Transverse-momentum resummation for inclusive vector boson production

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Standard Model @ LHC

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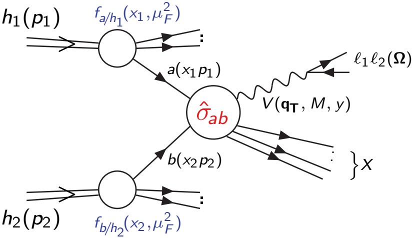
Drell–Yan $\mathbf{q_T}$ distribution



$$\mathbf{h_1(p_1) + h_2(p_2) \;\to\; V + X \;\to\; \ell_1 + \ell_2 + X}$$

where $\quad V = Z^0/\gamma^*, W^\pm$

QCD factorization formula:

$$\frac{d\sigma}{d^2\mathbf{q_T}\, dM^2\, dy\, d\mathbf{\Omega}} = \sum_{a,b} \int_0^1 dx_1 \int_0^1 dx_2\; f_{a/h_1}(x_1,\mu_F^2)\; f_{b/h_2}(x_2,\mu_F^2)\; \frac{d\hat{\sigma}_{ab}}{d^2\mathbf{q_T}\, dM^2\, d\hat{y}\, d\mathbf{\Omega}}(\alpha_S(\mu_R^2),\mu_R^2,\mu_F^2).$$

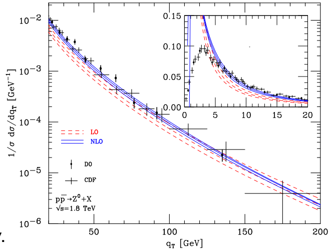
Fixed-order perturbative expansion reliable only for $q_T \sim M$. When $q_T \ll M$:

$$\int_0^{q_T^2} d\bar{q}_T^2\, \frac{d\hat{\sigma}_{q\bar{q}}}{d\bar{q}_T^2} \sim 1 + \alpha_S \left[c_{12} L_{q_T}^2 + c_{11} L_{q_T} + \cdots \right]$$

$$+\alpha_S^2 \left[c_{24} L_{q_T}^4 + \cdots + c_{21} L_{q_T} + \cdots \right] + \mathcal{O}(\alpha_S^3)$$

with $\alpha_S^n L_{q_T}^m \equiv \alpha_S^n \log^m(M^2/q_T^2) \gtrsim 1$.

Resummation of logarithmic corrections mandatory.



q_T resummation formalisms

- Resummation of large q_T logarithms achieved in Fourier conjugated space [Parisi,Petronzio('79)],[Kodaira,Trentadue('82)],[Collins,Soper,Sterman('85)], [Altarelli et al.('84)],[Catani,d'Emilio,Trentadue('88)],[Catani,de Florian, Grazzini('01)],[Catani,Grazzini('10)],[Balasz,Yuan,Nadolsky et al.('97,'02)], [Kulesza et al.('02)],[Banfi et al.('12)], [Guzzi et al.('13)].
- Results for q_T resummation also in the framework of Effective Theories, transverse-momentum dependent (TMD) parton densities and within p_T space formalisms: [Gao,Li,Liu('05)],[Idilbi,Ji,Yuan('05)],[Mantry,Petriello('10)], [García,Idibli,Scimemi('11)],[Becher,Neubert('10)],[Chiu et al.('12)], [Dokshitzer,Diakonov,Troian('78)],[Ellis et al.('97)],[Frixione,Nason,Ridolfi('99) [Erbert,Tackmann('17)],[Monni,Re,Torrielli('16)],[Bizon et al.('17,'18)], [D'Alesio,Murgia('04)],[Roger,Mulders('10)],[Collins('11)],[D'Alesio et al.('14)].
- Effective q_T-resummation obtained with Parton Shower algorithms combined with higher orders: [Alioli et al.('13),[Hoeche et al.('14)],[Karlberg et al.('14)].

$\mathbf{q_T}$ resummation in QCD [Catani,de Florian,Grazzini('01)] [Bozzi,Catani,de Florian,Grazzini('03,'06)]

$$\frac{d\hat{\sigma}}{dq_T^2} = \frac{d\hat{\sigma}^{(res)}}{dq_T^2} + \frac{d\hat{\sigma}^{(fin)}}{dq_T^2};$$

In the impact parameter space: $q_T \ll M \Leftrightarrow Mb \gg 1, \quad \log M/q_T \gg 1 \Leftrightarrow \log Mb \gg 1$

$$\frac{d\hat{\sigma}^{(res)}}{dq_T^2} = \frac{M^2}{\hat{s}} \int \frac{d^2\mathbf{b}}{4\pi} e^{i\mathbf{b}\cdot\mathbf{q_T}} \, \mathcal{W}(b, M),$$

In the Mellin space (with respect to $z = M^2/\hat{s}$) we have:

$$\mathcal{W}_N(b,M) = \hat{\sigma}^{(0)} \, \mathcal{H}_N(\alpha_S) \times \exp\left\{\mathcal{G}_N(\alpha_S, L)\right\}$$

with $\quad L \equiv \log(M^2 b^2)$

$$\mathcal{G}(\alpha_S, L) = L\, g^{(1)}(\alpha_S L) + g^{(2)}(\alpha_S L) + \frac{\alpha_S}{\pi} g^{(3)}(\alpha_S L) + \cdots \qquad \mathcal{H}(\alpha_S) = 1 + \frac{\alpha_S}{\pi}\mathcal{H}^{(1)} + \left(\frac{\alpha_S}{\pi}\right)^2 \mathcal{H}^{(2)} + \cdots$$

LL ($\sim \alpha_S^n L^{n+1}$): $g^{(1)}$, $(\hat{\sigma}^{(0)})$; NLL ($\sim \alpha_S^n L^n$): $g^{(2)}$, $\mathcal{H}^{(1)}$; $\cdots$ N^{k}LL ($\sim \alpha_S^n L^{n+k-1}$): $g^{(k+1)}$, $\mathcal{H}^{(k)}$;

Resummed result at small q_T *matched* with corresponding fixed "finite" part at large q_T: *uniform accuracy* for $q_T \ll M$ and $q_T \sim M$.

- Resummed effects exponentiated in a universal of Sudakov form factor, process-dependence factorized in the hard-virtual factor $H_c^F(\alpha_S)$ via all-order formula [Catani,Cieri,de Florian,G.F.,Grazzini('14)].
- Resummation performed at partonic cross section level: (collinear) PDF evaluated at $\mu_F \sim M$, $f_N(b_0^2/b^2) = \exp\left\{-\int_{b_0^2/b^2}^{\mu_F^2} \frac{dq^2}{q^2}\,\gamma_N(\alpha_S(q^2))\right\} f_N(\mu_F^2)$: no PDF extrapolation in the non perturbative region, study of μ_R and μ_F dependence as in fixed-order calculations.
- No need for NP models: Landau singularity of α_S regularized using a *Minimal Prescription* without power-suppressed corrections [Laenen et al.('00)],[Catani et al.('96)].
- Introduction of resummation scale $Q \sim M$: variations give an estimate of the uncertainty from uncalculated logarithmic corrections.

$$\ln\left(M^2 b^2\right) \to \ln\left(Q^2 b^2\right) + \ln\left(M^2/Q^2\right)$$

- Perturbative unitarity constraint: recover *exactly* the total cross-section (upon integration on q_T)

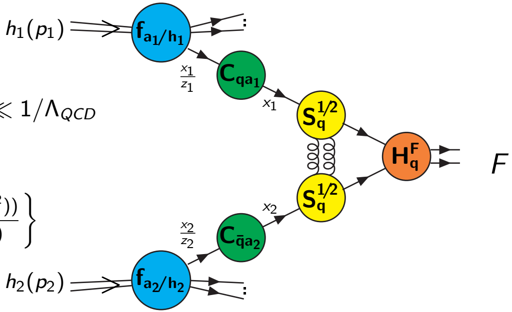
$$\ln\left(Q^2 b^2\right) \to \widetilde{L} \equiv \ln\left(Q^2 b^2 + 1\right) \quad \Rightarrow \quad \exp\left\{\alpha_S^n \widetilde{L}^k\right\}\Big|_{b=0} = 1 \;\Rightarrow\; \int_0^\infty dq_T^2 \left(\frac{d\hat{\sigma}}{dq_T^2}\right) = \hat{\sigma}^{(tot)};$$

- General procedure to treat the q_T recoil [Catani,de Florian,G.F.,Grazzini('15)]:

$$\frac{d\hat{\sigma}^{(0)}}{d\mathbf{\Omega}} = \hat{\sigma}^{(0)}(M^2)\, F(\mathbf{q_T}; M^2, \mathbf{\Omega}) \;\; with \;\; F(\mathbf{q_T}; M^2, \mathbf{\Omega}) = F(\mathbf{0}; M^2, \mathbf{\Omega}) + \mathcal{O}(\mathbf{q_T^2}/M^2)$$

Connection with CSS and TMD formalisms

[Collins,Soper,Sterman('85)]



$$M \gg \Lambda_{QCD}\ , \quad b \gg 1/M\ , \quad b \ll 1/\Lambda_{QCD}$$

$$C(\alpha_S(b_0^2/b^2)) = C(\alpha_S(M^2))$$
$$\times\ \exp\left\{ -\int_{b_0^2/b^2}^{M^2} \frac{dq^2}{q^2}\ \beta(\alpha_S(q^2))\ \frac{d\ln C(\alpha_S(q^2))}{d\ln \alpha_S(q^2)} \right\}$$

$$\frac{d\sigma_F^{(res)}}{dq_T^2} = \frac{M^2}{s}\ \sigma_{q\bar{q},F}^{(0)}\ H_q^F(\alpha_S(M^2)) \sum_{a_1,a_2} \int \frac{d^2\mathbf{b}}{2\pi}\ e^{i\mathbf{b}\cdot\mathbf{q_T}}\ S_q(M,b)$$
$$\times \int_{x_1}^1 \frac{dz_1}{z_1}\ C_{qa_1}(z_1;\alpha_S(b_0^2/b^2))\ f_{a_1/h_1}(x_1/z_1, b_0^2/b^2) \int_{x_2}^1 \frac{dz_2}{z_2}\ C_{\bar{q}\,a_2}(z_2;\alpha_S(b_0^2/b^2))\ f_{a_2/h_2}(x_2/z_2, b_0^2/b^2)$$

$$\tilde{F}_{q_f/h}(x,b,M) = \sum_a \int_x^1 \frac{dz}{z}\ \sqrt{S_q(M,b)}\, C_{q_f\,a}(z;\alpha_S(b_0^2/b^2))\ f_{a/h}(x/z, b_0^2/b^2)$$

q_T resummation: perturbative accuracy

- We have implemented the calculation in the publicly available code:

  DYTurbo: computes resummed and fixed-order fiducial cross section and related distributions it retains full kinematics of the vector boson and of its leptonic decay products [Camarda,Boonekamp,Bozzi,Catani,Cieri,Cuth,G.F.,de Florian,Glazov, Grazzini,Vincter,Schott('20)]

  https://dyturbo.hepforge.org.

- We have explicitly included in DYTurbo up to:
  - N^4LL logarithmic contributions to all orders (i.e. up to $\exp(\sim\alpha_S^n L^{n-3})$);
  - Approximated N^4LO corrections (i.e. up to $\mathcal{O}(\alpha_S^4)$) at small q_T;
  - NLO corrections (i.e. up to $\mathcal{O}(\alpha_S^2)$) at large q_T;
- Matching with NNLO corrections (i.e. up to $\mathcal{O}(\alpha_S^3)$) at large q_T from results in [Boughezal et al.('16)], [Gehrmann-De Ridder et al.('16)], [MCFM ('23)];
- Results up to N^3LO (i.e. up to $\mathcal{O}(\alpha_S^3)$) recovered for the total cross section (from unitarity).

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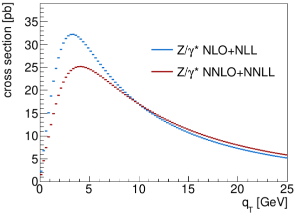
Fast predictions for Drell-Yan processes: DYTurbo

[Camarda, Boonekamp, Bozzi, Catani, Cieri, Cuth, G.F., de Florian, Glazov, Grazzini, Vincter, Schott ('20)]

Example calculation



- Example calculation for Z p_T spectrum at 13 TeV
  - No cuts on the leptons
  - Full rapidity range
  - 100 p_T bins
  - 20 parallel threads

Time required	RES	CT	V+jet
NLO+NLL	6 s	0.2 s	4 min
NNLO+NNLL	10 s	0.7 s	3.4 h

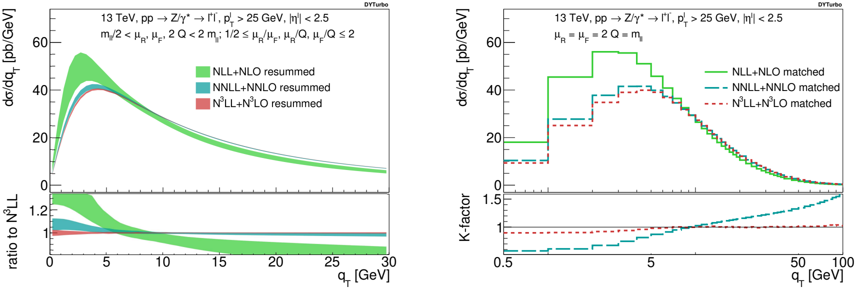
- The most demanding calculation is V+jet

  → can use APPLgrid/FASTnlo for this term

Stefano Camarda 11

Z/γ^* production at N^{3}LL+N^{3}LO (resummed and matched)

[Camarda,Cieri,G.F.('21)]

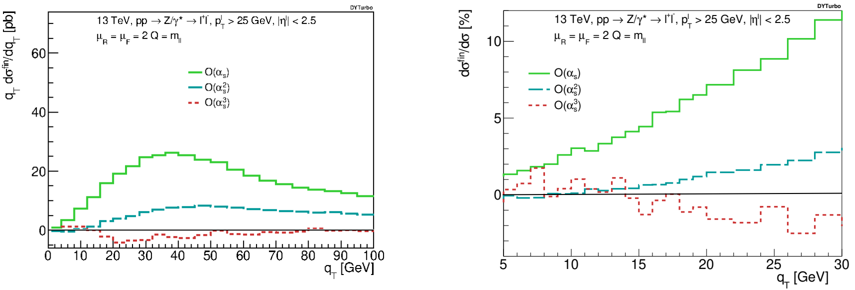


DYTurbo results. Resummed (left) and matched (right) NLL, NNLL and N^{3}LL bands for Z/γ^* q_T spectrum.

Lower panel: ratio with respect to the N^{3}LL central value.

Z/γ^* production: finite part

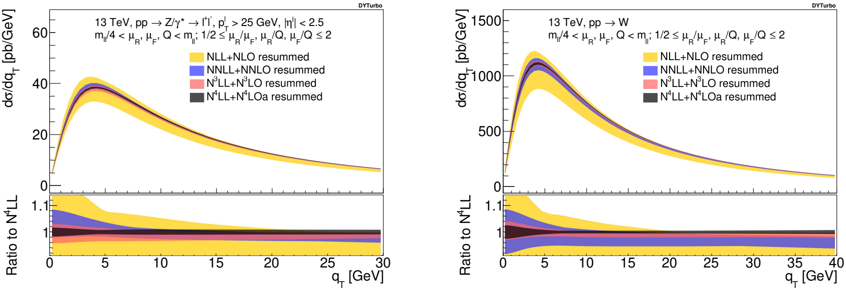
[Camarda,Cieri,G.F.('21)]



Finite part at $\mathcal{O}(\alpha_S)$, $\mathcal{O}(\alpha_S^2)$ and $\mathcal{O}(\alpha_S^3)$ (left) and ratio wrt matched results (right).

Z/γ^* and W production at N^{4}LL+N^{4}LOa resummed

[Camarda,Cieri,G.F.('23)]

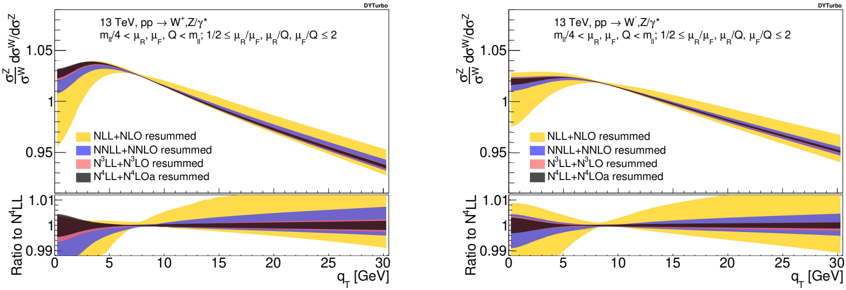


DYTurbo results. Resummed NLL, NNLL, N^{3}LL and N^{4}LLa bands for Z/γ^* (left) and W (right) q_T spectrum.

Lower panel: ratio with respect to the N^{4}LLa central value.

Z/γ^* and W ratio at N^{4}LL+N^{4}LOa resummed

[Camarda,Cieri,G.F.('23)]

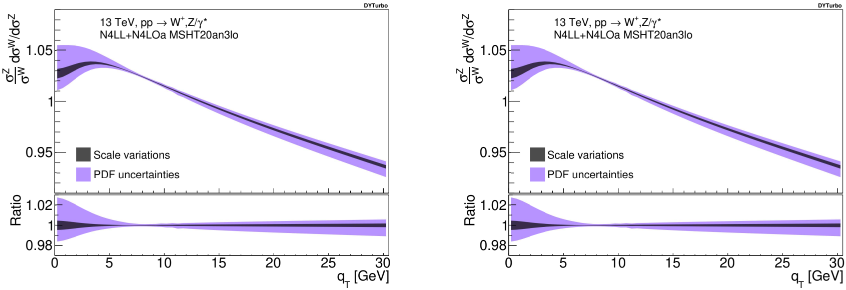


DYTurbo results. Resummed NLL, NNLL, N^{3}LL and N^{4}LLa bands for q_T spectrum of W^+ (left) and W^- (right) over Z/γ^* ratio.

Lower panel: ratio with respect to the N^{4}LLa central value.

Z/γ^* and W ratio at N^{4}LL+N^{4}LOa resummed

[Camarda,Cieri,G.F.('23)]



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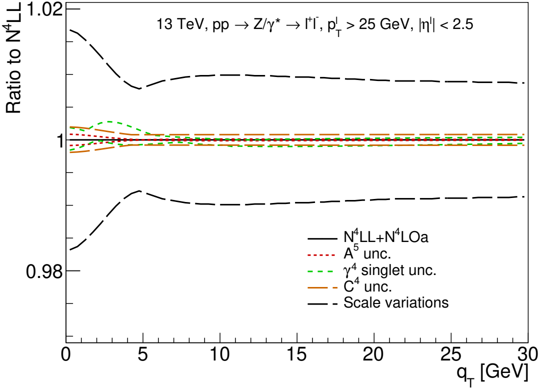
Lower panel: ratio with respect to the N^{4}LLa central value.

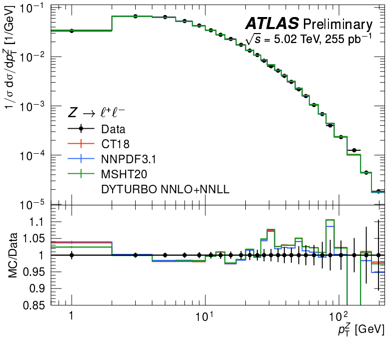
Z/γ^* at N^{4}LL+N^{4}LOa: numerical uncertainties

[Camarda,Cieri,G.F.('23)]

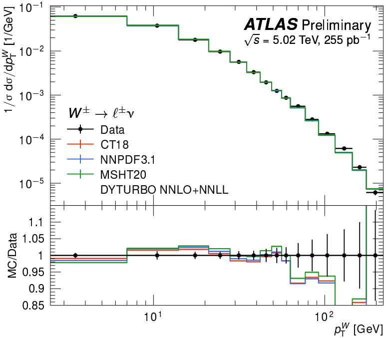


Uncertainties from approximations of the perturbative coefficients at N4LL+N4LOa compared to scale variations.

DYTurbo vs LHC data comparison



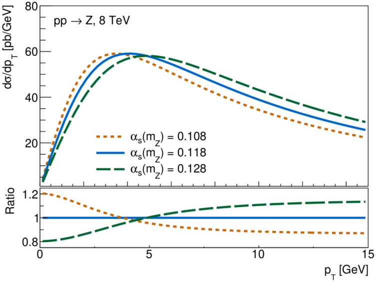
NNLL+NNLO `DYTurbo` predictions for Z qt spectrum with different PDF sets compared with data [ATLAS Coll.('23)]



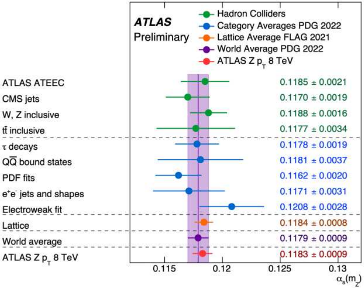
NNLL+NNLO `DYTurbo` predictions for W qt spectrum with different PDF sets compared with data [ATLAS Coll.('23)]

Modelling Z production for α_S and determination

→ see F. Giuli talk

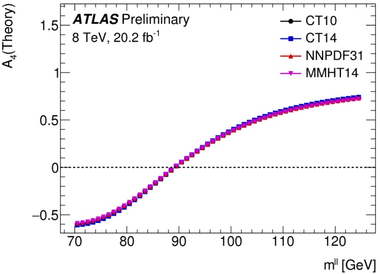


Z q_T distribution `DYTurbo` at different values of $\alpha_S(m_Z)$.

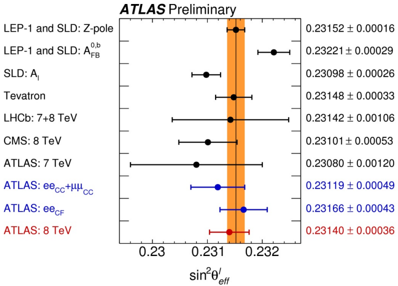


Comparison of determinations of $\alpha_S(m_Z)$.

Modelling Z production for $\sin^2\theta^l_{eff}$ determination



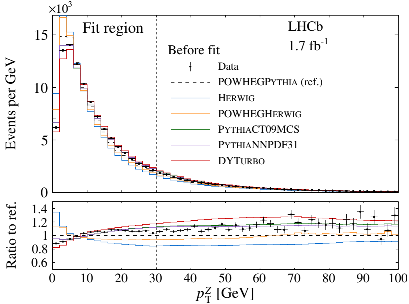
M_{ll} distribution for angular coefficient A_4 predicted with DYTurbo.



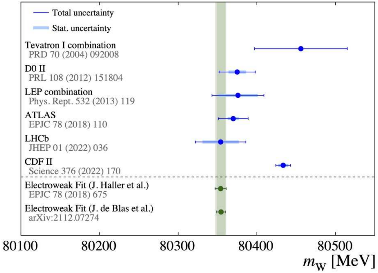
Comparison of the measurements of the $\sin^2\theta^l_{eff}$.

Modelling W and Z production for M_W determination

→ see H. Yin talk



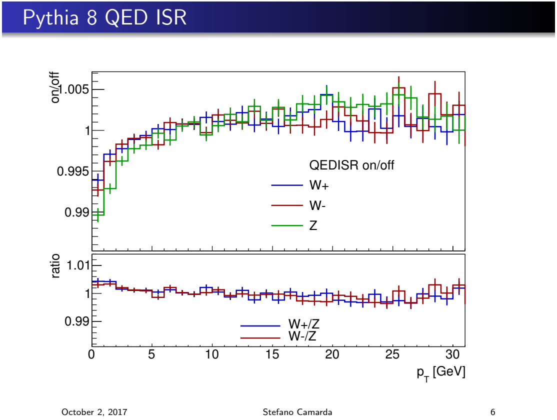
Z production at the LHC [LHCb Coll. ('22)]. LHCb data and Z q_T distribution for the different candidate models compared with LHCb data.



Measured values of M_W compared with the prediction of from the global electroweak fit

Combining QED and QCD q_T resummation

W and Z q_T distributions sensitive to QED effects.



Combining QED and QCD q_T resummation

[Cieri,G.F.,Sborlini('18)]

We start considering QED contributions to the q_T spectrum in the case of colourless and **neutral** high mass systems, e.g. on-shell Z boson production

$$h_1 + h_2 \;\to\; Z^0 + X$$

In the impact parameter and Mellin spaces resummed partonic cross section reads:

$$\mathcal{W}_N(b,M) = \hat{\sigma}^{(0)}\, \mathcal{H}'_N(\alpha_S,\alpha) \times \exp\left\{\mathcal{G}'_N(\alpha_S,\alpha,L)\right\}$$

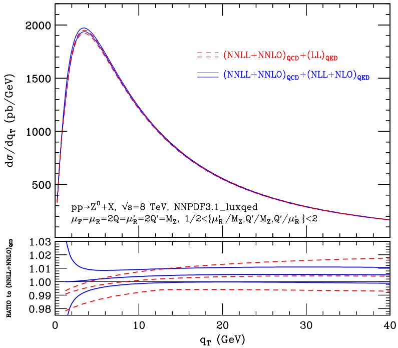
$$\mathcal{G}'(\alpha_S,\alpha,L) = \mathcal{G}(\alpha_S,L) + \; L\, g'^{(1)}(\alpha L) + g'^{(2)}(\alpha L) + \sum_{n=3}^{\infty} \left(\frac{\alpha}{\pi}\right)^{n-2} g'^{(n)}(\alpha L)$$

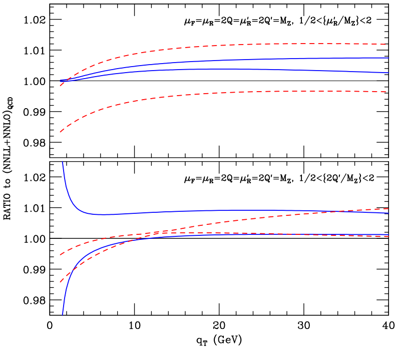
$$+\; g'^{(1,1)}(\alpha_S L,\alpha L) + \sum_{\substack{n,m=1 \\ n+m\neq 2}}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^{n-1} \left(\frac{\alpha}{\pi}\right)^{m-1} g_N'^{(n,m)}(\alpha_S L,\alpha L)$$

$$\mathcal{H}'(\alpha_S,\alpha) \;=\; \mathcal{H}(\alpha_S) + \frac{\alpha}{\pi}\mathcal{H}'^{(1)} + \sum_{n=2}^{\infty} \left(\frac{\alpha}{\pi}\right)^{n} \mathcal{H}_N'^{(n)} + \sum_{n,m=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^{n} \left(\frac{\alpha}{\pi}\right)^{m} \mathcal{H}_N'^{F\,(n,m)}$$

LL QED ($\sim\alpha^n L^{n+1}$): $g'^{(1)}$; NLL QED ($\sim\alpha^n L^n$): $g'^{(2)}$, $\mathcal{H}'^{(1)}$;
LL mixed QCD-QED ($\sim\alpha_S^n\alpha^n L^{2n}$): $g'^{(1,1)}$;

Combined QED and QCD q_T resummation for Z production at the LHC [Cieri,G.F.,Sborlini ('18)]

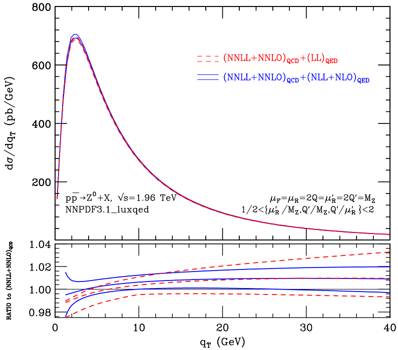


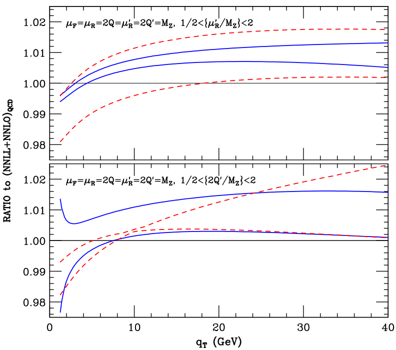


Z qT spectrum at the LHC. NNLL+NNLO QCD combined with the LL (red dashed) and NLL+NLO (blue solid) QED with the corresponding QED uncertainty bands.

Ratio of the resummation (upper panel) and renormalization (lower panel) QED scale-dependent results with respect to the central value NNLL+NNLO QCD result.

Combined QED and QCD q_T resummation for Z production at the Tevatron [Cieri,G.F.,Sborlini ('18)]





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Combining QED and QCD q_T resummation for W production

[Autieri,Cieri,G.F.,Sborlini ('23)]

We next consider QED contributions to the q_T spectrum in the case of colourless and **charged** high mass systems, e.g. on-shell $W^\pm$ boson production

$$h_1 + h_2 \ \rightarrow \ W^\pm + X$$

- Initial state QED emissions sensitive to different quark charges ($q\bar{q}' \rightarrow W^\pm$):

$$2e_q^2 \rightarrow e_q^2 + e_{\bar{q}'}^2$$

- Final state QED emissions: *abelianizion* of QCD resummation formula q_T resummation for $t\bar{t}$ production [Catani,Grazzini,Torre('14)]:

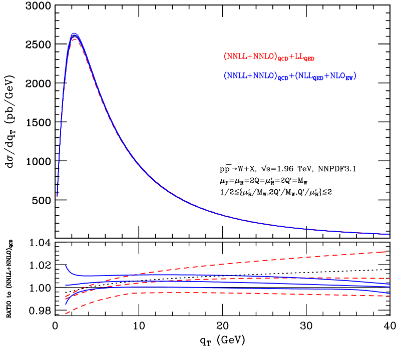
$$\Delta'(b, M) = \exp\left\{ - \int_{b_0^2/b^2}^{M^2} \frac{dq^2}{q^2} D'(\alpha(q^2)) \right\}$$

$$\text{with} \quad D'(\alpha) = \sum_{n=1}^{\infty} \left(\frac{\alpha}{\pi}\right)^n D'^{(n)} \,, \qquad \text{and} \qquad D'^{(1)} = -\frac{e^2}{2} \,.$$

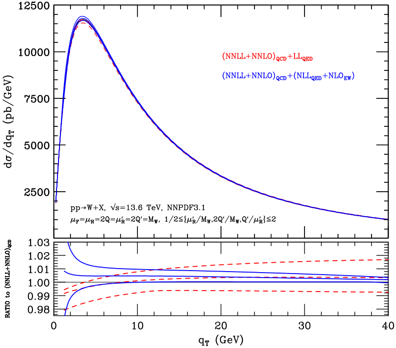
- Factor $\Delta'(b, M)$ resums soft (non collinear) QED emissions from final state (and from initial-final interference). Effects from $D'(\alpha)$ start to contribute at NLL. Same functional dependence, in terms of $g'^{(i)}$ functions, as the $B'(\alpha)$ term.

Combined QED and QCD q_T resummation on W production

[Autieri,Cieri,G.F.,Sborlini ('23)]



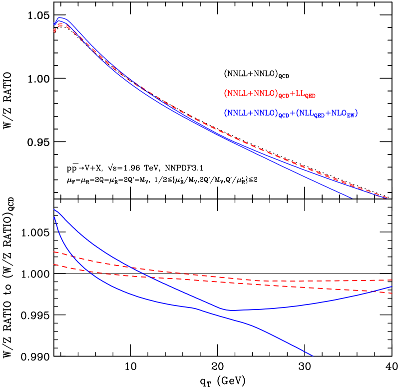
W qT spectrum at the Tevatron. NNLL+NNLO QCD results combined with the LL (red dashed) and NLL+NLO (blue solid) QED effects together with corresponding QED uncertainty bands.



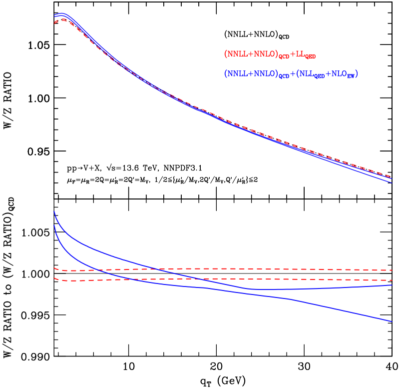
W qT spectrum at the LHC. NNLL+NNLO QCD results combined with the LL (red dashed) and NLL+NLO (blue solid) QED effects together with the corresponding QED uncertainty bands.

Combined QED and QCD q_T resummation on W/Z spectrum

[Autieri,Cieri,G.F.,Sborlini ('23)]



W over *Z qT* spectrum at the Tevatron. NNLL+NNLO QCD results combined with the LL (red dashed) and NLL+NLO (blue solid) QED effects together with the corresponding QED uncertainty bands.



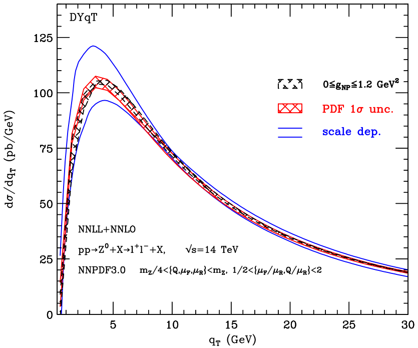
W over *Z qT* spectrum at the LHC. NNLL+NNLO QCD results combined with the LL (red dashed) and NLL+NLO (blue solid) QED effects together with the corresponding QED uncertainty bands.

Conclusions

- Discussed formalism to perform q_T resummed predictions and presented results for Drell–Yan production at the Tevatron and the LHC.
- Discussed perturbative uncertainty performing μ_R, μ_F and Q scale variation and its reduction going to higher accuracy.
- Discussed formalism to perform combined QCD and QED q_T resummation for Z and W production at hadron colliders.
- Presented a fast and numerically precise publicly available code `DYTurbo`: `https://dyturbo.hepforge.org`

Back up slides

PDFs uncertainties and NP effects



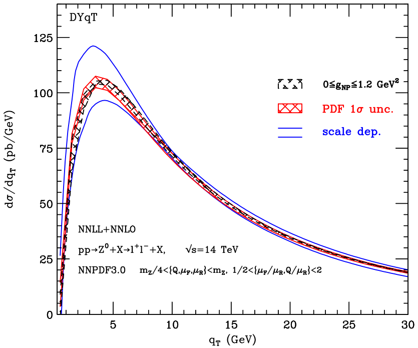
NNLL+NNLO result for Z q_T spectrum at the LHC at $\sqrt{s} = 14$ TeV. Perturbative scale dependence, PDF uncertainties and impact of NP effects.

- PDF uncertainty is smaller than the scale uncertainty and it is approximately independent on q_T (around the 3% level).
- Non perturbative *intrinsic* k_T effects parametrized by a NP form factor $S_{NP} = \exp\{-g_{NP} b^2\}$ with $0 < g_{NP} < 1.2\, GeV^2$:

$$\exp\{\mathcal{G}_N(\alpha_S, \widetilde{L})\} \quad \to \quad \exp\{\mathcal{G}_N(\alpha_S, \widetilde{L})\}\, S_{NP}$$

- NP effects increase the hardness of the q_T spectrum at small values of q_T. Non trivial interplay of perturbative and NP effects (higher-order contributions at small q_T can be mimicked by NP effects).
- NNLL+NNLO result with NP effects very close to perturbative result except for $q_T < 3GeV$ (i.e. below the peak).

PDFs uncertainties and NP effects



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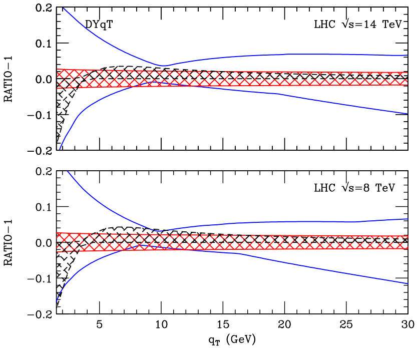

NNLL+NNLO result for Z q_T spectrum at the LHC at $\sqrt{s} = 14$ TeV (up) $\sqrt{s} = 8$ TeV (down). Perturbative scale dependence, PDF uncertainties and impact of NP effects normalized to central NNLL+NNLO prediction.

- PDF uncertainty is smaller than the scale uncertainty and it is approximately independent on q_T (around the 3% level).
- Non perturbative *intrinsic* k_T effects parametrized by a NP form factor $S_{NP} = \exp\{-g_{NP} b^2\}$ with $0 < g_{NP} < 1.2\ GeV^2$:

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Soft gluon exponentiation

Sudakov resummation feasible when:
dynamics AND kinematics factorize
$\Rightarrow$ exponentiation.

- Dynamics factorization: general propriety of QCD matrix element for soft emissions.

$$dw_n(q_1,\ldots,q_n) \simeq \frac{1}{n!}\prod_{i=1}^{n} dw_i(q_i)$$

- Kinematics factorization: not valid in general. For q_T distribution it holds in the impact parameter space (Fourier transform) [Parisi,Petronzio('79)]

$$\int d^2\mathbf{q_T}\,\exp(-i\mathbf{b}\cdot\mathbf{q_T})\,\delta^{(2)}\left(\mathbf{q_T}-\sum_{j=1}^{n}\mathbf{q_{T_j}}\right) = \exp(-i\mathbf{b}\cdot\sum_{j=1}^{n}\mathbf{q_{T_j}}) = \prod_{j=1}^{n}\exp(-i\mathbf{b}\cdot\mathbf{q_{T_j}})\,.$$

- Exponentiation holds in the impact parameter space. Results have then to be transformed back to the physical space: $q_T \ll M \Leftrightarrow Mb \gg 1$, $\log M/q_T \gg 1 \Leftrightarrow \log Mb \gg 1$.

The LL and NLL QED functions $g'^{(1)}$ and $g'^{(2)}$ has the same *functional* form of the QCD ones:

$$g'^{(1)}(\alpha L) = \frac{A_q'^{(1)}}{\beta_0'} \frac{\lambda' + \ln(1-\lambda')}{\lambda'} \quad ,$$

$$g_N'^{(2)}(\alpha L) = \frac{\widetilde{B}_{q,N}'^{(1)}}{\beta_0'} \ln(1-\lambda') - \frac{A_q'^{(2)}}{\beta_0'^2} \left(\frac{\lambda'}{1-\lambda'} + \ln(1-\lambda') \right)$$
$$+ \frac{A_q'^{(1)} \beta_1'}{\beta_0'^3} \left(\frac{1}{2} \ln^2(1-\lambda') + \frac{\ln(1-\lambda')}{1-\lambda'} + \frac{\lambda'}{1-\lambda'} \right) \quad ,$$

the *novel* LL mixed QCD-QED function reads:

$$g'^{(1,1)}(\alpha_S L, \alpha L) = \frac{A_q^{(1)} \beta_{0,1}}{\beta_0^2 \beta_0'} \, h(\lambda, \lambda') + \frac{A_q'^{(1)} \beta_{0,1}'}{\beta_0'^2 \beta_0} \, h(\lambda', \lambda) \quad ,$$

$$h(\lambda, \lambda') = -\frac{\lambda'}{\lambda - \lambda'} \ln(1-\lambda) + \ln(1-\lambda') \left[\frac{\lambda(1-\lambda')}{(1-\lambda)(\lambda-\lambda')} + \ln \left(\frac{-\lambda'(1-\lambda)}{\lambda - \lambda'} \right) \right]$$
$$- \mathrm{Li}_2 \left(\frac{\lambda}{\lambda - \lambda'} \right) + \mathrm{Li}_2 \left(\frac{\lambda(1-\lambda')}{\lambda - \lambda'} \right) ,$$

where $\lambda = \frac{1}{\pi} \beta_0 \, \alpha_S \, L$, $\lambda' = \frac{1}{\pi} \beta_0' \, \alpha \, L$, and β_0, β_0', β_1', $\beta_{0,1}$, $\beta_{0,1}'$ are the coefficients of the QCD and QED β functions.

Abelianization procedure

$$\frac{d\ln\alpha_S(\mu^2)}{d\ln\mu^2} = \beta(\alpha_S(\mu^2),\alpha(\mu^2)) = -\sum_{n=0}^{\infty}\beta_n\left(\frac{\alpha_S}{\pi}\right)^{n+1} - \sum_{n,m+1=0}^{\infty}\beta_{n,m}\left(\frac{\alpha_S}{\pi}\right)^{n+1}\left(\frac{\alpha}{\pi}\right)^{m},$$

$$\frac{d\ln\alpha(\mu^2)}{d\ln\mu^2} = \beta'(\alpha(\mu^2),\alpha_S(\mu^2)) = -\sum_{n=0}^{\infty}\beta'_n\left(\frac{\alpha}{\pi}\right)^{n+1} - \sum_{n,m+1=0}^{\infty}\beta'_{n,m}\left(\frac{\alpha}{\pi}\right)^{n+1}\left(\frac{\alpha_S}{\pi}\right)^{m}.$$

Novel QED coefficients obtained through an Abelianization algorithm

$$A_q'^{(1)} = e_q^2\,, \qquad A_q'^{(2)} = -\frac{5}{9}\,e_q^2\,N^{(2)} \qquad \widetilde{B}_{q,N}'^{(1)} = B_q'^{(1)} + 2\gamma_{qq,N}'^{(1)}\,,$$

$$\text{with}\quad B_q'^{(1)} = -\frac{3}{2}\,e_q^2\,,\quad N^{(n)} = N_c\sum_{q=1}^{n_f}e_q^n + \sum_{l=1}^{n_l}e_l^n\,,$$

$$\gamma_{qq,N}'^{(1)} = e_q^2\left(\frac{3}{4} + \frac{1}{2N(N+1)} - \gamma_E - \psi_0(N+1)\right)\,,\quad \gamma_{q\gamma,N}'^{(1)} = \frac{3}{2}\,e_q^2\,\frac{N^2+N+2}{N(N+1)(N+2)}\,.$$

$$\mathcal{H}'^{F\,(1)}_{q\bar{q}\leftarrow q\bar{q},N} = \frac{e_q^2}{2}\left(\frac{2}{N(N+1)} - 8 + \pi^2\right)\,,\quad \mathcal{H}'^{F\,(1)}_{q\bar{q}\leftarrow\gamma q,N} = \frac{3\,e_q^2}{(N+1)(N+2)}\,,$$

Resummed result *matched* with corresponding finite $\mathcal{O}(\alpha)$ term.

$\mathbf{q_T}$ recoil and lepton angular distribution

- The dependence of the resummed cross section on the leptonic variable $\mathbf{\Omega}$ is

$$\frac{d\hat{\sigma}^{(0)}}{d\mathbf{\Omega}} = \hat{\sigma}^{(0)}(M^2)\; F(\mathbf{q_T}/M; M^2, \mathbf{\Omega}) \;\;,\;\; \text{with} \;\; \int d\mathbf{\Omega}\; F(\mathbf{q_T}/M; \mathbf{\Omega}) = 1\;.$$

  the q_T dependence arise as a *dynamical* q_T-recoil of the vector boson due to *soft* and *collinear* multiparton emissions.

- This dependence cannot be *unambiguously* calculated through resummation (it is not singular)

$$F(\mathbf{q_T}/M; M^2, \mathbf{\Omega}) = F(\mathbf{0}/M; M^2, \mathbf{\Omega}) + \mathcal{O}(\mathbf{q_T}/M) \;\;,$$

  - After matching between *resummed* and *finite* component the $\mathcal{O}(\mathbf{q_T^2}/M^2)$ ambiguity start at $\mathcal{O}(\alpha_S^3)$ $\left(\mathcal{O}(\alpha_S^2)\right)$ at NNLL+NNLO (NLL+NLO).
  - After integration over leptonic variable $\mathbf{\Omega}$ the ambiguity *completely cancel*.

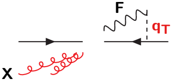
- A general procedure to treat the q_T recoil in q_T resummed calculations introduced in [Catani,de Florian,G.F.,Grazzini('15)].

- This procedure is directly related to the choice of a particular (among the infinite ones) vector boson rest frame to generate the lepton momenta: e.g. the Collins–Soper rest frame.

The q_T-subtraction method

[Catani,Grazzini('07)]

$$h_1(p_1) + h_2(p_2) \rightarrow F(M, q_T) + X$$



- **Observation:** at LO the q_T of the F is exactly zero.

$$d\sigma^{F}_{N^nLO}|_{q_T \neq 0} = d\sigma^{F+jets}_{N^{n-1}LO} \quad ,$$

for $q_T \neq 0$ the N^{n}LO IR sing. cancelled with the N^{n-1}LO subtraction method.

- **Key point**: treat $q_T = 0$ exploiting universality q_T resummation formalism

$$d\sigma^{F}_{N^nLO} = \mathcal{H}^{F}_{N^nLO} \otimes d\sigma^{F}_{LO} + \left[d\sigma^{F+jets}_{N^{n-1}LO} - d\sigma^{CT}_{N^{n-1}LO} \right] \quad ,$$

where

$$d\sigma^{CT}_{N^nLO} \xrightarrow{q_T \to 0} d\sigma^{F}_{LO} \otimes \sum_{n=1}^{\infty} \sum_{k=1}^{2n} \left(\frac{\alpha_S}{\pi} \right)^n \Sigma^{(n,k)} \frac{M^2}{q_T^2} \ln^{k-1} \frac{M^2}{q_T^2} d^2 q_T$$

$\mathcal{H}^{F}_{N^nLO}(\alpha_S)$ contains *multi-loop virtual* corrections via an *universal* factorization formula [Catani,Cieri,de Florian,G.F.,Grazzini('14)].

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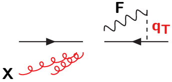

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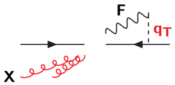

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Fiducial power corrections within the q_T subtraction

[Camarda,Cieri,G.F.('21)]

$$\sigma^F_{fid} = \int_{cuts} \mathcal{H}^F \otimes d\sigma^F_{LO} + \int_{cuts} \left[d\sigma^{F+\text{jets}}_{q_T > q_T^{cut}} - d\sigma^{CT}_{q_T > q_T^{cut}} \right] + \mathcal{O}\left((q_T^{cut}/M)^p \right)$$

- $d\sigma^{F+\text{jets}}$ and $d\sigma^{CT}$ are *separately* divergent, their sum is finite. A lower limit $q_T > q_T^{cut}$ is necessary with a power correction ambiguity, typically (standard fiducial cuts) linear $\mathcal{O}\left((q_T^{cut}/M) \right)$ [Alekhin et al.('21)]..
- The limit $q_T^{cut} \to 0$ leads to large cancellations and numerical uncertainties.
- **Key point**: "Fiducial" power corrections (FPC) absent including with

$$d\sigma^{FPC} = \left[d\widetilde{\sigma}^{CT}_{q_T < q_T^{cut}} - d\sigma^{CT}_{q_T < q_T^{cut}} \right]$$

  where $d\widetilde{\sigma}^{CT}$ is the q_T subtraction counteterm evaluated with q_T recoil defined in [Catani,de Florian,G.F.,Grazzini('15)].
- $d\sigma^{FPC}$ is *universal* and IR finite and can be treated as a *local* subtraction: integration for $q_T < q_T^{cut}$ extended at arbitrary small q_T (e.g. $q_T/M \sim 10^{-6}$ GeV)
- ("Recoil") q_T subtraction implemented in DYTurbo.
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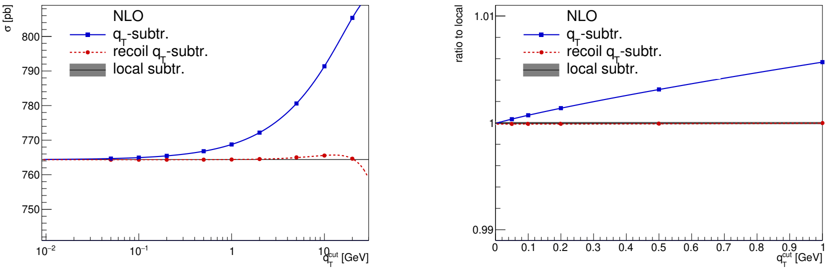
Fiducial power corrections at NLO

Z/γ^* production and decay at the LHC (13 TeV).
CUTS on leptons: $p_T > 25$ GeV, $|\eta| < 2.5$, $66 < M_{ll} < 116$ GeV,
$q_T < 100$ GeV.



NLO results with the q_T subtraction method (blue squared points) and q_T subtraction method without FPC (red circled points) at various values of q_T^{cut}, and with a local subtraction method (black line).

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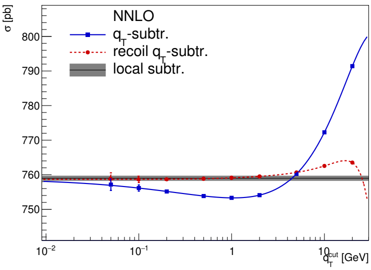

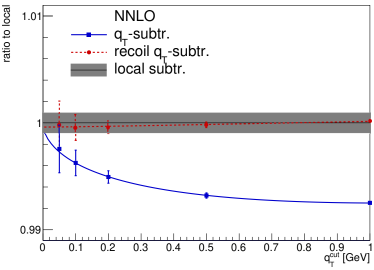

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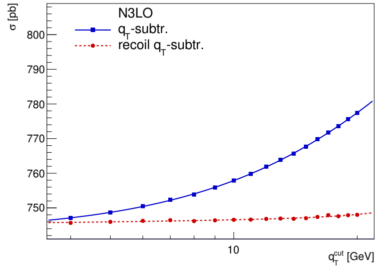

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CUTS on leptons: $p_T > 25$ GeV,

$|\eta| < 2.5$, $66 < M_{ll} < 116$ GeV,

$q_T < 100$ GeV.

N^{3}LO results with the q_T subtraction method (blue squared points) and q_T subtraction method without FPC (red circled points) at various values of q_T^{cut}.

Universality in q_T resummation

The resummation formula is invariant under the *resummation scheme* transformations [Catani, de Florian, Grazzini('01)] (for $h_c(\alpha_S) = 1 + \sum_{n=1}^{\infty} \alpha_S^n h_c^{(n)}$):

$$
\begin{aligned}
H_c^F(\alpha_S) &\rightarrow H_c^F(\alpha_S)\ [\, h_c(\alpha_S)\,]^{-1} \ , \\
B_c(\alpha_S) &\rightarrow B_c(\alpha_S) - \beta(\alpha_S)\ \frac{d \ln h_c(\alpha_S)}{d \ln \alpha_S} \ , \\
C_{cb}(z, \alpha_S) &\rightarrow C_{cb}(z, \alpha_S)\ [\, h_c(\alpha_S)\,]^{1/2} \ .
\end{aligned}
$$

- This implies that H_c^F, S_c (B_c) and C_{cb} not unambiguously computable separately.
- Resummation scheme: define H_c^F (or C_{ab}) for *single* processes (one for $q\bar{q} \to F$ one for $gg \to F$) and unambiguously determine the process-dependent H_c^F and the universal (process-independent) S_c and C_{ab} for any other process.
- *DY/H resummation scheme:* $H_q^{DY}(\alpha_S) \equiv 1\,,\ \ H_g^H(\alpha_S) \equiv 1\,.$
  Hard resummation scheme: $C_{ab}^{(n)}(z)$ for $n \geq 1$ do not contain any $\delta(1-z)$ term (other than plus distributions).
- $H_c^F(\alpha_S) = 1$ (i.e. $h_c(\alpha_S) = H_c^F(\alpha_S)$) *does not* correspond to a resummation scheme (S_c^F and C_{ab}^F would be process dependent, [de Florian, Grazzini('00)]).

Hard-collinear functions

The function $\mathcal{H}^F(\alpha_S)$ (process dependent and resummation-scheme independent) includes the hard-collinear contributions and it can be written in Mellin space as:

$$\mathcal{H}^F(\alpha_S) = H^F(\alpha_S)\, C(\alpha_S)\, C(\alpha_S)\,.$$

The functions $H^F(\alpha_S)$ and $C(\alpha_S)$ are scheme dependent (a simple resummation scheme is: $H^{DY}(\alpha_S) \equiv 1$, i.e. $H^{DY(n)} = 0$ for $n > 0$):

$$\mathcal{H}^F(\alpha_S) = 1 + \sum_{n=1}^{\infty} \alpha_S^n\, \mathcal{H}^{F(n)}, \quad H^F(\alpha_S) = 1 + \sum_{n=1}^{\infty} \alpha_S^n\, H^{F(n)}, \quad C(\alpha_S) = 1 + \sum_{n=1}^{\infty} \alpha_S^n\, C^{(n)}\,,$$

therefore

$$\begin{aligned}
\mathcal{H}^{F(1)} &= H^{F(1)} + C^{(1)} + C^{(1)} \\
\mathcal{H}^{F(2)} &= H^{F(2)} + C^{(2)} + C^{(2)} + H^{F(1)}(C^{(1)} + C^{(1)}) + C^{(1)}C^{(1)} \\
\mathcal{H}^{F(3)} &= H^{F(3)} + C^{(3)} + C^{(3)} + H^{F(2)}(C^{(1)} + C^{(1)}) + H^{F(1)}(C^{(2)} + C^{(2)} + C^{(1)}C^{(1)}) \\
&\quad + C^{(2)}C^{(1)} + C^{(2)}C^{(1)} \\
\mathcal{H}^{F(4)} &= H^{F(4)} + C^{(4)} + C^{(4)} + H^{F(3)}(C^{(1)} + C^{(1)}) + H^{F(2)}(C^{(2)} + C^{(2)} + C^{(1)}C^{(1)}) \\
&\quad + H^{F(1)}(C^{(3)} + C^{(3)} + C^{(2)}C^{(1)} + C^{(2)}C^{(1)}) + C^{(3)}C^{(1)} + C^{(3)}C^{(1)} + C^{(2)}C^{(2)}
\end{aligned}$$

Universality of hard factors at all orders

[Catani,Cieri,de Florian,G.F.,Grazzini('14)]

- *Process-dependence* is fully encoded in the hard-virtual factor $H_c^F(\alpha_S)$.
- However $H_c^F(\alpha_S)$ has an *all-order universal* structure: it can be directly related to the virtual amplitude of the corresponding process $c(\hat{p}_1) + \bar{c}(\hat{p}_2) \to F(\{q_i\})$.

$$\mathcal{M}_{c\bar{c}\to F}(\hat{p}_1, \hat{p}_2; \{q_i\}) = \alpha_S^k \sum_{n=0}^{\infty} \left(\frac{\alpha_S}{2\pi}\right)^n \mathcal{M}^{(n)}_{c\bar{c}\to F}(\hat{p}_1, \hat{p}_2; \{q_i\}) \,,$$

renormalized virtual amplitude (UV finite but IR divergent).

$$\tilde{I}_c(\epsilon, M^2) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{2\pi}\right)^n \tilde{I}_c^{(n)}(\epsilon) \,,$$

IR subtraction *universal* operators (contain IR ϵ-poles and IR finite terms)

$$\widetilde{\mathcal{M}}_{c\bar{c}\to F}(\hat{p}_1, \hat{p}_2; \{q_i\}) = \left[1 - \tilde{I}_c(\epsilon, M^2)\right] \mathcal{M}_{c\bar{c}\to F}(\hat{p}_1, \hat{p}_2; \{q_i\}) \,,$$

hard-virtual subtracted amplitude (IR finite).

Hard factor is directly related to the all-loop virtual amplitude:

$$\alpha_S^{2k}(M^2)\, H_q^F(x_1 p_1, x_2 p_2; \mathbf{\Omega}; \alpha_S(M^2)) = \frac{|\widetilde{\mathcal{M}}_{q\bar{q}\to F}(x_1 p_1, x_2 p_2; \{q_i\})|^2}{|\mathcal{M}^{(0)}_{q\bar{q}\to F}(x_1 p_1, x_2 p_2; \{q_i\})|^2} \,,$$

This formula extended up to N3LL in the case of threshold resummation.

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