SM prediction for the CP asymmetries in two-body hadronic charm decays

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In collaboration with Antonio Pich & Luiz Vale Silva

Based on hep-ph/2305.11951 and upcoming publication

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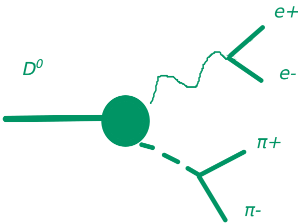
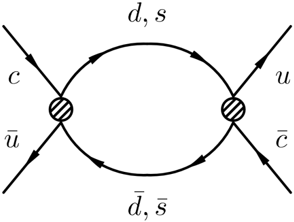
Introduction

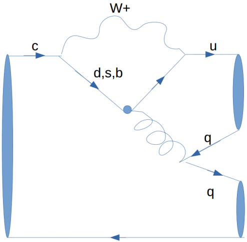
Charm Physics in the limelight

- Complementary to K and B Physics *(CKM parameters)* but different *(masses)*
- Experimental programme is growing (LHCb, Belle II, BESIII)

Rare decays | Mixing | CP violation in decays







- CPV in hadronic D modes: only discovery of CPV in the charm sector
- Plus new result of KK has puzzling implications

A new Flavour Physics 'anomaly' or an incomplete theory prediction?

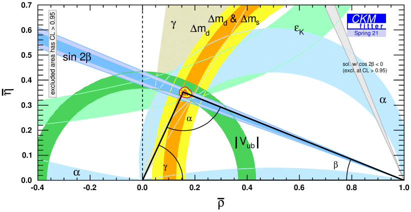
$$\Delta A_{CP}^{exp} \equiv A_{CP}(D^0 \to K^+K^-) - A_{CP}(D^0 \to \pi^+\pi^-) = [-1.54 \pm 0.29] \cdot 10^{-3}$$

$$\Delta A_{CP}^{dir,exp} = [-1.57 \pm 0.29] \cdot 10^{-3}$$ [LHCb 2019]

$$A_{CP}(D^0 \to K^+K^-) = [6.8 \pm 5.4(\text{stat}) \pm 1.6(\text{syst})] \cdot 10^{-4}$$ [LHCb 2022]

$$A_{CP}^{dir}(D^0 \to \pi^+\pi^-) = [23.2 \pm 6.1] \cdot 10^{-4}$$

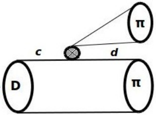
- Is the SM theoretical prediction in agreement?
  Is it NP? [see e.g. 2210.16330]
- Weak sector (CKM parameters) probed by $K\&B$ physics



- Strong sector (hadronic uncertainties) problematic

Weak and strong, short and long distance

Tree topology

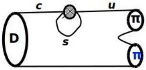


$$\mathscr{A}(D^0 \to f) = A(f) + i r_{CKM} B(f)$$

$$\mathscr{A}(\overline{D^0} \to f) = A(f) - i r_{CKM} B(f)$$

$$a_{CP}^{dir} \approx 2 r_{CKM} \frac{|B(f)|}{|A(f)|} \cdot \sin \arg \frac{A(f)}{B(f)}$$

Penguin topology



From the short distance front:

$$\mathscr{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} \left[\Sigma_{i=1}^{2} C_i(\mu) \left(\lambda_d Q_i^d(\mu) + \lambda_s Q_i^s(\mu) \right) - \lambda_b (\Sigma_{i=3}^{6} C_i(\mu) Q_i(\mu) + C_{8g}(\mu) Q_{8g}(\mu)) \right]$$

$\lambda_q = V_{cq}^* V_{uq}\,, \quad q = d, s, b.$
$|\lambda_d| \approx |\lambda_s| = \mathscr{O}(\lambda)$

$$r_{CKM} = Im \frac{V_{cb}^* V_{ub}}{V_{cd}^* V_{ud}} \approx 6.2 \cdot 10^{-4}$$

Penguin operators

Current-current operators

$$Q_1^q = (\bar{q}c)_{V-A} (\bar{u}q)_{V-A}$$

$$Q_2^d = (\bar{q}_j c_i)_{V-A} (\bar{u}_i q_j)_{V-A}$$

$$(q = d, s)$$

$$Q_3 = (\bar{u}c)_{V-A} \Sigma_q (\bar{q}q)_{V-A}$$

$$Q_4 = (\bar{u}_j c_i)_{V-A} \Sigma_q (\bar{q}_i q_j)_{V-A}$$

$$Q_5 = (\bar{u}c)_{V-A} \Sigma_q (\bar{q}q)_{V+A}$$

$$Q_6 = (\bar{u}_j c_i)_{V-A} \Sigma_q (\bar{q}_i q_j)_{V+A}$$

$C_{4,6} < 0.1\, C_2,\ 0.03\, C_1$ *(GIM mechanism at play)*

Weak and strong, short and long distance

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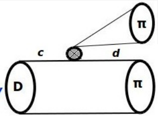

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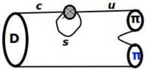

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Problem: hadronic matrix elements

$$\langle hh | Q_i | D^0 \rangle$$

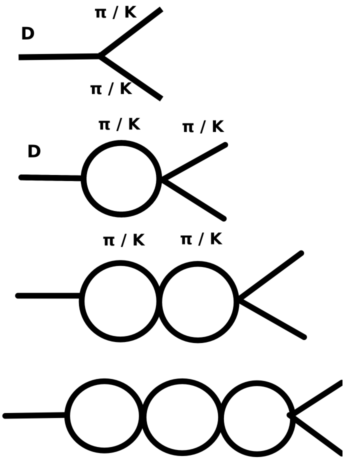
Charm scale is special!

$\Lambda_{\chi PT} \approx m_\rho < m_D = 1865$ MeV
$\frac{\Lambda_{QCD}}{m_c} = \mathscr{O}(1)$

See also: Khodjamirian, Petrov Phys. Lett. B, 774:235–242, 2017, Brod, Kagan, Zupan Phys. Rev. D, 86:014023, 2012, Schacht, Soni Phys. Lett. B, 825:136855, 2022

A way to look at the problem: rescattering

- Strong process, blind to the weak phase
- Isospin (u↔d) is a good symmetry of strong interactions
- In I=0, two channels:



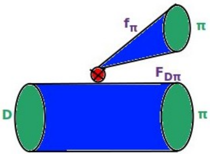
$$S_{strong} = \begin{pmatrix} \pi\pi \to \pi\pi & \pi\pi \to KK \\ KK \to \pi\pi & KK \to KK \end{pmatrix}$$

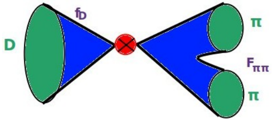
Rescattering & what we learn about strong phases

- S matrix is **unitary**, as well as strong sub-matrix
- For I=0: $$\begin{pmatrix} A(D\to\pi\pi) \\ A(D\to KK) \end{pmatrix} = \underbrace{\begin{pmatrix} \eta e^{i2\delta_1} & i\sqrt{1-\eta^2}e^{i(\delta_1+\delta_2)} \\ i\sqrt{1-\eta^2}e^{i(\delta_1+\delta_2)} & \eta e^{i2\delta_2} \end{pmatrix}}_{S_{strong}} \cdot \begin{pmatrix} A^*(D\to\pi\pi) \\ A^*(D\to KK) \end{pmatrix}$$
- The phases are related to the rescattering phases **which are known from data/nuclear experiments**
- Watson's theorem (elastic rescattering limit):
  $argA(D\to\pi\pi) = argA(\pi\pi\to\pi\pi)mod\pi$
- With inelasticities: more complicated, phase-shifts dependent on magnitudes of the amplitudes too

Magnitudes of matrix elements *without* rescattering

At the limit of $N_c \to \infty$, we are only left with the matrix elements from *factorisation*





(Same for $D \to KK$)

- Non-rescattering "bare" amplitudes:

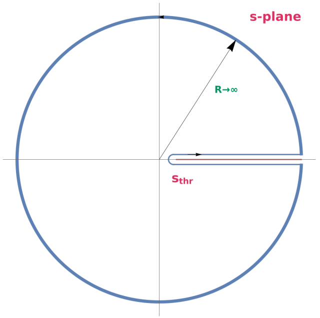
  $T^B(D^0 \to \pi^+\pi^-) \propto \lambda_d C_1 \langle \pi^+\pi^- | Q_1 | D^0 \rangle_{fac} - \lambda_b (C_4 \langle \pi^+\pi^- | Q_4 | D^0 \rangle_{fac} + C_6 \langle \pi^+\pi^- | Q_6 | D^0 \rangle_{fac})$
- Form factors are at the *non-rescattering* limit!
- Decay constant and form factor come from lattice and data (through χPT)
- Internal gluon exchanges at each current are naturally included (but internal quark loops are suppressed)

Basic property of scattering amplitudes: analyticity

- Fundamental, model-independent property related to **causality**
- Cauchy's theorem: $A(s) = \frac{1}{2\pi i} \oint_C ds' \frac{A(s')}{s'-s}$ leads to

$$ReA(s) = \frac{1}{\pi} \int_{s_{thr}}^{\infty} ds' \frac{ImA(s')}{s'-s}$$

(Dispersion relation)



- Unitarity of S-matrix & dispersion relation:

$$\underbrace{ReA(s)}_{\text{Re at a point}} = \frac{1}{\pi} \underbrace{\int_{s_{thr}}^{\infty} ds' \frac{\tan \delta(s')}{s'-s} ReA(s')}_{\text{integral of Re along the physical region}}$$

Analyticity & what we learn about magnitudes

- Integral equation, studied by Muskhelishvili-Omnes
- One subtraction: needs one piece of physical information
- Single channel case (& one subtraction at s_0), **physical** solution:

$$|A_I(s)| = \underbrace{A_I(s_0)}_{\text{ampl. when } \Omega = 1} \underbrace{exp\{\frac{s-s_0}{\pi} PV \int_{4M_\pi^2}^{\infty} dz \frac{\delta_I(z)}{(z-s_0)(z-s)}\}}_{\text{Omnes factor } \Omega}$$

We need more than just large N_C!

$|A_I(s = m_D^2)| = (\text{large } N_C \text{ result}) \times (\text{Omnes factor})_I$

- More channels: Equally more solutions. No analytical solution

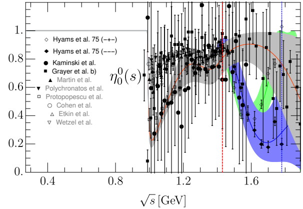
What we do

Summary of our method

- Factor out weak phases
- Flavour basis to isospin
- Isospin blocks:
  - I=0, unitarity with 2 channels: $\pi\pi$ and KK
  - I=1 with KK elastic rescattering
  - I=2 with $\pi\pi$ elastic rescattering
- Isospin amplitudes treated with dispersion relations calculated numerically (based on Moussallam et al. [hep-ph/9909292])
- Use inelasticity and phase-shift parametrisations [Pelaez et al., 1907.13162],[Pelaez et al., 2010.11222] up to energies $\sim m_D$ - extrapolate for higher & consider uncertainties
- For I=1 and 2, extract |Omnes factors| from Br's of $A(D^+ \to \pi^+\pi^0) \sim A_{I=2}, A(D^+ \to K^+\overline{K^0}) \sim A_{I=1}$, phases left unconstrained
- Decay-specific physical input: large N_C limit (for subtraction constant)

Choice of Omnes factors

For the isospin=0 channels we **calculate** numerically the Omnes matrix at $s = m_D^2$



	solution I	solution II	solution III
$\eta_0^0,\, m_\eta^* = 1$	$\Omega^{(0)} = \begin{pmatrix} 0.80\, e^{+1.60\, i} & 1.01\, e^{-1.69\, i} \\ 0.56\, e^{-1.50\, i} & 0.59\, e^{+2.07\, i} \end{pmatrix}$	$\Omega^{(0)} = \begin{pmatrix} 0.39\, e^{+1.64\, i} & 0.59\, e^{-2.13\, i} \\ 0.51\, e^{-1.31\, i} & 0.56\, e^{+2.43\, i} \end{pmatrix}$	$\Omega^{(0)} = \begin{pmatrix} 0.71\, e^{+0.53\, i} & 1.35\, e^{-2.67\, i} \\ 0.38\, e^{-0.98\, i} & 0.42\, e^{+2.65\, i} \end{pmatrix}$
$\eta_0^0 - \delta\eta_0^0,\, m_\eta^* = 1$	$\Omega^{(0)} = \begin{pmatrix} 0.56\, e^{+1.84\, i} & 0.61\, e^{-1.73\, i} \\ 0.57\, e^{-1.41\, i} & 0.58\, e^{+2.25\, i} \end{pmatrix}$	$\Omega^{(0)} = \begin{pmatrix} 0.42\, e^{+1.75\, i} & 0.54\, e^{-2.05\, i} \\ 0.51\, e^{-1.33\, i} & 0.55\, e^{+2.43\, i} \end{pmatrix}$	$\Omega^{(0)} = \begin{pmatrix} 0.35\, e^{+1.13\, i} & 0.74\, e^{-2.47\, i} \\ 0.50\, e^{-1.18\, i} & 0.55\, e^{+2.48\, i} \end{pmatrix}$
$\eta_0^0 - \delta\eta_0^0,\, m_\eta^* = 2$	$\Omega^{(0)} = \begin{pmatrix} 0.58\, e^{+1.80\, i} & 0.64\, e^{-1.74\, i} \\ 0.58\, e^{-1.37\, i} & 0.61\, e^{+2.26\, i} \end{pmatrix}$	$\Omega^{(0)} = \begin{pmatrix} 0.43\, e^{+1.64\, i} & 0.58\, e^{-2.10\, i} \\ 0.52\, e^{-1.25\, i} & 0.57\, e^{+2.48\, i} \end{pmatrix}$	$\Omega^{(0)} = \begin{pmatrix} 0.40\, e^{+1.01\, i} & 0.80\, e^{-2.50\, i} \\ 0.50\, e^{-1.11\, i} & 0.56\, e^{+2.53\, i} \end{pmatrix}$
$\eta_0^0 - \delta\eta_0^0,\, m_\eta^* = 3$	$\Omega^{(0)} = \begin{pmatrix} 0.60\, e^{+1.76\, i} & 0.66\, e^{-1.74\, i} \\ 0.60\, e^{-1.33\, i} & 0.63\, e^{+2.26\, i} \end{pmatrix}$	$\Omega^{(0)} = \begin{pmatrix} 0.44\, e^{+1.53\, i} & 0.61\, e^{-2.16\, i} \\ 0.52\, e^{-1.17\, i} & 0.59\, e^{+2.53\, i} \end{pmatrix}$	$\Omega^{(0)} = \begin{pmatrix} 0.45\, e^{+0.91\, i} & 0.86\, e^{-2.53\, i} \\ 0.50\, e^{-1.04\, i} & 0.57\, e^{+2.58\, i} \end{pmatrix}$
sol. B': $\lvert g_0^0 \rvert$	$\Omega^{(0)} = \begin{pmatrix} 2.01\, e^{+1.39\, i} & 2.47\, e^{-1.76\, i} \\ 0.37\, e^{-0.33\, i} & 0.54\, e^{+3.05\, i} \end{pmatrix}$	$\Omega^{(0)} = \begin{pmatrix} 1.91\, e^{+0.60\, i} & 2.78\, e^{-2.55\, i} \\ 0.31\, e^{-0.23\, i} & 0.45\, e^{+3.30\, i} \end{pmatrix}$	$\Omega^{(0)} = \begin{pmatrix} 2.20\, e^{+0.43\, i} & 3.55\, e^{-2.72\, i} \\ 0.35\, e^{+0.03\, i} & 0.57\, e^{+3.40\, i} \end{pmatrix}$
sol. C': $\lvert g_0^0 \rvert$	$\Omega^{(0)} = \begin{pmatrix} 1.83\, e^{+1.38\, i} & 2.65\, e^{-1.76\, i} \\ 0.34\, e^{-0.40\, i} & 0.57\, e^{+3.00\, i} \end{pmatrix}$	$\Omega^{(0)} = \begin{pmatrix} 1.80\, e^{+0.59\, i} & 3.11\, e^{-2.56\, i} \\ 0.29\, e^{-0.24\, i} & 0.49\, e^{+3.24\, i} \end{pmatrix}$	$\Omega^{(0)} = \begin{pmatrix} 2.09\, e^{+0.43\, i} & 3.94\, e^{-2.72\, i} \\ 0.32\, e^{-0.03\, i} & 0.61\, e^{+3.34\, i} \end{pmatrix}$

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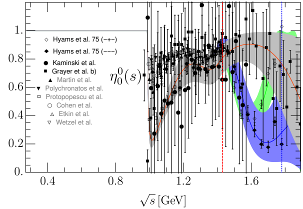
For the isospin=0 channels we **calculate** numerically the Omnes matrix at $s = m_D^2$



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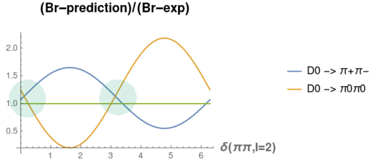
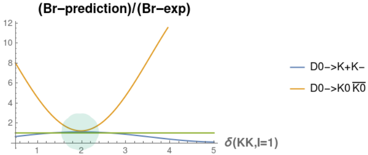
- We examine the branching fraction predictions for the decays $\pi^+\pi^-$, $\pi^0\pi^0$, K^+K^-, $K^0\overline{K^0}$ based on each Omnes matrix separately
- Only a few of them give simultaneously correct *Br* values for all channels:





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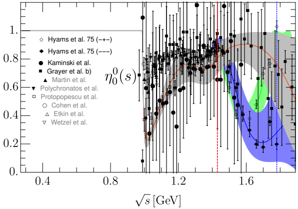

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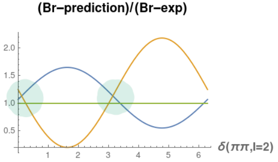
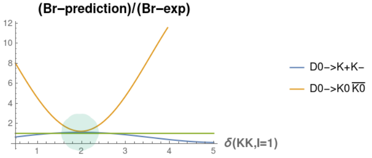
$\eta_0^0 - \delta\eta_0^0,\ m_\eta^* = 1$	$\Omega^{(0)} = \begin{pmatrix} 0.56\,e^{+1.84\,i} & 0.61\,e^{-1.73\,i} \\ 0.57\,e^{-1.41\,i} & 0.58\,e^{+2.25\,i} \end{pmatrix}$	$\Omega^{(0)} = \begin{pmatrix} 0.42\,e^{+1.75\,i} & 0.54\,e^{-2.05\,i} \\ 0.51\,e^{-1.33\,i} & 0.55\,e^{+2.43\,i} \end{pmatrix}$	$\Omega^{(0)} = \begin{pmatrix} 0.35\,e^{+1.13\,i} & 0.74\,e^{-2.47\,i} \\ 0.50\,e^{-1.18\,i} & 0.55\,e^{+2.48\,i} \end{pmatrix}$
$\eta_0^0 - \delta\eta_0^0,\ m_\eta^* = 2$	$\Omega^{(0)} = \begin{pmatrix} 0.58\,e^{+1.80\,i} & 0.64\,e^{-1.74\,i} \\ 0.58\,e^{-1.37\,i} & 0.61\,e^{+2.26\,i} \end{pmatrix}$	$\Omega^{(0)} = \begin{pmatrix} 0.43\,e^{+1.64\,i} & 0.58\,e^{-2.10\,i} \\ 0.52\,e^{-1.25\,i} & 0.57\,e^{+2.48\,i} \end{pmatrix}$	$\Omega^{(0)} = \begin{pmatrix} 0.40\,e^{+1.01\,i} & 0.80\,e^{-2.50\,i} \\ 0.50\,e^{-1.11\,i} & 0.56\,e^{+2.53\,i} \end{pmatrix}$
$\eta_0^0 - \delta\eta_0^0,\ m_\eta^* = 3$	$\Omega^{(0)} = \begin{pmatrix} 0.60\,e^{+1.76\,i} & 0.66\,e^{-1.74\,i} \\ 0.60\,e^{-1.33\,i} & 0.63\,e^{+2.26\,i} \end{pmatrix}$	$\Omega^{(0)} = \begin{pmatrix} 0.44\,e^{+1.53\,i} & 0.61\,e^{-2.16\,i} \\ 0.52\,e^{-1.17\,i} & 0.59\,e^{+2.53\,i} \end{pmatrix}$	$\Omega^{(0)} = \begin{pmatrix} 0.45\,e^{+0.91\,i} & 0.86\,e^{-2.53\,i} \\ 0.50\,e^{-1.04\,i} & 0.57\,e^{+2.58\,i} \end{pmatrix}$
sol. B': $\lvert g_0^0 \rvert$	$\Omega^{(0)} = \begin{pmatrix} 2.01\,e^{+1.39\,i} & 2.47\,e^{-1.76\,i} \\ 0.37\,e^{-0.33\,i} & 0.54\,e^{+3.05\,i} \end{pmatrix}$	$\Omega^{(0)} = \begin{pmatrix} 1.91\,e^{+0.60\,i} & 2.78\,e^{-2.55\,i} \\ 0.31\,e^{-0.23\,i} & 0.45\,e^{+3.30\,i} \end{pmatrix}$	$\Omega^{(0)} = \begin{pmatrix} 2.20\,e^{+0.43\,i} & 3.55\,e^{-2.72\,i} \\ 0.35\,e^{+0.03\,i} & 0.57\,e^{+3.40\,i} \end{pmatrix}$
sol. C': $\lvert g_0^0 \rvert$	$\Omega^{(0)} = \begin{pmatrix} 1.83\,e^{+1.38\,i} & 2.65\,e^{-1.76\,i} \\ 0.34\,e^{-0.40\,i} & 0.57\,e^{+3.00\,i} \end{pmatrix}$	$\Omega^{(0)} = \begin{pmatrix} 1.80\,e^{+0.59\,i} & 3.11\,e^{-2.56\,i} \\ 0.29\,e^{-0.24\,i} & 0.49\,e^{+3.24\,i} \end{pmatrix}$	$\Omega^{(0)} = \begin{pmatrix} 2.09\,e^{+0.43\,i} & 3.94\,e^{-2.72\,i} \\ 0.32\,e^{-0.03\,i} & 0.61\,e^{+3.34\,i} \end{pmatrix}$

- We examine the branching fraction predictions for the decays $\pi^+\pi^-$, $\pi^0\pi^0$, K^+K^-, $K^0\overline{K^0}$ based on each Omnes matrix separately
- Only a few of them give simultaneously correct Br values for all channels:





Results

Rescattering quantified

With the branching fractions correctly reproduced
(old $D \to \pi\pi$, KK puzzle seems to be solved!)
the Omnes matrix looks like:

$$\Omega_{I=0} = \begin{pmatrix} 0.58e^{1.8i} & 0.64e^{-1.7i} \\ 0.58e^{-1.4i} & 0.61e^{2.3i} \end{pmatrix}$$

The **physical solution** is

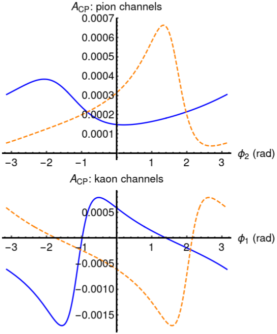
$$\begin{pmatrix} \mathbf{A}(D \to \pi\pi) \\ \mathbf{A}(D \to KK) \end{pmatrix} = \Omega_{I=0} \cdot \begin{pmatrix} \mathbf{A}_{\text{factorisation}}(D \to \pi\pi) \\ \mathbf{A}_{\text{factorisation}}(D \to KK) \end{pmatrix}$$

It turns out:

Significant rescattering between the two final states!

penguin insertions $\approx$ tree insertions
(of curr.-curr. operators, for I=0 reduced matrix elements)

CP asymmetries



charged meson channels
neutral meson channels

$$\Delta A_{CP}^{dir,exp} = (-1.57 \pm 0.29) \cdot 10^{-3}$$

With $\delta(I=2,\pi\pi)$, $\delta(I=1,KK)$ around the chosen values, we predict:

$$\Delta A_{CP}^{dir,theo} \sim 5 \cdot (10^{-4})!!$$

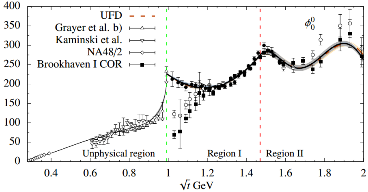
and $a_{CP}^{dir}(D^0 \to \pi^+\pi^-) \approx 3 \cdot 10^{-4}$,
$a_{CP}^{dir}(D^0 \to K^+K^-) \approx -2 \cdot 10^{-4}$

$$a_{CP}^{dir} \approx 2 \underbrace{r_{CKM}}_{\sim 6\cdot 10^{-4}} \underbrace{\frac{|B(f)|}{|A(f)|}}_{\sim 1/3} \cdot \underbrace{\sin \arg \frac{A(f)}{B(f)}}_{\sim 1}$$

NB: Short-distance penguins also not negligible for the CP asymmetries: C_6 small but annihilation insertion very large so that $C_6 \langle Q_6 \rangle_{fac} \sim C_1 \langle Q_1 \rangle_{fac}$

With fewer uncertain strong parameters (preliminary)

- $\pi\pi$, KK inelasticity has large uncertainties
- Use only one low-energy strong phase for isospin 0: $\pi\pi + KK$ phase



- Assumption: 2-channel unitarity → CPT/unitarity theorem also applying
- We manage to constrain:

$$
\begin{aligned}
0 &< a_{CP}(\pi\pi)(0-0) \lesssim 5 \times 10^{-4} \\
-3 \times 10^{-4} &\lesssim a_{CP}(KK)(0-0) < 0
\end{aligned}
$$

- The CP asymmetry from $I = 2/0$ interference is not constrained, but would require very large values of isospin-0 Omnes matrix elements

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- CPV in $D^0 \rightarrow \pi^0\pi^0$ should be of similar magnitude *(could experiments look there?)*
- Future directions: diferent isospin-2 scenarios, more channels in isospin-0?
- But these are naively not expected to change the picture...

Thank you very much!

BACKUP

Isospin-2 and -1 fixing

$$\mathscr{A}(D^+ \to \pi^+\pi^0) = \frac{3}{2\sqrt{2}} A^{\pi}_{I2}$$

$$\mathscr{A}(D^+ \to K^+\overline{K^0}) = A^{K}_{I1}$$

We fix $|A^{\pi}_{I2}|$, $|A^{K}_{I1}|$ from the Br's and use them in e.g.

$$\mathscr{A}(D^0 \to \pi^+\pi^-) = -\frac{1}{2\sqrt{3}} A^{\pi}_{I2} + \frac{1}{\sqrt{6}} A^{\pi}_{I0}$$

If I=2 elastic then $A^{\pi}_{I2} = \Omega_{I=2} A_{fac,I=2}$
If inelastic $A^{\pi}_{I2} = \Omega_{I=2} A_{fac,I=2} +$ (mixing) but we use directly
$A^{\pi}_{I2} = |A^{\pi}_{I2}| exp\{i\delta^{\pi\pi}_{I=2}\}$, phase left free

Naive estimate of final state interaction effects

$$\begin{pmatrix} A^{I=0}_{\pi\pi} \\ A^{I=0}_{KK} \end{pmatrix} = S_S^{1/2} \cdot \begin{pmatrix} A^{I=0}_{\pi\pi,\text{bare}} \\ A^{I=0}_{KK,\text{bare}} \end{pmatrix}$$

bare amplitudes: from factorisation (no strong phases)
Reproduces correctly Watson's theorem
What unitarity gives:

$$\begin{pmatrix} A^{I=0}_{\pi\pi} \\ A^{I=0}_{KK} \end{pmatrix} = S_S \cdot \begin{pmatrix} (A^{I=0}_{\pi\pi})^* \\ (A^{I=0}_{KK})^* \end{pmatrix}$$

No direct solution for the amplitudes, just relates them to the phases:

$$argA^{I=0}_{\pi\pi} = \delta_1 + arccos\sqrt{\frac{(1+\eta)^2 - \left(\frac{|A^{I=0}_{KK}|}{|A^{I=0}_{\pi\pi}|}\right)^2 (1-\eta^2)}{4\eta}}$$

$$argA^{I=0}_{KK} = \delta_2 + arccos\sqrt{\frac{(1+\eta)^2 - \left(\frac{|A^{I=0}_{\pi\pi}|}{|A^{I=0}_{KK}|}\right)^2 (1-\eta^2)}{4\eta}}$$

Numerical solution of 2-channel case

$$\begin{pmatrix} ReA^{\pi}(s) \\ ReA^{K}(s) \end{pmatrix} = \frac{s-s_0}{\pi} PV \int_{s_{thr}}^{\infty} ds' \frac{(ReT)^{-1}(ImT)(s')}{(s'-s)(s'-s_0)} \begin{pmatrix} ReA^{\pi}(s') \\ ReA^{K}(s') \end{pmatrix} + \begin{pmatrix} ReA_0^{\pi}(s_0) \\ ReA_0^{K}(s_0) \end{pmatrix}$$

- We discretise following the method from [Moussallam et al. hep-ph/9909292] into

$$\begin{pmatrix} ReA^{\pi}(s_i) \\ ReA^{K}(s_i) \end{pmatrix} = \frac{s_i-s_0}{\pi} \sum_j \hat{w}_j \frac{(ReT)^{-1}(ImT)(s_j)}{(s_j-s_i)(s_j-s_0)} \begin{pmatrix} ReA^{\pi}(s_j) \\ ReA^{K}(s_j) \end{pmatrix} + \begin{pmatrix} ReA_0^{\pi}(s_0) \\ ReA_0^{K}(s_0) \end{pmatrix}$$

- This creates an **invertible** matrix which gives *a* (discrete) solution
- Subtleties taken care of as in [Moussallam et al. hep-ph/9909292]
- To pick the *fundamental* solutions, we fix the vector at an unphysical point $s < 0$ and
  - check they behave as $\frac{1}{s}$ for large s
  - make sure the numerical determinant behaves as the (known) analytical determinant

Isospin decomposition

- $\pi\pi$ states can have isospin=0,2. KK can have isospin=0,1.

$$\begin{pmatrix} A(\pi^+\pi^-) \\ A(\pi^0\pi^0) \\ A(K^+K^-) \\ A(K^0\overline{K}^0) \end{pmatrix} = \begin{pmatrix} -\frac{1}{2\sqrt{3}} & -\frac{1}{\sqrt{6}} & 0 & 0 \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{6}} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & -\frac{1}{2} & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} A_\pi^2 \\ A_\pi^0 \\ A_K^1 \\ A_K^0 \end{pmatrix}$$

CPV in I=0

$$\begin{pmatrix} A^\pi \\ A^K \end{pmatrix} = \begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix} \begin{pmatrix} Re\lambda_d T^\pi + ... \\ Re\lambda_s T^K + ... \end{pmatrix}$$

$$\begin{pmatrix} B^\pi \\ B^K \end{pmatrix} = \begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix} \begin{pmatrix} Im\lambda_d T^\pi + \sum_i Im\lambda_{d_i} P_i^\pi \\ Im\lambda_s T^K + \sum_i Im\lambda_{d_i} P_i^K \end{pmatrix}$$

Can consider either $Im\lambda_d = 0$ or $Im\lambda_s = 0$, not both simultaneously

$\Rightarrow$ In a_{CP}^{dir} there always exists a term $\sim T^\pi T^K$, both for $\pi\pi$ and for KK

Large N_C limit & effective operators

- $Q_1(i) = (\overline{d_i}c)_{V-A}(\overline{u}d_i)_{V-A}$, $Q_2(i) = (\overline{d}d)_{V-A}(\overline{u}c)_{V-A}$,
  $Q_{5,3} = (\overline{u}c)_{V-A}\sum_q(\overline{q}q)_{V\pm A}$,
  $Q_4 = \sum_q(\overline{u}q)_{V-A}(\overline{q}c)_{V-A}$, $Q_6 = -2\sum_q(\overline{u}q)_{S+P}(\overline{q}c)_{S-P}$
- $C_1 = 1.18$, $C_2 = -0.32$, $C_3 = 0.011$, $C_4 = -0.031$, $C_5 = 0.0068$, $C_6 = -0.032$ ($\mu = 2$ GeV)
- $\lambda_d = V_{cd}^* V_{ud} \approx 0.22$
- $\overline{m_c}(2GeV) = 1.045GeV$
- Compare $m_D = 1865$ MeV to $\Lambda_{\chi PT} \approx m_\rho = 775$ MeV