University of Zurich[UZH]

Associated production of a W boson with a top-quark pair in NNLO QCD

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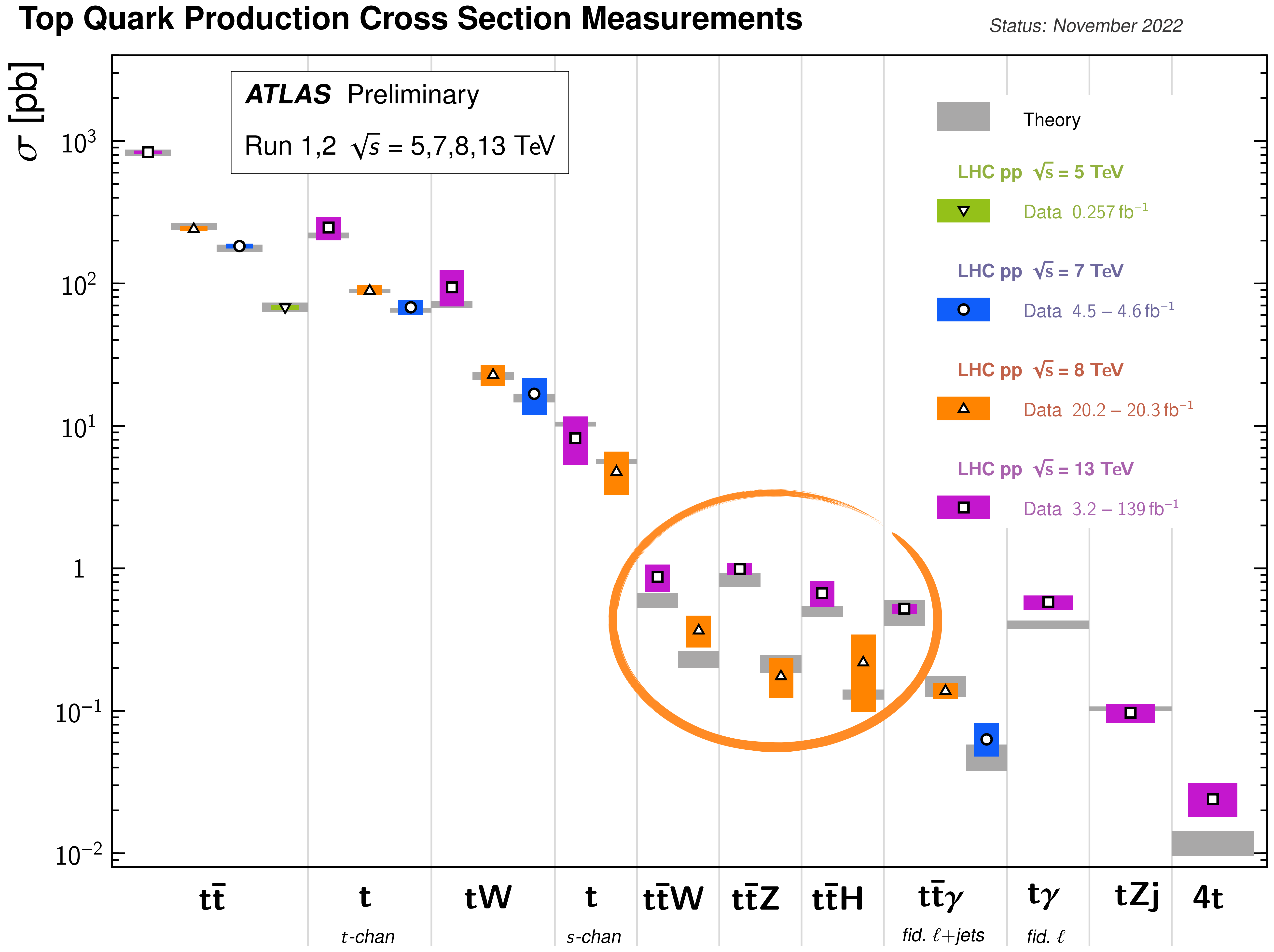
in collaboration with S. Devoto, M. Grazzini, S. Kallweit, J. Mazzitelli, L. Rottoli and C. Savoini
[arXiv:2306.16311]

SM@LHC 2023
FermiLab - 11th July 2023

Introduction

The production of a top-quark pair together with a vector or Higgs boson is among **the most massive SM signatures** at hadron colliders



Small cross sections, but already observed and **measured with** $10 - 20\,\%$ **uncertainties**

Crucial to characterise the top-quark interactions, in particular with the Higgs boson

Introduction

Among the other $t\bar{t}X$ processes, the $t\bar{t}W$ process is **rather peculiar**

- Complex final-state signature characterised by two b-jets and three W bosons: **irreducible SM source of same sign dilepton pairs**

→ Relevant for BSM searches in **multi-lepton signature**

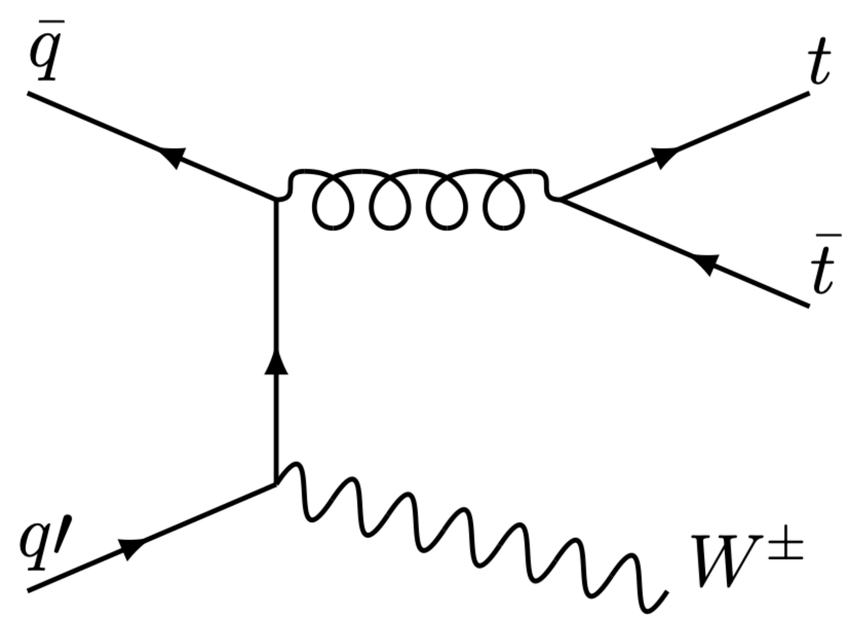
- It represents a **relevant background** also for SM processes like $t\bar{t}H$ and $t\bar{t}t\bar{t}$ production

see talk by Zhi Zheng

Introduction

Among the other $t\bar{t}X$ processes, the $t\bar{t}W$ process is **rather peculiar**

- The W boson can only be emitted off an initial-state light quark: **no gluon fusion channel at LO**



Different pattern of radiative corrections

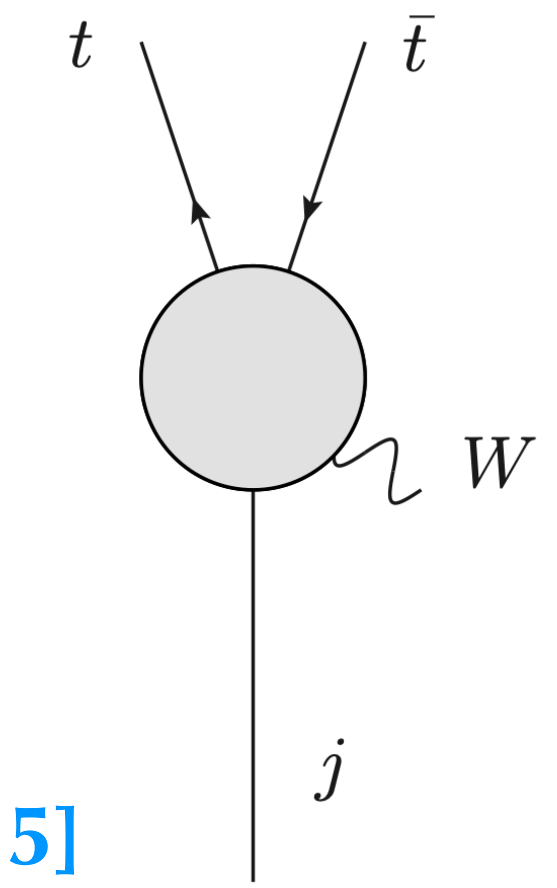
$\alpha_S^2\alpha$ LO QCD | ~~$\alpha_S\alpha^2$~~ | α^3

$\alpha_S^3\alpha$ NLO QCD | $\alpha_S^2\alpha^2$ | $\alpha_S\alpha^3$ | α^4

Large NLO QCD corrections: $\mathcal{O}(50-90\%)$
Giant K-factor in the region of high transverse momenta of the top-quark pair, which recoils against a hard jet while the W boson is relatively soft

quark-gluon channel opening

[Maltoni, Pagani, Tsinikos, 2015]



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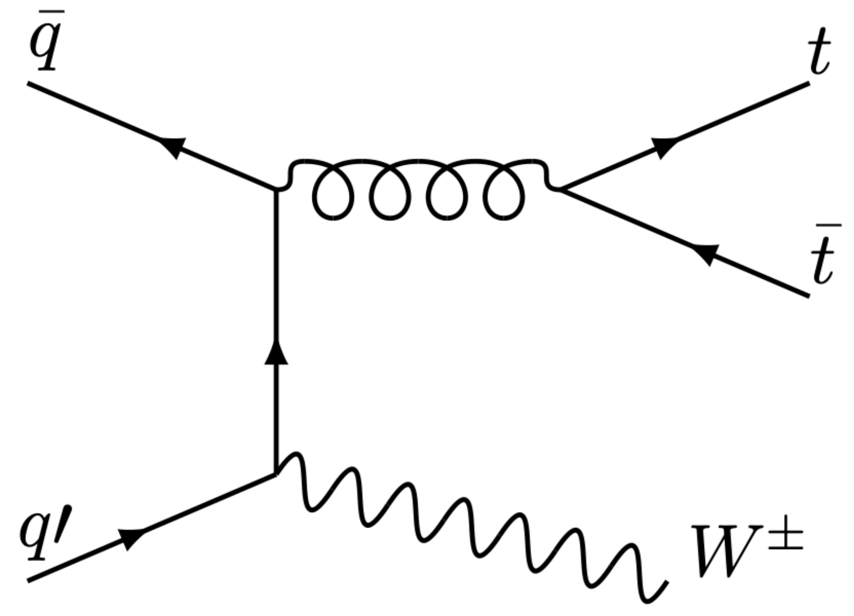


Different pattern of radiative corrections

$\alpha_S^2\alpha$ LO QCD

~~$\alpha_S\alpha^2$~~

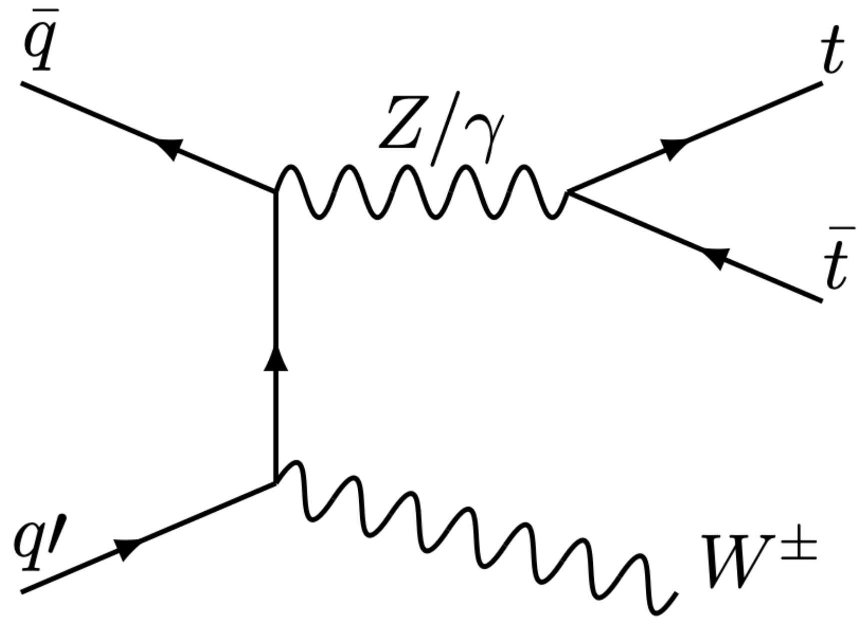
α^3

$\alpha_S^3\alpha$ NLO QCD

$\alpha_S^2\alpha^2$ NLO EW

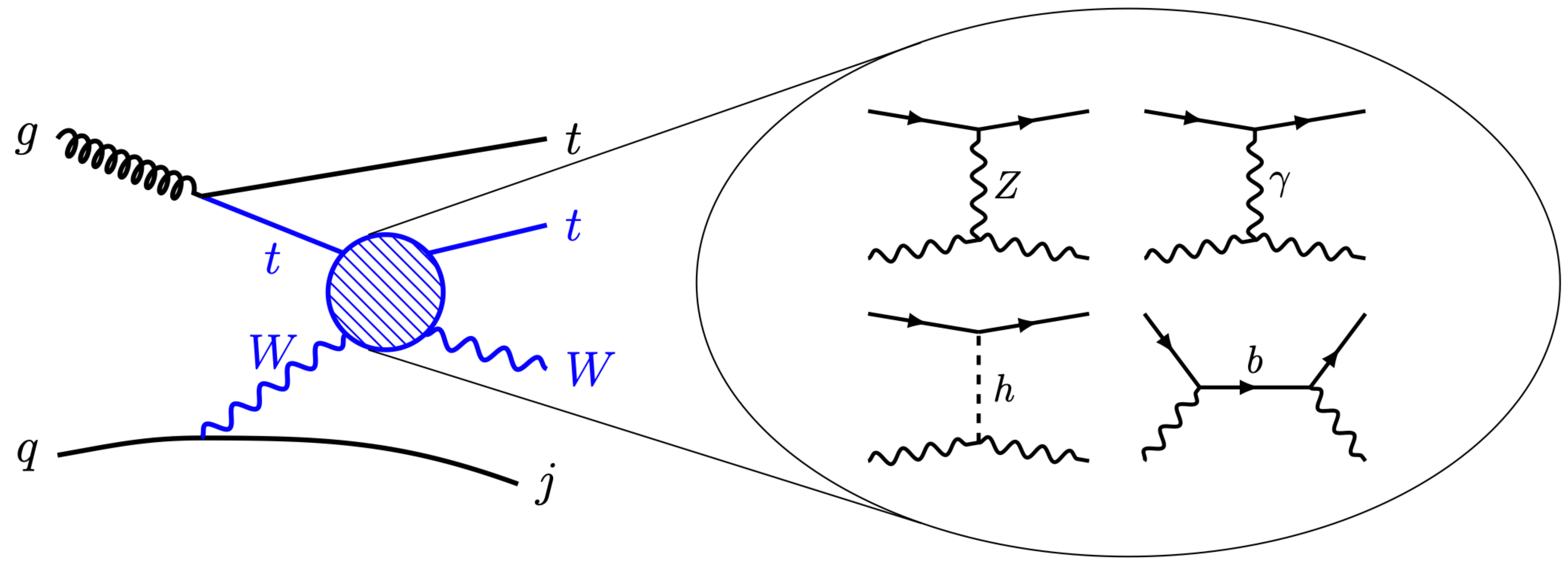
$\alpha_S\alpha^3$

α^4



Subleading EW

[Frederix, Pagani, Zaro, 2017]



Large **positive** subleading EW $\mathcal{O}(+10\%)$ (at the LHC) which partially cancels against the **negative** NLO EW $\mathcal{O}(-5\%)$
Dominated by configuration involving the $tW \to tW$ scattering process and enhanced by the gluon luminosity

State of the art: theory

NLO QCD corrections

[Badger, Campbell, Ellis, 2010] [Campbell, Ellis, 2012]

NLO QCD + EW corrections (on-shell top quarks and *W*)

[Frixione, Hirschi, Pagani, Shao, Zaro, 2015] [Frederix, Pagani, Zaro, 2017]

inclusion of soft gluon resummation at NNLL

[Li, Li, Li, 2014] [Broggio, Ferroglia, Ossola, Pecjak, 2016] [Kulesza, Motyka, Schwartlaender, Stebel, Theeuwes, 2019]

NLO QCD corrections (full off-shell process, three charged lepton signature)

[Bevilacqua, Bi, Hartanto, Kraus, Nasuti, Worek, 2020-2021] [Denner, Pellicciolі, 2020]

combined NLO QCD + EW corrections (full off-shell process, three charged lepton signature)

[Denner, Pellicciolі, 2020]

NLO QCD + EW (on-shell) predictions supplemented with multi-jet merging as la FxFx

[Frixione, Frederix, 2012] [Frederix, Tsinikos, 2021]

Current theory reference in comparison with data

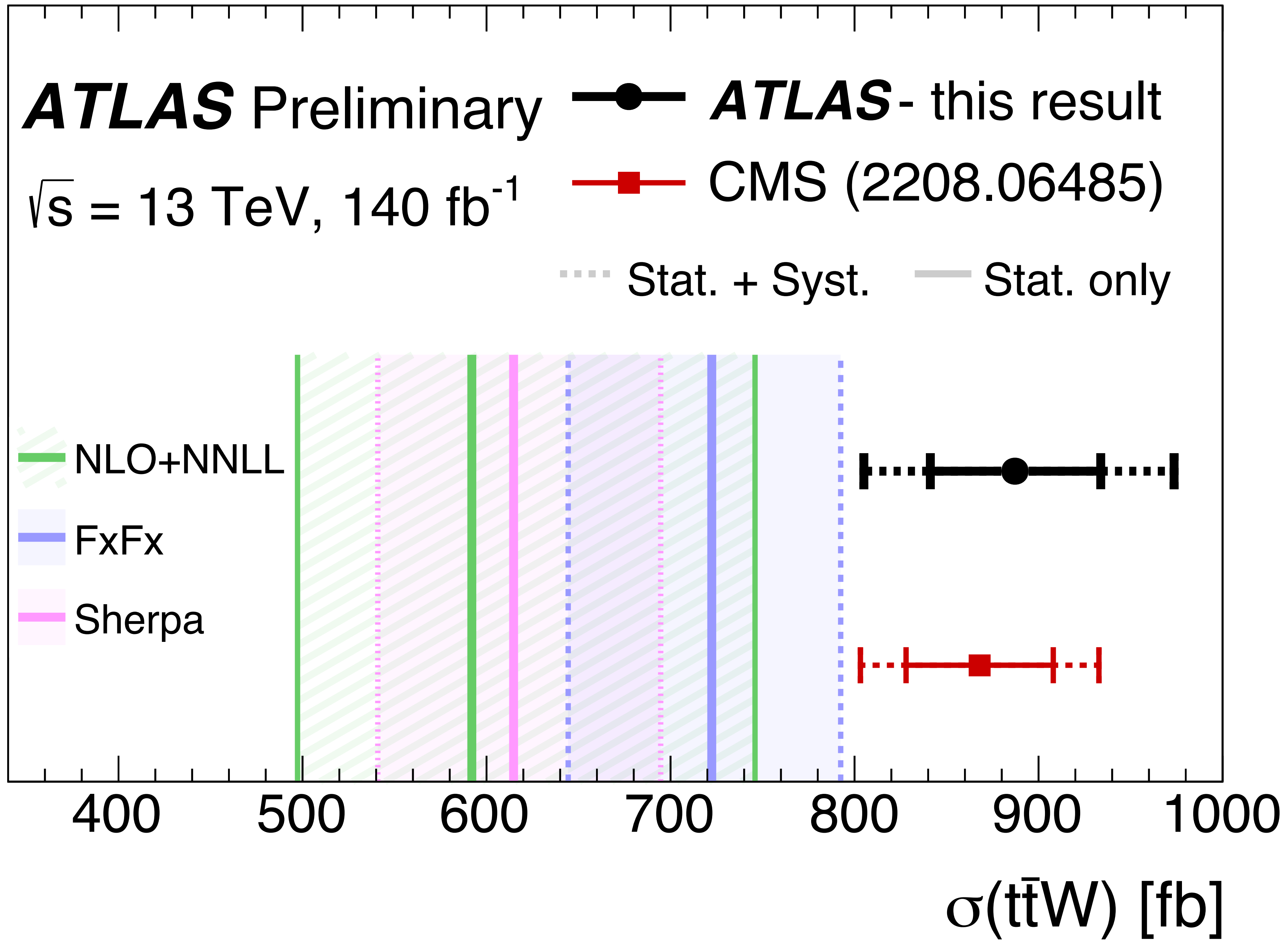
State of the art: data-theory comparison

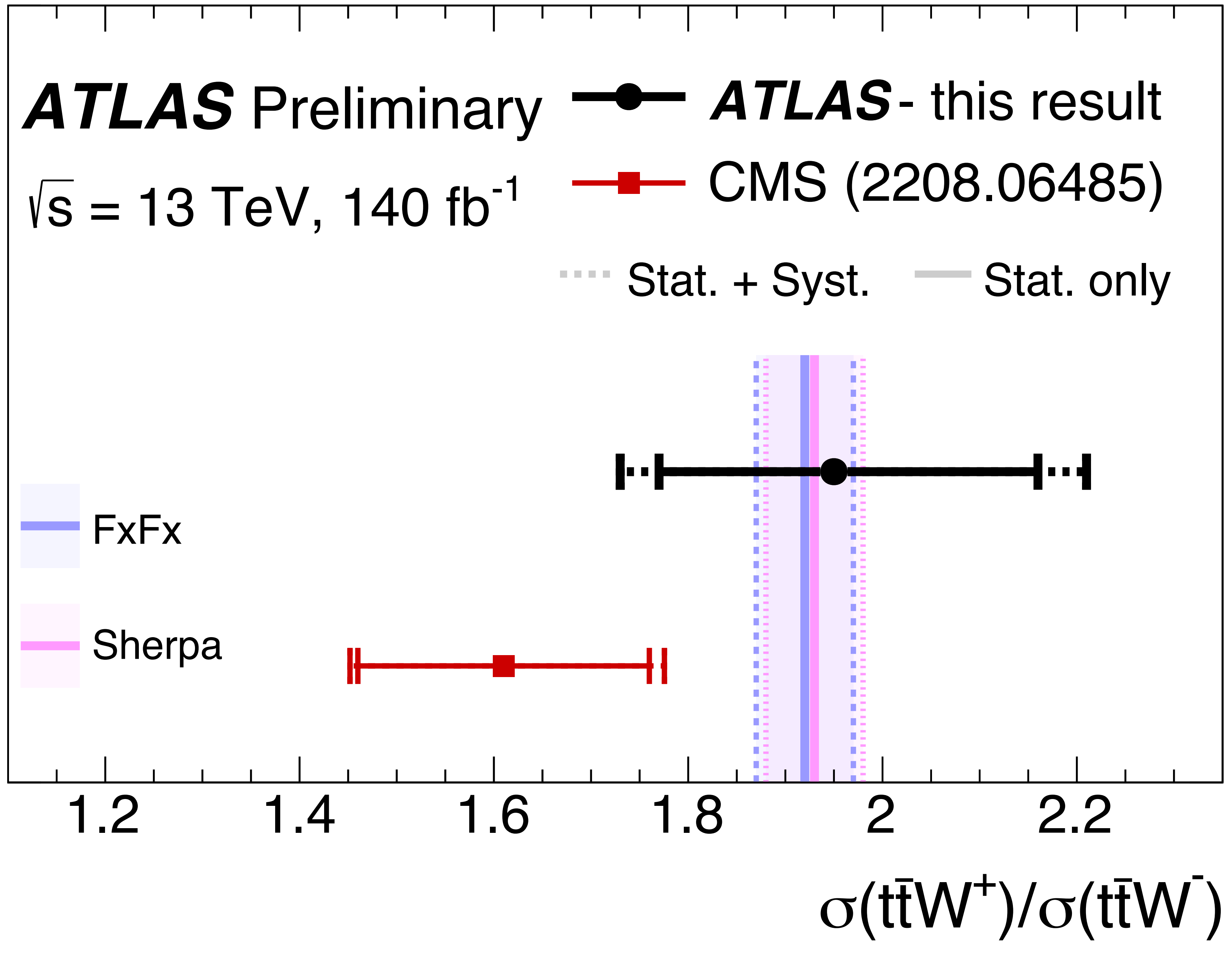
[CMS, 2208.06485, 2022]
[ATLAS-CONF-2023-019, 2023]

- FxFx multi-jet merging (including NLO QCD corrections to $t\bar{t}Wj$) and EW corrections increase the NLO QCD cross sections

- Nonetheless, measured $t\bar{t}W$ rates by ATLAS and CMS at $\sqrt{s} = 8$ TeV and $\sqrt{s} = 13$ TeV are consistently higher than the SM predictions. This tension is also confirmed by indirect measurements of $t\bar{t}W$ in the context of $t\bar{t}H$ and $t\bar{t}t\bar{t}$ analyses

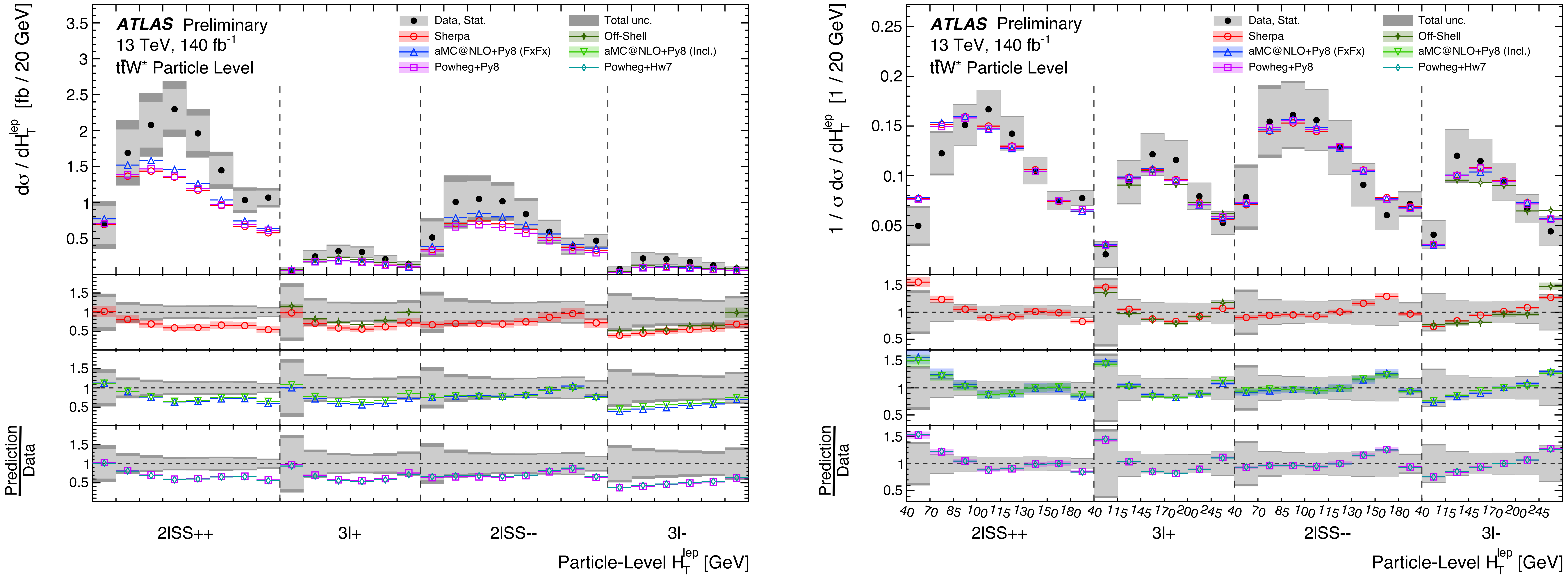
- **The most recent measurements confirm this picture with a slightly excess at the $1\sigma - 2\sigma$ level**





State of the art: data-theory comparison

- ATLAS measured also **differential distributions**, finding a disagreement in the overall normalisation consistent with the inclusive measurement result
- The latest off-shell fixed-order predictions give indications that this disagreement **is not predominantly due to missing singly-resonant contributions** which are not included in the reference on-shell predictions



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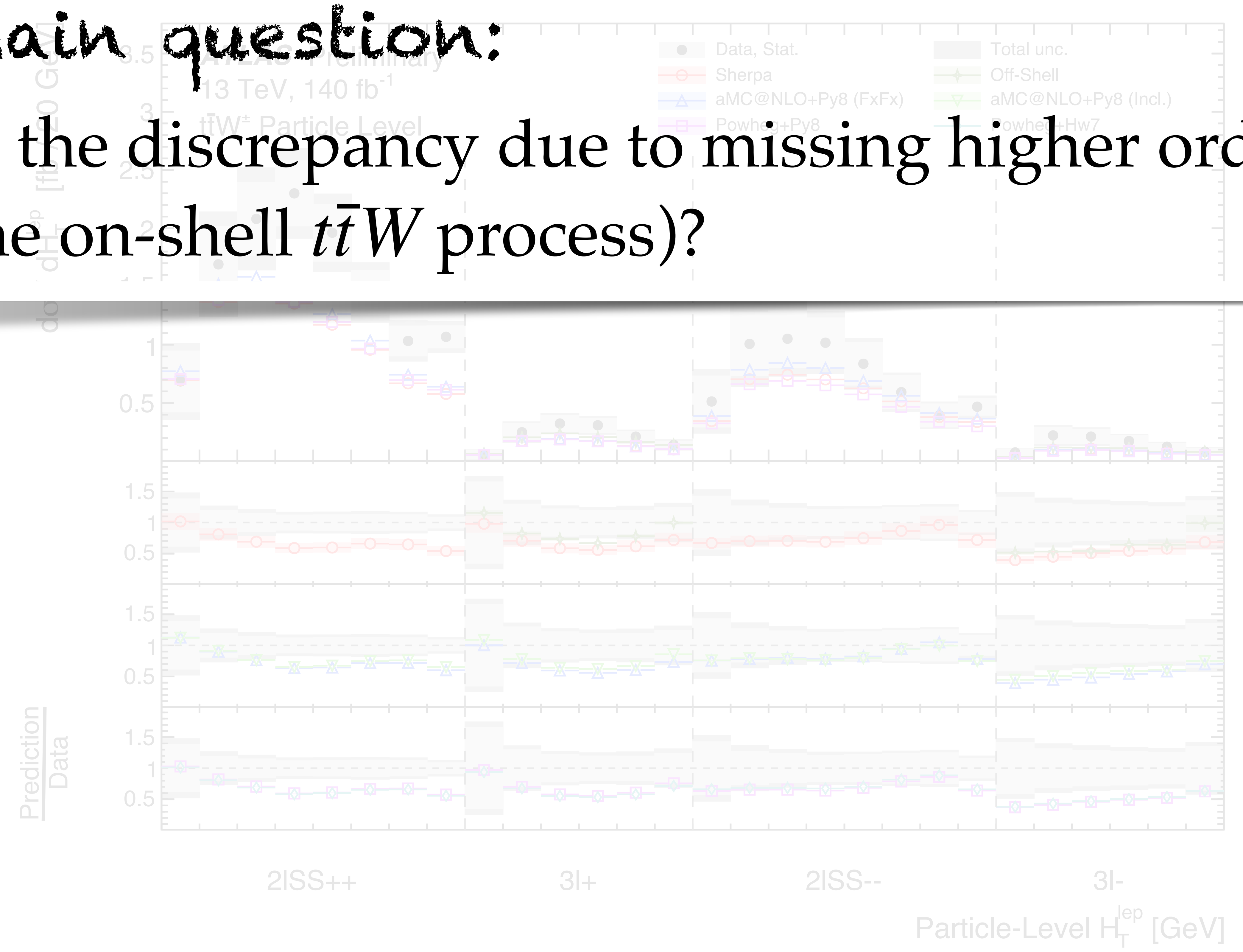
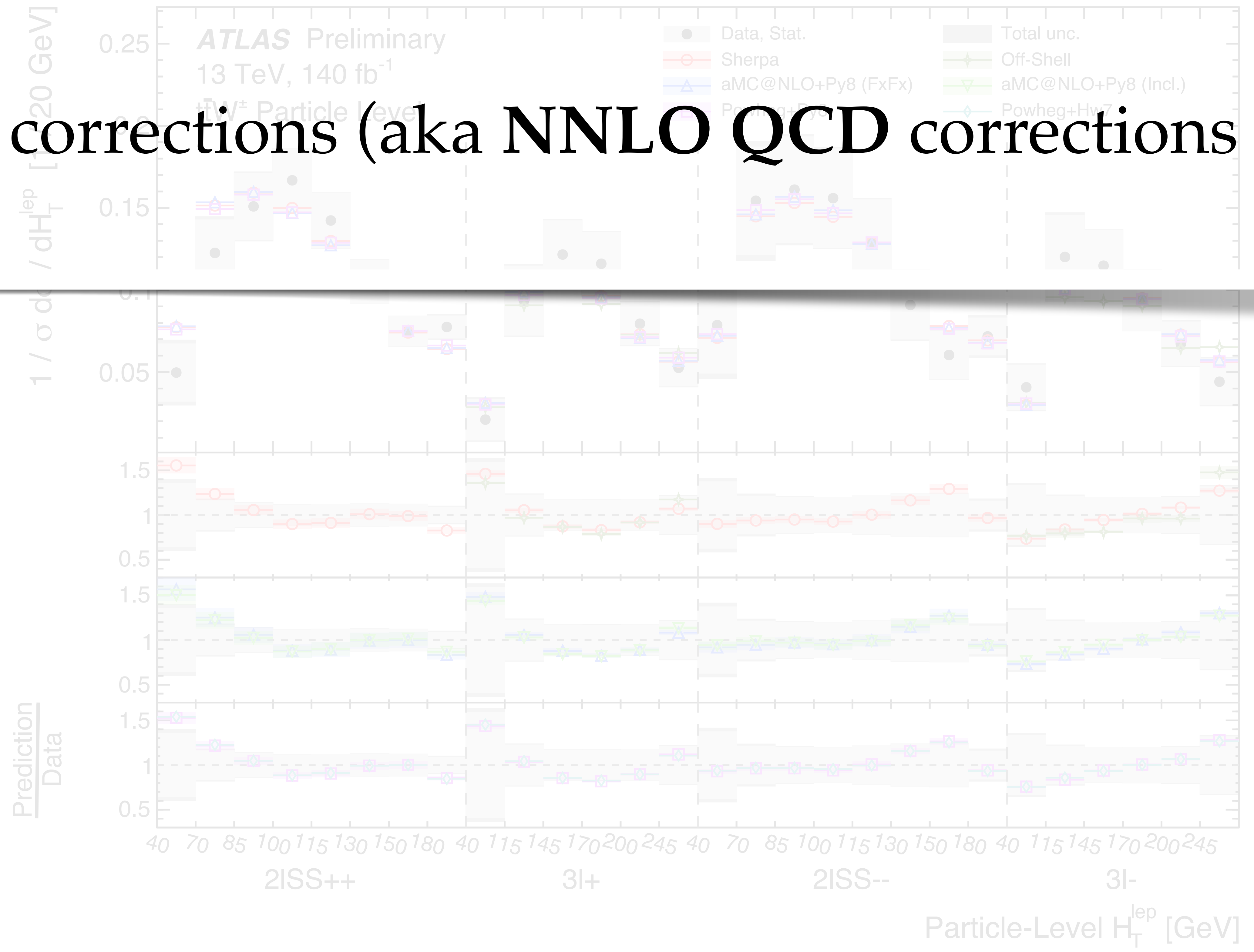
Main question:

Is the discrepancy due to missing higher order corrections (aka **NNLO QCD** corrections to the on-shell $t\bar{t}W$ process)?





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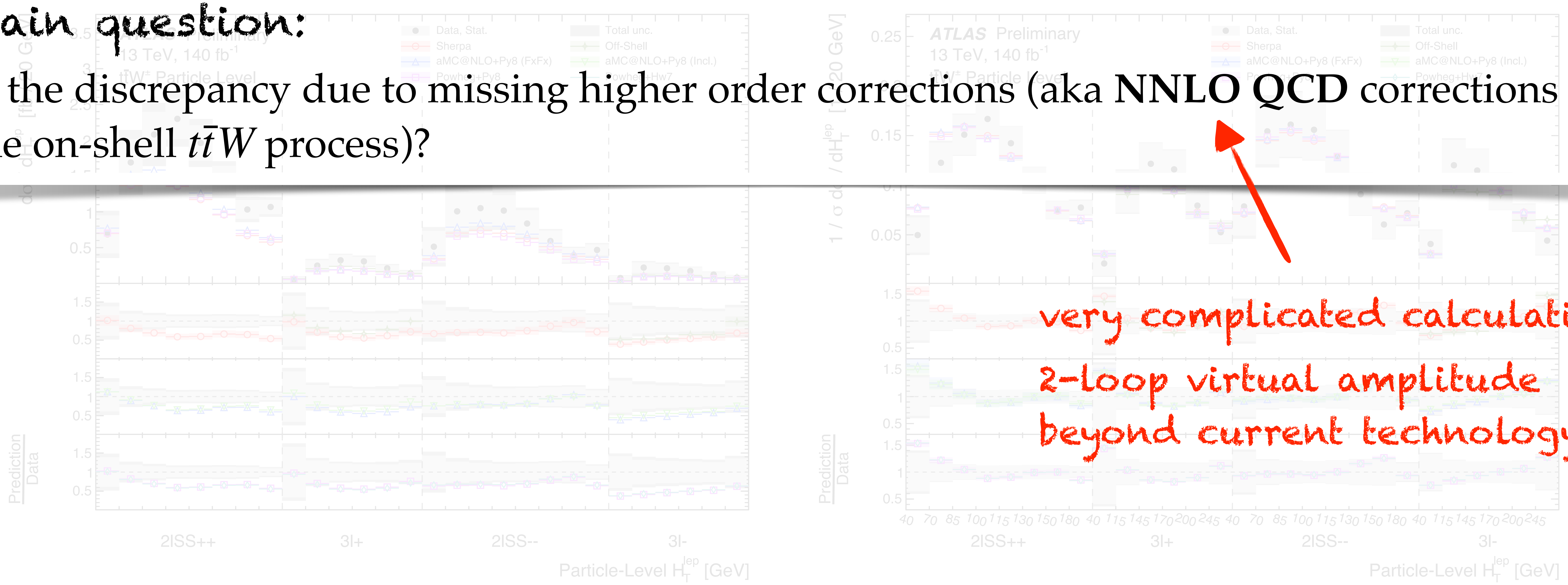

Main question:

Is the discrepancy due to missing higher order corrections (aka **NNLO QCD** corrections to the on-shell $t\bar{t}W$ process)?

very complicated calculation!

2-loop virtual amplitude beyond current technology

Outline

- Methodology: slicing formalism
- Methodology: two-loop virtual amplitude
- Phenomenological results
- Conclusions

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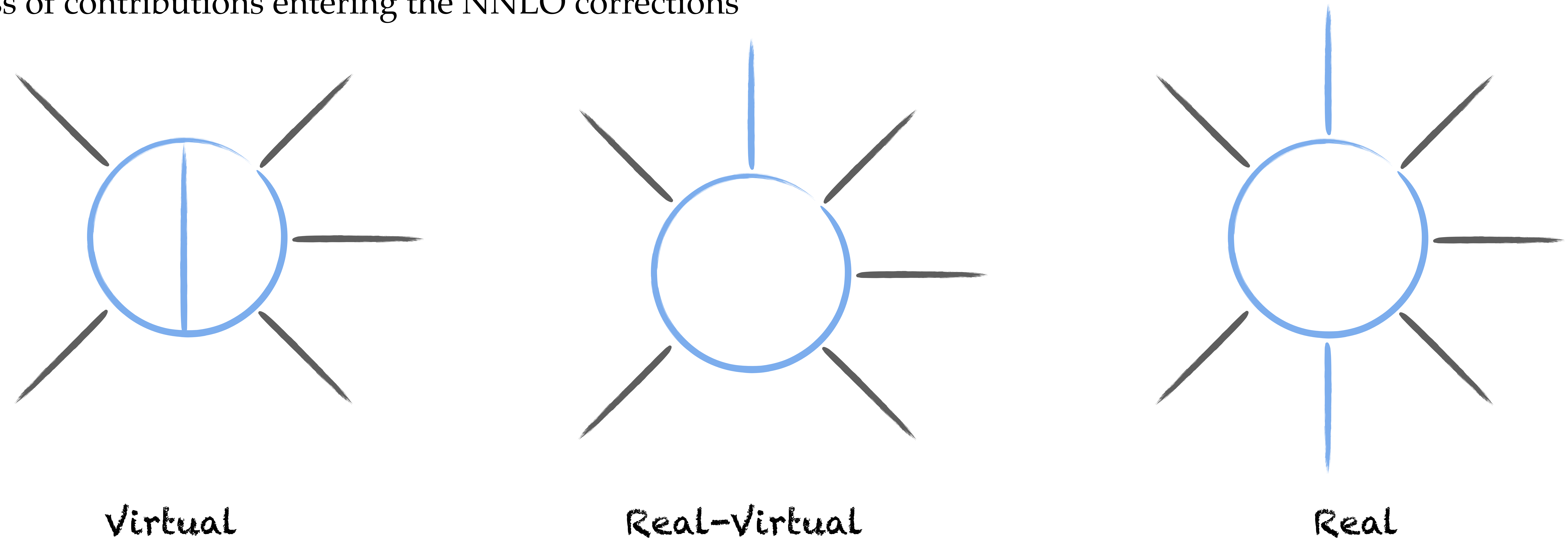
Infrared singularities

Class of contributions entering the NNLO corrections



KLN theorem and collinear factorisation ensure the cancellation of singularities for any infrared safe observables, but virtuals, real-virtual and reals live on different phase spaces and are separately divergent …

Subtraction/Slicing scheme required!

q_T-subtraction formalism

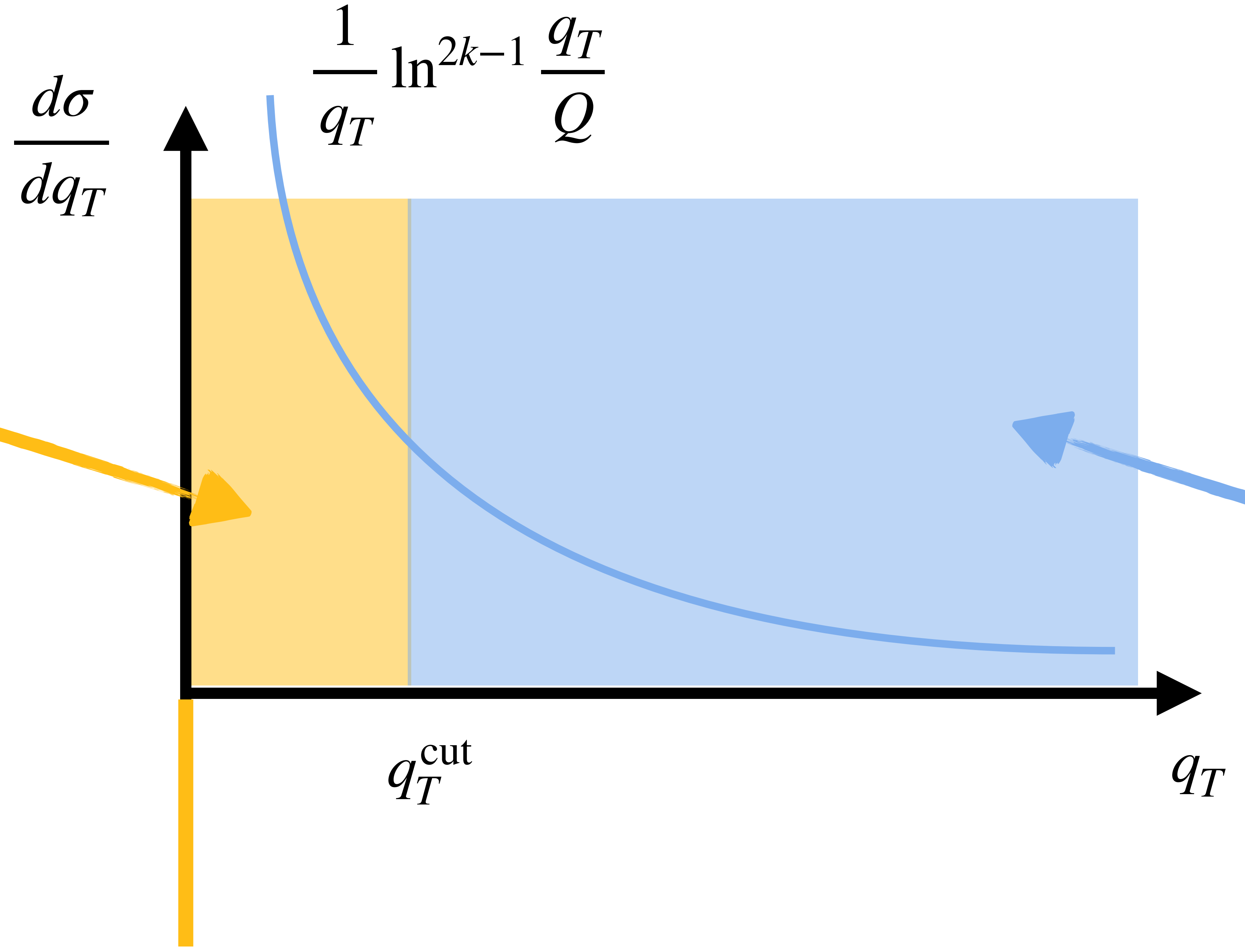
[Catani, Grazzini, 2007]

Cross section for the production of a triggered final state F at N^{k}LO

All emission unresolved; approximate the cross section with its singular part in the soft and/or collinear limits

q_T resummation

- expand to fixed order
- $\mathcal{O}(\alpha_s^k)$ ingredient required



1 emission always resolved

$F + j$ @ N^{k-1}LO

complexity of the calculation reduced by one order!

$$\int d\sigma_{N^kLO} = \mathcal{H} \otimes d\sigma_{LO} + \int \left[d\sigma^{R}_{N^{k-1}LO} - d\sigma^{CT}_{N^kLO} \right]_{q_T > q_T^{\rm cut}} + \mathcal{O}\left((q_T^{\rm cut})^{\ell} \right)$$

q_T-subtraction formalism: extension to massive final states

$$\int d\sigma_{N^kLO} = \mathscr{H} \otimes d\sigma_{LO} + \int \left[d\sigma^{R}_{N^{k-1}LO} - d\sigma^{CT}_{N^kLO} \right]_{q_T > q_T^{\rm cut}} + \mathcal{O}\left((q_T^{\rm cut})^{\ell} \right)$$

All ingredients for $t\bar{t}W + j$ @ NLO available:

Required matrix elements implemented in public libraries such as OpenLoops2

[Buccioni, Lang, Lindert, Maierhöfer, Pozzorini, Zhang, Zoller '19]

Local subtraction scheme available, for example dipole subtraction

[Catani, Seymour, '98] [Catani, Dittmaier, Seymour, Trocsanyi '02]

Automatised implementation in the **MATRIX framework**, which relies on the efficient multi-channel Monte Carlo integrator MUNICH

[Grazzini, Kallweit, Wiesemann '17] [Kallweit in preparation]

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$\mathscr{H}$ contains virtual correction after subtraction of IR singularities and contribution of soft/collinear origin

- Beam functions ✓
- Soft function

[Catani, Cieri, de Florian, Ferrera, Grazzini '12]
[Gehrmann, Luebbert, Yang '14]
[Echevarria, Scimemi, Vladimirov '16]
[Luo, Wang, Xu, Yang, Yang, Zhu '19]
[Ebert, Mistlberger, Vita]

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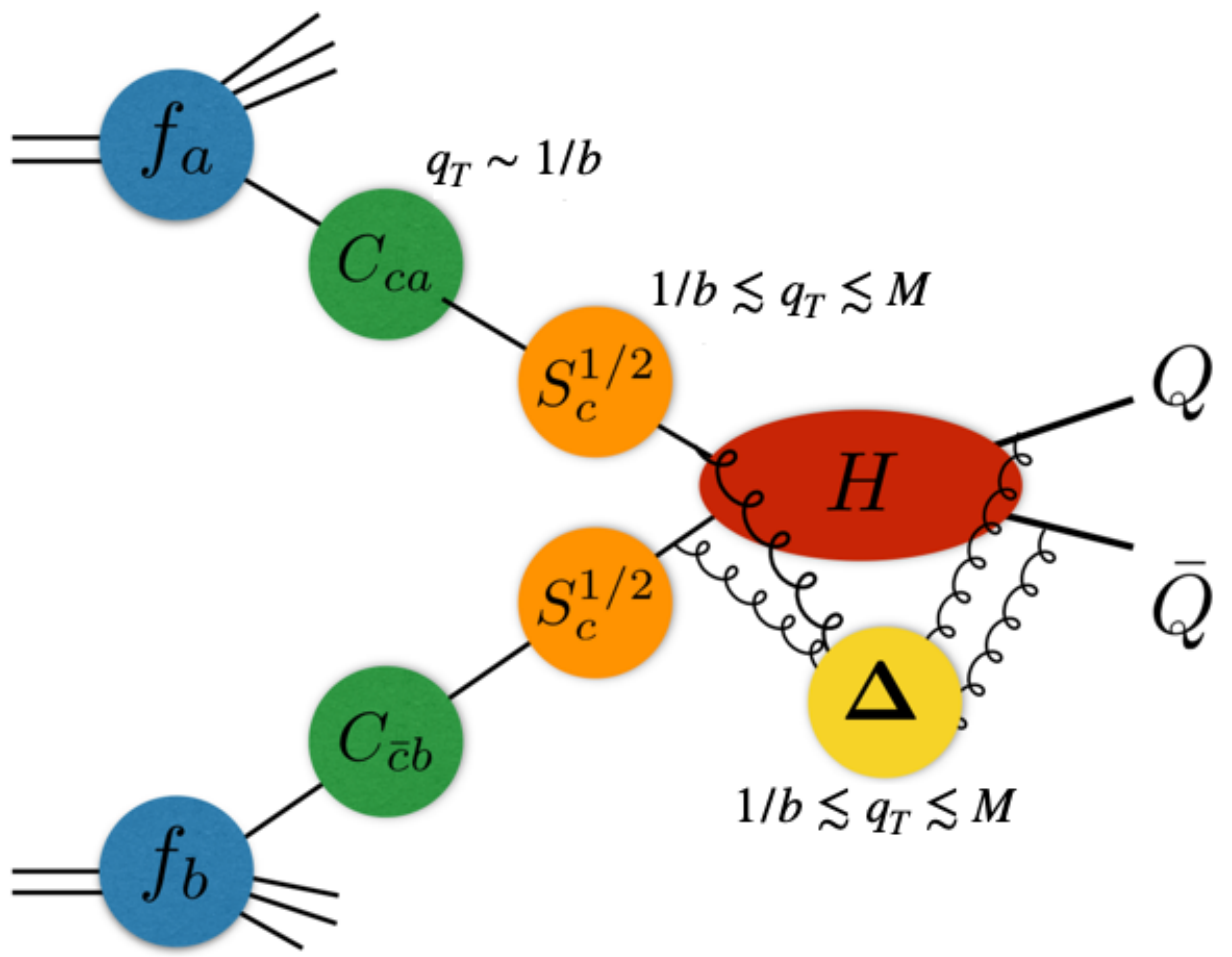

The resummation formula shows a **richer structure** because of additional soft singularities

- Soft logarithms controlled by the **transverse momentum anomalous dimension $\boldsymbol{\Gamma}_t$** known up to NNLO **[Mitov, Sterman, Sung, 2009], [Neubert, et al 2009]**
- Hard coefficient gets a **non-trivial** colour structure (matrix in colour-space)
- Non trivial azimuthal correlations

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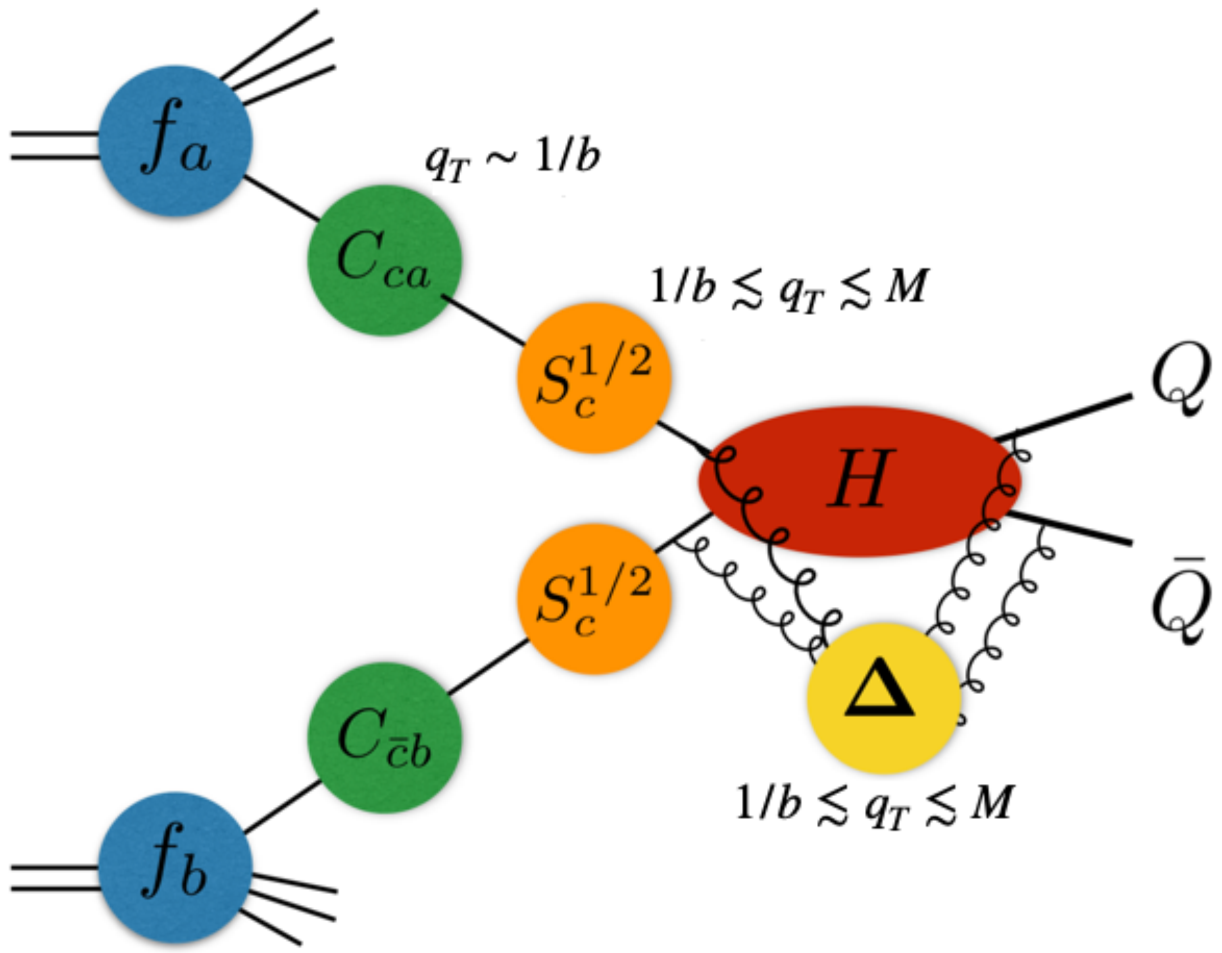

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q_T subtraction formalism extended to the case of **heavy quarks** production [Catani, Grazzini, Torre, 2014]

Successful employed for the computation of NNLO QCD corrections to the production of

- a **top pair** [Catani, Devoto, Grazzini, Kallweit, Mazzitelli, Sargsyan 2019]
- a **bottom pair** production [Catani, Devoto, Grazzini, Kallweit, Mazzitelli, 2021]

q_T-subtraction formalism: extension to massive final states

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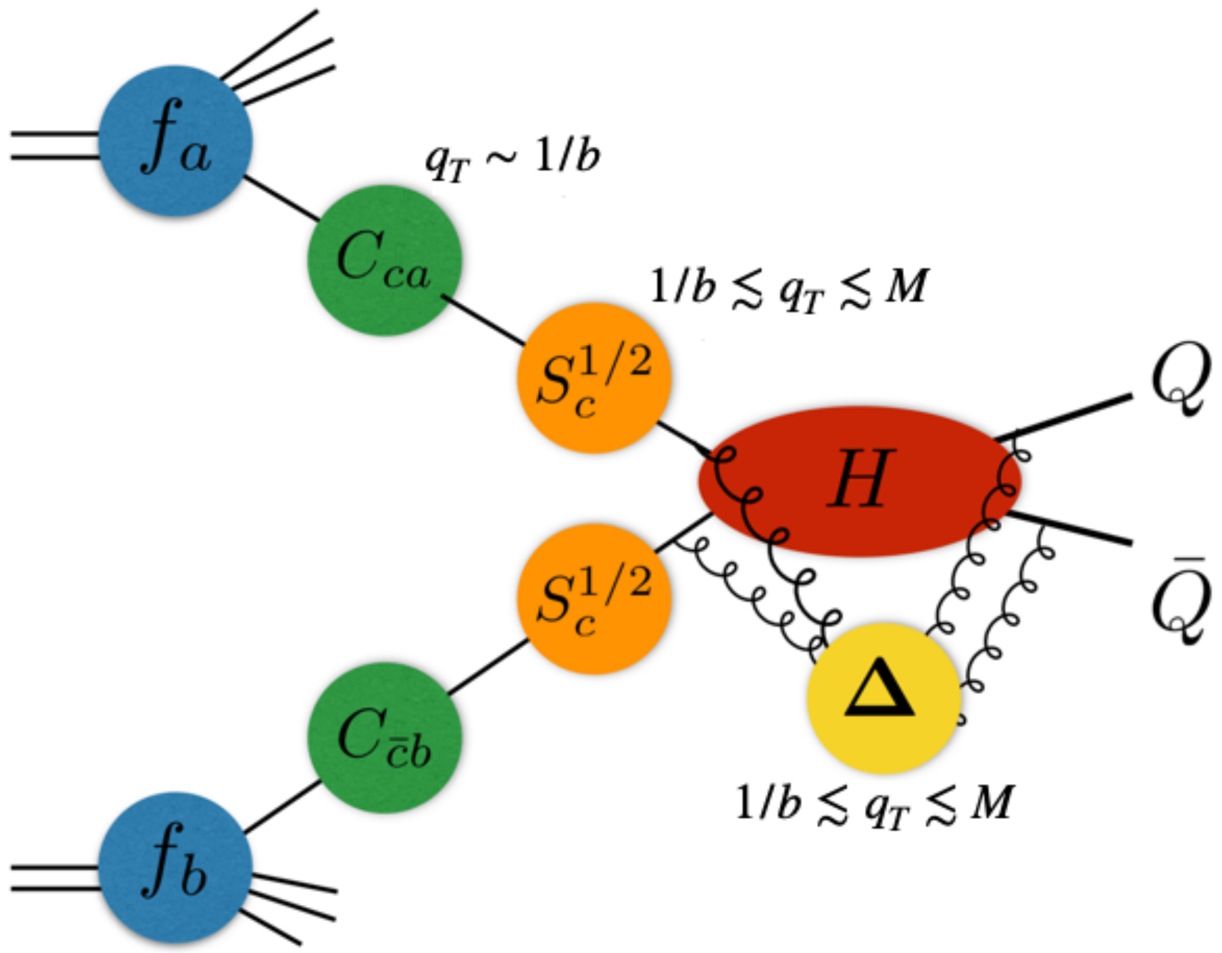

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Non trivial ingredient

- **Two-loop soft function** for heavy-quark (back-to-back Born kinematic) [Catani, Devoto, Grazzini, Mazzitelli,2023]
- Recently generalised to **arbitrary kinematics** [Devoto, Mazzitelli in preparation]

q_T-subtraction formalism: extension to massive final states

$$\int d\sigma_{N^kLO} = \mathscr{H} \otimes d\sigma_{LO} + \int \left[d\sigma^{R}_{N^{k-1}LO} - d\sigma^{CT}_{N^kLO} \right]_{q_T > q_T^{\text{cut}}} + \mathcal{O}\left((q_T^{\text{cut}})^{\ell}\right)$$

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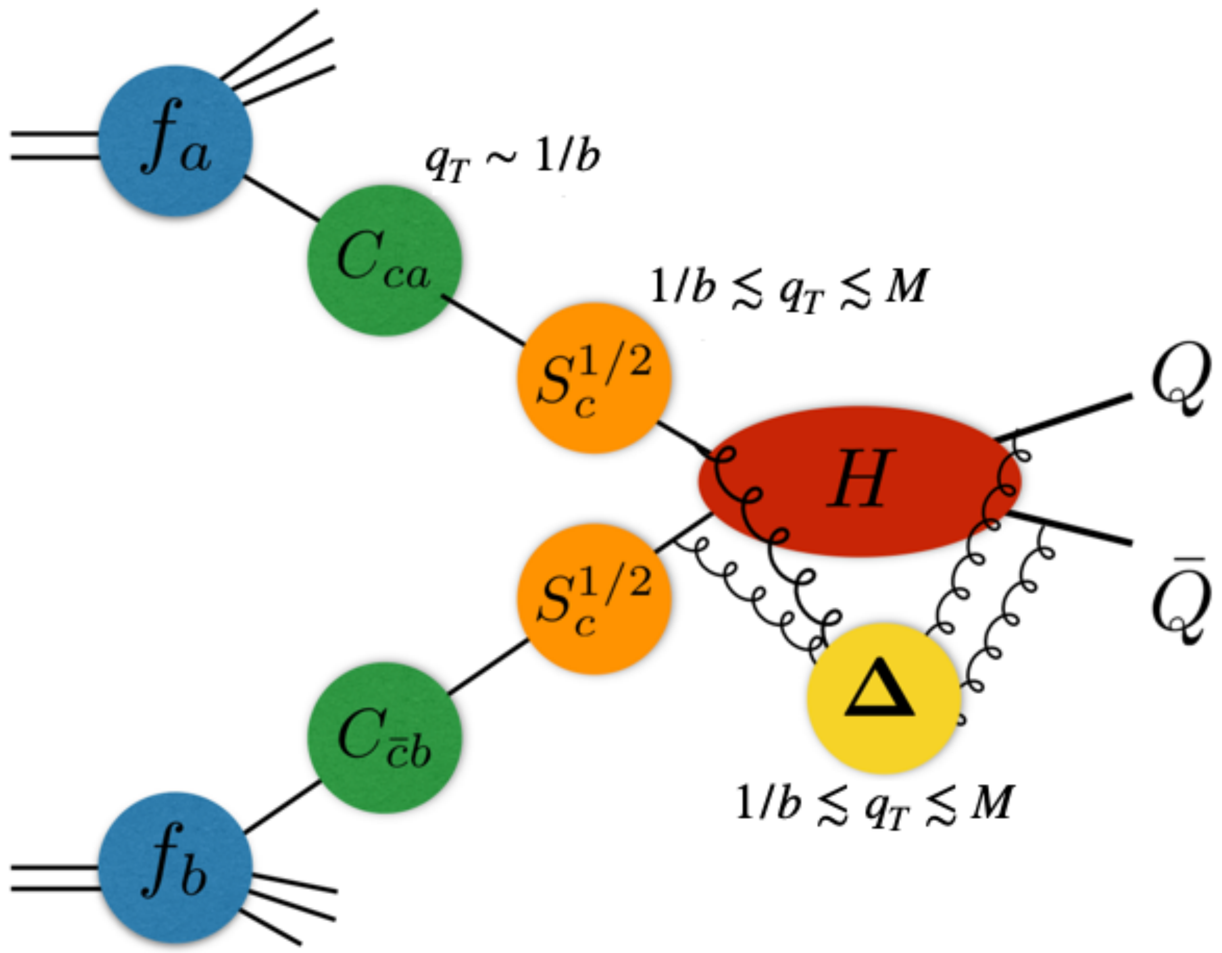

The resummation formula shows a **richer structure** because of additional soft singularities

Once the corresponding two-loop amplitude is available, the framework allows the calculation of the NNLO correction to the production of **a massive heavy-quark pair and a generic color singlet process**

- **First applications:** $t\bar{t}H$, $b\bar{b}W$
  [Catani, Devoto, Grazzini, Kallweit, Mazzitelli, Savoini, 2022]
  [Buonocore, Devoto, Kallweit, Mazzitelli, Rottoli, Savoini, 2022]

q_T-subtraction formalism: hard-virtual coefficient

All the ingredients are available and implemented in MATRIX **except for the two-loop virtual amplitude** entering $\mathcal{H}$

$$\mathcal{H} = H\delta(1-z_1)\delta(1-z_1) + \delta H(z_1, z_2)$$

in terms of the perturbatively computable **hard-virtual function**

$$H = 1 + \frac{\alpha_S(\mu_R)}{2\pi} H^{(1)} + \left(\frac{\alpha_S(\mu_R)}{2\pi}\right)^2 H^{(2)} + \ldots$$

$$H^{(n)} = \frac{2\Re < \mathcal{M}_{\text{fin}}^{(n)} | \mathcal{M}^{(0)} >}{|\mathcal{M}^{(0)}|^2}$$

$$|\mathcal{M}_{\text{fin}}(\mu_{\text{IR}}) > = Z^{-1}(\mu_{\text{IR}})|\mathcal{M} >$$

IR subtraction at subtraction scale μ_{IR}
[Ferroglia, Neubert, Pecjak, Yang, 2008]

At NNLO, the only missing ingredient is then contained in the $H^{(2)}$ contribution

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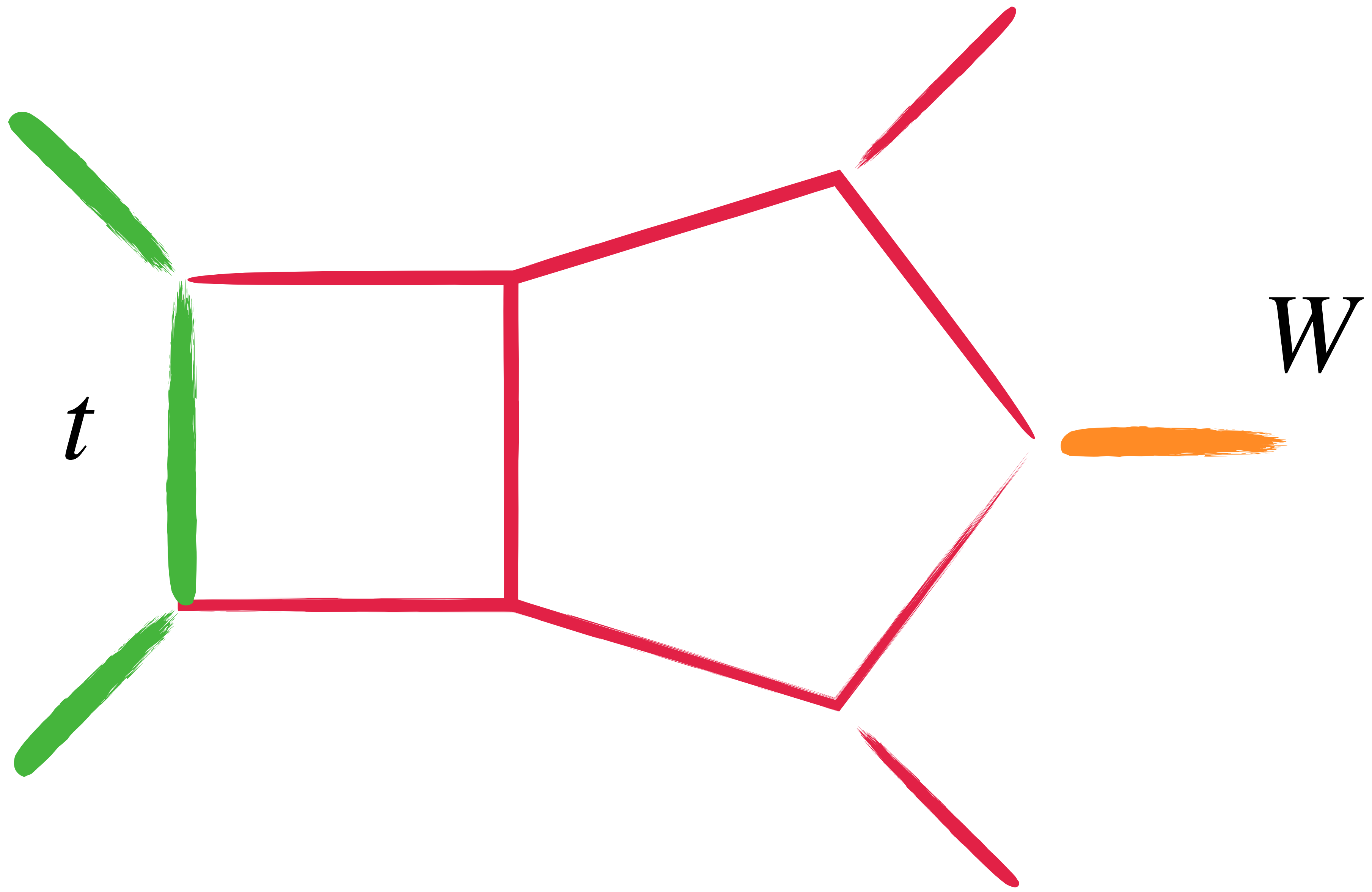
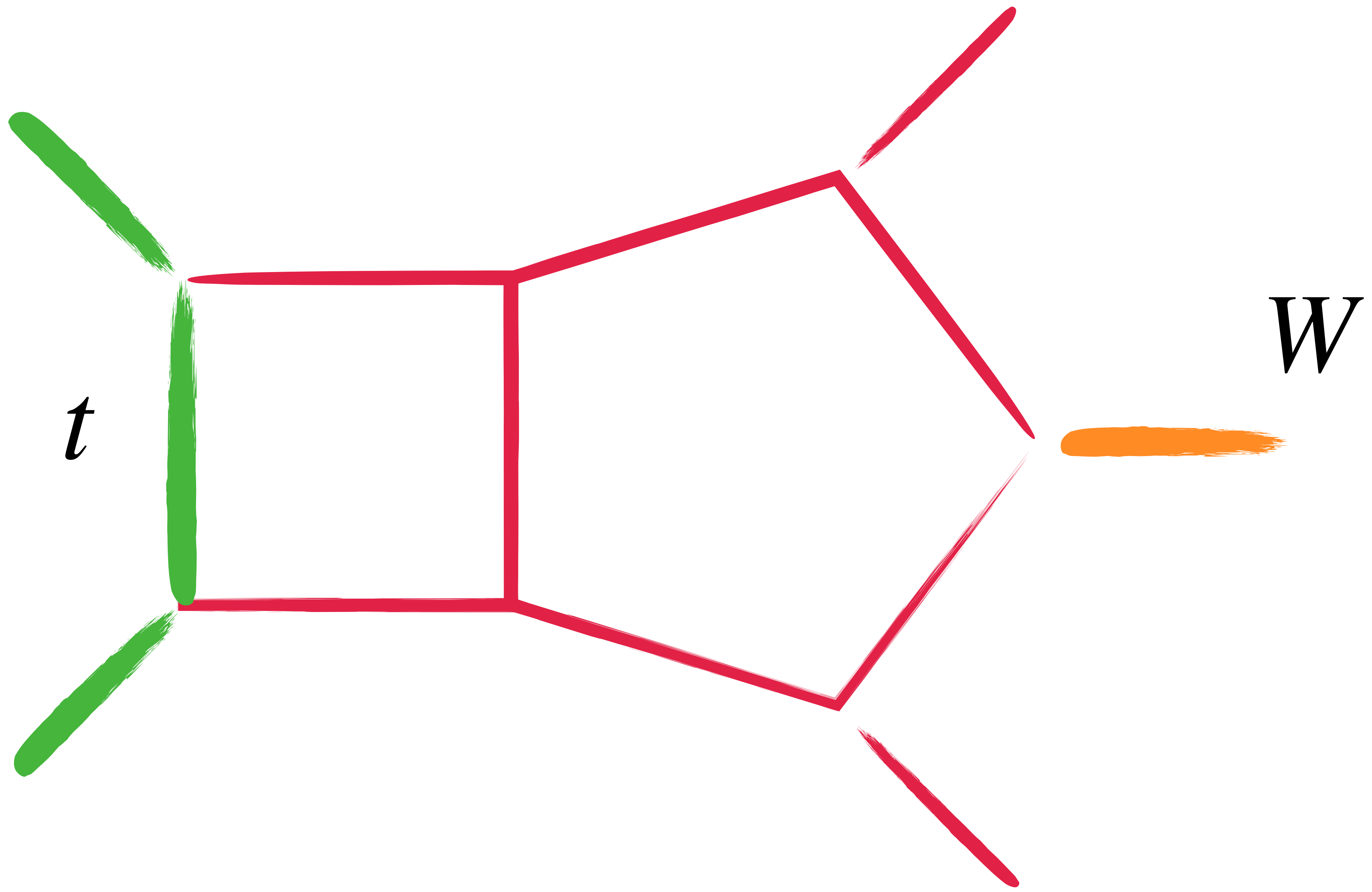
Two-loop virtual amplitude



5-point amplitude with 1 massive particle
current state of the art, more massive legs out of reach!

[Badger, Hartanto, Zoia, 2021]
[Abreu, Febres Cordero, Ita, Klinkert, Page, Sotnikov, 2021]

Two-loop virtual amplitude



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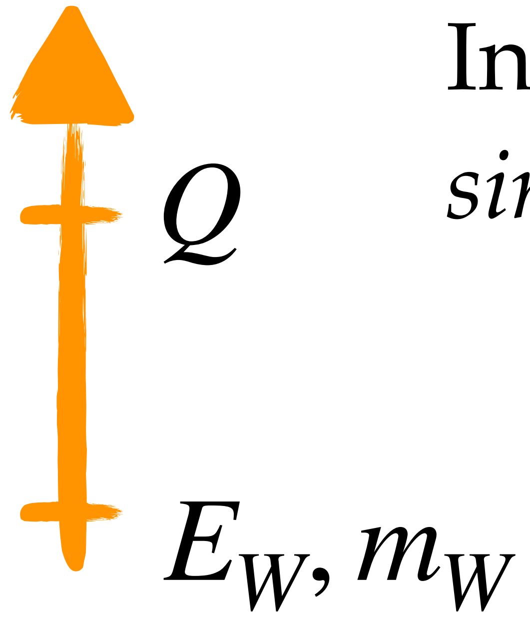
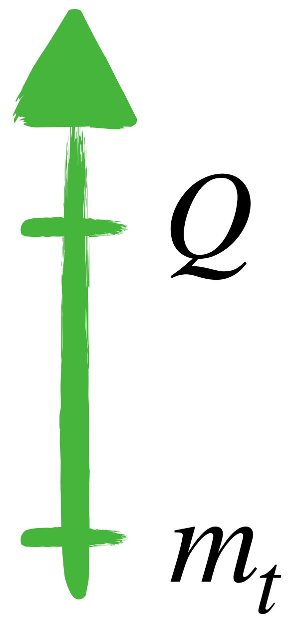
Smart idea: look for reliable approximation(s) based on **factorisation theorems**



In some kinematical regimes, the amplitude "**factorises**" into a *calculable factor* and a *simpler (available) amplitude*

- the energy and mass of the W boson are smaller than the other relevant scales
  soft W approximation
- the mass of $t/\bar{t}$ is negligible compared to their energy (ultra relativist tops) boson
  massification



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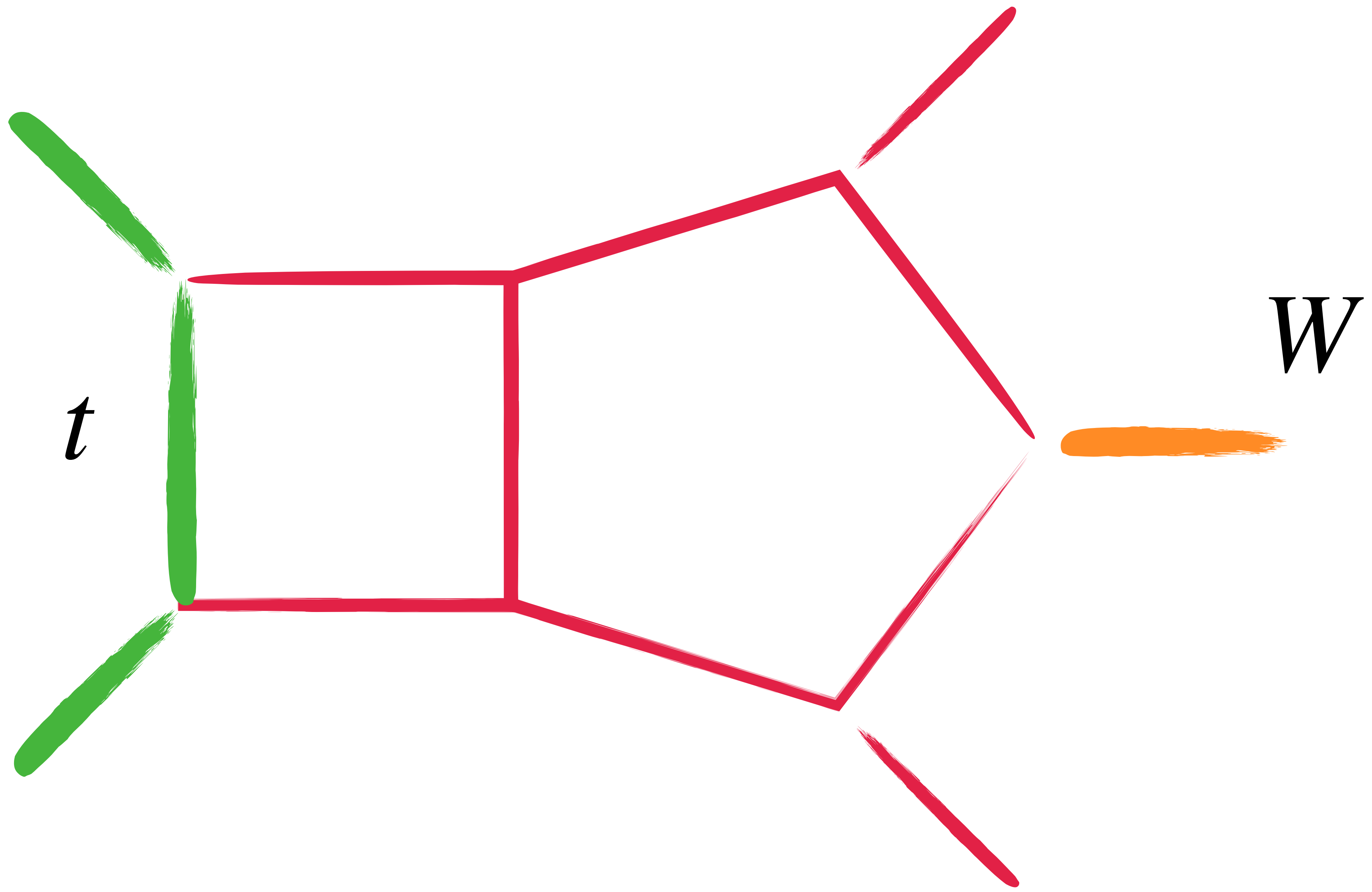

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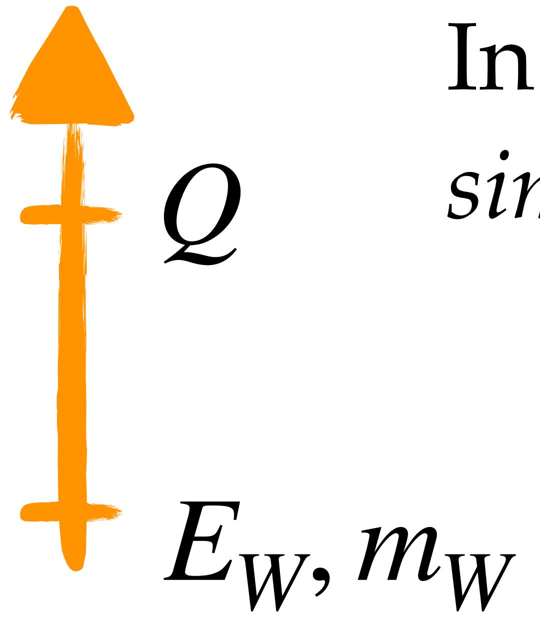
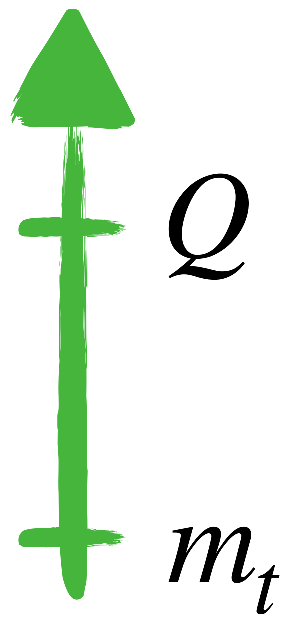


In some kinematical regimes, the amplitude "**factorises**" into a *calculable factor* and a *simpler (available) amplitude*

Disclaimer: None of the two regimes is obviously reasonable for the bulk of the events. The quality of the approximation **must be carefully assessed**

Good starting point: two largely complementary approximations!



Soft approximation

In the limit in which the incoming $q\bar{q}'$ pair emits a soft W, the multi-loop QCD amplitude factorises as

$$|\mathcal{M}^{[p,k]}_{q\bar{q}'\to t\bar{t}W}> \simeq \frac{g}{\sqrt{2}}\left(\frac{p_2\cdot\varepsilon^*(k)}{p_2\cdot k}-\frac{p_1\cdot\varepsilon^*(k)}{p_1\cdot k}\right)\times|\mathcal{M}^{[p]}_{q_L\bar{q}'_R\to t\bar{t}}>$$

Eikonal factor
(analogous to soft photon/gluon)

"reduced" polarised $t\bar{t}$ amplitude

Remarks

- the soft W emission **selects a particular helicity configuration**
- the required NNLO QCD $q\bar{q}'\to t\bar{t}$ amplitude is **available** [Bärnreuther, Czakon, Fiedler, 2013] [Chen, Czakon, Poncelet, 2017] [Mandal, Mastrolia, Ronca, Bobadilla Torres, 2022]
- the use of the formula for a generic phase point required a **momentum mapping**: we adopt a recoil scheme in which the momentum of the W is absorbed by the top quark pair preserving the invariant mass of the event

Soft approximation

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Eikonal factor
(analogous to soft photon/gluon)

"reduced" polarised $t\bar{t}$ amplitude

Remarks

- We apply the approximation for estimating the hard-virtual coefficient

$$H^{(n)} = \frac{2\Re < \mathscr{M}^{(n)}_{\rm fin}|\mathscr{M}^{(0)} >}{|\mathscr{M}^{(0)}|^2}$$

both on numerator and denominator: in this way we are effectively reweighing by the exact LO result!

Application of soft approximation: $t\bar{t}H$

[Catani, Devoto, Grazzini, Kallweit, Mazzitelli, Savoini, 2022]

In the case of soft H emission, we have a similar factorisation formula (for soft scalars)

$$|\mathscr{M}^{[p,k]}_{t\bar{t}H}> \simeq F(\alpha_s(\mu)R); m_t/\mu_R) \times J(k) \times |\mathscr{M}^{[p]}_{t\bar{t}}>$$

Normalisation correction factor beyond LO factorisation Calculable in perturbation theory

Eikonal factor

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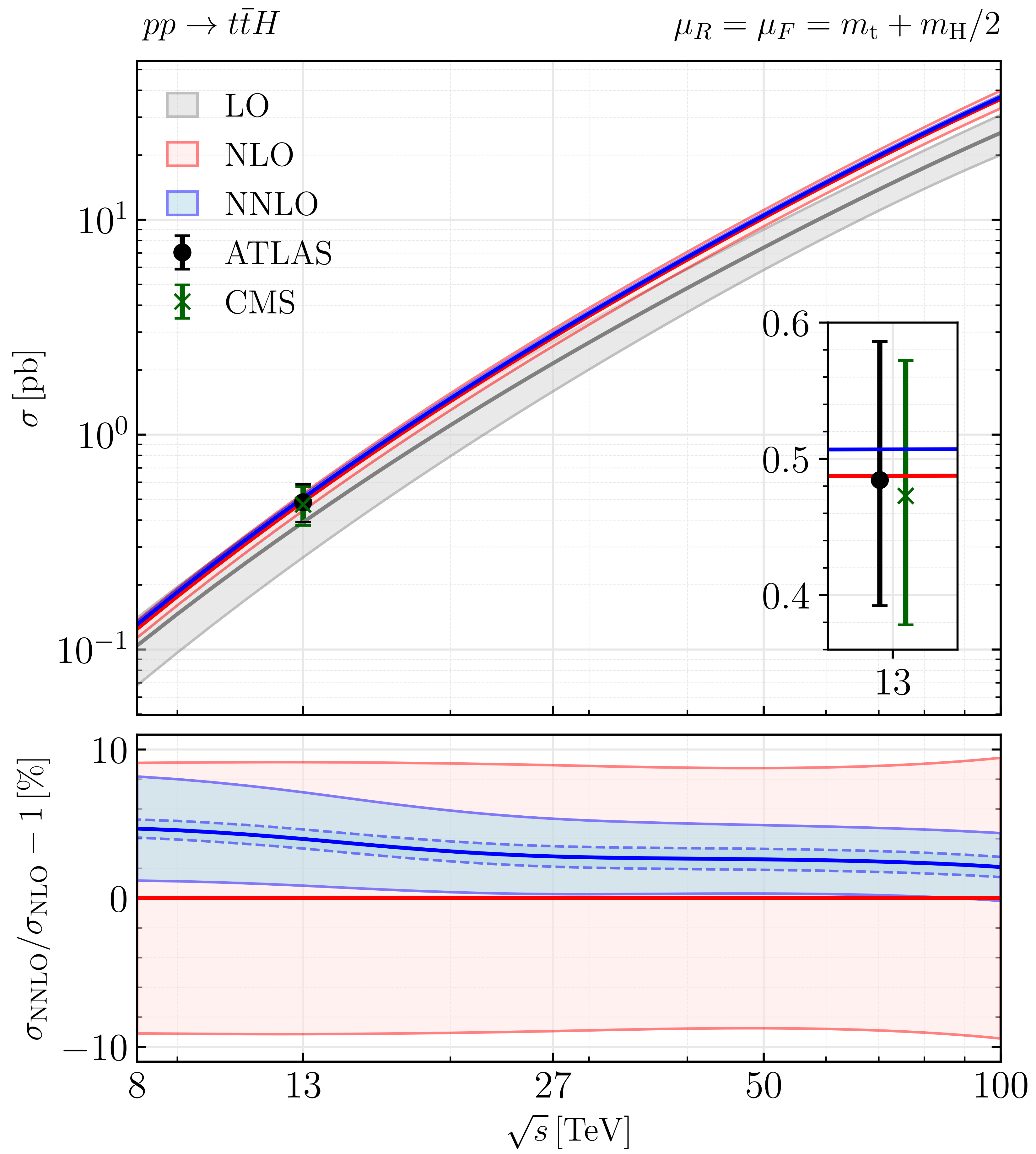
Successfully applied to $t\bar{t}H$ production at hadron colliders

- Careful assessment of the uncertainties associated to the soft approximation
  $\sim 100\,\%$ uncertainty in gg, $\sim 15\,\%$ uncertainty in $q\bar{q}$
  it works better for the $q\bar{q}$ channel

- Relative size of the hard contribution $\Delta\sigma_{\mathrm{NNLO,H}}$ wrt the σ_{LO}
  $\sim 1\,\%$ in gg, $\sim 3\,\%$ in $q\bar{q}$

FINAL UNCERTAINTY:
$\pm 0.6\,\%$ on σ_{NNLO}, $\pm 15\,\%$ on $\Delta\sigma_{\mathrm{NNLO}}$

subdominant wrt scale variations!



Massification procedure in a nutshell

[Mitov, Moch, 2007]

Amplitude factorisation in massless QCD

[Catani, 1998][Sterman, Tejeda-Yeomans, 2003]

$$|\mathcal{M}^{[p]}> = \mathcal{J}^{[p]}\left(\frac{Q^2}{\mu^2}, \alpha_S(\mu^2), \epsilon\right) \times \mathcal{S}^{[p]}\left(\{k_i\}\frac{Q^2}{\mu^2}, \alpha_S(\mu^2), \epsilon\right) \times |\mathcal{H}^{[p]}>$$

Jet function: collinear contributions

Soft function: coherent soft radiation

Hard function: short-distance dynamics

Massification procedure in a nutshell

[Mitov, Moch, 2007]

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Amplitude factorisation in QCD with a **massive** parton of mass $m^2 \ll Q^2$

$$|\mathscr{M}^{[p],(m)}> = \mathscr{J}^{[p]}\left(\frac{Q^2}{\mu^2}, \frac{m_i^2}{\mu^2}\alpha_S(\mu^2), \epsilon\right) \times \mathscr{S}^{[p]}\left(\{k_i\}\frac{Q^2}{\mu^2}, \alpha_S(\mu^2), \epsilon\right) \times |\mathscr{H}^{[p]}> + \mathscr{O}\left(\frac{m^2}{Q^2}\right)$$

$$\mathscr{J}^{[p]}\left(\frac{Q^2}{\mu^2}, \frac{m_i^2}{\mu^2}\alpha_S(\mu^2), \epsilon\right) = \prod_i \mathscr{J}^i\left(\frac{Q^2}{\mu^2}, \frac{m_i^2}{\mu^2}\alpha_S(\mu^2), \epsilon\right) = \prod_i \left(\mathscr{F}^i\left(\frac{Q^2}{\mu^2}, \frac{m_i^2}{\mu^2}\alpha_S(\mu^2), \epsilon\right)\right)^{1/2}$$

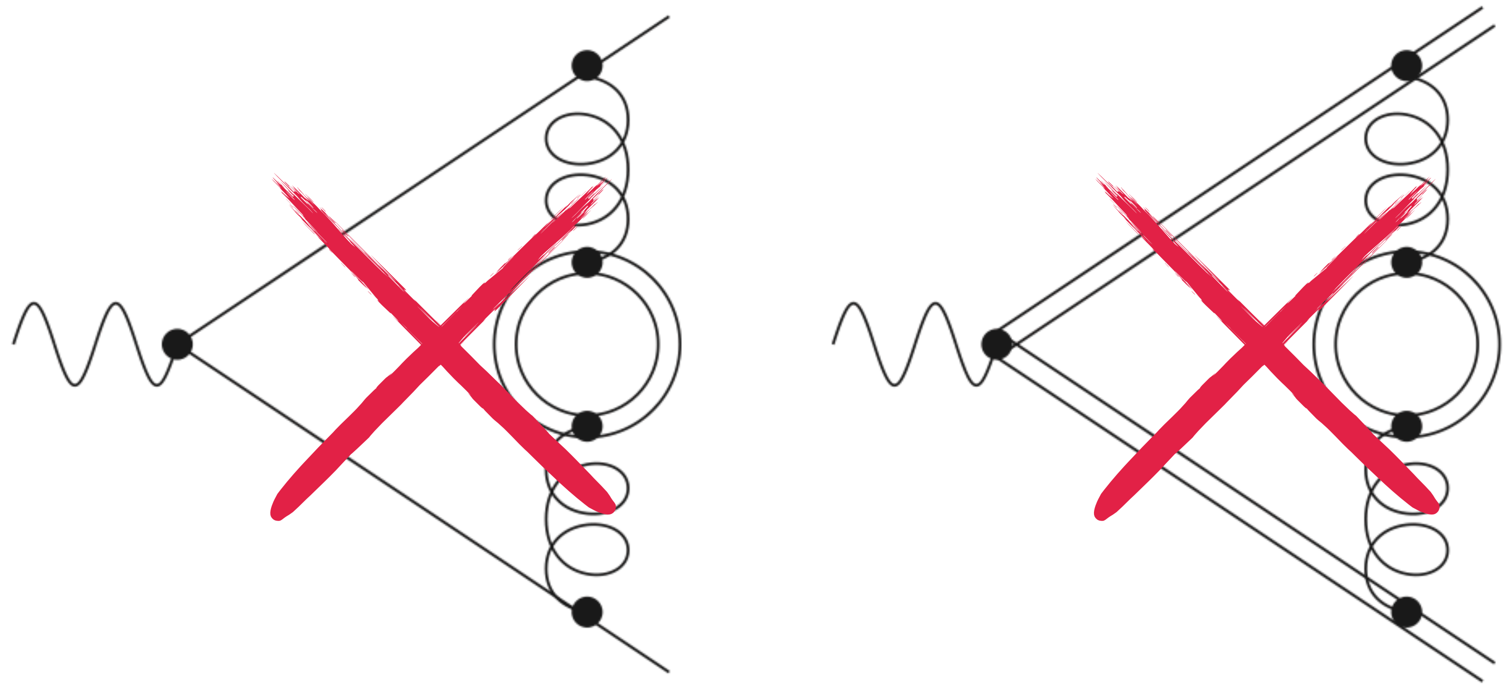
space-like massive form factor

Massification procedure in a nutshell

[Mitov, Moch, 2007]

Caveat: starting from NNLO, heavy quark loop insertions **break** this simple “collinear” factorisation picture

We estimate that they have a negligible impact by inspecting the tree-level emission process of four tops and by removing heavy quark loop diagrams from the real-virtual contribution



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Massification procedure in a nutshell

[Mitov, Moch, 2007]

Master formula of "massification"

$$|\mathcal{M}^{[p],(m)}> = \prod_i \left[Z_{[i]}\left(\frac{m^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right)\right]^{1/2} \times |\mathcal{M}^{[p]}> + \mathcal{O}\left(\frac{m^2}{Q^2}\right)$$

$$Z_{[i]}\left(\frac{m^2}{\mu^2}, \alpha_S(\mu^2), \epsilon\right) = \mathcal{F}^i\left(\frac{Q^2}{\mu^2}, \frac{m_i^2}{\mu^2}, \alpha_S(\mu^2), \epsilon\right)\left[\mathcal{F}^i\left(\frac{Q^2}{\mu^2}, 0, \alpha_S(\mu^2), \epsilon\right)\right]^{-1}$$

History & Remarks

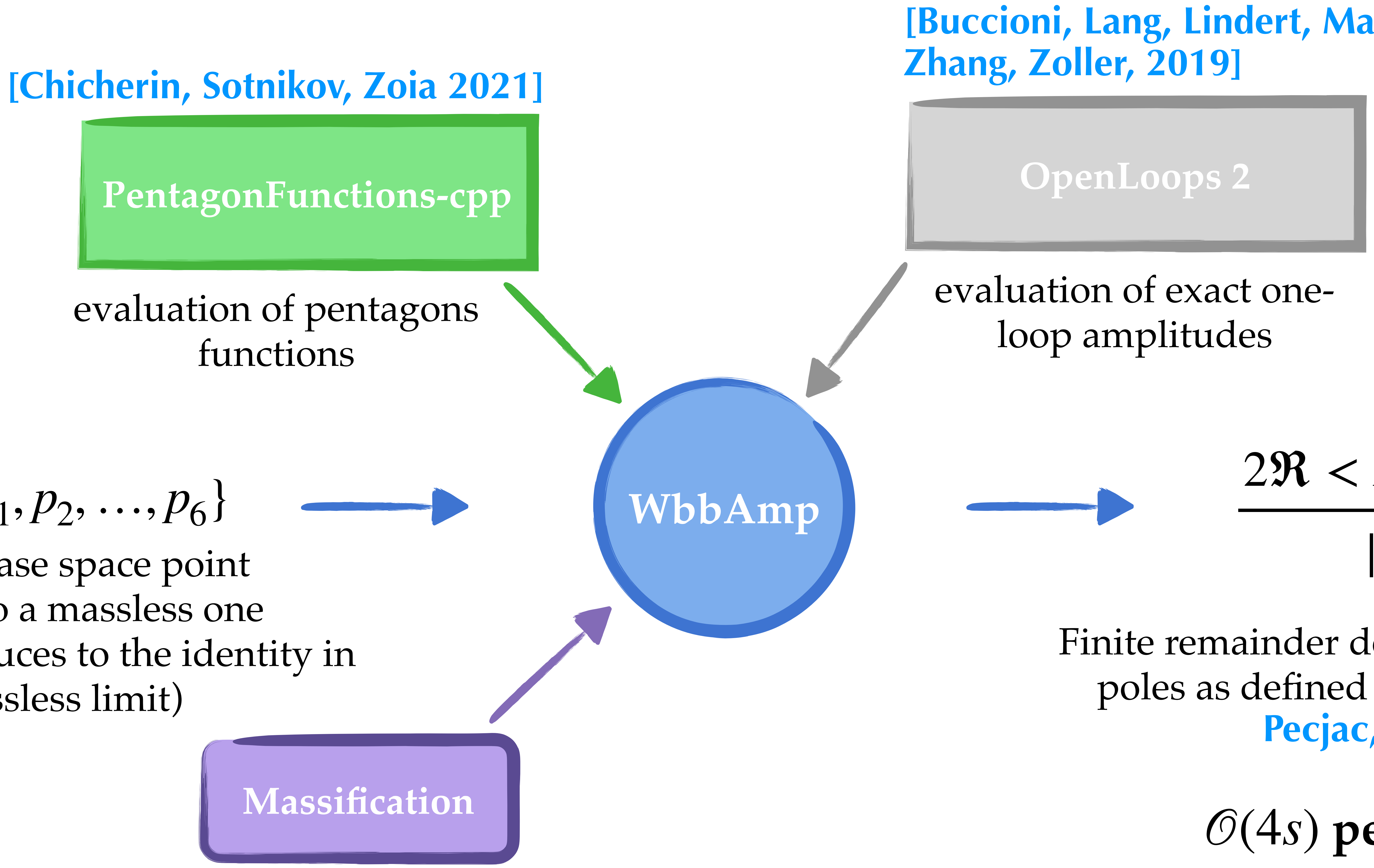
- Neglecting heavy quark insertions, the formula retrieves **mass logarithms** and **constant terms**
- Consistent with previous results for NNLO QED correction to Bhabha scattering [Glover, TauskandJ, VanderBij, 2001] [Penin 2005-2006]
- Successfully employed to derive and cross check results for $q\bar{q} \to Q\bar{Q}$ and $gg \to Q\bar{Q}$ amplitudes [Czakon, Mitov, Moch, 2007]
- Recently extended to the case of two different external masses ($M \gg m$)

[Engel, Gnendiger, Signer, Ulrich 2019]

WQQAmp: a massive C++ implementation

[Buonocore, Rottoli, Savoini, https://gitlab.com/lrottoli/WQQAmp]

We have implemented the one-loop and two-loop amplitudes of [Abreu et al, 2022] in a **C++ library** for the efficient numerical evaluation of the **massive amplitudes**



$$PS = \{p_1, p_2, \ldots, p_6\}$$

massive phase space point **mapped** into a massless one (the mapping reduces to the identity in the massless limit)

$$\frac{2\Re < M_0 | M_2^{\text{fin}} >}{|M_0|^2}$$

Finite remainder defined subtracting the IR poles as defined in [Ferroglia, Neubert, Pecjac, Yang, 2009]

$\mathcal{O}(4s)$ **per phase space point**

Application of massification: $b\bar{b}W$

[Buonocore, Devoto, Kallweit, Mazzitelli, Rottoli, Savoini, 2022]

$b\bar{b}W$ **ideal candidate** to apply the massification procedure: clear hierarchy between the bottom mass and the characteristic hard scale

The calculation with massive bottom quarks (4FS) **reduces ambiguity related to flavour tagging** beyond NLO associated to a massless one (5FS)

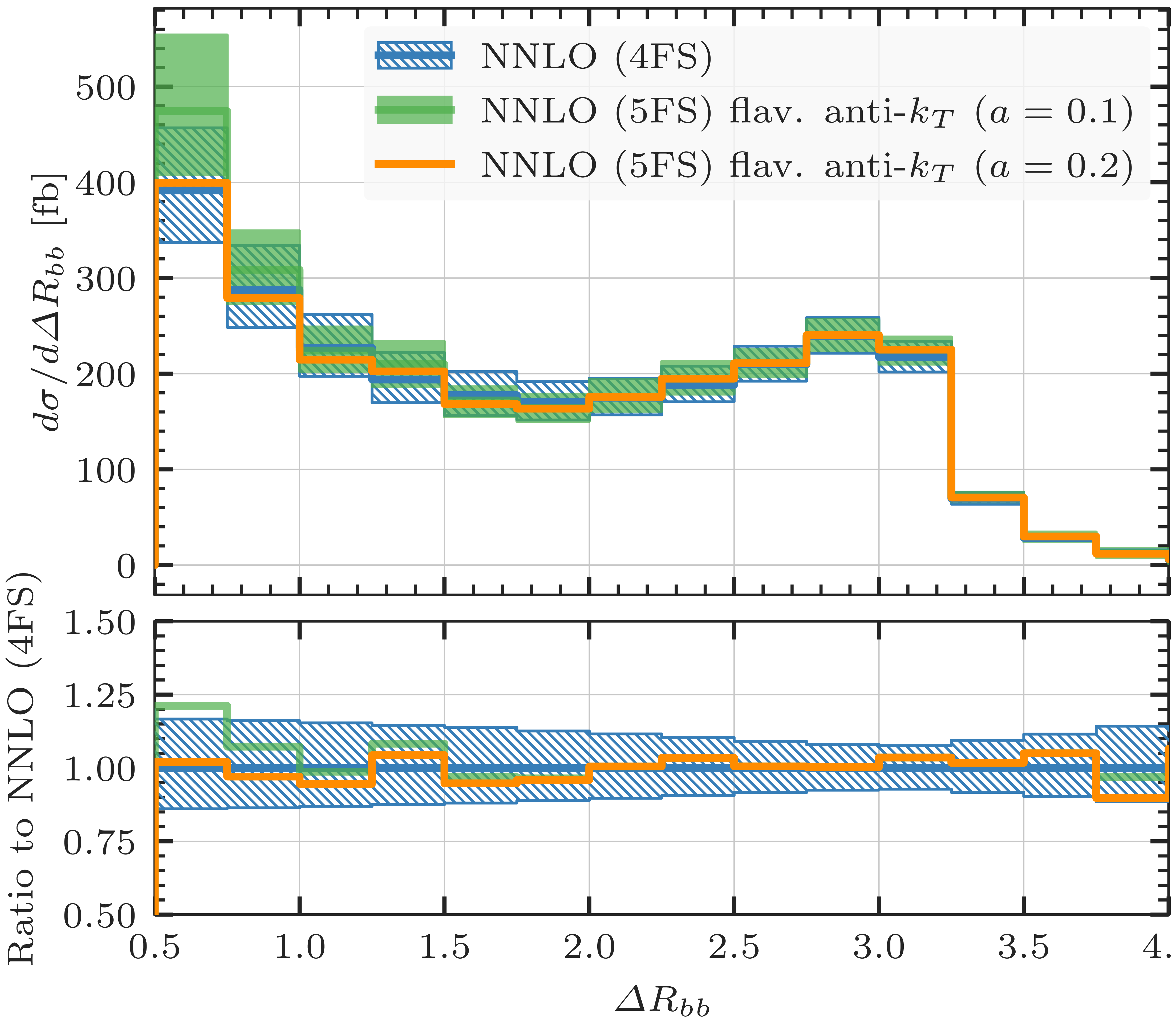
Massless calculation

[Hartanto, Poncelet, Popescu, Zoia, 2022]

Jet algorithm	σ_{NNLO} [fb]	K_{NNLO}
flavour-k_{T}	$445(5)^{+6.7\%}_{-7.0\%}$	1.23
flavour anti-k_{T} ($a = 0.05$)	$690(7)^{+10.9\%}_{-9.7\%}$	1.38
flavour anti-k_{T} ($a = 0.1$)	$677(7)^{+10.4\%}_{-9.4\%}$	1.36
flavour anti-k_{T} ($a = 0.2$)	$647(7)^{+9.5\%}_{-8.9\%}$	1.33

$\mathcal{O}(50\%)$ difference when using flavour k_T algorithm

[Czakon, Mitov, Poncelet, 2022]



Quality of the approximations for $t\bar{t}W$

Observations

- in $t\bar{t}H$, relatively large uncertainty due soft approximation but the corresponding hard contribution represent a small fraction of the NNLO QCD correction

  but the approximation works better for the $q\bar{q}$ channel!

- massification approach fully justified for $b\bar{b}W$

  does it still work for a very heavy quark as the top?

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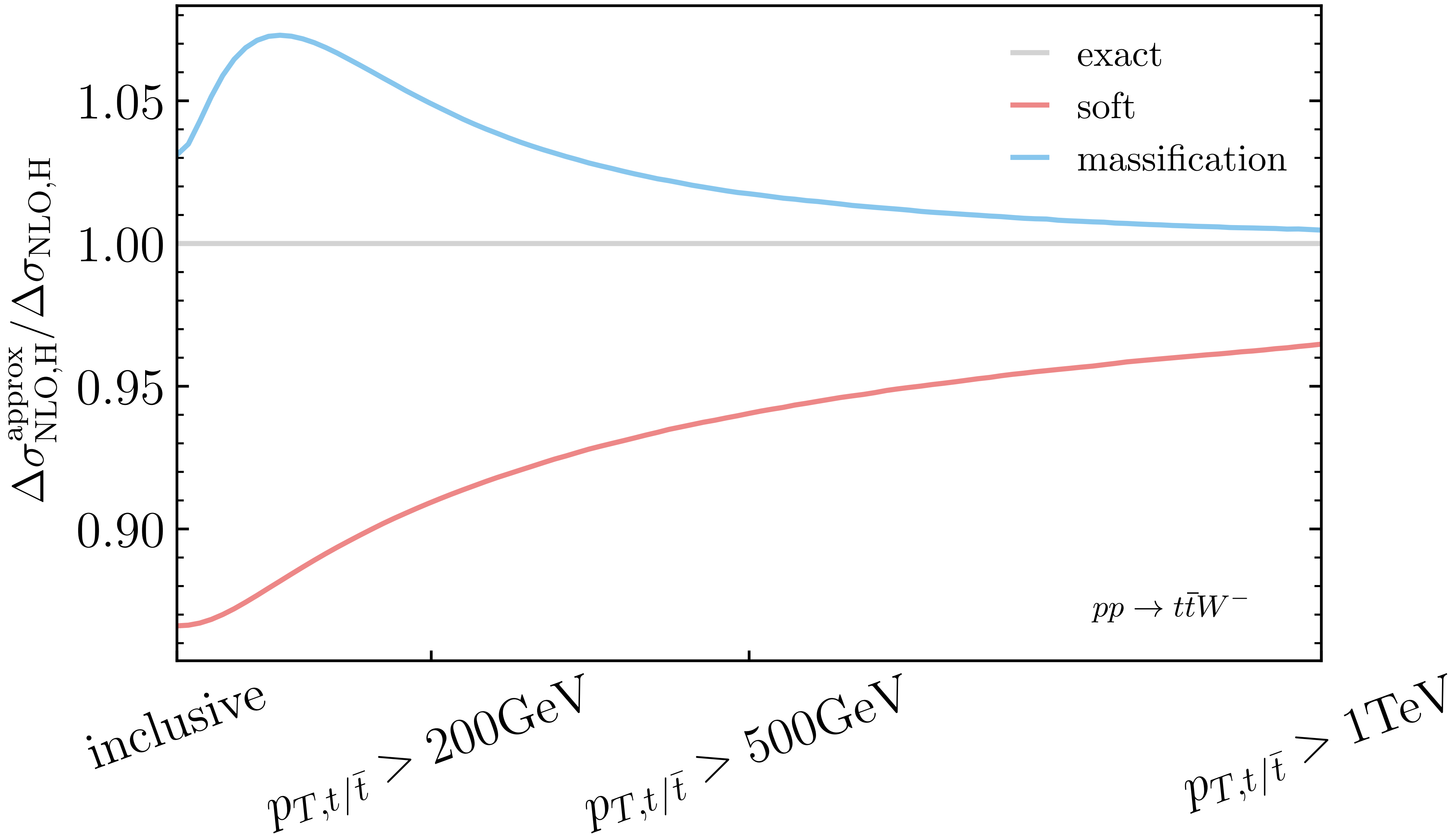
Analysis at NLO (comparison with the exact result!)



- **Both** approximations **provide a good estimate** of the exact one-loop contribution!
- Clear pattern: soft approximation tends to undershoot the exact result while massification tends to overshoot it
- Convergence in the asymptotic limit for high p_T top quarks where both approximation are expected to work

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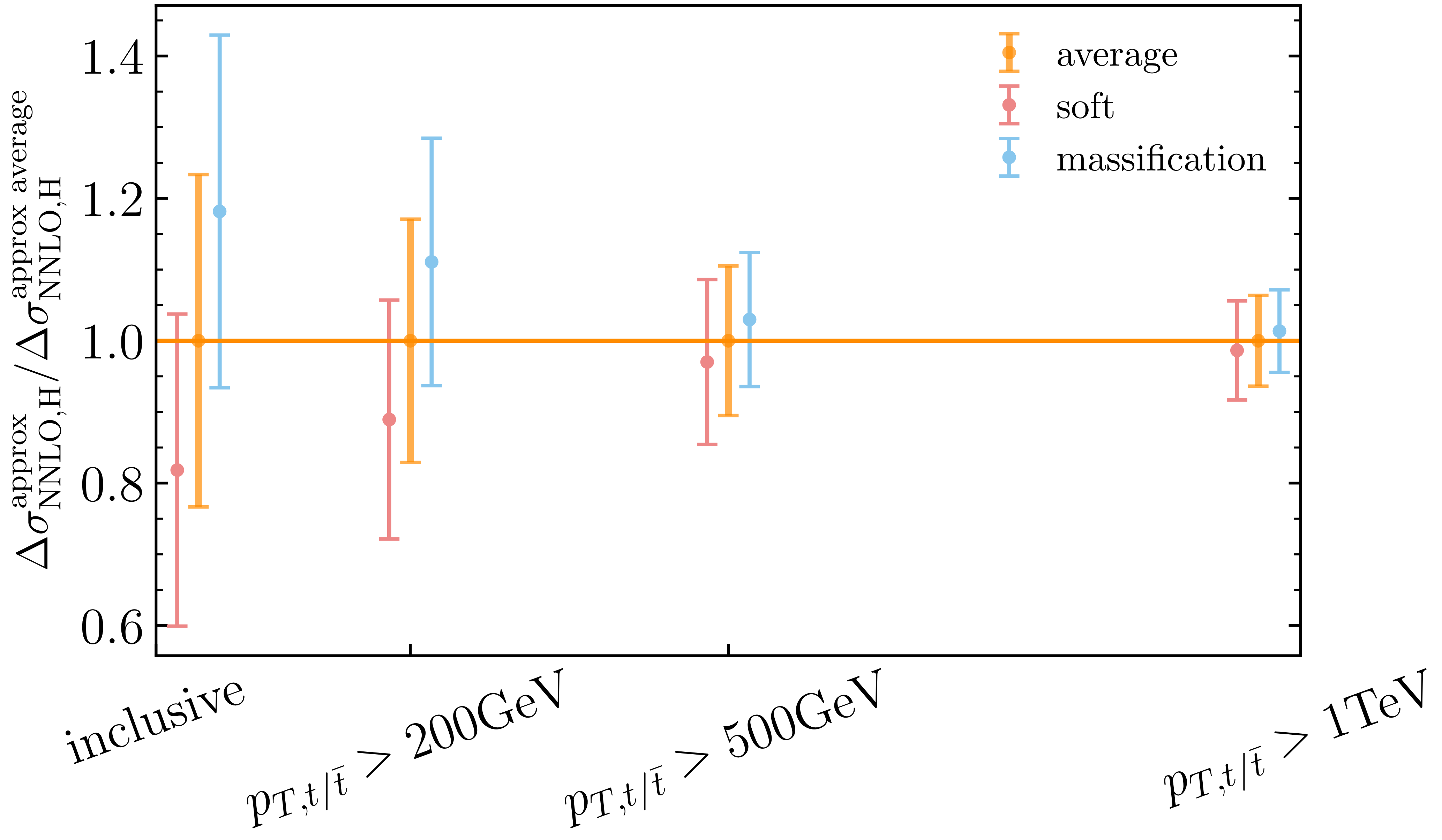
Analysis at NNLO



- **Similar pattern** as at NLO
- **Uncertainties** estimated as the maximum between what we obtain varying the subtraction scale $1/2 \leq \mu_{\text{IR}}/Q \leq 2$ and twice the NLO deviation
- Soft approximation and massification are consistent within their uncertainties!

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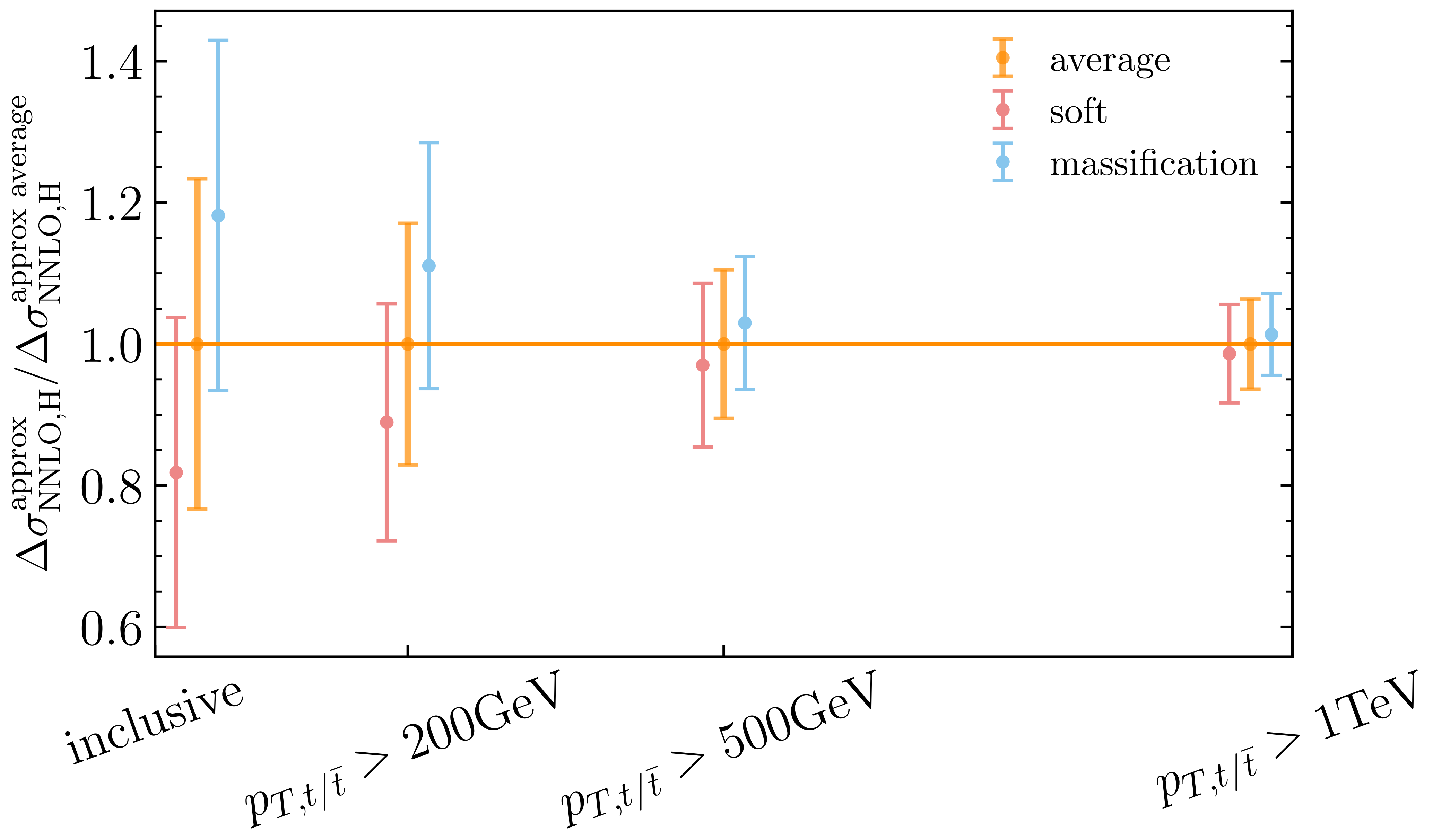
Analysis at NNLO



Best prediction obtained as average of the two with linear combination of uncertainties

Relatively large impact of two-loop virtual contribution: $\sim 7\,\%$ of NNLO cross section

FINAL UNCERTAINTY:

$\pm 1.8\,\%$ on σ_{NNLO}, $\pm 25\,\%$ on $\Delta\sigma_{\text{NNLO,H}}$

similar to what obtained in recent $2 \to 3$ in leading colour approximation

see e.g. [Abreu, De Laurentis, Ita, Klinkert, Page, Sotnikov 2023]

Outline

- Methodology: slicing formalism
- Methodology: two-loop virtual amplitude
- Phenomenological results
- Conclusions

Setup

$$t\bar{t}W + X \ @ \sqrt{s} = 13\,\text{TeV}$$

EW	G_μ-scheme, CKM diagonal
pdf sets	`NNPDF31_nnlo_as_0118_luxqed`
α_s	3-loop running with $n_f = 5$ light quarks
scale variations	7-point $(1/2 < \mu_R/\mu_F < 2)$

Main input values

$$m_t = 172.2 \text{ GeV}$$

$$m_W = 80.385 \text{ GeV}$$

$$m_Z = 91.1876 \text{ GeV}$$

$$G_\mu = 1.6639 \times 10^{-5} \text{ GeV}^{-2}$$

Reference scale

$$\mu_0 = m_t + \frac{m_W}{2} \equiv \frac{M}{2}$$

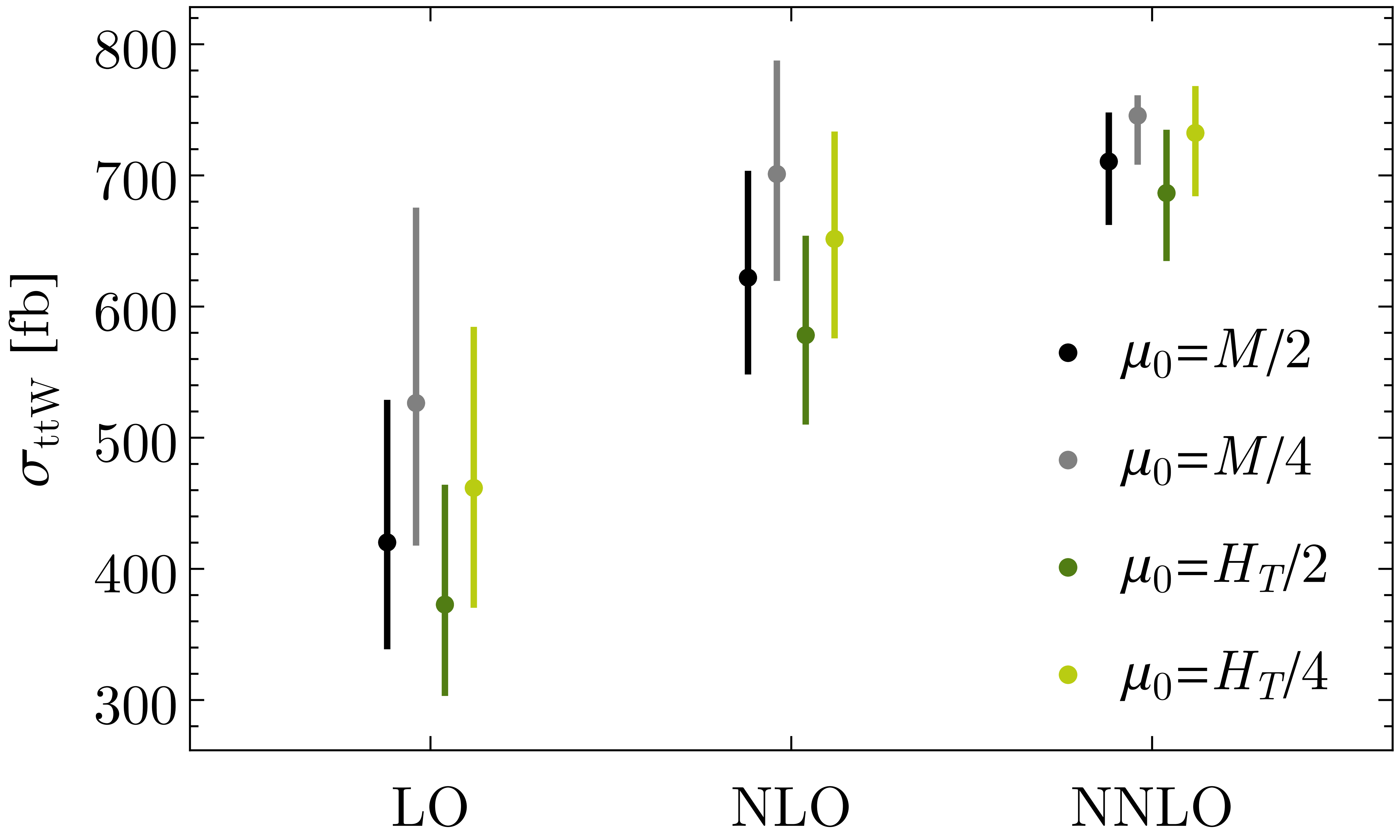
Other scales

$$\mu_0 = \frac{m_T(W) + m_T(t) + m_T(\bar{t})}{2} \equiv \frac{H_T}{2}$$

Scale variations and perturbative uncertainties

We estimate the **perturbative uncertainties** (due to missing higher order corrections) on the basis of

- scale variations
- behaviour of the perturbative series
- different scale choices: $M/2$, $M/4$, $H_T/2$, $H_T/4$
- breakdown of the corrections in different channels



First evidence of the convergence of the perturbative expansion starts at NNLO. Preference for smaller scale choices

The four predictions are fully consistent within their uncertainties

Scale variations and perturbative uncertainties

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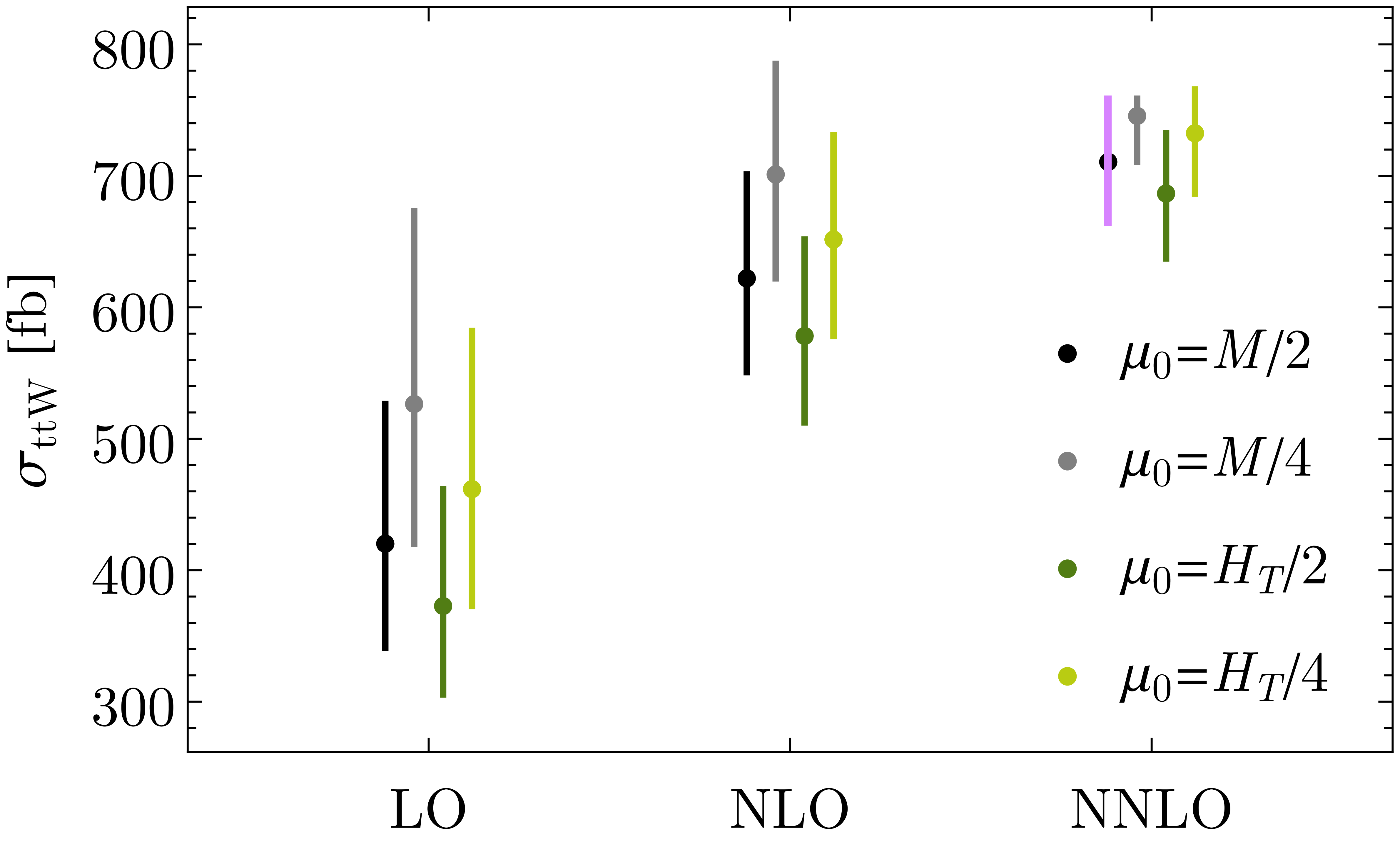

First evidence of the convergence of the perturbative expansion starts at NNLO. Preference for smaller scale choices

Using the predictions with $\mu_0 = M/2$ and **symmetrising its scale uncertainty**, we obtain an interval that almost encompasses also the predictions obtained with $\mu_0 = M/4$ and $\mu_0 = H_T/4$.

Scale variations and perturbative uncertainties

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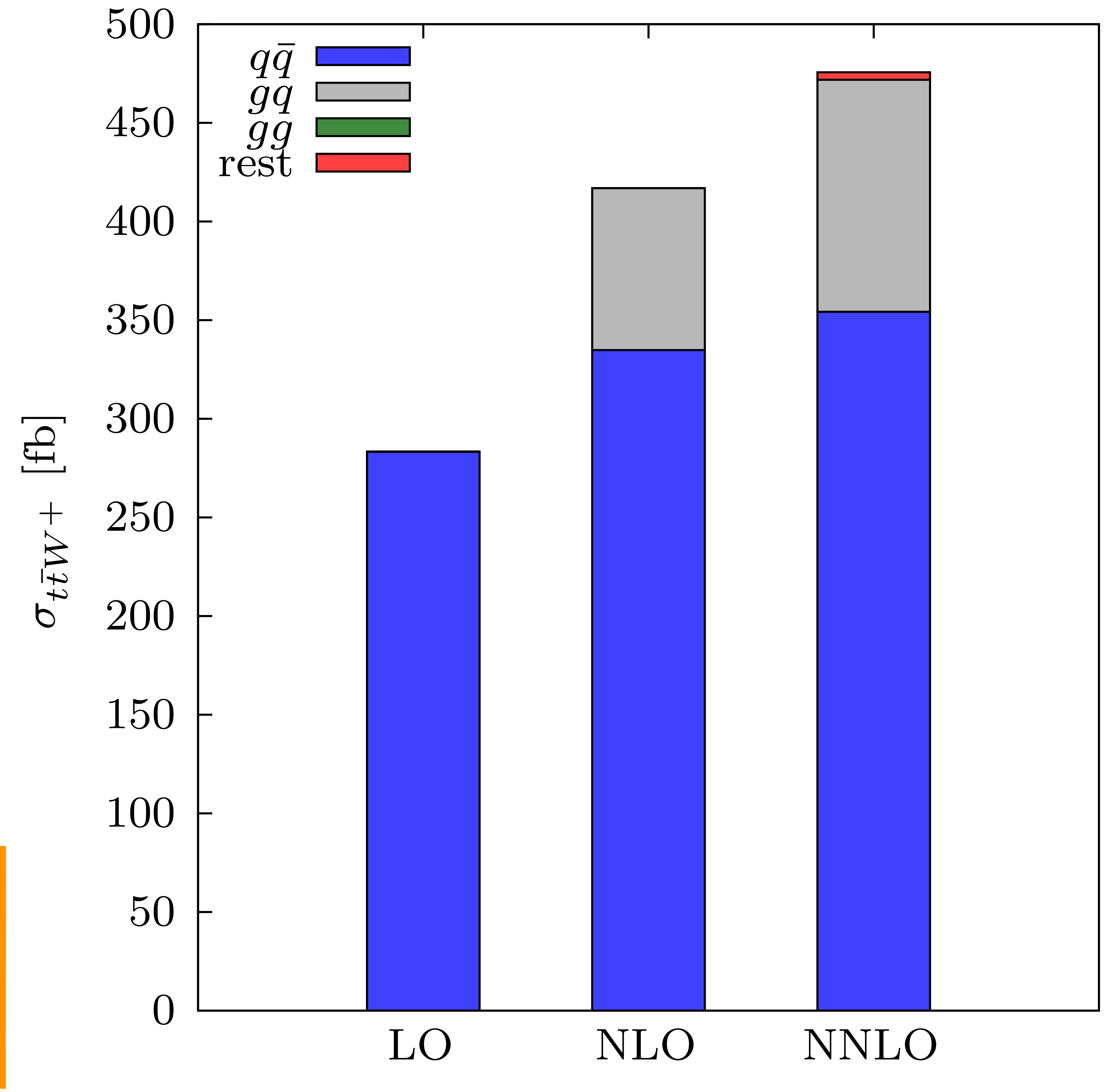
- behaviour of the perturbative series
- different scale choices: $M/2$, $M/4$, $H_T/2$, $H_T/4$
- **breakdown** of the corrections in **different channels**

No new large contribution from channels opening up at NNLO

NNLO corrections dominated by virtual and real correction to the gq channel (NLO accurate)

→ We use as central scale $\mu_0 = M/2$ and estimate perturbative uncertainties through **symmetrised scale variations**



$t\bar{t}W$: inclusive cross sections

	$\sigma_{t\bar{t}W^+}$ [fb]	$\sigma_{t\bar{t}W^-}$ [fb]	$\sigma_{t\bar{t}W}$ [fb]	$\sigma_{t\bar{t}W^+}/\sigma_{t\bar{t}W^-}$
$\mathrm{LO_{QCD}}$	$283.4^{+25.3\%}_{-18.8\%}$	$136.8^{+25.2\%}_{-18.8\%}$	$420.2^{+25.3\%}_{-18.8\%}$	$2.071^{+3.2\%}_{-3.2\%}$
$\mathrm{NLO_{QCD}}$	$416.9^{+12.5\%}_{-11.4\%}$	$205.1^{+13.2\%}_{-11.7\%}$	$622.0^{+12.7\%}_{-11.5\%}$	$2.033^{+3.0\%}_{-3.4\%}$
$\mathrm{NNLO_{QCD}}$	$475.2^{+4.8\%}_{-6.4\%} \pm 1.9\%$	$235.5^{+5.1\%}_{-6.6\%} \pm 1.9\%$	$710.7^{+4.9\%}_{-6.5\%} \pm 1.9\%$	$2.018^{+1.6\%}_{-1.2\%}$
$\mathrm{NNLO_{QCD}+NLO_{EW}}$	$497.5^{+6.6\%}_{-6.6\%} \pm 1.8\%$	$247.9^{+7.0\%}_{-7.0\%} \pm 1.8\%$	$745.3^{+6.7\%}_{-6.7\%} \pm 1.8\%$	$2.007^{+2.1\%}_{-2.1\%}$
ATLAS	$585^{+6.0\%+8.0\%}_{-5.8\%-7.5\%}$	$301^{+9.3\%+11.6\%}_{-9.0\%-10.3\%}$	$890^{+5.6\%+7.9\%}_{-5.6\%-7.9\%}$	$1.95^{+10.8\%+8.2\%}_{-9.2\%\ -6.7\%}$
CMS	$553^{+5.4\%+5.4\%}_{-5.4\%-5.4\%}$	$343^{+7.6\%+7.3\%}_{-7.6\%-7.3\%}$	$868^{+4.6\%+5.9\%}_{-4.6\%-5.9\%}$	$1.61^{+9.3\%+4.3\%}_{-9.3\%-3.1\%}$

Best prediction

Uncertainty associated to the approximation of the 2-loop virtual amplitude

Impact of radiative corrections

- Large positive NLO QCD corrections: +50 %
- Moderate positive NNLO QCD corrections: +14 – 15 %
- Relatively sizeable positive corrections from all LO and NLO contributions at $O(\alpha^3)$, $O(\alpha_S^2\alpha^2)$, $O(\alpha\alpha^3)$, $O(\alpha^4)$: +5 %
- The ratio $\sigma_{t\bar{t}W^+}/\sigma_{t\bar{t}W^-}$ is rather stable and only slightly decreases increasing the perturbative order

$t\bar{t}W$: inclusive cross sections

	$\sigma_{t\bar{t}W^+}$ [fb]	$\sigma_{t\bar{t}W^-}$ [fb]	$\sigma_{t\bar{t}W}$ [fb]	$\sigma_{t\bar{t}W^+}/\sigma_{t\bar{t}W^-}$
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Best prediction

Uncertainty associated to the approximation of the 2-loop virtual amplitude

Other uncertainties

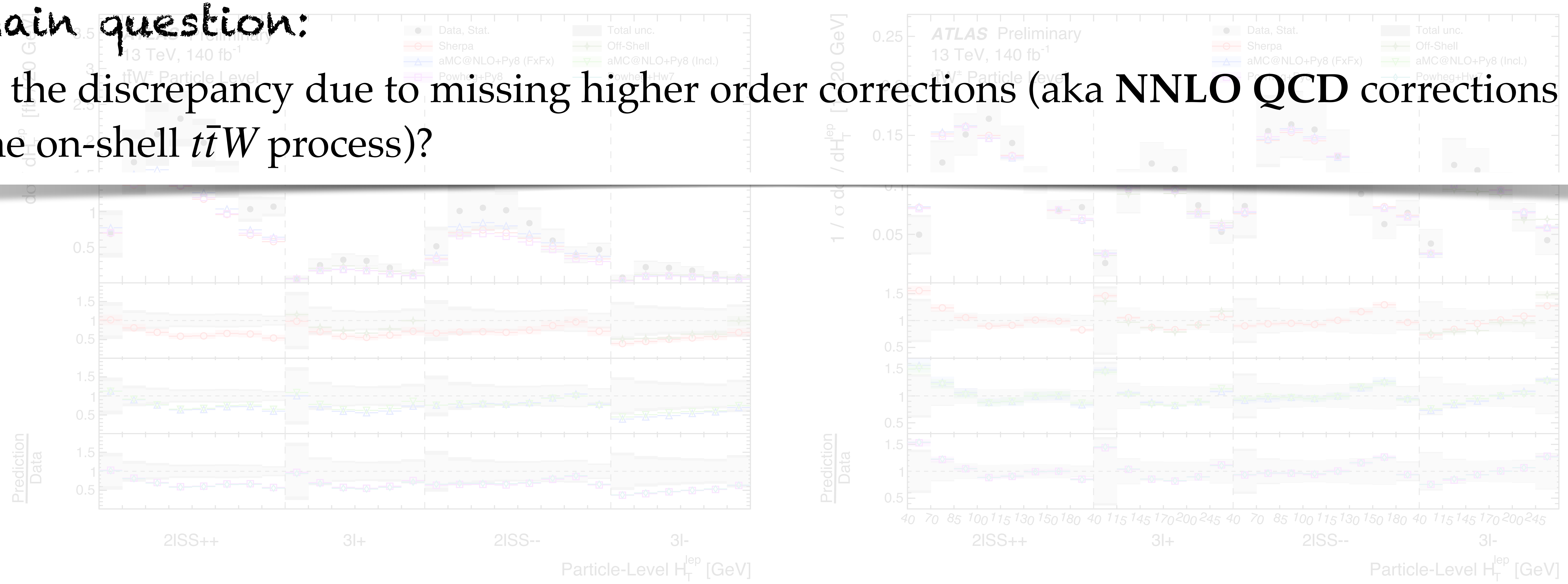
- PDF uncertainties: $\pm 1.8\,\%$ ($\pm 1.8\,\%$ ratio) computed with new MATRIX+PINEAPPL implementation

  [S. Devoto, T. Jezo, S. Kallweit and C. Schwan in preparation]

- α_s uncertainties (half the difference between pdf sets for $\alpha_s(m_Z) = 0.118 \pm 0.001$) $\pm 1.8\,\%$ (negligible for ratio)
- Systematics of the q_T-subtraction method ($r_{\mathrm{cut}} \to 0$ extrapolation) are negligible

State of the art: data-theory comparison

- ATLAS measured also **differential distributions**, finding a disagreement in the overall normalisation consistent with the inclusive measurement result
- The latest off-shell fixed-order predictions give indications that this disagreement **is not predominantly due to missing singly-resonant contributions** which are not included in the reference on-shell predictions



Main question:

Is the discrepancy due to missing higher order corrections (aka **NNLO QCD** corrections to the on-shell $t\bar{t}W$ process)?

$t\bar{t}W$: updated comparison with data

The inclusion of newly computed NNLO QCD corrections leads to

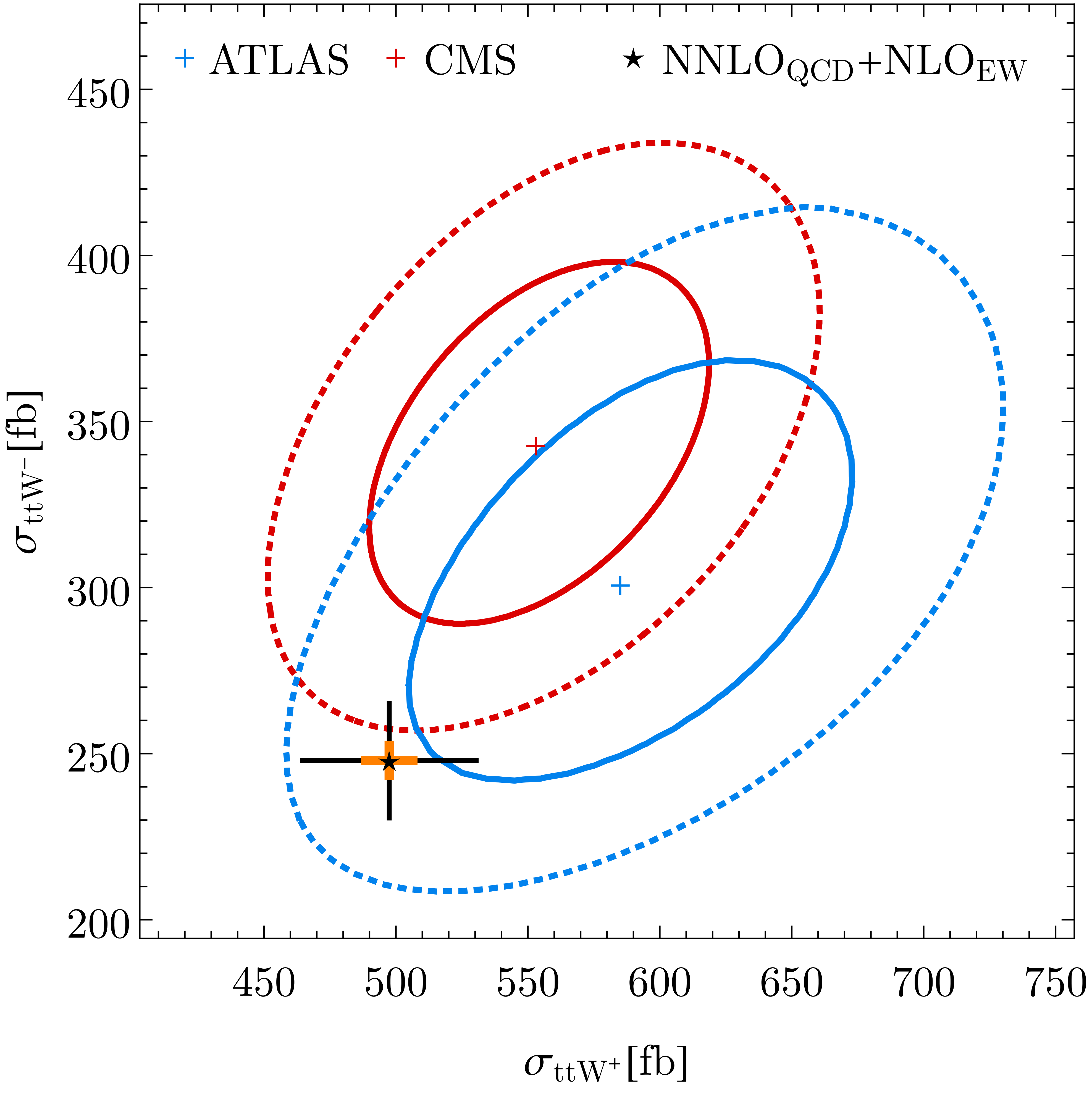
- moderately higher rates
- reduction of perturbative uncertainties

Comparing to the NLO QCD + EW prediction supplemented with FxFx multijet merging, we find good agreement within the quoted uncertainties

$$\sigma_{t\bar{t}W} = 745.3^{+6.7\%}_{-6.7\%}$$ Our best prediction

$$\sigma^{FxFx}_{t\bar{t}W} = 722.3^{+9.7\%}_{-10.8\%}$$

Tension stays at the level of
1σ (ATLAS) - 2σ (CMS)



Conclusions

We have presented the first calculation of the NNLO QCD radiative corrections to (on-shell) $t\bar{t}W$ based on

- the q_T subtraction formalism for the production of **a coloured massive final state + a colour singlet system** (thanks to the progress in the calculation of the corresponding soft function)
- a **reliable approximation of the missing two-loop virtual amplitude** based on two factorisation approaches: the soft W boson approximation and the massification procedure.
  The two-loop virtual contribution is **not negligible** (7% of σ_{NNLO}) and we have achieved **a good control** (at the level of 1.8%, smaller than the scale uncertainty)

We have studied their impact on $t\bar{t}W$ rates at the LHC

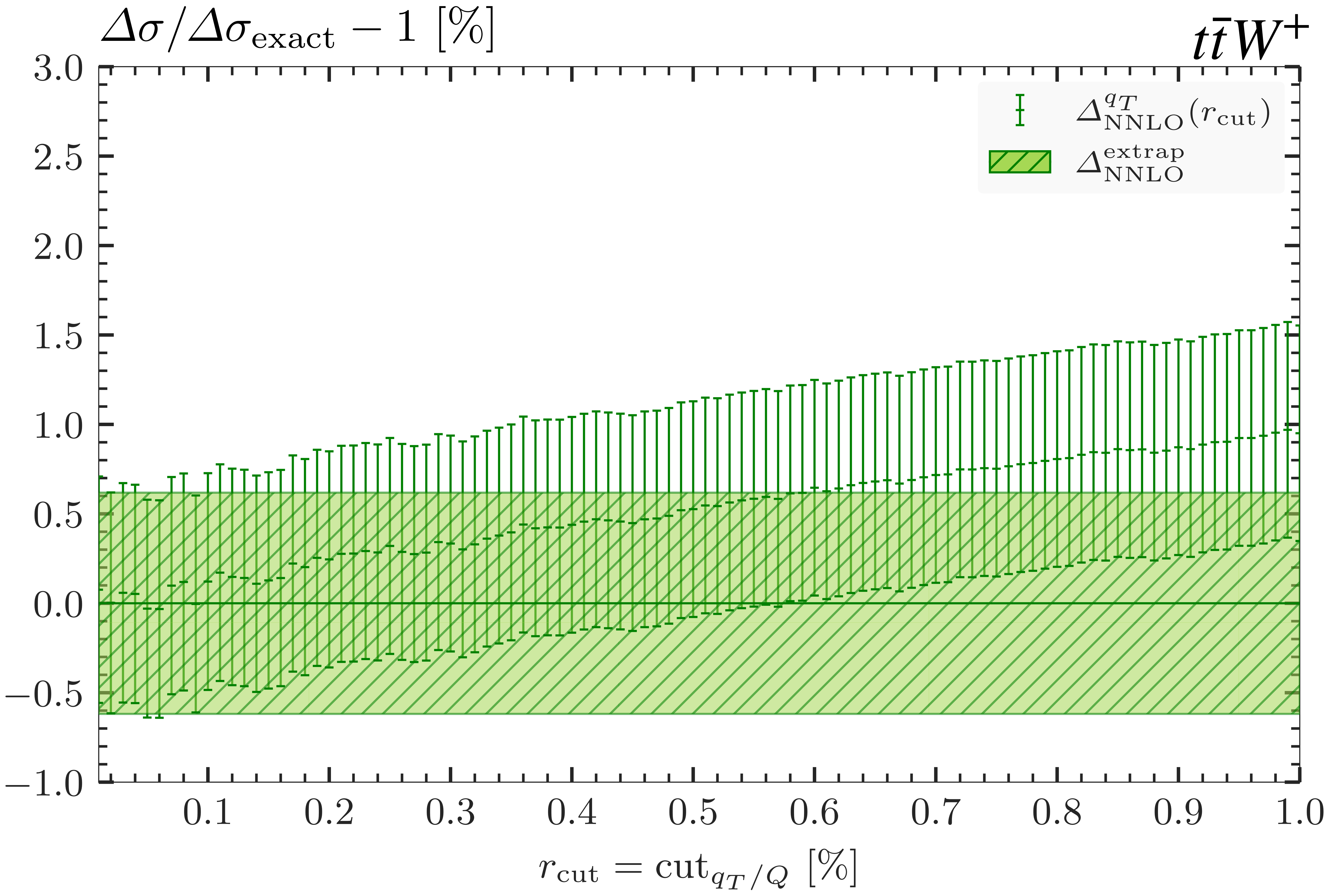
- NNLO QCD radiative corrections leads to **moderately higher rates** (around +15%) and **reduce the perturbative uncertainties** (around 7%)
- **the tension with data stays at the $1\sigma - 2\sigma$ level**

BACKUP

q_T subtraction systematics

$$d\sigma_{N^kLO} = \mathcal{H} \otimes d\sigma_{LO} + \left[d\sigma^{R}_{N^{k-1}LO} - d\sigma^{CT}_{N^kLO}\right]_{q_T/Q > r_{\text{cut}}} + \mathcal{O}(r^{\ell}_{\text{cut}}) \qquad r_{\text{cut}} = \frac{q_{T,\text{cut}}}{m_{t\bar{t}W}}$$

residual power corrections



Behaviour of the power corrections compatible with a **linear scaling** as expected from processes with massive final state

Overall very mild power corrections

Control of the NNLO correction at $\mathcal{O}(0.6\%)$ → sub permille effect at the level of the total cross section

Soft H approximation

$$|\mathscr{M}_{t\bar{t}H}^{[p,k]}> \simeq F(\alpha_s(\mu)R); m_t/\mu_R) \times J(k) \times |\mathscr{M}_{t\bar{t}}^{[p]}>$$

$$J(k) = \sum_i \frac{m_t}{v}\frac{m_t}{p_i \cdot k}$$

The perturbative function $F\left(\alpha_S(\mu_R); m_t/\mu_R\right)$ can be extracted from the soft limit of the scalar form factor of the heavy quark

[Bernreuther et al, 2005] [Blümlein et al, 2017]

$$F(\alpha_s(\mu)R); m_t/\mu_R) = 1 + \frac{\alpha_S}{2\pi}(-3C_F) + \left(\frac{\alpha_S}{2\pi}\right)^2 \left(\frac{33}{4}C_F^2 - \frac{185}{12}C_FC_A + \frac{13}{6}C)F(n_l+1) - 6C_F\beta_0 \ln\frac{\mu_R^2}{m_t^2}\right) + \mathcal{O}\left(\alpha_S^3\right)$$

Alternatively, it can be derived by using Higgs low-energy theorems

see e.g. [Kniehl, Spira, 1995]

$t\bar{t}H$: quality of the soft H approximation

At LO, the soft H approximation overestimates the exact result by

- gg channel: a factor of 2.3 at $\sqrt{s} = 13$ TeV and a factor of 2 at $\sqrt{s} = 100$ TeV
- $q\bar{q}$ channel: a factor of 1.11 at $\sqrt{s} = 13$ TeV and a factor of 1.06 at $\sqrt{s} = 100$ TeV

	$\sqrt{s} = 13$ TeV		$\sqrt{s} = 100$ TeV	
σ [fb]	gg	$q\bar{q}$	gg	$q\bar{q}$
σ_{LO}	261.58	129.47	23055	2323.7
$\Delta\sigma_{\mathrm{NLO,H}}$	88.62	7.826	8205	217.0
$\Delta\sigma_{\mathrm{NLO,H}}\vert_{\mathrm{soft}}$	61.98	7.413	5612	206.0
$\Delta\sigma_{\mathrm{NNLO,H}}\vert_{\mathrm{soft}}$	$-2.980(3)$	2.622(0)	$-239.4(4)$	65.45(1)

At NLO, the approximation performs better than at LO because of the LO re-weighting

$t\bar{t}H$: quality of the soft H approximation & uncertainties

Uncertainties estimates by

- varying the momentum mapping used to absorb the recoil of the H boson
- varying the infrared μ_{IR} subtraction scale at which the $H^{(2)}$ is evaluated from the central value $m_{t\bar{t}H}$ to $m_{t\bar{t}H}/2$ and $2m_{t\bar{t}H}$
  When evaluating $H^{(2)}$ at a subtraction scale different from the central value, we added the contribution stemming from the running from the μ_{IR} to $m_{t\bar{t}H}$ using the exact matrix elements

Uncertainties estimated by multiplying by a **tolerance factor of 3** the deviations found at NLO:
30% for the gg channel and 5% for the $q\bar{q}$ channel.
This encompasses the uncertainties associated to the variations above

Finally uncertainties obtained by combining linearly the gg and the $q\bar{q}$ channel

0.6% on σ_{NNLO}