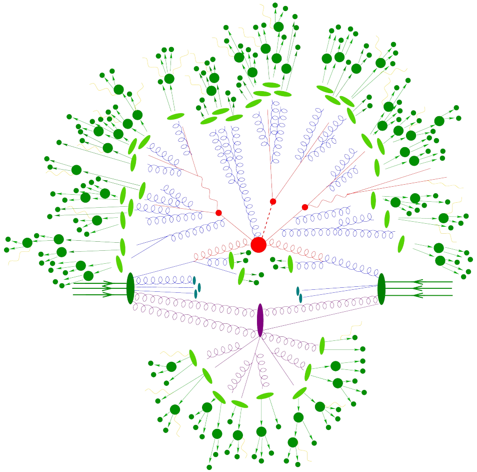
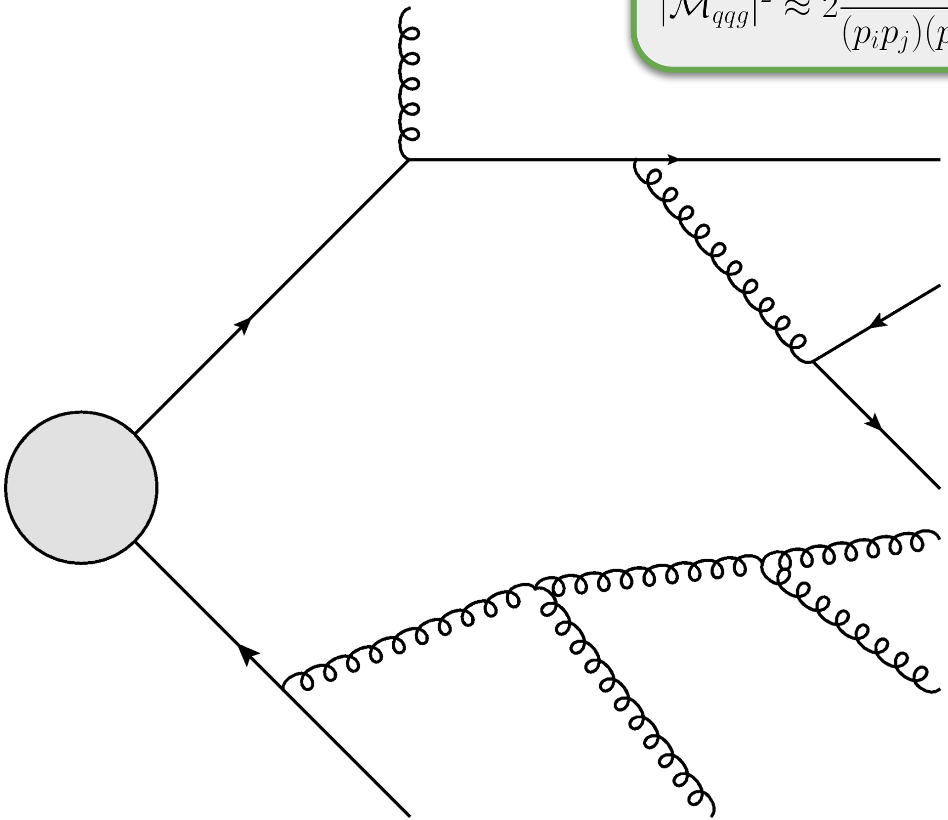


Alexander von Humboldt
Stiftung/Foundation

Progress on Parton Showers

Florian Herren

Parton Showers



PS dresses hard process with soft and collinear gluons/quarks;
Probabilities given by soft/collinear splitting kernels:

$$|\mathcal{M}_{qqg}|^2 \approx 2\frac{p_i p_k}{(p_i p_j)(p_j p_k)}|\mathcal{M}_{qq}|^2 \qquad |\mathcal{M}_{qqg}|^2 \approx \frac{P_{qq}(z)}{2(p_i p_j)}|\mathcal{M}_{qq}|^2$$

Momentum conservation must be enforced! Need on-shell momentum mapping from n+1 to n parton configurations

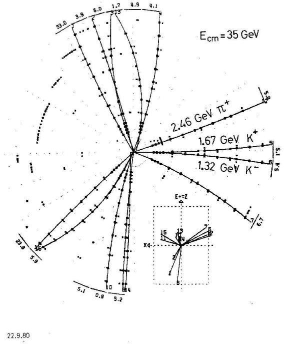
Interfacing to (N)NLO calculation needs PS matching to remove double counting $\longrightarrow$ See Talks by Emmanuele Re and Marius Wiesemann

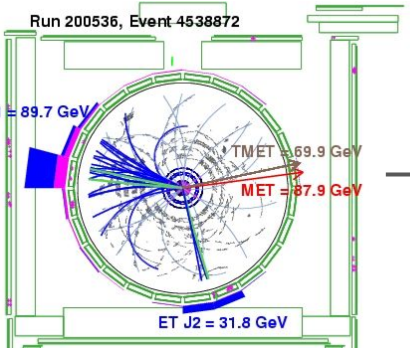
PS resums large logarithms, but to what order?

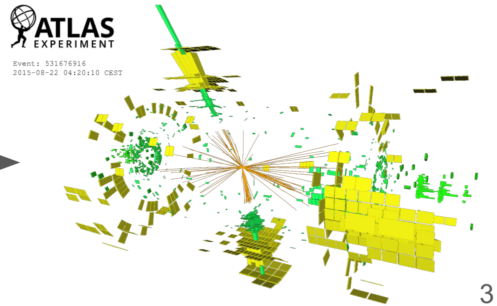
Parton Showers

Color coherent evolution
Backward evolution
ME corrections
Color dipole formalism
Spin correlations
Color dipole showers
LO merging
NLO matching
Dipole-like showers
Antenna showers
NLO merging
Color ME corrections
NNLO matching
Threshold effects
NLO splitting kernels
Higher-order PS framework
Accuracy quantification
Amplitude evolution
NLL & NGLs in one framework
Efficient 1/Nc corrections

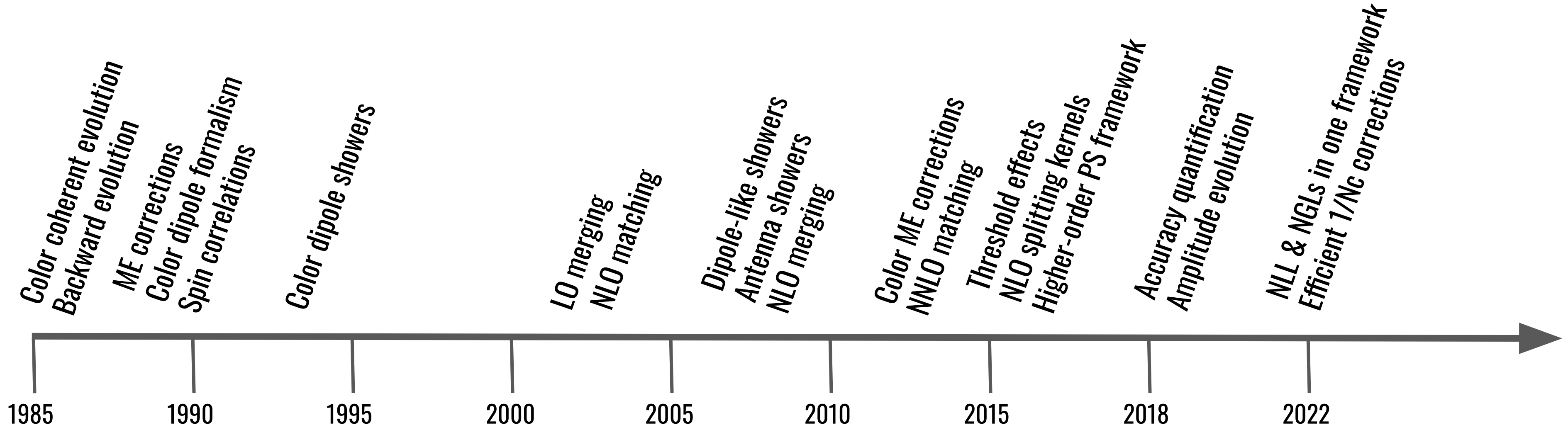
1985 1990 1995 2000 2005 2010 2015 2018 2022







Parton Showers



Many developments, but the basics are still the same!

Algorithms used in Practice

We have a good selection of Parton Showers for LHC simulations, allowing for cross-checks and some uncertainty estimates

Project	Evolution variable	Coherence	References
Herwig++	Angle or Dipole-$k_\perp$	Angular Ordering / Dipole	[Marchesini, Webber Nucl. Phys. B (1988), 461] [Corcella et al. arXiv:hep-ph/0011363]
Pythia	(Dipole-)$k_\perp$	Dipole	[Sjöstrand, Skands hep-ph/0408302] [Höche, Prestel 1506.05057] (Dire)
Sherpa	(Dipole-)$k_\perp$	Dipole	[Schumann, Krauss 0709.1027] [Höche, Prestel 1506.05057] (Dire)
Vincia	Dipole-$k_\perp$	(Sector) Antenna	[Giele, Kosower, Skands 0707.3652, 1102.2126]

The agreement between Vincia's antenna shower and the more standard dipole showers validates the dipole approximation

Comparisons

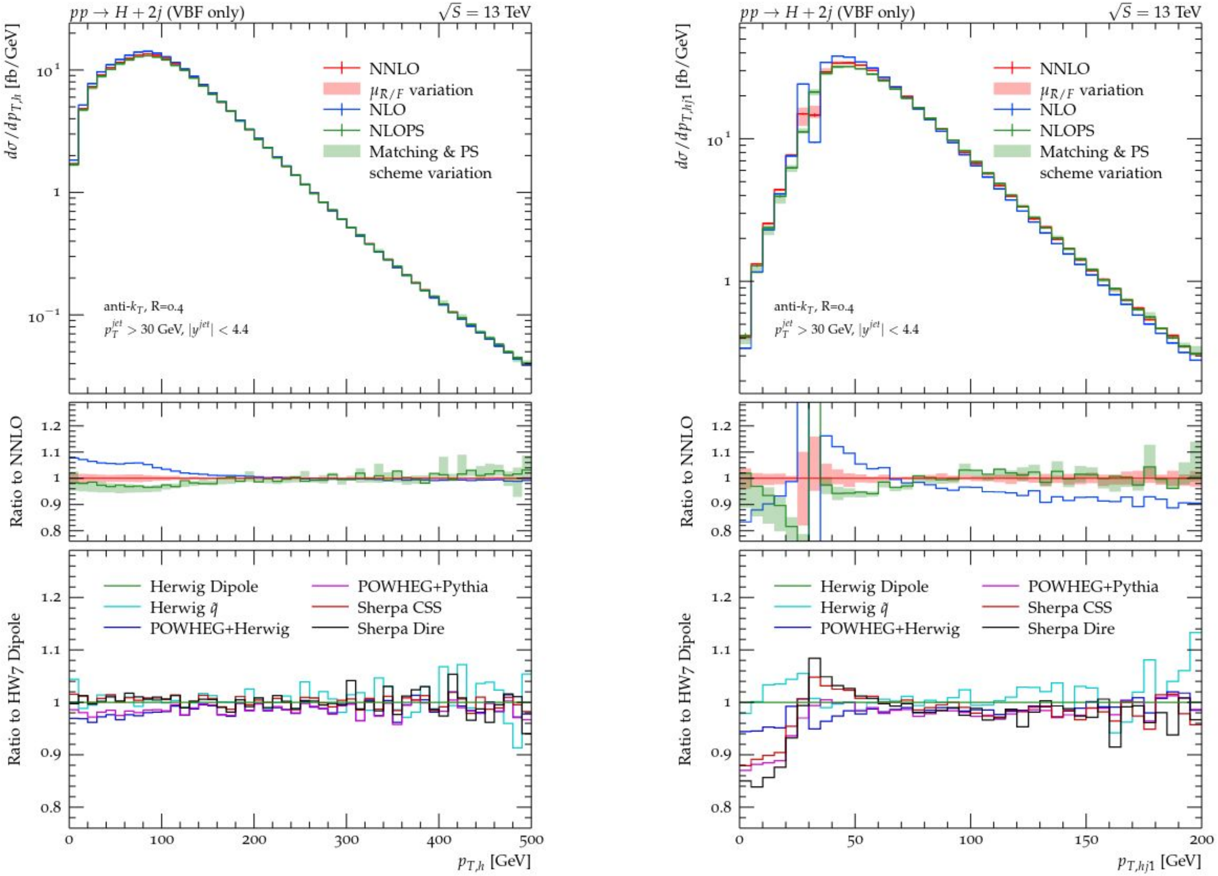
One way to assess the reliability of parton showers is to compare their predictions
[Buckley et al. 2105.11399]
[Bellm et al. 1903.12563]

→ Non-trivial, all parameters and hidden assumptions must be the same

NLOPS and NNLO agree reasonably well in most of the phase space

Throughout most of the phase space, different showers show reasonable agreement

PS variation are of similar size as scale variations



Where would we like to be in 10 years from now?

Comparing different parton showers to each other is not a good way to estimate uncertainties

→ We need (parametric) uncertainty bands!

We need to make use of the plethora of fixed-order calculations

→ Matching!

Parton Showers naively only capture the leading soft and collinear behaviour correctly

→ We need to study next-to-leading power corrections!

Where would we like to be in 10 years from now?

Comparing different parton showers to each other is not a good way to estimate uncertainties

→ We need (parametric) uncertainty bands!

→

The way to go is to construct parton showers at NLO and (N)NLL

→ Lot's of recent developments

We need to make use of the plethora of fixed-order calculations

→ Matching!

→

People are working towards NNLO matching and even N3LO!

→ Lot's of recent developments

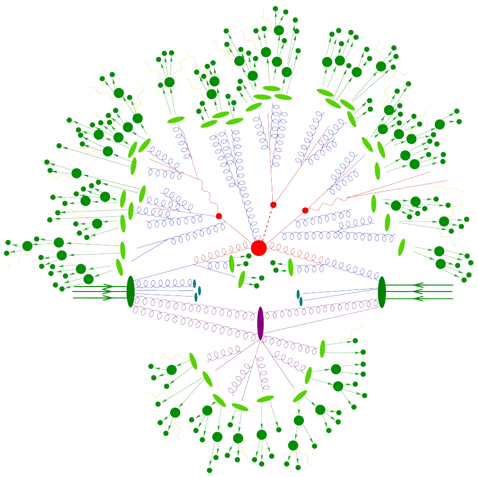
Parton Showers naively only capture the leading soft and collinear behaviour correctly

→ We need to study next-to-leading power corrections!

→

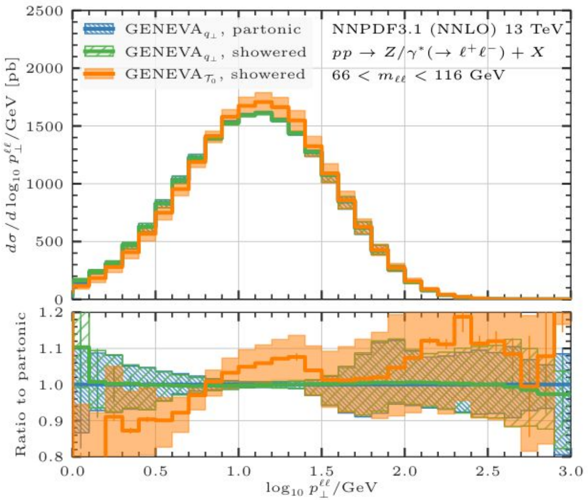
Systematic studies of subleading power effects in different kinematics mappings need to be conducted

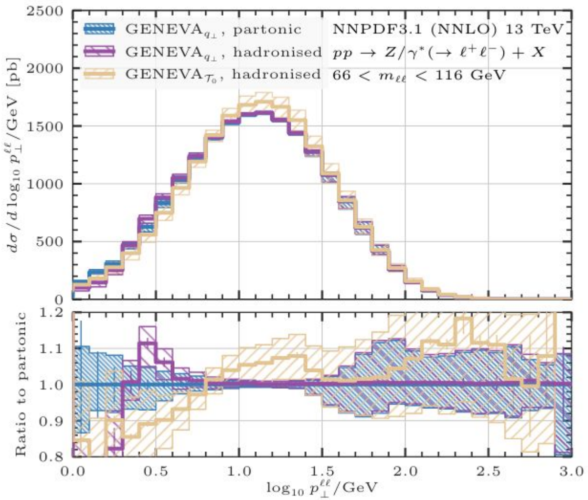
→ Sadly, not much progress

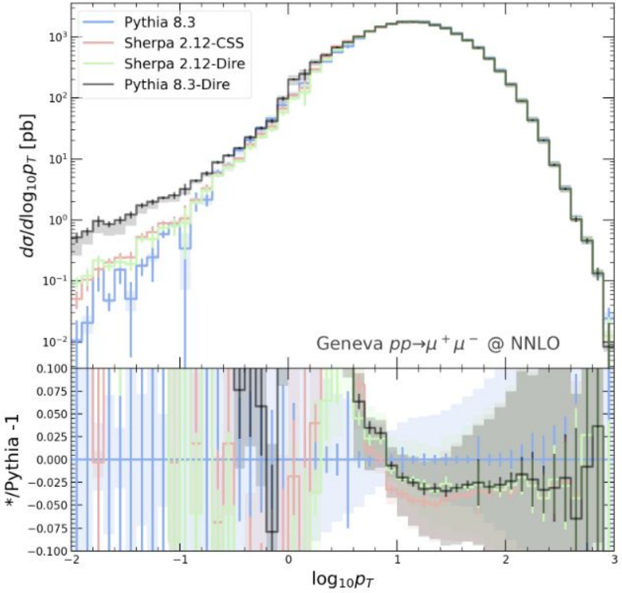


(N)NNLO Matching Updates

Matching Updates





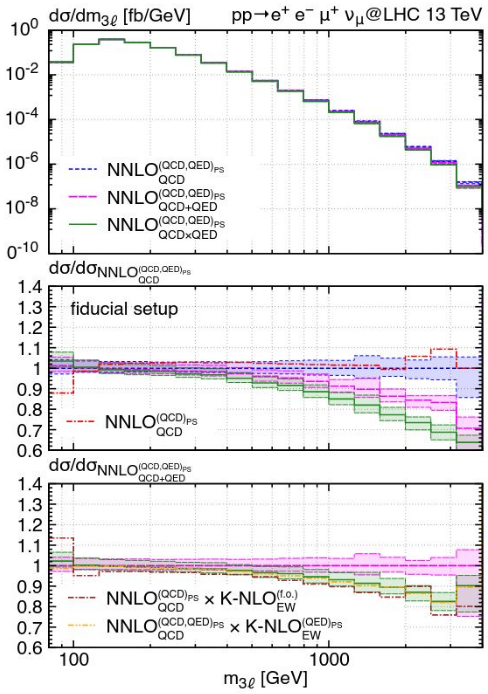


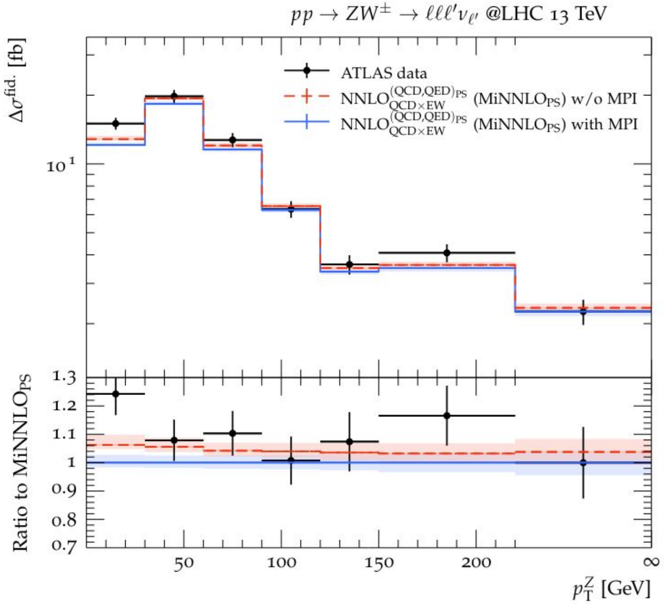
Geneva uses known resummation in jettiness/qT and matches to NNLO
[Aioli, Broggio, Gavardi, Kallweit, Lim, Nagar, Napoletano, Rottoli 2102.08390]

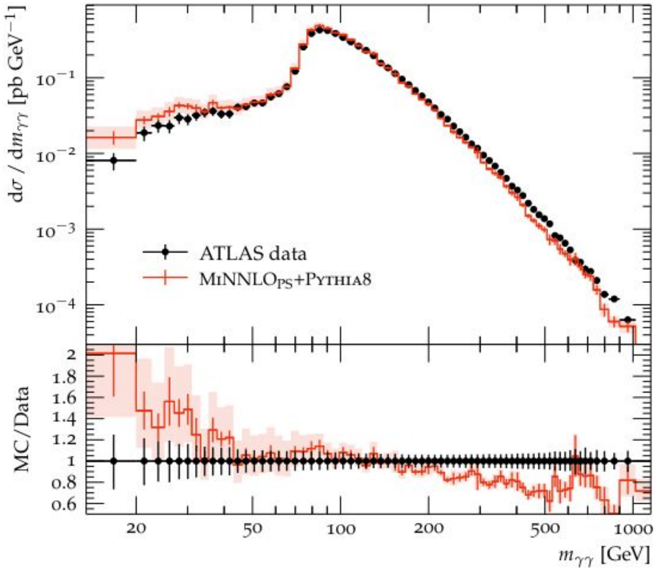
Allows choice of resolution variable and assessment of shower scheme uncertainty

See Davide Napoletano's Talk at HP2

Matching Updates







New results on di-photon production at NNLO [Galvari, Oleari, Re 2204.12602]

and WZ production at NNLO QCD/NLO EW using the MiNNLOPS Method [Lindert, Lombardi, Wiesemann, Zanderighi, Zanoli 2208.12660]

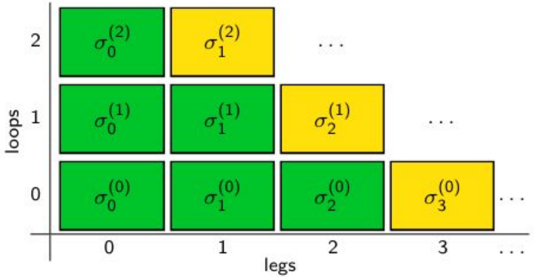
Matching Updates

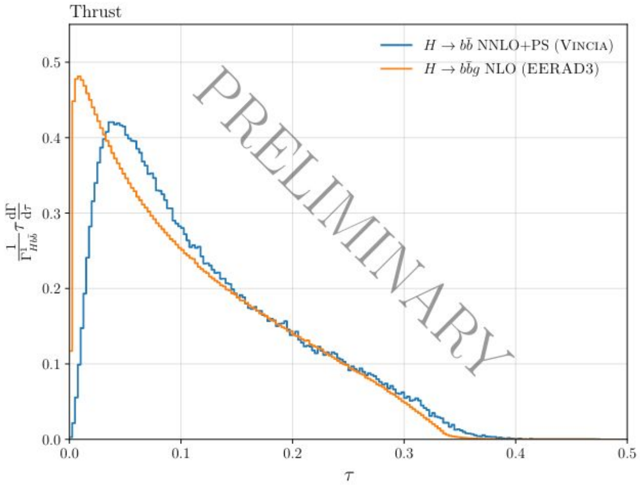
$$\langle O \rangle^{\text{VINCIA}}_{\text{NNLO+PS}} = \int \mathrm{d}\Phi_2 \, \mathrm{B}(\Phi_2) \, \underbrace{k_{\text{NNLO}}(\Phi_2)}_{\text{local } K\text{-factor}} \, \underbrace{\mathcal{S}_2(t_0, O)}_{\text{shower operator}}$$

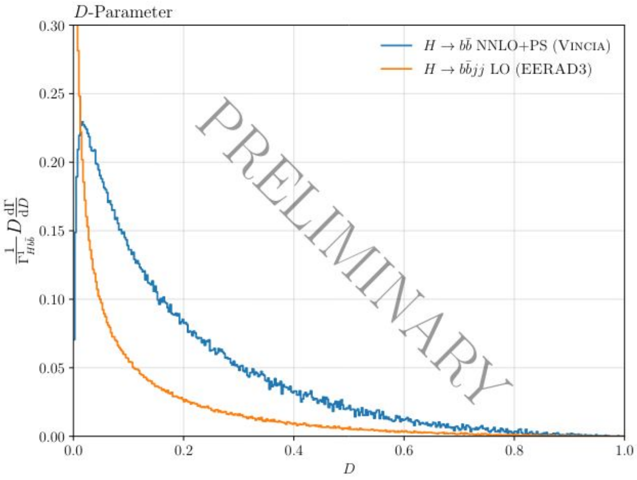
Work towards fully differential matching at NNLO in Vincia
[Campbell, Höche, Li, Preuss, Skands 2108.07133]

Shower matches NNLO singularity structure, “POWHEG @NNLO”

See Christian Preuss’ Talk at HP2





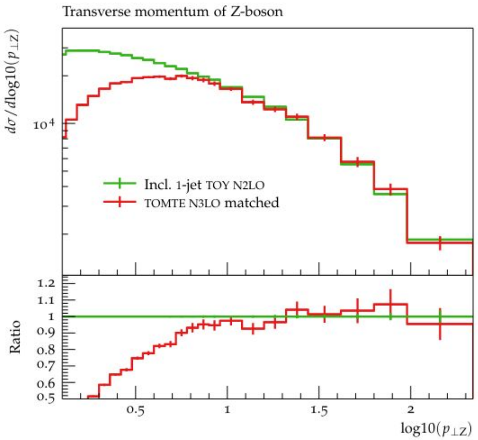


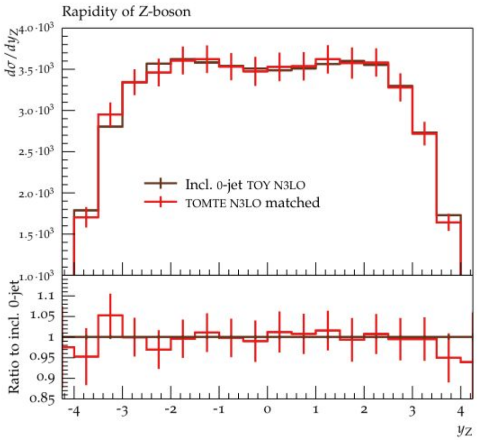
Matching Updates

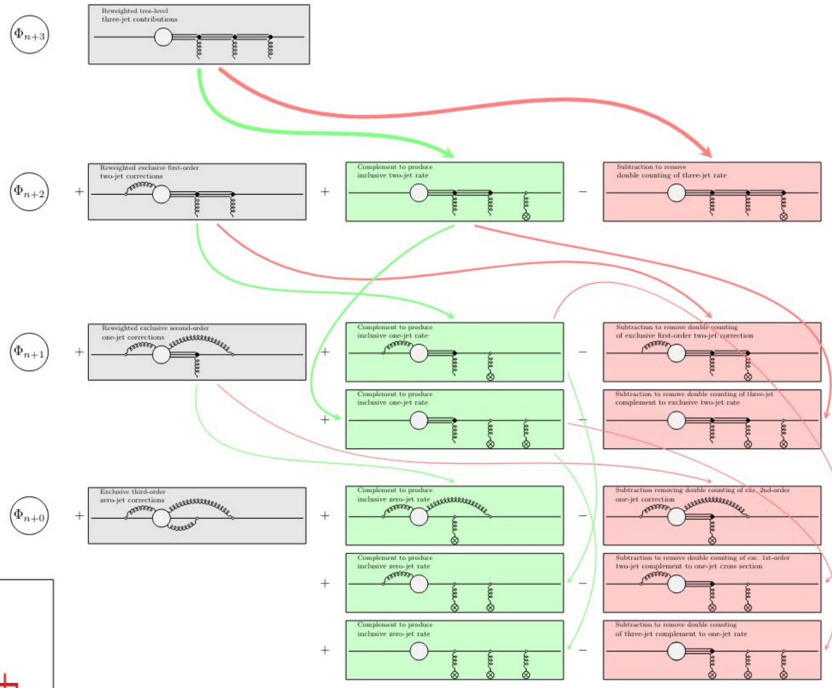
First N3LO parton shower matching:

TOMTE
[Prestel 2106.03206], [Bertone, Prestel 2202.01082]

- Matching to inclusive results
- Extension of UN2LOPS

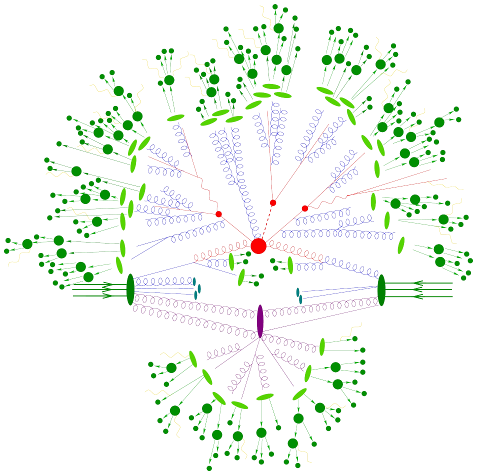






Shower precision not there yet, but we would like to use the best perturbative precision available

TOMTE can work with any parton shower!



NLL Parton Showers

NLL Showers

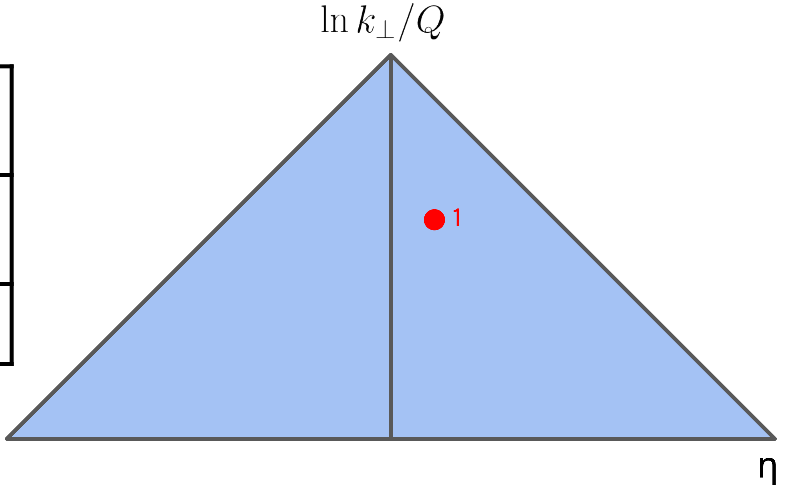
Criteria for NLL accuracy at leading color outlined in:
[Dasgupta, Dreyer, Hamilton, Monni, Salam, Soyez 2002.11114]

Where do the logarithms come from?
(see also [Banfi, Salam, Zanderighi hep-ph/0407286])

Shower needs to reproduce results of analytic resummation of rIRC observables

Depends on logarithmic variables of emission pairs:

Energies/Angles	Distinctly different	Comparable
Distinctly different	LL	NLL
Comparable	NLL	NNLL



Shower needs to reproduce the correct tree-level ME squared in these regions

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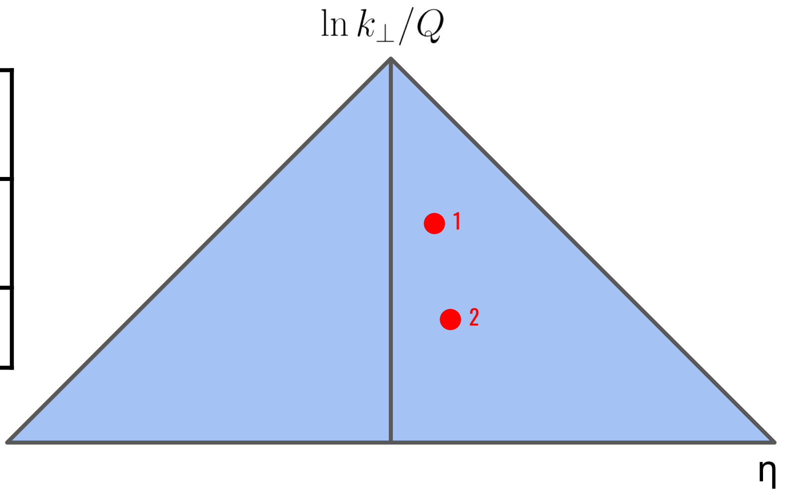

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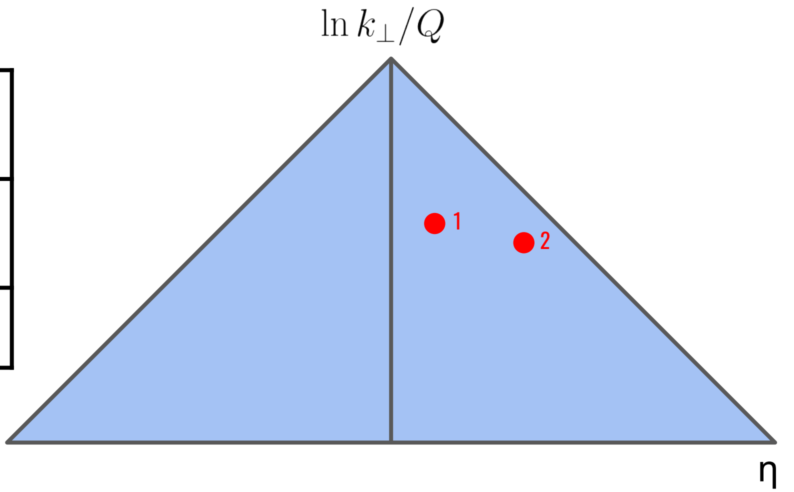

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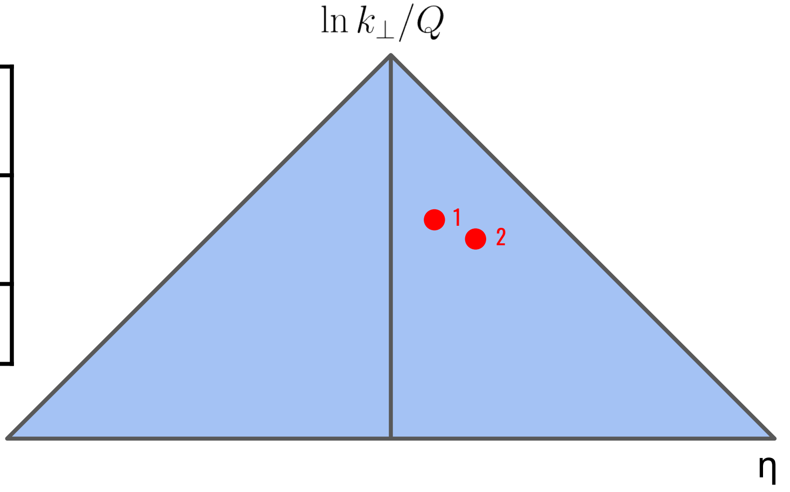


Shower needs to reproduce the correct tree-level ME squared in these regions

NLL Showers

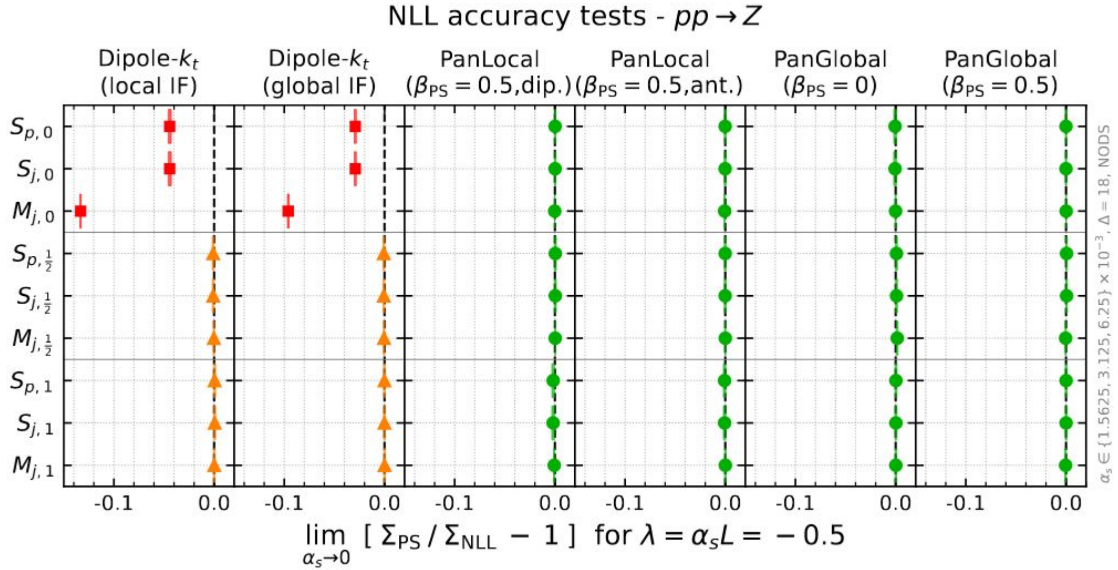
$$k = zp_+ + 2\frac{|k_\perp^2|}{2zp_+ \cdot p_-}p_- + k_\perp \qquad t \sim |k_\perp^2|e^{-2\beta y}$$

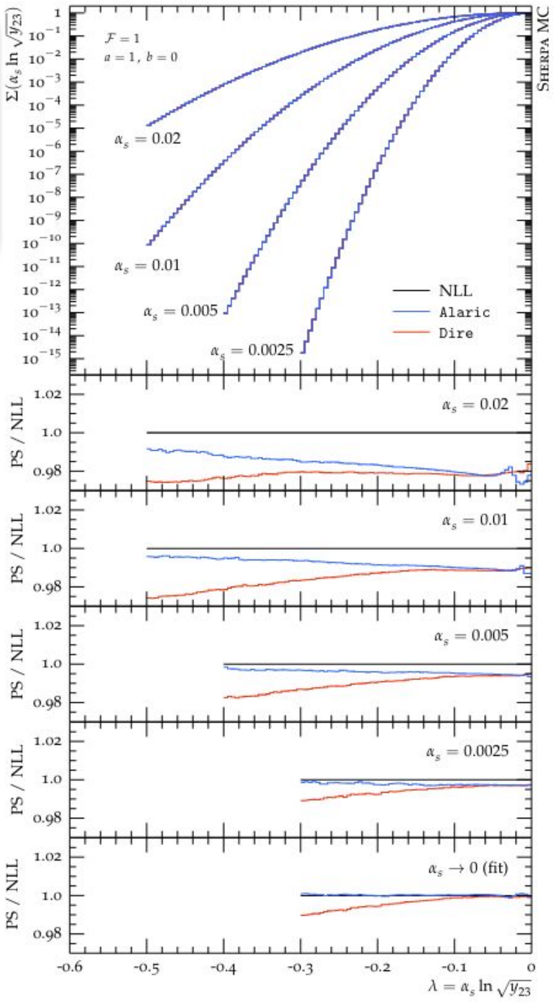
Project	Ordering	Recoil -	Recoil ⊥	Tests	Refs.
Herwig	Angle	Global	Local	Analytical for global observables, Phase space not covered in non-global case	[Marchesini, Webber Nucl. Phys. B (1988), 461]
PanLocal	$0 < \beta < 1$	Local	Local	Numerical tests in e+ e-, pp (colour singlet), DIS for a variety of global and non-global observables	[Dasgupta et al. 2002.11114, ...]
PanGlobal	$0 \leq \beta < 1$	Local	Global		
Deductor	$\beta = 1$	Global	Local	Analytical and numerical for thrust	[Nagy, Soper 2011.04777]
FHP	$\beta = 0$	Global	Global	Analytical and numerical for thrust, multiplicity	[Forshaw, Holguin, Plätzer 2003.06400]
Alaric	$\beta = 0$	Global	Global	General, analytical proof for any global rIRC safe observable; Numerical tests for LEP event shapes and y_{23}	[Herren et al. 2208.06057]

NLL Showers

PanScales has demonstrated NLL accuracy for a wide range of observables in colour singlet production, $e^+ e^-$ -> Hadrons and recently DIS/VBF

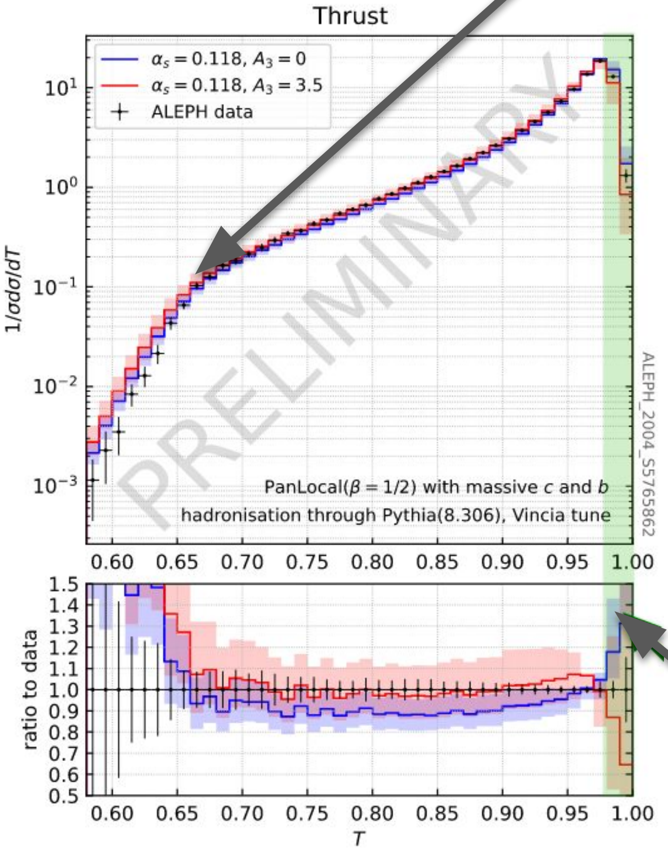
For Alaric an analytic proof of NLL accuracy for global observables exists (both for IS and FS evolution) & numerical tests in $e^+ e^-$ -> Hadrons

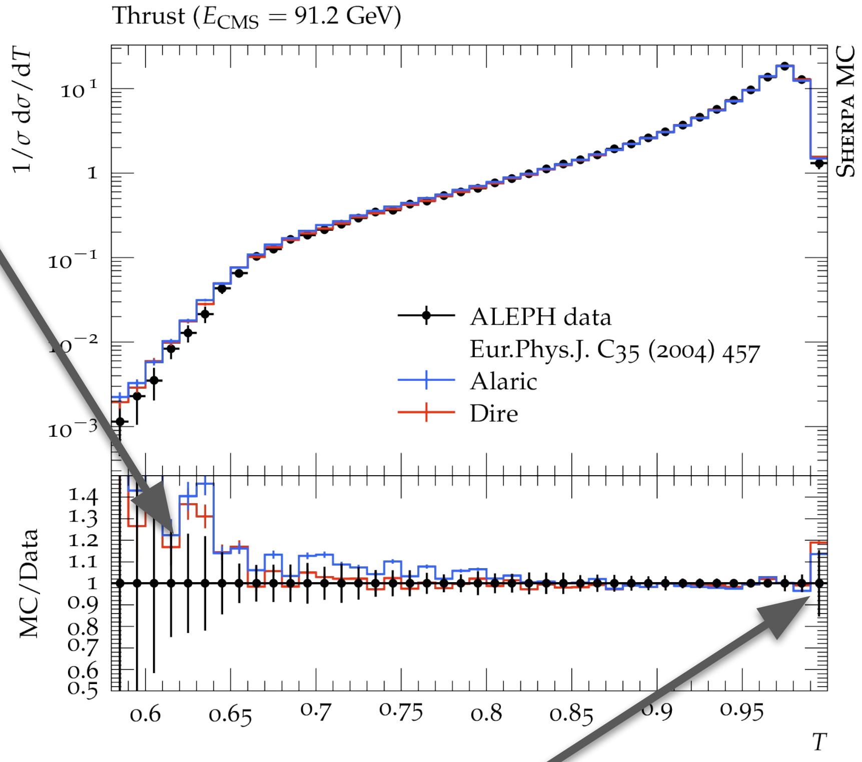




NLL Showers

Multijets

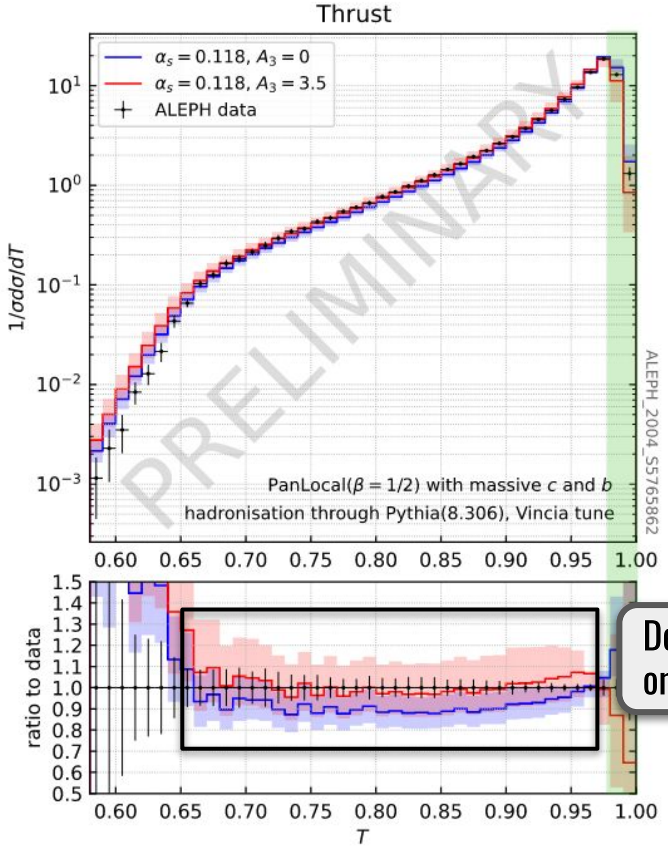


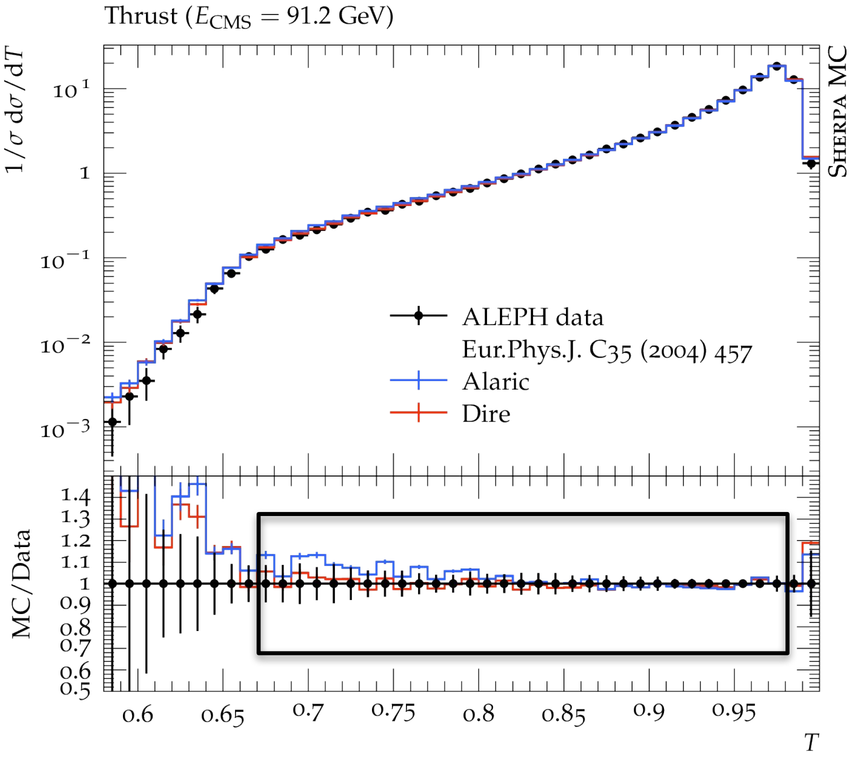


Hadronisation region

For PanLocal result see Melissa van Beekveld's Talk at [FCC-ee PS Workshop]

NLL Showers





Despite both, Alaric and PanLocal, being NLL, only one of describes experimental data well

NLL precision is a requirement for a next-generation parton shower, but it is not sufficient

Massive Improvements

ALARIC has been extended to handle massive quarks as emitters [Assi, Höche 2307.00728]

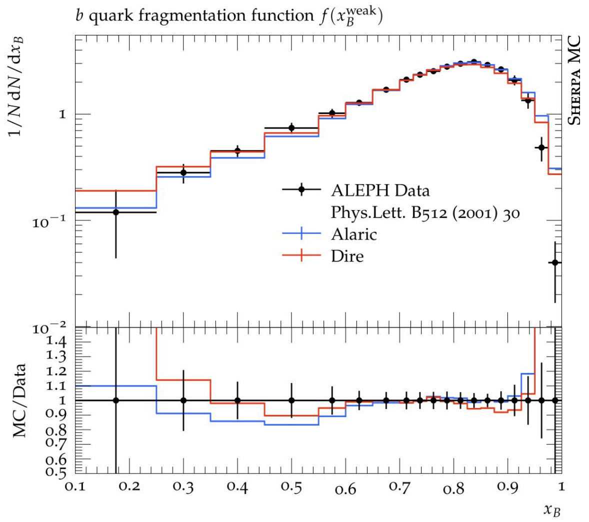
Number of important technical steps:

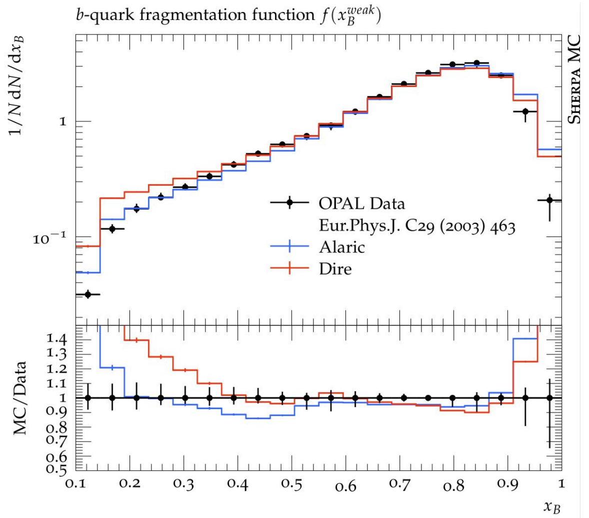
- Matching massive Eikonal to quasi-collinear limit
- Demonstrate NLL safety of momentum mappings
- Computation of all counterterms for MC@NLO matching

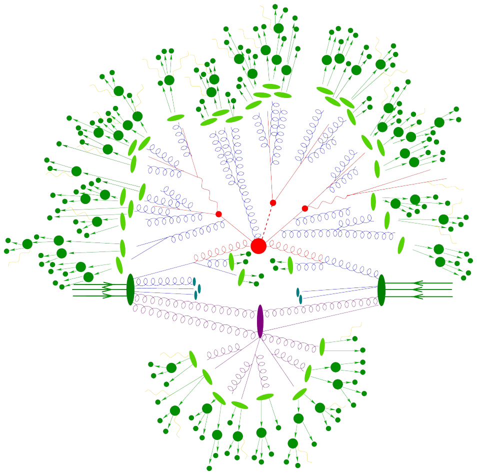
Important for phenomenology:

- Processes with top quarks
- b-jets
- b-hadron decays in Sherpa

Initial state evolution requires matching to PDFs and still WIP, but conceptually clear







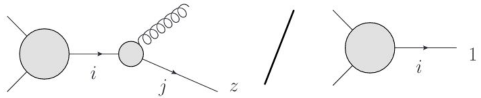
Towards NLO/NNLL Parton Showers

NLO Status

At NLO, various effects have to be included correctly, avoiding any double counting:

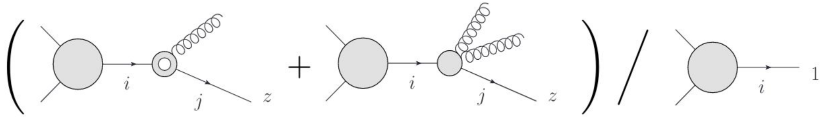
- Iterated LO splittings
- Virtual corrections to splittings
- Genuine triple collinear splittings
- Genuine double soft emissions

$$D^{(1)}_{ji}(z,\mu) = -\frac{1}{\varepsilon}P^{(0)}_{ji}(z) \quad \leftrightarrow$$



$$D^{(2)}_{ji}(z,\mu) = -\frac{1}{2\varepsilon}P^{(1)}_{ji}(z) + \frac{\beta_0}{4\varepsilon^2}P^{(0)}_{ji}(z) + \frac{1}{2\varepsilon^2}\int_z^1 \frac{\mathrm{d}x}{x}P^{(0)}_{jk}(x)P^{(0)}_{ki}(z/x)$$

$$\leftrightarrow$$



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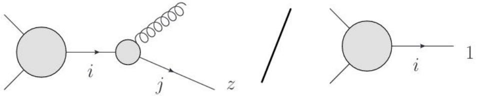

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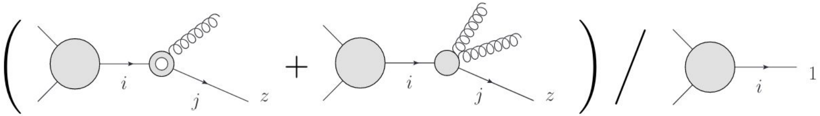

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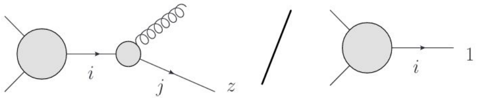
Should go hand in hand with fully differential NNLO matching (“Parton Shower as NNLO subtraction scheme”)

→ Integrated counterterms need to be computed

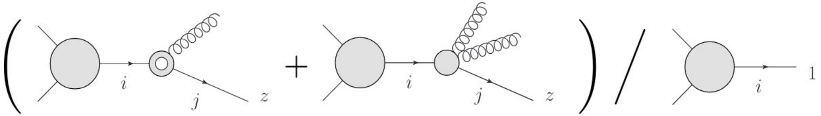
NLO Status

$$D_{ji}^{(1)}(z,\mu) = -\frac{1}{\varepsilon}P_{ji}^{(0)}(z) \quad \leftrightarrow$$



$$D_{ji}^{(2)}(z,\mu) = -\frac{1}{2\varepsilon}P_{ji}^{(1)}(z) + \frac{\beta_0}{4\varepsilon^2}P_{ji}^{(0)}(z) + \frac{1}{2\varepsilon^2}\int_z^1 \frac{\mathrm{d}x}{x} P_{jk}^{(0)}(x) P_{ki}^{(0)}(z/x)$$

$\leftrightarrow$



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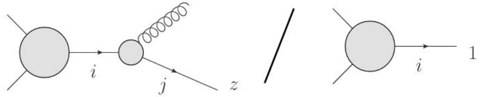
For double soft/triple collinear we need to learn from NNLO subtraction schemes

For ALARIC the work on the MC@NLO counterterms for massive particles allows for easy computation of NNLO counterterms in iterated limits [Assi, Höche 2307.00728]

NLO Status

$$D^{(1)}_{ji}(z,\mu) = -\frac{1}{\varepsilon} P^{(0)}_{ji}(z) \quad \leftrightarrow$$



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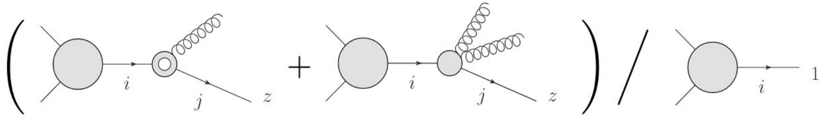
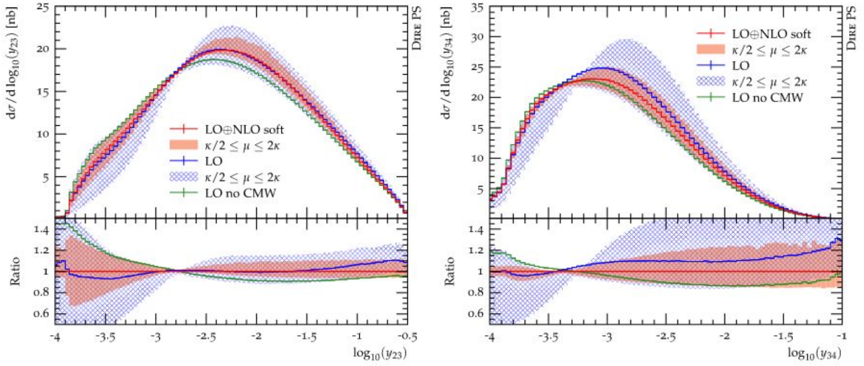
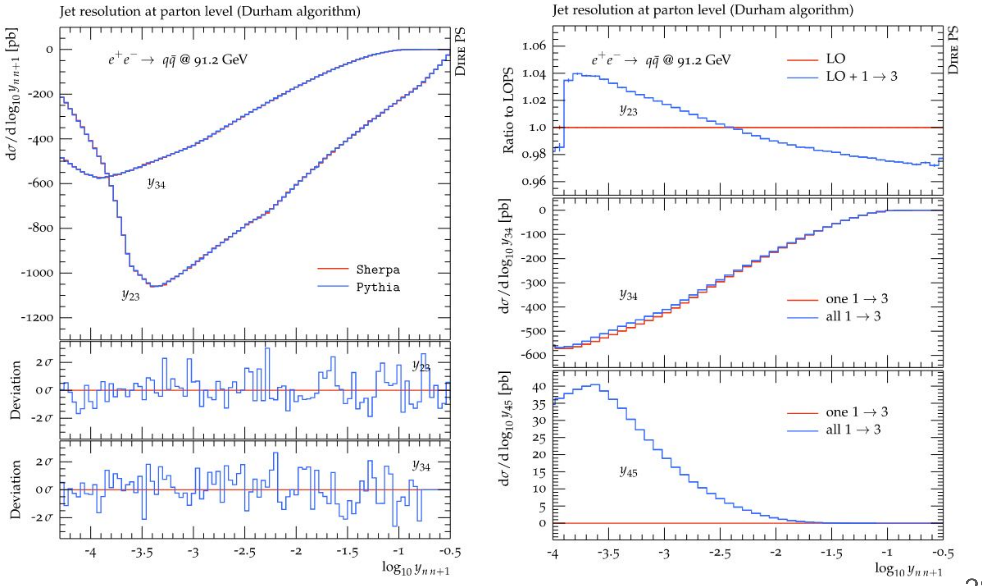


Studies of triple collinear and double soft effects in Dire:

[Höche, Prestel 1705.00742]
[Dulat, Höche, Prestel 1805.03757]
[Gellersen, Höche, Prestel 2110.05964]

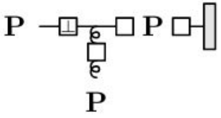




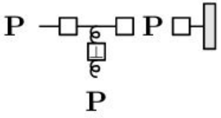
NLO Status

Work on higher order splittings on amplitude level: [Löschner, Plätzer, Simpson-Dore 2112.14454]

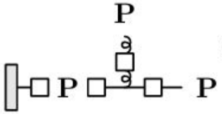
NNLL studies to go beyond CMW coupling: [Dasgupta, El-Menoufi 2109.07496]



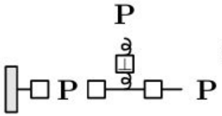
$$\simeq -\frac{g_s}{S_{ij}}\sqrt{\frac{z_i+z_j}{z_i}}\,\frac{\bar{u}_{\lambda_1}}{\sqrt{2n\cdot p_i}}\left(\not{k}_{\perp,i}\not{\epsilon}_{\lambda_3}\not{p}_i\right)\frac{u_{\lambda_2}}{\sqrt{2n\cdot p_i}},$$



$$\simeq 2\frac{g_s}{S_{ij}}\frac{\sqrt{z_i(z_i+z_j)}}{z_j}\,\frac{\bar{u}_{\lambda_1}}{\sqrt{2n\cdot p_i}}\not{p}_i\frac{u_{\lambda_2}}{\sqrt{2n\cdot p_i}}\,k_{\perp,j}\cdot\epsilon_{\lambda_3},$$



$$\simeq -2\frac{g_s}{S_{jk}}\,z_k\,\frac{\bar{u}_{\bar{\lambda}_1}}{\sqrt{2n\cdot p_k}}\left(\not{p}_k\, p_k\cdot\epsilon_{\bar{\lambda}_3}\right)\frac{u_{\bar{\lambda}_2}}{\sqrt{2n\cdot p_k}},$$



$$\simeq 2\frac{g_s}{S_{jk}}\,\frac{z_k}{z_j}\,\frac{n\cdot p_k}{n\cdot p_i}\,\frac{\bar{u}_{\bar{\lambda}_1}}{\sqrt{2n\cdot p_k}}\left(\not{p}_k k_{\perp,j}\cdot\epsilon_{\bar{\lambda}_3}\right)\frac{u_{\bar{\lambda}_2}}{\sqrt{2n\cdot p_k}}\,.$$

$$\mathcal{B}_2^{q,(\mathrm{ab.})}(z) = \left(\frac{\theta^2}{\sigma_0}\frac{d^2\sigma}{dz\,d\theta^2}\right)^{\text{d-r}} - \left(\frac{\theta^2}{\sigma_0}\frac{d^2\sigma}{dz\,d\theta^2}\right)^{\text{s-o}} + \left(\frac{\theta^2}{\sigma_0}\frac{d^2\sigma}{dz\,d\theta^2}\right)^{\text{r-v}} \tag{3.46}$$

$$= \left(\frac{C_F\alpha_s}{2\pi}\right)^2\left(\frac{1+z^2}{1-z}\left(-3\ln z + 2\mathrm{Li}_2\left(\frac{z-1}{z}\right) - 2\ln z\ln(1-z)\right) - 1 + H^{\text{fin.}}(z)\right). \tag{3.47}$$

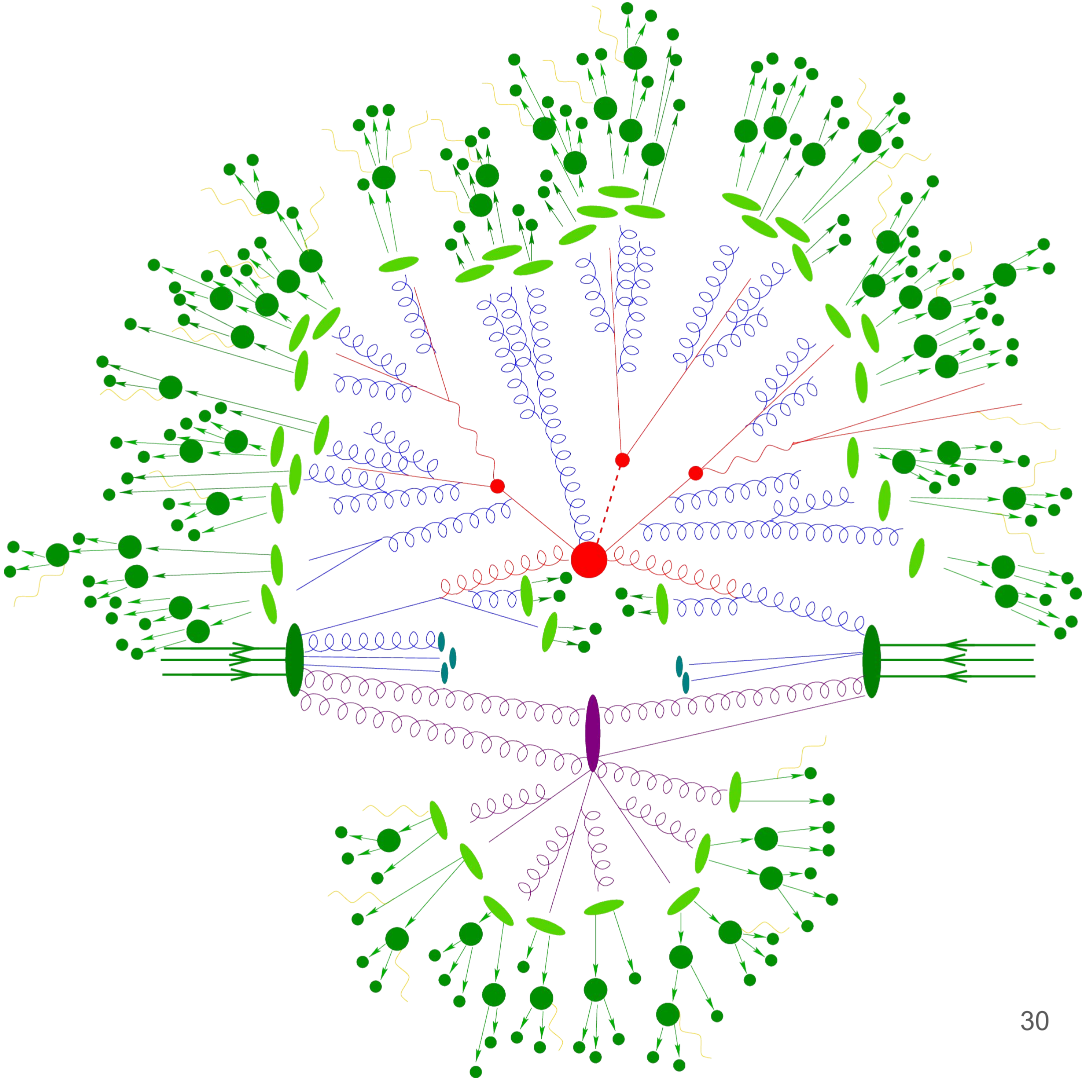
Conclusions

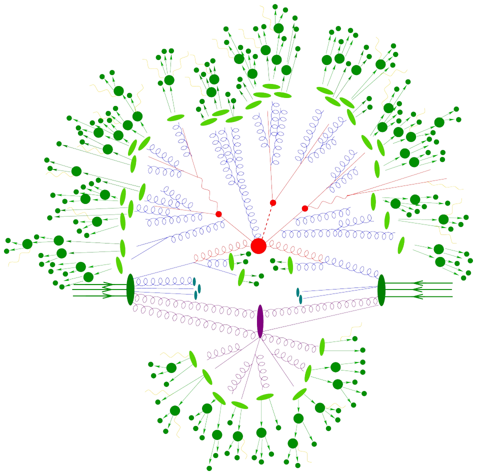
There have been a lot of exciting developments on parton showers in recent years!

- We are getting close to fully differential NNLO matching
- We are on a good way to understand Parton Showers at NLL

However, a lot of work remains:

- Showers at NLO are not quite there yet
- Massive quarks need to be included consistently, velocity logarithms, Quarkonia?
- NLP corrections might be necessary
- Can we do MC@NNLO?
- Most importantly: Implementation and Validation for use by experiments





Backup

Soft Radiation in Alaric

Factorisation in the soft limit:

$$ {}_n\langle 1,\ldots,n|1,\ldots,n\rangle_n = -8\pi\alpha_s \sum_{i,k\neq j} {}_{n-1}\langle 1,\ldots,\not{j},\ldots,n\big|\mathbf{T}_i\mathbf{T}_k\, w_{ik,j}\big|1,\ldots,\not{j},\ldots,n\rangle_{n-1} $$

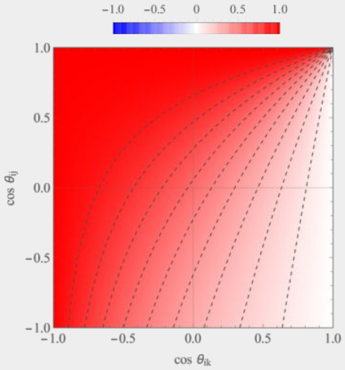
Multiplicative matching of singularities:

$$ W_{ik,j} = \bar{W}^i_{ik,j} + \bar{W}^k_{ki,j} $$

$$ \bar{W}^i_{ik,j} = W_{ik,j}\frac{1-\cos\theta_{jk}}{2-\cos\theta_{ij}-\cos\theta_{jk}} $$

[Catani, Seymour] hep-ph/9605323

Implement radiator differentially



$$ \frac{1}{2p_ip_j}P_{(ij)i}(z) \rightarrow \frac{1}{2p_ip_j}P_{(ij)i}(z)+\delta_{(ij)i}\left[\frac{\bar{W}^i_{ik,j}}{E_j^2} - w^{(\mathrm{coll})}_{ik,j}(z)\right] $$

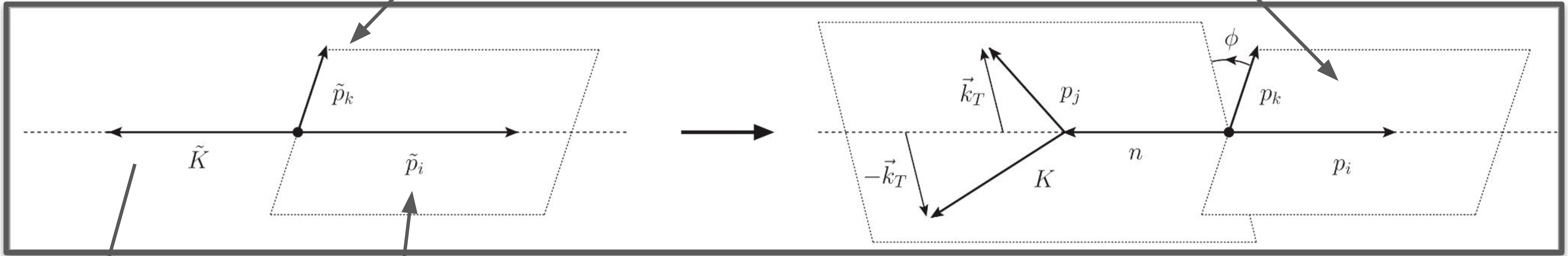
Splitting functions depend on direction of color spectator! N.b.: only leading color

Recoil in Alaric

Main Idea:
maintain directions of hard particles exactly

$$p_i = z\tilde{p}_i \qquad z = \frac{p_i n}{(p_i + p_j)n}$$
$$p_k = \tilde{p}_k$$

Color Spectator



Hard system

Emitter

$$v = \frac{p_i p_j}{p_i \tilde{K}} \qquad \kappa = \frac{\tilde{K}^2}{2\tilde{p}_i \tilde{K}}$$

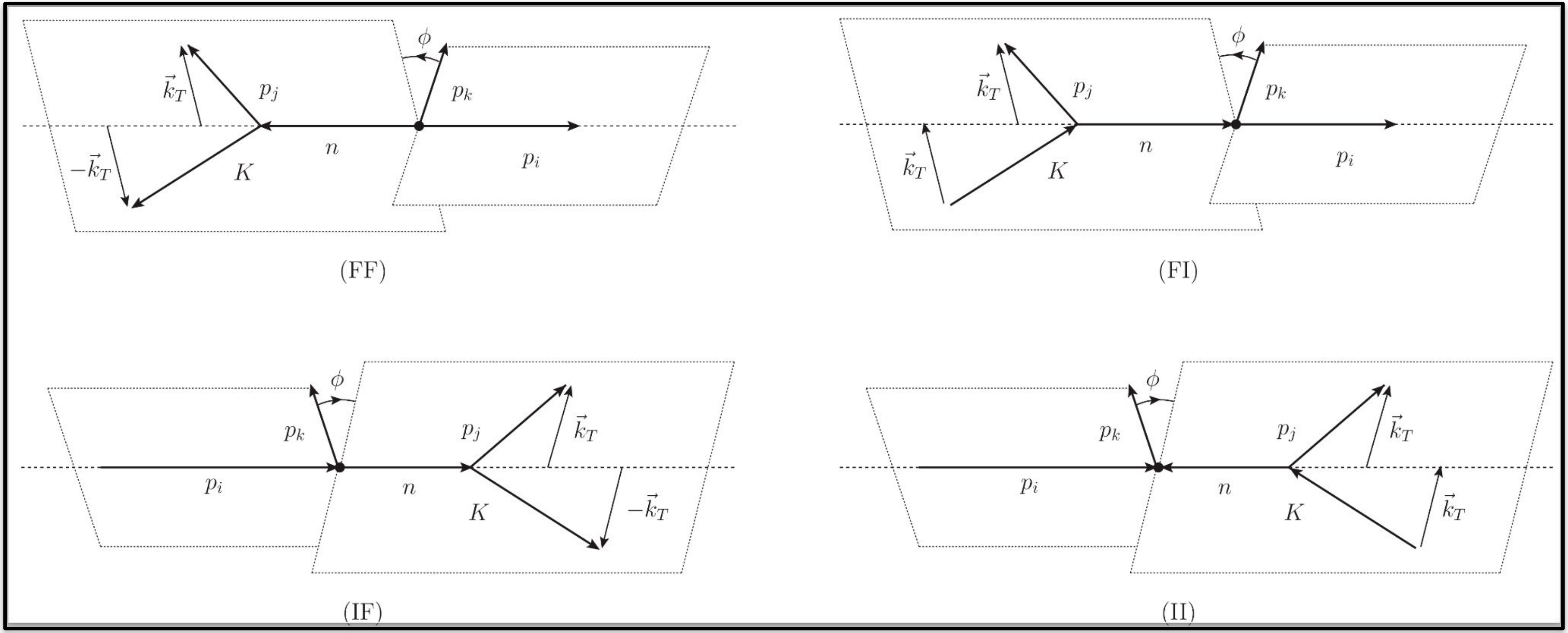
$$p_j = (1-z)\tilde{p}_i + v\left(\tilde{K} - (1-z+2\kappa)\tilde{p}_i\right) + k_\perp$$
$$K = \tilde{K} - v\left(\tilde{K} - (1-z+2\kappa)\tilde{p}_i\right) - k_\perp$$

Recoil distributed to remaining momenta through Lorentz Transformation:

$$p_l^\mu \to \Lambda^\mu_\nu(K, \tilde{K}) p_l^\nu$$

Recoil in Alaric

Momentum mapping works for initial and final state emitters/spectator
→ e+ e-, pp, DIS, … all treated on same footing



NLL Proof for Alaric

Recoil distributed to remaining momenta through Lorentz Transformation:

$$p_l^\mu \to \Lambda^\mu_\nu(K, \tilde{K}) p_l^\nu$$

Define

$$X^\mu = p_j^\mu - (1-z)\,\tilde{p}_i^\mu$$
$$= v\left(\tilde{K}^\mu - (1-z+2\kappa)\,\tilde{p}_i^\mu\right) + k_\perp^\mu$$

$$\Lambda^\mu{}_\nu(K, \tilde{K}) = g^\mu_\nu + \tilde{K}^\mu A_\nu + X^\mu B_\nu$$

$$A^\nu = 2\left[\frac{(\tilde{K}-X)^\nu}{(\tilde{K}-X)^2} - \frac{(\tilde{K}-X/2)^\nu}{(\tilde{K}-X/2)^2}\right] \qquad B^\nu = \frac{(\tilde{K}-X/2)^\nu}{(\tilde{K}-X/2)^2}$$

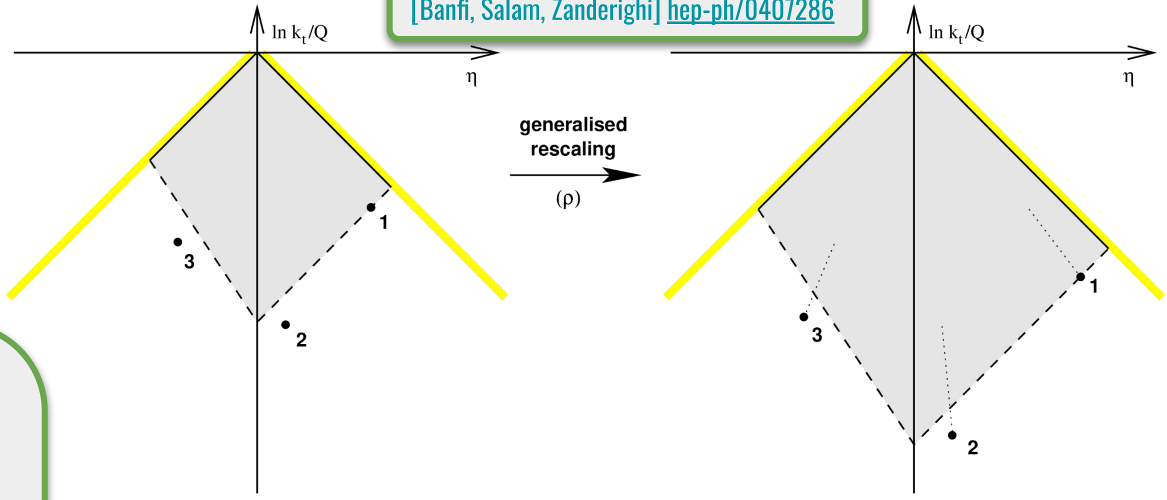
Suppressed by

$$\mathcal{O}(k_\perp/K)$$

$$\Lambda^\mu_\nu \approx g^\mu_\nu + \frac{K_\rho X_\sigma}{K^2} T_\nu^{\mu\rho\sigma} + \mathcal{O}(k_\perp^2)$$

NLL Proof for Alaric

[Banfi, Salam, Zanderighi] hep-ph/0407286



For one emission kinematic variables in the Lund plane scale like:

$$k_{t,l} \to k'_{t,l} = k_{t,l}\rho^{(1-\xi_l)/a+\xi_l/(a+b)}$$

$$\eta_l \to \eta'_l = \eta - \xi_l \frac{\ln \rho}{a+b}$$

$$\xi_l = \frac{\eta_l}{\eta_{l,\max}}$$

where a = 1 and b = 0 for Alaric

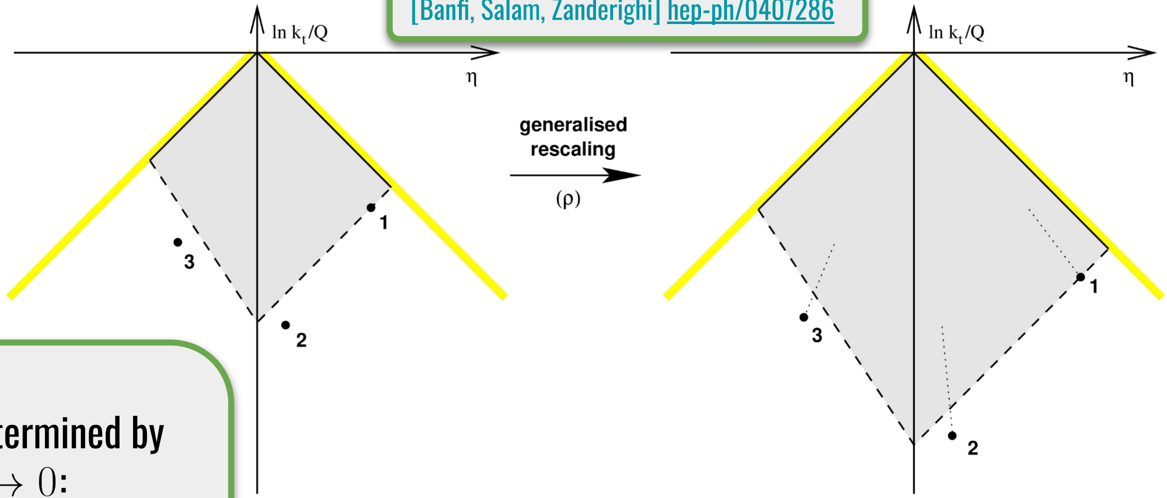
Working in the rest frame of the color dipole, the other momenta scale like:

$$\tilde{p}_l^0 \sim \rho^{1-\xi_l}$$
$$\tilde{p}_l^{1,2} \sim \rho$$
$$\tilde{p}_l^3 \sim \rho^{1-\xi_l}$$

for $\rho \to 0$

NLL Proof for Alaric

[Banfi, Salam, Zanderighi] hep-ph/0407286



Scaling under an additional emission is determined by the Lorentz transformation in the limit $\rho \to 0$:

$$\Delta p_l^\mu = 2\,\frac{\tilde{K}X}{\tilde{K}^2}\,\frac{\tilde{p}_l\tilde{K}}{\tilde{K}^2}\,\tilde{K}^\mu - \frac{\tilde{p}_l X}{\tilde{K}^2}\,\tilde{K}^\mu + \frac{\tilde{p}_l\tilde{K}}{\tilde{K}^2}\,X^\mu$$

Scaling becomes:

$$\Delta p_l^0 \sim \;\rho^{1-\xi_l}X^0 + \rho^{2-\xi_l-\max(\xi_i,\xi_j)}\tilde{K}^0 + \rho^{1-\xi_l}X^0 \;\sim \rho^{2-\xi_l-\max(\xi_i,\xi_j)} \qquad \tilde{p}_l^0 \sim \;\rho^{1-\xi_l}$$

$$\Delta p_l^{1,2} \sim \;\rho^{1-\xi_l}X^{1,2} \sim \rho^{2-\xi_l} \qquad \tilde{p}_l^{1,2} \sim \;\rho$$

$$\Delta p_l^3 \sim \;\rho^{1-\xi_l}X^3 \sim \rho^{2-\xi_l-\max(\xi_i,\xi_j)} \qquad \tilde{p}_l^3 \sim \;\rho^{1-\xi_l}$$