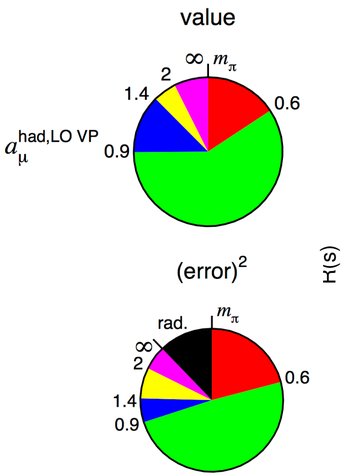




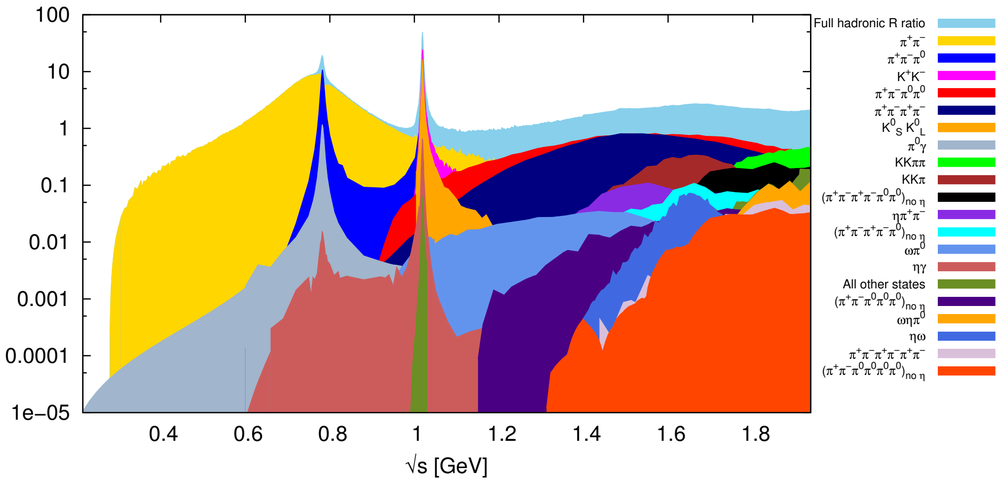
HVP: data-driven

[M. Davier et al, arXiv:1908.00921]





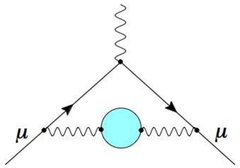
[A. Keshavarzi et al, arXiv:1802.02995]



- total hadronic cross section σ_{had} from > 100 data sets in more than 35 channels summed up to ~ 2GeV
- @ > 2 GeV: inclusive data + pQCD + narrow resonances
- σ_{had} defined to include real & virtual photons
- direct integration method: no need to specify resonances (ρ,…)
- two independent compilations (DHMZ, KNT)

Tensions between BaBar and KLOE data sets:

- Cross checks using analyticity and unitarity relating pion form factor to $\pi\pi$ scattering
- Combinations of data sets affected by tensions
  ➠ conservative merging procedure



HVP: data-driven

Conservative merging procedure to obtain a realistic assessment of the underlying uncertainties:

[B. Malaescu @ INT g-2 workshop]

Detailed comparisons by-channel and energy range between direct integration results:

	DHMZ19	KNT19	Difference
$\pi^+\pi^-$	507.85(0.83)(3.23)(0.55)	504.23(1.90)	3.62
$\pi^+\pi^-\pi^0$	46.21(0.40)(1.10)(0.86)	46.63(94)	−0.42
$\pi^+\pi^-\pi^+\pi^-$	13.68(0.03)(0.27)(0.14)	13.99(19)	−0.31
$\pi^+\pi^-\pi^0\pi^0$	18.03(0.06)(0.48)(0.26)	18.15(74)	−0.12
K^+K^-	23.08(0.20)(0.33)(0.21)	23.00(22)	0.08
$K_S K_L$	12.82(0.06)(0.18)(0.15)	13.04(19)	−0.22
$\pi^0\gamma$	4.41(0.06)(0.04)(0.07)	4.58(10)	−0.17
Sum of the above	626.08(0.95)(3.48)(1.47)	623.62(2.27)	2.46
[1.8, 3.7] GeV (without $c\bar{c}$)	33.45(71)	34.45(56)	−1.00
J/ψ, $\psi(2S)$	7.76(12)	7.84(19)	−0.08
[3.7, ∞) GeV	17.15(31)	16.95(19)	0.20
Total $a_\mu^{\mathrm{HVP,\,LO}}$	$694.0(1.0)(3.5)(1.6)(0.1)_\psi(0.7)_{\mathrm{DV+QCD}}$	692.8(2.4)	1.2

Energy range	ACD18	CHS18	DHMZ19	DHMZ19′	KNT19
≤ 0.6 GeV		110.1(9)	110.4(4)(5)	110.3(4)	108.7(9)
≤ 0.7 GeV		214.8(1.7)	214.7(0.8)(1.1)	214.8(8)	213.1(1.2)
≤ 0.8 GeV		413.2(2.3)	414.4(1.5)(2.3)	414.2(1.5)	412.0(1.7)
≤ 0.9 GeV		479.8(2.6)	481.9(1.8)(2.9)	481.4(1.8)	478.5(1.8)
≤ 1.0 GeV		495.0(2.6)	497.4(1.8)(3.1)	496.8(1.9)	493.8(1.9)
[0.6, 0.7] GeV		104.7(7)	104.2(5)(5)	104.5(5)	104.4(5)
[0.7, 0.8] GeV		198.3(9)	199.8(0.9)(1.2)	199.3(9)	198.9(7)
[0.8, 0.9] GeV		66.6(4)	67.5(4)(6)	67.2(4)	66.6(3)
[0.9, 1.0] GeV		15.3(1)	15.5(1)(2)	15.5(1)	15.3(1)
≤ 0.63 GeV	132.9(8)	132.8(1.1)	132.9(5)(6)	132.9(5)	131.2(1.0)
[0.6, 0.9] GeV		369.6(1.7)	371.5(1.5)(2.3)	371.0(1.6)	369.8(1.3)
$[\sqrt{0.1}, \sqrt{0.95}]$ GeV		490.7(2.6)	493.1(1.8)(3.1)	492.5(1.9)	489.5(1.9)

Include constraints using unitarity & analyticity for $\pi\pi$ and $\pi\pi\pi$ channels

[CHS 2018, Colangelo et al, arXiv:1810.00007; HHKS19, Hoferichter et al, arXiv:1907.01556]

$$\Rightarrow a_\mu^{\mathrm{HVP,LO}} = 693.1\,(2.8)_{\mathrm{exp}}\,(2.8)_{\mathrm{sys}}\,(0.7)_{\mathrm{DV+pQCD}} \times 10^{-10} = 693.1\,(4.0) \times 10^{-10}$$

Efforts on Radiative Corrections for low energy $e^+e^- \to$ hadrons

Workstop+Conference in Zurich 5-9 June 2023 (LOC: A. Signer, G. Stagnitto, Y. Ulrich)

STRONG 2020

RADIO MONTECARLO

Radiative corrections and Monte Carlo tools for low-energy hadronic cross sections in e^+e^- collisions

5–9 Jun 2023
University of Zurich
Europe/Zurich timezone

Enter your search term

- Overview
- Timetable
- Contribution List
- Code of Conduct

Contact
yannick.ulrich@durham....

In this workstop, we will discuss radiative corrections and Monte Carlo tools for low-energy hadronic cross sections in e^+e^- collisions. This is to be seen as part of the Strong 2020 effort. We will cover

- leptonic processes at NNLO and beyond
- processes with hadrons
- parton shower
- experimental input

Each area will be given at least half a day, starting with an open 1h seminar followed by a lengthy discussion.

Just like previous workstops, we try to gather a small number of theorists who actively work on this topic to make very concrete progress. It should not just be about giving talks, but to actually learn from each other and put together the jigsaw pieces.

Additionally to the workstop that is only by-invite only, there is a broader conference directly following the workstop.

Three-day in-person (Workstop) + a three half day hybrid conference

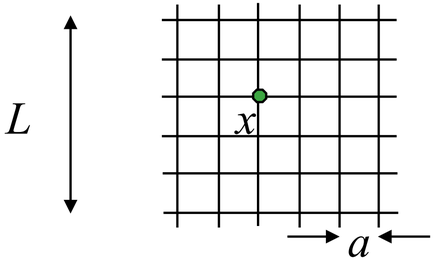
5 Working Groups:

- WP1: Leptonic processes at NNLO
- WP2: Form factor contributions at N3LO
- WP3: Processes with hadrons
- WP4: Parton showers
- WP5: Experimental input

Final goal: full NNLO MC. Aim to write a report by Autumn 2023

Lattice QCD Introduction

$$\mathcal{L}_{\mathrm{QCD}} = \sum_f \bar{\psi}_f (\not{D} + m_f)\psi_f + \frac{1}{4}\mathrm{tr} F_{\mu\nu}F^{\mu\nu}$$

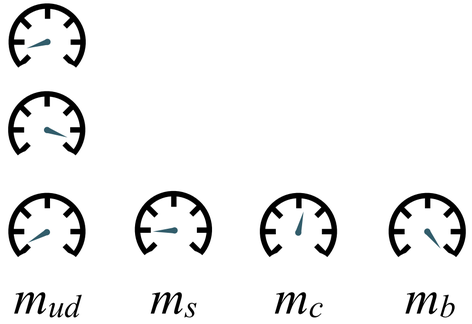


- discrete Euclidean space-time (spacing a)
  derivatives ➞ difference operators, etc…
- finite spatial volume (L)
- finite time extent (T)

Integrals are evaluated numerically using monte carlo methods.

adjustable parameters

- lattice spacing: $a \to 0$
- finite volume, time: $L \to \infty,\ T > L$
- quark masses (m_f): $M_{H,\mathrm{lat}} = M_{H,\mathrm{exp}}$
  tune using hadron masses $m_f \to m_{f,\mathrm{phys}}$
  extrapolations/interpolations



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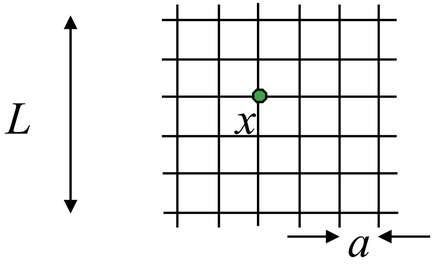

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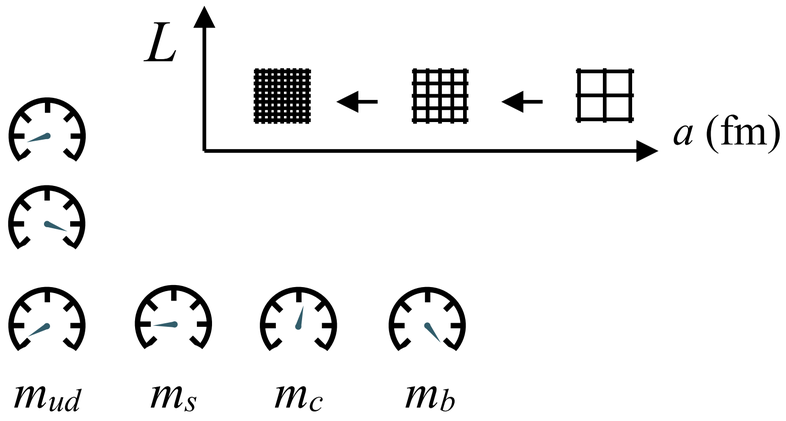

Lattice QCD Introduction

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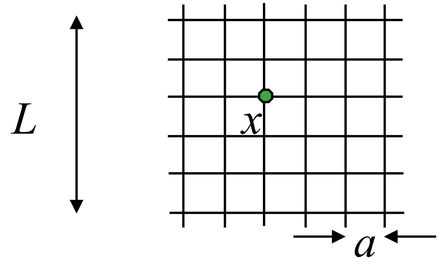

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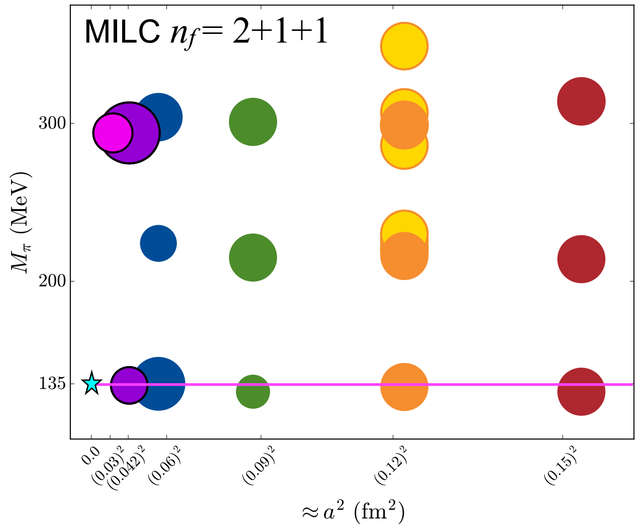

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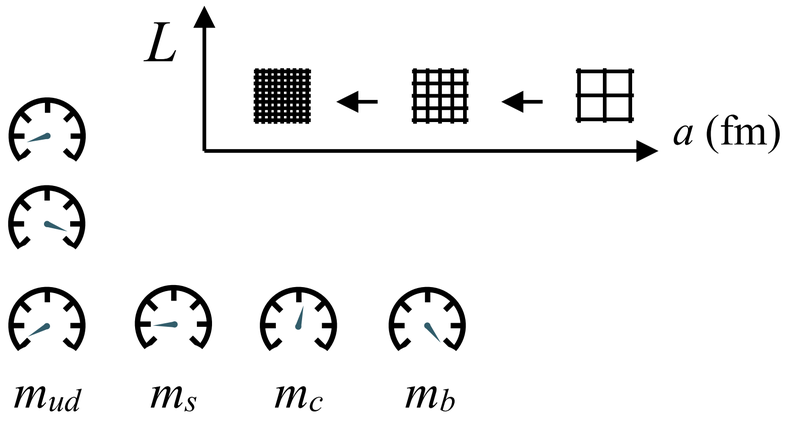
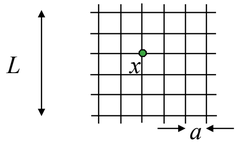




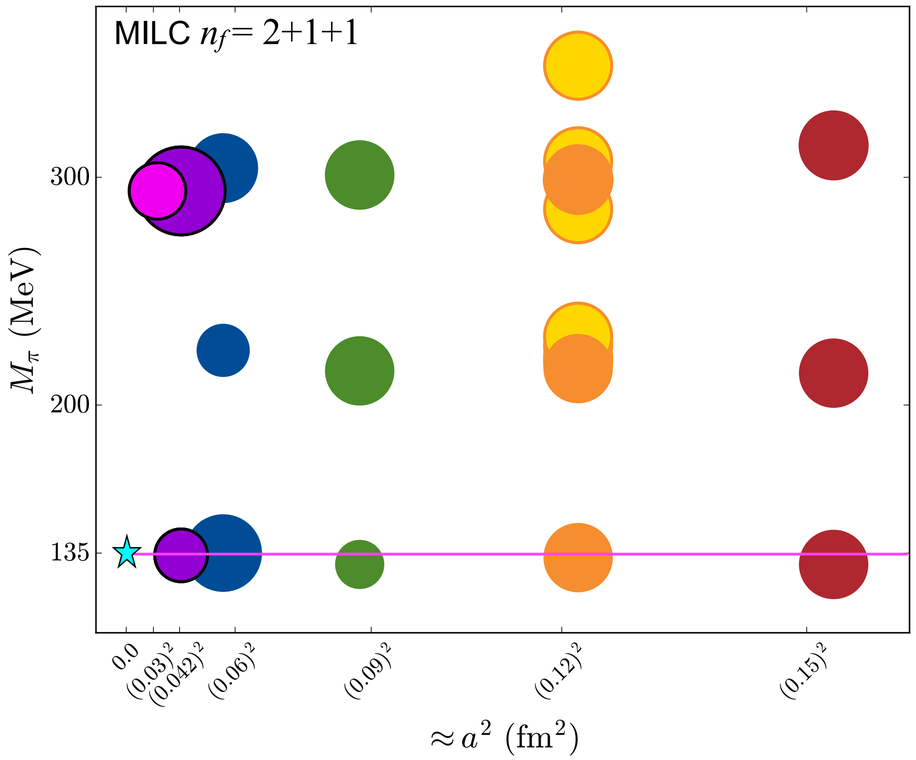
Lattice QCD Introduction

systematic error analysis

...of lattice spacing, chiral, heavy quark, and finite volume effects is based on EFT (Effective Field Theory) descriptions of QCD

➙ ab initio

- finite a: Symanzik EFT
- light quark masses: Chiral Perturbation Theory
- heavy quarks: HQET
- finite L: finite volume EFT

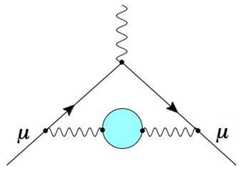


Snowmass 2013 ➠ present

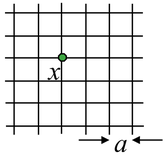
https://www.usqcd.org/documents/13flavor.pdf and [J. Butler et al, arXiv:1311.1076]

Quantity	CKM element	Present expt. error	2007 forecast lattice error	Present lattice error	2018 lattice error	2021 FLAG Average
f_K/f_π	$\lvert V_{us}\rvert$	0.2%	0.5%	0.4%	0.15%	0.18 %
$f_+^{K\pi}(0)$	$\lvert V_{us}\rvert$	0.2%	–	0.4%	0.2%	0.18 %
f_D	$\lvert V_{cd}\rvert$	4.3%	5%	2%	$< 1\%$	0.3 %
f_{D_s}	$\lvert V_{cs}\rvert$	2.1%	5%	2%	$< 1\%$	0.2 %
$D \to \pi\ell\nu$	$\lvert V_{cd}\rvert$	2.6%	–	4.4%	2%	0.7 % [from FNAL/MILC, 2212.12648]
$D \to K\ell\nu$	$\lvert V_{cs}\rvert$	1.1%	–	2.5%	1%	0.6 %
$B \to D^*\ell\nu$	$\lvert V_{cb}\rvert$	1.3%	–	1.8%	$< 1\%$	1.7 % [from FNAL/MILC, 2105.14019]
$B \to \pi\ell\nu$	$\lvert V_{ub}\rvert$	4.1%	–	8.7%	2%	~3 %
f_B	$\lvert V_{ub}\rvert$	9%	–	2.5%	$< 1\%$	0.7 % (0.6 % for f_{B_s})
ξ	$\lvert V_{ts}/V_{td}\rvert$	0.4%	2–4%	4%	$< 1\%$	1.3 %
Δm_s	$\lvert V_{ts}V_{tb}\rvert^2$	0.24%	7–12%	11%	5%	4.5 %
B_K	$\mathrm{Im}(V_{td}^2)$	0.5%	3.5–6%	1.3%	$< 1\%$	1.3 %

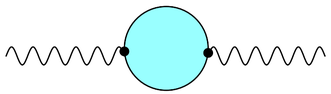
QED corrections important/dominant source of theory error



Lattice HVP: Introduction



[B. Lautrup, A. Peterman, E. de Rafael, Phys. Rep 1972; E. de Rafael, Phys. Let. B 1994; T. Blum, PRL 2002]



$\hat{\Pi}(q^2)$

Leading order HVP correction:

$$a_\mu^{\mathrm{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int dq^2 \omega(q^2)\, \hat{\Pi}(q^2)$$

- Calculate $a_\mu^{\mathrm{HVP,LO}}$ in Lattice QCD

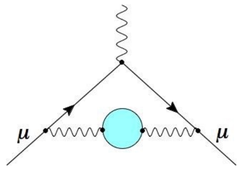
Compute correlation function: $C(t) = \frac{1}{3}\sum_{i,x} \langle j_i(x,t) j_i(0,0)\rangle$

and $\hat{\Pi}(Q^2) = 4\pi^2 \int_0^\infty dt\, C(t) \left[t^2 - \frac{4}{Q^2}\sin^2\left(\frac{Qt}{2}\right)\right]$

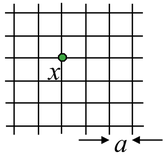
[D. Bernecker and H. Meyer, arXiv:1107.4388, EPJA 2011]

Obtain $a_\mu^{\mathrm{HVP,LO}}$ from an integral over Euclidean time:

$$a_\mu^{\mathrm{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt\, \tilde{w}(t)\, C(t)$$

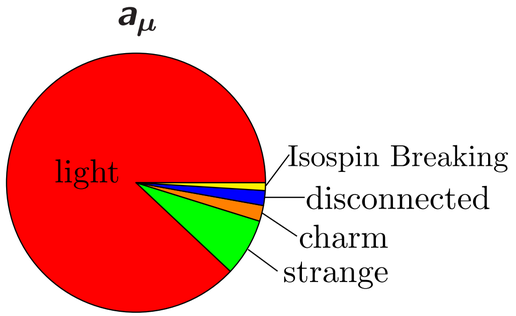


Lattice HVP: Introduction



Calculate $a_\mu^{\rm HVP}$ in Lattice QCD:

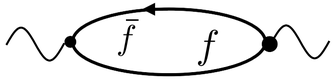
$$a_\mu^{\rm HVP,LO} = \sum_f a_{\mu,f}^{\rm HVP,LO} + a_{\mu,\rm disc}^{\rm HVP,LO}$$



- Separate into connected for each quark flavor + disconnected contributions (gluon and sea-quark background not shown in diagrams)
  Note: almost always $m_u = m_d$

f= ud, s, c, b

$\sum_f$



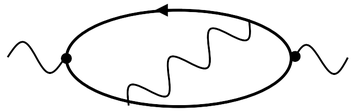
+

- light-quark connected contribution:
  $a_\mu^{\rm HVP,LO}(ud)$ ~90% of total
- s,c,b-quark contributions
  $a_\mu^{\rm HVP,LO}(s,c,b)$ ~8%, 2%, 0.05% of total

- disconnected contribution:
  $a_{\mu,\rm disc}^{\rm HVP,LO}$ ~2% of total

- need to add QED and strong isospin breaking ($\sim m_u - m_d$) corrections:

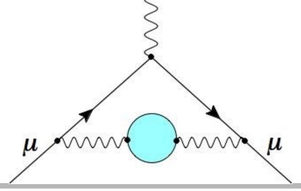
+



+ …

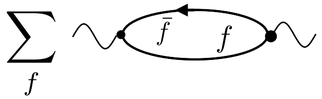
- Isospinbreaking (QED + $m_u \neq m_d$) corrections:
  $\delta a_\mu^{\rm HVP,LO}$ ~1% of total

$$a_\mu^{\rm HVP,LO} = a_\mu^{\rm HVP,LO}(ud) + a_\mu^{\rm HVP,LO}(s) + a_\mu^{\rm HVP,LO}(c) + a_{\mu,\rm disc}^{\rm HVP,LO} + \delta a_\mu^{\rm HVP,LO}$$

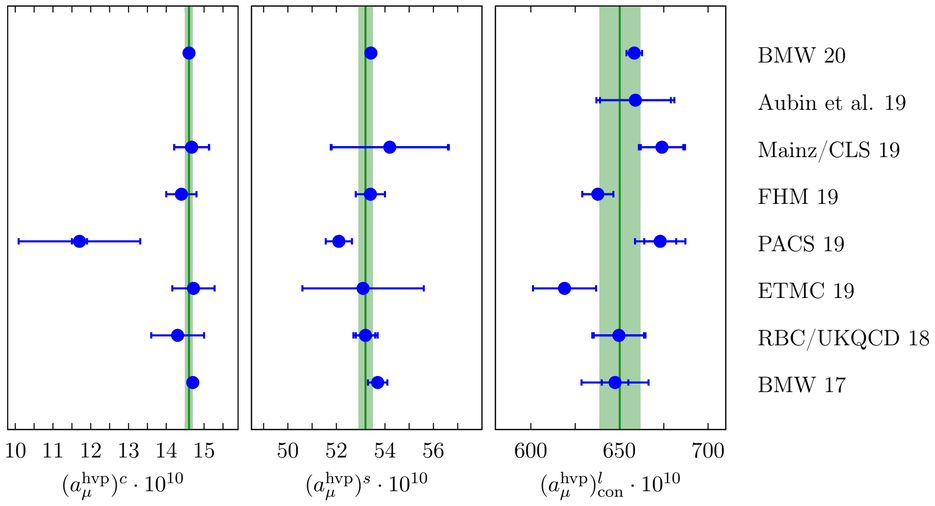


Lattice HVP: subleading corrections

H. Wittig @ Lattice HVP workshop

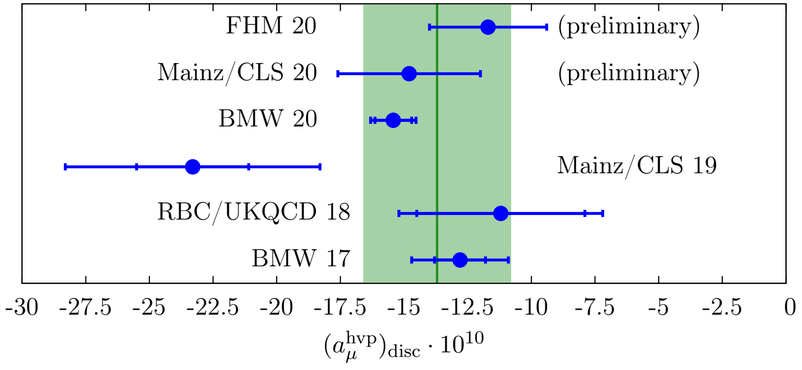


- Charm, strange contributions already well determined.
- Mild tensions for light contribution

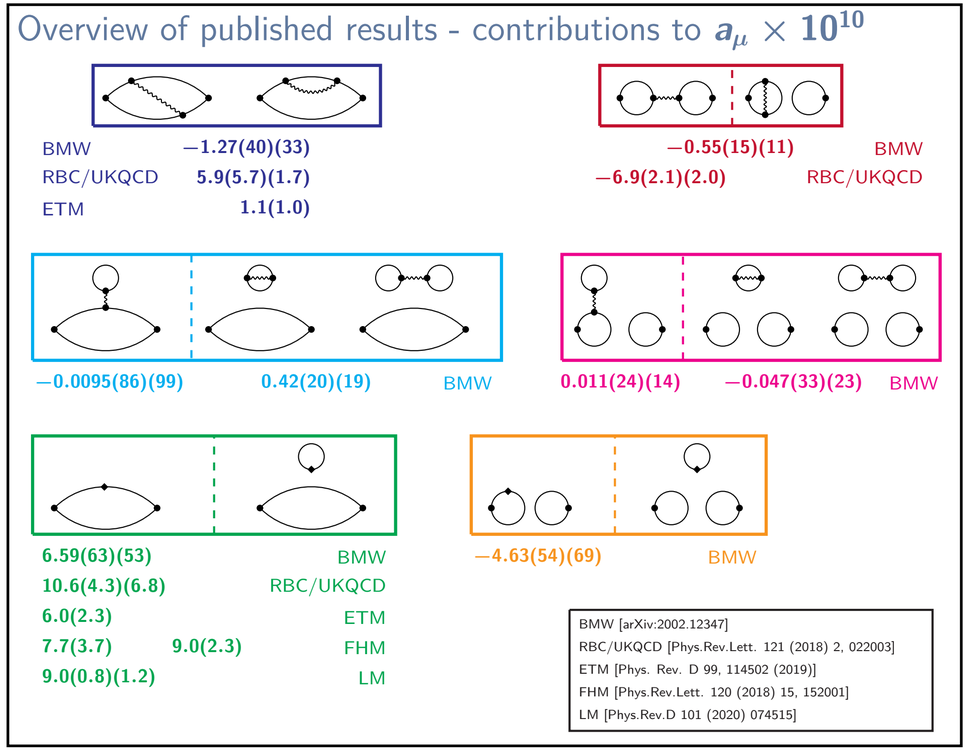


Ongoing efforts by FNAL-HPQCD-MILC RBC/UKQCD, Mainz

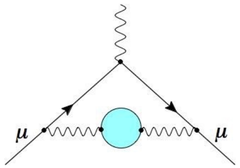
Consistent results with increasing precision



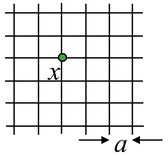
V. Gülpers @ Lattice HVP workshop



- Some tensions between lattice results for individual contributions.
- Large cancellations between individual contributions: $\delta a_\mu^{\rm IB} \lesssim 1\,\%$



HVP: lattice



In 2020 WP:

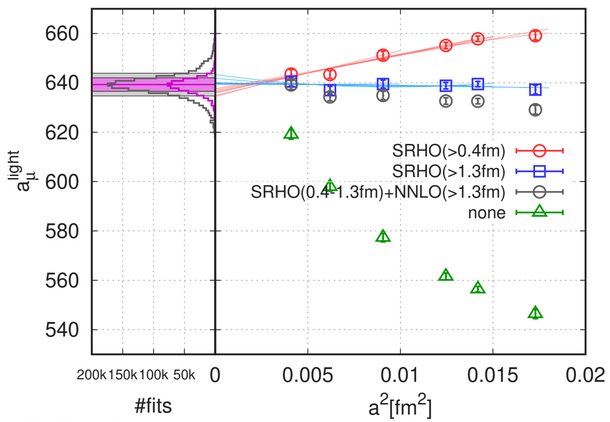
- Lattice HVP average at $2.6\,\%$ total uncertainty: $a_\mu^{\mathrm{HVP,LO}} = 711.6\,(18.4) \times 10^{10}$
- BMW 20 (published in 2021) first LQCD calculation with sub-percent ($0.8\,\%$) error in tension with data-driven HVP (2.1σ)
- total error in BMW calculation is dominated by light-quark connected contribution.
- Large taste-breaking effects with BMW set-up
  ➠ uncorrected data not easily fit to power series, i.e.

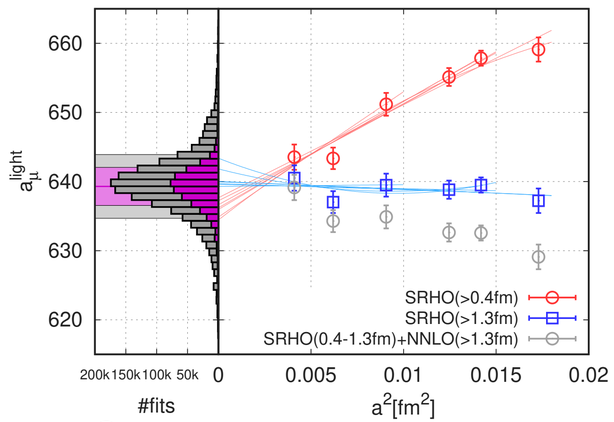
1. $A_0 + A_1\left[a^2\right] + A_2\left[a^2\right]^2$
2. $A_0 + A_1\left[a^2\alpha_s^3(\frac{1}{a})\right] + A_2\left[a^2\alpha_s^3(\frac{1}{a})\right]^2$

Kalman Szabo (BMW) @ Lattice 2021

Taste improvement II

- $a_\mu(a) \to a_\mu(a) - a_\mu^{\mathrm{SRHO}}(a) + a_\mu^{\mathrm{RHO}}$
- reduces lattice artefact, also makes a^2 dependence linear





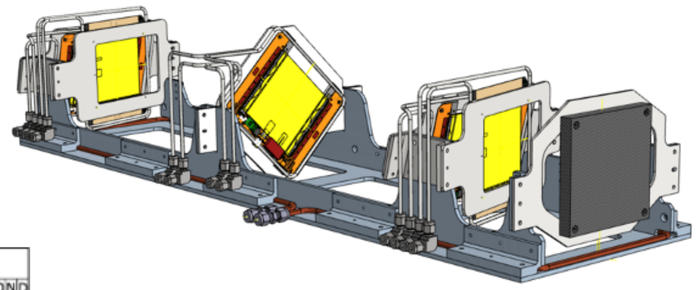
SRHO improvement gives central value. Systematic errors by:

1. change starting point of improvement $t = 0.4 \to 1.3$ fm
2. skip coarse lattices
3. change $\Gamma = 0$ and $\Gamma = 3$
4. replace SRHO by NNLO SXPT above 1.3 fm

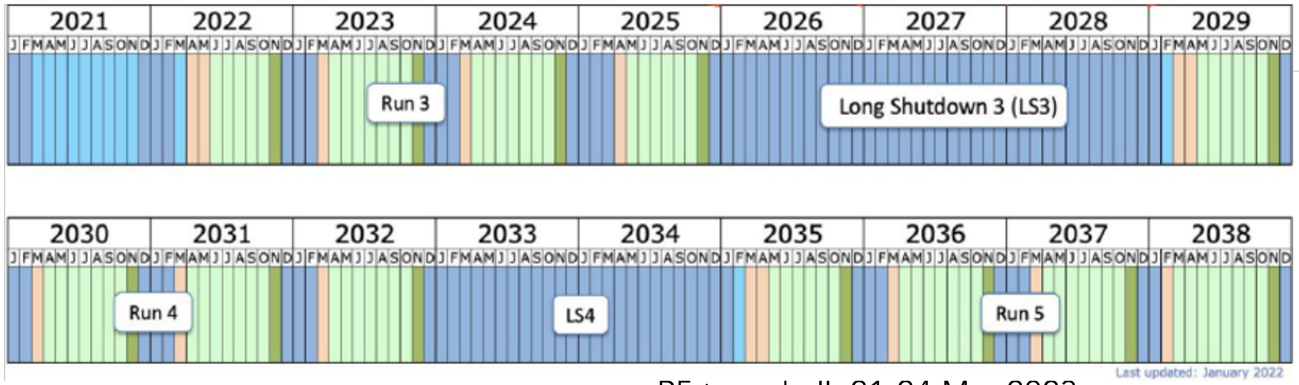
Alternative measurement of HVP for a_μ^{HVP} : MUonE at CERN

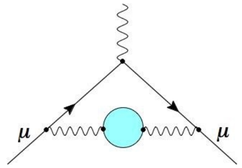
- Space-like determination of a_μ^{HVP} at <0.5% through the scattering of 160 GeV muons on electron target
- Much progress in the last years, inc. detector optimization and development of $\mu - e$ (N)NLO MC
- Staged approach towards the full experiment: one station (2022), two stations (2023); possible 10 stations before LS3 (2026) (2% accuracy)
- Technical proposal towards full experiment in preparation
- Growing interest from both experimental and theory community

-C. M. Carloni Calame et al PLB 746 (2015) 325
-G. Abbiendi et al Eur.Phys.J.C 77 (2017) 3, 139
-LoI https://cds.cern.ch/record/2677471/files/SPSC-I-252.pdf



LHC schedule

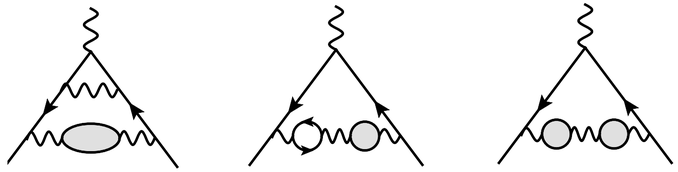




HVP: higher order (NLO, NNLO)

Figures by T. Teubner

$a_\mu^{\mathrm{HVP,NLO}} = -9.83(7) \times 10^{-10}$ [based on KNT 2019]



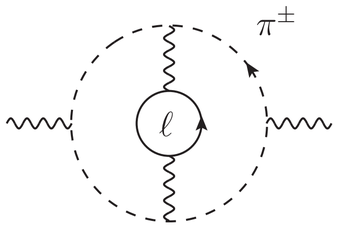
$a_\mu^{\mathrm{HVP,NNLO}} = 1.24(1) \times 10^{-10}$ [Kurz et al, arXiv:1403.6400, PLB 2014]

space-like NLO and NNLO HVP kernels for LQCD evaluations and MUonE

[Balsani et al, arXiv:2112.05704; Nesterenko, arXiv:2209.03217, arXiv: 2112.05009]

mixed leptonic, hadronic (double bubble) contributions to a_μ are $< 10^{-11}$

[Hoferichter + Teubner, arXiv:2112.06929]



Connections

$$\sigma(e^+e^- \to \text{hadrons}) \quad \Leftrightarrow \quad a_\mu^{\text{HVP}} \quad \Leftrightarrow \quad \Delta\alpha_{\text{had}}(M_Z^2)$$

- $\Delta\alpha_{\text{had}}(M_Z^2)$ also depends on the hadronic vacuum polarization function, and can be written as an integral over $\sigma(e^+e^- \to \text{hadrons})$, but weighted towards higher energies.
- a shift in a_μ^{HVP} also changes $\Delta\alpha_{\text{had}}(M_Z^2)$: ➠ EW fits
  [Passera, et al, 2008, Crivellin et al 2020, Keshavarsi et al 2020, Malaescu & Scott 2020]
  If the shift in a_μ^{HVP} is in the low-energy region ($\lesssim 1\,\text{GeV}$), the impact on $\Delta\alpha_{\text{had}}(M_Z^2)$ and EW fits is small.
- A shift in a_μ^{HVP} from low ($\lesssim 2\,\text{GeV}$) energies
  ➠ $\sigma(e^+e^- \to \pi\pi)$
  must satisfy unitarity & analyticity constraints ➠ $F_\pi^V(s)$
  can be tested with lattice calculations
  [Colangelo, Hoferichter, Stoffer, arXiv:2010.07943]

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Martin Hoferichter @ Lattice HVP workshop

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Hadronic running of α and global EW fit

	e^+e^- KNT, DHMZ	EW fit HEPFit	EW fit GFitter	guess based on BMWc
$\Delta\alpha_{\text{had}}^{(5)}(M_Z^2) \times 10^4$	276.1(1.1)	270.2(3.0)	271.6(3.9)	277.8(1.3)
difference to e^+e^-		-1.8σ	-1.1σ	$+1.0\sigma$

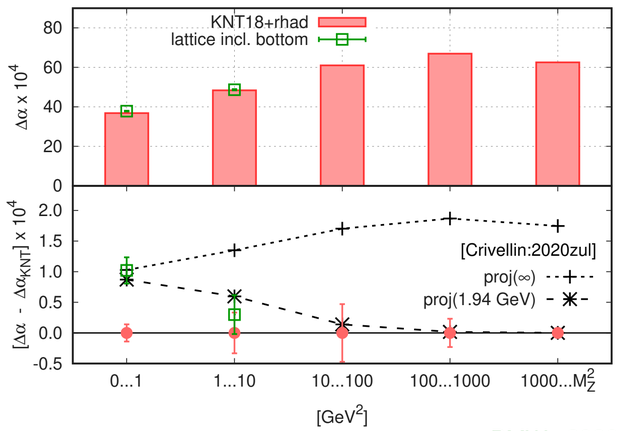
- **Time-like formulation**:

$$\Delta\alpha_{\text{had}}^{(5)}(M_Z^2) = \frac{\alpha M_Z^2}{3\pi} P \int_{s_{\text{thr}}}^{\infty} ds \frac{R_{\text{had}}(s)}{s(M_Z^2 - s)}$$

- **Space-like formulation**:

$$\Delta\alpha_{\text{had}}^{(5)}(M_Z^2) = \frac{\alpha}{\pi}\hat{\Pi}(-M_Z^2) + \frac{\alpha}{\pi}\left(\hat{\Pi}(M_Z^2) - \hat{\Pi}(-M_Z^2)\right)$$

- Global EW fit
  - Difference between HEPFit and GFitter implementation mainly treatment of M_W
  - Pull goes into **opposite direction**



BMWc 2020

More in talks by M. Passera, B. Malaescu (phenomenology) and K. Miura, T. San José (lattice)

Connections

$$\sigma(e^+e^- \to \text{hadrons}) \quad \Leftrightarrow \quad a_\mu^{\text{HVP}} \quad \Leftrightarrow \quad \Delta\alpha_{\text{had}}(M_Z^2)$$

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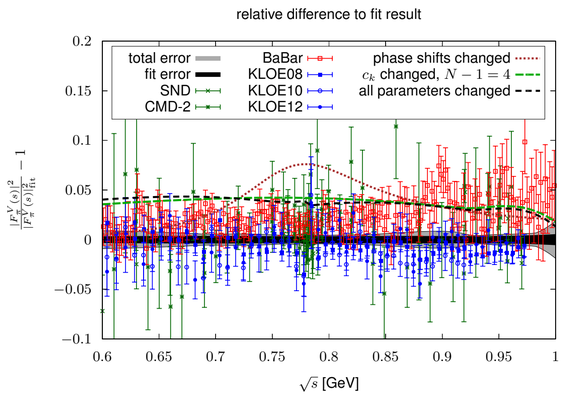
Peter Stoffer @ Lattice HVP workshop

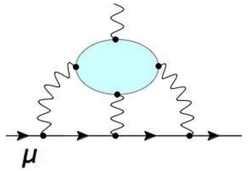
Constraints on the two-pion contribution to HVP — arXiv:2010.07943 [hep-ph]

Modifying $a_\mu^{\pi\pi}|_{\leq 1\,\text{GeV}}$

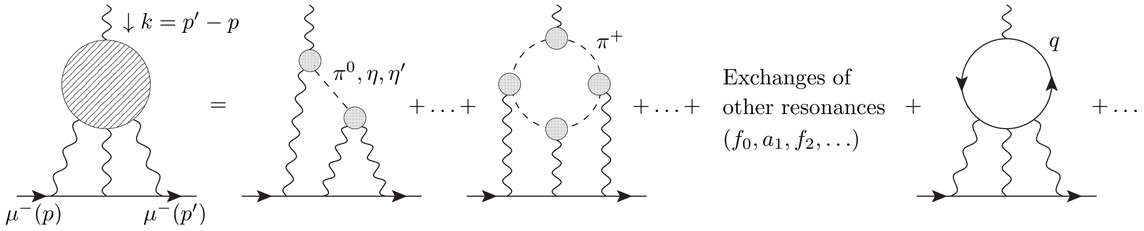
- "low-energy" scenario: local changes in cross section of $\sim 8\%$ **around ρ**
- "high-energy" scenario: impact on **pion charge radius** and space-like VFF $\Rightarrow$ chance for **independent lattice-QCD checks**
- requires **factor ~ 3 improvement** over χQCD result: $\langle r_\pi^2 \rangle = 0.433(9)(13)\,\text{fm}^2$ → arXiv:2006.05431 [hep-ph]



- Can new physics hide in the low-energy $\sigma(e^+e^- \to \pi\pi)$ cross section? ➠ No [Luzio, et al, arXiv:2112.08312]
- Neutral, long-lived hadrons, heretofore undetected? [Farrar, arXiv:2206.13460]



Hadronic Light-by-light

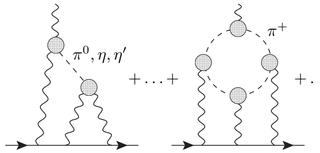


Dispersive approach:

[Colangelo at al, 2014; Pauk & Vanderhaegen 2014; …]

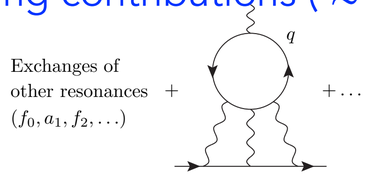
- ✦ model independent
- ✦ significantly more complicated than for HVP
- ✦ provides a framework for data-driven evaluations
- ✦ can also use lattice results as inputs

Dominant contributions ($\approx 75\,\%$ of total):



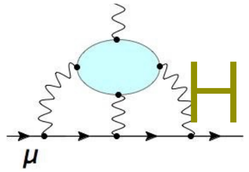
- ✦ Well quantified with $\approx 6\,\%$ uncertainty
- ✦ η, η' pole contributions: Canterbury approximants only
- ✦ Ongoing work: consolidation of η, η' pole contributions using disp. relations and LQCD

Subleading contributions ($\approx 25\,\%$ of total):



- ✦ Not yet well known
  ➠ dominant contribution to total uncertainty
- ✦ Ongoing work:
  - Implementation of short-distance constraints (now at 2-loop)
  - DR implementation for axial vector contributions
  - new $q_4 = 0$ DR program for higher spin intermediate states
    [Luedtke @ Higgscentre workshop with Procura and Stoffer, in progress]
  - Mainz and BESIII ramping up $\gamma^{(*)}\gamma^*$ programs
    [A. Denig and C. Redmer @ Higgscentre workshop]

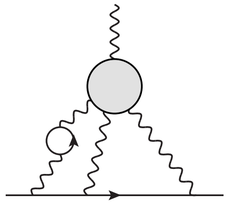
Dispersive, data-driven evaluation of HLbL with $\leq 10\,\%$ total uncertainty feasible by ~2025.



HLbL: dispersive

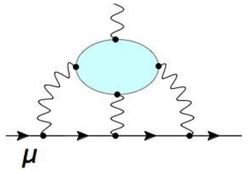
Comparison:

Contribution	PdRV(09) [471]	N/JN(09) [472, 573]	J(17) [27]	Our estimate
π^0, η, η'-poles	114(13)	99(16)	95.45(12.40)	93.8(4.0)
π, K-loops/boxes	−19(19)	−19(13)	−20(5)	−16.4(2)
S-wave $\pi\pi$ rescattering	−7(7)	−7(2)	−5.98(1.20)	−8(1)
subtotal	88(24)	73(21)	69.5(13.4)	69.4(4.1)
scalars	–	–	–	} − 1(3)
tensors	–	–	1.1(1)	
axial vectors	15(10)	22(5)	7.55(2.71)	6(6)
u, d, s-loops / short-distance	–	21(3)	20(4)	15(10)
c-loop	2.3	–	2.3(2)	3(1)
total	105(26)	116(39)	100.4(28.2)	92(19)

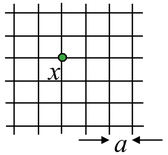


NLO HLbL contribution:

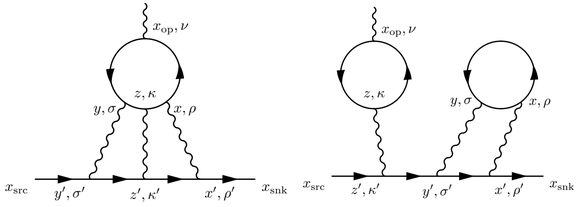
$$a_\mu^{\mathrm{HLbL,NLO}} = 2\,(1) \times 10^{-11}$$

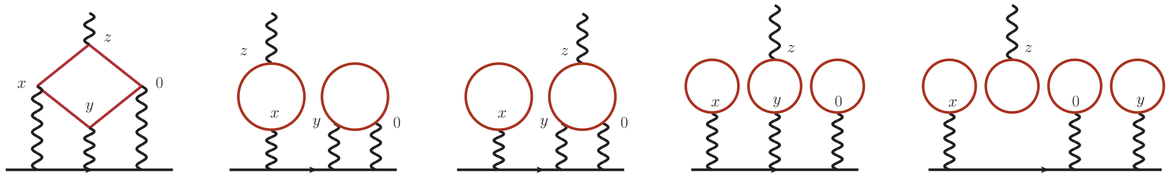


Hadronic Light-by-light



Lattice QCD+QED: Two independent and complete direct calculations of a_μ^{HLbL}





- RBC/UKQCD
  [T. Blum et al, arXiv:1610.04603, 2016 PRL; arXiv:1911.08123, 2020 PRL]
- QCD + $\mathrm{QED_L}$ (finite volume)

  DWF ensembles at/near phys mass,
  $a \approx 0.08 - 0.2\,\mathrm{fm}$, $L \sim 4.5 - 9.3\,\mathrm{fm}$

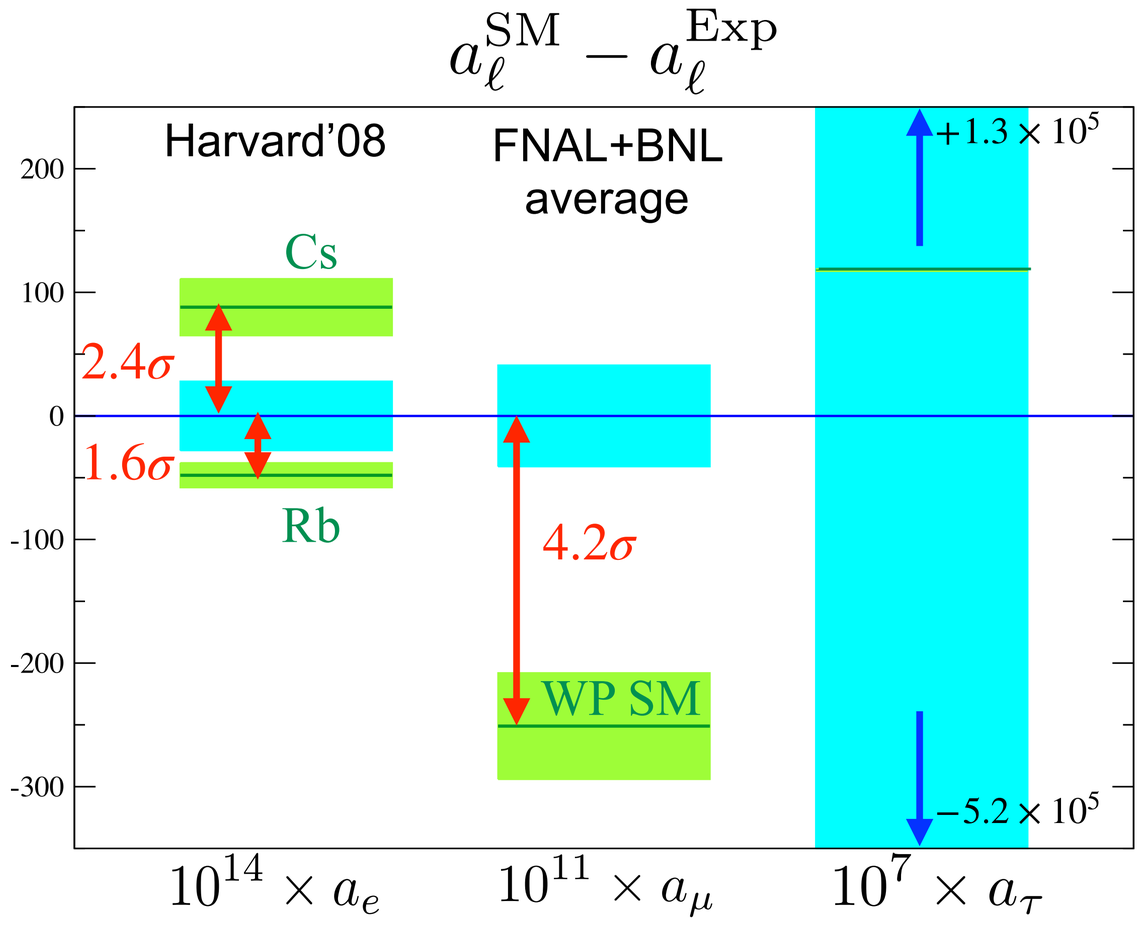
- Mainz group
  [E. Chao et al, arXiv:2104.02632]
- QCD + QED (infinite volume & continuum)

  CLS (2+1 Wilson-clover) ensembles
  $m_\pi \sim 200 - 430\,\mathrm{MeV}$, $a \approx 0.05 - 0.1\,\mathrm{fm}$, $m_\pi L > 4$

- Cross checks between RBC/UKQCD & Mainz approaches in White Paper at unphysical pion mass
- Both groups are continuing to improve their calculations, adding more statistics, lattice spacings, physical mass ensemble (Mainz)
- update from RBC/UKQCD [T. Blum @ Higgscentre workshop] preliminary results from QCD + QED (inf.)

Lattice HLbL results with $10\,\%$ total uncertainty feasible by ~2025

Lepton moments summary



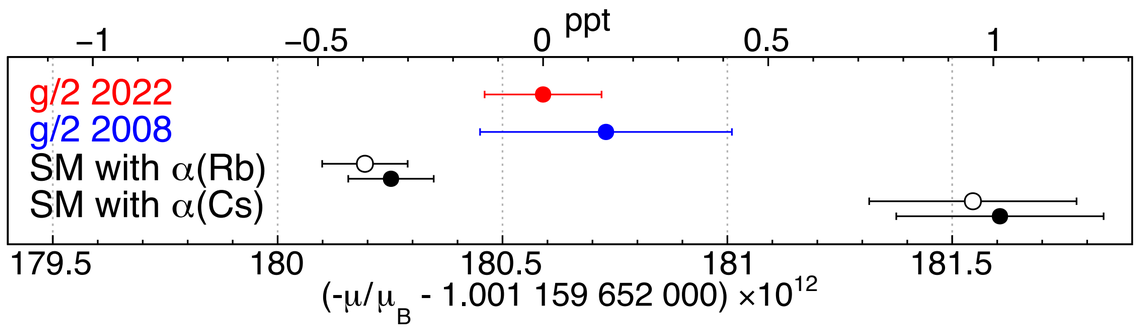
Cs: α from Berkeley group [Parker et al, Science 360, 6385 (2018)]

Rb: α from Paris group [Morel et al, Nature 588, 61–65(2020)]

Sensitivity to heavy new physics: $a_\ell^{\rm NP} \sim \frac{m_\ell^2}{\Lambda^2}$

$(m_\mu/m_e)^2 \sim 4\times10^4$

[X. Fan et al (Gabrielse group), arXiv:2209.13084]



Prospects for tau moment measurement:

Chiral Belle arXiv:2205.12847

- ☆ use polarized e^- beam
- ☆ with $40ab^{-1}$ measurement of a_τ at 10^{-5} feasible
- ☆ with more statistics measurement at 10^{-6} possible