Sterile neutrinos in $0\nu\beta\beta$

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with
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INSTITUTE for NUCLEAR THEORY

Sterile neutrinos

- ν_R's could help solve several SM deficiencies:
  - Neutrino masses
  - Leptogenesis
  - Dark matter candidate
- Appear in Left-Right/Leptoquarks/GUTs

Canetti et al. '13

Boyarski et al. '19

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$$\mathscr{L} = \mathscr{L}_{SM} - \frac{1}{2}\bar{\nu}_R M_R \nu_R^c - \bar{L}\tilde{H}Y_\nu \nu_R + \mathrm{h.c.}$$

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- In the mass basis

$$\mathscr{L} = -\frac{1}{2}\bar{\nu}_i^c m_i \nu_i$$

$$U^T M_\nu U = \text{diag}(m_1, m_2 \ldots m_{3+n}) , \qquad \nu_{\text{mass}} = U N_{\text{flavor}}$$

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PMNS mixing matrix

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Generally Majorana neutrinos

$\implies 0\nu\beta\beta$

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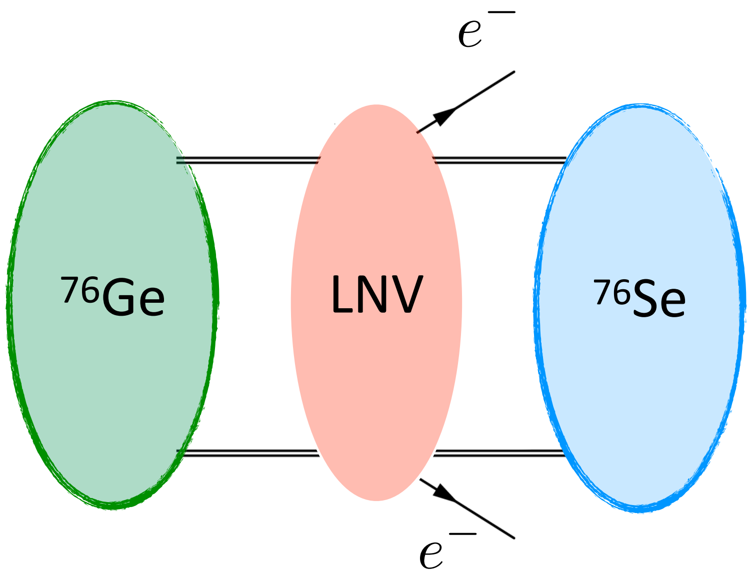
PMNS mixing matrix

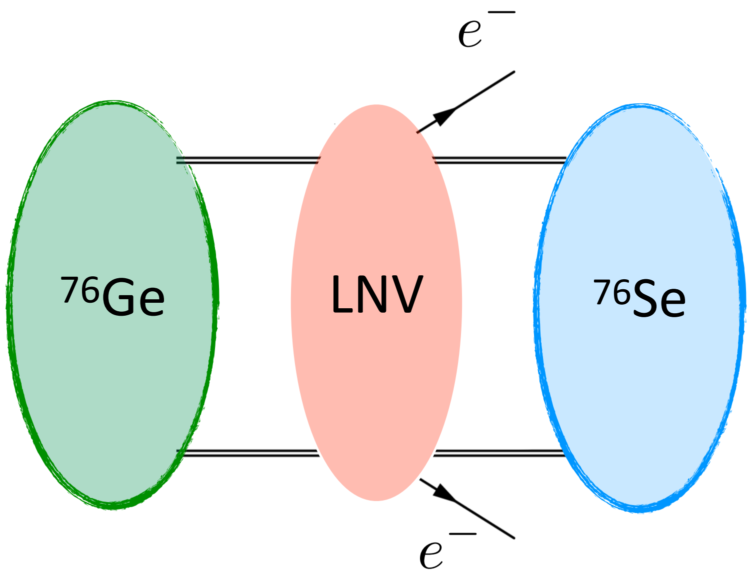
$0\nu\beta\beta$



- Violates lepton number, ΔL=2

Schechter, Valle, `82

$0\nu\beta\beta$



- Violates lepton number, ΔL=2

- Stringently constrained experimentally
    - To be improved by 1-2 orders

$T^{0\nu}_{1/2}(^{76}\mathrm{Ge})$ Gerda	$T^{0\nu}_{1/2}(^{130}\mathrm{Te})$ Cuore	$T^{0\nu}_{1/2}(^{136}\mathrm{Xe})$ KamLAND-zen
$> 9 \cdot 10^{25}\,\mathrm{yr}$	$> 3.2 \cdot 10^{25}\,\mathrm{yr}$	$> 1.1 \cdot 10^{26}\,\mathrm{yr}$

Future reach:
(LEGEND, nEXO, CUPID)

$T^{0\nu}_{1/2} > 10^{28}\mathrm{yr}$

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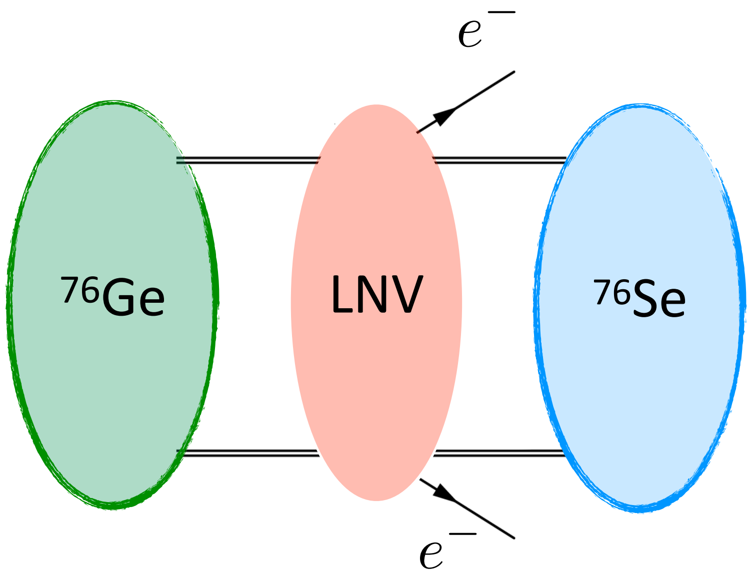

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- Would imply that
  - Neutrino’s are Majorana particles
  - Physics beyond the SM

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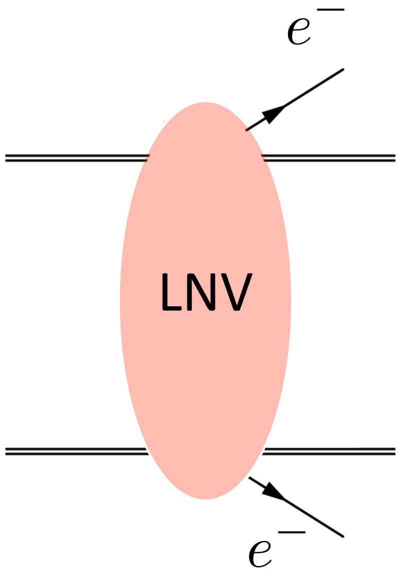
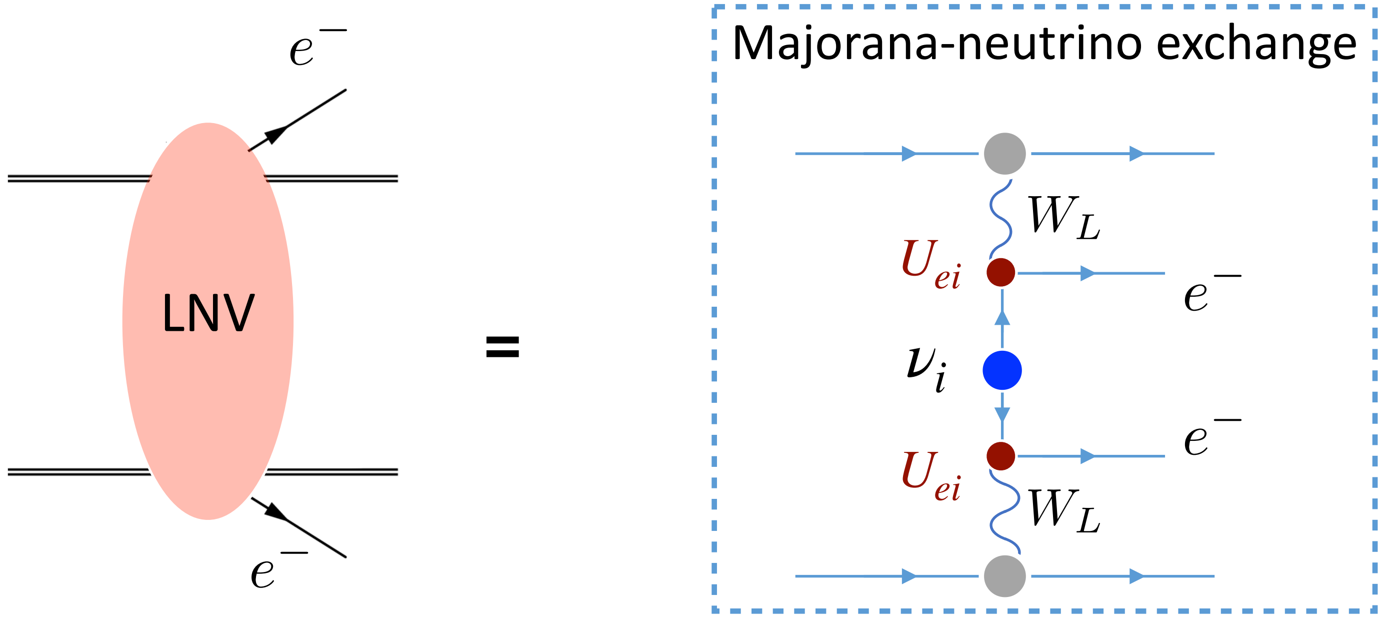
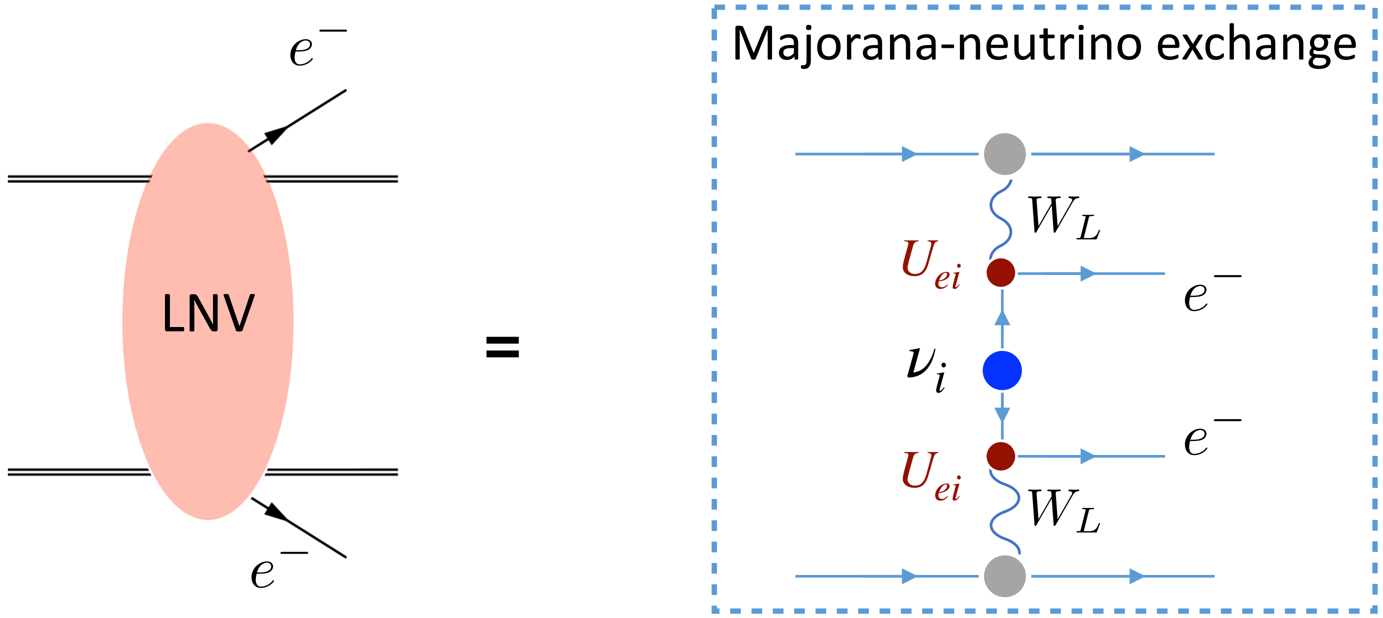


= **BSM physics**

$0\nu\beta\beta$



$0\nu\beta\beta$



Amplitude

$$\mathscr{M} \sim G_F^2 \bar{u}(p_{e1}) u^c(p_{e2}) \sum_{i=1}^{3+n} m_i U_{ei}^2 \int_{x,y} \langle {}^{136}\mathrm{Ba} \,|\, T\{(\bar{u}_L \gamma^\mu d_L)_x (\bar{u}_L \gamma_\mu d_L)_y\} \,|\, {}^{136}\mathrm{Xe} \rangle \int_q \frac{e^{iq\cdot(x-y)}}{q^2 - m_i^2}$$

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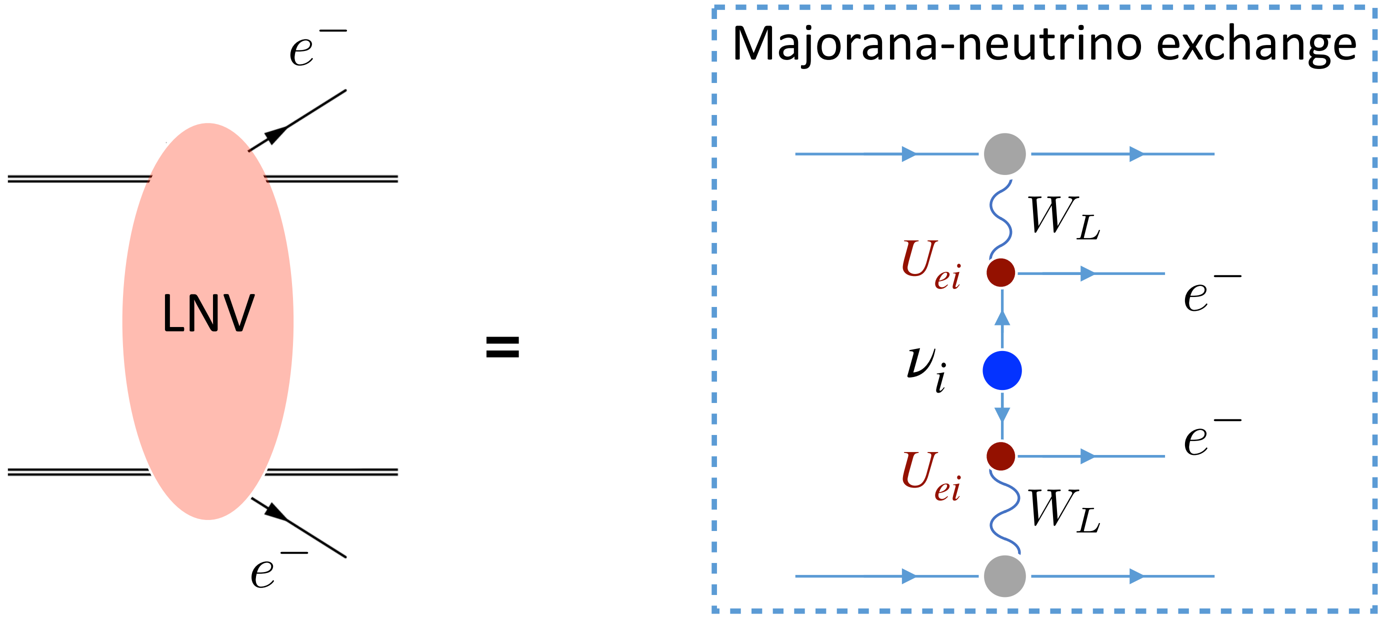

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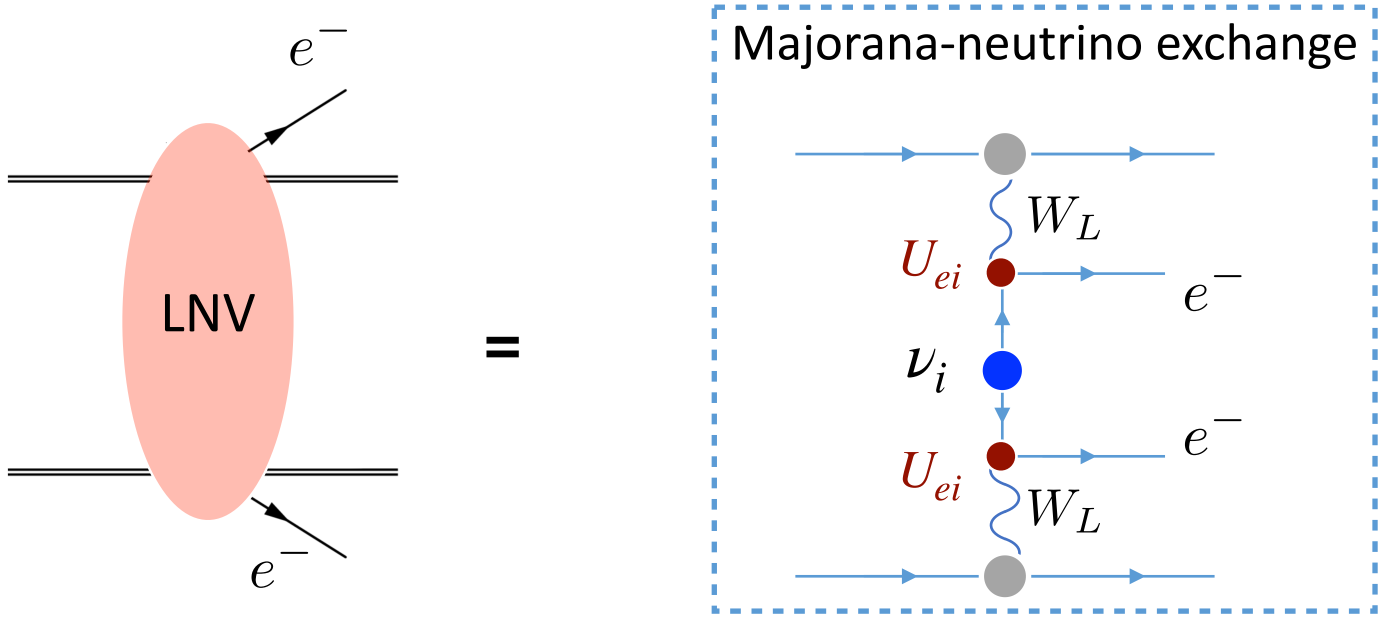
Decay rate

$$\Gamma \sim \int |\mathscr{M}|^2 \sim G_{01} \left| \sum_{i=1}^{3+n} m_i U_{ei}^2 A_\nu(m_i) \right|^2$$

$0\nu\beta\beta$



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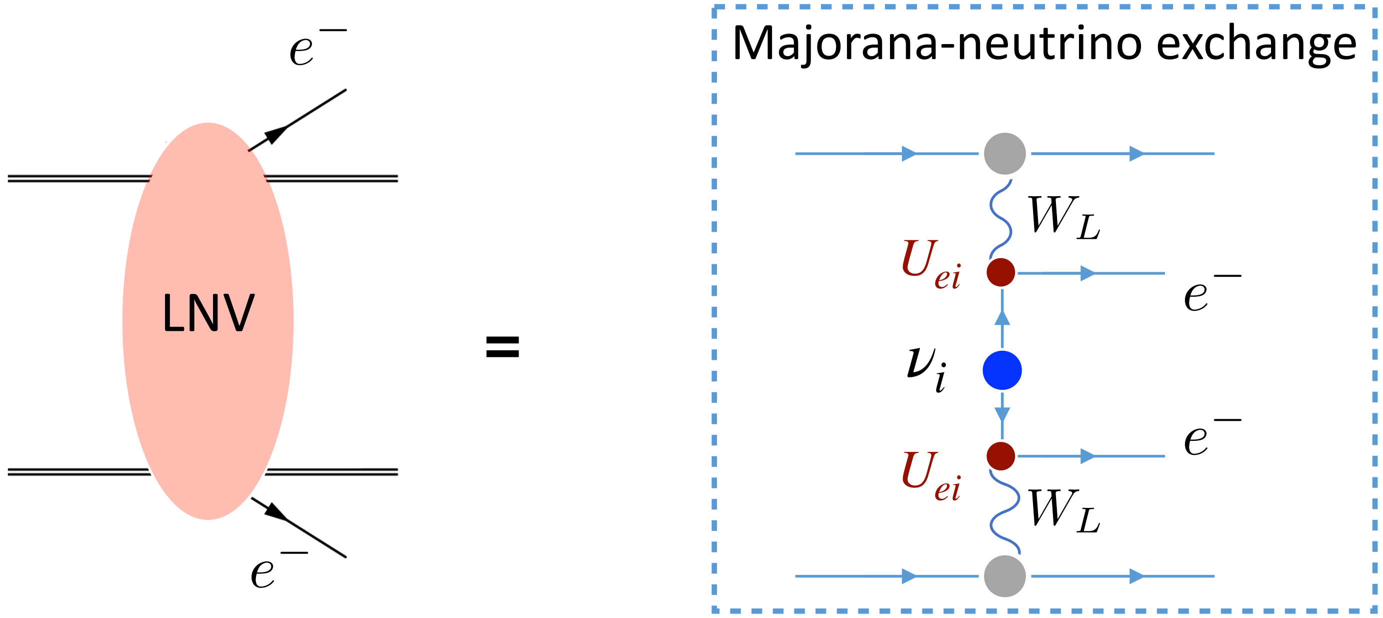
Phase space

$0\nu\beta\beta$



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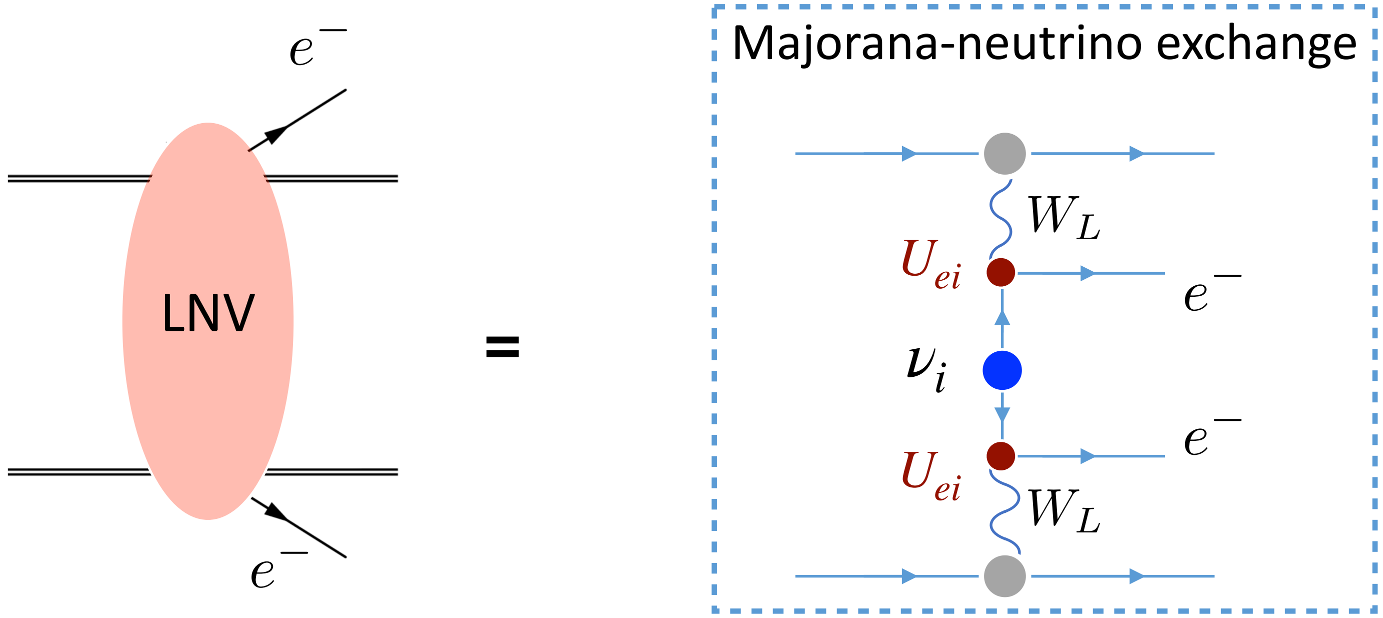
Phase space

model parameters

$0\nu\beta\beta$



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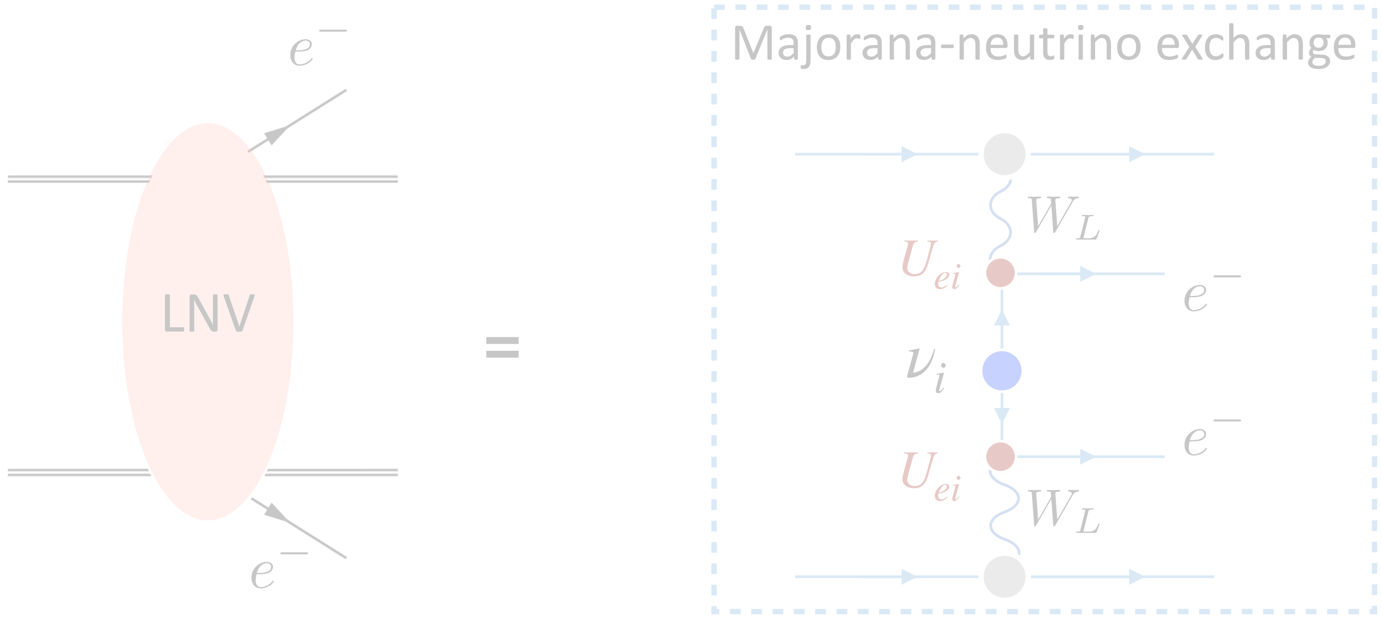
Hadronic/nuclear physics

$0\nu\beta\beta$



m_i dependence of A_ν
required for $0\nu\beta\beta$

Using: $A_\nu(m_i) = \text{constant}$

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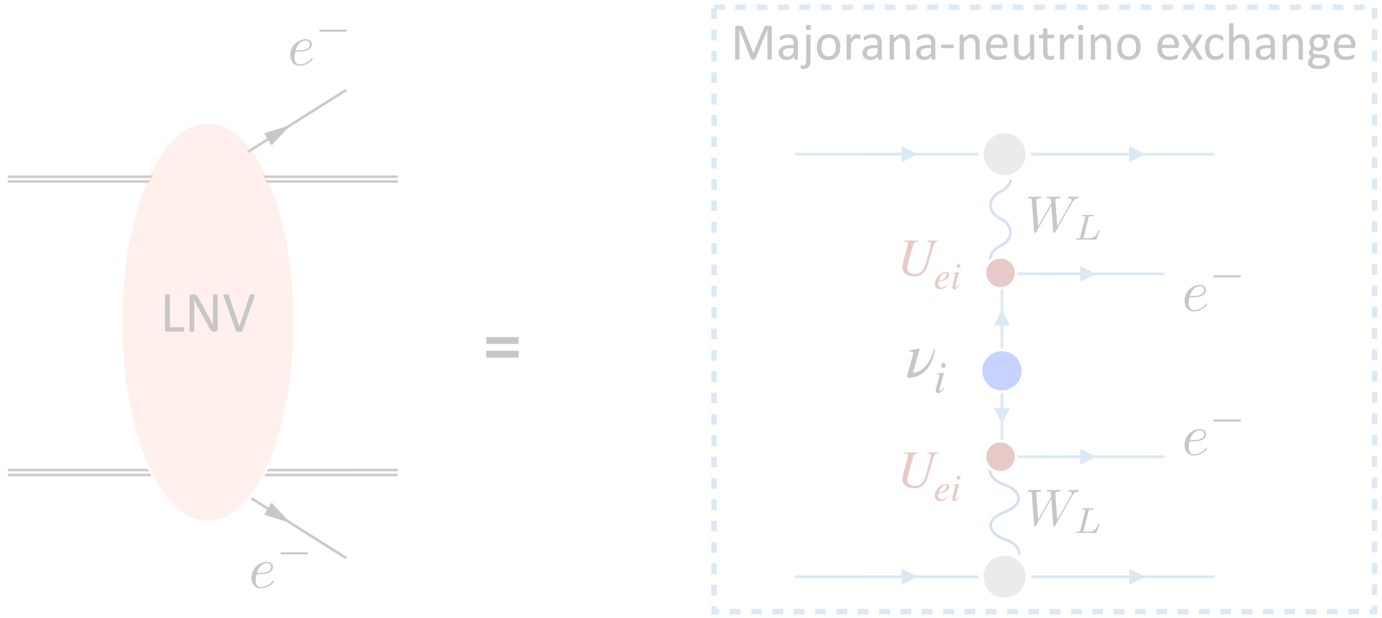

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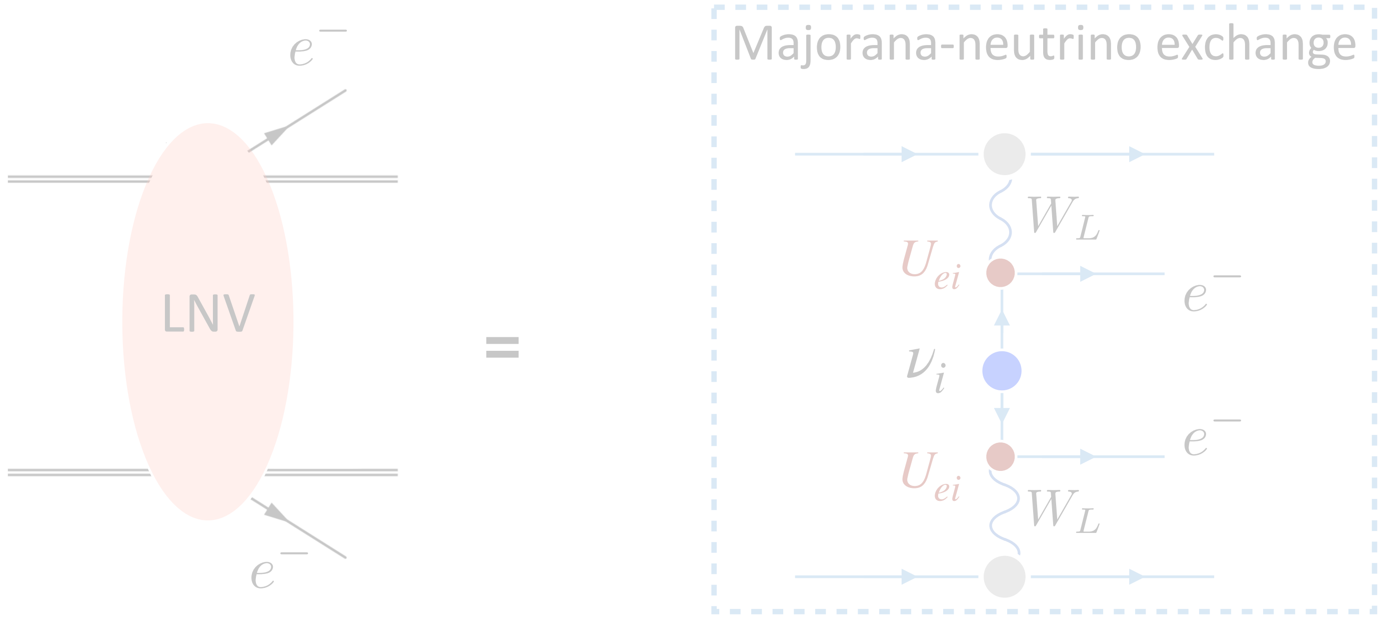

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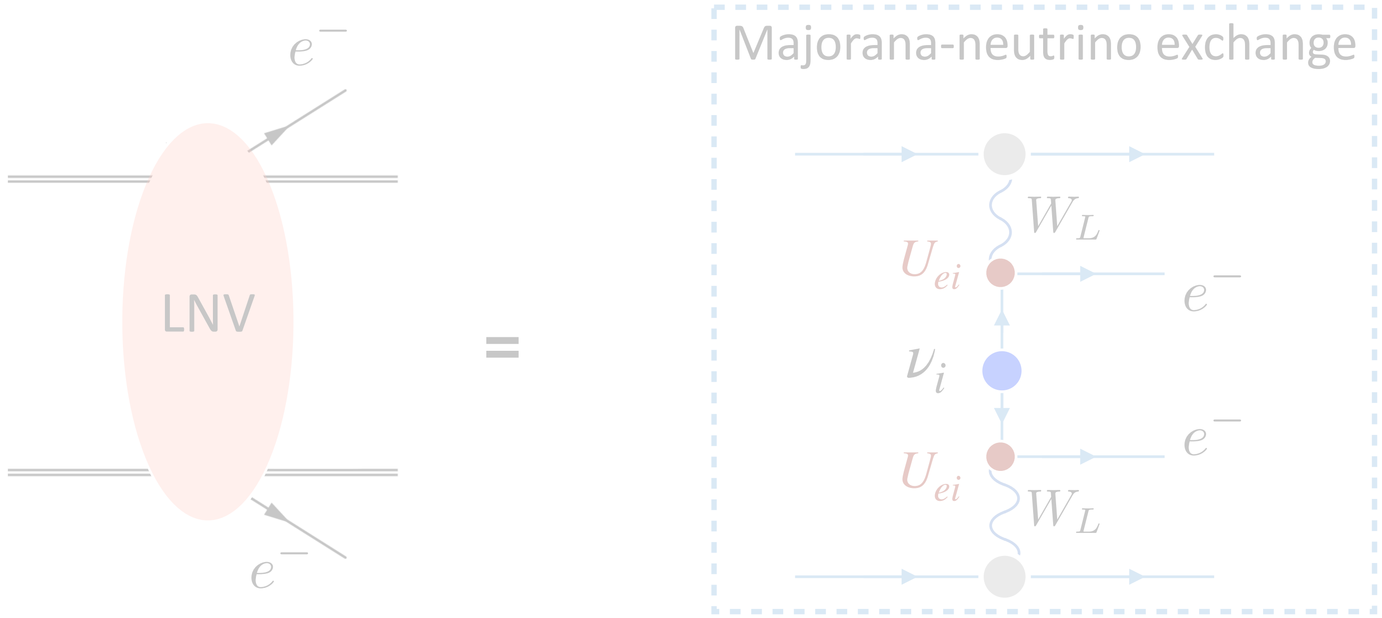
$0\nu\beta\beta$



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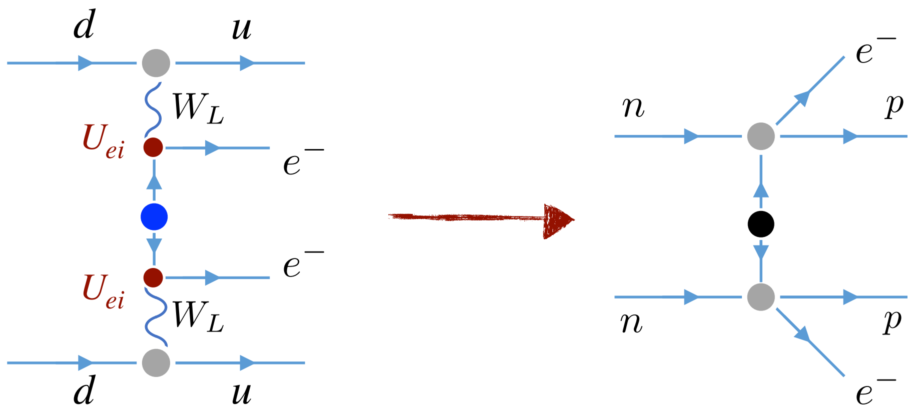
$0\nu\beta\beta$

Commonly used approach:

- Assume quark currents factorize:



$= V(q)$

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$0\nu\beta\beta$

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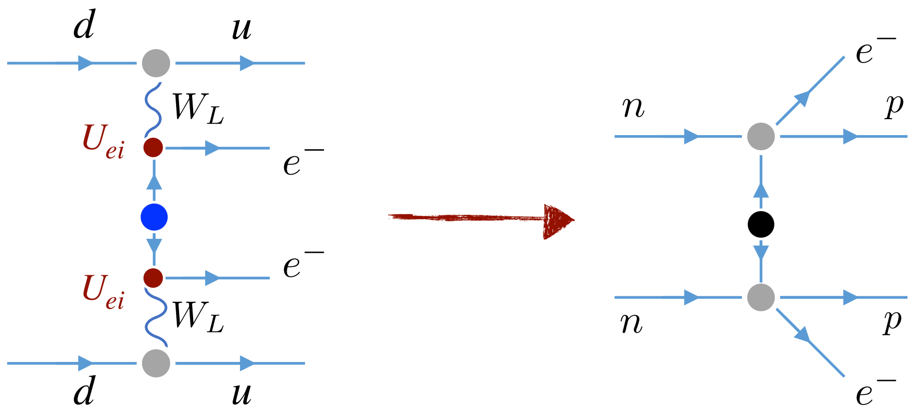

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Faessler et al, ’14; Asaka et al ’16; Bolton et al ’20,’22; Fang et al, ’22;

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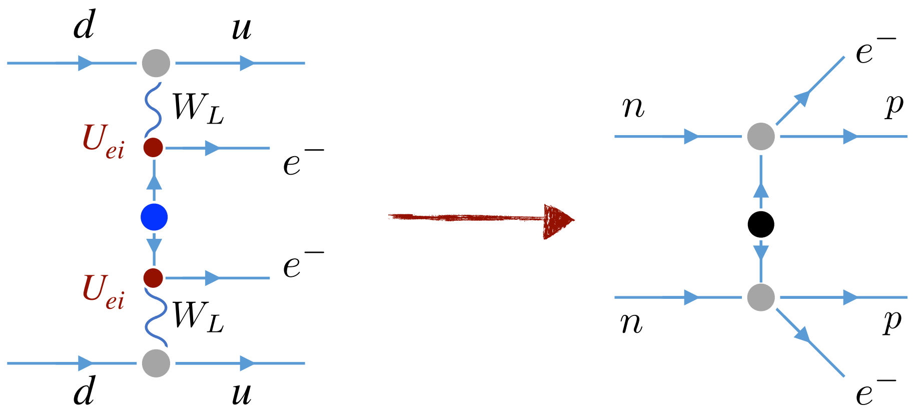

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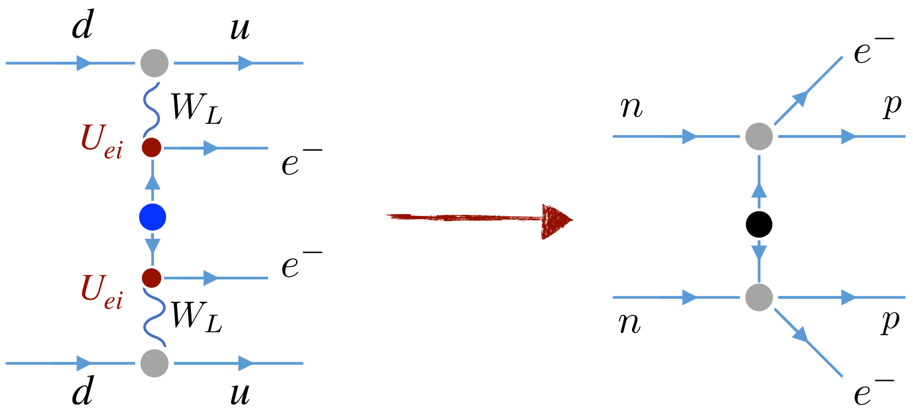

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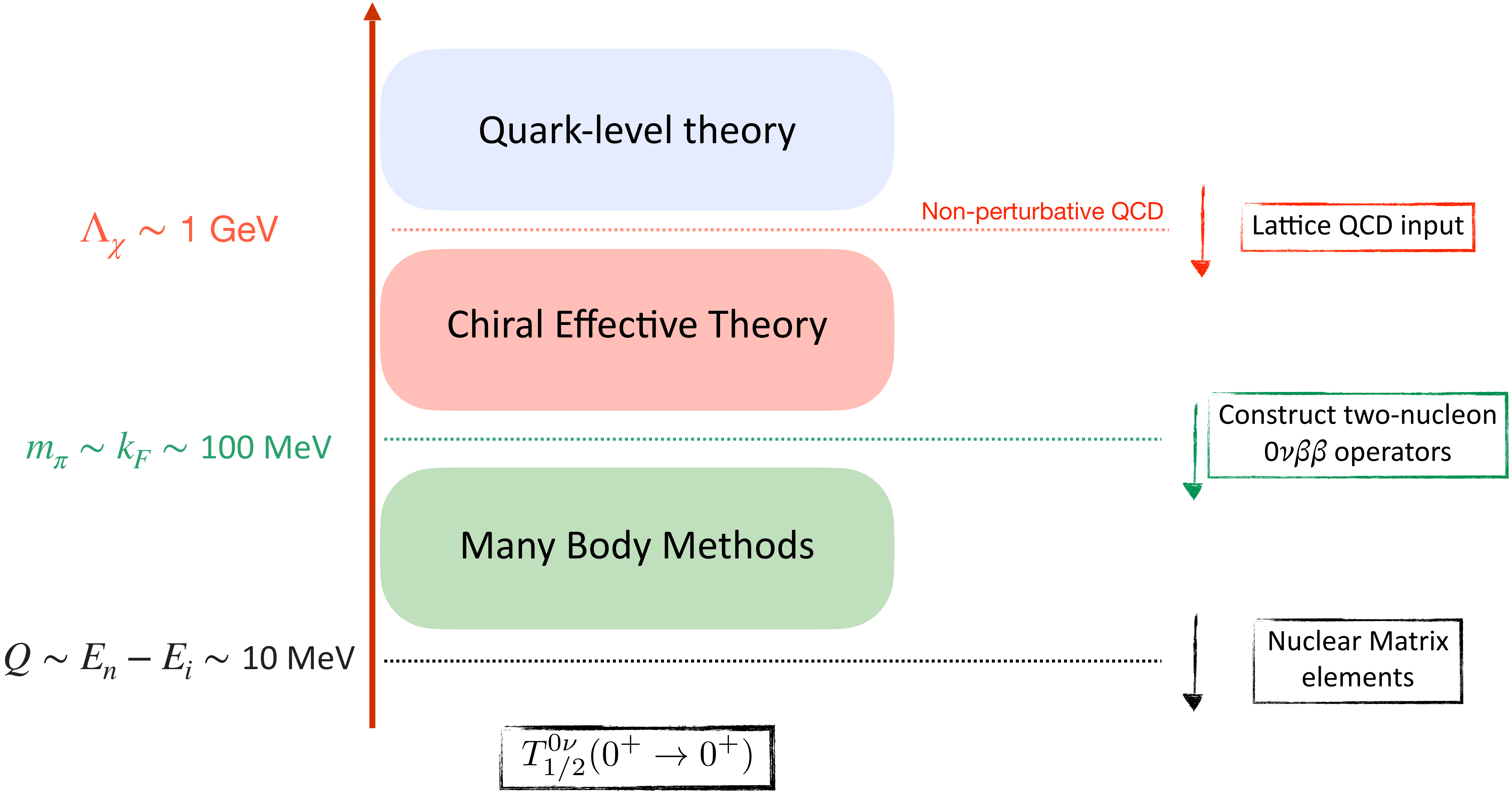
This talk:
Obtain A_ν using EFT approach

Faessler et al, ’14; Asaka et al ’16; Bolton et al ’20,’22; Fang et al, ’22;

EFT approach

One scale at a time



Quark-level theory

$\Lambda_\chi \sim$ 1 GeV

Non-perturbative QCD

Lattice QCD input

Chiral Effective Theory

$m_\pi \sim k_F \sim$ 100 MeV

Construct two-nucleon $0\nu\beta\beta$ operators

Many Body Methods

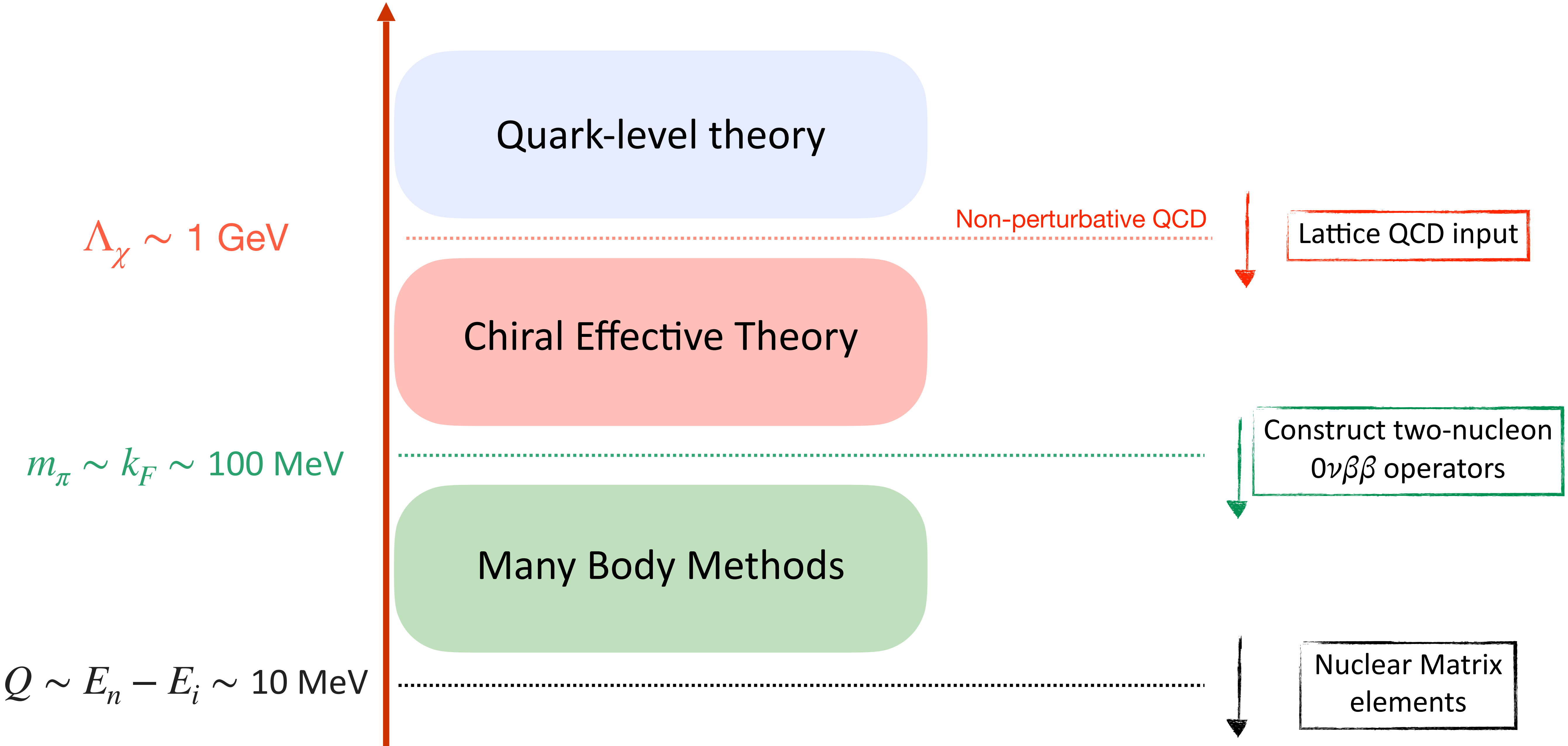
$Q \sim E_n - E_i \sim$ 10 MeV

Nuclear Matrix elements

$\boxed{T^{0\nu}_{1/2}(0^+ \to 0^+)}$

EFT approach

One scale at a time

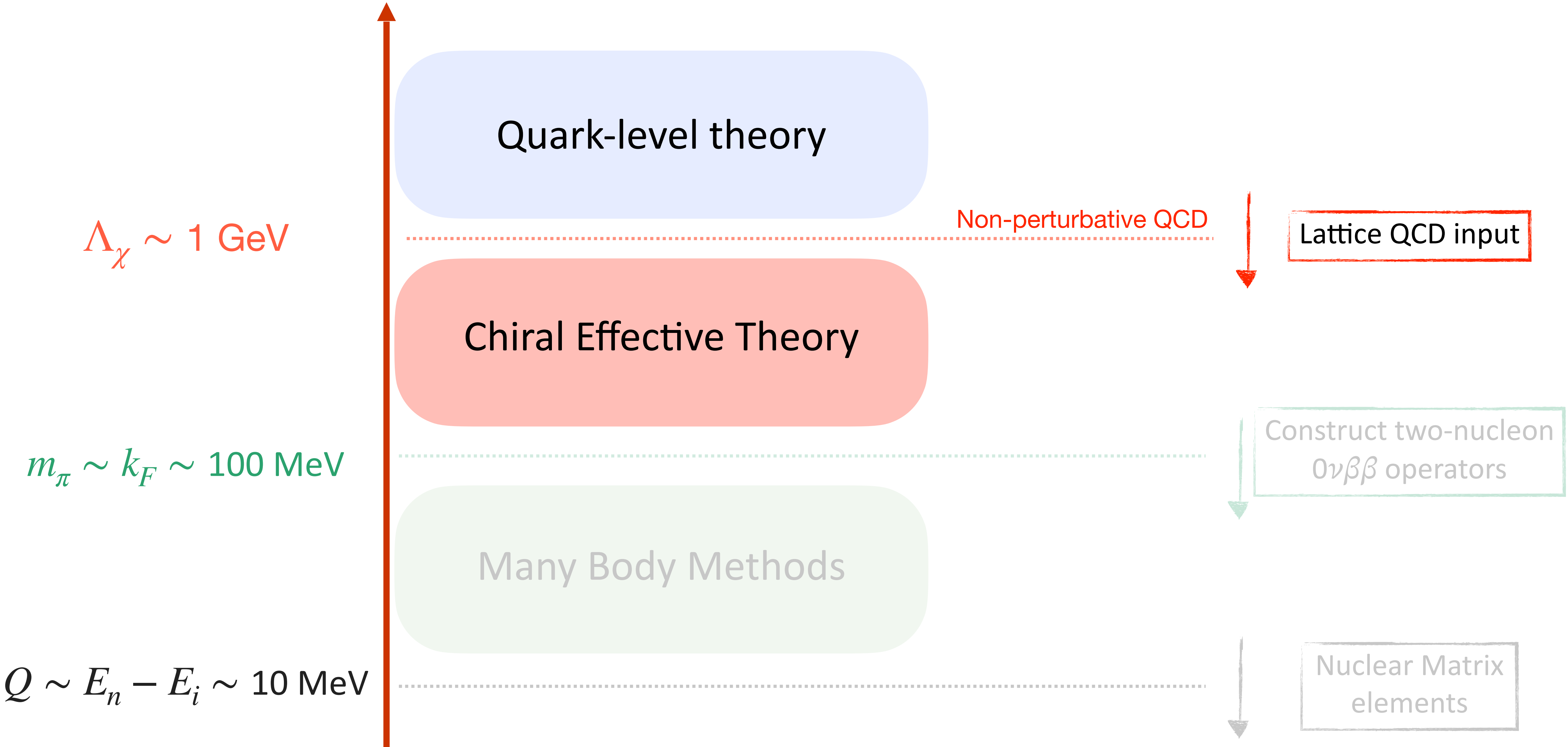


- Warm up exercise: simpler case of active neutrinos

  - $m_i \ll k_F$
  - $A_\nu(m_i) \simeq A_\nu$
  - So-called `Standard mechanism'

EFT approach

One scale at a time



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Matching to Chiral EFT

SM EFT'

SU(3)xU(1)$_{em}$ invariant

In terms of quarks, gluons, leptons $\mathcal{L}(u, d, g, \nu, e)$

M_{QCD}
1 GeV

Chiral Effective Theory

In terms of nucleons, pions, leptons $\mathcal{L}_{\chi PT}(N, \pi, \nu, e)$

Form of operators determined by chiral symmetry

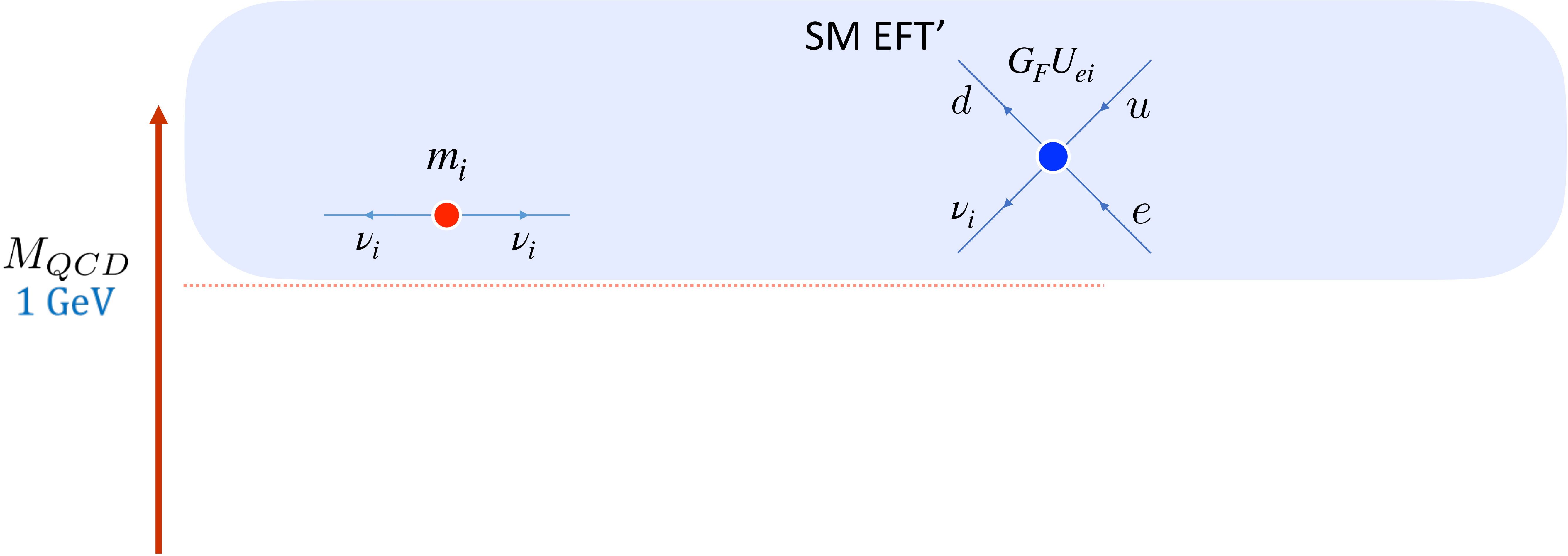
Each operator comes with an unknown constant (LEC)

Need a power-counting scheme

- Usually Naive dimensional analysis (NDA)/Weinberg power counting

Manohar, Georgi, `84; Weinberg, `90, `91

Matching to Chiral EFT



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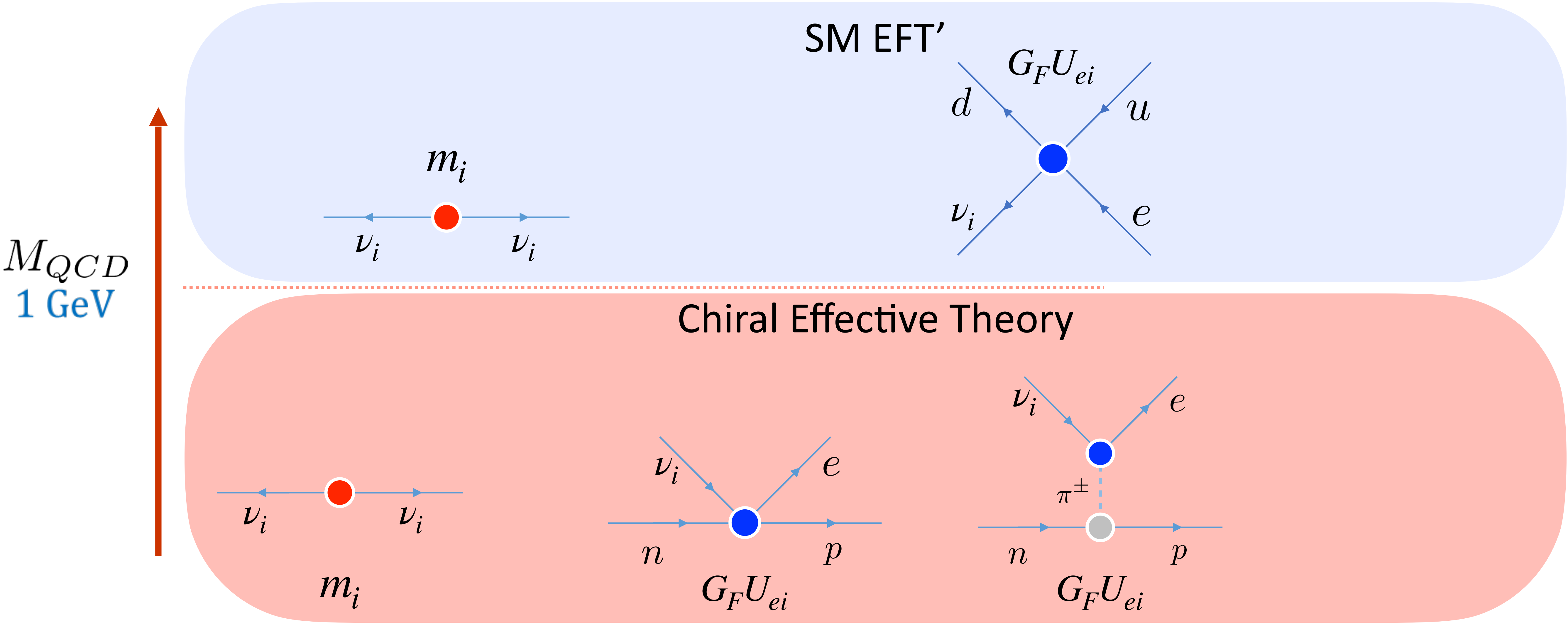
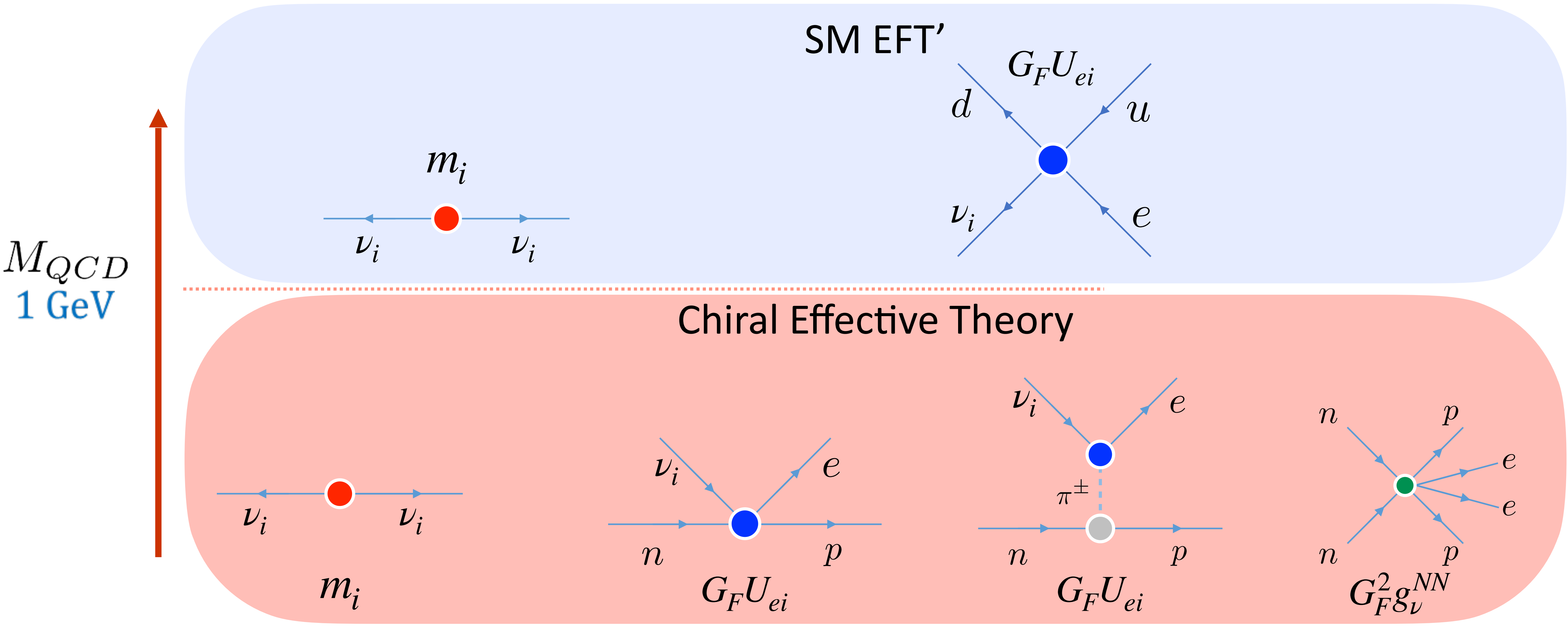


- At LO in Weinberg counting, only need the nucleon one-body currents
  - All needed low-energy constants are known

Matching to Chiral EFT



- At LO in Weinberg counting, only need the nucleon one-body currents
  - All needed low-energy constants are known

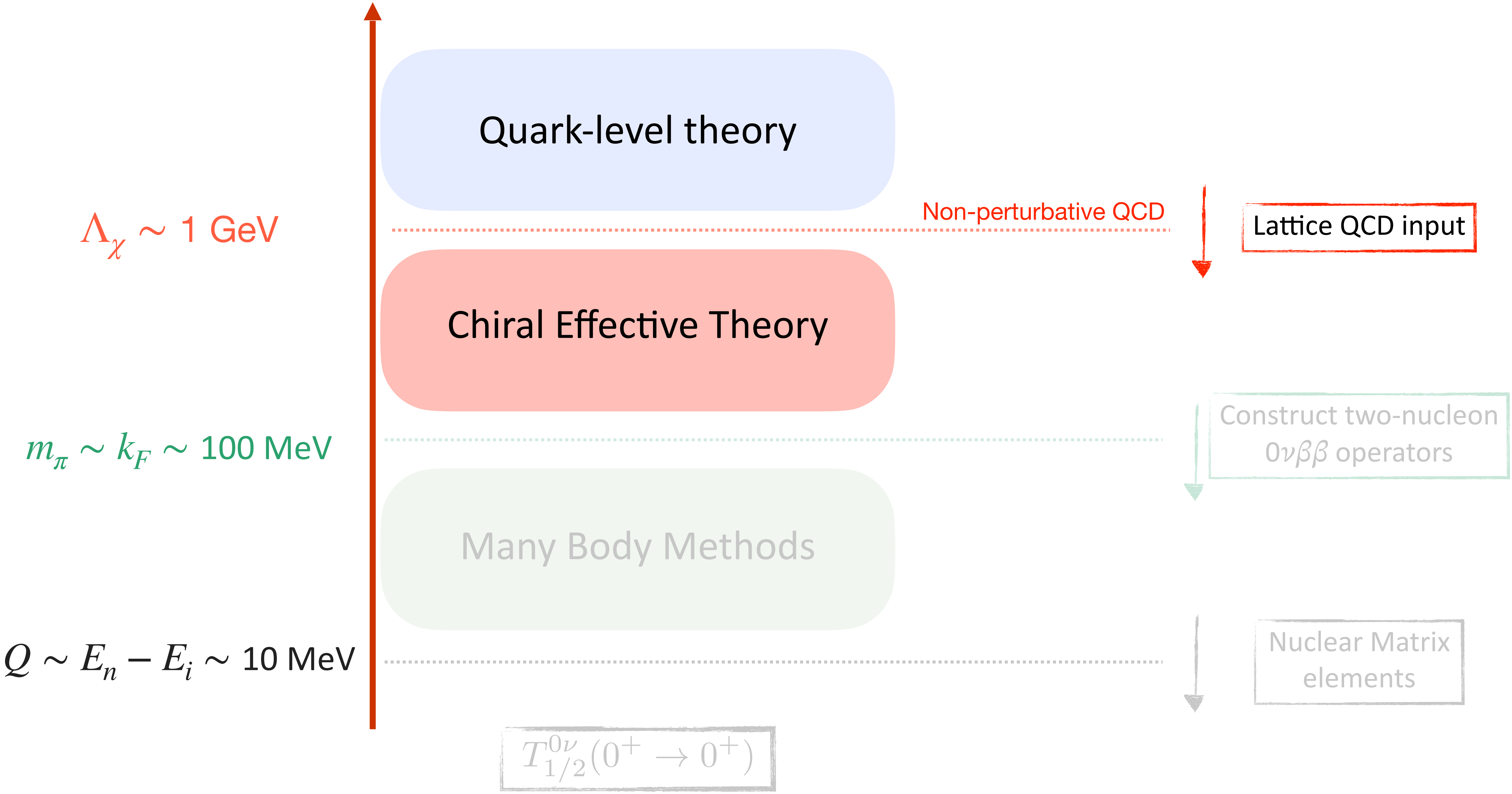
- Additional `non-NDA' contact interaction needed for renormalization
  - New LEC, g_ν^{NN}.
  - Currently unknown only model estimates

Cirigliano et al '18,'19

Cirigliano, et al, '21; Richardson et al, '21

EFT approach

One scale at a time



Quark-level theory

$\Lambda_\chi \sim$ 1 GeV

Non-perturbative QCD

Lattice QCD input

Chiral Effective Theory

$m_\pi \sim k_F \sim$ 100 MeV

Construct two-nucleon $0\nu\beta\beta$ operators

Many Body Methods

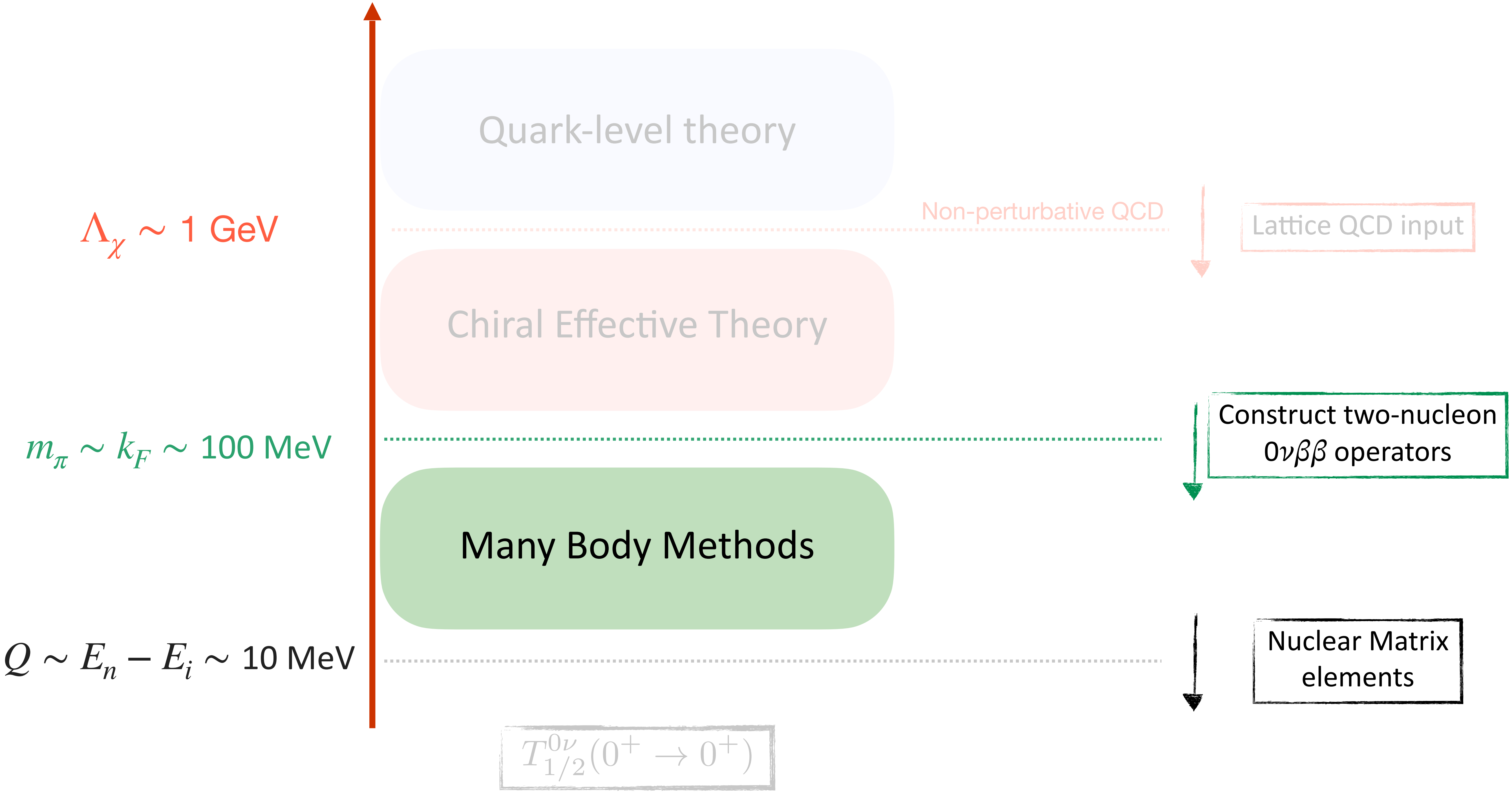
$Q \sim E_n - E_i \sim$ 10 MeV

Nuclear Matrix elements

$T^{0\nu}_{1/2}(0^+ \to 0^+)$

EFT approach

One scale at a time



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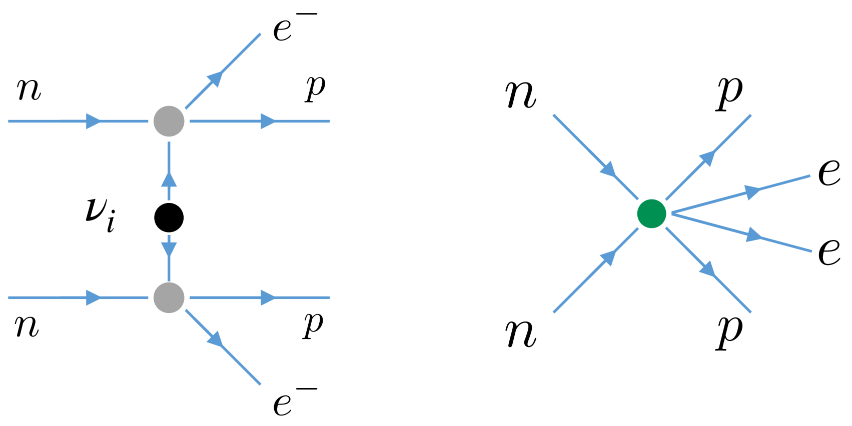
Chiral EFT

Active $\nu's$: leading order

Leading order

$V_{\Delta L=2} =$

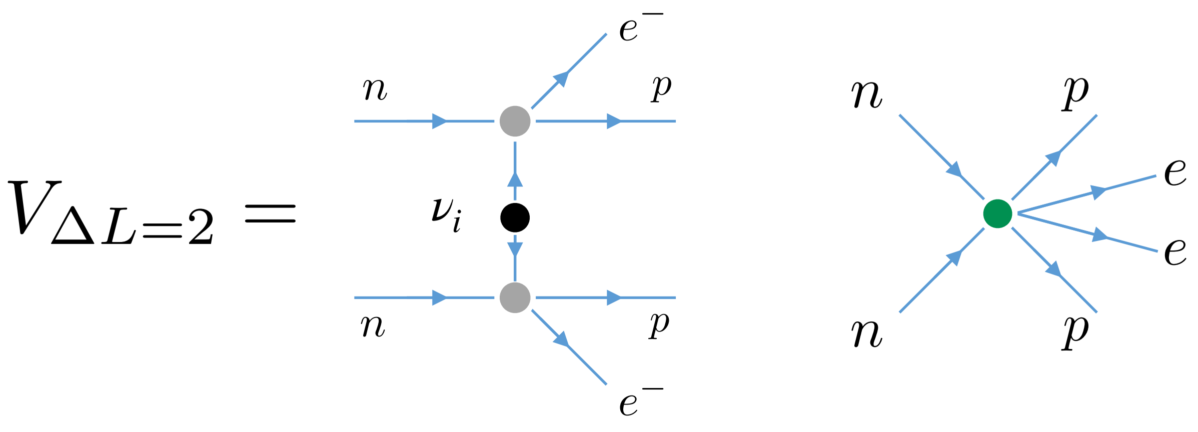


Need to evaluate $A_\nu = \langle {}^{136}\mathrm{Ba} | V | {}^{136}\mathrm{Xe} \rangle$

Chiral EFT

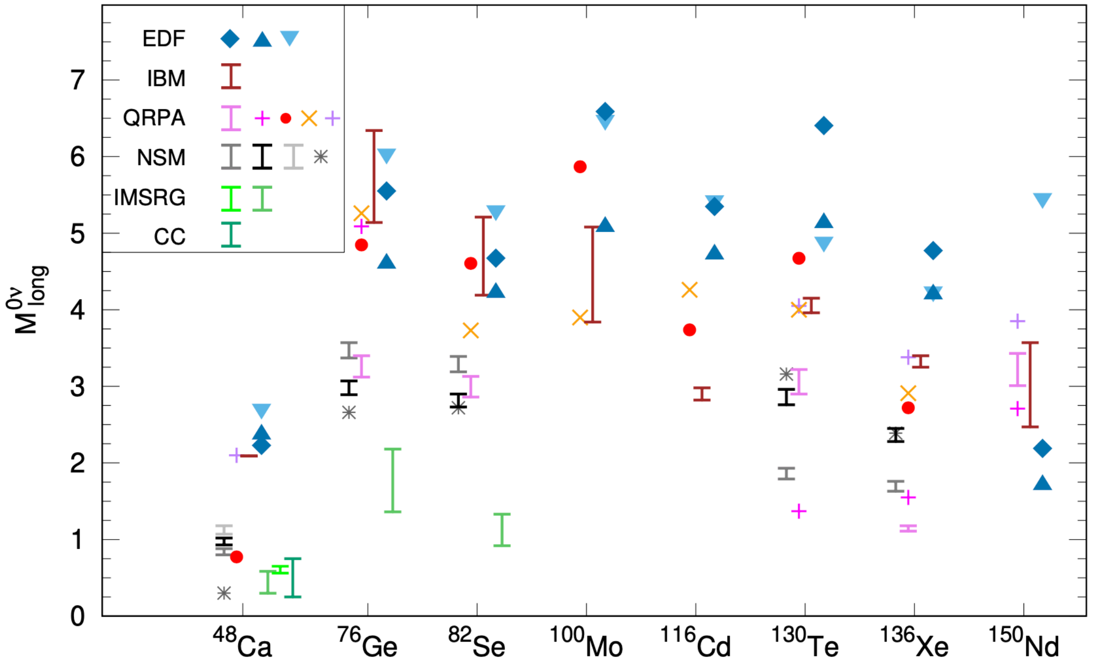
Active $\nu's$: leading order

Leading order



Need to evaluate $A_\nu = \langle {}^{136}\mathrm{Ba} | V | {}^{136}\mathrm{Xe} \rangle$

- Requires many-body methods
- Matrix elements differ factor 2-3 between methods
- *Ab initio* NMEs for $A \geq 48$ are starting to appear
- Including estimates of g_ν^{NN} effects



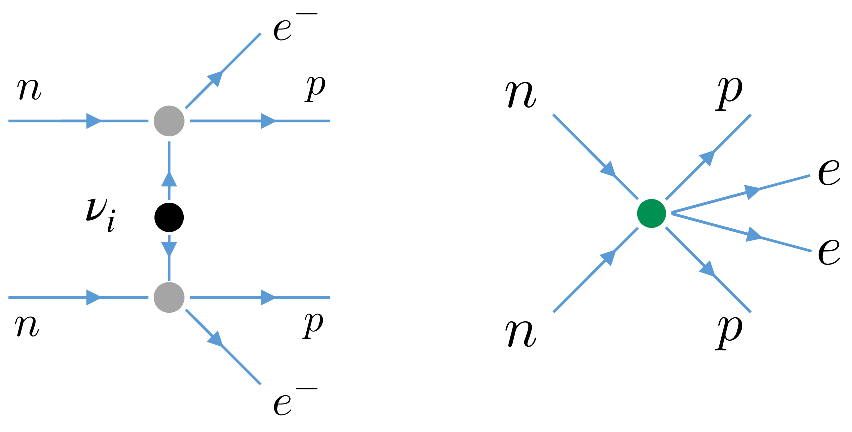
Agostini et al, '22; Belley et al '20; Yao et al '20; Wirth, Yao, Hegert '21

Chiral EFT

Active $\nu's$: beyond leading order

Leading order

$$V_{\Delta L=2} =$$

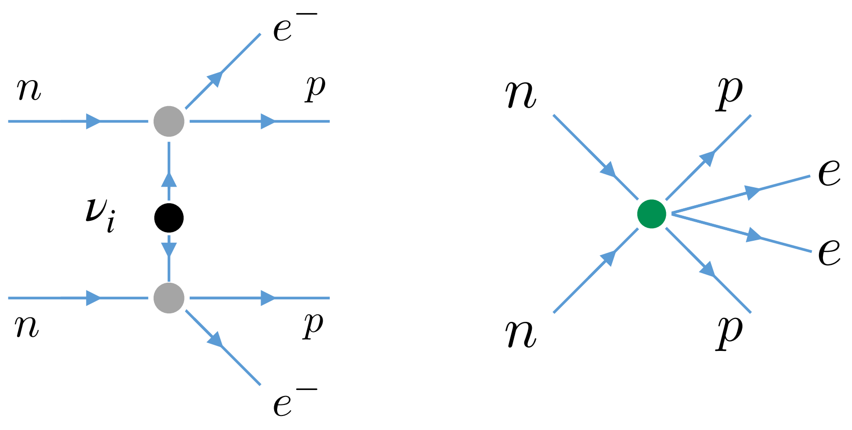


Chiral EFT

Active $\nu's$: beyond leading order

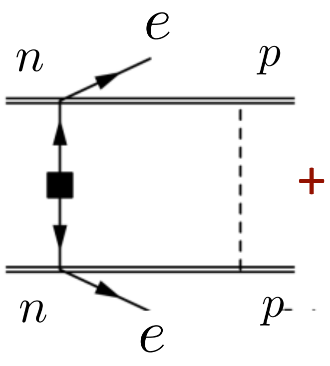
Leading order

$$V_{\Delta L=2} =$$



Next-to-next-to-leading order

Cirigliano et al ‘17



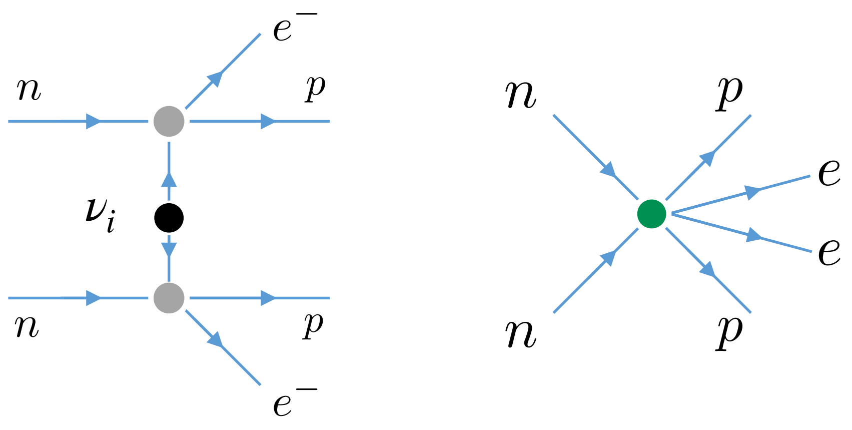
+ form factors & counterterms

Chiral EFT

Active $\nu's$: beyond leading order

Leading order

$$V_{\Delta L=2} =$$



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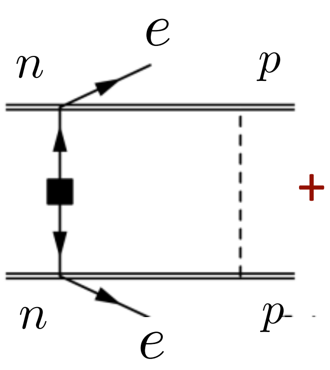

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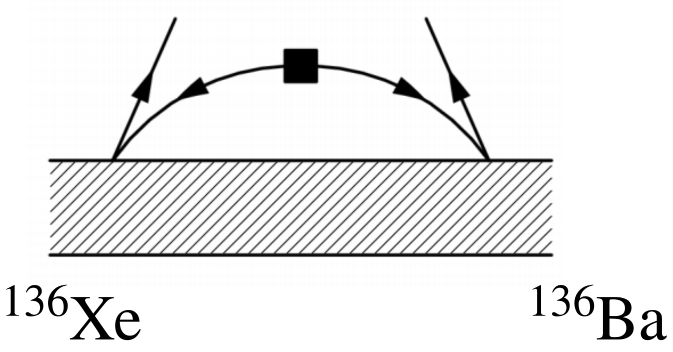
Next-to-next-to leading order



$$A_\nu^{\text{usoft}} \sim \sum_N \langle f\,|\,J_\mu\,|\,n\rangle\langle n\,|\,J^\mu\,|\,i\rangle \Delta E \left(\ln \frac{\mu}{2\Delta E} + 1\right)$$

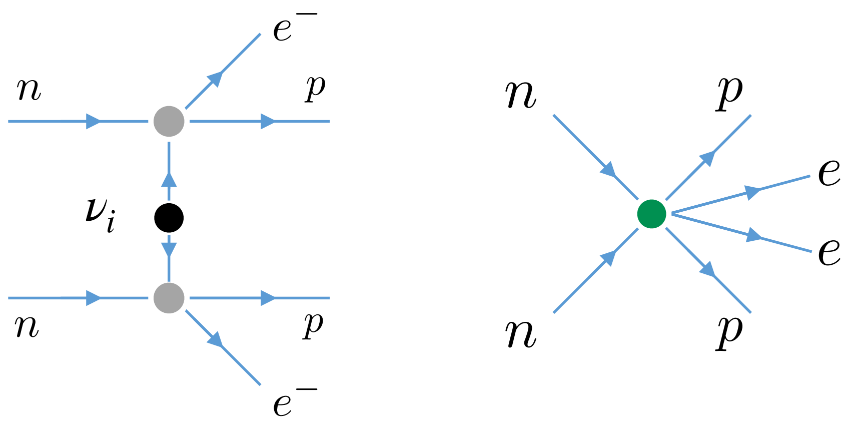
$$\Delta E = E_n - E_i + E_e$$

Chiral EFT

Active $\nu's$: beyond leading order

Leading order

$$V_{\Delta L=2} =$$



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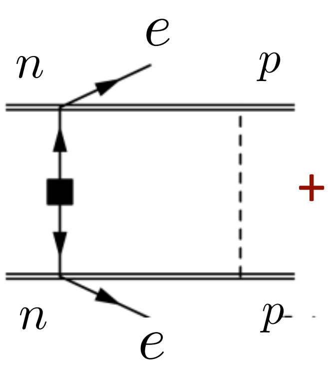
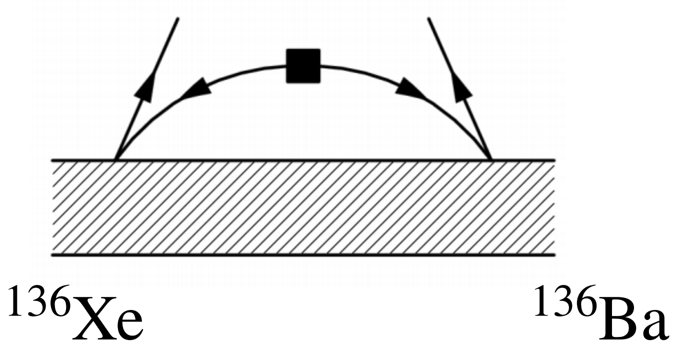


+ form factors & counterterms

Next-to-next-to leading order



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$$\Delta E = E_n - E_i + E_e$$

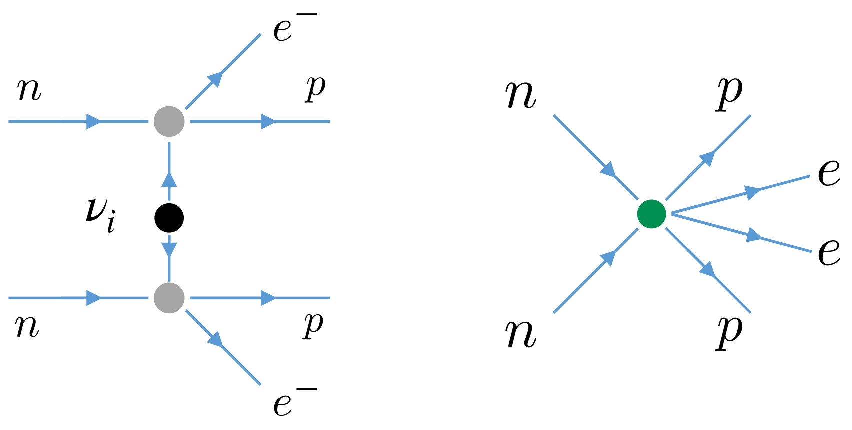
- Total: $A_\nu = \langle {}^{136}\text{Ba} | V | {}^{136}\text{Xe} \rangle \qquad + A_\nu^{\text{usoft}}$

Chiral EFT

Active $\nu's$: beyond leading order

Leading order

$$V_{\Delta L=2} =$$



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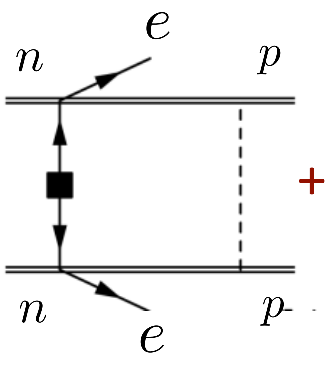

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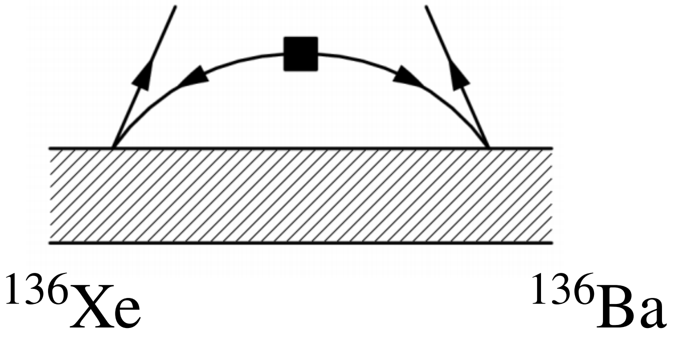

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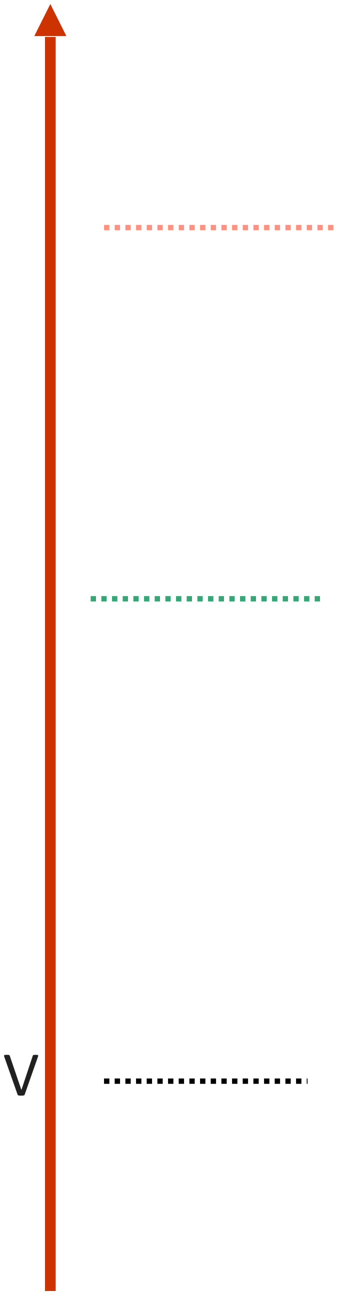
- Total: $A_\nu = \langle {}^{136}\mathrm{Ba} | V | {}^{136}\mathrm{Xe} \rangle \quad + A_\nu^{\mathrm{usoft}}$

- N2LO effects:
  - Estimated to be $\lesssim \mathcal{O}(10\%)$
  - Become sensitive to intermediate states

Pastore et al '17

Momentum scales

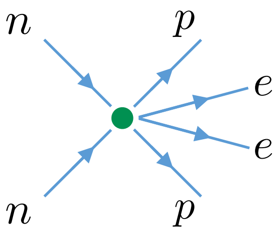
Active $\nu's$



$\Lambda_\chi \sim$ 1 GeV

$m_\pi \sim k_F \sim$ 100 MeV

$Q \sim E_n - E_i \sim$ 10 MeV



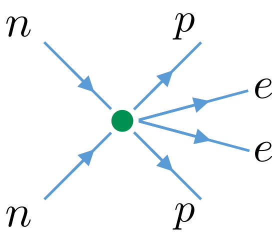
`hard $\nu's$':

$q_0 \sim \vec{q} \sim \Lambda_\chi$

Momentum scales

Active $\nu's$

$\Lambda_\chi \sim$ 1 GeV



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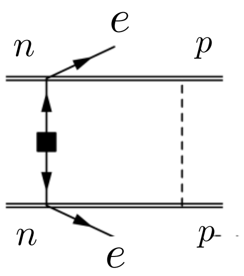


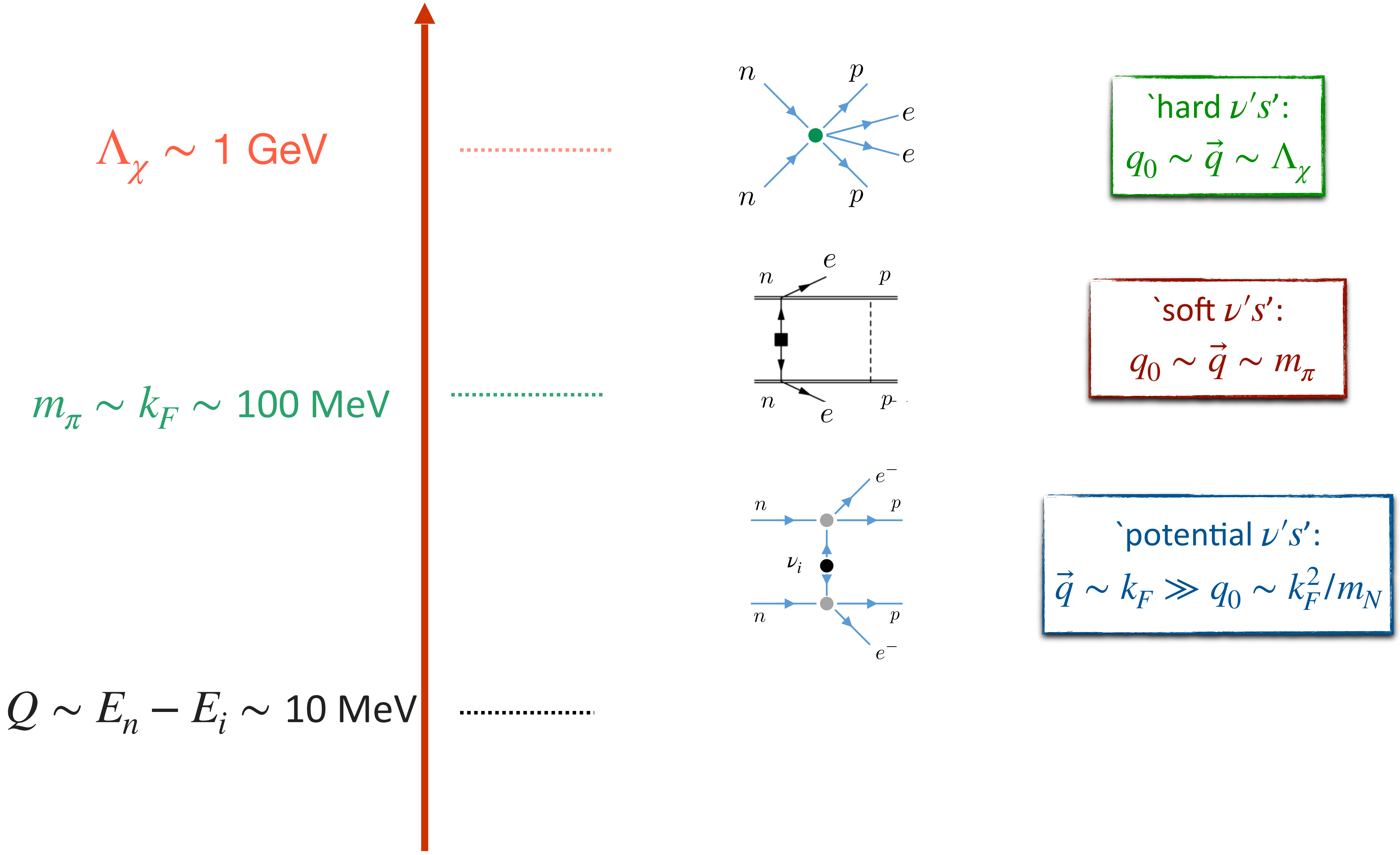
\`soft $\nu's$':

$q_0 \sim \vec{q} \sim m_\pi$

$Q \sim E_n - E_i \sim$ 10 MeV

Momentum scales

Active $\nu's$



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$m_\pi \sim k_F \sim$ 100 MeV

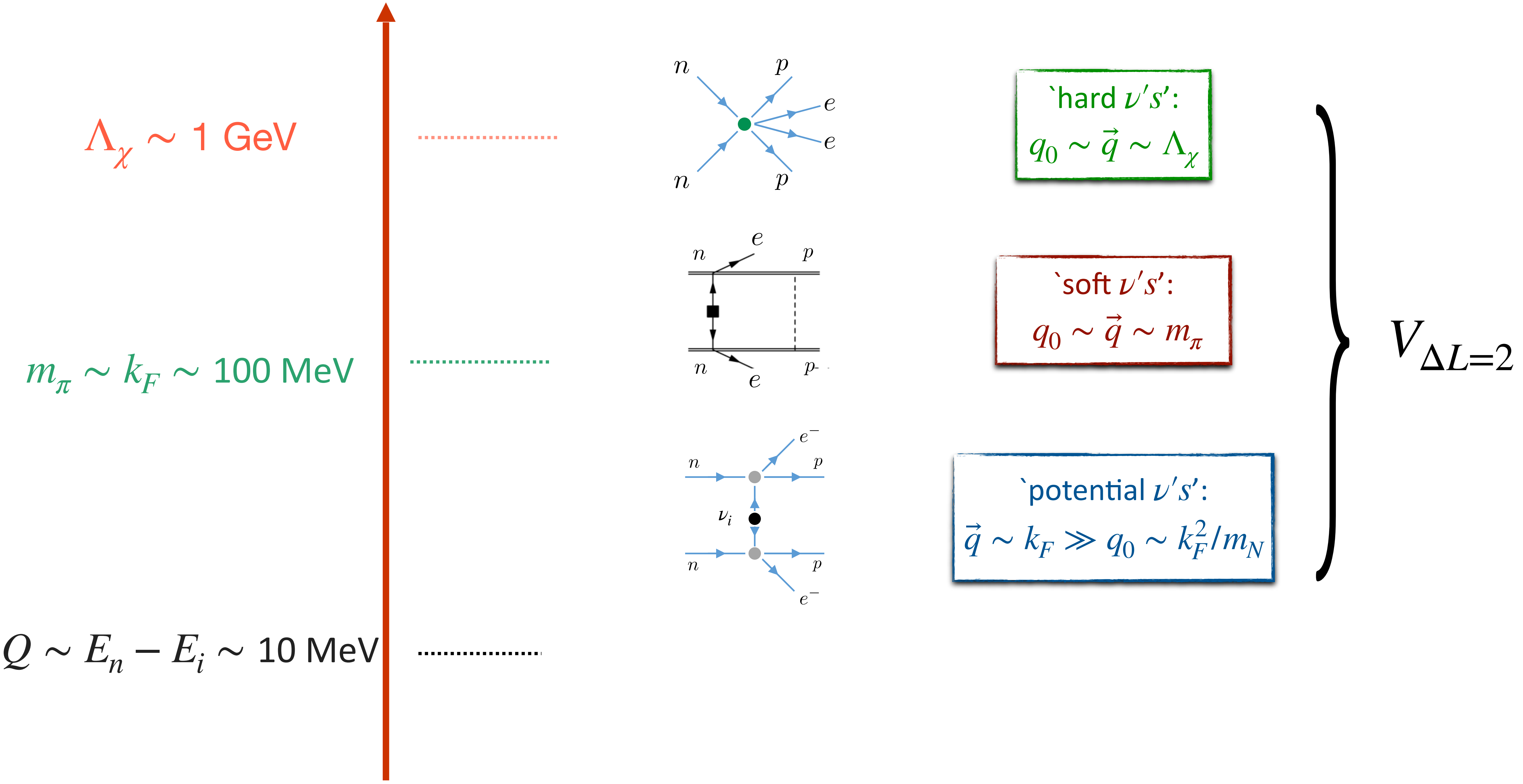
`soft $\nu's$':
$q_0 \sim \vec{q} \sim m_\pi$

`potential $\nu's$':
$\vec{q} \sim k_F \gg q_0 \sim k_F^2/m_N$

$Q \sim E_n - E_i \sim$ 10 MeV

Momentum scales

Active $\nu's$



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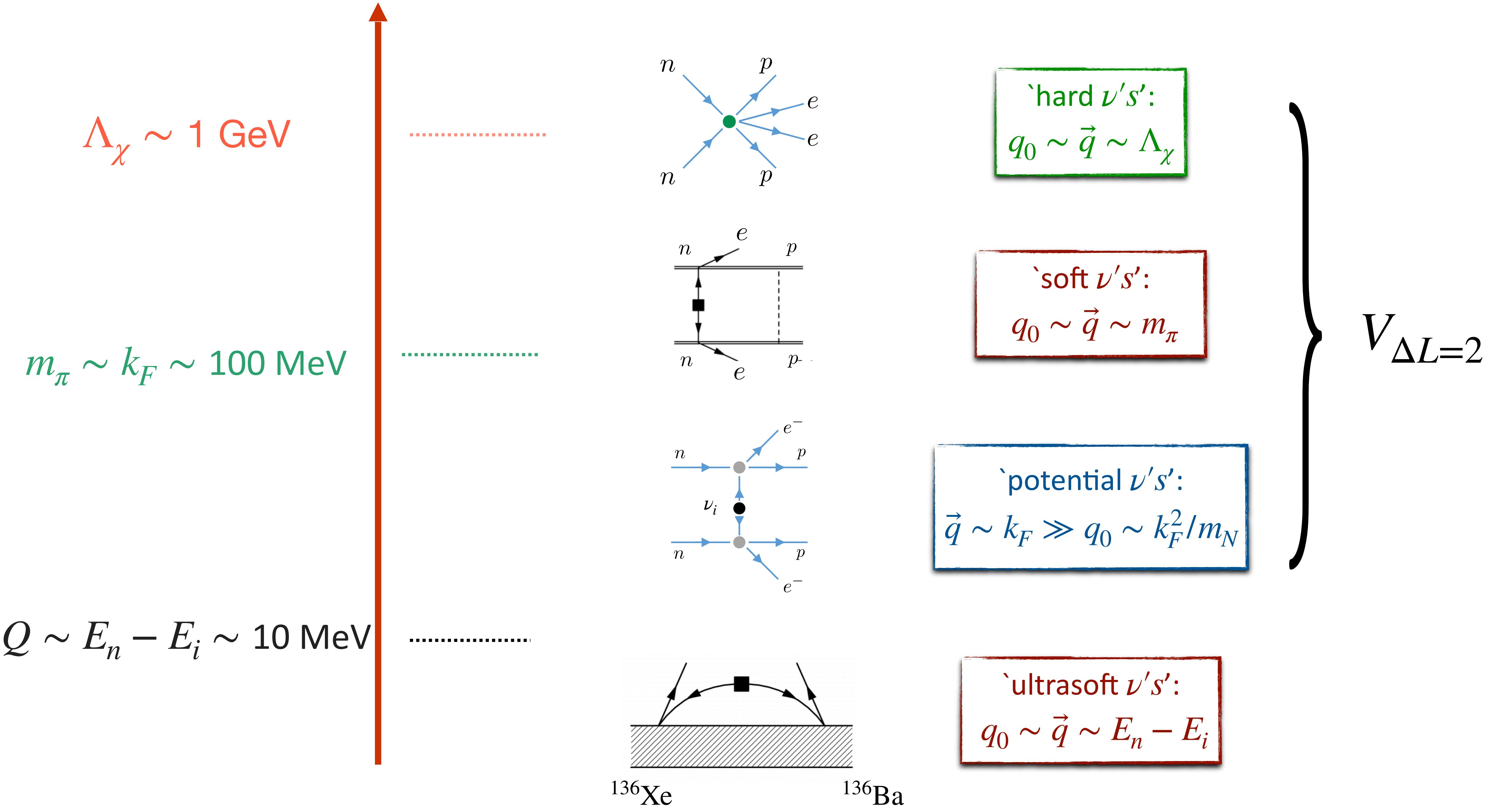
$\left.\right\} V_{\Delta L=2}$

$$A_\nu = \langle {}^{136}\mathrm{Ba} \,|\, V_{\Delta L=2} \,|\, {}^{136}\mathrm{Xe} \rangle$$

Momentum scales

Active $\nu' s$



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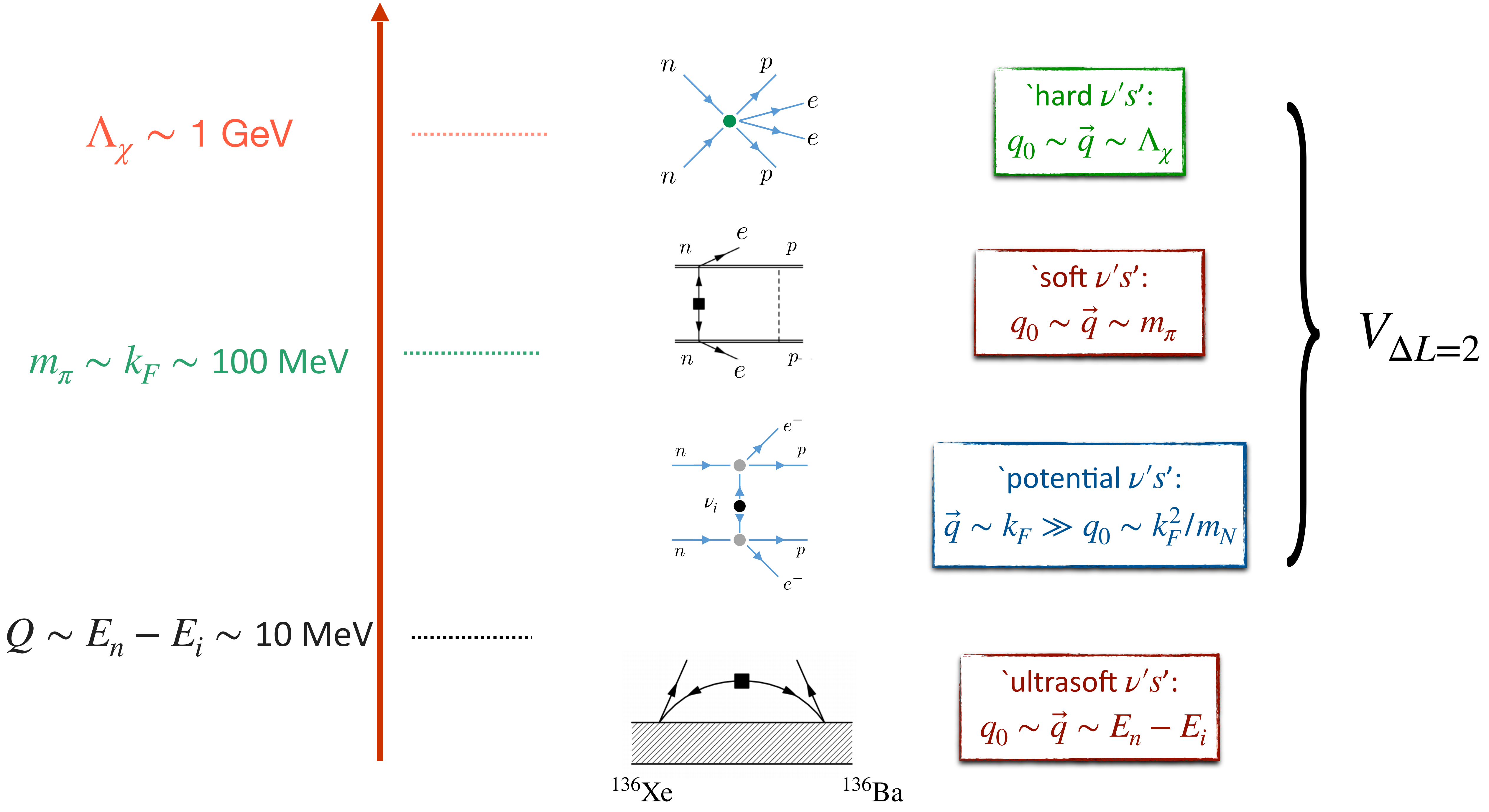
`ultrasoft $\nu' s$': $q_0 \sim \vec{q} \sim E_n - E_i$

$V_{\Delta L=2}$

$$A_\nu = \langle ^{136}\text{Ba} | V_{\Delta L=2} | ^{136}\text{Xe} \rangle \qquad + \qquad A_\nu^{\text{usoft}}$$

Momentum scales

Active + sterile $\nu's$



$\Lambda_\chi \sim$ 1 GeV

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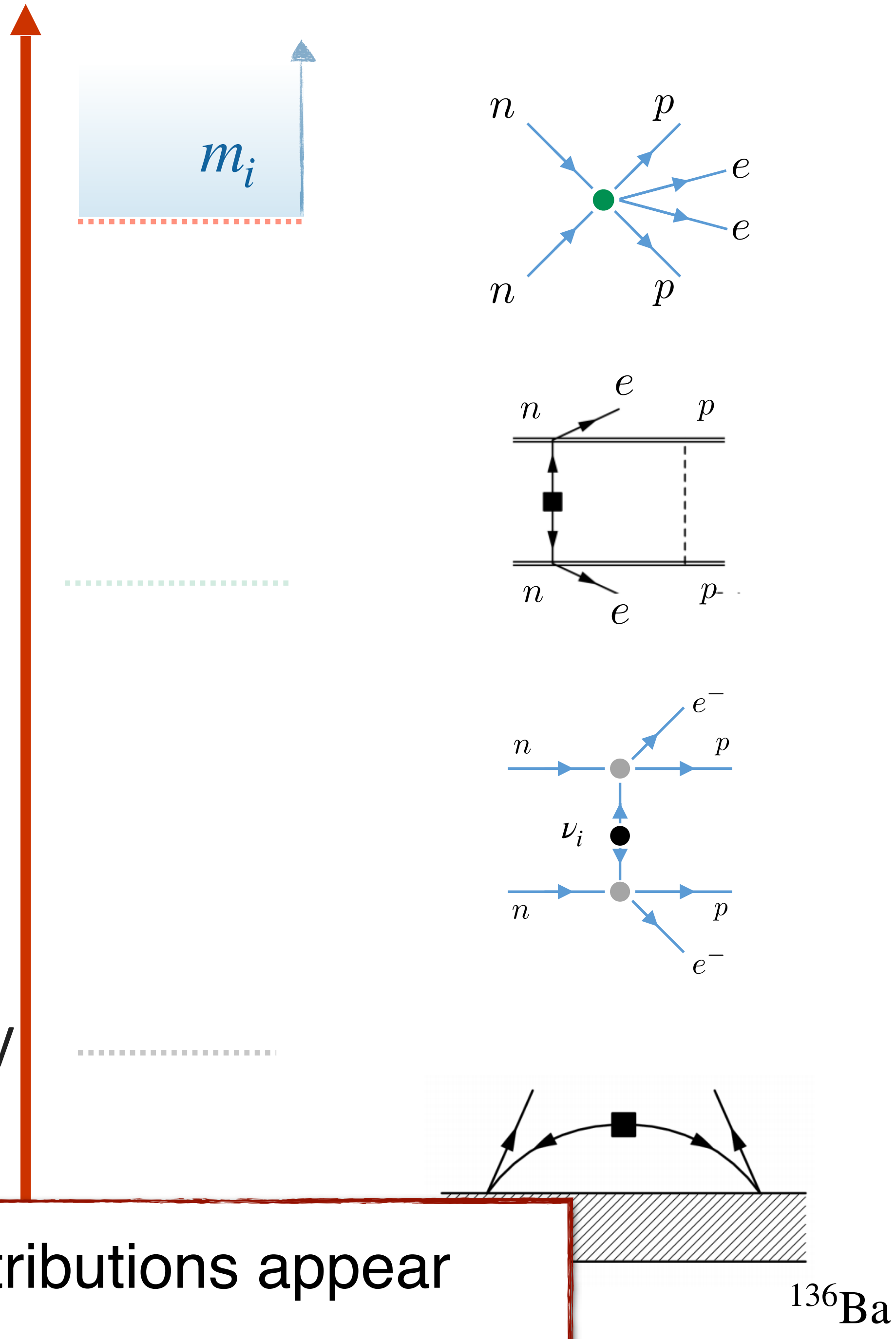
$$A_\nu = \langle {}^{136}\mathrm{Ba} | V_{\Delta L=2} | {}^{136}\mathrm{Xe} \rangle \quad + \quad A_\nu^{\mathrm{usoft}}$$

Momentum scales

Active + sterile $\nu's$



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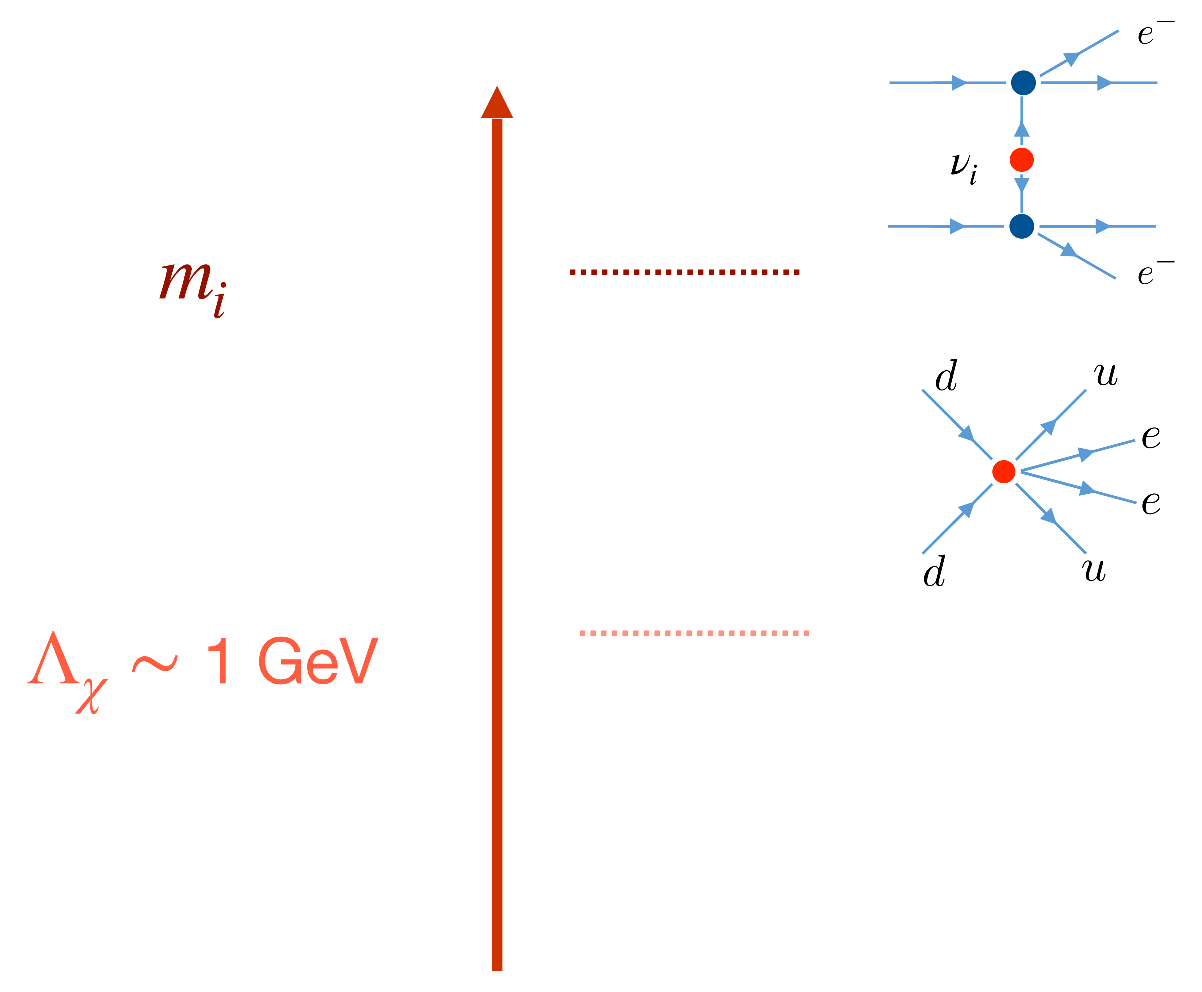
\`ultrasoft $\nu's$': $q_0 \sim \vec{q} \sim E_n - E_i$

$$\left.\right\} V_{\Delta L=2}$$

- Same types of contributions appear
- How to include ν_i depends on m_i

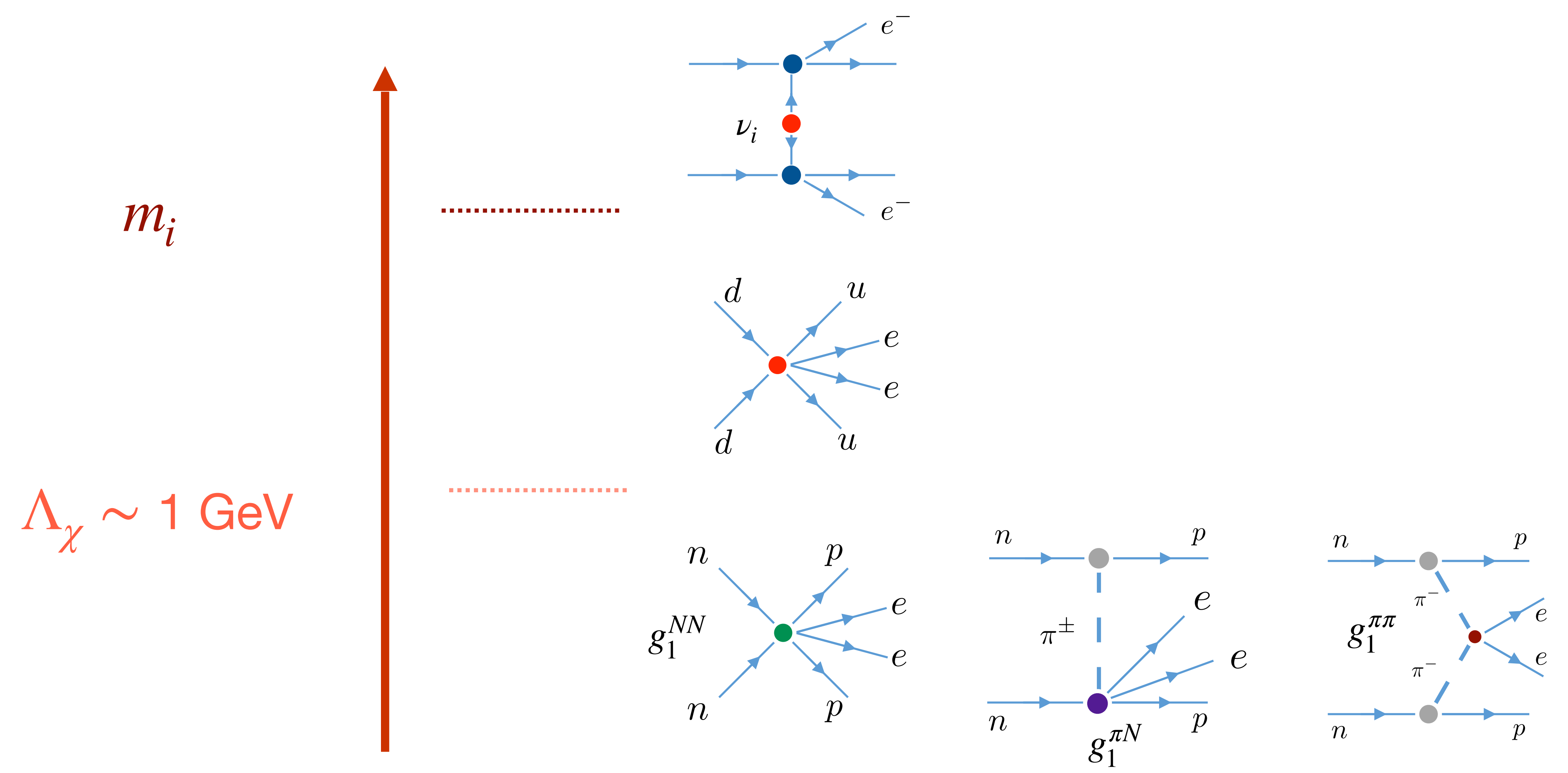
$$A_\nu = \langle ^{136}\mathrm{Ba} | V_{\Delta L=2} | ^{136}\mathrm{Xe} \rangle \quad + \quad A_\nu^{\mathrm{usoft}}$$

$m_i \gg \Lambda_\chi$



- ν_i can be integrated-out at quark level
  - Determines m_i dependence: $A_\nu(m_i) \sim U_{ei}^2/m_i^2$

$m_i \gg \Lambda_\chi$



- ν_i can be integrated-out at quark level
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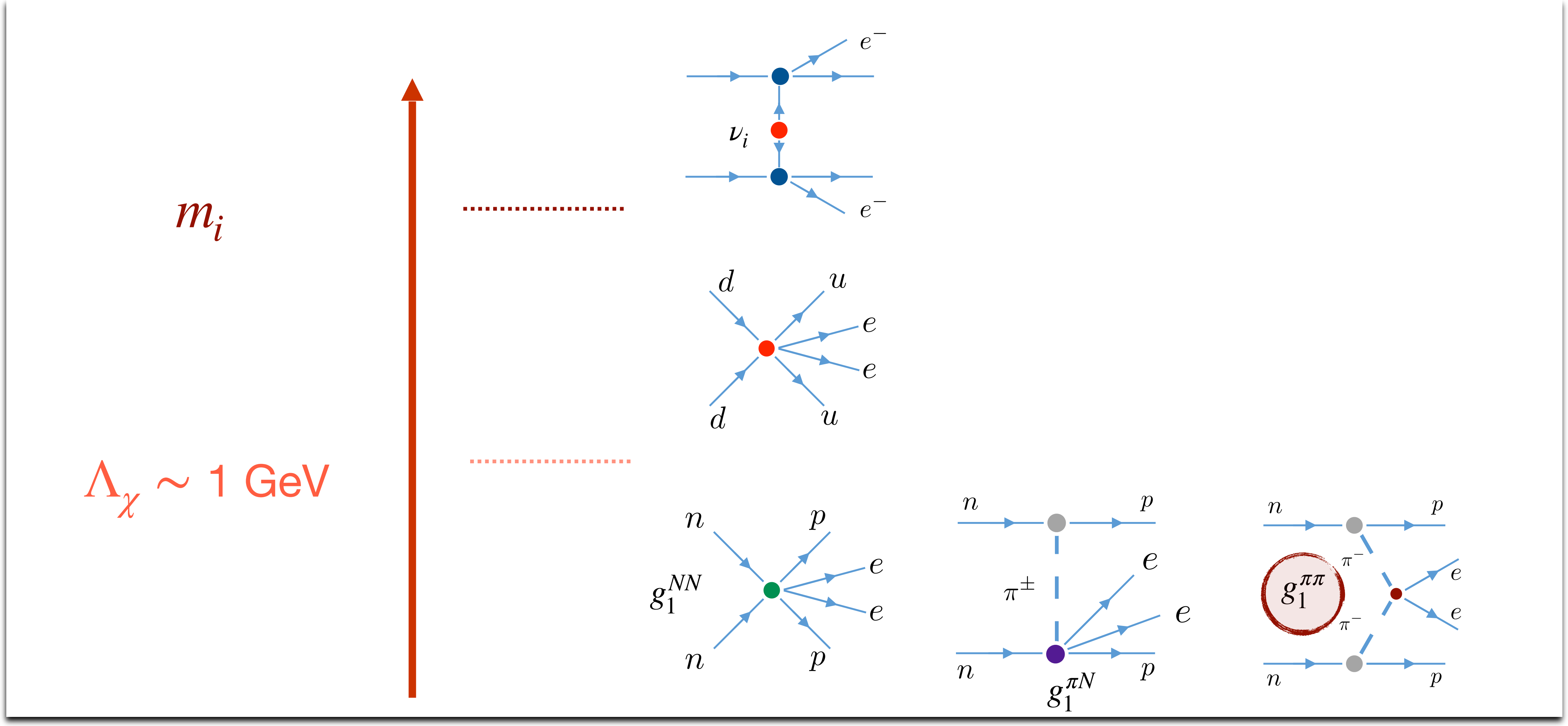
- Match to chiral EFT without ν_i
- Involves several LECs

$m_i \gg \Lambda_\chi$



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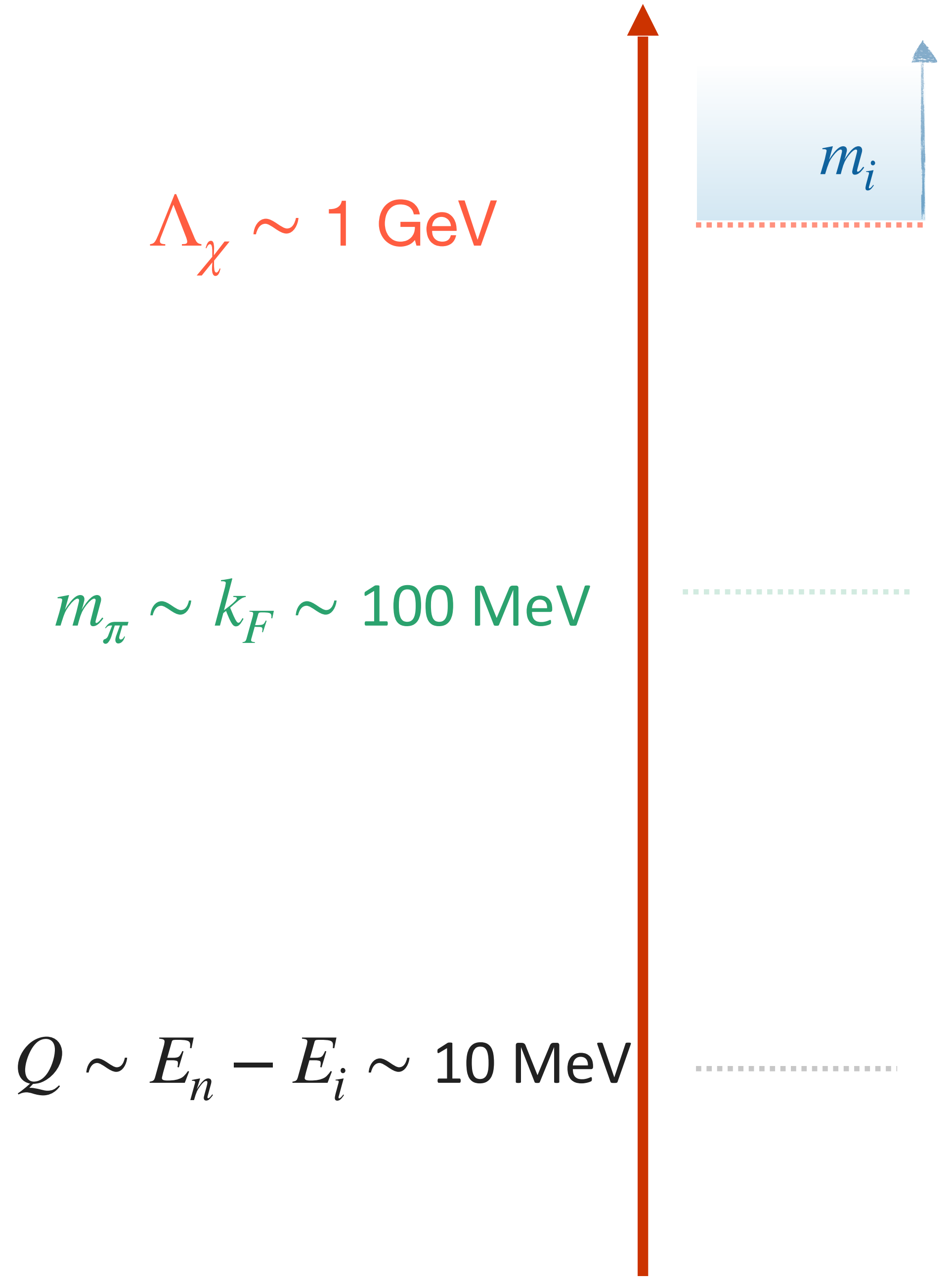
  - Determines m_i dependence: $A_\nu(m_i) \sim U_{ei}^2/m_i^2$

- Match to chiral EFT without ν_i
- Involves several LECs
  - Only $g_1^{\pi\pi}$ known

Nicholson et al '18; Detmold et al '22

EFT approach

One momentum scale at a time



m_i

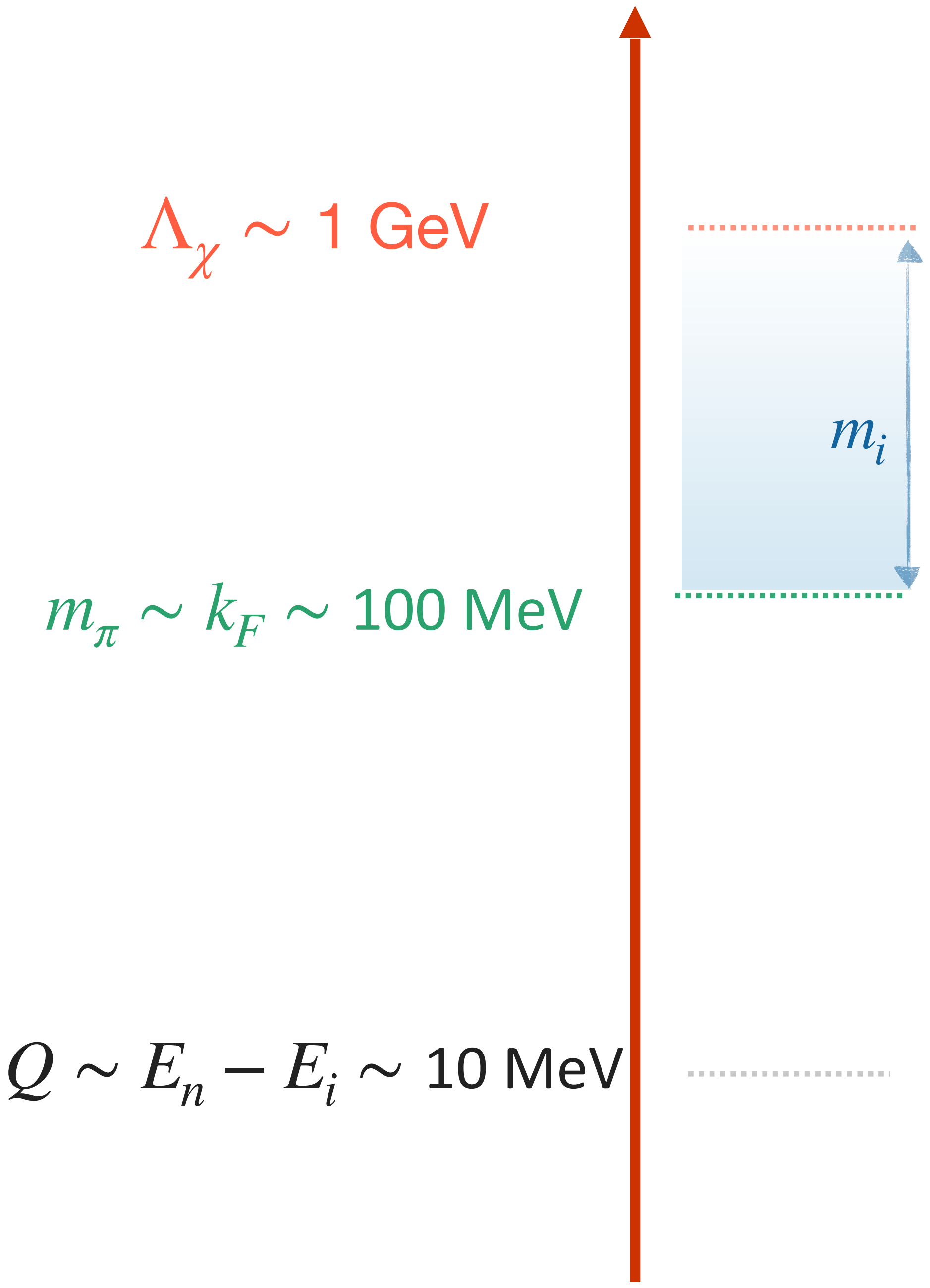
$\Lambda_\chi \sim$ 1 GeV

$m_\pi \sim k_F \sim$ 100 MeV

$Q \sim E_n - E_i \sim$ 10 MeV

EFT approach

One momentum scale at a time



$\Lambda_\chi \sim 1$ GeV

m_i

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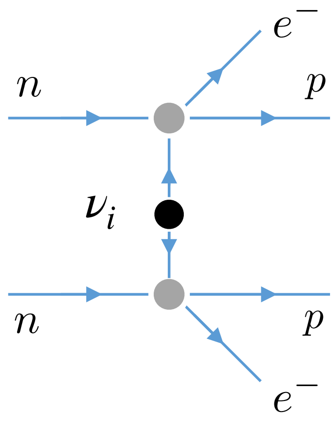
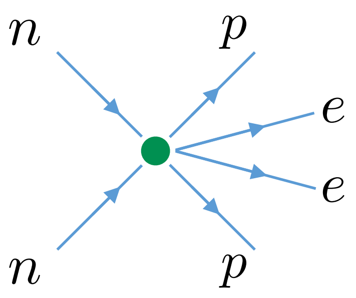
$Q \sim E_n - E_i \sim 10$ MeV

$\Lambda_\chi \gtrsim m_i \gtrsim k_F$

$V_{\Delta L=2} =$

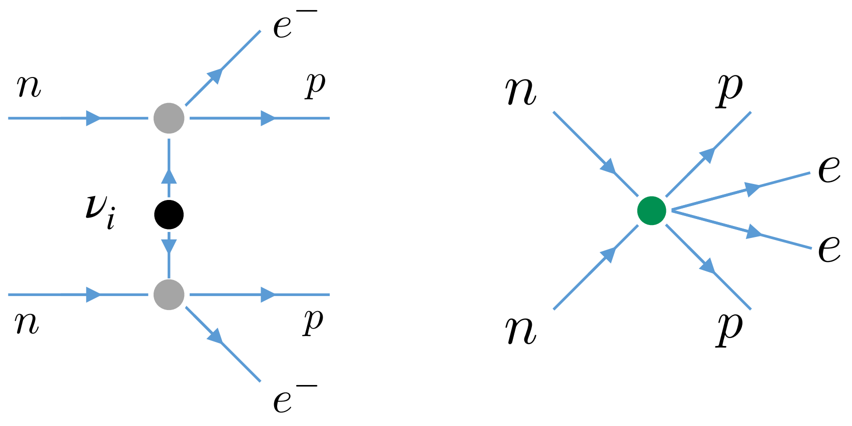




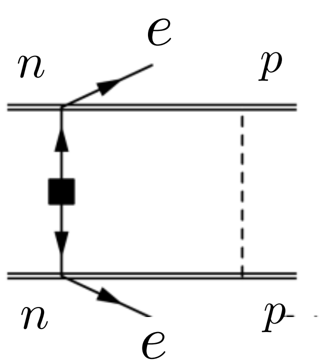
- Have to keep ν_i in the chiral theory
- Again have `potential’ + `hard’ contributions
- m_i dependence in NMEs and g_ν^{NN}

$\Lambda_\chi \gtrsim m_i \gtrsim k_F$

$V_{\Delta L=2} =$



Soft contributions $\mathcal{O}\left(\frac{m_i^2}{\Lambda_\chi^2}\right)$



- Have to keep ν_i in the chiral theory
- Again have \`potential' + \`hard' contributions
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- \`soft' contributions can be significant

$\Lambda_\chi \gtrsim m_i \gtrsim k_F$

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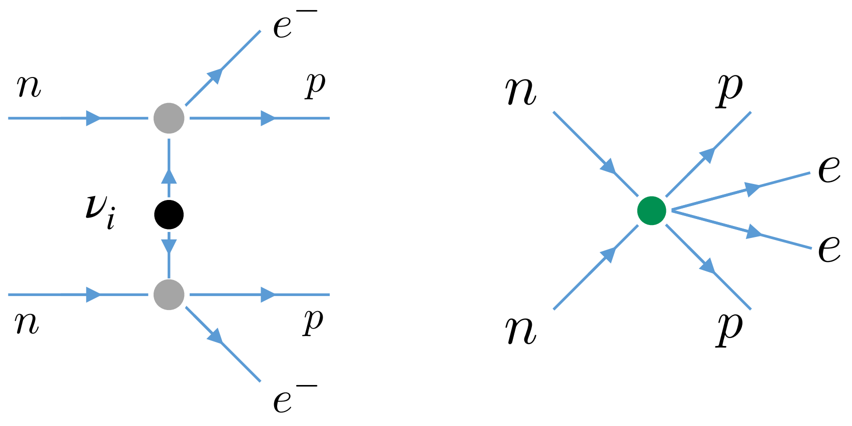

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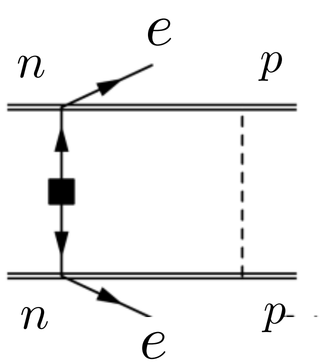
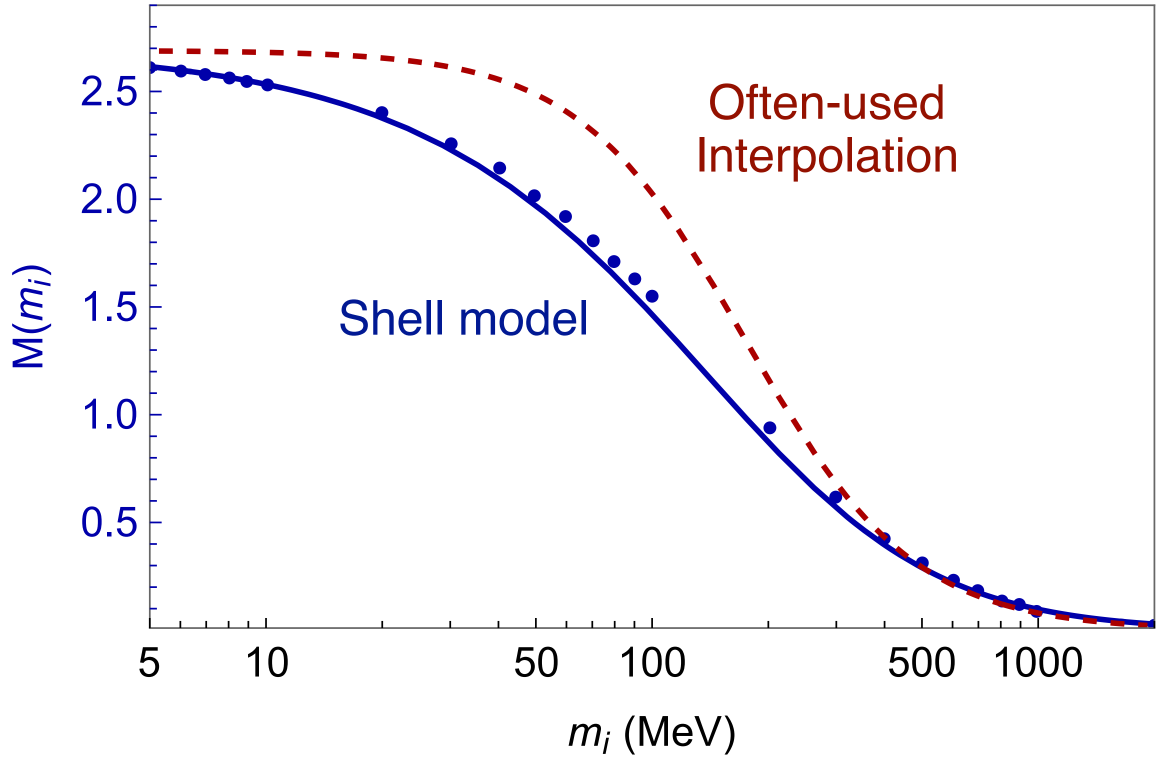


- Have to keep ν_i in the chiral theory
- Again have `potential' + `hard' contributions
- m_i dependence in NMEs and g_ν^{NN}

- `soft' contributions can be significant

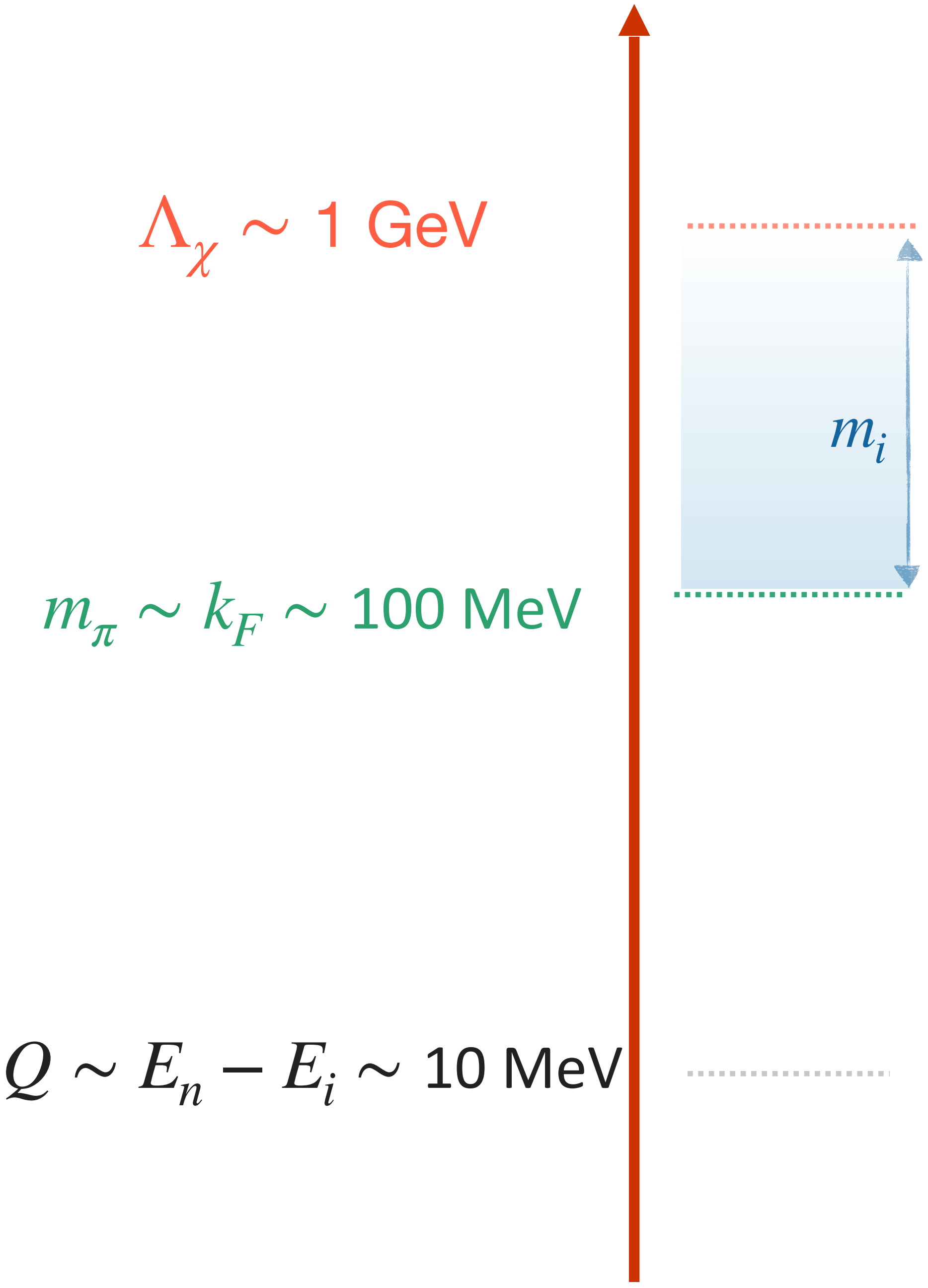
Present in usual approach

$$A_\nu(m_i) = A_\nu(0)\frac{\langle p^2\rangle}{\langle p^2\rangle + m_i^2}$$



EFT approach

One momentum scale at a time



$\Lambda_\chi \sim 1$ GeV

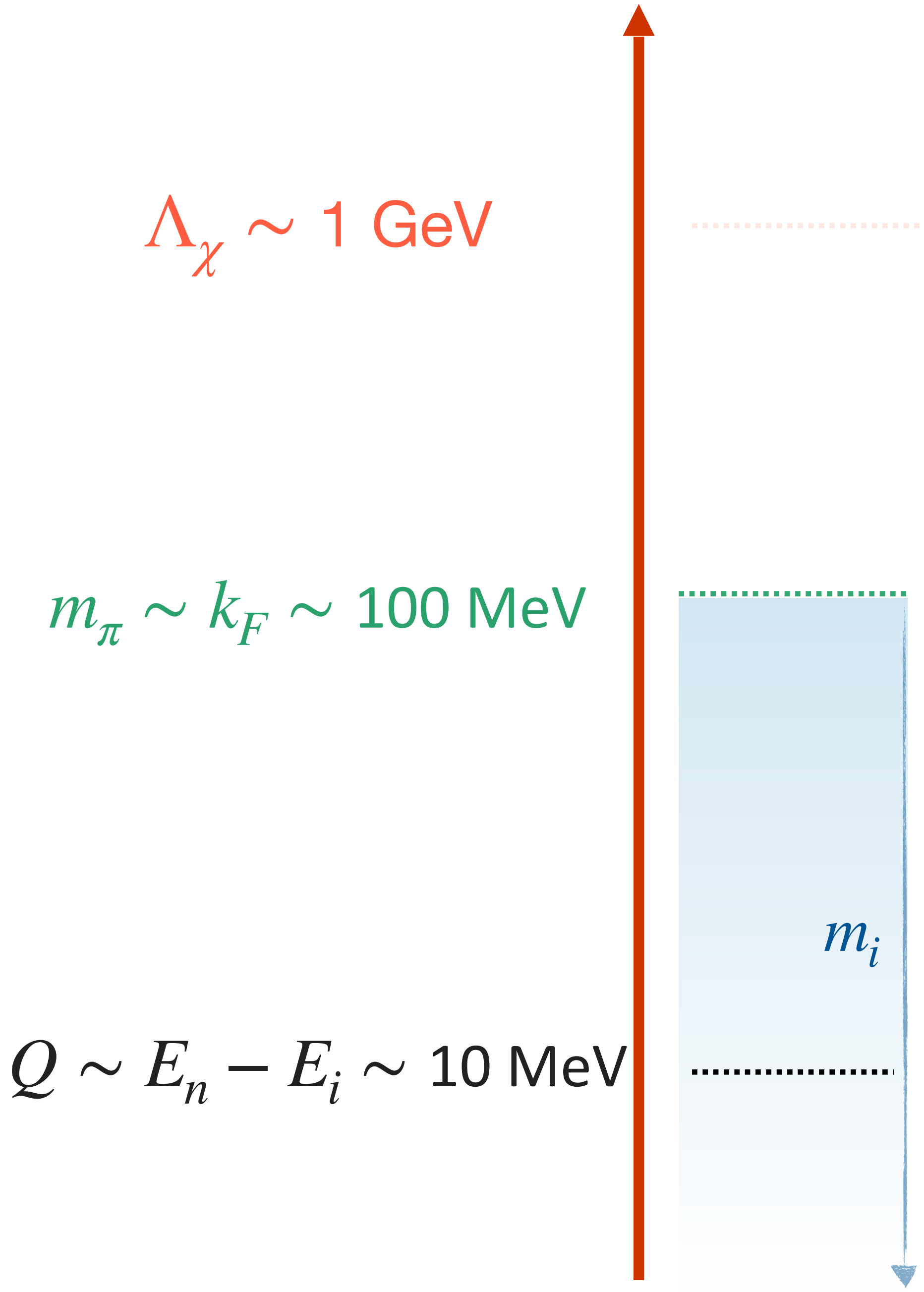
m_i

$m_\pi \sim k_F \sim 100$ MeV

$Q \sim E_n - E_i \sim 10$ MeV

EFT approach

One momentum scale at a time



$\Lambda_\chi \sim$ 1 GeV

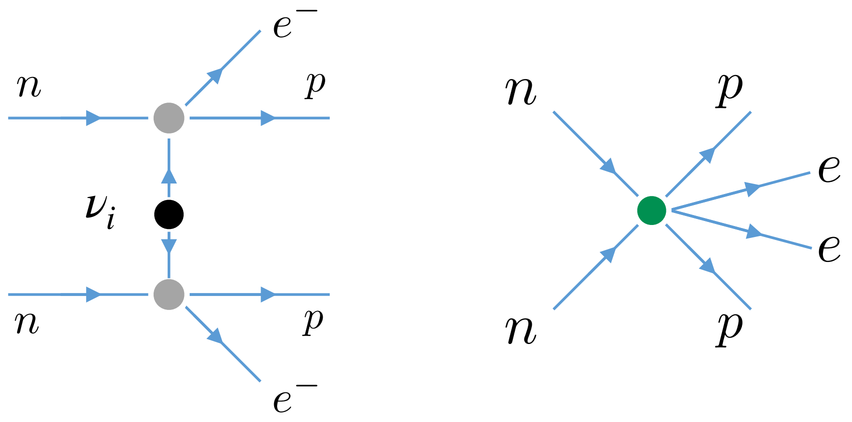
$m_\pi \sim k_F \sim$ 100 MeV

m_i

$Q \sim E_n - E_i \sim$ 10 MeV

$k_F \gtrsim m_i$

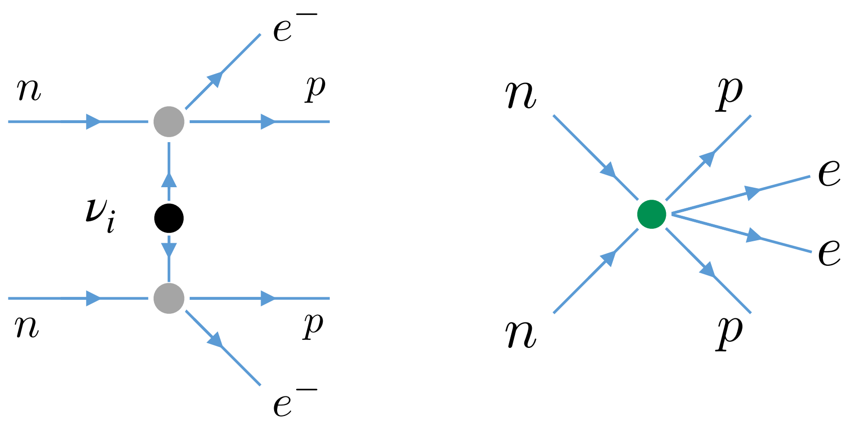
$V_{\Delta L=2} =$



- Similar to previous case:
  - Contributions from potential + hard regions
  - Soft contributions are now negligible

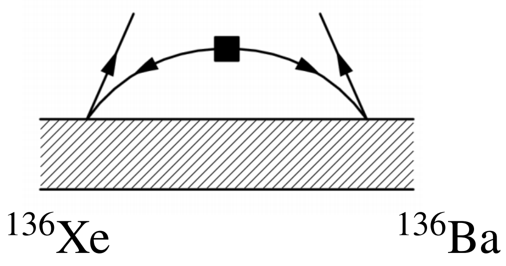
$k_F \gtrsim m_i$

$V_{\Delta L=2} =$



- Similar to previous case:
    - Contributions from potential + hard regions
    - Soft contributions are now negligible

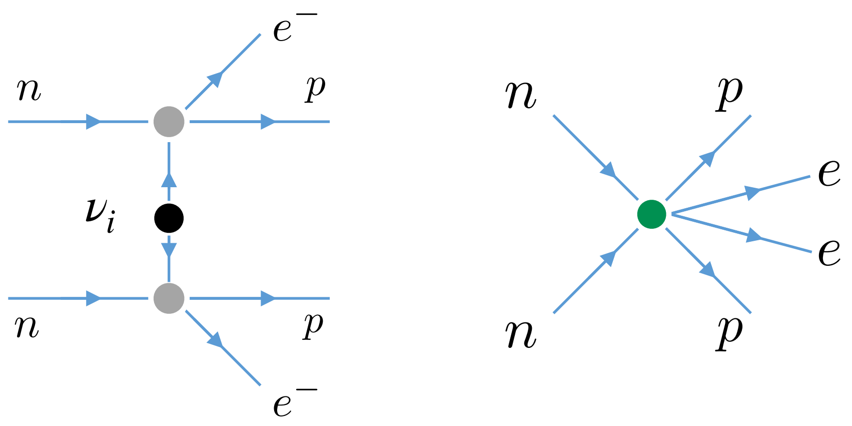
Ultrasoft contributions



- Ultrasoft $\nu's$ start to contribute
    - Dominant effect for small m_i
    - Not captured by often-used interpolation

$k_F \gtrsim m_i$

$$V_{\Delta L=2} =$$



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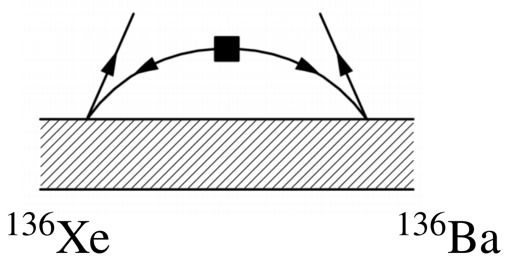

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  - Not captured by often-used interpolation

$$A_\nu^{\text{usoft}} \sim \sum_N \langle f | J_\mu | n \rangle \langle n | J^\mu | i \rangle \times \begin{cases} \dfrac{m_i}{k_F}, & \Delta E \lesssim m_i \lesssim k_F \\ & \end{cases}$$

$k_F \gtrsim m_i$

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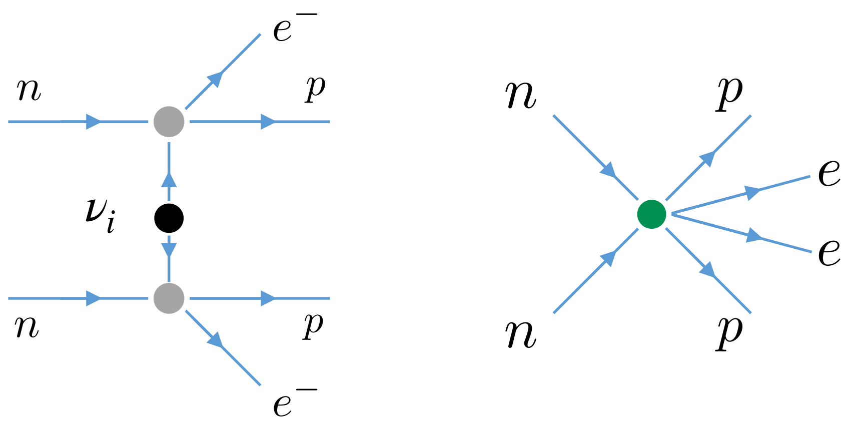

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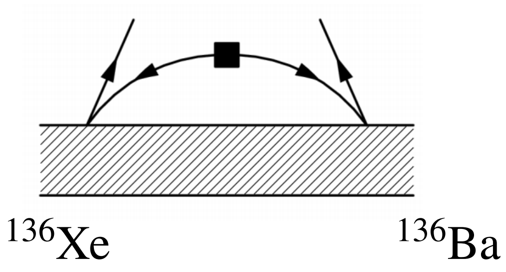

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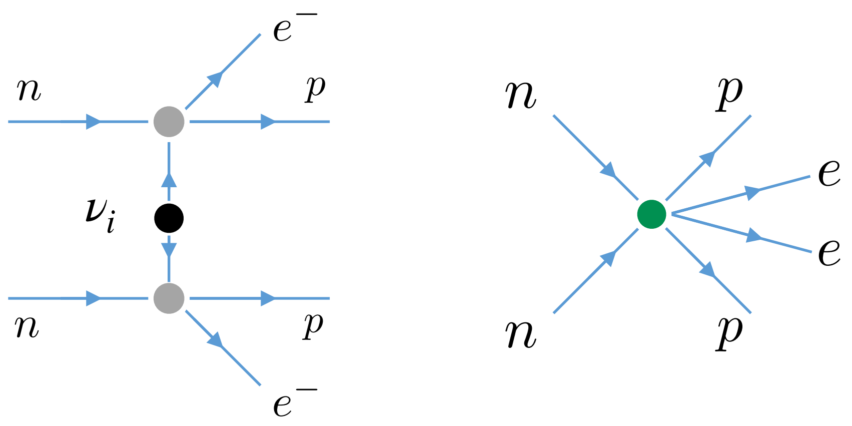

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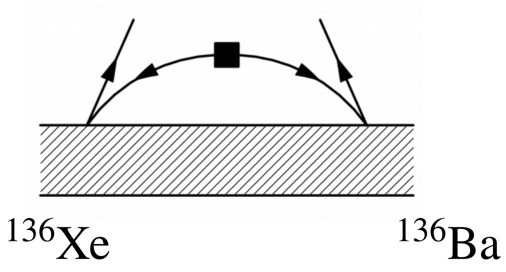

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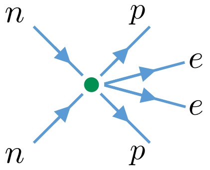
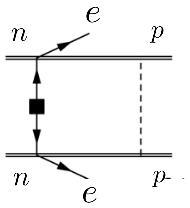
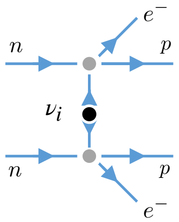
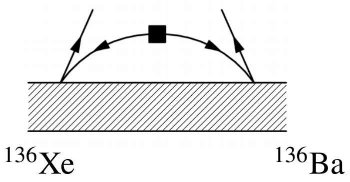
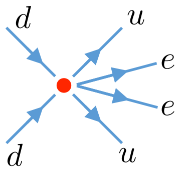
Larger m_i dependence than interpolation

$$A_\nu^{\text{int}}(m_i) \sim 1 + m_i^2/k_F^2$$

Overview

Leading m_i dependence	$m_i \ll \Delta E$	$\Delta E \ll m_i \ll k_F$	$k_F \ll m_i \ll \Lambda_\chi$	$\Lambda_\chi \ll m_i$
 Hard			$\frac{k_F^2}{m_i^2}$	
 Soft			$\frac{m_i^2}{\Lambda_\chi^2}$	
 Potential	$\frac{m_i^2}{k_F^2}$		$\frac{k_F^2}{m_i^2}$	
 Ultrasoft	$\frac{m_i^2}{4\pi\Delta E k_F}\ln\frac{m_i}{\Delta E}$	$\frac{m_i}{k_F}$		
 Perturbative				$\frac{k_F^2}{m_i^2}$

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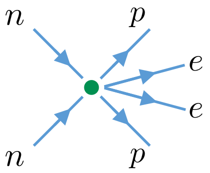
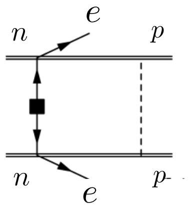
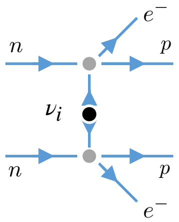
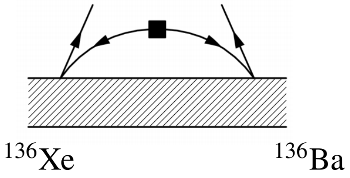
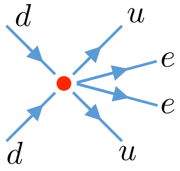
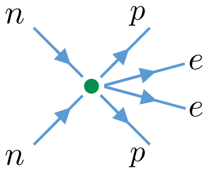
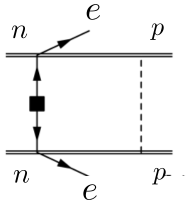
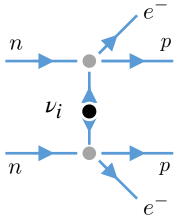
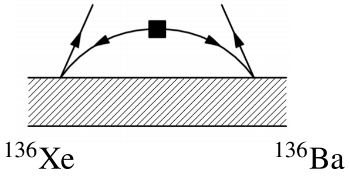
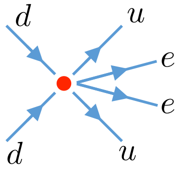
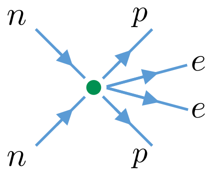
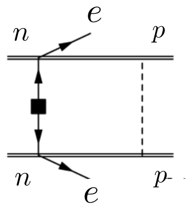
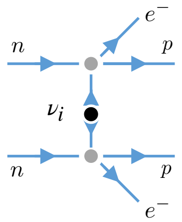
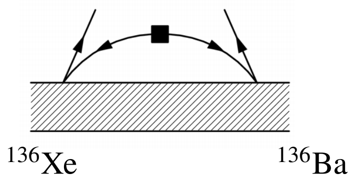
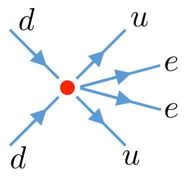
	Hard			$\frac{k_F^2}{m_i^2}$	
	Soft			$\frac{m_i^2}{\Lambda_\chi^2}$	
	Potential	$\frac{m_i^2}{k_F^2}$		$\frac{k_F^2}{m_i^2}$	
	Ultrasoft	$\frac{m_i^2}{4\pi \Delta E k_F} \ln \frac{m_i}{\Delta E}$	$\frac{m_i}{k_F}$		
	Perturbative				$\frac{k_F^2}{m_i^2}$

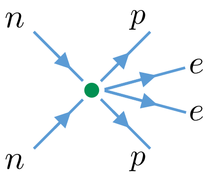
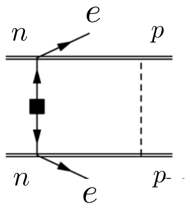
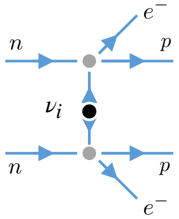
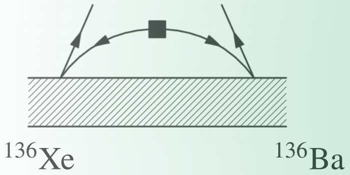
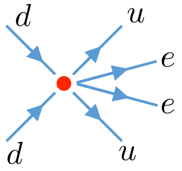
Overview

Leading m_i dependence	$m_i \ll \Delta E$	$\Delta E \ll m_i \ll k_F$	$k_F \ll m_i \ll \Lambda_\chi$	$\Lambda_\chi \ll m_i$
 Hard			$\frac{k_F^2}{m_i^2}$	
 Soft			$\frac{m_i^2}{\Lambda_\chi^2}$	
 Potential	$\frac{m_i^2}{k_F^2}$		$\frac{k_F^2}{m_i^2}$	
 Ultrasoft	$\frac{m_i^2}{4\pi\Delta E k_F}\ln\frac{m_i}{\Delta E}$	$\frac{m_i}{k_F}$		
 Perturbative				$\frac{k_F^2}{m_i^2}$

Overview

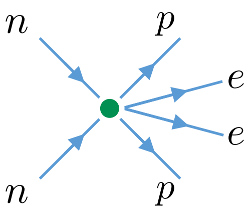
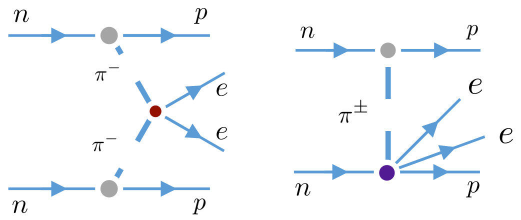
Leading m_i dependence		$m_i \ll \Delta E$	$\Delta E \ll m_i \ll k_F$	$k_F \ll m_i \ll \Lambda_\chi$	$\Lambda_\chi \ll m_i$
	Hard			$\frac{k_F^2}{m_i^2}$	
	Soft			$\frac{m_i^2}{\Lambda_\chi^2}$	
	Potential	$\frac{m_i^2}{k_F^2}$		$\frac{k_F^2}{m_i^2}$	
	Ultrasoft	$\frac{m_i^2}{4\pi\Delta E k_F}\ln\frac{m_i}{\Delta E}$	$\frac{m_i}{k_F}$		
	Perturbative				$\frac{k_F^2}{m_i^2}$

Overview

Leading m_i dependence		$m_i \ll \Delta E$	$\Delta E \ll m_i \ll k_F$	$k_F \ll m_i \ll \Lambda_\chi$	$\Lambda_\chi \ll m_i$
	Hard			$\frac{k_F^2}{m_i^2}$	
	Soft			$\frac{m_i^2}{\Lambda_\chi^2}$	
	Potential	$\frac{m_i^2}{k_F^2}$		$\frac{k_F^2}{m_i^2}$	
	Ultrasoft	$\frac{m_i^2}{4\pi\Delta E k_F} \ln \frac{m_i}{\Delta E}$	$\frac{m_i}{k_F}$		
	Perturbative				$\frac{k_F^2}{m_i^2}$

Ultrasoft dominant for small masses

Overview

Required input	$m_i \ll \Delta E$	$\Delta E \ll m_i \ll k_F$	$k_F \ll m_i \ll \Lambda_\chi$	$\Lambda_\chi \ll m_i$
Low-energy constants			$g_\nu^{NN}(m_i)$  	$g_1^{\pi\pi}, g_1^{\pi N}, g_1^{NN}$ 
Nuclear matrix elements	$\langle f \vert \tau^+ \sigma \vert n \rangle$ $\Delta E \sim E_n - E_i$	$M_\nu(m_i) = \langle f \vert V \vert i \rangle$	$M_\nu^{\text{short-distance}}$	

Overview

Required input	$m_i \ll \Delta E$	$\Delta E \ll m_i \ll k_F$	$k_F \ll m_i \ll \Lambda_\chi$	$\Lambda_\chi \ll m_i$
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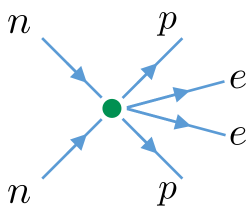
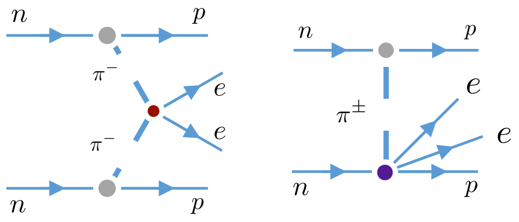
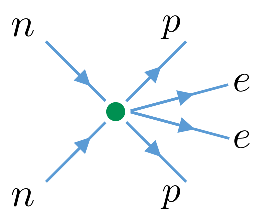
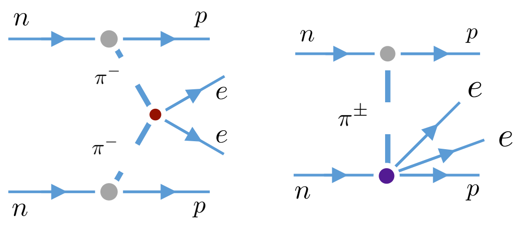




$M_\nu(m_i) = \langle f | V | i \rangle$

- Known from LQCD

Overview

Required input	$m_i \ll \Delta E$	$\Delta E \ll m_i \ll k_F$	$k_F \ll m_i \ll \Lambda_\chi$	$\Lambda_\chi \ll m_i$
Low-energy constants			$g_\nu^{NN}(m_i)$ 	$g_1^{\pi\pi}, g_1^{\pi N}, g_1^{NN}$ 
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- Interpolate g_ν^{NN} between $m_i = 0$ and $m_i \gg \Lambda_\chi$ regions

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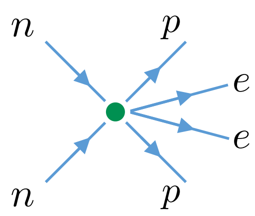
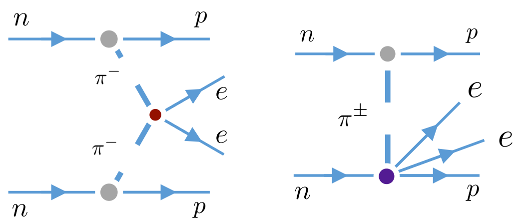
Low-energy constants			$g_\nu^{NN}(m_i)$  	$g_1^{\pi\pi}, g_1^{\pi N}, g_1^{NN}$ 
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- Shell model calculations for the NMEs

Phenomenology

Toy model: 3+1

- Add just one sterile neutrino to the SM
  - Assume mass matrix of the form

$$M_\nu = \begin{pmatrix} 0 & 0 & 0 & M_D \\ 0 & 0 & 0 & M_D \\ 0 & 0 & 0 & M_D \\ M_D & M_D & M_D & M_R \end{pmatrix}$$

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- Not realistic:
  - Only two nonzero ν masses
  - Does not reproduce mixing angles

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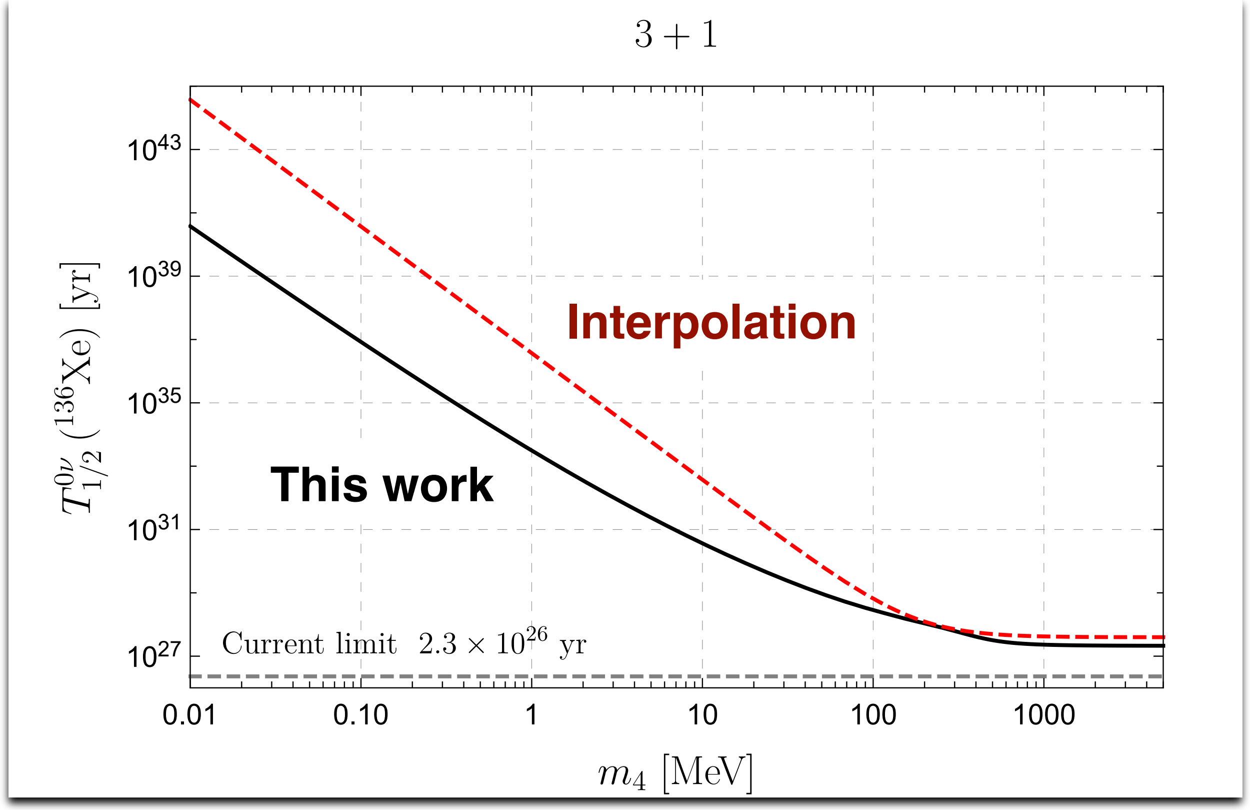
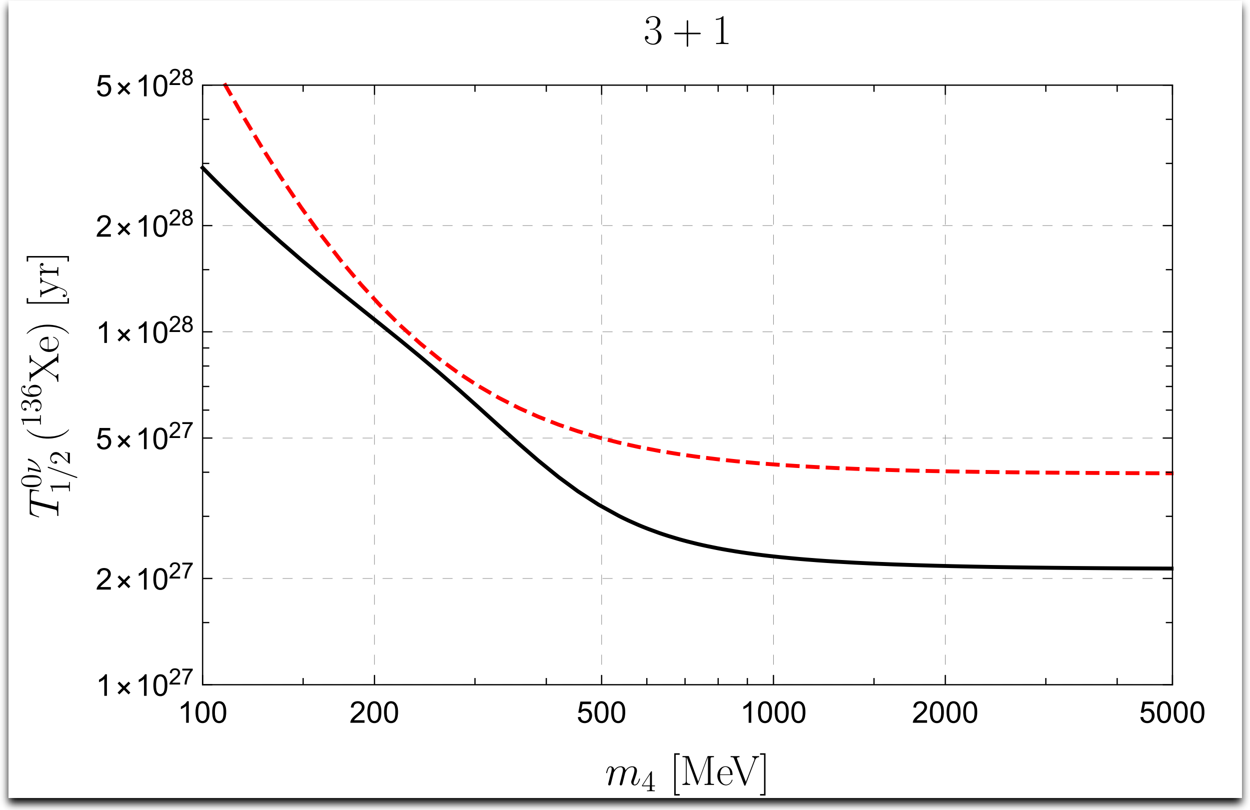


$3+1$



$3+1$

Another toy model

Toy model: 1+1+1 pseudo-Dirac

- Assume 1 active, two sterile neutrinos
  - Assume mass matrix of the form
  - $m_S \gg m_D, \mu_{S,X}$

$$M_\nu = \begin{pmatrix} 0 & m_D & 0 \\ m_D & \mu_X & m_S \\ 0 & m_S & \mu_S \end{pmatrix}$$

Bolton et al, 2020; Mohapatra et al, ’86; Nandi and Sarkar ’86;

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- Interesting aspects:
  - Two heavier ν's, form a pseudo-Dirac pair with $M_2 - M_1 \sim \mu_S \ll M_2$
  - Light neutrino mass proportional to LNV parameter (opposite to seesaw)

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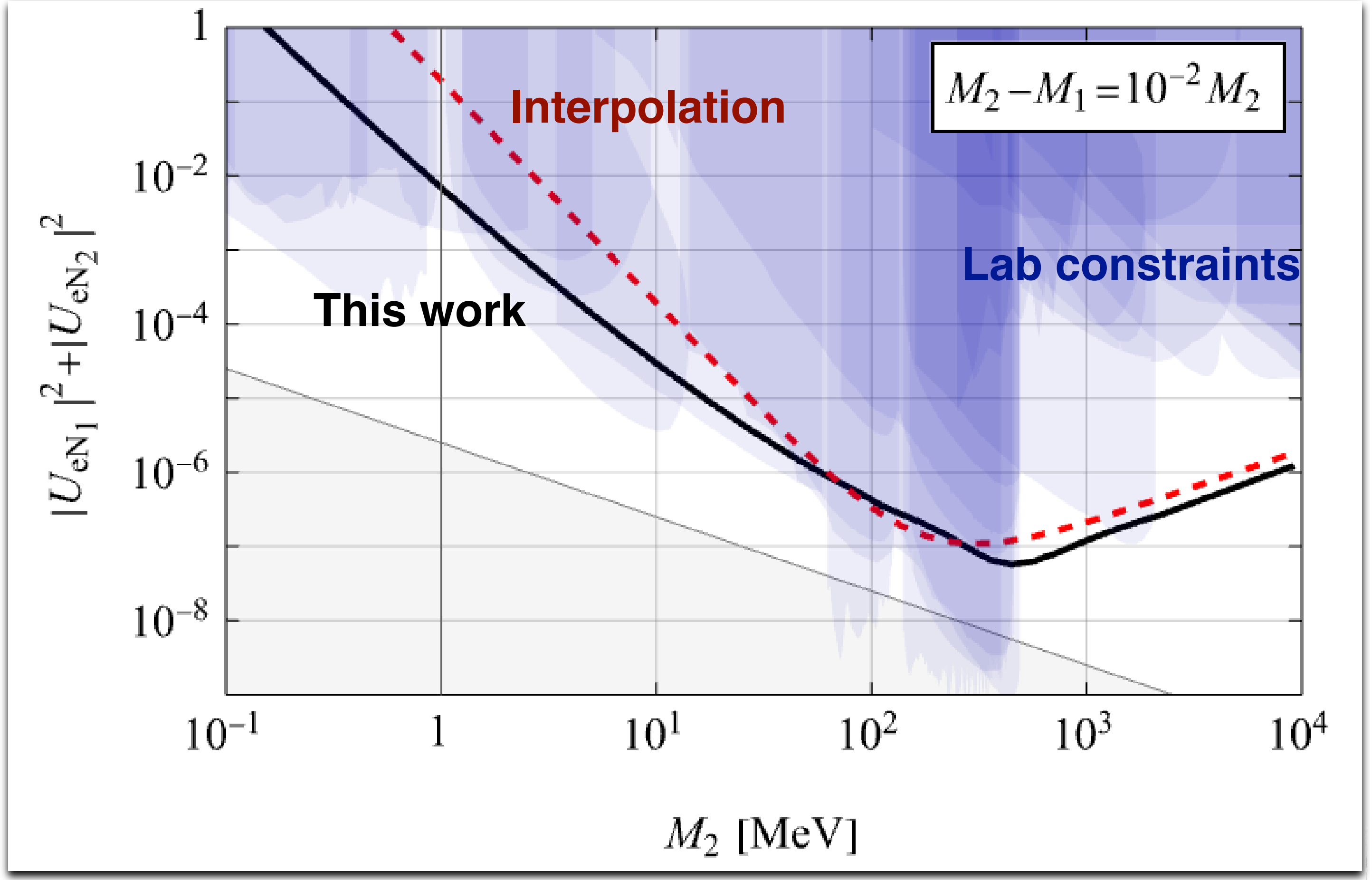

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Summary

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  - Neutrino masses
  - Leptogenesis
  - Dark matter candidate

- Generally lead to $0\nu\beta\beta$

  - Minimal extension induces cancellations in $0\nu\beta\beta$

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  - Neutrino masses
  - Leptogenesis
  - Dark matter candidate
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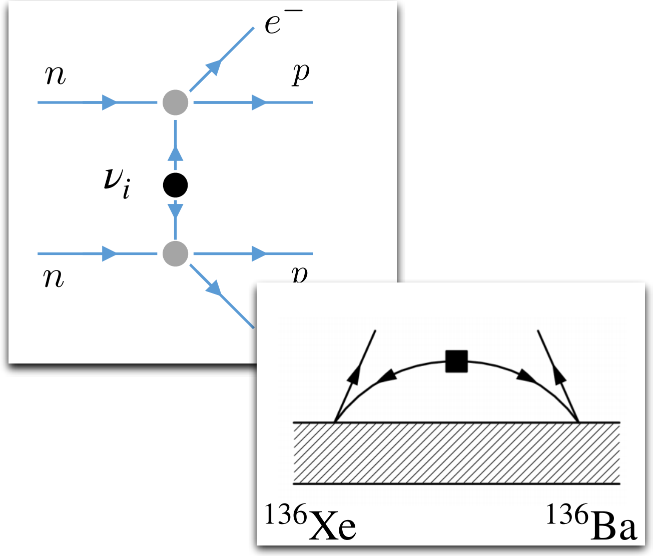
  - Minimal extension induces cancellations in $0\nu\beta\beta$

- m_i dependence can be captured in an EFT framework
  - Systematically track ν's momenta scalings
- Usually subleading contributions can become important
  - Ultrasoft contributions promoted from N2LO to LO for $m_i \lesssim k_F$

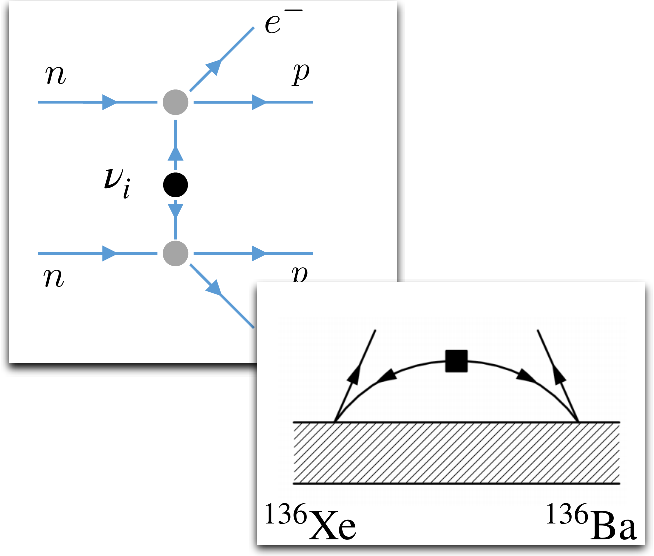


Summary

- Sterile neutrinos are motivated by
  - Neutrino masses
  - Leptogenesis
  - Dark matter candidate

- Generally lead to $0\nu\beta\beta$

  - Minimal extension induces cancellations in $0\nu\beta\beta$



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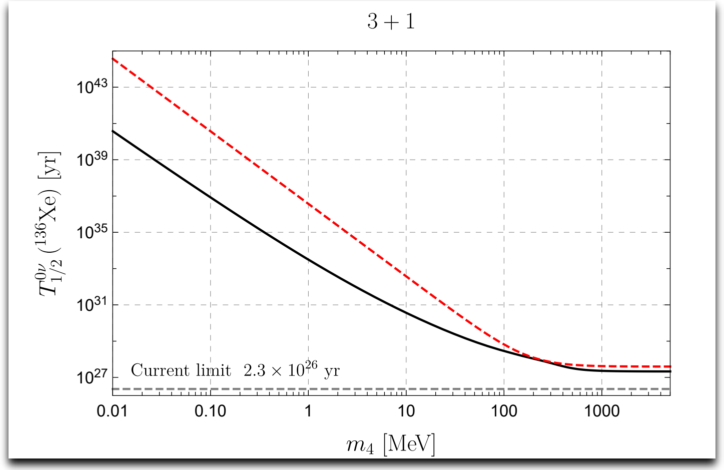


- Significant changes compared to usual approach

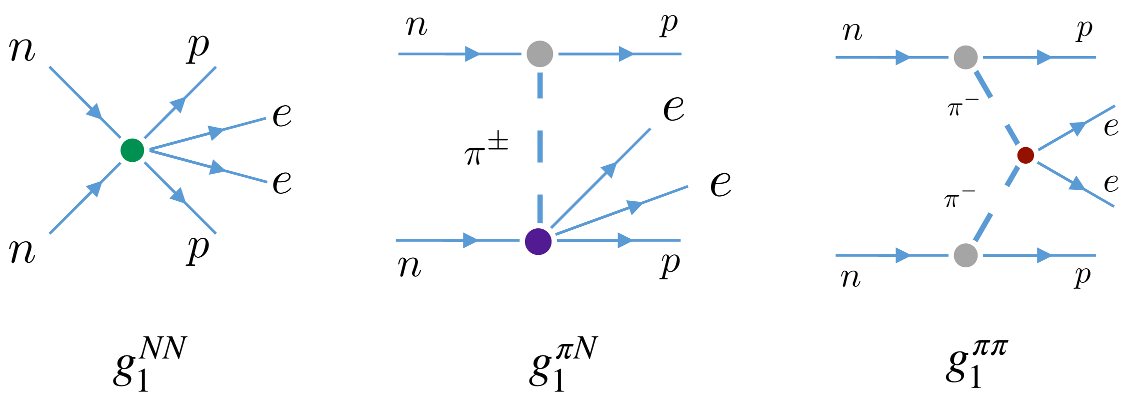
  - Can already be seen in simple toy models

Back up slides

Hadronic matrix elements

Required LECs

$m_i \gg \Lambda_\chi$



NDA	$\mathcal{O}(1)$	$\mathcal{O}(1)$	$\mathcal{O}(1)$
Used value	$\frac{1+3g_A^2}{4}$	0	0.36

LQCD: Nicholson et al ‘18

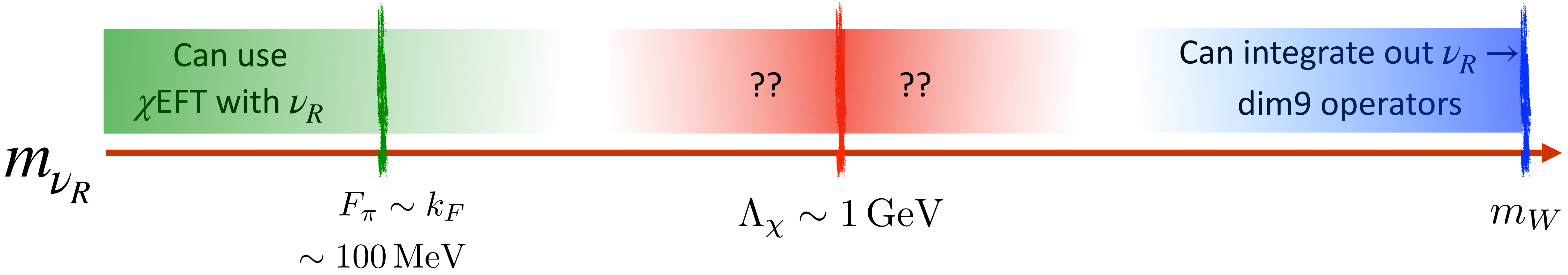
Required LECs

$m_i \lesssim \Lambda_\chi$

Interpolation

$$g_\nu^{NN}(m_i) = g_\nu^{NN}(0)\frac{1+(m_i/m_c)^2}{1+(m_i/m_c)^2(m_i/m_d)^2}\,,$$

- NDA gives $m_c \sim 1\,\mathrm{GeV}^2$
- Model esteems imply $g_\nu^{NN}(0) \sim -\,\mathrm{fm}^2$



Can use χEFT with ν_R

??

??

Can integrate out $\nu_R \rightarrow$ dim9 operators

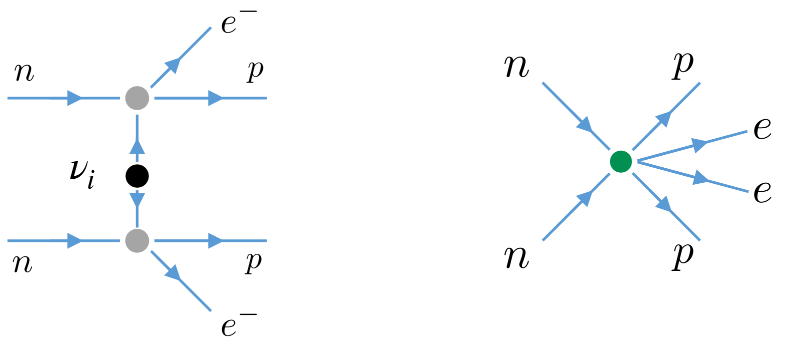
m_{ν_R}

$F_\pi \sim k_F$
$\sim 100\,\mathrm{MeV}$

$\Lambda_\chi \sim 1\,\mathrm{GeV}$

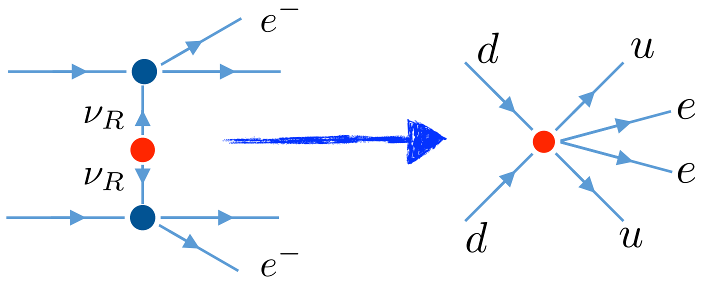
m_W

- Chiral EFT involving ν_R



Match for $m_i \sim 2$ GeV
$\Longrightarrow\ m_d$

- Integrate out ν_R



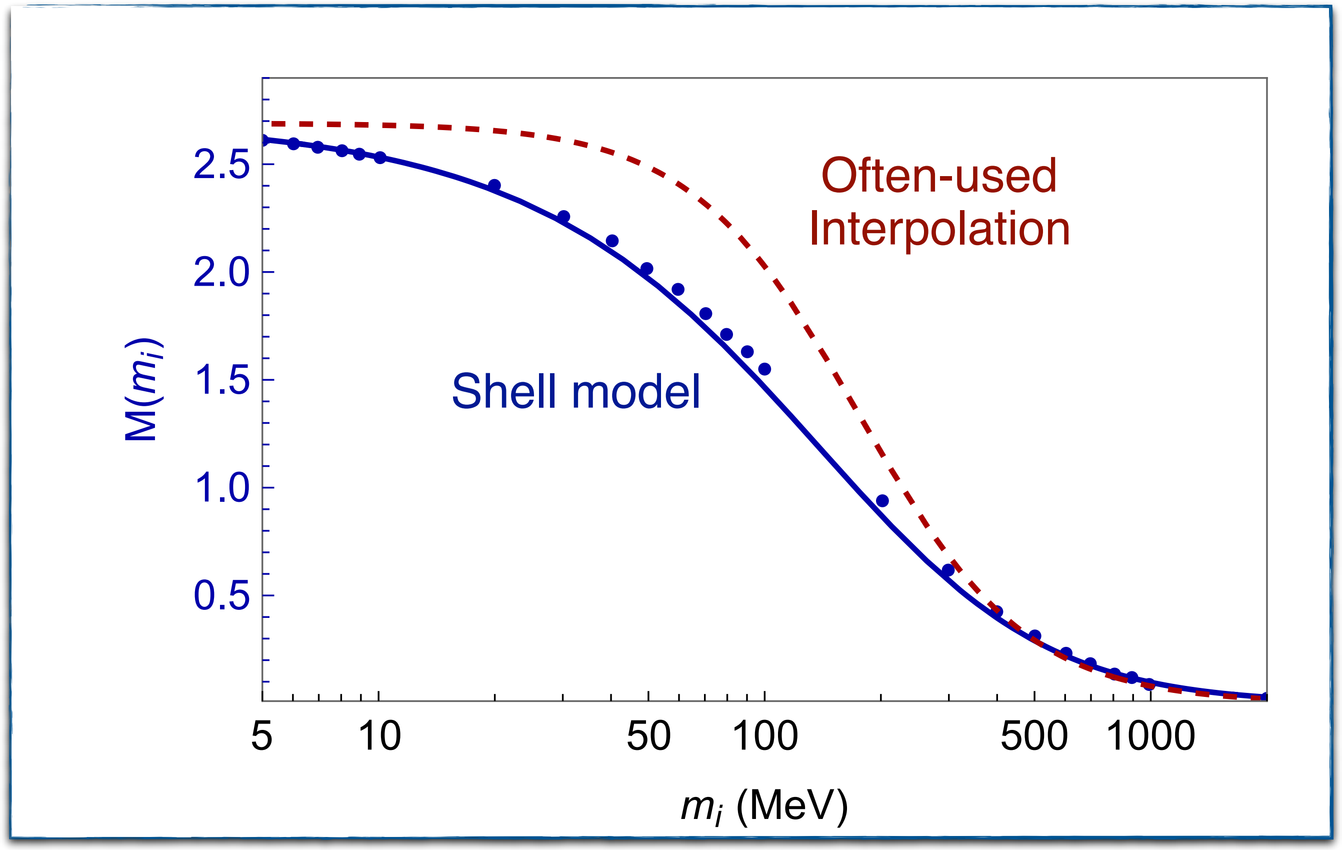
$\rightarrow$ Chiral EFT without ν_R

Nuclear matrix elements

Required NMEs

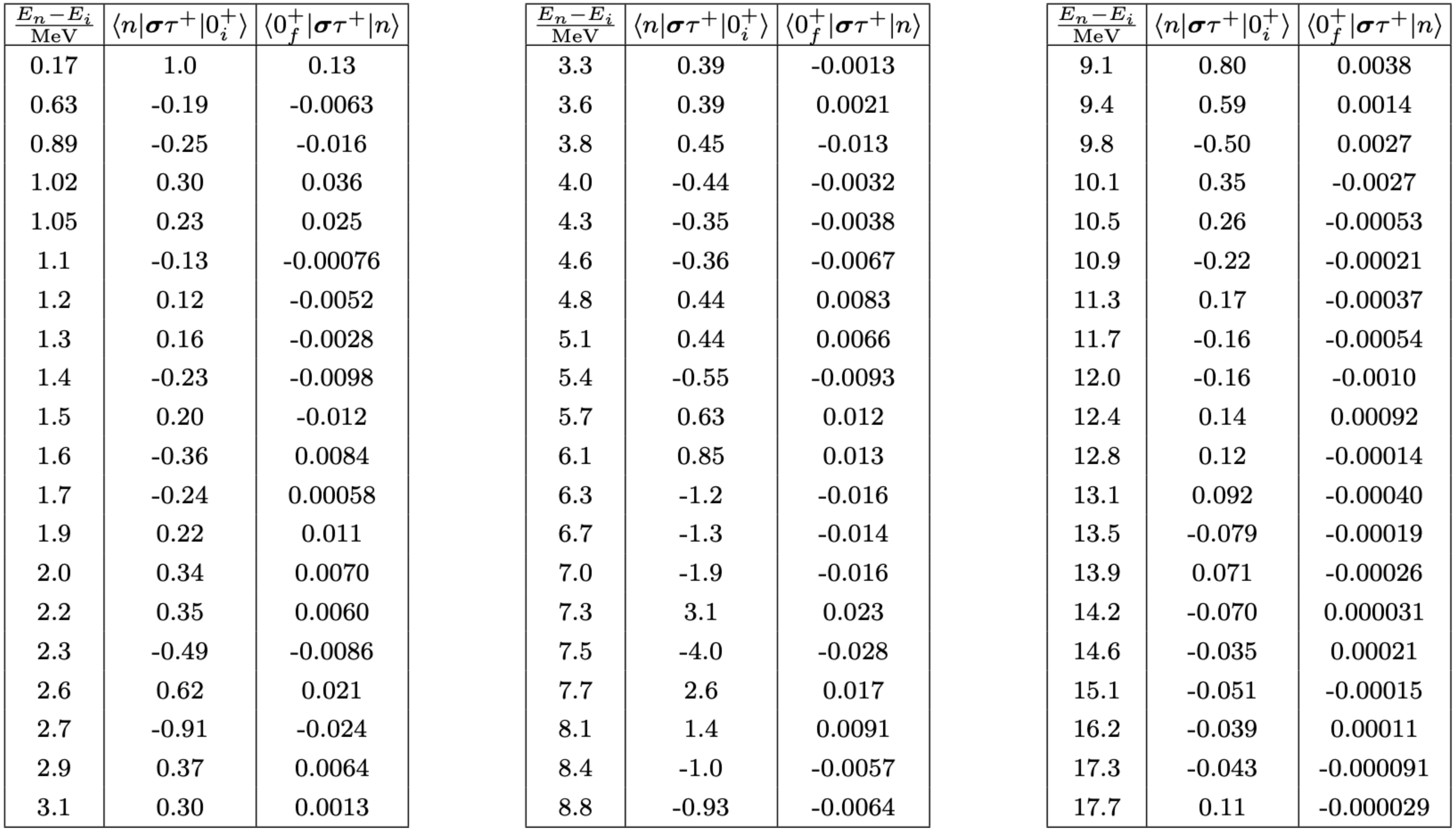
Potential contribution

$$A_\nu = \langle {}^{136}\mathrm{Ba}\,|\,V(m_i)\,|\,{}^{136}\mathrm{Xe}\rangle$$

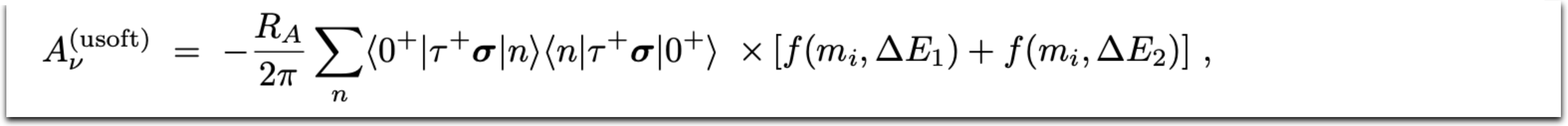


Required NMEs

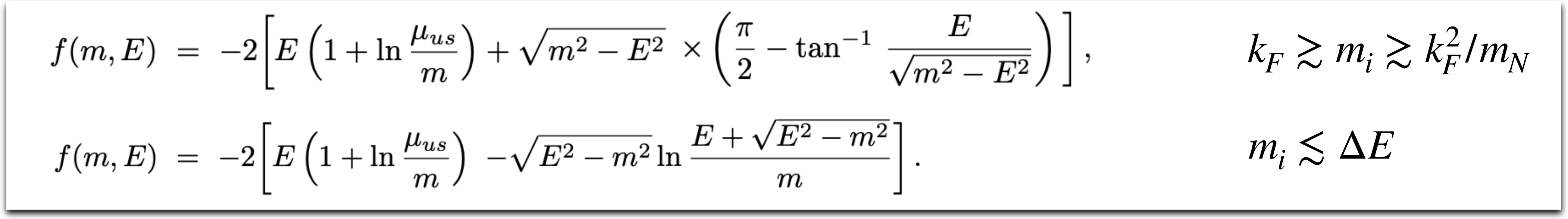
Ultrasoft contribution



$\frac{E_n-E_i}{\text{MeV}}$	$\langle n\vert\boldsymbol{\sigma}\tau^+\vert 0_i^+\rangle$	$\langle 0_f^+\vert\boldsymbol{\sigma}\tau^+\vert n\rangle$
0.17	1.0	0.13
0.63	-0.19	-0.0063
0.89	-0.25	-0.016
1.02	0.30	0.036
1.05	0.23	0.025
1.1	-0.13	-0.00076
1.2	0.12	-0.0052
1.3	0.16	-0.0028
1.4	-0.23	-0.0098
1.5	0.20	-0.012
1.6	-0.36	0.0084
1.7	-0.24	0.00058
1.9	0.22	0.011
2.0	0.34	0.0070
2.2	0.35	0.0060
2.3	-0.49	-0.0086
2.6	0.62	0.021
2.7	-0.91	-0.024
2.9	0.37	0.0064
3.1	0.30	0.0013
3.3	0.39	-0.0013
3.6	0.39	0.0021
3.8	0.45	-0.013
4.0	-0.44	-0.0032
4.3	-0.35	-0.0038
4.6	-0.36	-0.0067
4.8	0.44	0.0083
5.1	0.44	0.0066
5.4	-0.55	-0.0093
5.7	0.63	0.012
6.1	0.85	0.013
6.3	-1.2	-0.016
6.7	-1.3	-0.014
7.0	-1.9	-0.016
7.3	3.1	0.023
7.5	-4.0	-0.028
7.7	2.6	0.017
8.1	1.4	0.0091
8.4	-1.0	-0.0057
8.8	-0.93	-0.0064
9.1	0.80	0.0038
9.4	0.59	0.0014
9.8	-0.50	0.0027
10.1	0.35	-0.0027
10.5	0.26	-0.00053
10.9	-0.22	-0.00021
11.3	0.17	-0.00037
11.7	-0.16	-0.00054
12.0	-0.16	-0.0010
12.4	0.14	0.00092
12.8	0.12	-0.00014
13.1	0.092	-0.00040
13.5	-0.079	-0.00019
13.9	0.071	-0.00026
14.2	-0.070	0.000031
14.6	-0.035	0.00021
15.1	-0.051	-0.00015
16.2	-0.039	0.00011
17.3	-0.043	-0.000091
17.7	0.11	-0.000029



$$A_\nu^{(\text{usoft})} = -\frac{R_A}{2\pi}\sum_n \langle 0^+|\tau^+\boldsymbol{\sigma}|n\rangle\langle n|\tau^+\boldsymbol{\sigma}|0^+\rangle \times [f(m_i,\Delta E_1)+f(m_i,\Delta E_2)]\ ,$$



$$f(m,E) = -2\left[E\left(1+\ln\frac{\mu_{us}}{m}\right)+\sqrt{m^2-E^2}\times\left(\frac{\pi}{2}-\tan^{-1}\frac{E}{\sqrt{m^2-E^2}}\right)\right],$$

$k_F \gtrsim m_i \gtrsim k_F^2/m_N$

$$f(m,E) = -2\left[E\left(1+\ln\frac{\mu_{us}}{m}\right)-\sqrt{E^2-m^2}\ln\frac{E+\sqrt{E^2-m^2}}{m}\right].$$

$m_i \lesssim \Delta E$

Required NMEs

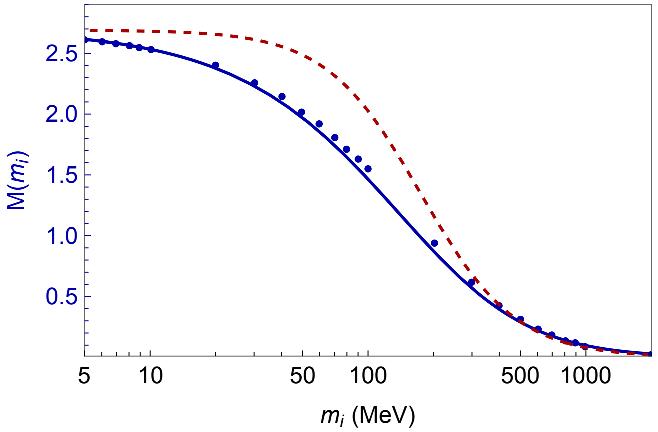
Ultrasoft/potential contributions

- Part of the ultrasoft and potential contributions are related:
  - For $m_\pi \gtrsim m_i \gtrsim \Delta E$

$$A_\nu^{\text{usoft}} \simeq \frac{R_A}{2\pi} m_i \sum_n \langle 0^+ | \tau^+ \sigma | n \rangle \langle n | \tau^+ \sigma | 0^+ \rangle$$

- This linear term is also present in

$$A_\nu^{\text{pot}} = \langle {}^{136}\text{Ba} | V(m_i) | {}^{136}\text{Xe} \rangle$$



- Have to make sure not to double count
  - In practice we remove the linear term from the potential contributions
- Allows for a cross check of the form

$$A_\nu^{\text{usoft}} \simeq m_i \frac{d}{dm_i} A_\nu^{\text{pot}}$$

  - Numerically works to $\sim 20\,\%$

Renormalization arguments

Checking the power counting

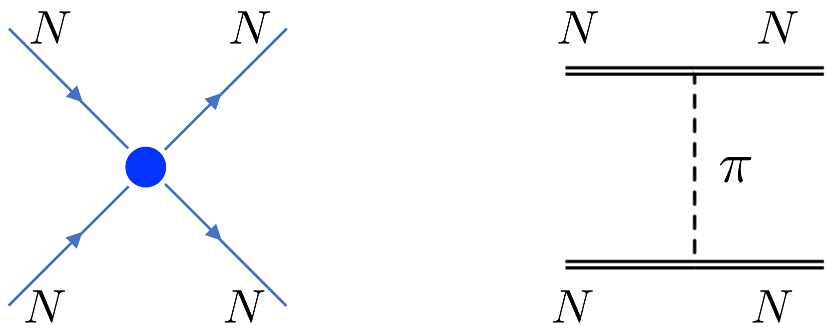
Check that $\mathcal{A}(nn \to ppee)$ is finite

Checking the power counting

Check that $\mathcal{A}(nn \to ppee)$ is finite

- Requires inclusion of the strong interaction

$$\mathcal{L}_\chi = C \left(N^T P_{^1S_0} N\right)^\dagger N^T P_{^1S_0} N - \frac{g_A}{2F_\pi} \nabla\pi \cdot \bar{N} \tau \boldsymbol{\sigma} N$$



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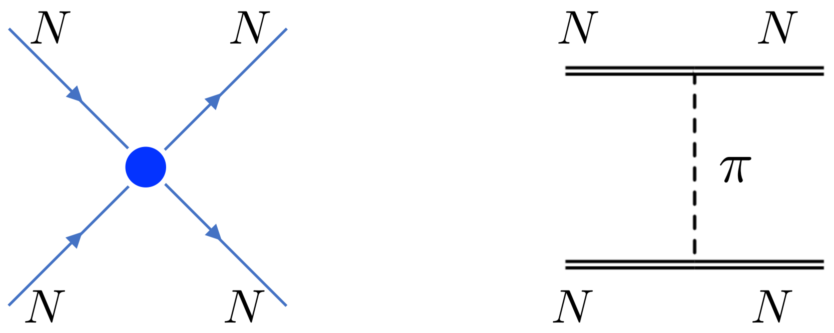
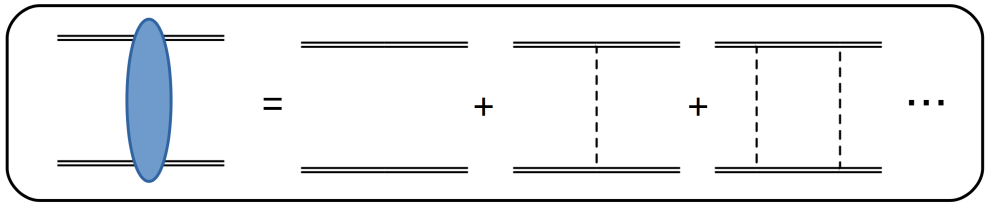
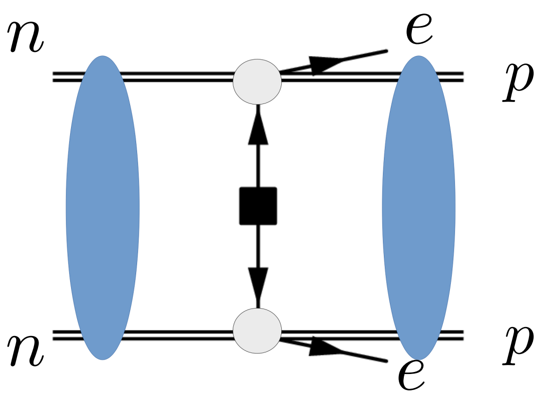


Dress the ΔL=2 potential with (renormalized) strong interactions:





✓ finite

Checking the power counting

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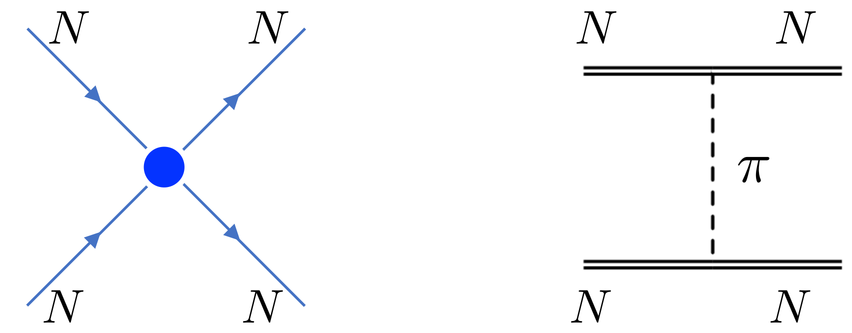
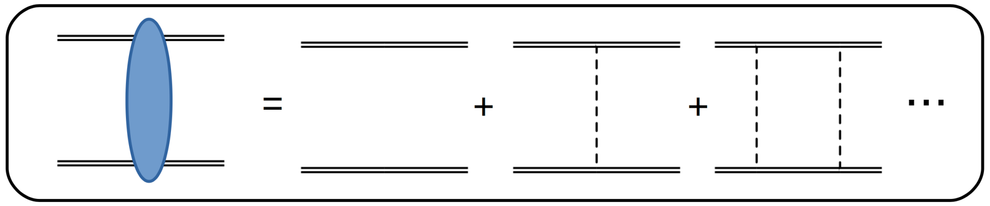
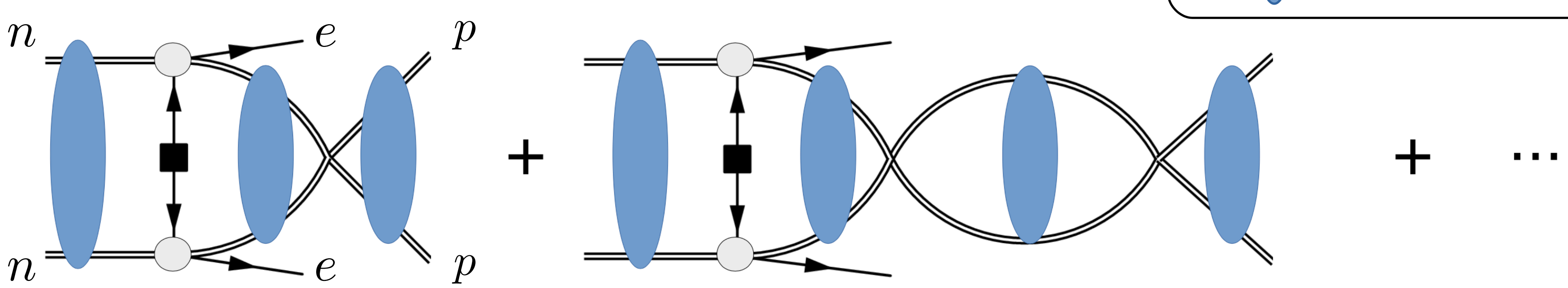


Dress the ΔL=2 potential with (renormalized) strong interactions:





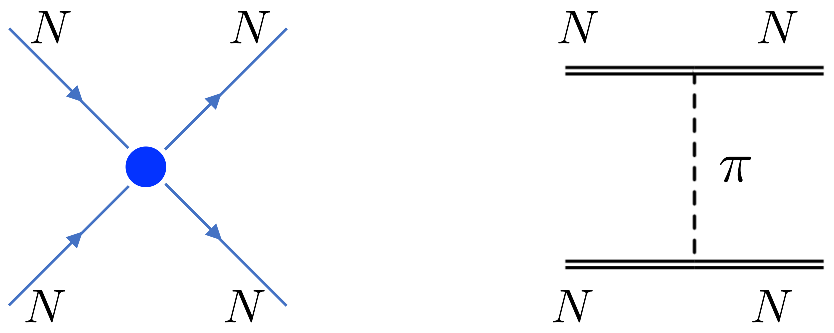
✓ **finite**

Checking the power counting

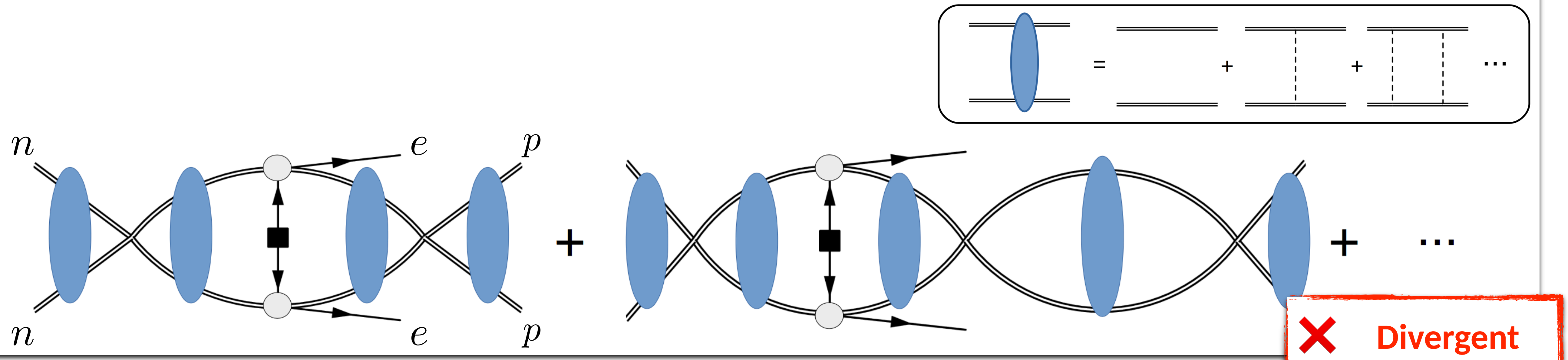
Check that $\mathcal{A}(nn \to ppee)$ is finite

- Requires inclusion of the strong interaction

$$\mathcal{L}_\chi = C\left(N^T P_{^1S_0} N\right)^\dagger N^T P_{^1S_0} N - \frac{g_A}{2F_\pi}\nabla\pi \cdot \bar{N}\tau\boldsymbol{\sigma}N$$



Dress the ΔL=2 potential with (renormalized) strong interactions:



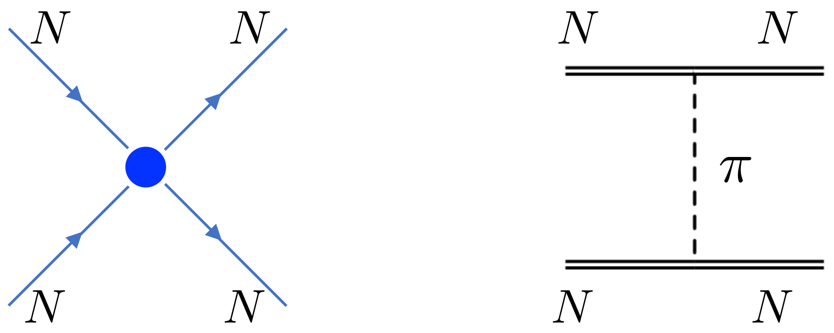
❌ **Divergent**

Checking the power counting

Check that $\mathcal{A}(nn \to ppee)$ is finite

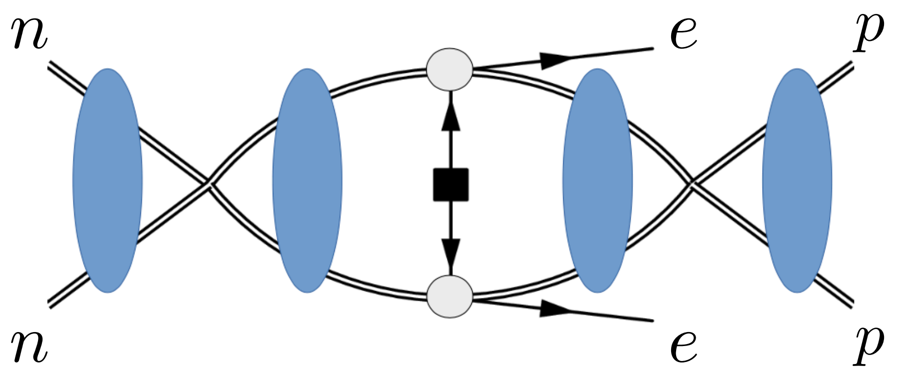
- Requires inclusion of the strong interaction

$$\mathcal{L}_\chi = C\left(N^T P_{{}^1S_0} N\right)^\dagger N^T P_{{}^1S_0} N - \frac{g_A}{2F_\pi}\nabla\pi\cdot\bar{N}\tau\boldsymbol{\sigma}N$$



Dress the ΔL=2 potential with (renormalized) strong interactions:

In MS-bar:

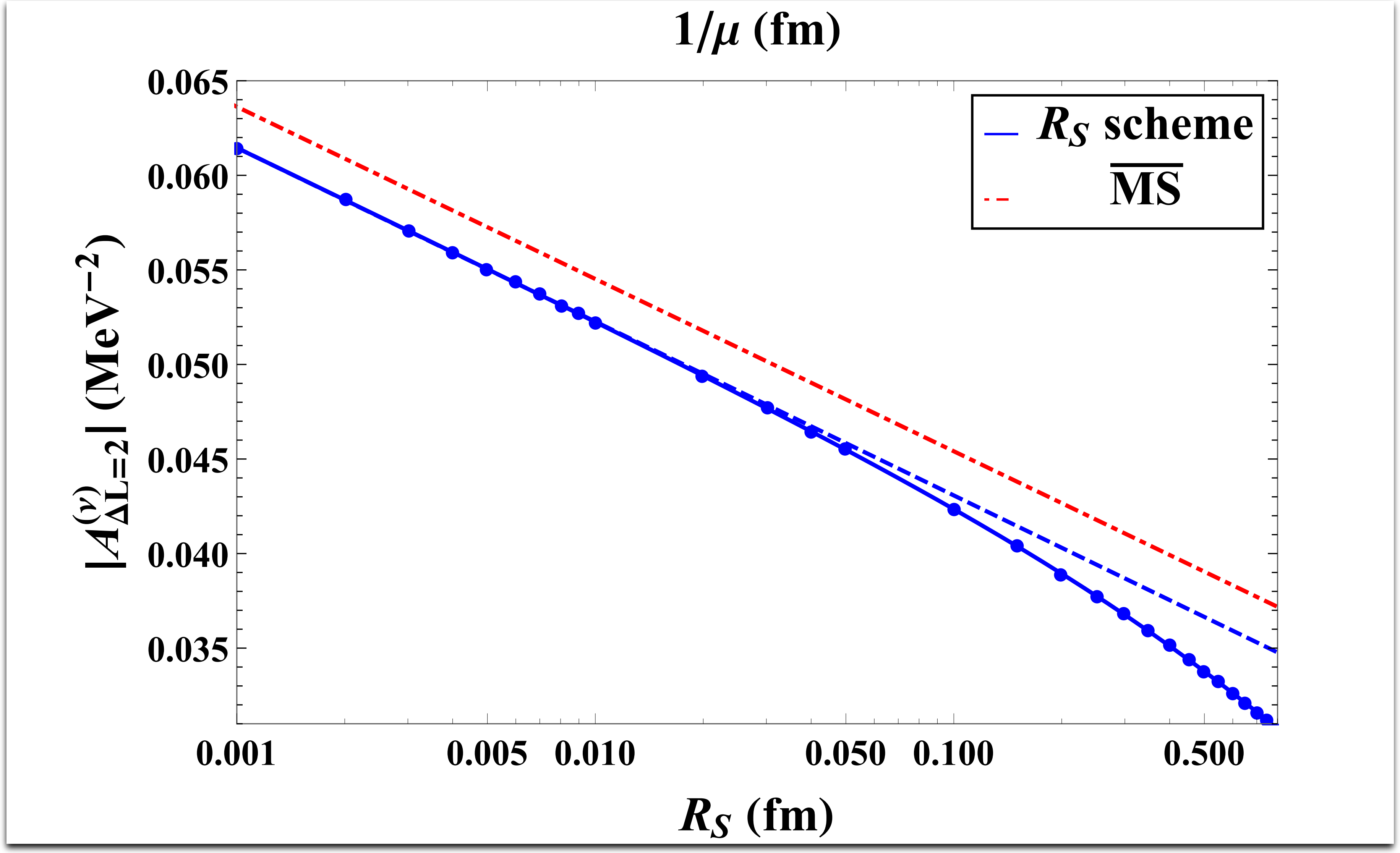


$$= -\left(\frac{m_N}{4\pi}\right)^2 \left(1+2g_A^2\right)\frac{1}{2}\left(\log\frac{\mu^2}{-(|\mathbf{p}|+|\mathbf{p}'|)^2+i0^+}+1\right)$$

$+\text{finite}$

Regulator dependent

Numerical results



- Amplitudes obtained using
  - MS-bar
  - Coordinate-space cut-off

$$\tilde{C}\,\delta^{(3)}(\mathbf{r}) \rightarrow \frac{\tilde{C}(R_S)}{(\sqrt{\pi}R_S)^3}\exp\left(-\frac{r^2}{R_S^2}\right)$$

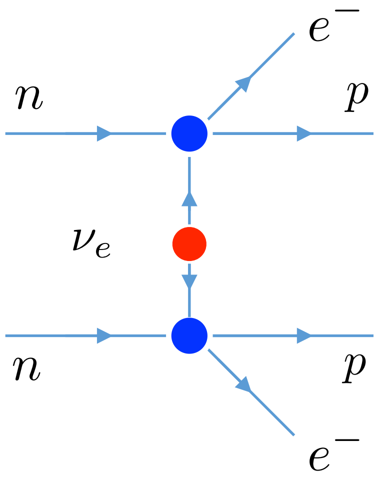
- Clear μ or R_S dependence

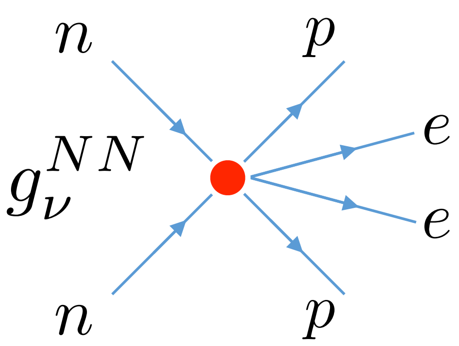
Need for a counter term

New interaction needed at leading order to get physical amplitudes:

$$\mathcal{L}_{CT} = 2G_F^2 V_{ud}^2 m_{\beta\beta}\, g_\nu^{NN}\, \bar{p}n\, \bar{p}n\, \bar{e}_L C \bar{e}_L^T$$

$$V_{\Delta L=2} = V_\nu + V_{\nu,CT} =$$





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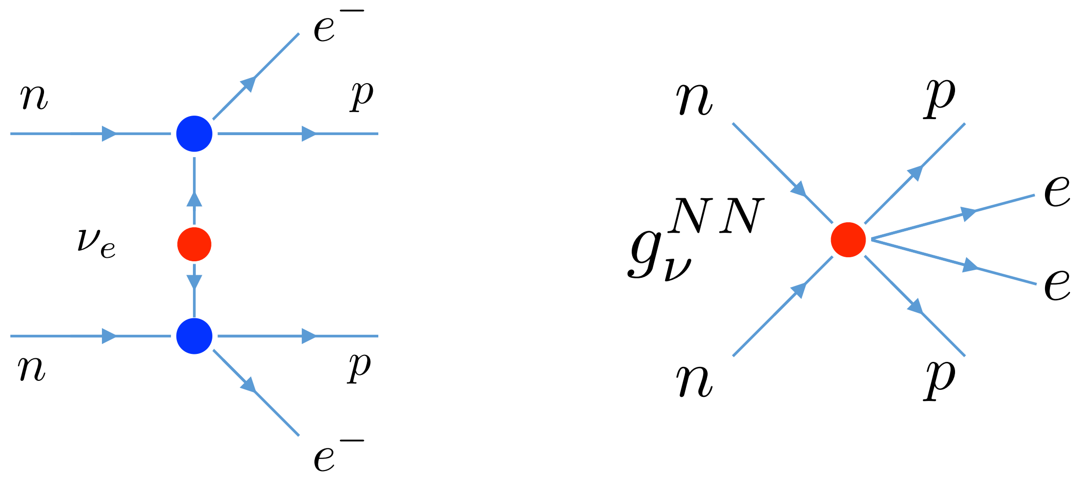

- g_ν^{NN} to be determined from a lattice calculation of $\mathcal{A}(nn \to ppe^-e^-)$
  - Area of active research

Davoudi and Kadam, '20, '21
Feng et al, '20

Need for a counter term

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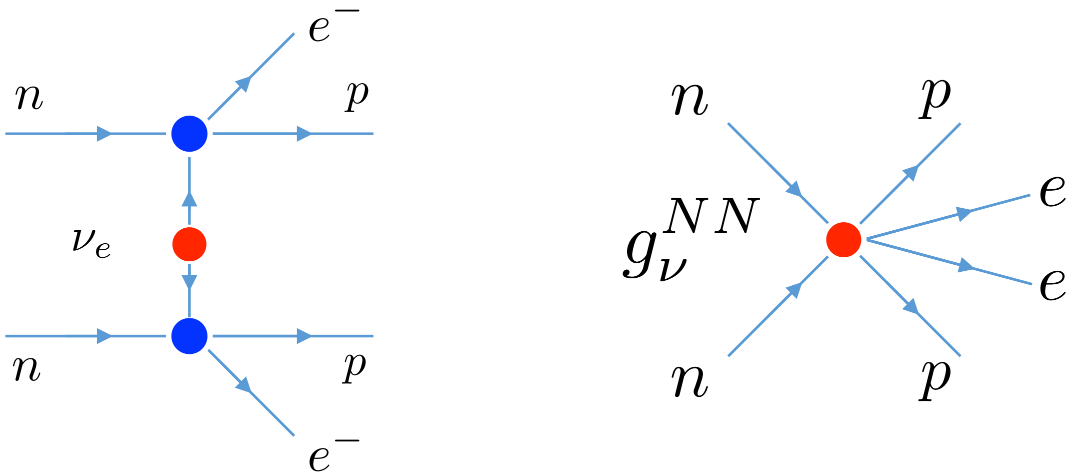


- g_ν^{NN} to be determined from a lattice calculation of $\mathcal{A}(nn \to ppe^-e^-)$
  - Area of active research

Davoudi and Kadam, '20, '21
Feng et al, '20

- Several estimates give $\tilde{g}_\nu^{NN} = \mathcal{O}(1)$
  - Comparison with isospin-breaking observables
  - Model (Cottingham) estimate
  - Large-Nc estimate

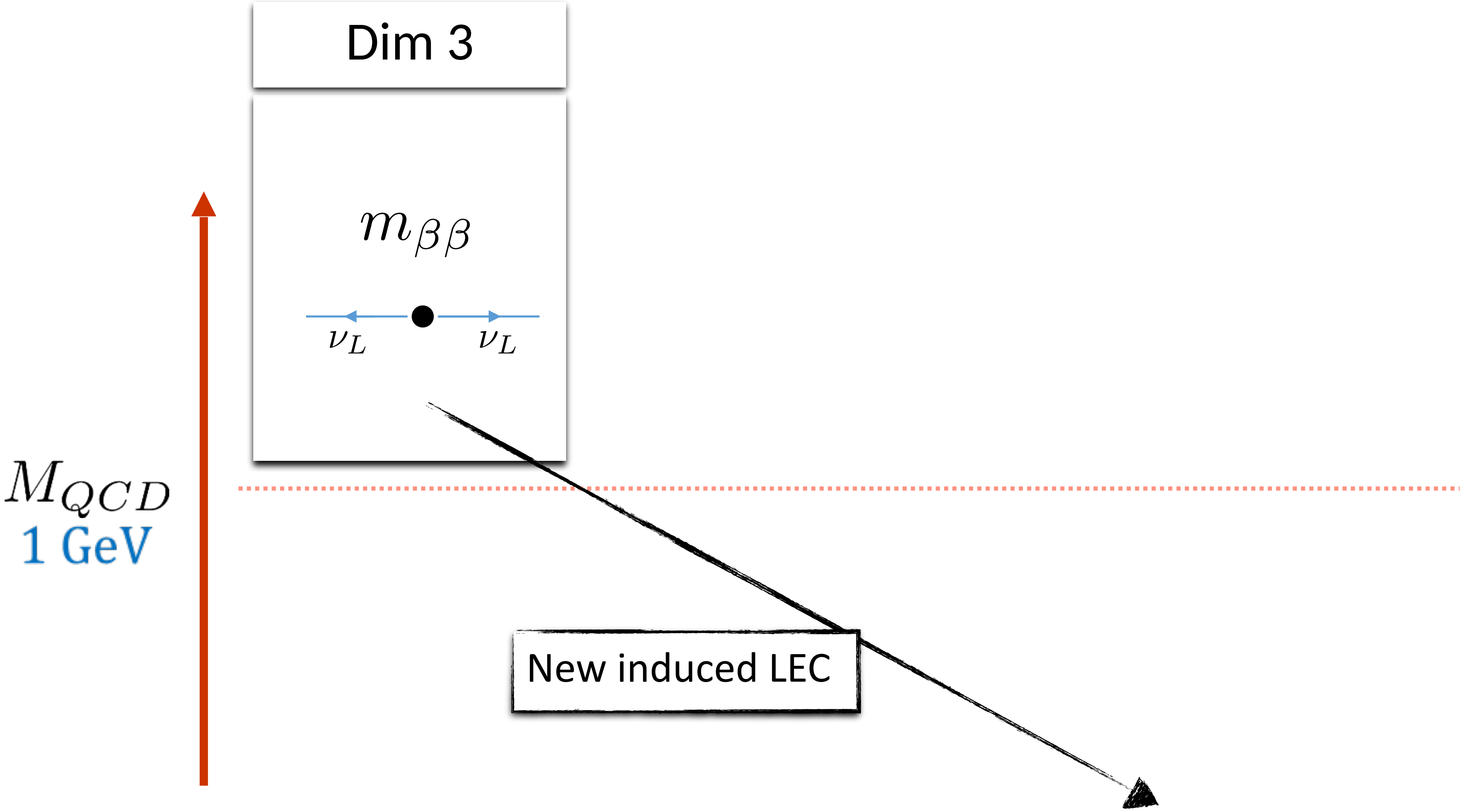
Cirigliano, et al, '19,'20, '21

Richardson et al, '21

See backup

Chiral EFT

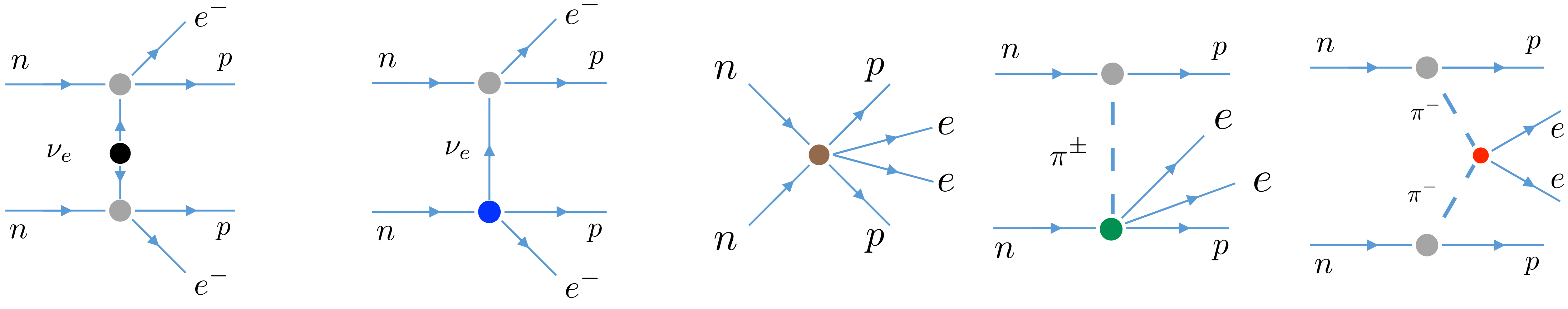
Non-Weinberg counting



Dim 3

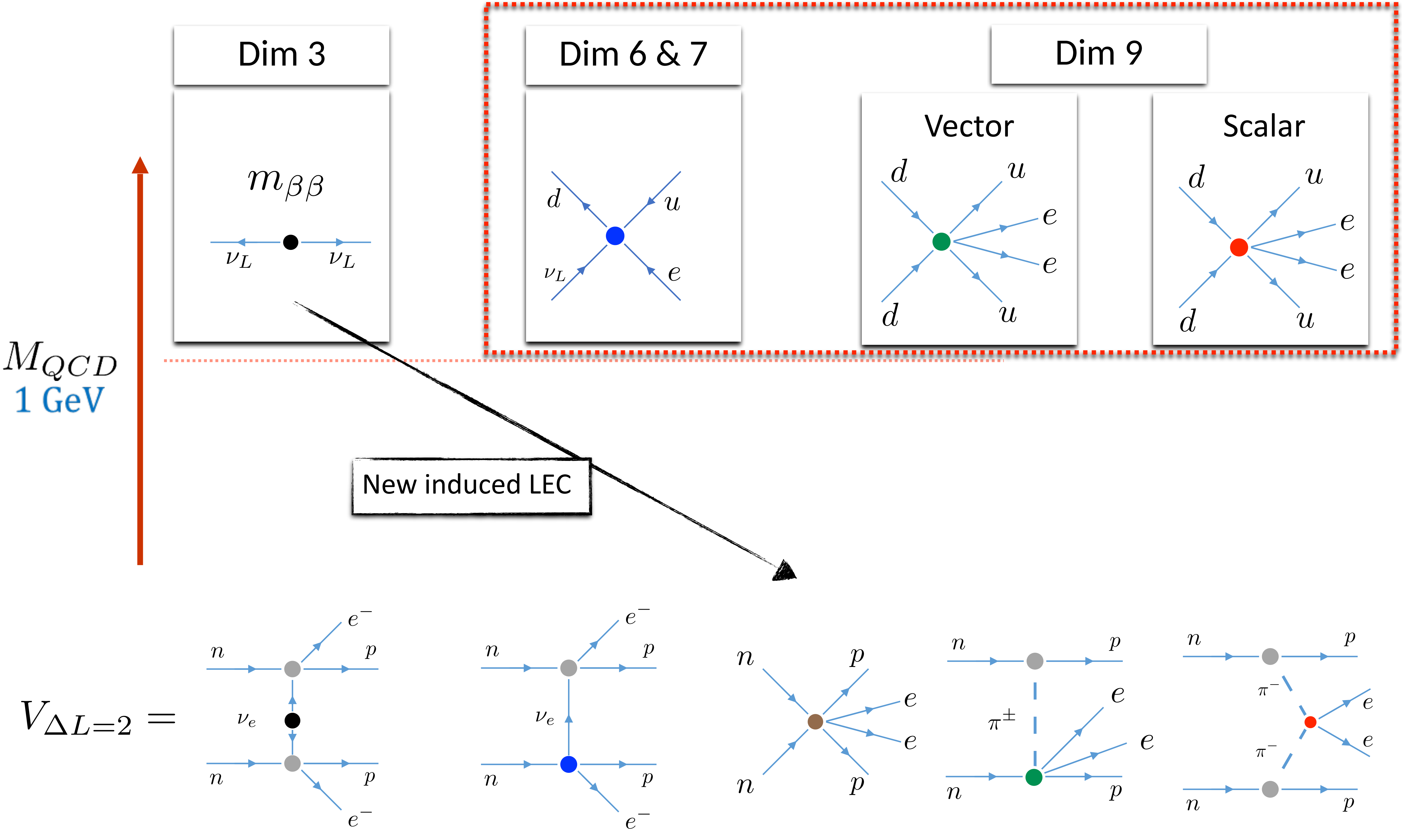
New induced LEC

$V_{\Delta L=2} =$



Chiral EFT

Non-Weinberg counting affects higher dimensional interactions as well



Dim 3

Dim 6 & 7

Dim 9

Vector

Scalar

New induced LEC

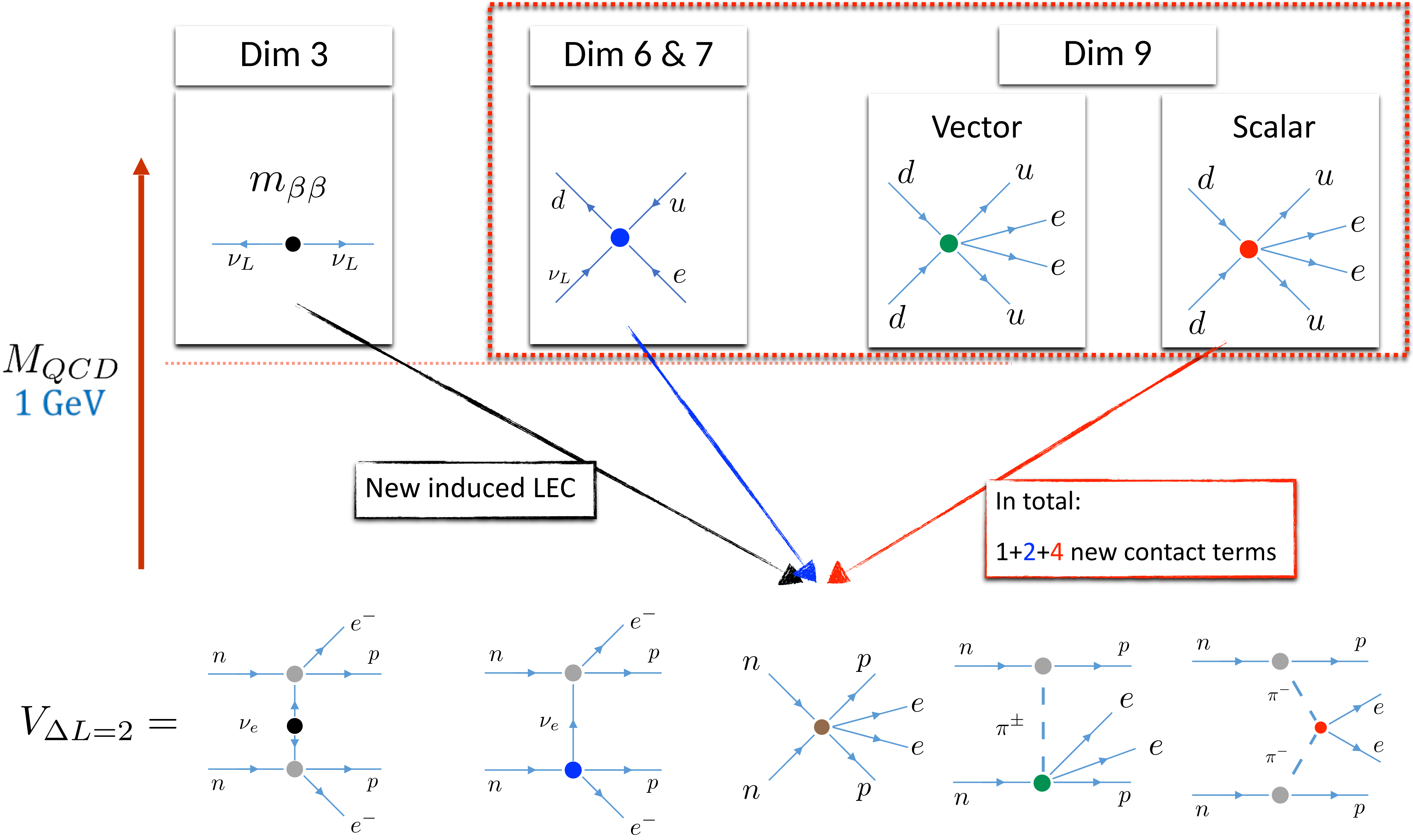
M_{QCD}

1 GeV

$V_{\Delta L=2} =$

Chiral EFT

Non-Weinberg counting affects higher dimensional interactions as well



Dim 3

Dim 6 & 7

Dim 9

Vector

Scalar

New induced LEC

In total:

1+2+4 new contact terms

Estimate of impact in light nuclei

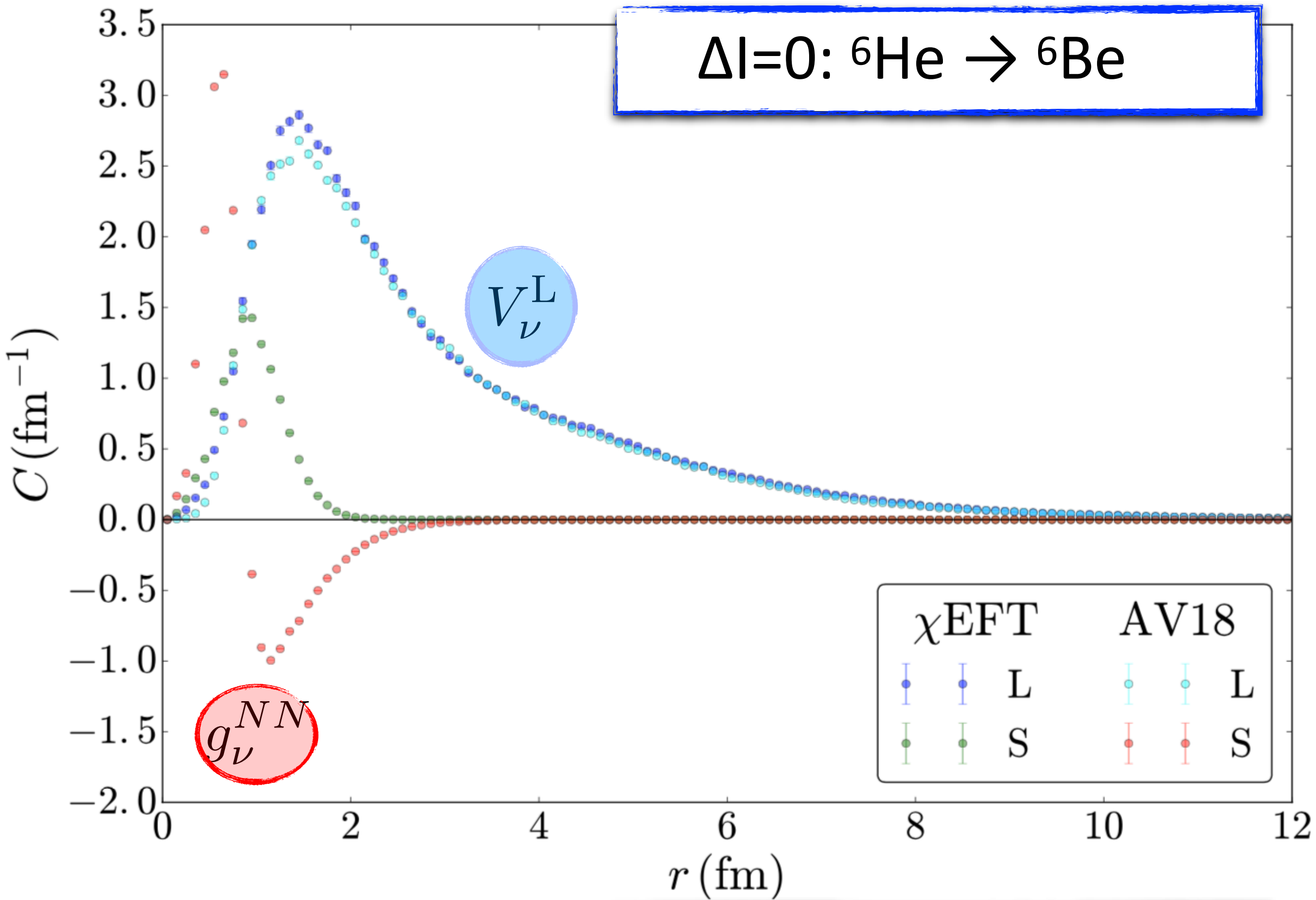
Estimate of impact

Light nuclei

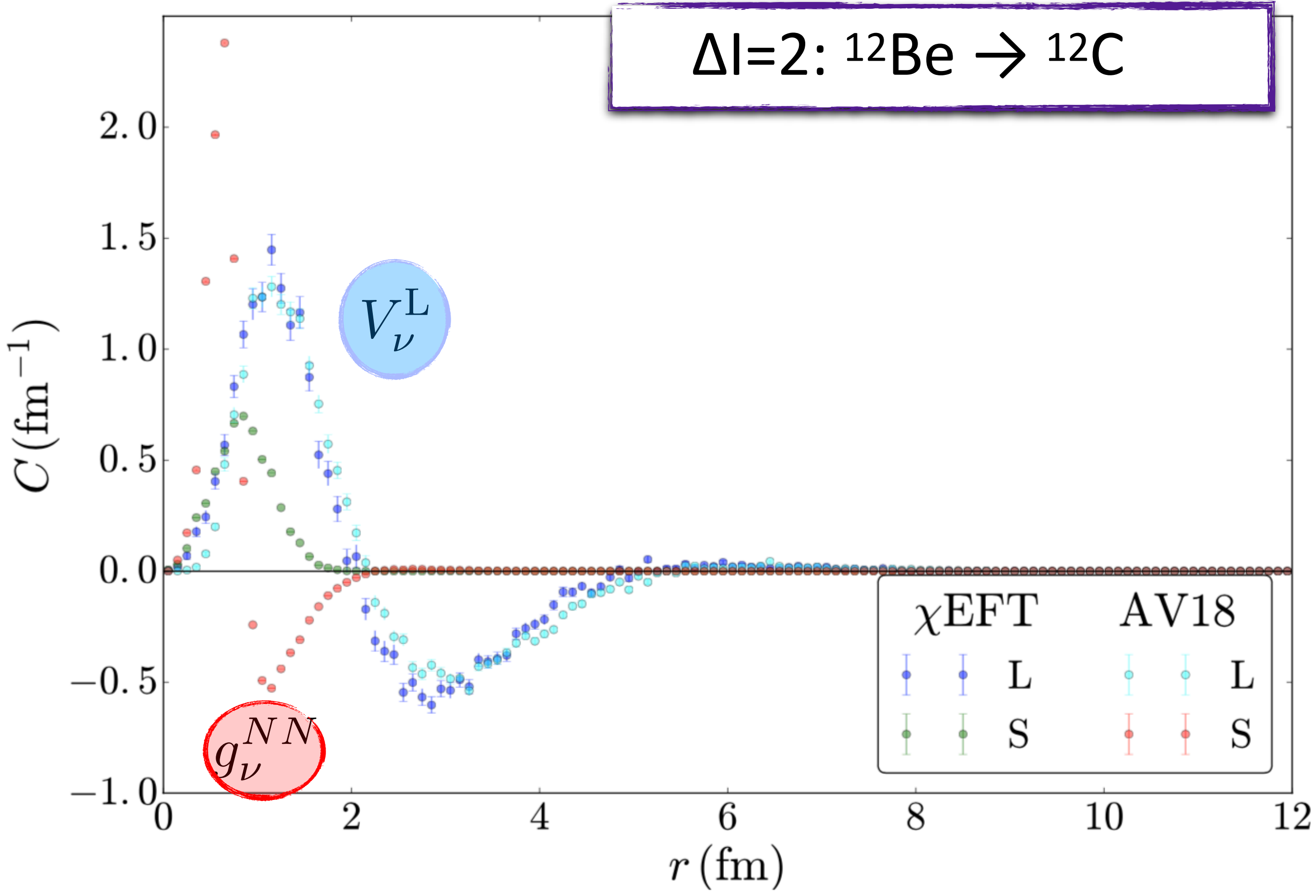
M. Piarulli, R. Wiringa, S. Pastore

- Combine estimate $g_\nu = (C_1 + C_2)/2$
- With wavefunctions:
  - From Chiral potential
    M. Piarulli et. al. '16
  - Obtained from AV18 potential
    R. Wiringa, Stoks, Schiavilla, '95

- ~10% effect in ^{6}He → ^{6}Be
- ~60% effect in ^{12}Be → ^{12}C
  - Due to presence of a node
  - Feature in realistic 0νββ candidates



ΔI=0: ^{6}He → ^{6}Be



ΔI=2: ^{12}Be → ^{12}C