Resummation beyond leading power

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LHC is a precision machine!

- Experiments at the LHC will reach 1% precision for some observables
- Run III & High-luminosity upgrade will deliver a lot of luminosity — enormous increase in statistics
- Some future measurements will be limited by the theoretical accuracy
- Energy frontier is becoming now the **Precision Frontier** and relies strongly on the theoretical input

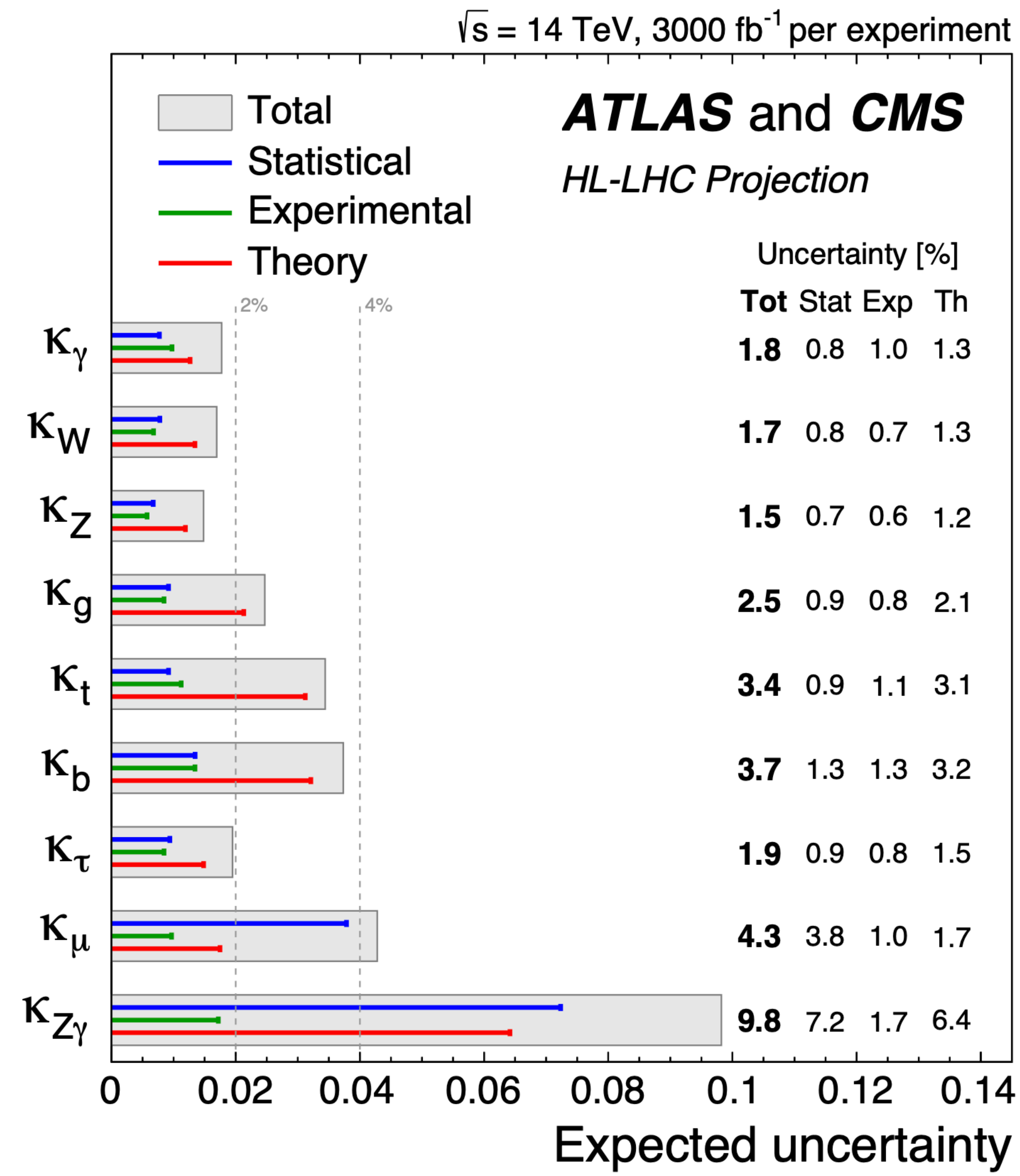

Figure from arXiv:1902.00134

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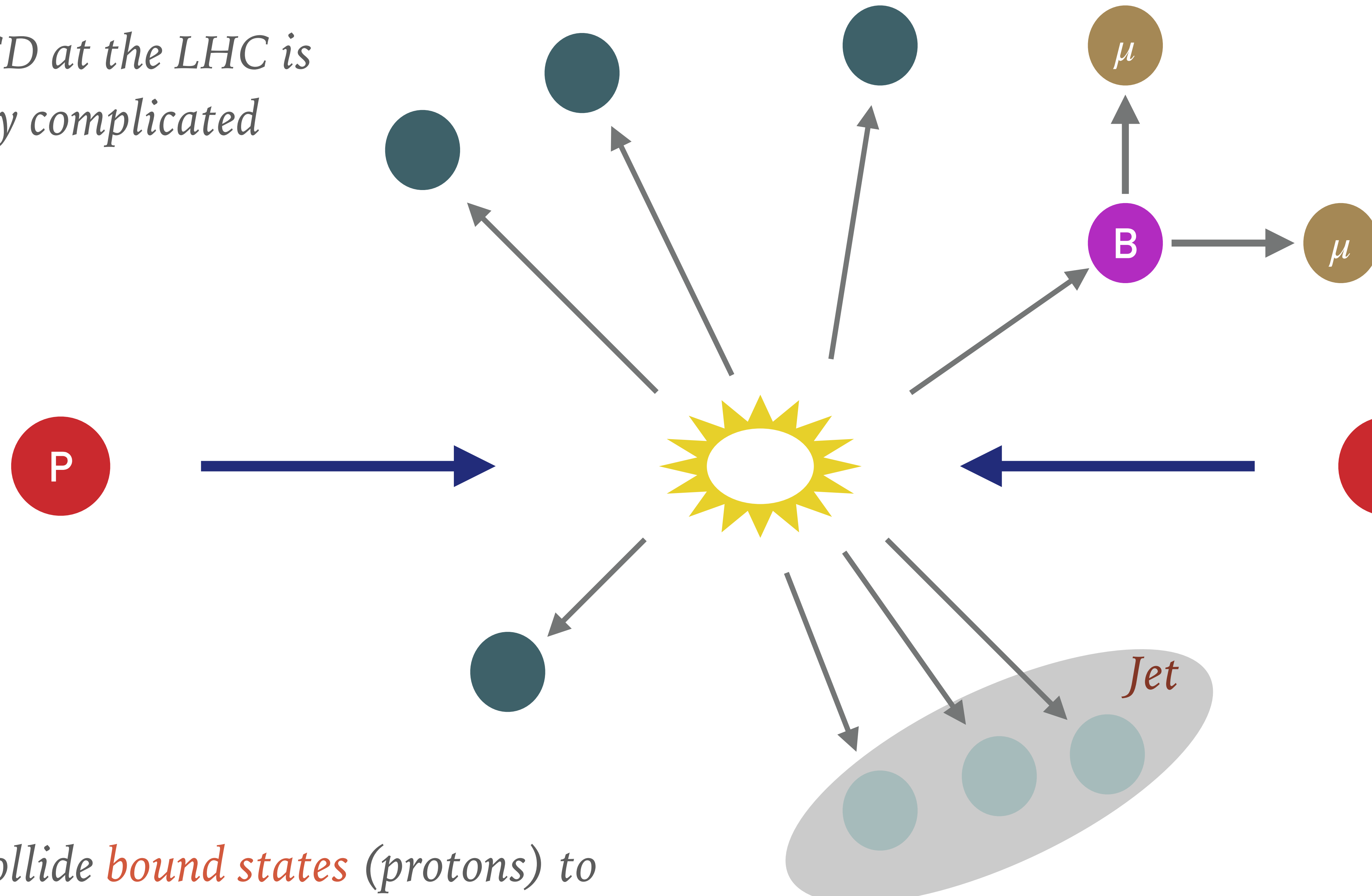
HOW DO WE MAKE PREDICTIONS FOR THE LHC?

QCD at the LHC is very complicated

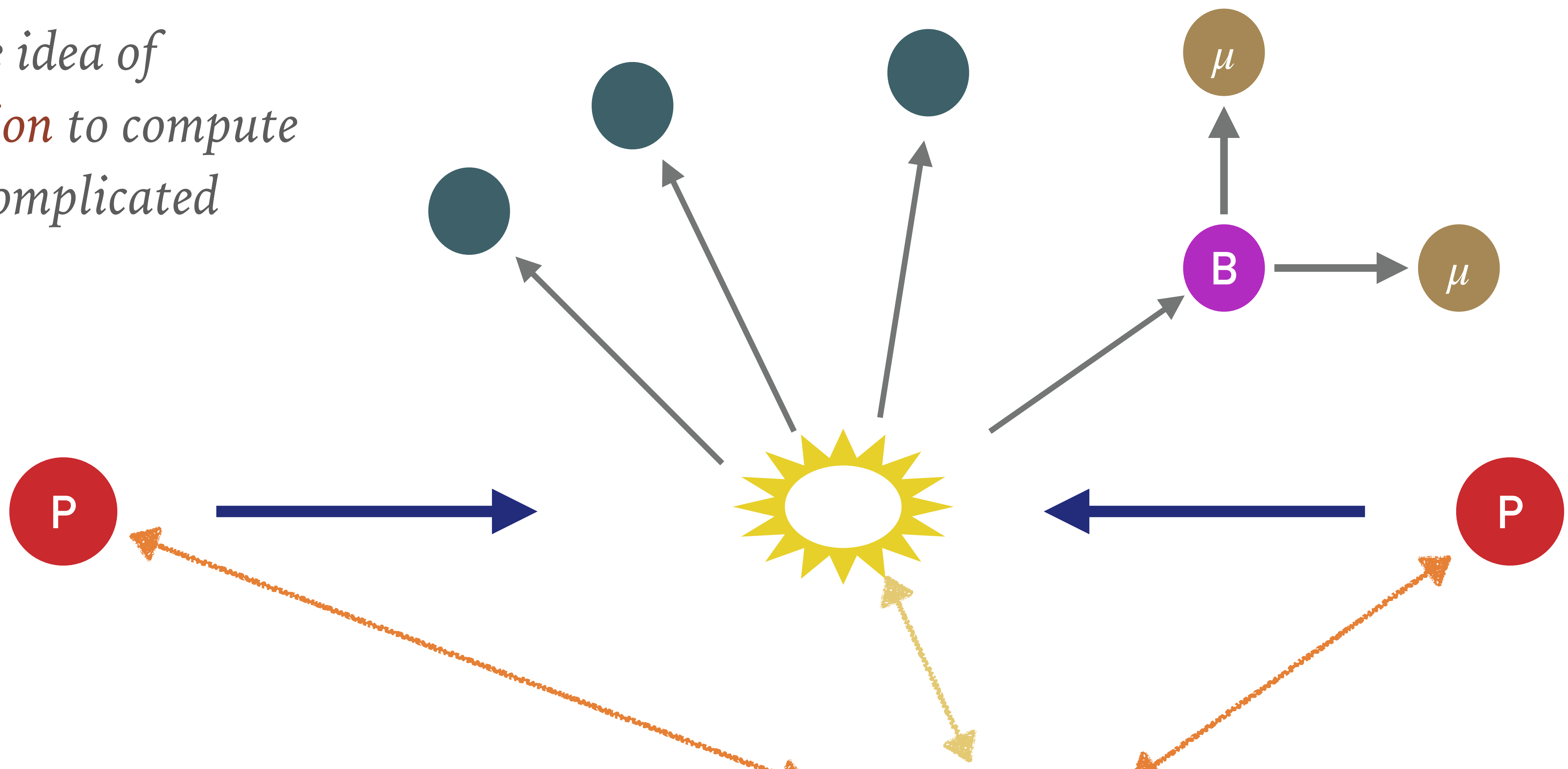


We collide *bound states* *(protons) to produce a plethora of new particles, many of which are bound states themselves*

Bound states → internal structure

FACTORIZATION

We use the idea of factorization to compute rates for complicated processes



Long and short distance physics can be described separately

$$d\sigma \sim \sum_{a,b} \int_0^1 dx_a dx_b \, f_{a/A}(x_a) \, \hat{\sigma}_{ab}(z) \, f_{b/B}(x_b) + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{Q}\right)$$

PERTURBATIVE CALCULATIONS

$$d\hat{\sigma}_{N_1N_2\to X} = d\hat{\sigma}^{(0)}_{N_1N_2\to X} + \alpha_s\, d\hat{\sigma}^{(1)}_{N_1N_2\to X} + \alpha_s^2\, d\hat{\sigma}^{(2)}_{N_1N_2\to X} + \alpha_s^3\, d\hat{\sigma}^{(3)}_{N_1N_2\to X} + \ldots$$

	LO	NLO	NNLO	N3LO
$\alpha_S(m_Z) = 0.118$		↓	↓	↓
Naive expectation		10%	1%	0.1%

Do we need N3LO?

ASSOCIATED HIGGS PRODUCTION (HIGGSSTRAHLUNG @ N3LO)

$$\mu_0 = M_H + M_V$$

Process	$\sigma^{\rm LO}$ [pb]	$\sigma^{\rm NLO}$ [pb]	$\rm K^{NLO}$	$\sigma^{\rm NNLO}$ [pb]	$\rm K^{NNLO}$	$\sigma^{\rm N^3LO}$ [pb]	$\rm K^{N^3LO}$
W^+H	$0.753^{+2.70\%}_{-3.49\%}$	$0.886^{+1.54\%}_{-1.27\%}$	1.177	$0.891^{+0.24\%}_{-0.29\%}$	1.006	$0.883^{+0.29\%}_{-0.34\%}$	0.991
W^-H	$0.480^{+2.79\%}_{-3.63\%}$	$0.562^{+1.49\%}_{-1.23\%}$	1.170	$0.564^{+0.24\%}_{-0.29\%}$	1.004	$0.558^{+0.31\%}_{-0.36\%}$	0.990
ZH	$0.673^{+2.66\%}_{-3.44\%}$	$0.788^{+1.48\%}_{-1.23\%}$	1.171	$0.792^{+0.22\%}_{-0.27\%}$	1.005	$0.785^{+0.28\%}_{-0.32\%}$	0.991

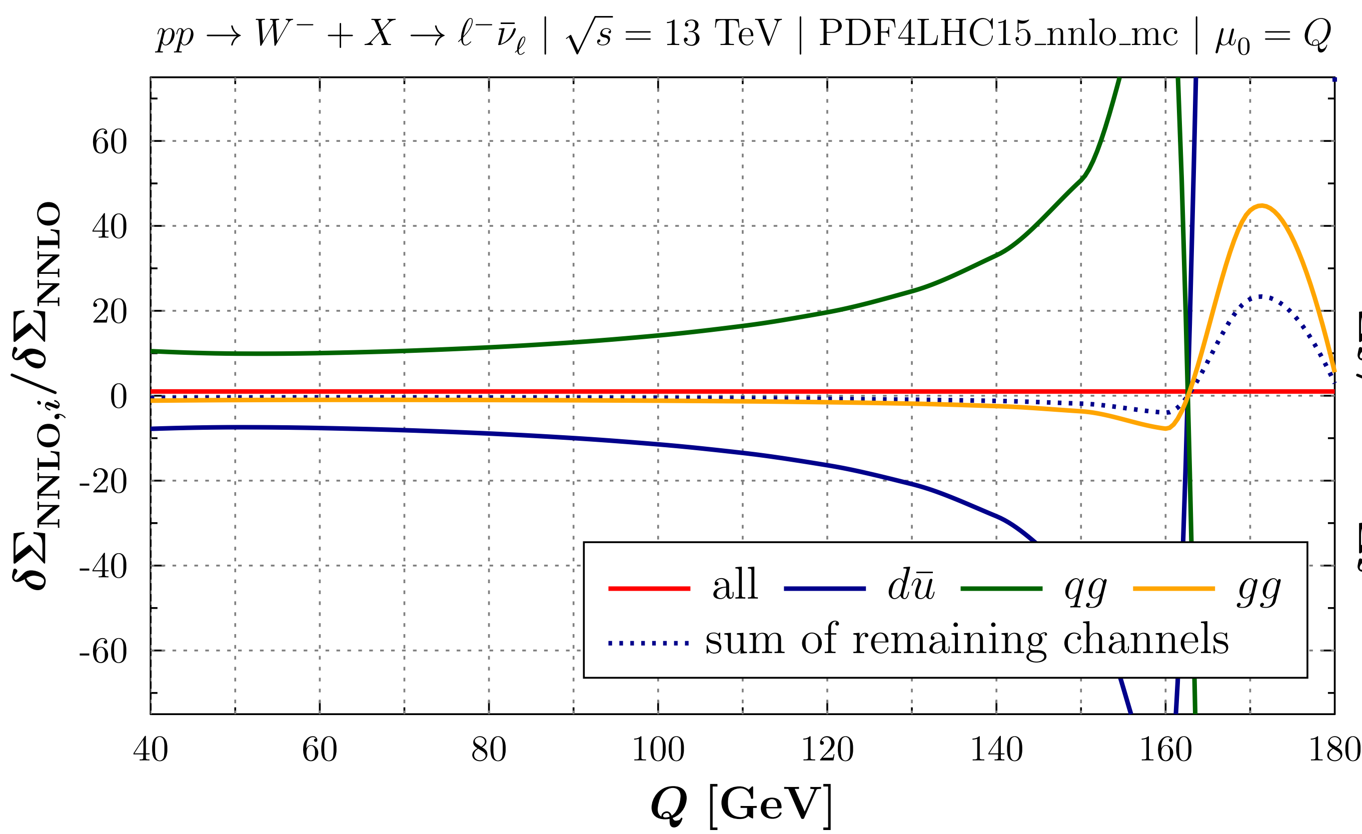
$$\mu_0 = M_{HV}$$

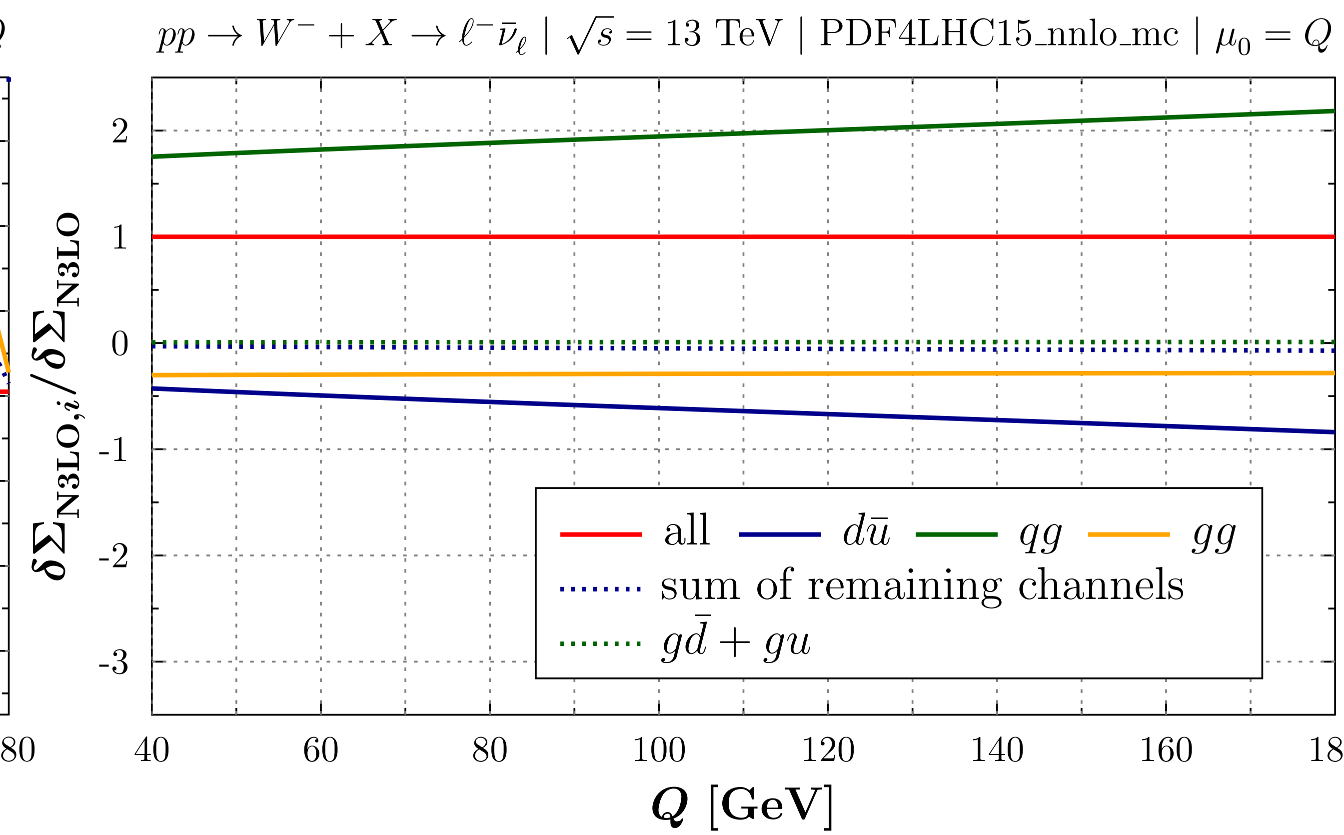
Process	$\sigma^{\rm LO}$ [pb]	$\sigma^{\rm NLO}$ [pb]	$\rm K^{NLO}$	$\sigma^{\rm NNLO}$ [pb]	$\rm K^{NNLO}$	$\sigma^{\rm N^3LO}$ [pb]	$\rm K^{N^3LO}$
W^+H	$0.758^{+2.43\%}_{-3.13\%}$	$0.883^{+1.38\%}_{-1.20\%}$	1.165	$0.891^{+0.28\%}_{-0.34\%}$	1.009	$0.884^{+0.27\%}_{-0.30\%}$	0.992
W^-H	$0.484^{+2.50\%}_{-3.26\%}$	$0.560^{+1.34\%}_{-1.23\%}$	1.158	$0.564^{+0.27\%}_{-0.34\%}$	1.007	$0.559^{+0.30\%}_{-0.33\%}$	0.991
ZH	$0.678^{+2.40\%}_{-3.11\%}$	$0.786^{+1.33\%}_{-1.16\%}$	1.159	$0.792^{+0.25\%}_{-0.32\%}$	1.008	$0.786^{+0.26\%}_{-0.29\%}$	0.992

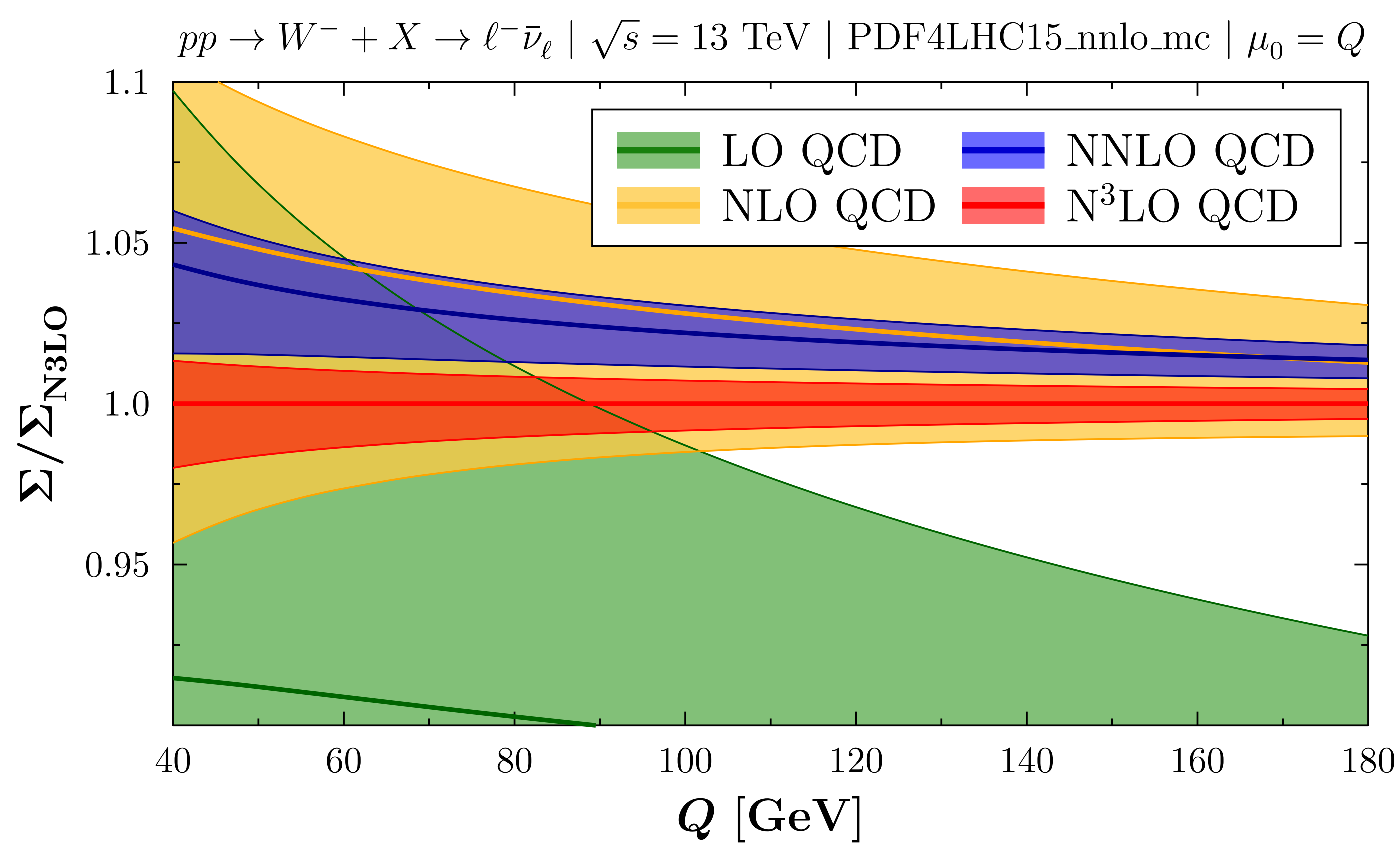
- *N3LO and NNLO corrections are comparable and partially cancel*
- *Scale uncertainty does not decrease when going from NNLO to N3LO*

J. Baglio, C. Duhr, B. Mistlberger, R.S., 2022

PATH TOWARDS 1% ACCURACY







$PP \to W^-, \sqrt{s} = 13TeV$

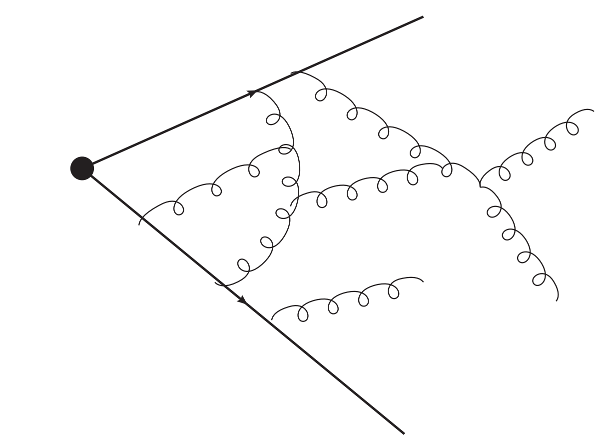
- *$\mathcal{O}(10)$ cancellation at NNLO*
- *N3LO cancelations less prominent*

J. Baglio, C. Duhr, B. Mistlberger, R.S., 2022

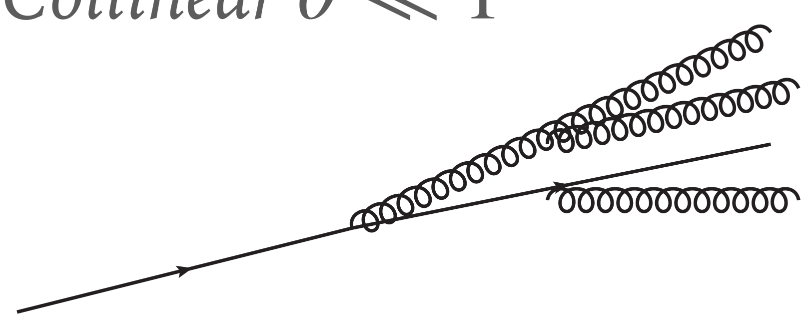
WHY WE WANT TO FACTORIZE?

Differential observables are more challenging!

Soft $E \ll Q$



Collinear $\theta \ll 1$



QCD singular limits lead to the appearance of large logarithms of a ratio of different scales

$$\hat{\sigma}(z) = \sum_{n=0}^{\infty} \alpha_s^n \left[c_n \delta(1-z) + \sum_{m=0}^{2n-1} \left(c_{nm} \left[\frac{\ln^m(1-z)}{1-z} \right]_+ + d_{nm} \ln^m(1-z) \right) + \ldots \right]$$

Expansion parameter is not α_s but $\alpha_s \ln^2(1-z)$

Large logarithmic corrections must be controlled to all orders when approaching singular limits

We must be sure that we understand the Standard Model!

POWER CORRECTIONS

Factorization theorems take many different forms and shapes and are essential for the LHC physics

$$d\sigma \sim \underbrace{\sum_{a,b} \int_0^1 dx_a dx_b \, f_{a/A}(x_a)\, \hat{\sigma}_{ab}(z)\, f_{b/B}(x_b)}_{\text{leading power}} + \underbrace{\mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{Q}\right)}_{\text{power corrections}}$$

$$gg \to H$$

Leading power (LP) terms are fairly well understood, especially thanks to the effective field theory approach (SCET) developed in early 2000's

Power corrections (NLP) are being systematically studied only recently, there are many conceptual challenges associated with them

WHY LOOK AT POWER CORRECTIONS?

Power corrections are needed to:

- *Extend the applicability of factorization*
- *Assess theoretical error*
- *Improve matching between fixed order and resumed results*
- *Understand power suppressed processes*

Other applications:

- ➤ *Improve fixed order subtraction schemes*
- ➤ *Bootstrap program*
- ➤ *Understanding all-order structure of QFT*

PERTURBATIVE POWER CORRECTIONS

Non-perturbative power corrections

$$d\sigma \sim \underbrace{\sum_{a,b} \int_0^1 dx_a dx_b\, f_{a/A}(x_a)\, \hat{\sigma}_{ab}(z) f_{b/B}(x_b)}_{\text{leading power}} + \underbrace{\mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{Q}\right)}_{\text{power corrections}}$$

Perturbative power corrections

$$\hat{\sigma}(z) = \sum_{n=0}^{\infty} \alpha_s^n \left[c_n \delta(1-z) + \sum_{m=0}^{2n-1} \left(c_{nm} \left[\frac{\ln^m(1-z)}{1-z} \right]_+ + d_{nm} \ln^m(1-z) \right) + \ldots \right]$$

Leading power

Next-to-leading power

Soft collinear effective field theory

RESUMMATION IN SCET

Unlike traditional diagrammatic approach, SCET allows for systematic, more intuitive derivation of factorization theorems. Each function is associated with an operatorial definition and resummation is performed using renormalization group equations

$$\sigma \sim H \otimes J_1 \otimes \ldots \otimes J_n \otimes S$$

Hard matching coefficient

Jet functions - matrix elements of collinear fields

Soft function - matrix element of soft fields

1. Define functions

2.Renormalize UV divergences of EFT operators

3.Solve RGE

Key point:
Each function is a single scale object - RGE resums large logs

SCET 101

Systematic expansion in $\lambda \sim \frac{p_\perp}{n_+p}$

Hard-Collinear momentum	$n_+p \sim Q$	$p_\perp \sim \lambda Q$	$n\_p \sim \lambda^2 Q$
Soft momentum	$n_+p \sim \lambda^2 Q$	$p_\perp \sim \lambda^2 Q$	$n\_p \sim \lambda^2 Q$

Every field has a well defined scaling

Hard-collinear quark: $\chi \sim \lambda$ *Soft quark:* $q \sim \lambda^3$

Hard-collinear gluon: $\mathscr{A}_\perp \sim \lambda$

Operators are non-local along the light-cone direction

$$\int dt_1 dt_2\, C^{B1}(t_1, t_2)\, \chi(t_1 n_+) \mathscr{A}_\perp(t_2 n_+)$$

$$\frac{d\sigma}{d\lambda} = \sum_{k=0} \sum_{l=0}^{2k-1} \left(\frac{1}{\lambda^2} c_{kl}^{\mathrm{LP}} + d_{kl}^{\mathrm{NLP}} \right) \alpha^k \ln^l \lambda$$

LAGRANGIAN AND DECOUPLING

$$\mathscr{L}_{\rm LP} = \bar{\chi} \left(i n_- D + i \not{D}_{\perp c} \frac{1}{i n_+ D_c} i \not{D}_{\perp c} \right) \frac{\not{n}_+}{2} \chi$$

$$i n_- D = i n_- \partial + g\, n_- A_c(x) + g\, n_- A_s(x_-)$$

Collinear

LAGRANGIAN AND DECOUPLING

$$\mathcal{L}_{\text{LP}} = \bar{\chi}\left(in\_D + i\not{D}_{\perp c}\frac{1}{in_+D_c}\, i\not{D}_{\perp c}\right)\frac{\not{n}_+}{2}\,\chi$$

Collinear

$$in\_D = in\_\partial + g\, n\_A_c(x) + g\, n\_A_s(x\_)$$

Field redefinition

$$\chi_c^{(0)}(x) = Y_+^\dagger(x\_)\chi_c(x)$$

$$Y_+(x) = \mathbf{P}\exp\left[ig_s \int_{-\infty}^{0} ds\, n\_A_s(x + sn\_)\right]$$

Soft Wilson line

LAGRANGIAN AND DECOUPLING

$$\mathscr{L}_{\rm LP} = \bar{\chi}\left(in_-D + i\not{D}_{\perp c}\frac{1}{in_+D_c}\, i\not{D}_{\perp c}\right)\frac{\not{n}_+}{2}\,\chi$$

Collinear

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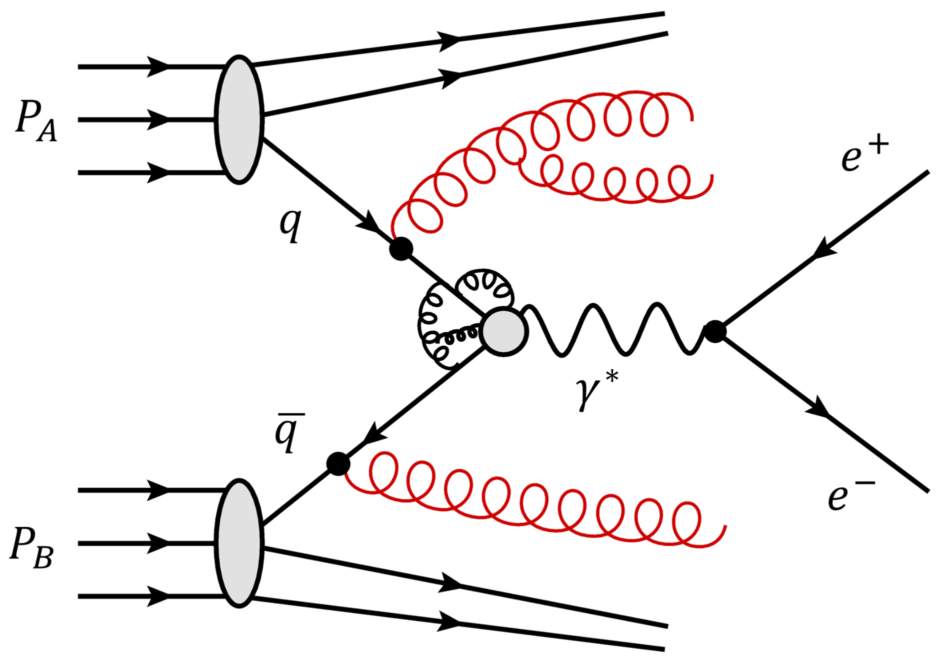
Soft Wilson line

$$\mathscr{L}_{\rm LP} = \bar{\chi}\left(in_-D_c + i\not{D}_{\perp c}\frac{1}{in_+D_c}\, i\not{D}_{\perp c}\right)\frac{\not{n}_+}{2}\,\chi$$

No soft at LP - Hilbert space factorizes

$$in_-D_c = in_-\partial + g\, n_-A_c(x)$$

LEADING POWER FACTORIZATION FOR DRELL-YAN

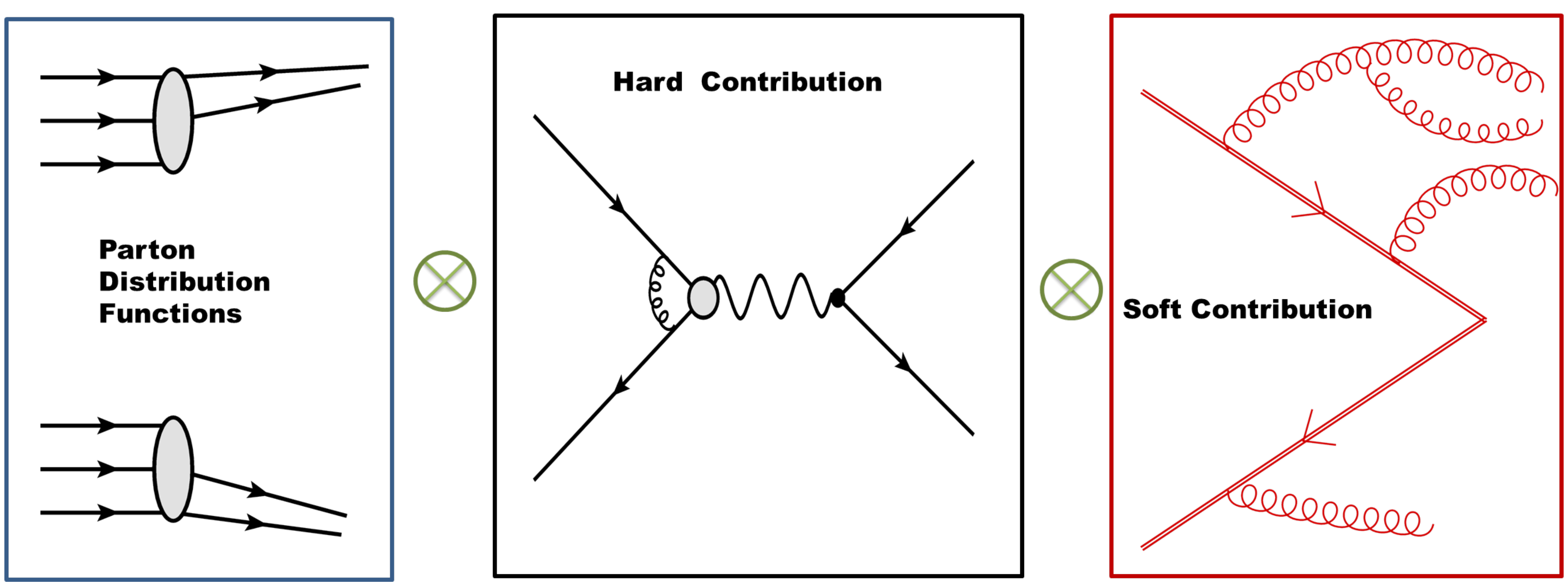


$$\hat{\sigma}(z) = |C(Q^2)|^2 S_{\rm DY}(Q(1-z))$$

C is the hard Wilson matching coefficient $\mu_h \sim Q$

S is the soft function

$$S_{\rm DY}(\Omega) = \int \frac{dx^0}{4\pi} e^{ix^0\Omega/2} \frac{1}{N_c} Tr \langle 0 | \bar{\mathbf{T}}(Y_+^\dagger(x^0) Y_-(x^0)) \, \mathbf{T}(Y_-^\dagger(0) Y_+(0)) | 0 \rangle$$



Next-to-soft radiation

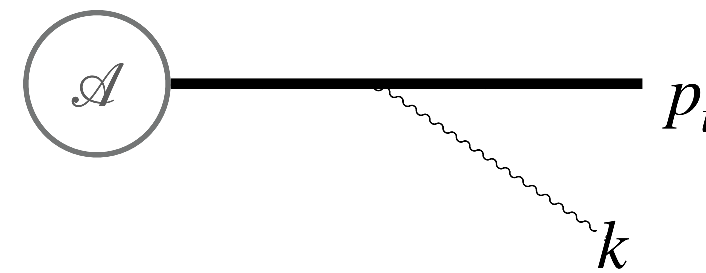
SUBLEADING SOFT THEOREM

LBK theorem

Low, 1958

Burnett, Kroll, 1968

$$\mathscr{A}_{\text{rad}} = -g_s \sum_{i=1}^{n} t_i^a \overline{u}(p_i) \left(\frac{p_i \cdot \varepsilon^a(k)}{p_i \cdot k} + \frac{k_\nu \varepsilon_\mu^a(k) J_i^{\mu\nu}}{p_i \cdot k} \right) \mathscr{A}$$



Radiative amplitude for soft gluon/photon	Is related in a universal way, up to order $(k)^0$	To the non-radiative amplitude

$$J_i^{\mu\nu} = L_i^{\mu\nu} + \Sigma_i^{\mu\nu} = p_i^\mu \frac{\partial}{\partial p_{i\nu}} - p_i^\nu \frac{\partial}{\partial p_{i\mu}} + \Sigma_i^{\mu\nu}$$

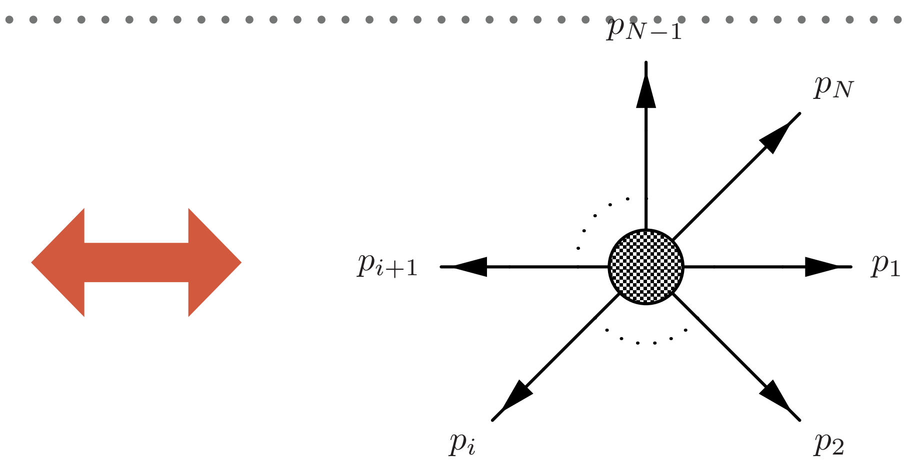
Works for any spin

Leading power: coupling proportional to momentum

Next-to-leading power: coupling proportional to angular momentum

SOFT THEOREM IN QCD SCET

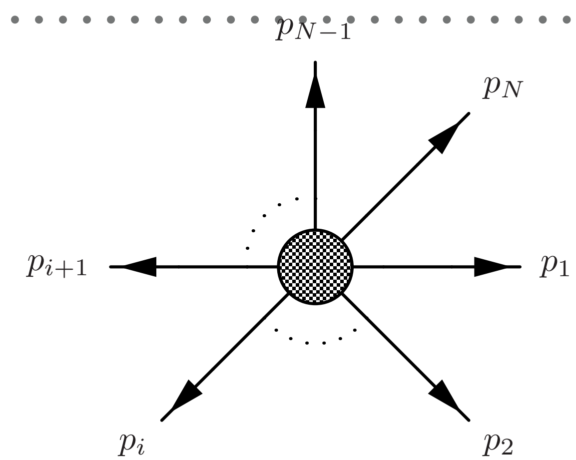
1st step: non-radiative matching $\mathscr{A}$



N-jet operator in SCET

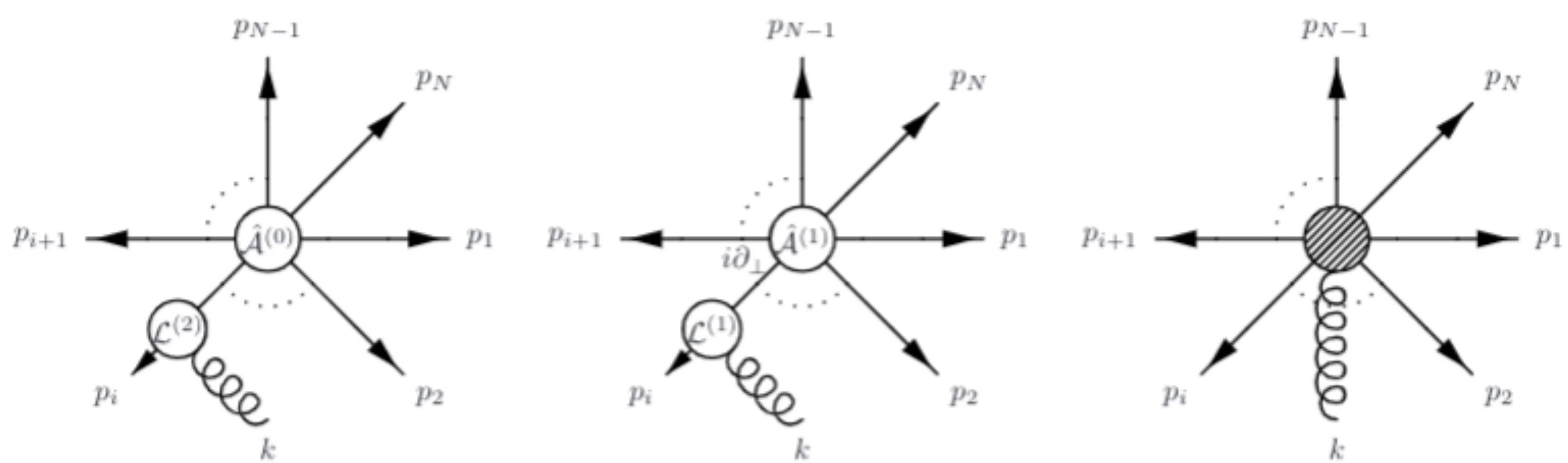
SOFT THEOREM IN QCD SCET

1st step: non-radiative matching $\mathscr{A}$ ⟷



N-jet operator in SCET

2st step: time-ordered products reproduce LBK terms



SCET derivation: First two terms come from time-ordered product of the Lagrangian and the operator representing non-radiative amplitude

At $\mathcal{O}(\lambda^4)$ we can write for the first time soft gluon building block $F_s^{\mu\nu} \sim \lambda^4$ so this term is no longer universal

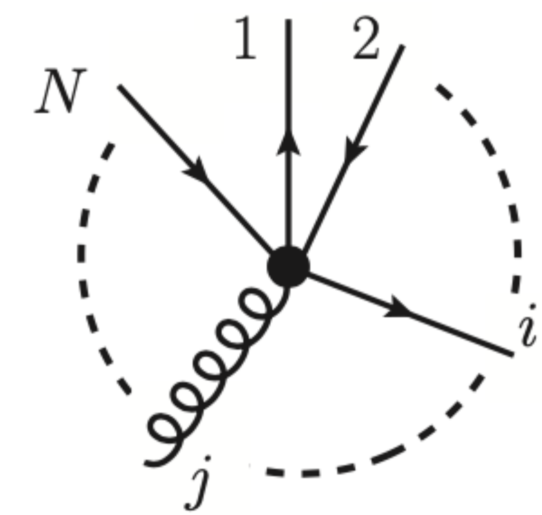
BEYOND LEADING POWER

SCET allows for systematic expansion

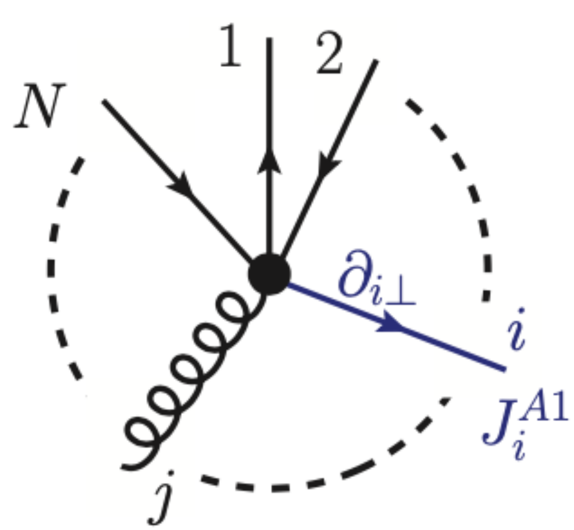
1. Power-suppressed operators

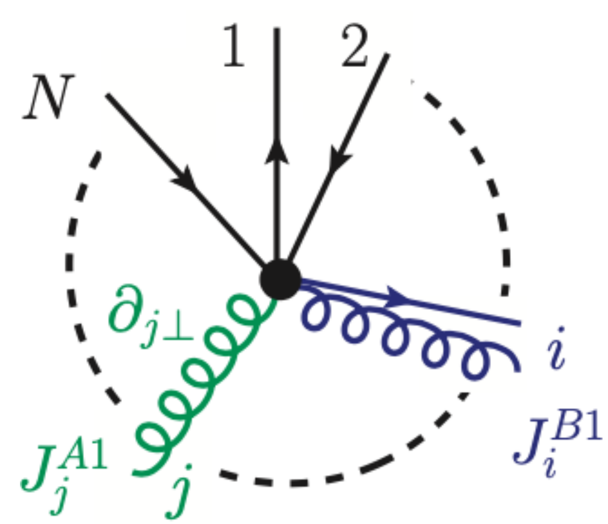
2. Subleading Lagrangian insertions

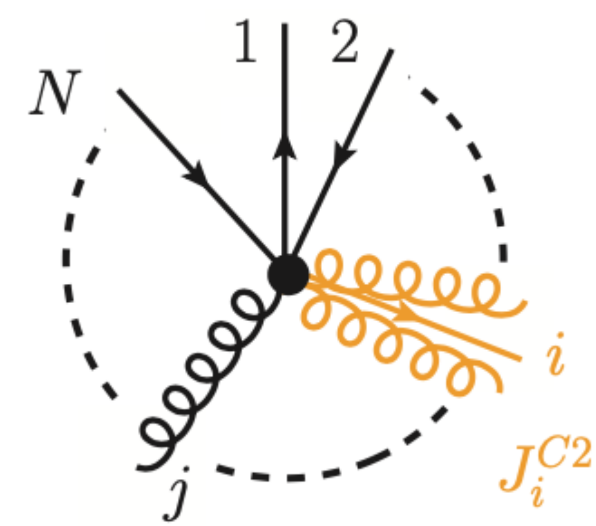
LP-N-jet



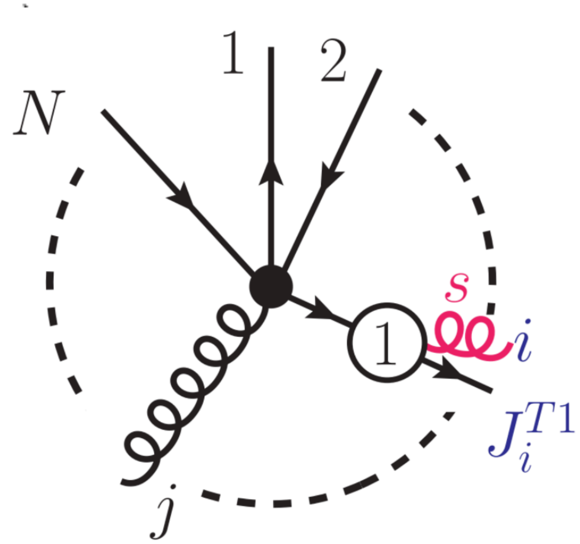
NLP-N-jet Current





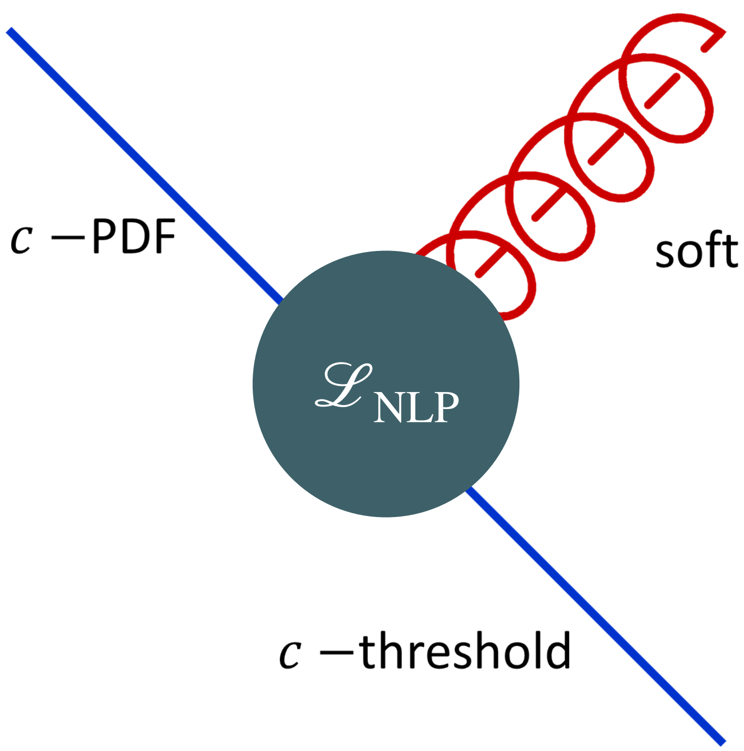


NLP-N-jet Lagrangian insertion



[M. Beneke, M. Garny, **RS**, J. Wang: 2017-2019]

FACTORIZATION BEYOND LP



$$S_{\rm DY}(\Omega) = \int \frac{dx^0}{4\pi}\, e^{ix^0\Omega/2}\, \frac{1}{N_c}\, Tr\, \langle 0 |\, \bar{\mathbf{T}}(Y_+^\dagger(x^0) Y_-(x^0))\, \mathbf{T}(Y_-^\dagger(0) Y_+(0))\, | 0 \rangle$$

Power counting limits the number of subleasing soft-collinear interactions

$$i \int d^4z\, e^{i\omega \frac{n_+z}{2}} \times \mathbf{T}\left[\chi_c(tn_+) \times \mathscr{L}_c^{(n)}(z)\right] = J(t;\omega)\, \chi_c^{\rm PDF}(tn_+)$$

$J(t,\omega)$ is a new object - radiative jet function

Soft functions have now explicit soft filed insertions

$$\widetilde{\mathcal{S}}_{2\xi}(x, z_-) = \bar{\mathbf{T}}\left[Y_+^\dagger(x) Y_-(x)\right] \mathbf{T}\left[Y_-^\dagger(0) Y_+(0) \frac{i\partial_\perp^\nu}{in_-\partial} \mathscr{B}^+_{\perp\nu}(z_-)\right]$$

$$S_{2\xi}(\Omega,\omega) = \int \frac{dx^0}{4\pi} \int \frac{d(n_+z)}{4\pi}\, e^{ix^0\Omega/2 - i\omega(n_+z)/2}\, \frac{1}{N_c}\, Tr\, \langle 0 |\, \widetilde{\mathcal{S}}_{2\xi}(x^0, z_-)\, | 0 \rangle$$

M. Beneke, A. Broggio, M. Garny, S. Jaskiewicz, RS, L. Vernazza, J. Wang, 2018

M. Beneke, A. Broggio, S. Jaskiewicz, L. Vernazza, 2019

LL RESULT FOR DIAGONAL CHANNEL

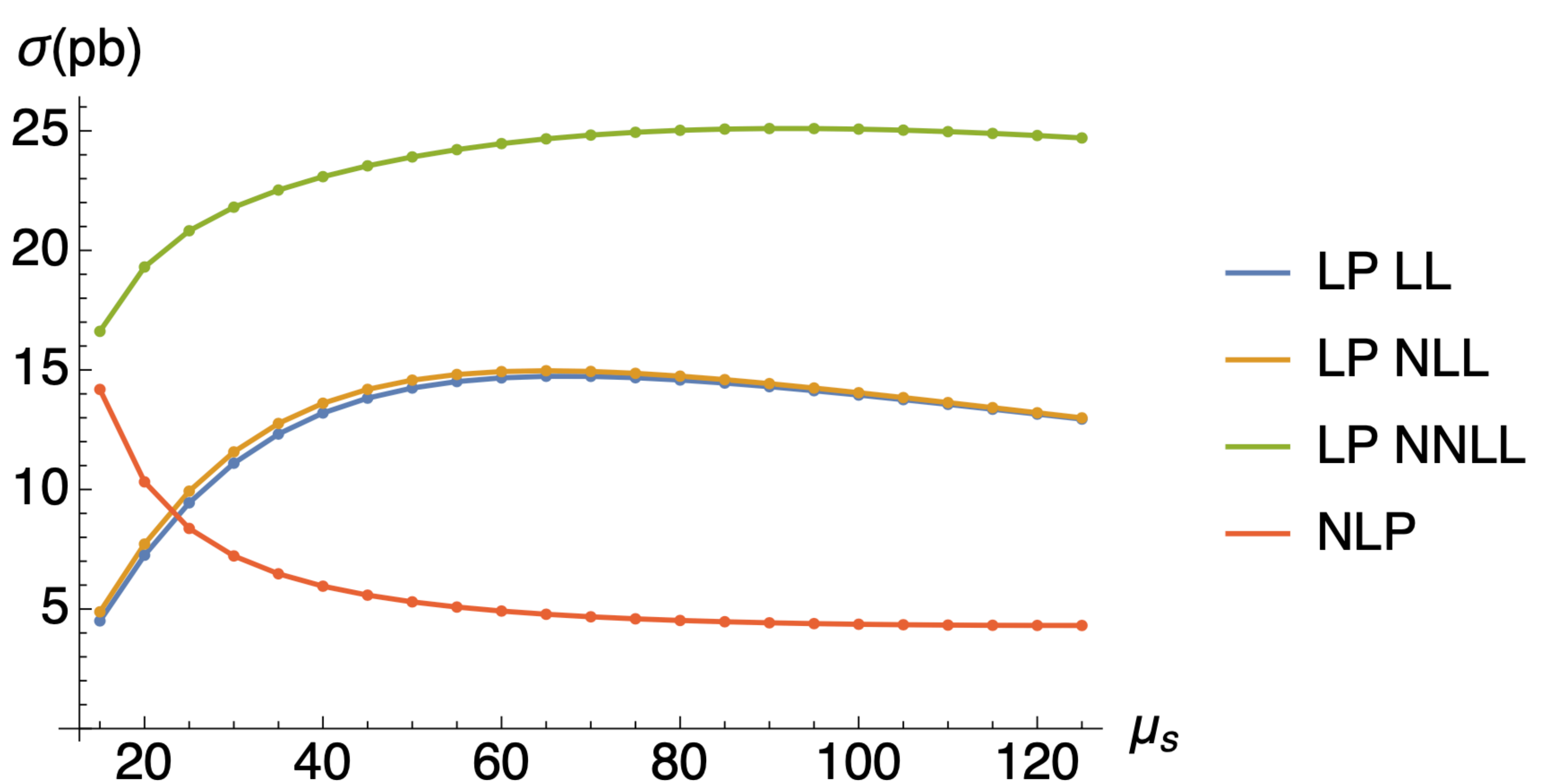
$$\hat{\sigma}(z) = H(\hat{s}) \times Q^2 \int \frac{d^3\vec{q}}{(2\pi)^3\, 2\sqrt{Q^2+\vec{q}^2}} \frac{1}{2\pi} \int d^4x\, e^{i(x_a p_A + x_b p_B - q)\cdot x} \times \left\{ \widetilde{S}_0(x) + 2 \cdot \frac{1}{2} \int d\omega\, J^{(O)}_{2\xi}(x_a n_+ p_A;\omega) \widetilde{S}_{2\xi}(x,\omega) + \bar{c} - term \right\}$$

NLP factorization for threshold Drell-Yan

$$\Delta^{LL}_{NLP}(z,\mu) = \frac{\hat{\sigma}^{LL}_{NLP}(z,\mu)}{z} = exp\left[2\frac{\alpha_s C_F}{\pi}\ln^2\frac{\mu}{\mu_s} - 2\frac{\alpha_s C_F}{\pi}\ln^2\frac{\mu}{\mu_h}\right] \times (-4)\frac{\alpha_s C_F}{\pi}\ln\frac{\mu_s}{\mu}\,\theta(1-z)$$

NLP resumed results

σ (pb)	$\mu_s = \mu_s^{\rm dyn}$	
	$\mu_h^2 = m_H^2$	$\mu_h^2 = -m_H^2$
$\sigma_{\rm LP}^{\rm NNLL}$	24.12	28.04
$\sigma_{\rm LP}^{\rm NNLO}$	28.93	
$\sigma_{\rm LP}^{\rm N^3LO}$	29.24	
$\sigma_{\rm NLP}^{\rm LL}$ (A)	7.18	12.76
$\sigma_{\rm NLP}^{\rm LL}$ (B)	8.82	15.68
$\sigma_{\rm non\,LP}^{\rm NNLO}$	11.90	
$\sigma_{\rm non\,LP}^{\rm N^3LO}$	16.27	
$\sigma_{\rm LP}^{\rm NNLL} + \sigma_{\rm NLP}^{\rm LL}$ (A)	31.30	40.80
$\sigma_{\rm LP}^{\rm NNLL} + \sigma_{\rm NLP}^{\rm LL}$ (B)	32.94	43.72
$\sigma^{\rm NNLO}$	40.82	
$\sigma^{\rm N^3LO}$	45.52	



M. Beneke, M. Garny, S. Jaskiewicz, RS, L. Vernazza, J. Wang, 2019

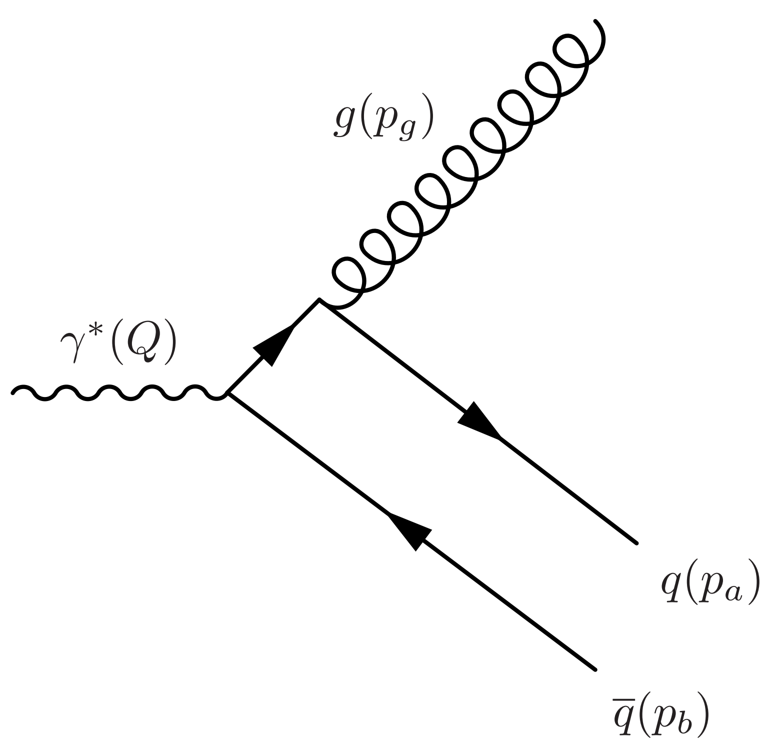
Soft quark radiation

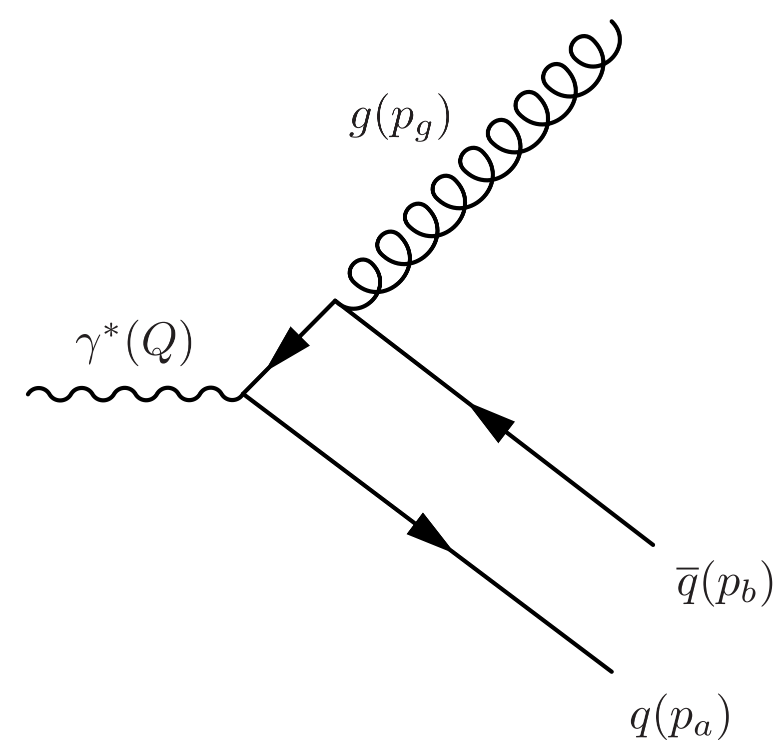
THRUST AT NLP

Diagonal channel similar to DY *I. Moult, I. Stewart, G. Vita, H. X. Zhu, 2018*

Off-diagonal contributions are more difficult

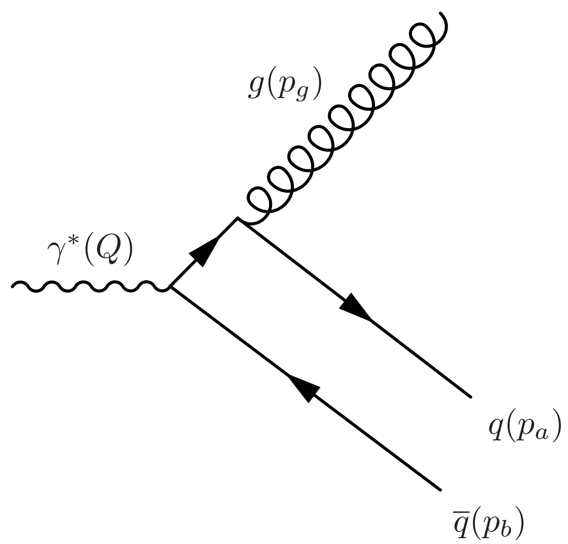
M. Beneke, M. Garny, S. Jaskiewicz, RS, L. Vernazza, J. Wang, 2022

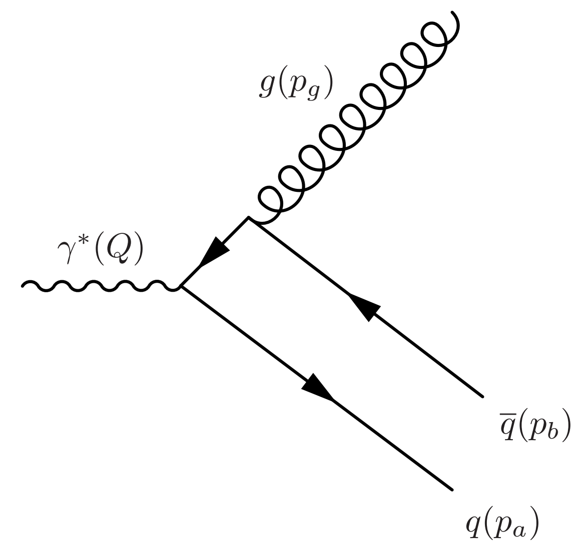




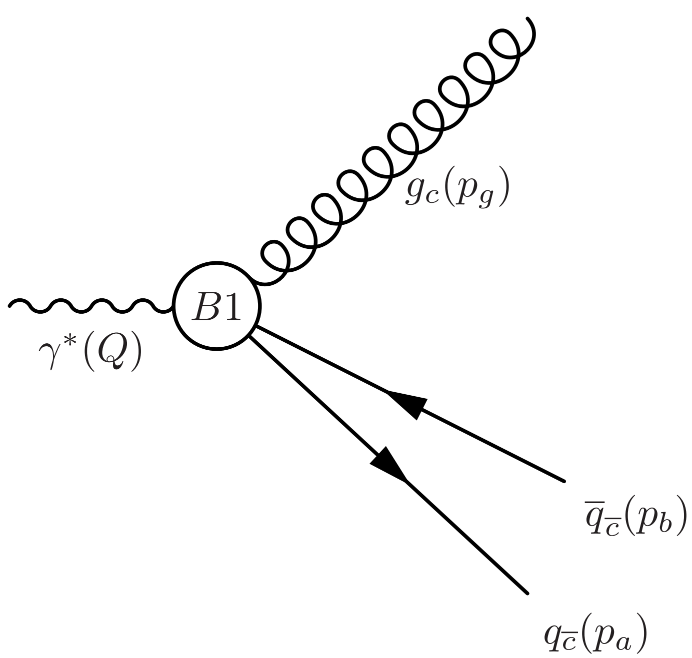
EMISSION OF SOFT QUARK VIOLATES FACTORIZATION AND LEADS TO ENDPOINT DIVERGENCIES *(This is also true beyond LL for gluons)*

THRUST AT NLP

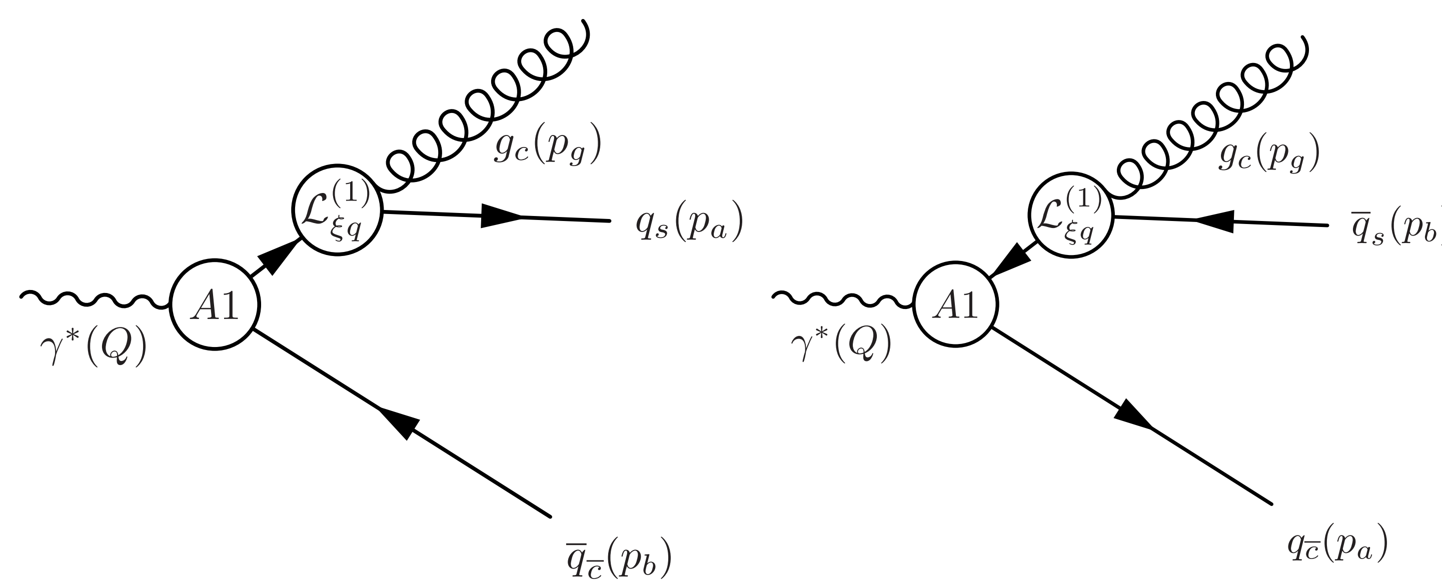




B-TYPE



A-TYPE



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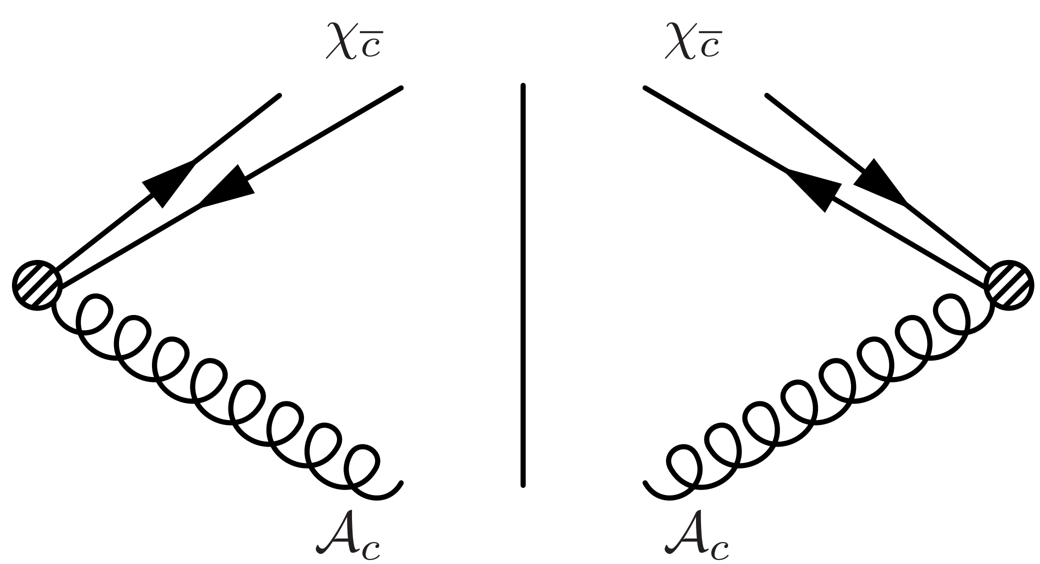


$$\frac{1}{\sigma_0}\frac{d\sigma}{dM_R^2 dM_L^2}\Big|_{\text{B-type}} \sim \int_0^1 dr dr' \, C^{B1}(r) C^{B1}(r')^* \times \mathcal{J}_{\bar{c}}^{q\bar{q}}\left(r, r'\right) \otimes \mathcal{J}_c^{(g)} \otimes S^{(g)}$$

Hard matching coefficients

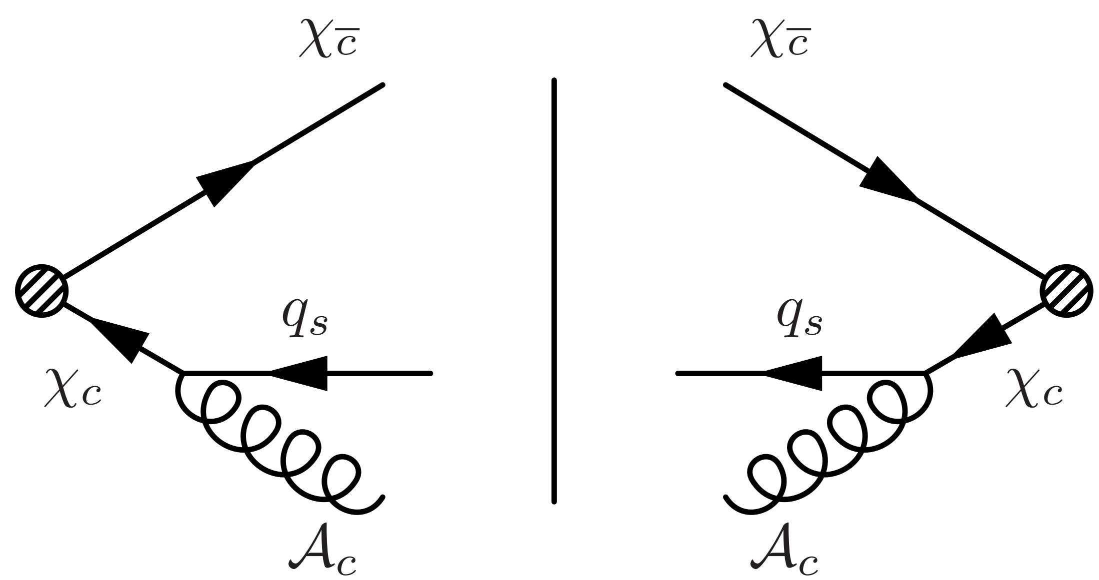
Jet functions

Soft function

NLP | NLP | LP | LP

Additional convolutions

A-TYPE



$$\frac{1}{\sigma_0}\frac{d\sigma}{dM_R^2 dM_L^2}\Big|_{\mathrm{A-type}} \sim \int_0^\infty d\omega d\omega' \left|C^{A0}\right|^2 \times \mathcal{J}_{\bar{c}}^{(\bar{q})} \otimes \mathcal{J}_c(\omega,\omega') \otimes S_{\mathrm{NLP}}(\omega,\omega')$$

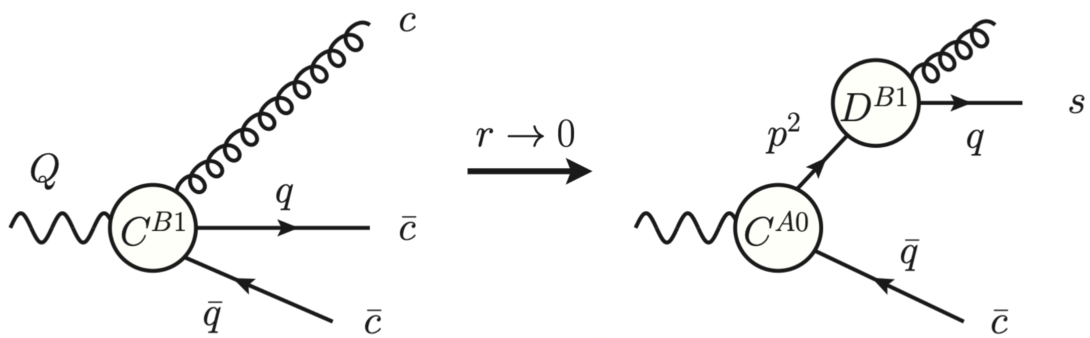
Hard matching coefficients — LP

Jet functions — LP, NLP

Soft function — NLP

Additional convolutions

ENDPOINT FACTORIZATION



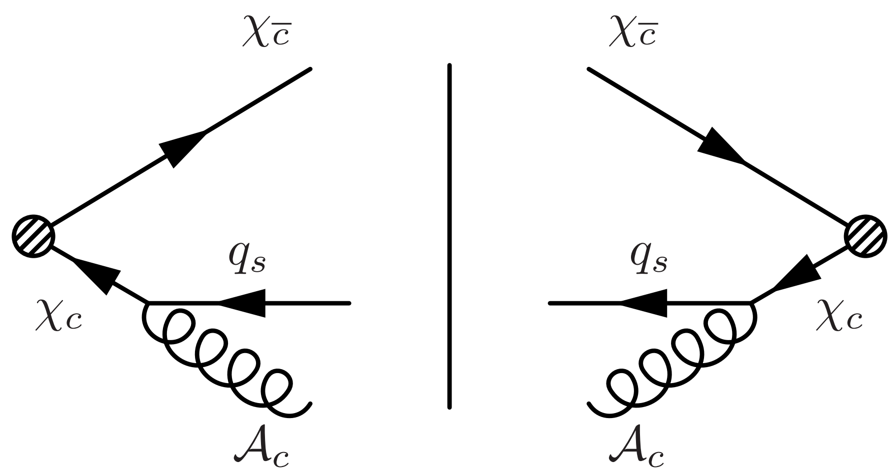
$$\left[\!\left[C_1^{\mathrm{B1}}\left(Q^2, r\right)\right]\!\right]_0 = C^{\mathrm{A0}}\left(Q^2\right) \times \frac{D^{\mathrm{B1}}\left(rQ^2\right)}{r}$$

When momentum fraction carried by the quark is small, the quark effectively becomes soft

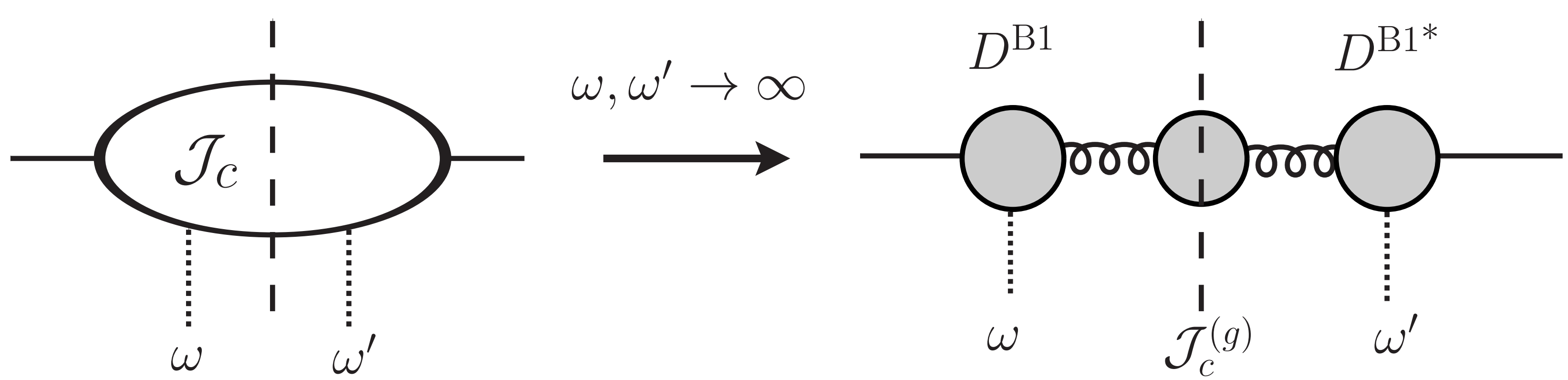
HARD MATCHING COEFFICIENT MUST FACTORIZE

M. Beneke, M. Garny, S. Jaskiewicz, RS, L. Vernazza, J. Wang, 2020

ENDPOINT FACTORIZATION



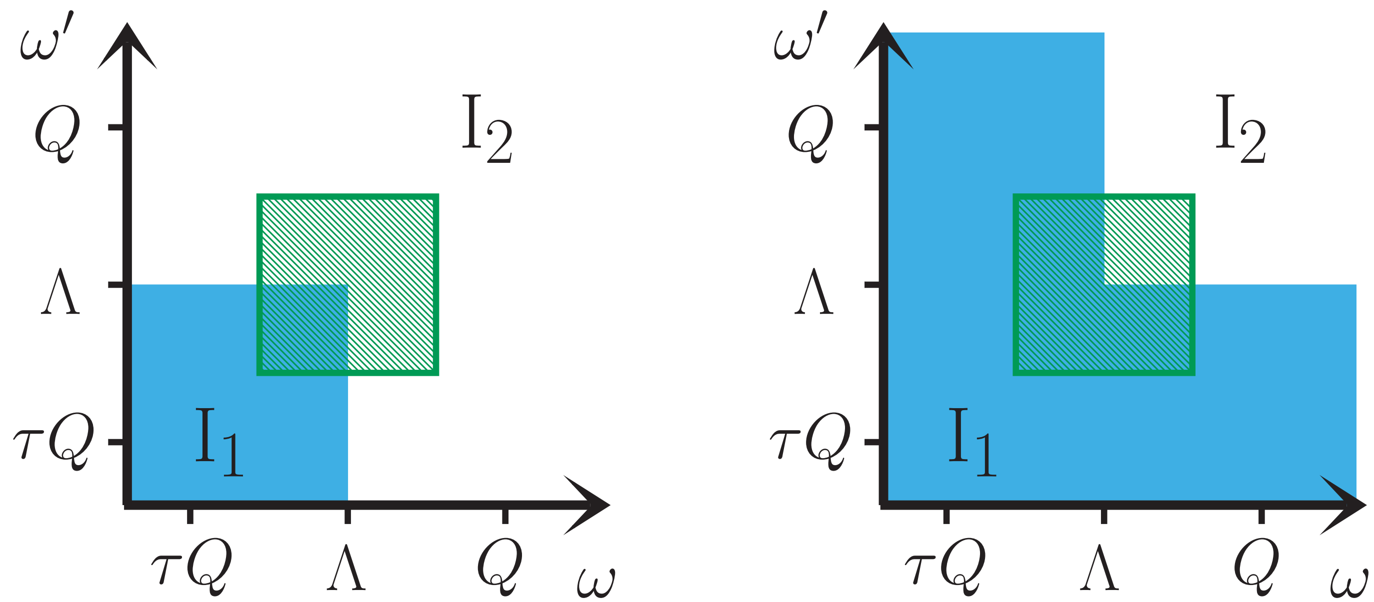
AND NLP JET FUNCTION MUST FACTORIZE



$$[\![\mathcal{J}_c\left(p^2,\omega,\omega'\right)]\!] = \mathcal{J}_c^{(g)}(p^2)\,\frac{D^{\mathrm{B1}}(\omega Q)}{\omega}\,\frac{D^{\mathrm{B1}*}(\omega' Q)}{\omega'}$$

M. Beneke, M. Garny, S. Jaskiewicz, RS, L. Vernazza, J. Wang, 2022

ENDPOINT SUBTRACTION



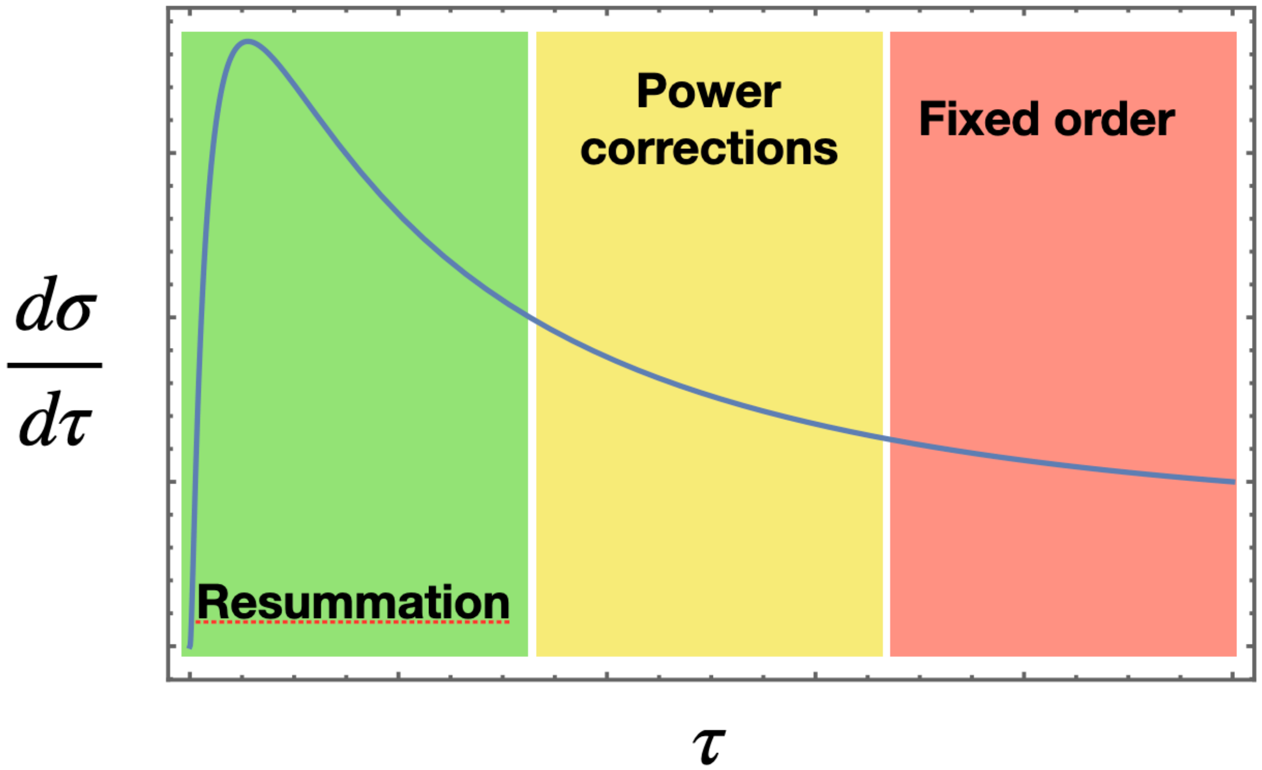
Endpoint factorization condition ensure that both A and B contributions agree on the green square

$$\frac{1}{\sigma_0}\frac{d\sigma}{dM_R^2 dM_L^2}\bigg|_{\mathrm{B-type}} \sim \int_0^1 dr dr' \; C^{B1}(r) C^{B1}(r')^* \times \mathcal{J}_{\bar{c}}^{q\bar{q}}\left(r, r'\right) \otimes \mathcal{J}_c^{(g)} \otimes S^{(g)}$$

$$\frac{1}{\sigma_0}\frac{d\sigma}{dM_R^2 dM_L^2}\bigg|_{\mathrm{A-type}} \sim \int_0^\infty d\omega d\omega' \; \left|C^{A0}\right|^2 \times \mathcal{J}_{\bar{c}}^{(\bar{q})} \otimes \mathcal{J}_c\left(\omega, \omega'\right) \otimes S_{\mathrm{NLP}}\left(\omega, \omega'\right)$$

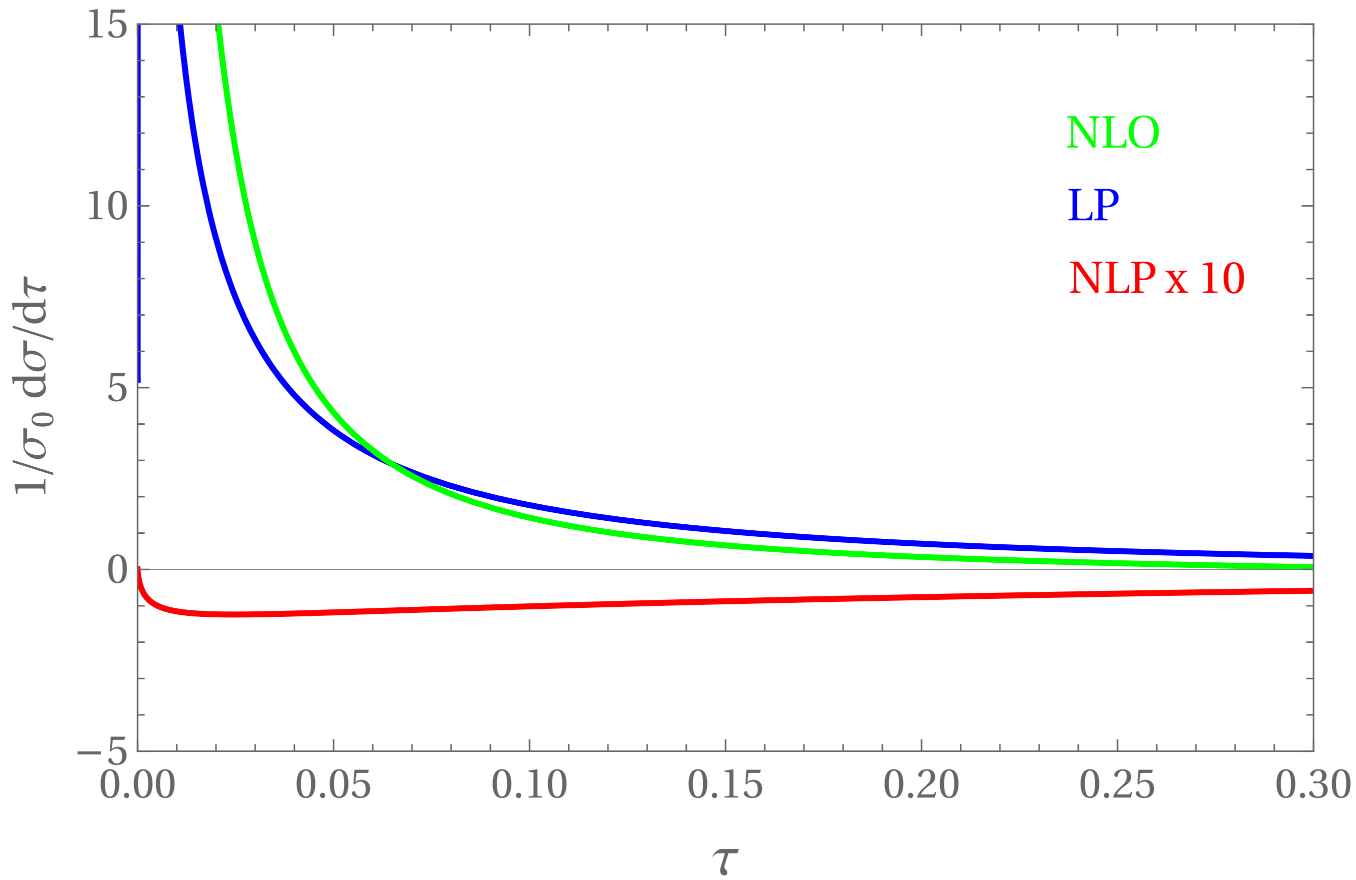
TO SOLVE THE PROBLEM WE SUBTRACT THE ENDPOINT CONTRIBUTION

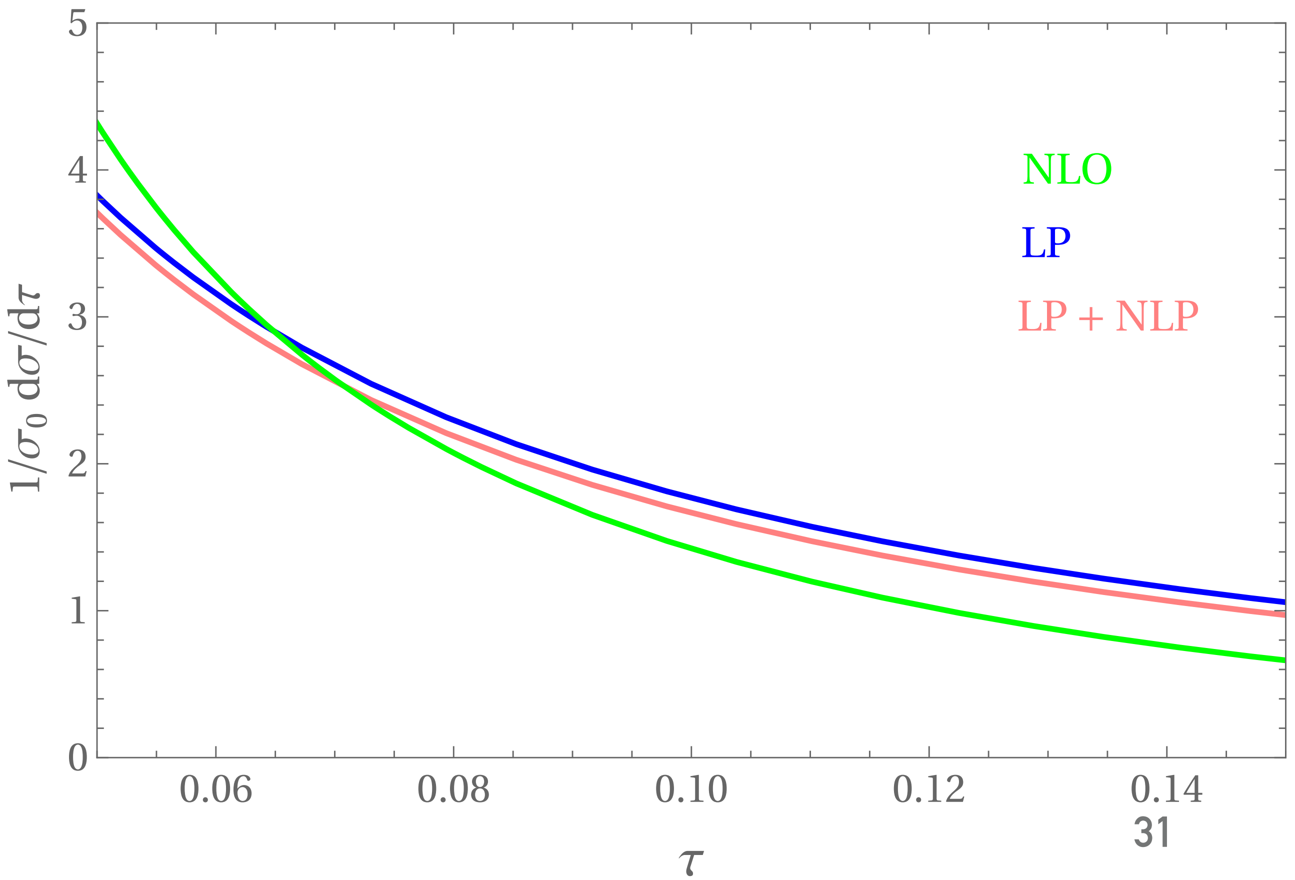
THRUST AT NLP



NLP LL CORRECTIONS EXCEED 5%

Next step: impact on α_s determination

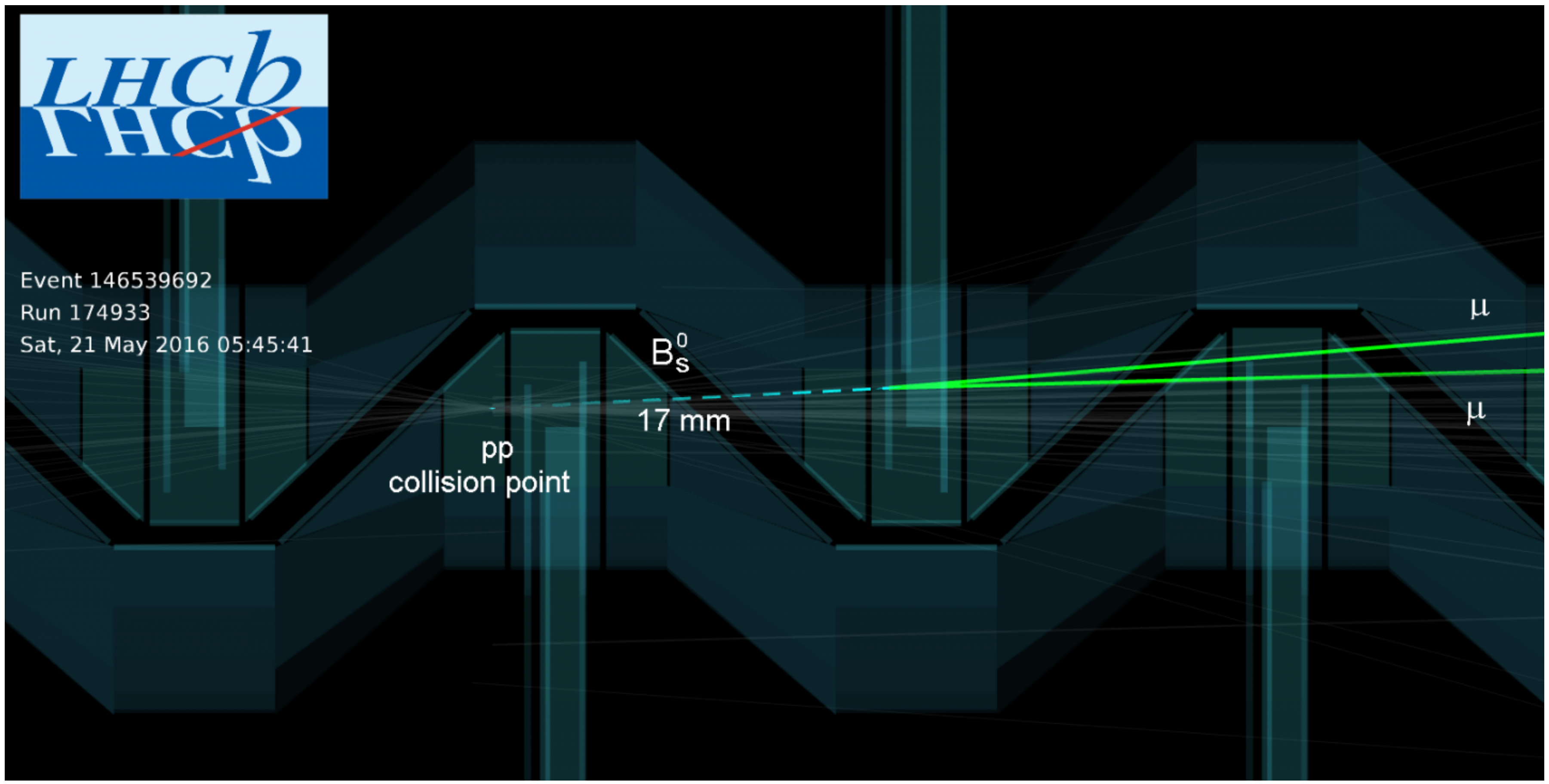




From collider physics to flavor

$B_q \to \mu^+\mu^-$

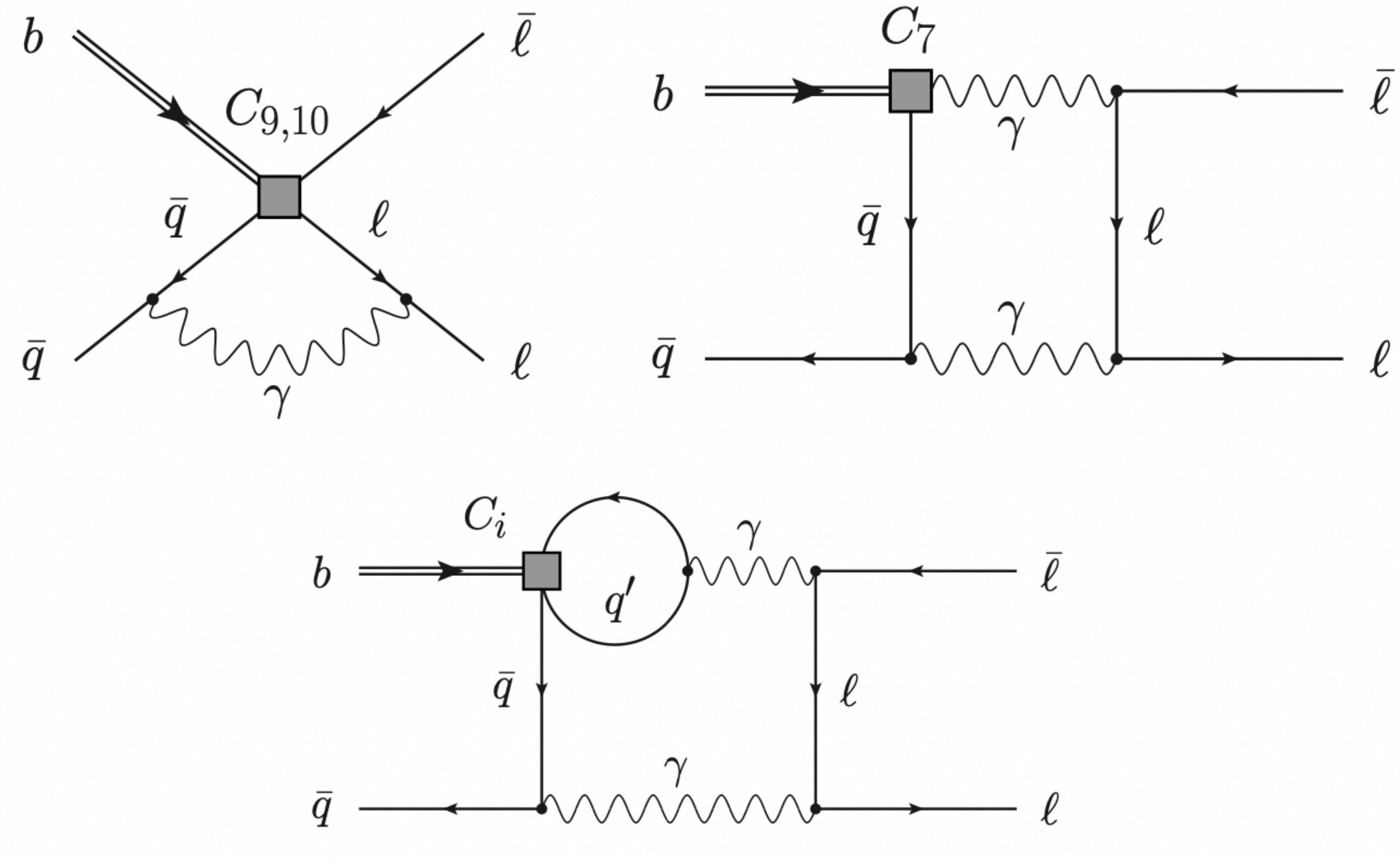
B_q *meson is a QCD bound state formed from heavy b-quark and a light quark* $q = s, d$



- *Standard Model decay rate for* $B_q \to \mu^+\mu^+$ *is tiny (doubly suppressed)*
- *The signal is very clean*
- *Theoretical uncertainties are under control*

Excellent test for the Standard Model and New Physics scenarios

THEORY PREDICTION



M. Beneke, C. Bobeth, and R. S., 2018,2019

We discovered unexpectedly large QED (%-level) correction due to power enhancement

Previous estimate $< 0.3\ \%$

Our most accurate prediction:

$$Br(B_d \to \mu^+\mu^-) = (1.027 \pm 0.050) \times 10^{-10}$$

$$Br(B_s \to \mu^+\mu^-) = (3.660 \pm 0.140) \times 10^{-9}$$

INCLUSIVE B DECAY MODES

- $\bar{B} \to X_{s,d}\,\gamma$ and $\bar{B} \to X_{s,d}\,\ell^+\ell^-$ are very clean tests of the Standard Model
- They will be extremely well measured at Belle-II
- Well understood at leading power in Λ/m_b expansion

C. W. Bauer, et. al., 2000

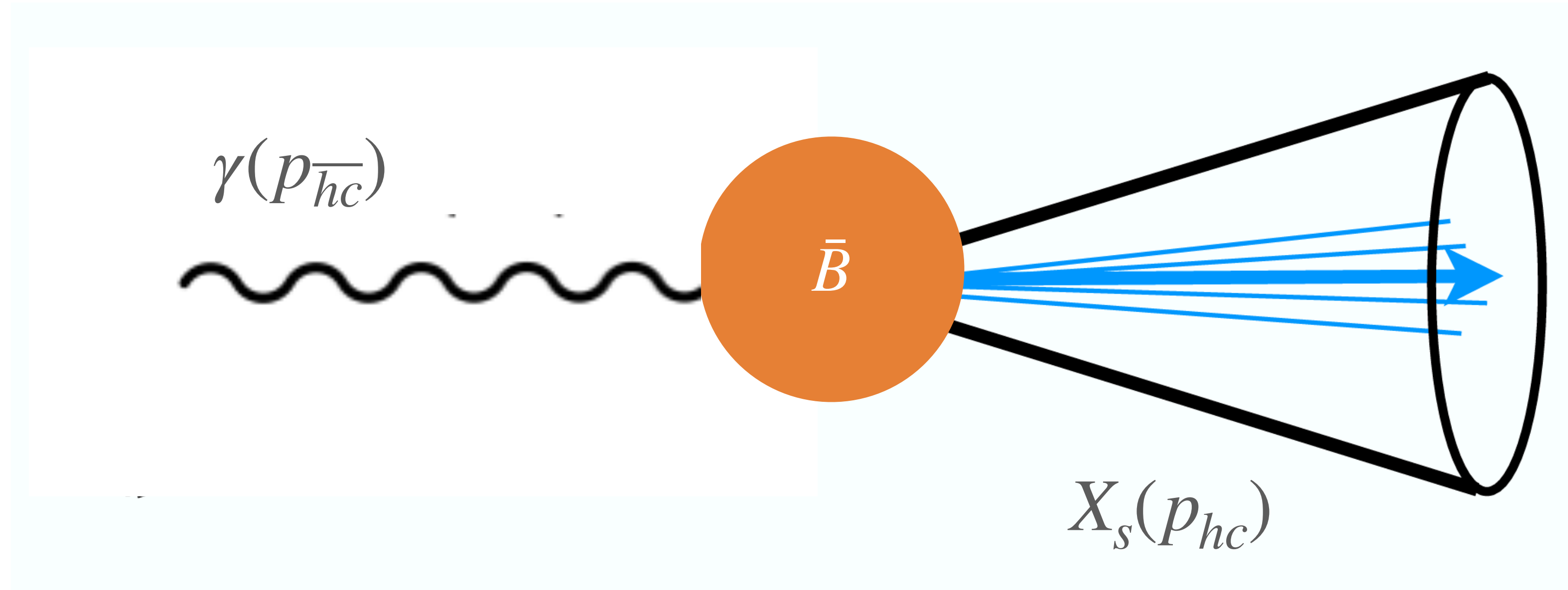
G. Korchemsky, et. al. 1994

$$\mathscr{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{q=u,c} \lambda_q \left(C_1\, O_1^q + C_2\, O_2^q + C_{7\gamma}\, O_{7\gamma} + C_{8g}\, O_{8g} + \sum_{i=3,\ldots,6} C_i\, O_i \right).$$

$$\hat{O}_7 = \frac{e}{16\pi^2} m_b\, \bar{s}\, \sigma^{\mu\nu} \frac{1}{2}(1+\gamma_5) b\, F_{\mu\nu}$$

POWER CORRECTIONS

I will focus on $\bar{B} \to X_s \gamma$

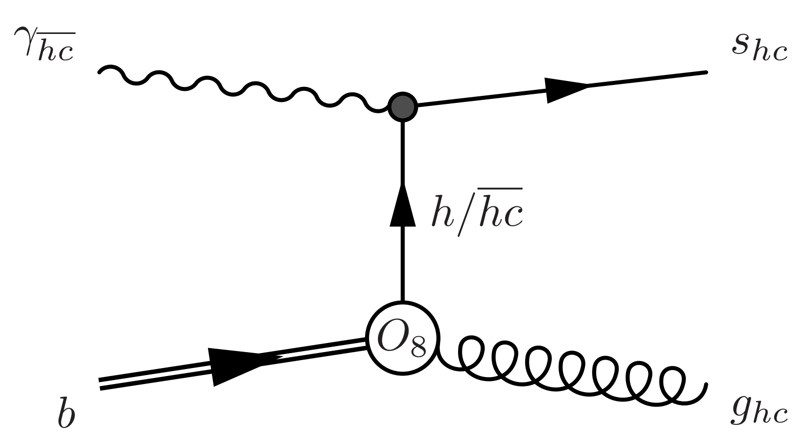


The naive factorization theorem

$$d\Gamma(\bar{B} \to X_s \gamma) = \sum_{n=0}^{\infty} \frac{1}{m_b^n} \sum_i H_i^{(n)} \otimes J_i^{(n)} \otimes S_i^{(n)}$$
$$+ \sum_{n=1}^{\infty} \frac{1}{m_b^n} \left[\sum_i H_i^{(n)} \otimes J_i^{(n)} \otimes S_i^{(n)} \otimes \bar{J}_i^{(n)} + \sum_i H_i^{(n)} \otimes J_i^{(n)} \otimes S_i^{(n)} \otimes \bar{J}_i^{(n)} \otimes \bar{J}_i^{(n)} \right]$$

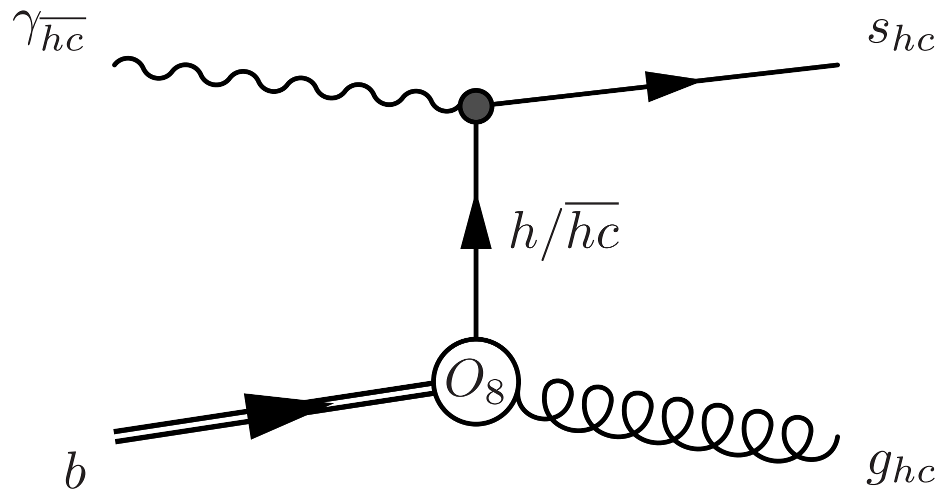
We typically distinguish direct and resolved contributions

Resolved: photon couples to light quarks (non-local)

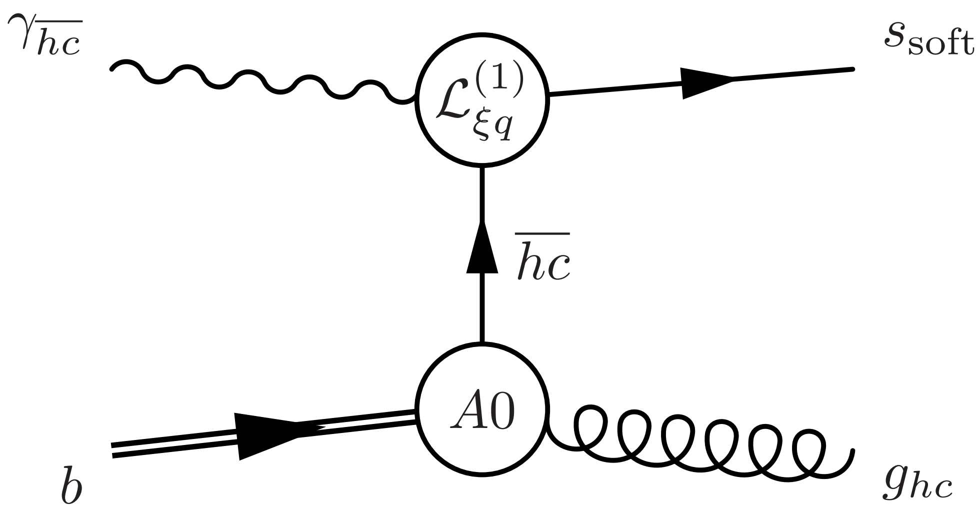


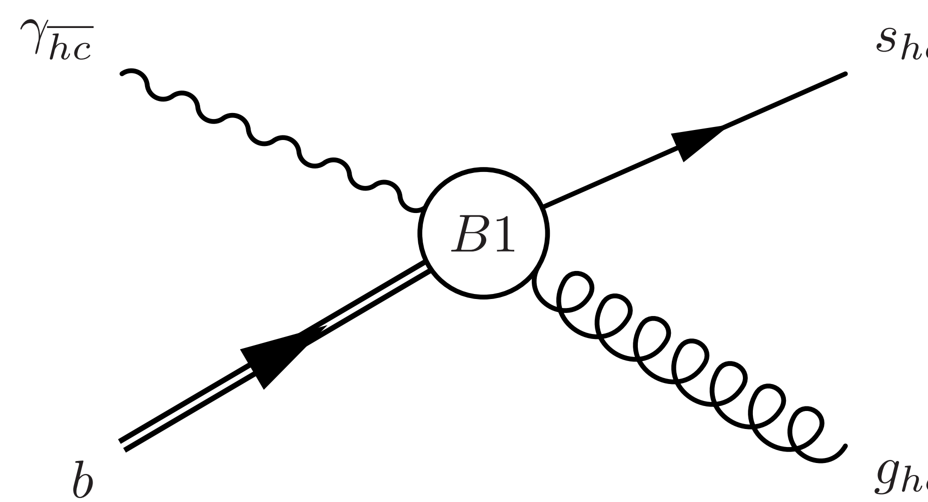
Contains anti-hard collinear radiate jet functions $\bar{J}$

$O_{8g} - O_{8g}$



Degeneracy in the EFT leads to divergences





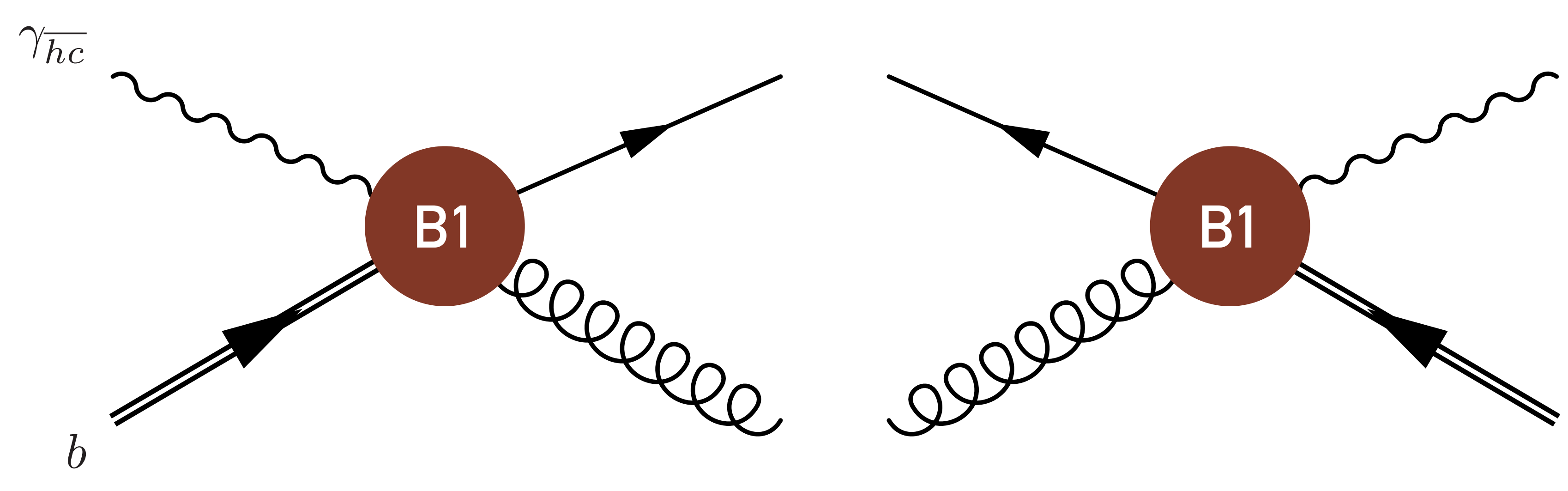
$$\mathcal{O}_{8g}^{A0}(0) = \bar{\chi}_{\overline{hc}}(0)\,\frac{\not{n}}{2}\gamma_{\mu\perp}\mathcal{A}^{\mu}{}_{hc\perp}(0)\left(1+\gamma_5\right)h(0)$$

$$\mathcal{O}_{8g}^{B1}(u) = \int \frac{dt}{2\pi} e^{-ium_b t}\bar{\chi}_{hc}(t\bar{n})\,\gamma_{\nu\perp}\,Q_s\,\mathcal{B}^{\nu}{}_{\overline{hc}\perp}(0)\,\gamma_{\mu\perp}\,\mathcal{A}^{\mu}{}_{hc\perp}(0)\left(1+\gamma_5\right)h(0)$$

DIRECT CONTRIBUTION

$$C_{LO}^{B1}\left(m_b, u\right) = (-1)\frac{\bar{u}}{u}\frac{m_b^2}{4\pi^2}\frac{G_F}{\sqrt{2}}\lambda_t C_{8g} = (-1)\frac{\bar{u}}{u}C_{LO}^{A0}\left(m_b\right)$$

$$\frac{d\Gamma}{dE_\gamma} = \mathcal{N}_B \int_0^1 du C^{B1}\left(m_b, u\right) \int_0^1 du' C^{B1*}\left(m_b, u'\right) \int_{-p_+}^{\bar{\Lambda}} d\omega J\left(M_B\left(p_+ + \omega\right), u, u'\right) \mathcal{S}\left(\omega\right)$$

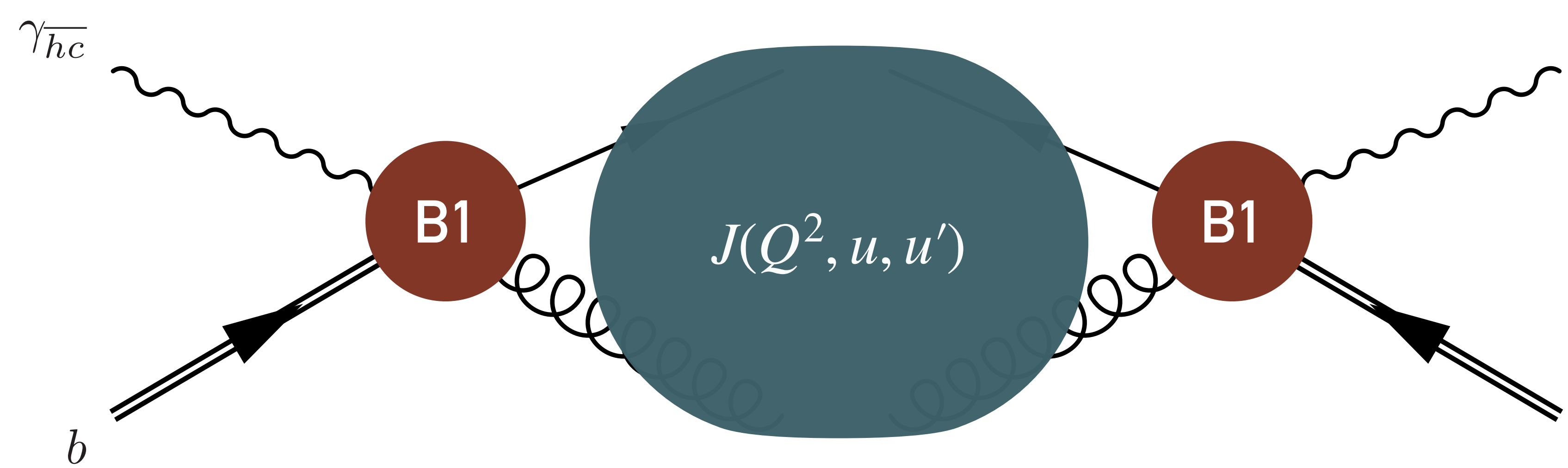


DIRECT CONTRIBUTION

$$J\left(p^2,u,u'\right)=\frac{(-1)}{2N_c}\frac{1}{2\pi}\int\frac{dtdt'}{(2\pi)^2}d^4x\,e^{-im_b(ut-u't')+ipx}(d-2)^2$$

$$\text{Disc}\left[\langle 0|\,tr\left[\frac{\not{\bar{n}}}{4}(1-\gamma_5)\mathcal{A}^{\mu}{}_{hc\perp}(x)\,\chi_{hc}(t'\bar{n}+x)\,\overline{\chi}_{hc}(t\bar{n})\,\mathcal{A}^{hc\perp}_{\mu}(0)\,(1+\gamma_5)\right]|0\rangle\right]$$

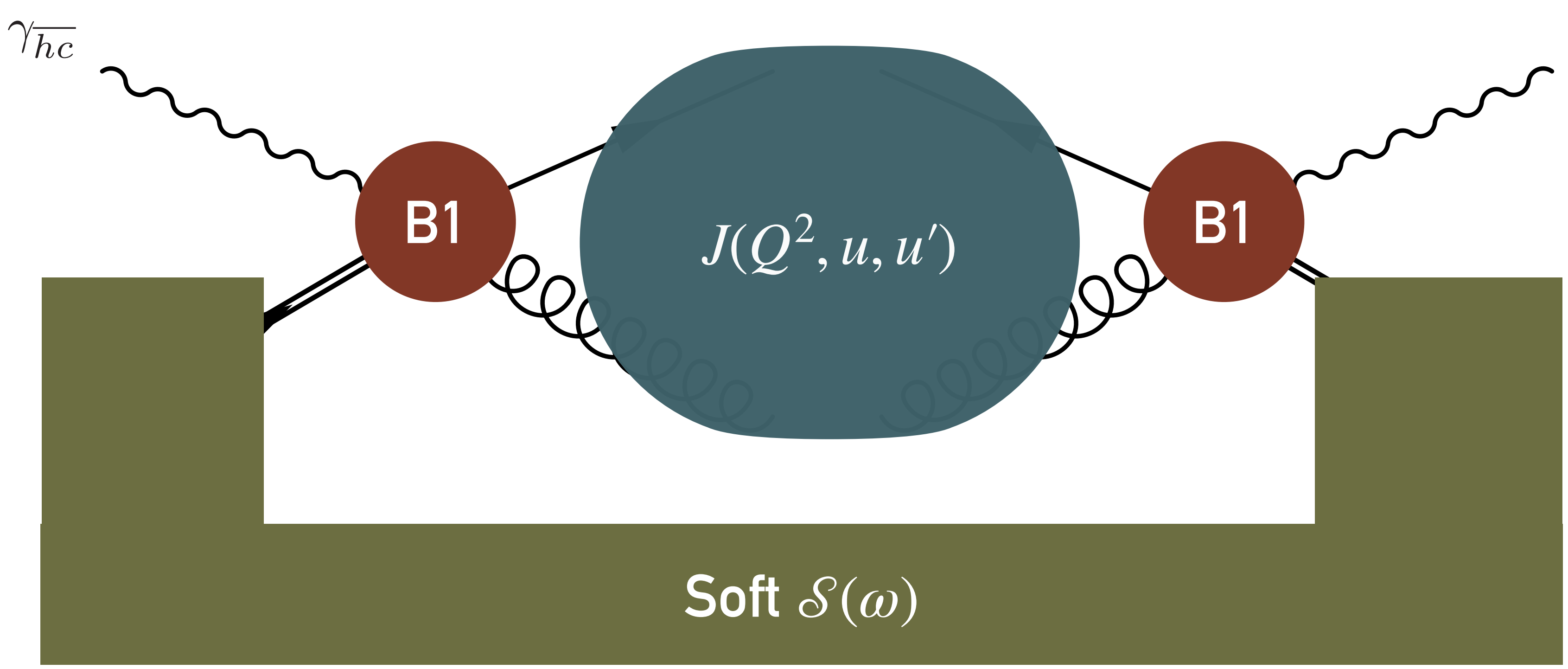
$$\frac{d\Gamma}{dE_\gamma}=\mathcal{N}_B\int_0^1 du\,C^{B1}(m_b,u)\int_0^1 du'\,C^{B1*}(m_b,u')\int_{-p_+}^{\bar{\Lambda}}d\omega\,J\left(M_B\left(p_++\omega\right),u,u'\right)\mathcal{S}(\omega)$$



DIRECT CONTRIBUTION

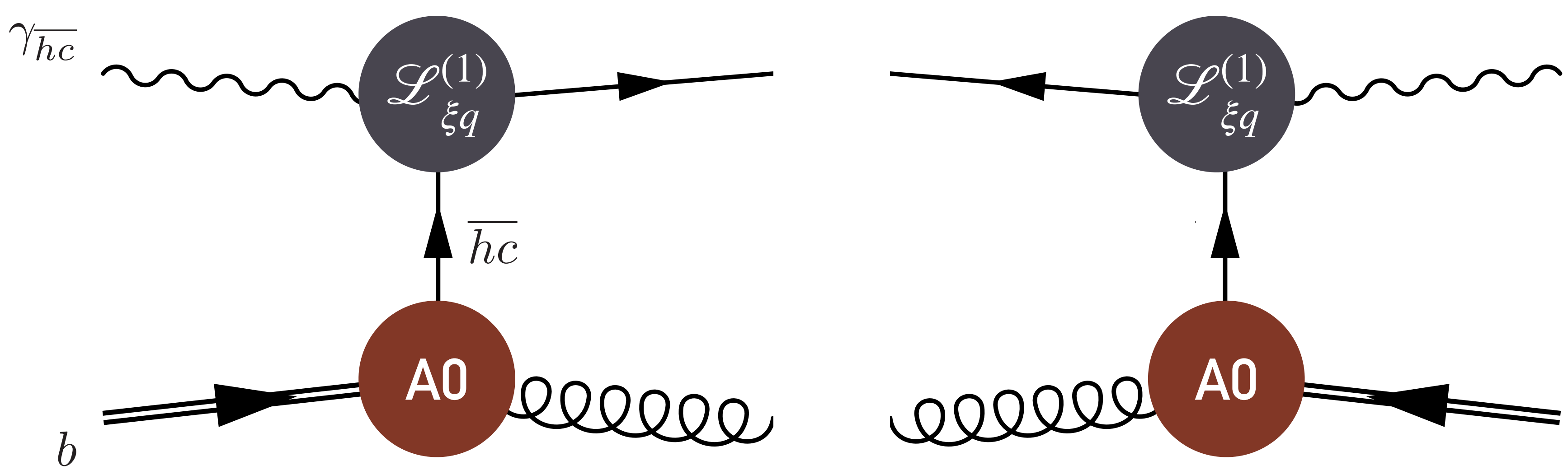
$$\mathcal{S}(\omega) = \frac{1}{2m_B}\int \frac{dt}{2\pi} e^{-i\omega t} \langle B \Big| \, h(tn)\, S_n(tn)\, S_n^\dagger(0)\, h(0) \, \Big| B \rangle$$

$$\frac{d\Gamma}{dE_\gamma} = \mathcal{N}_B \int_0^1 du\, C^{B1}(m_b, u) \int_0^1 du'\, C^{B1*}(m_b, u') \int_{-p_+}^{\overline{\Lambda}} d\omega\, J\Big(M_B(p_+ + \omega), u, u'\Big)\, \mathcal{S}(\omega)$$



RESOLVED CONTRIBUTION

$$\frac{d\Gamma}{dE_\gamma} = \mathcal{N}_A \left| C^{A0}\left(m_b\right) \right|^2 \int_{-p_+}^{\overline{\Lambda}} d\omega J_g \left(m_b \left(p_+ + \omega \right) \right) \int d\omega_1 \int d\omega_2 \bar{J}\left(\omega_1\right) \bar{J}^*\left(\omega_2\right) \mathcal{S}\left(\omega, \omega_1, \omega_2\right)$$



$$C^{A0}_{LO}\left(m_b\right) = \frac{m_b^2}{4\pi^2} \frac{G_F}{\sqrt{2}} \lambda_q C_{8g}$$

RESOLVED CONTRIBUTION

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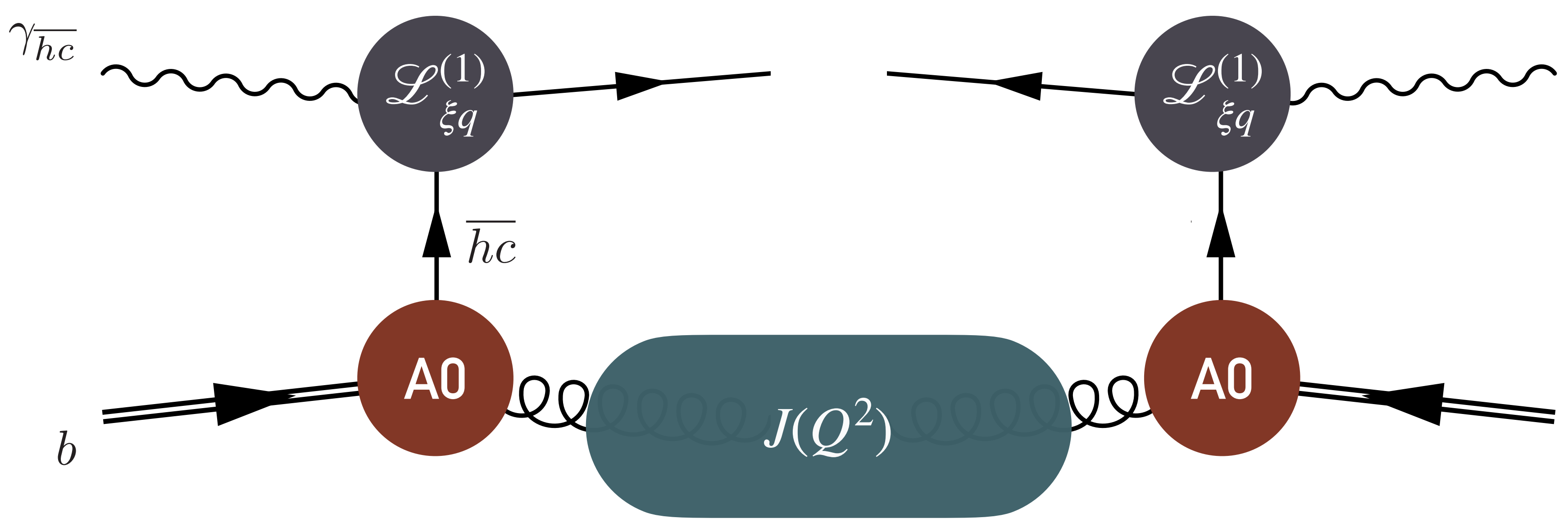
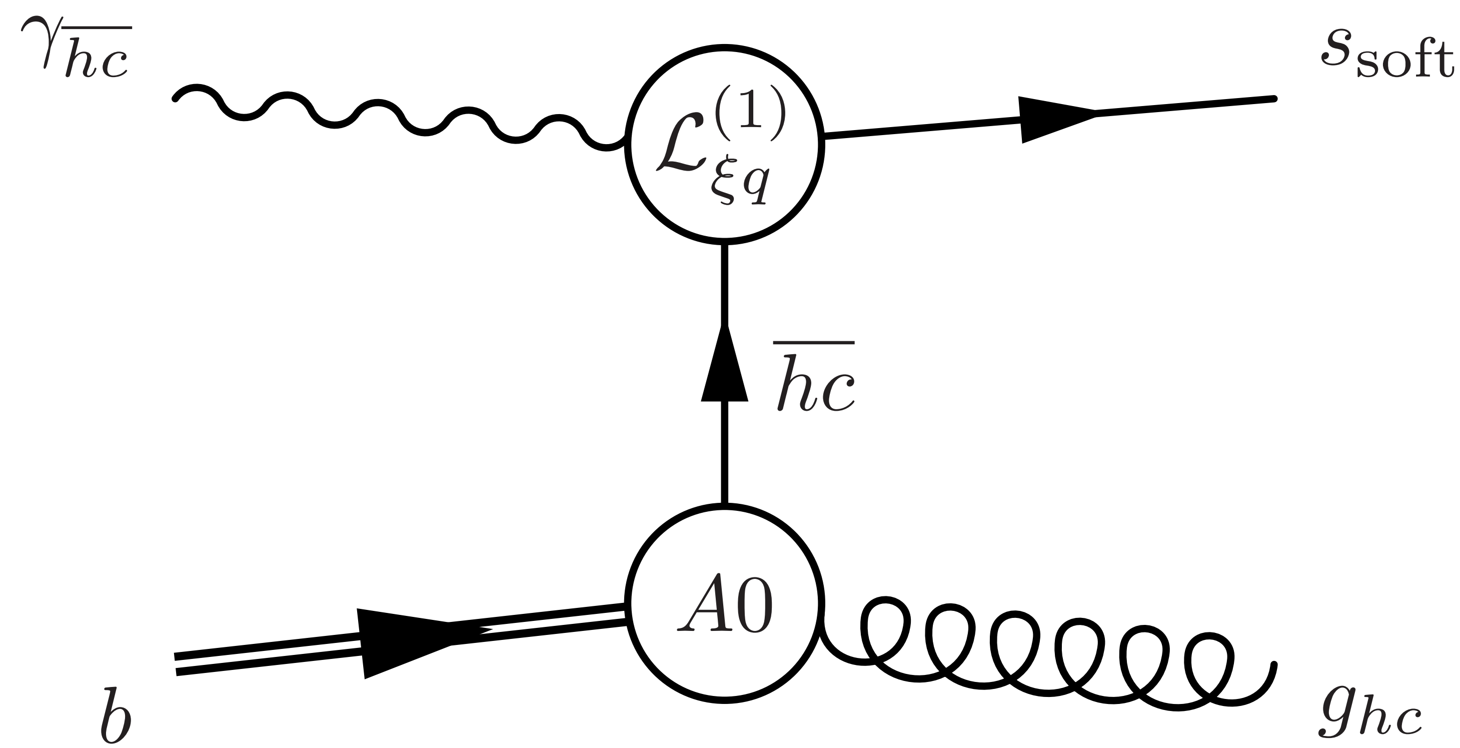


$$C_{LO}^{A0}\left(m_b\right) = \frac{m_b^2}{4\pi^2}\frac{G_F}{\sqrt{2}}\lambda_q C_{8g}$$

RADIATIVE JET FUNCTIONS

We integrate-out hard-anti-collinear QCD modes

$$\mathscr{O}_{T\xi q} = i\int d^d x T\left[\mathscr{L}^{(1)}_{\xi q}(x), \mathscr{O}^{A0}_{8g}(0)\right]$$



$\bar{J}(\omega)$ is formally identical to hard-collinear jet-functions typically defined when matching SCET I on SCET II

$$\mathscr{O}_{T\xi q} = \int d\omega \int \frac{dt}{2\pi} e^{-it\omega}\left[\overline{q_s}\right]_\alpha (tn)\left[\bar{J}(\omega)\right]^{a\,\nu\mu}_{\alpha\beta} Q_s\,\mathscr{B}^{\nu}{}_{\overline{hc}\perp}(0)\,\mathscr{A}^{\mu}_{hc\perp}(0)\left[h(0)\right]_\beta .$$

RADIATIVE JET FUNCTIONS

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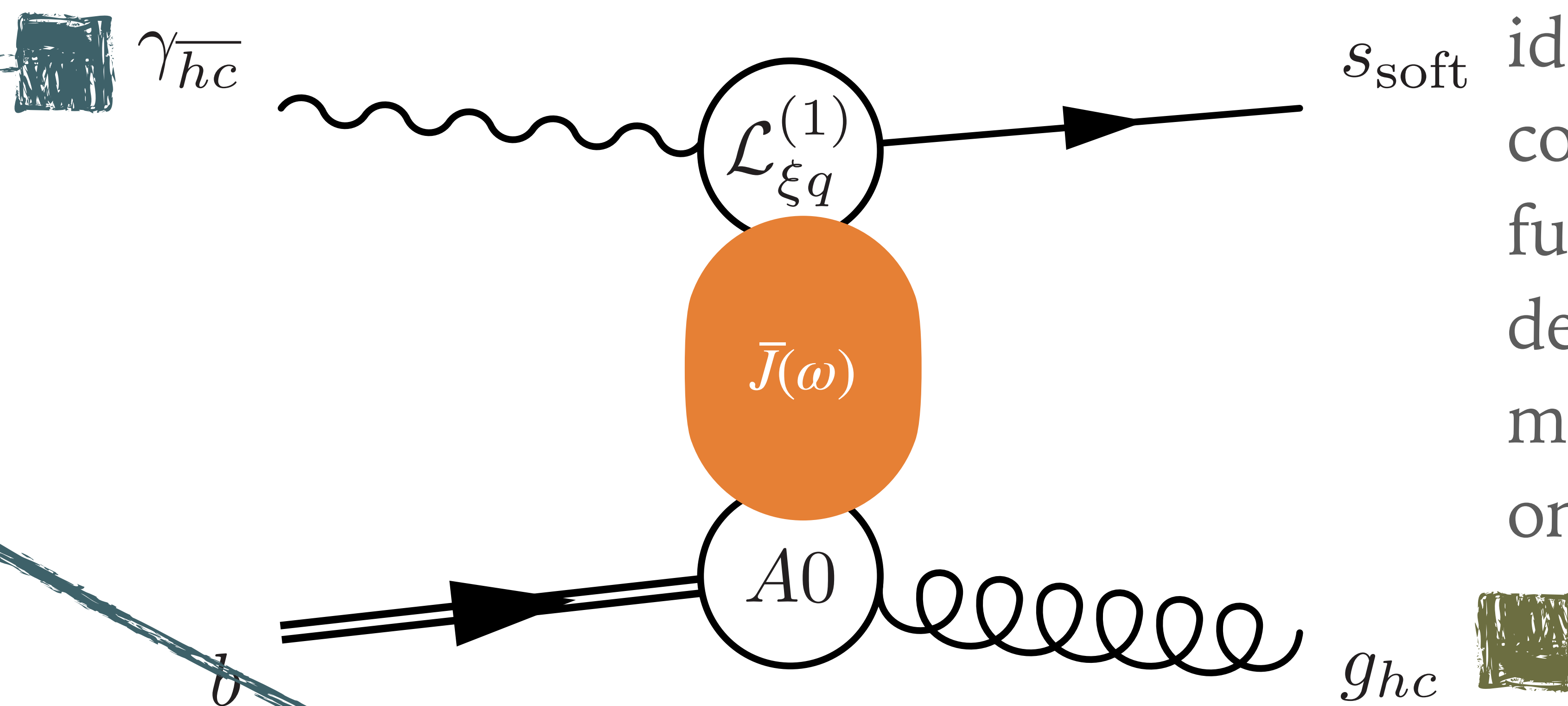

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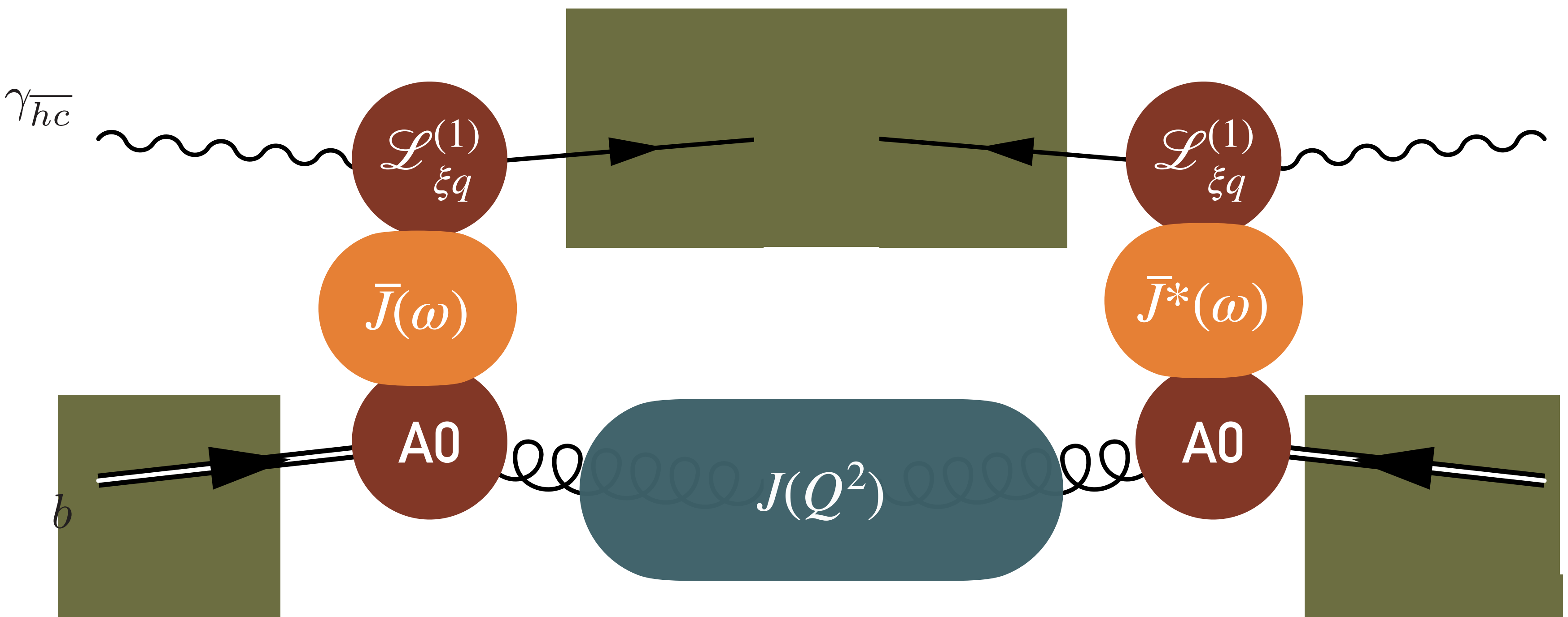
RESOLVED CONTRIBUTION

$$\frac{d\Gamma}{dE_\gamma} = \mathcal{N}_A \left| C^{A0}(m_b) \right|^2 \int_{-p_+}^{\bar{\Lambda}} d\omega J_g\left(m_b\left(p_+ + \omega\right)\right) \int d\omega_1 \int d\omega_2 \bar{J}(\omega_1) \bar{J}^*(\omega_2) \mathcal{S}(\omega, \omega_1, \omega_2)$$



$$\mathcal{S}(\omega, \omega_1, \omega_2) = \int \frac{du}{2\pi} e^{-iu\omega} \int \frac{dt}{2\pi} e^{-it\omega_1} \int \frac{dt'}{2\pi} e^{it'\omega_2} \mathcal{S}(u, t, t')$$

$$\mathcal{S}(u,t,t') = (d-2)^2 g_s^2 \langle B| \bar{h}(un)(1-\gamma_5)\left[S_n(un)\, t^a S_n^\dagger(un)\right] S_{\bar{n}}(un) S_{\bar{n}}^\dagger(t'\bar{n}+un) \frac{\not{n}\not{\bar{n}}}{4} q_s(t'\bar{n}+un)\, \bar{q}_s(t\bar{n}) \frac{\not{\bar{n}}\not{n}}{4} S_{\bar{n}}(t\bar{n}) S_{\bar{n}}^\dagger(0)\left[S_n(0)\, t^a S_n^\dagger(0)\right](1+\gamma_5) h(0)|B\rangle \,/\, (2m_B)$$

ENDPOINT DIVERGENCE IN RESOLVED CONTRIBUTION

$$\left[\bar{J}(\omega)\right]^{a\,\nu\mu}_{\alpha\beta} = \frac{t^a}{(\omega + i\,\epsilon)} \left[\gamma^{\nu}_{\perp}\gamma^{\mu}_{\perp}\frac{\bar{\not n}\not n}{4}\right]_{\alpha\beta}$$

T. Hurth, R.S.

$$\frac{d\Gamma}{dE_\gamma} = 2\mathcal{N}\, \left|C^{A0}_{LO}(m_b)\right|^2 \int_{-p_+}^{\bar{\Lambda}} d\omega\delta\left(m_b\left(p_+ + \omega\right)\right) \int_{-\infty}^{\infty} d\omega_1 \int_{-\infty}^{\omega_1} d\omega_2 \frac{1}{(\omega_1 - i\epsilon)} \frac{1}{(\omega_2 + i\epsilon)} \mathcal{S}\left(\omega, \omega_1, \omega_2\right)$$

For $\omega, \omega' \sim \Lambda$, the integral is well behaved

For $\omega, \omega' \gg \Lambda$ light quarks become hard-collinear and we can decouple soft gluons from them

$$\mathcal{S}\left(u,t,t'\right) = (d-2)^2 g_s^2 \left\langle B\right| \bar{h}\left(un\right)\left(1-\gamma_5\right)\left[S_n\left(un\right)t^a S_n^\dagger\left(un\right)\right] S_{\bar{n}}\left(un\right) S_{\bar{n}}^\dagger\left(t'\bar{n}+un\right)$$
$$\frac{\not n\bar{\not n}}{4} q_s\left(t'\bar{n}+un\right)\bar{q}_s\left(t\bar{n}\right)\frac{\bar{\not n}\not n}{4} S_{\bar{n}}\left(t\bar{n}\right) S_{\bar{n}}^\dagger(0)\left[S_n(0)\,t^a S_n^\dagger(0)\right]\left(1+\gamma_5\right)h(0)\left|B\right\rangle\,/\,(2m_B)$$

$$\mathcal{S}(u,t,t') \to \mathcal{S}\left(u\right) = \frac{1}{2m_B}\left\langle B\right|\, h\left(un\right) S_n\left(un\right) S_n^\dagger\left(0\right) h\left(0\right)\,\left|B\right\rangle$$

ENDPOINT DIVERGENCE IN RESOLVED CONTRIBUTION

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T. Hurth, R.S.

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$$\frac{\not{n}\not{\bar{n}}}{4} q_s(t'\bar{n}+un)\,\bar{q}_s(t\bar{n})\,\frac{\not{\bar{n}}\not{n}}{4} S_{\bar{n}}(t\bar{n})\,S_{\bar{n}}^\dagger(0)\left[S_n(0)\,t^a S_n^\dagger(0)\right](1+\gamma_5)\,h(0)\,|B\rangle\;/\,(2m_B)$$

Schematically: $\quad t, t' \to 0$

$$\mathcal{S}(u,t,t') \to \mathcal{S}\,(u) = \frac{1}{2m_B}\langle B\,|\;h\,(un)\,S_n\,(un)\,S_n^\dagger\,(0)\,h\,(0)\;|\,B\rangle$$

ENDPOINT DIVERGENCE IN RESOLVED CONTRIBUTION

$$\left[\bar{J}(\omega)\right]^{a\,\nu\mu}_{\alpha\beta} = \frac{t^a}{(\omega + i\,\epsilon)}\left[\gamma_\perp^\nu \gamma_\perp^\mu \frac{\not{\bar{n}}\not{n}}{4}\right]_{\alpha\beta}$$

T. Hurth, R.S.

$$\frac{d\Gamma}{dE_\gamma} = 2\mathcal{N}\,\left|C_{LO}^{A0}(m_b)\right|^2 \int_{-p_+}^{\bar{\Lambda}} d\omega\,\delta\left(m_b\left(p_+ + \omega\right)\right)\int_{-\infty}^{\infty} d\omega_1 \int_{-\infty}^{\omega_1} d\omega_2 \frac{1}{(\omega_1 - i\epsilon)}\frac{1}{(\omega_2 + i\epsilon)}\,\mathcal{S}\left(\omega, \omega_1, \omega_2\right)$$

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$$\frac{\not{n}\not{\bar{n}}}{4} q_s(t'\bar{n}+un)\,\bar{q}_s(t\bar{n})\,\frac{\not{\bar{n}}\not{n}}{4} S_{\bar{n}}(t\bar{n})\,S_{\bar{n}}^\dagger(0)\left[S_n(0)\,t^a S_n^\dagger(0)\right](1+\gamma_5)\,h(0)\,|B\rangle\;/\;(2m_B)$$

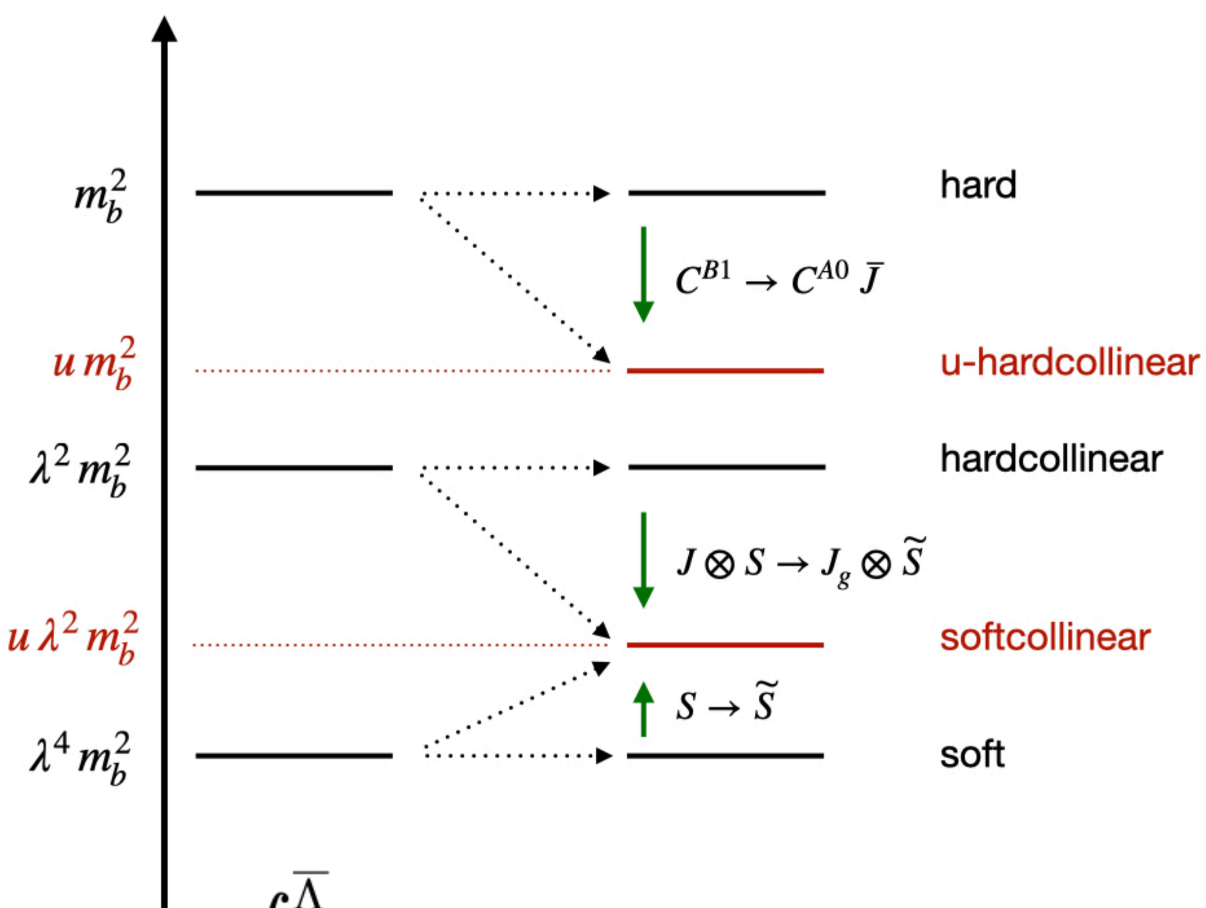
Schematically: $\quad t, t' \to 0 \qquad q_s(un) \to S_n(un) \qquad \bar{q}_s(0) \to S_n^\dagger(0)$

$$\mathcal{S}(u,t,t') \to \mathcal{S}(u) = \frac{1}{2m_B}\langle B|\,h(un)\,S_n(un)\,S_n^\dagger(0)\,h(0)\,|B\rangle$$

ENDPOINT RESHUFFLING

Endpoint relations can be formulated as operator statements — *T. Hurth, R.S.*

$$C^{B1}\left(m_b,u\right)\left\langle \mathcal{O}_{8g}^{B1}(u)\right\rangle\Big|_{u\to 0} = C^{A0}\left(m_b\right) i\int d^dx e^{-i(nx/2)um_b}\left\langle T\left\{\mathcal{L}^{(1)}_{\xi q_{sc}}(x), \mathcal{O}_{8g}^{A0-u}(0)\right\}\right\rangle$$



Factorization conditions

$$\lim_{u\to 0} C^{B1}\left(m_b,u\right) = (-1)C^{A0}\left(m_b\right) m_b\bar{J}\left(um_b\right)$$

M. Beneke et. al., 2020

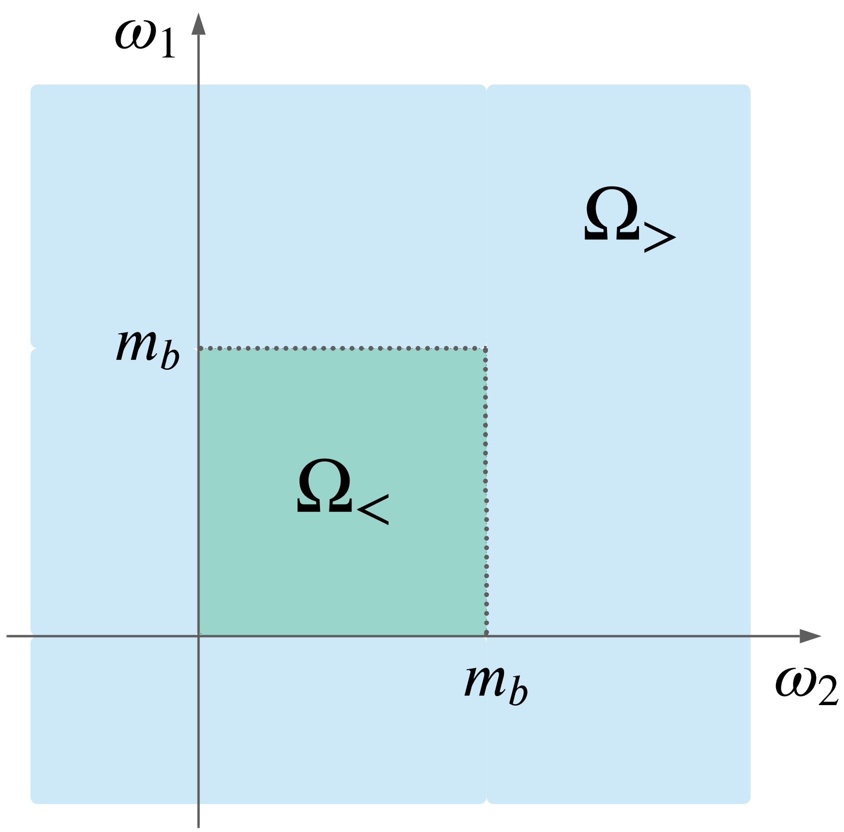
$$\int_{-p_+}^{\bar{\Lambda}} d\omega\, [\![J\left(m_b\left(p_+ + \omega\right), u, u'\right)\mathcal{S}(\omega)]\!] = \int_{-p_+}^{\bar{\Lambda}} d\omega J_g(m_b(p_+ + \omega))\widetilde{\mathcal{S}}(\omega, m_b u, m_b u')$$

ENDPOINT SUBTRACTION

T. Hurth, R.S.

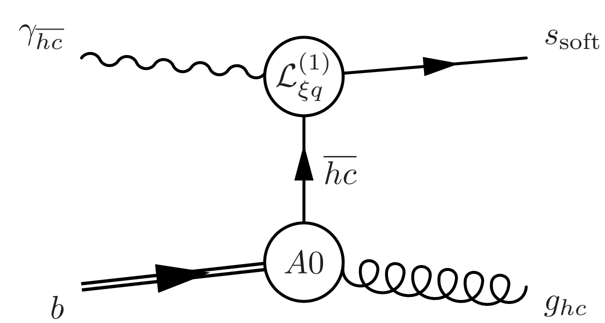
$$\int d\omega_1 d\omega_2 \widetilde{S}(\omega_1, \omega_2, \omega) \bar{J}(\omega_1) \bar{J}^*(\omega_2) = 0$$

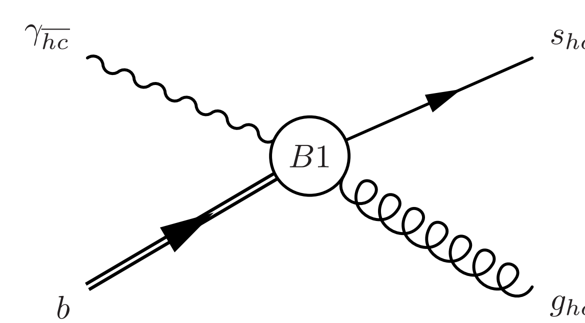
$$0 = \int_0^\infty d\omega_1 \int_0^\infty d\omega_2 = 2 \int_0^\infty d\omega_1 \int_0^{\omega_1} d\omega_2 = 2 \left(\int_0^{m_b} d\omega_1 + \int_{m_b}^\infty d\omega_1 \right) \int_0^{\omega_1} d\omega_2$$



We can subtract the endpoint contribution from both terms in the factorization theorem

Note, when $\omega_1 \gg \omega_2$ or $\omega_2 \gg \omega_1$, there is no divergence in SCET I problems

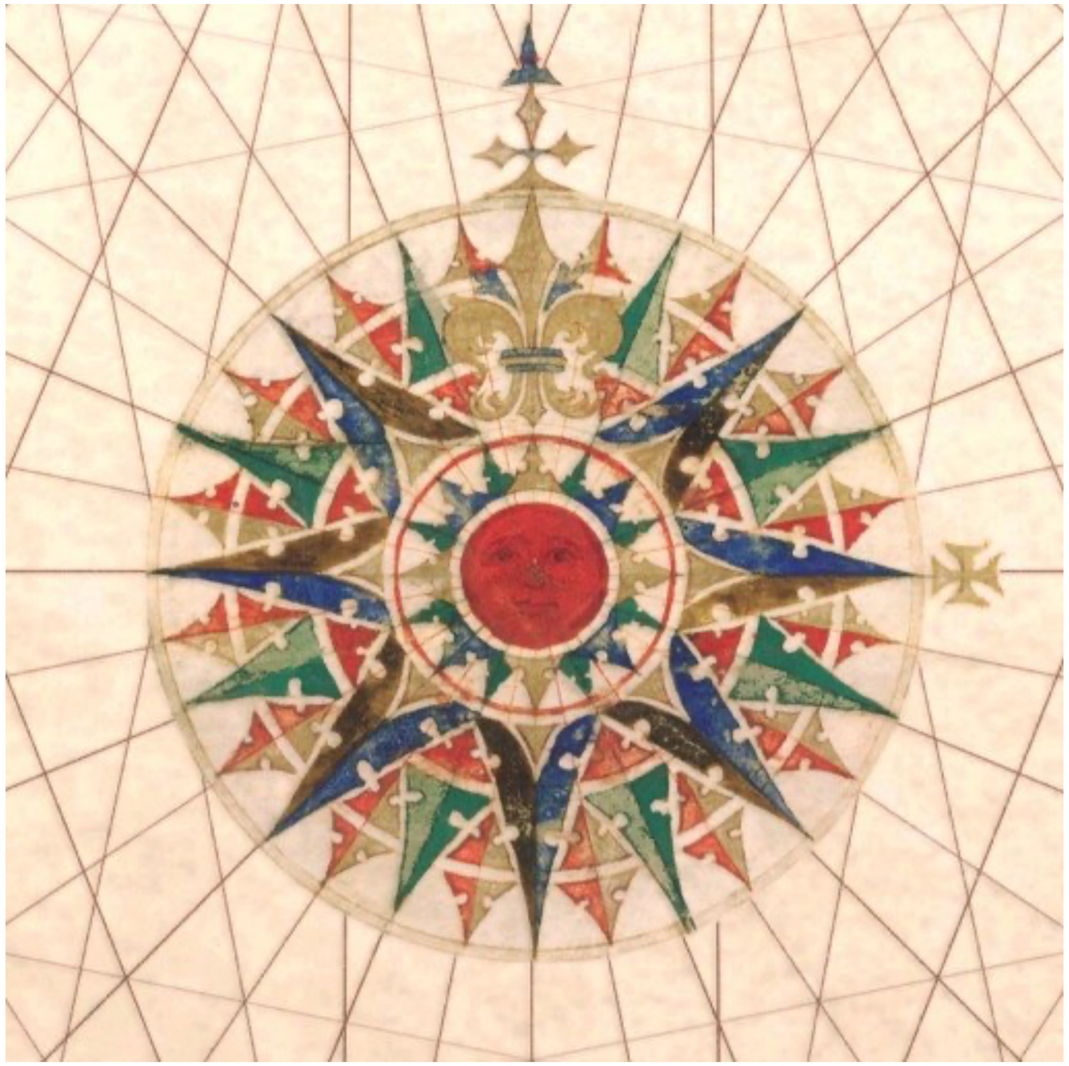




WITH THIS REARRANGEMENT WE ACHIEVED NLP FACTORIZATION!

SUMMARY

- World of power corrections is still unexplored and offers many new opportunities
- Exciting theoretical explorations of the all-order structure of QFT and non-local EFTs
- Strong phenomenological motivations in the precision era
  - Flavor physics
  - Collider physics
  - New physics searches



TIME TO EXPLORE THE UNKNOWN WITHIN THE STANDARD MODEL TO MAKE DISCOVERIES