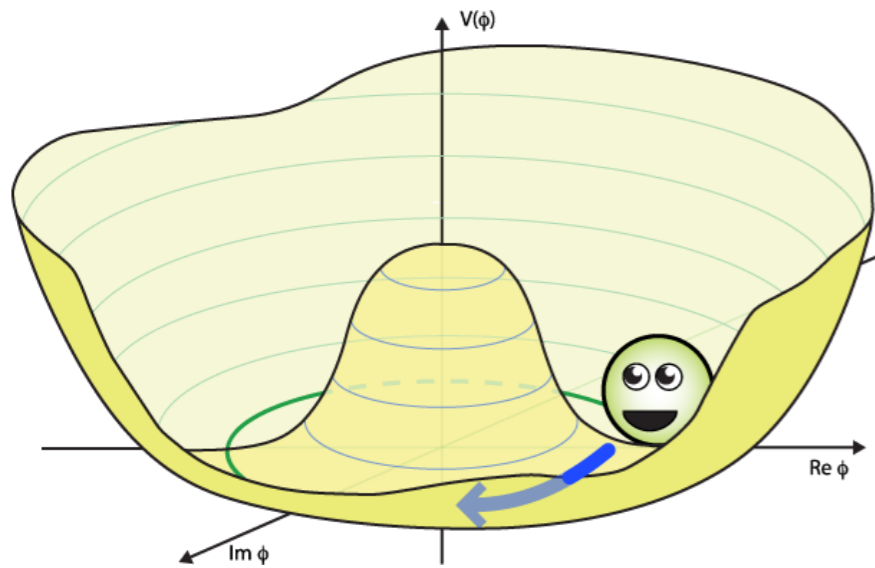


Higgs self-couplings

Pier Paolo Giardino
SM@LHC - 12/07/2023

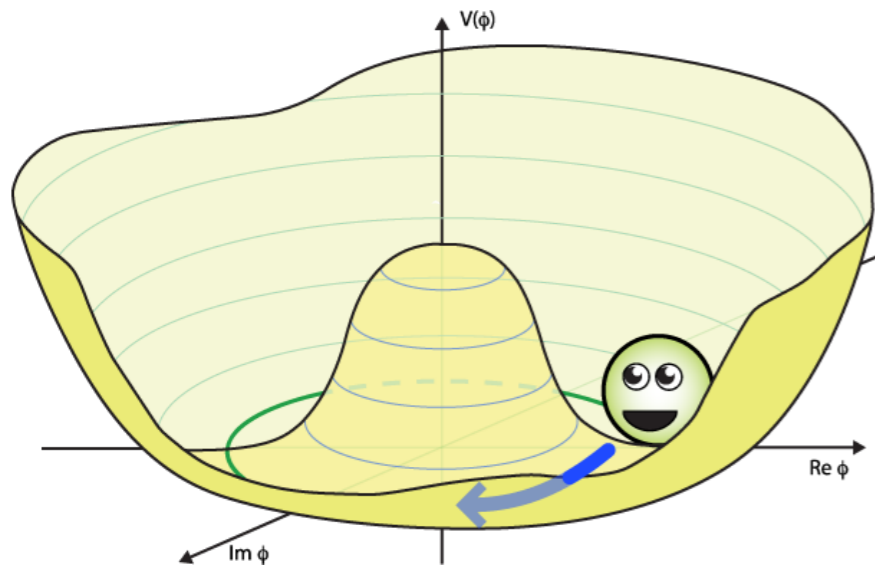




$$V(\phi) = -\mu^2(\phi^\dagger\phi) + \lambda(\phi^\dagger\phi)^2$$

$$V(H) = \frac{1}{2}M_H^2H^2 + \frac{M_H^2}{2v}H^3 + \frac{M_H^2}{8v^2}H^4$$

In the SM the Higgs potential is determined by 2 parameters:
 the **mass of the Higgs** (M_H) and the **Fermi Constant** ($G_\mu = 1/(\sqrt{2}v^2)$)
 which we know quite precisely



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Precise knowledge of the Higgs potential in the SM

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$$V(H) = \frac{1}{2}M_H^2 H^2 + \lambda_3 v H^3 + \lambda_4 H^4 + \dots$$

The relation between self-couplings and input parameters is modified by NP

Particularly if the EWSB is not linearly realized.

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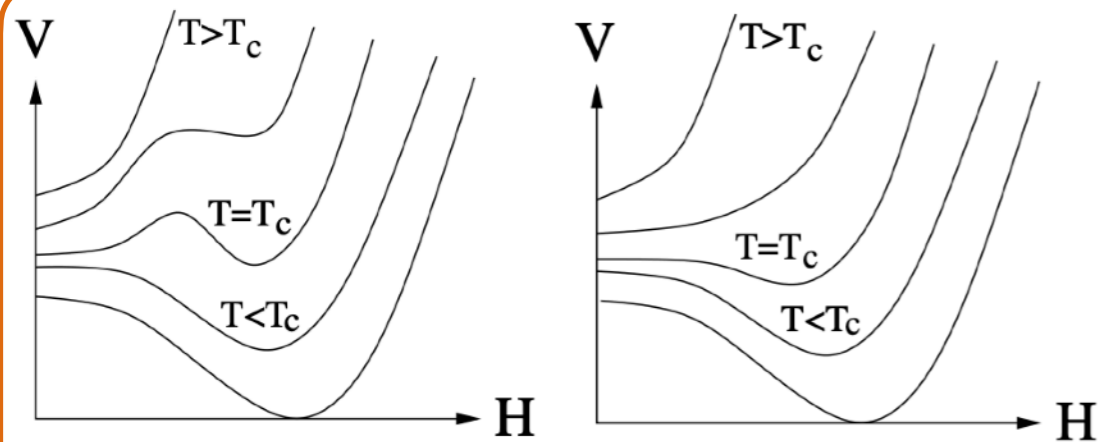
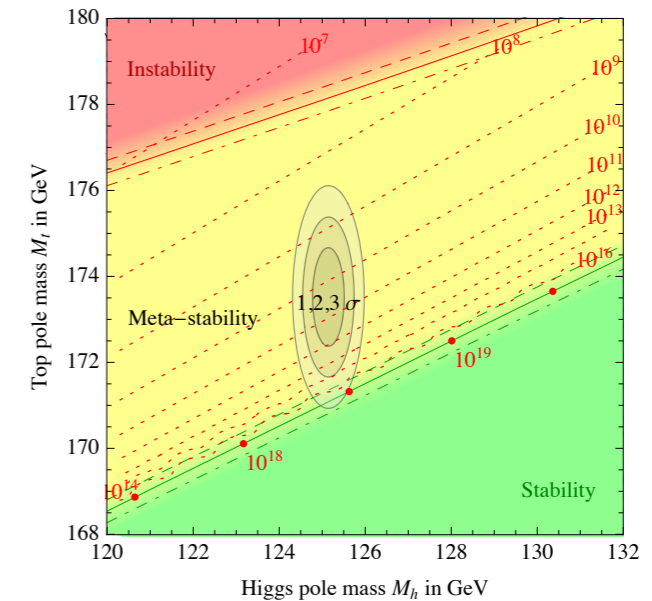
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Measuring the Higgs potential can give us information on NP

The shape of the Higgs potential (self-couplings) can have large phenomenological implications

D. Buttazzo, G. Degrassi, PPG, G. Giudice, F. Sala, A. Salvio, A. Strumia, arXiv:1307.3536 [hep-ph]

In the SM the EW vacuum is meta-stable.
This prediction is strongly model dependent.



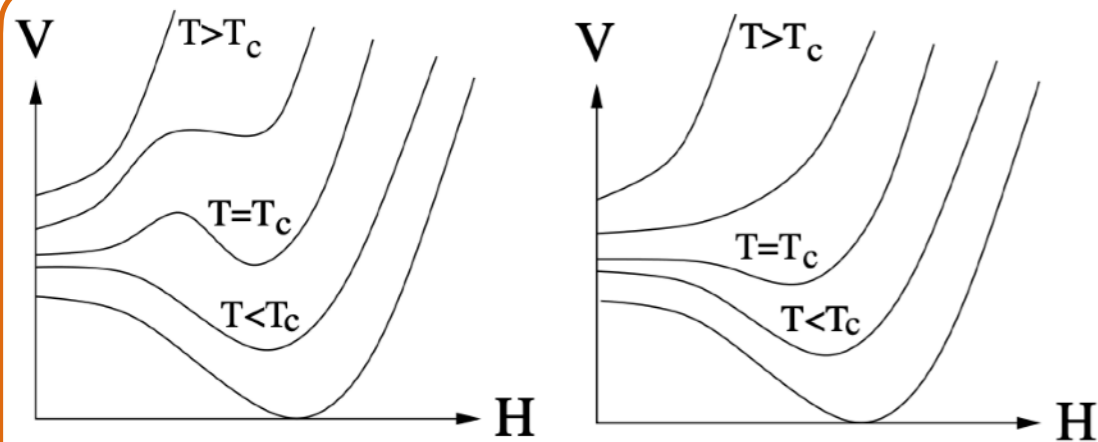
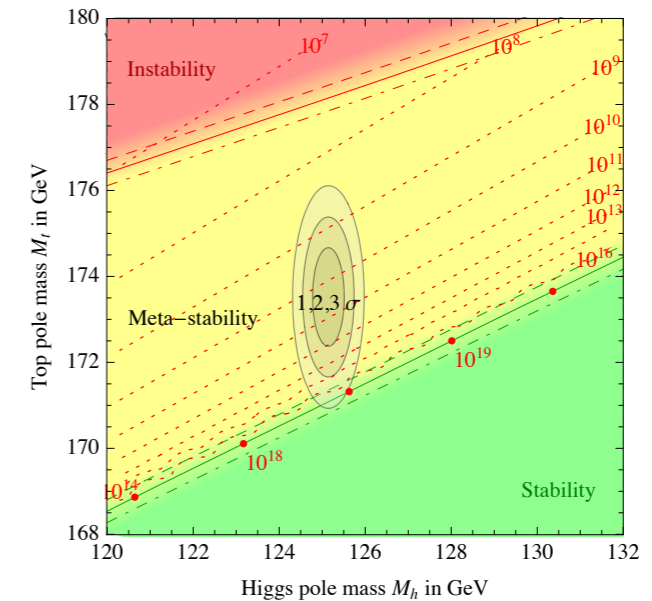
J. Cline, arXiv:hep-ph/0609145

1st order phase transition necessary
for EW baryogenesis. 2nd order in SM

See Henning Bahl's talk

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Measurement of Higgs self-couplings is crucial

See Henning Bahl's talk

While the title of this talk is “Higgs self-couplings”
I will concentrate on the Higgs trilinear self-coupling

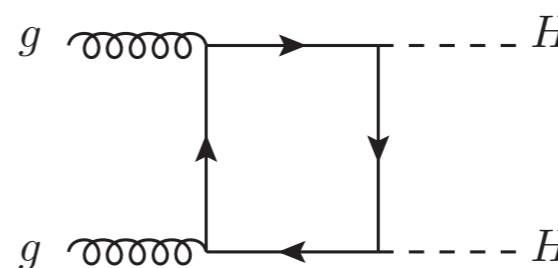
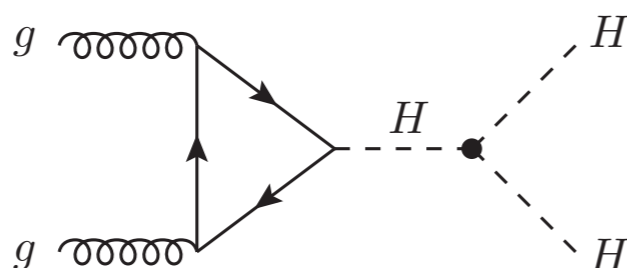
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I leave the quartic for when this conference will have changed name:

SM@LHC \longrightarrow SM@FCC

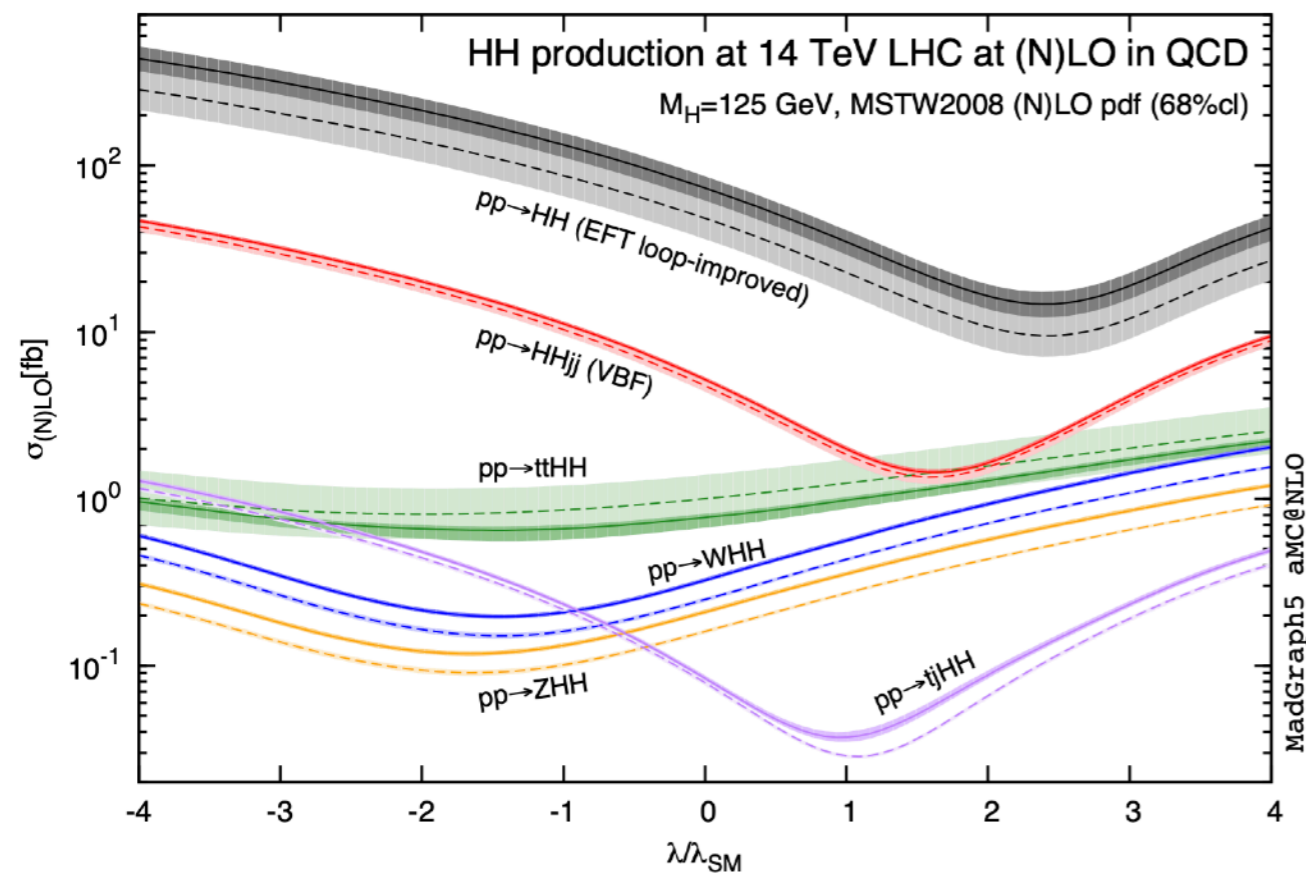
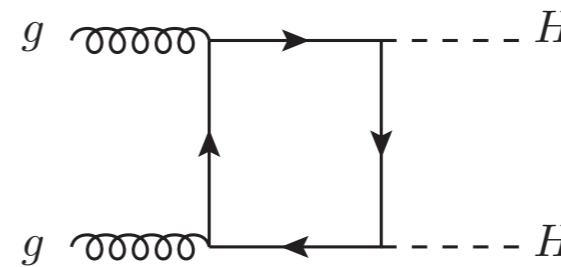
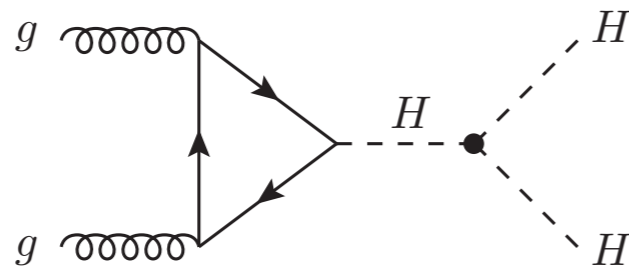
How do we measure the Higgs trilinear (λ_3)?

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The processes that depend on λ_3 at LO are the production of a pair of Higgs bosons

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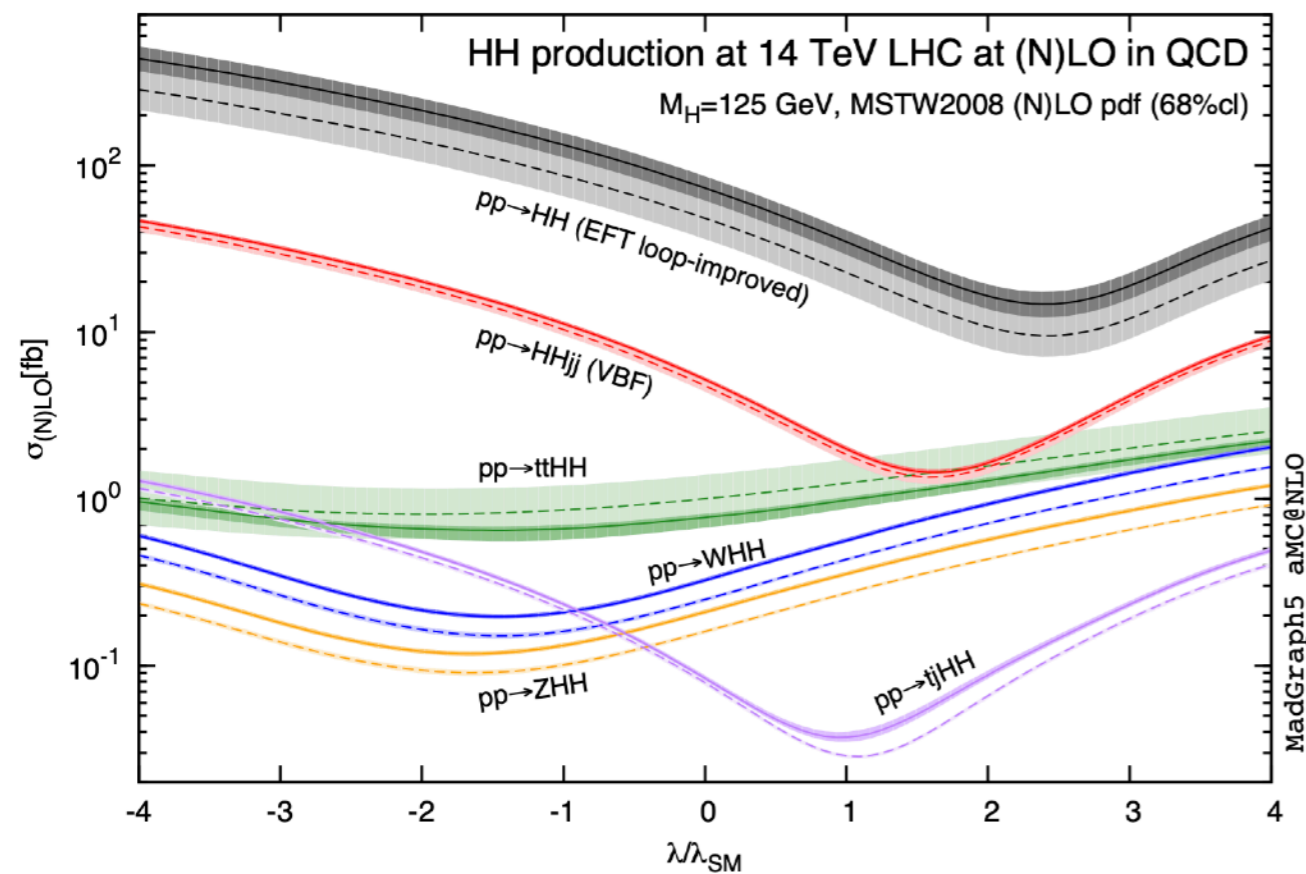
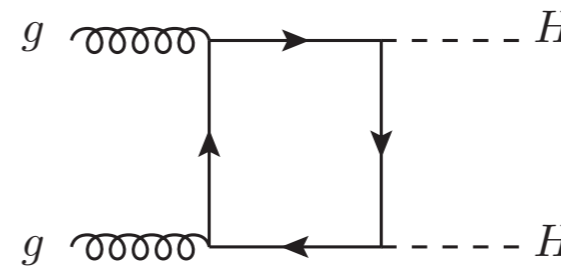
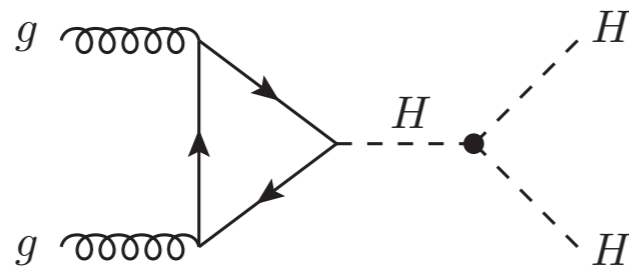


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Strong interference between the amplitudes around the SM value

R. Frederix, S. Frixione, V. Hirschi, F. Maltoni, O. Mattelaer, P. Torrielli, E. Vryonidou, M. Zaro, arXiv:1401.7340 [hep-ph]

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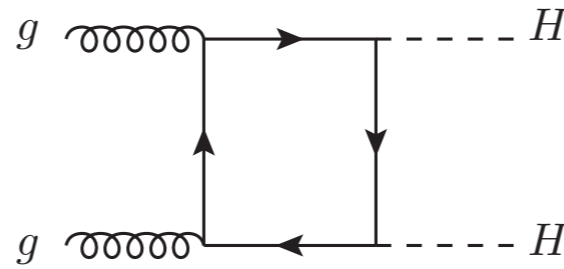
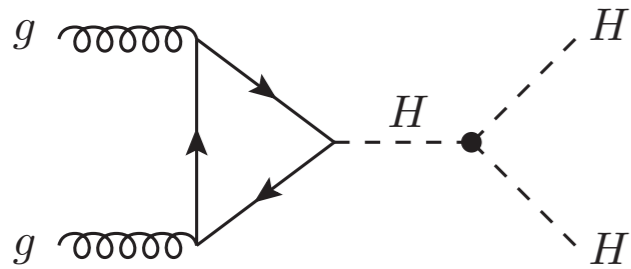
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Let's assume $\lambda_3/\lambda_{3,SM} = \kappa_\lambda = 1 + \delta_\lambda$

R. Frederix, S. Frixione, V. Hirschi, F. Maltoni, O. Mattelaer, P. Torrielli, E. Vryonidou, M. Zaro, arXiv:1401.7340 [hep-ph]

To extract λ_3 from the measurements, it is necessary to know the theory

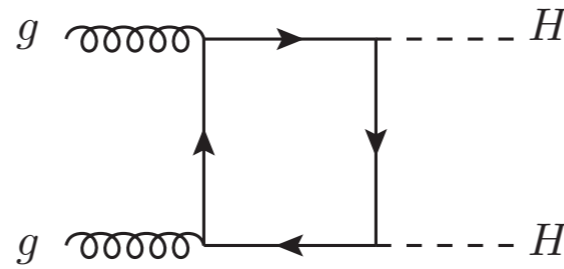
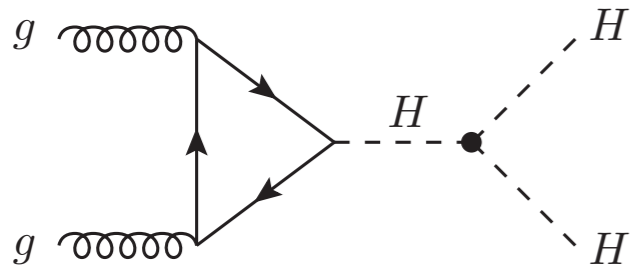
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At LO

Glover, van der Bij Nucl.Phys.B 309 (1988) 282

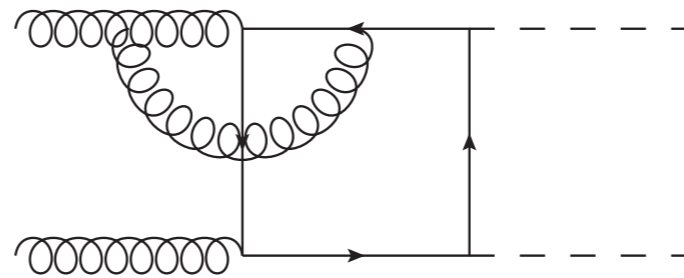
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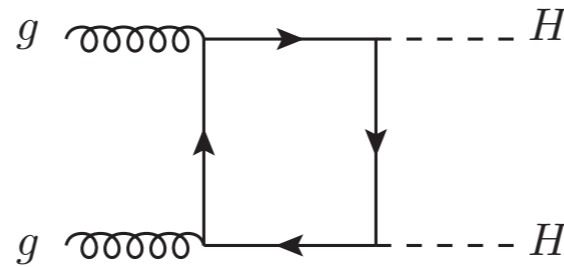
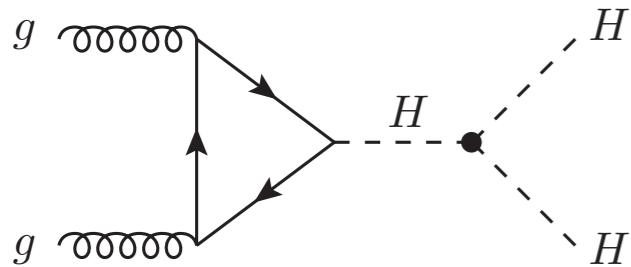
At LO

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At NLO the calculation is complicated by the structure of the box



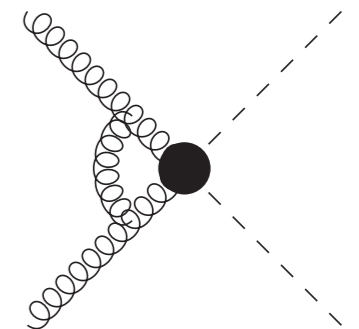
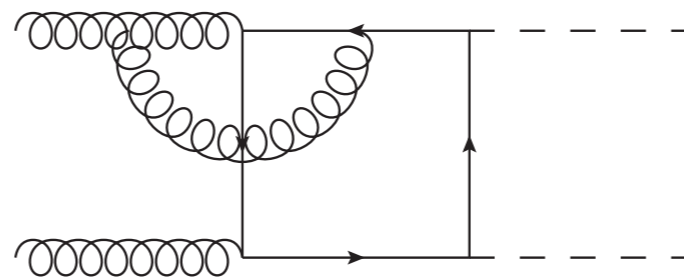
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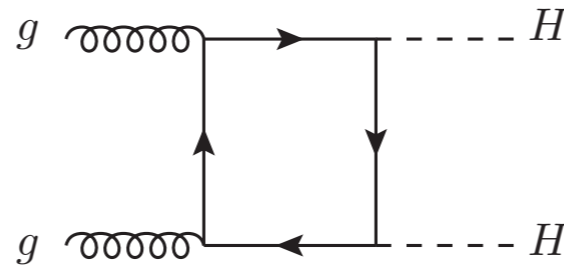
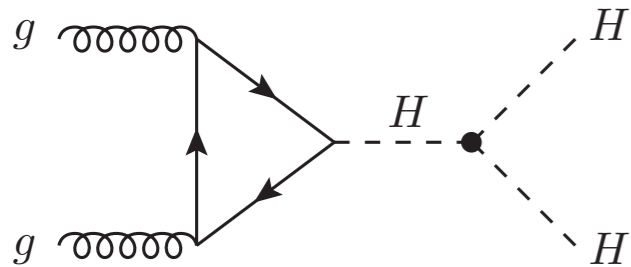
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First NLO calculation was done for $m_t \rightarrow \infty$ ($k \approx 2$)

S. Dawson, S. Dittmaier and M. Spira, Phys. Rev. D 58 (1998) 115012

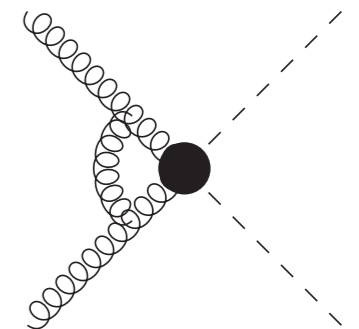
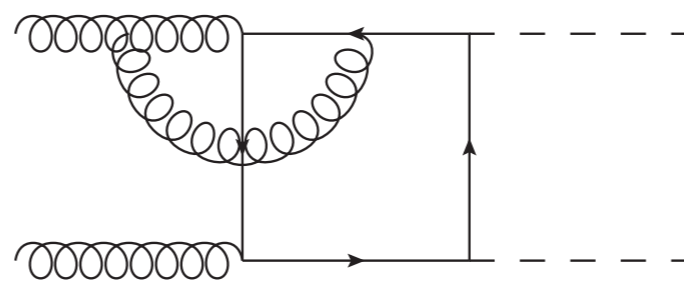
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Numerical results available for **NLO QCD** (finite top mass; $\delta_\sigma \sim -15\%$)

S. Borowka, N. Greiner, G. Heinrich, S. P. Jones, M. Kerner, J. Schlenk and T. Zirke arXiv:1608.04798 [hep-ph]
 J. Baglio, F. Campanario, S. Glaus, M. Mühlleitner, M. Spira and J. Streicher arXiv:1811.05692 [hep-ph]

Analytic results for **NLO QCD** exist for different approximations

Large Top Expansion $(1/m_T)^n$

J. Grigo, J. Hoff, K. Melnikov and M. Steinhauser, arXiv:1305.7340 [hep-ph]
 G. Degrandi, P. P. G. and R. Gröber, arXiv:1603.00385 [hep-ph]

High Energy expansion $(m_T)^n$

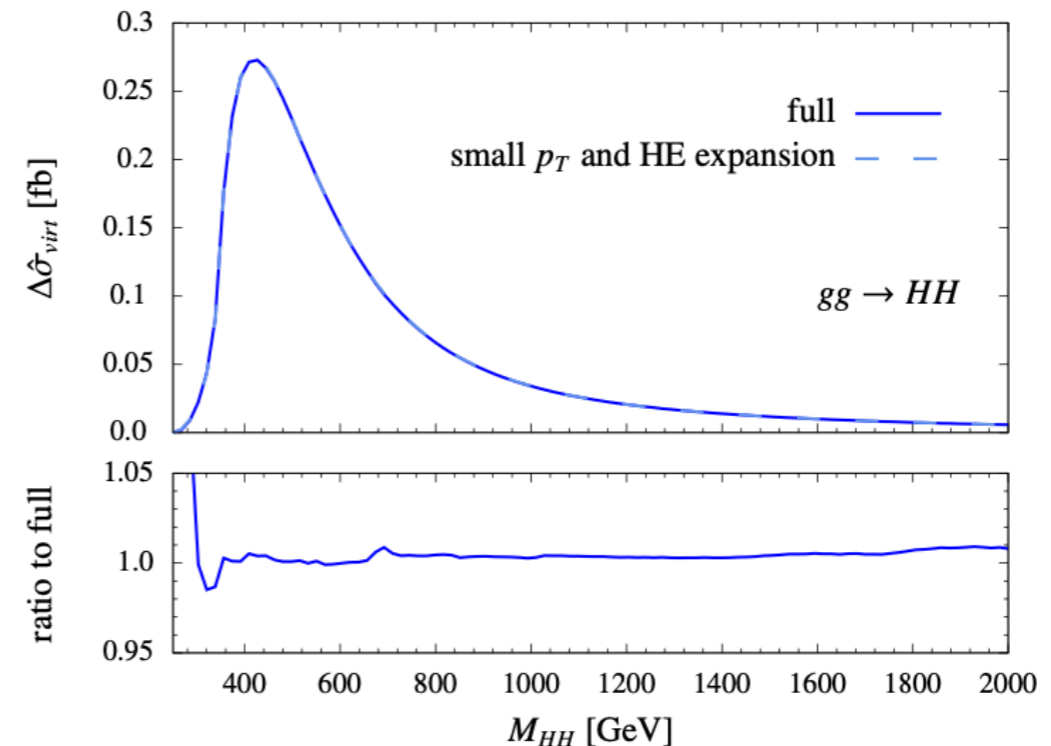
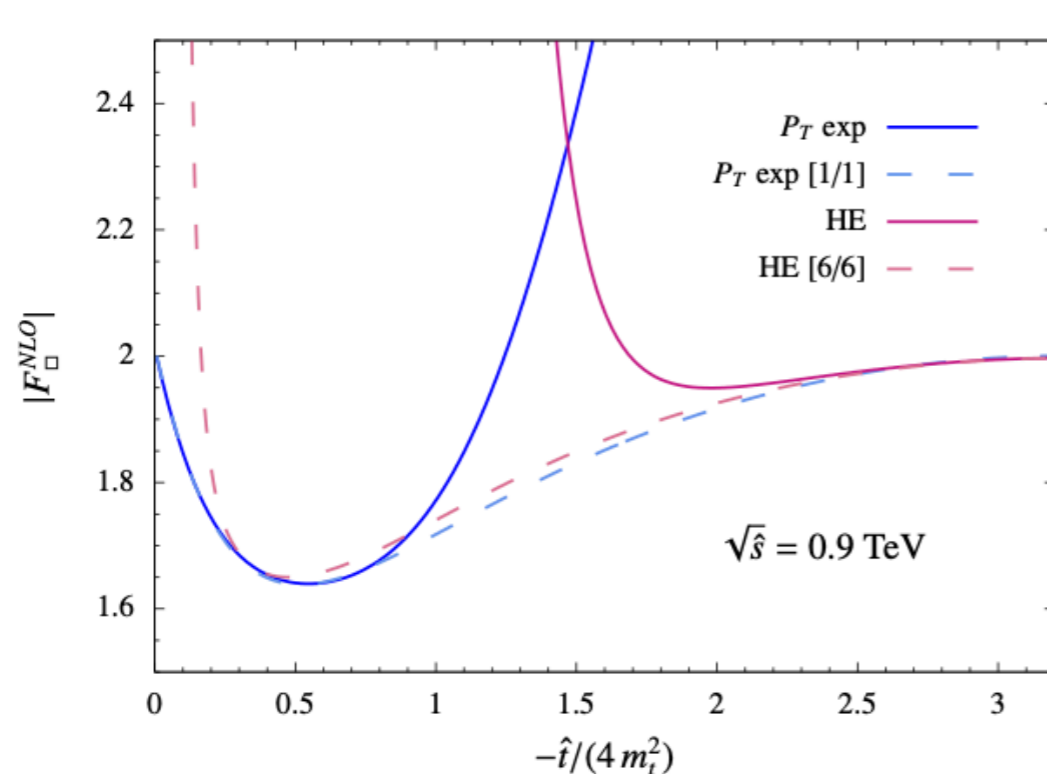
J. Davies, G. Mishima, M. Steinhauser and D. Wellmann,
 arXiv:1801.09696 [hep-ph], & arXiv:1811.05489 [hep-ph]

Small p_T expansion $(p_T)^n$

R. Bonciani, G. Degrandi, P. P. G. and R. Gröber, arXiv:1806.11564 [hep-ph]

Analytic results are valid only for specific regions of phase space,
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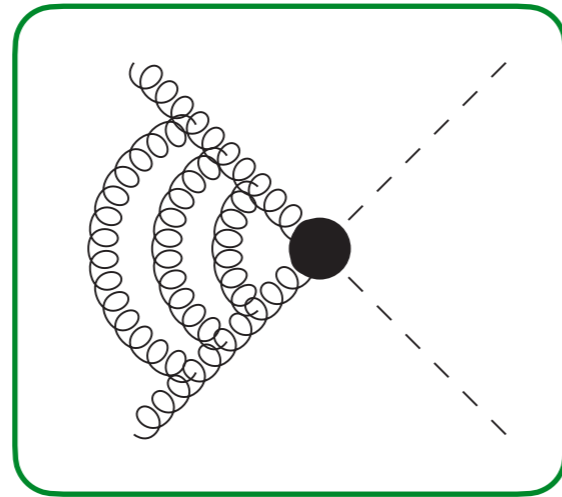
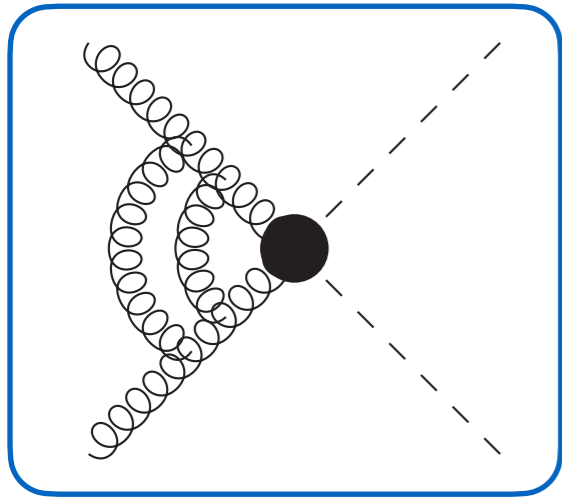


Reproduces full within 1%

The analytical results can be sewn to cover the entire phase space using the Padé approximant $[m/n] = \frac{p_0 + p_1x + \dots + p_mx^m}{1 + q_1x + \dots + q_nx^n}$

L. Bellafante, G. Degrossi, P. P. G., R. Gröber and M. Vitti, arXiv:2202.12157 [hep-ph]

Beyond NLO QCD

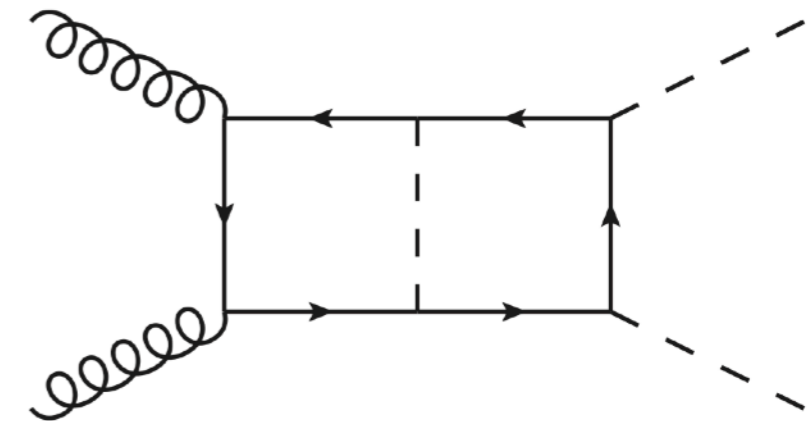


NNLO ($k \approx 1.2$) and
NNNLO QCD ($k \approx 1.03$)
 corrections in Heavy Top Limit

see for example D. de Florian and J. Mazzitelli, arXiv:1309.6594 [hep-ph]; J. Davies, F. Herren, G. Mishima and M. Steinhauser, arXiv:1904.11998 [hep-ph]; M. Spira, arXiv:1607.05548 [hep-ph]; L. B. Chen, H. T. Li, H. S. Shao and J. Wang, arXiv:1912.13001 [hep-ph]; M. Grazzini, G. Heinrich, S. Jones, S. Kallweit, M. Kerner, J. M. Lindert and J. Mazzitelli, arXiv:1803.02463 [hep-ph]

EW NLO corrections $\propto Y_T$; exp. \sim few %

Joshua Davies, Go Mishima, Kay Schönwald, Matthias Steinhauser, Hantian Zhang, arXiv:2207.02587 [hep-ph]
 Margarete Mühlleitner, Johannes Schlenk and Michael Spira: arXiv:2207.02524 [hep-ph]

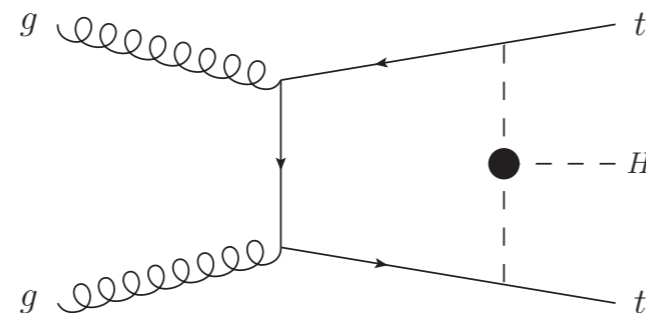
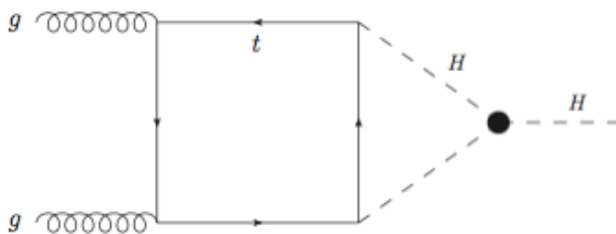
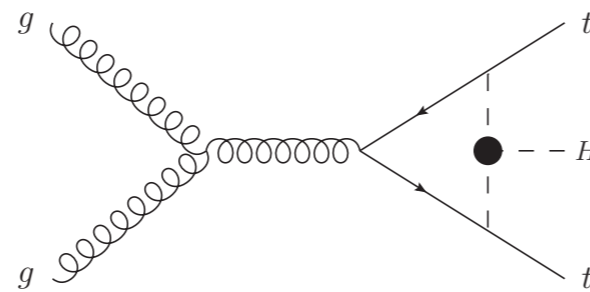
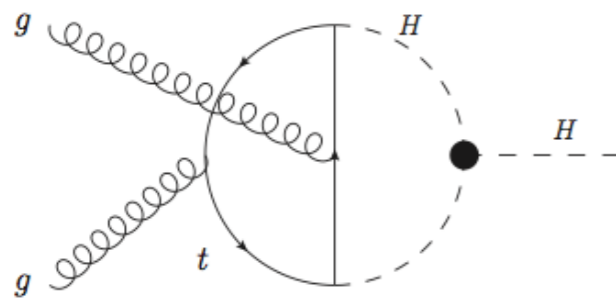


First steps towards QCD NNLO with light fermions for $t=0$

Joshua Davies, Kay Schönwald, Matthias Steinhauser, arXiv:2307.04796 [hep-ph]

If δ_λ is large we may see deviations coming from loops

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λ_3 appears at NLO in Single Higgs processes

Not trivial kinematic dependence

M. McCullough arXiv:1312.3322 [hep-ph]

M. Gorbahn and U. Haisch, arXiv:1607.03773 [hep-ph]

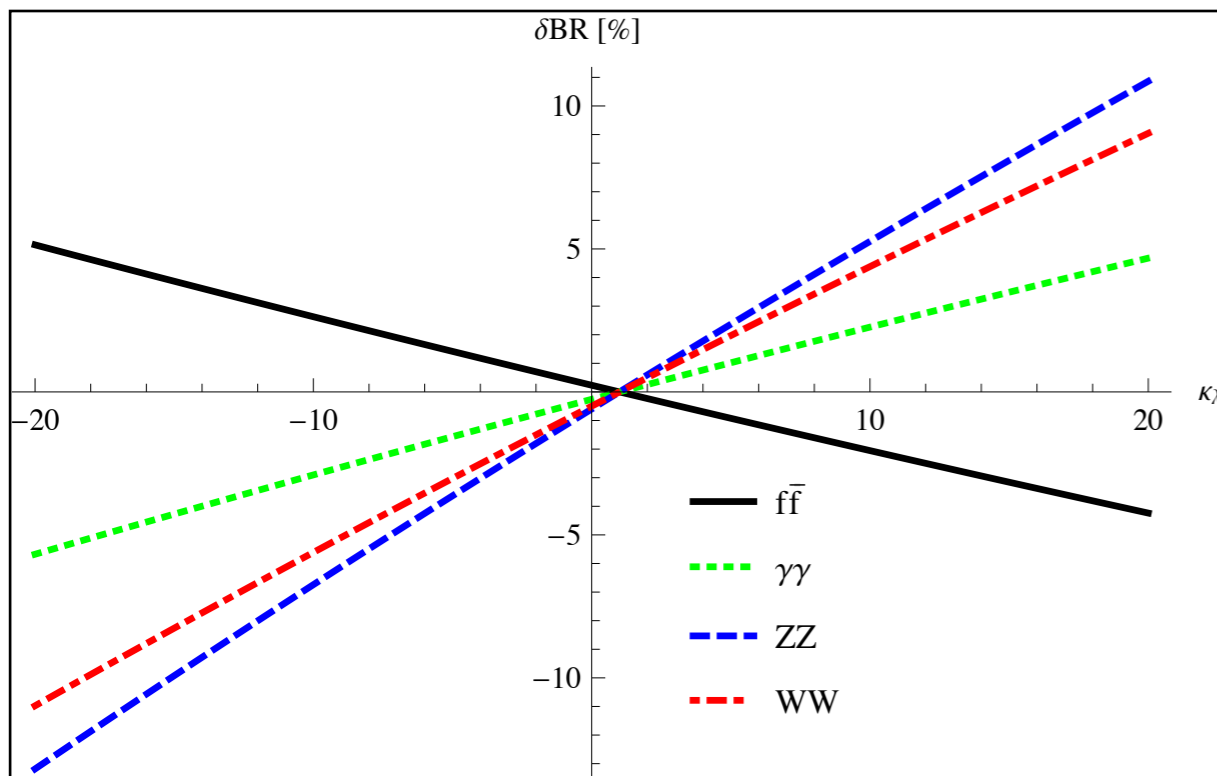
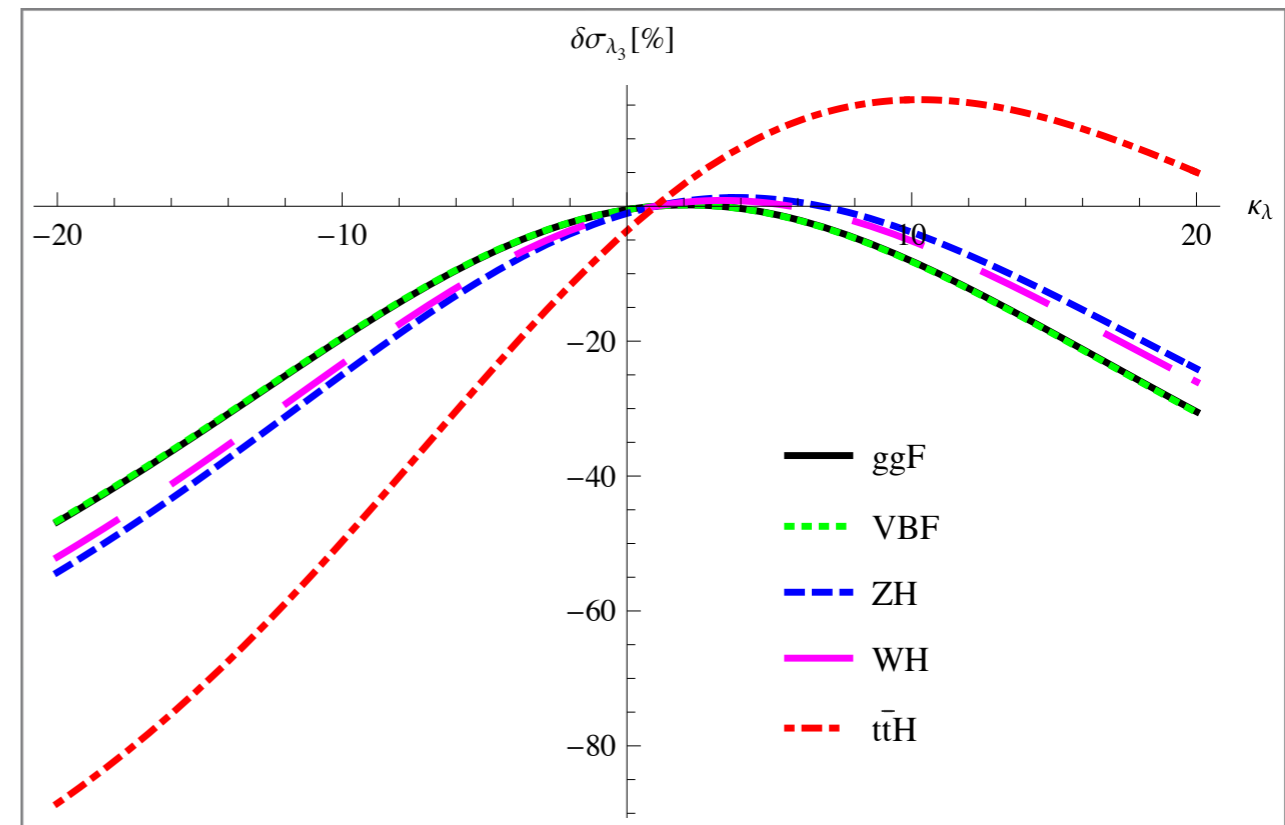
G. Degrandi, PPG, F. Maltoni, D. Pagani, arXiv:1607.04251 [hep-ph]

W. Bizon, M. Gorbahn, U. Haisch and G. Zanderighi, arXiv:1610.05771 [hep-ph].

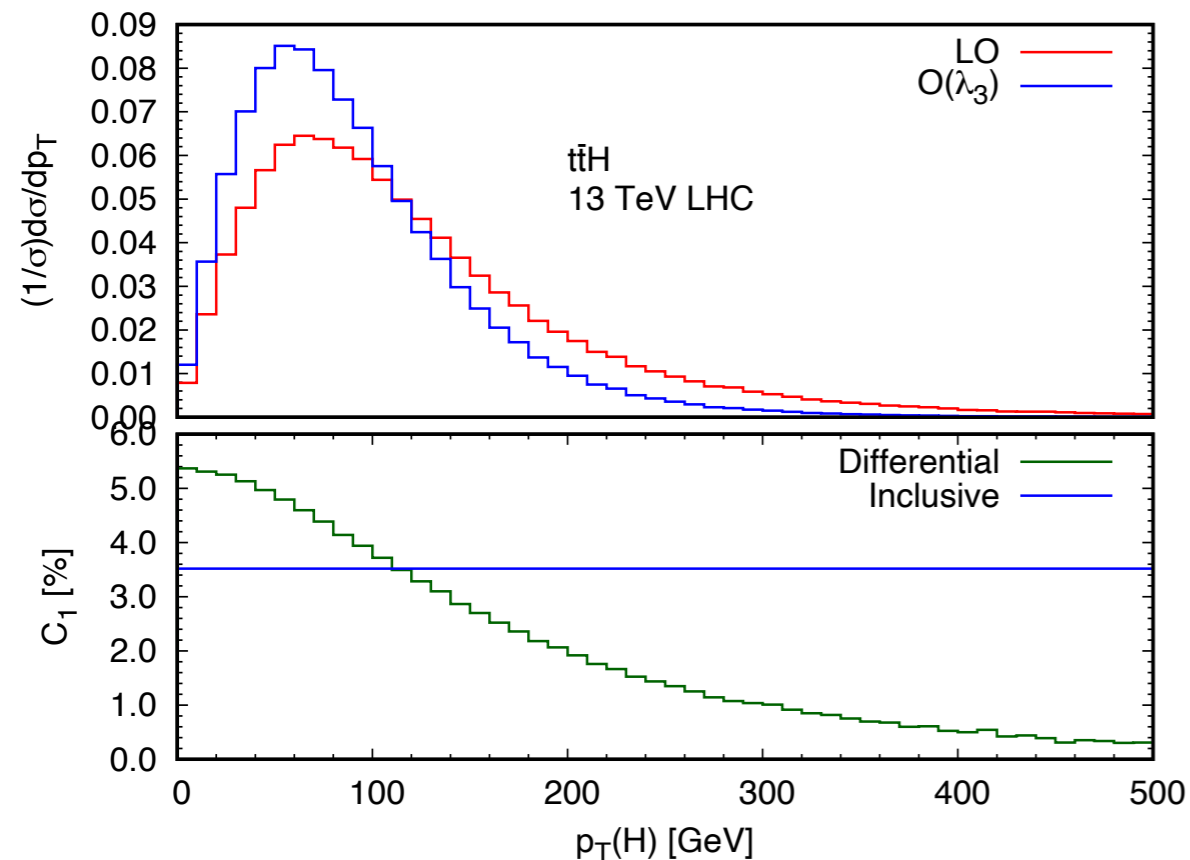
$$\Sigma_{NLO} = Z_H \Sigma_{LO} (1 + \kappa_\lambda C_1)$$

$$Z_H = \frac{1}{1 - \kappa_\lambda^2 \delta Z_H} \quad C_1 = \frac{\int 2\Re(\mathcal{M}^{0*} \mathcal{M}_{\lambda_3^{SM}}^1)}{\int |\mathcal{M}^0|^2}$$

Modifications in principle
observables at LHC



In the range close to the SM, the
decays are more sensitive to λ_3
than the production processes



Further information can be obtained from differential distributions (not trivial dependence on kinematics due to loop structure)

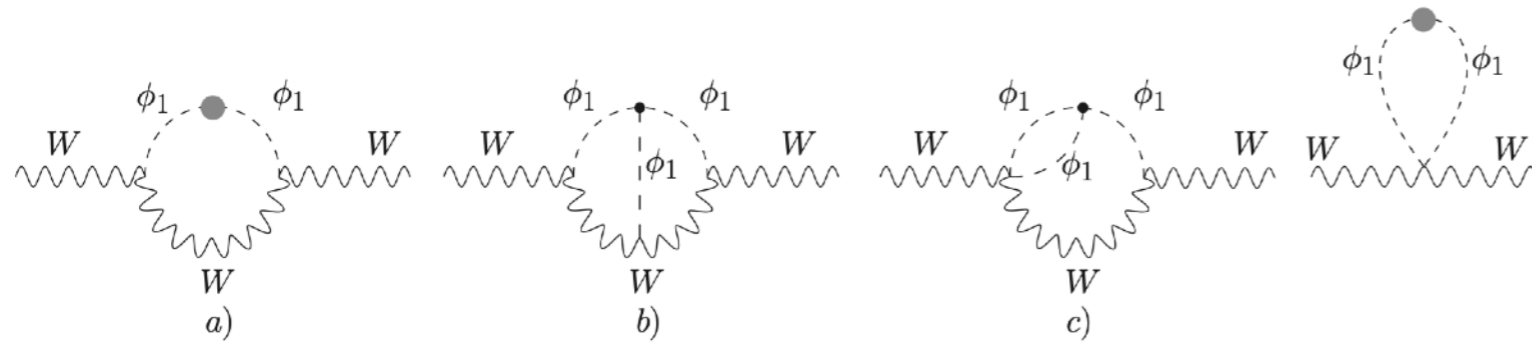
F. Maltoni, D. Pagani, A. Shivaji, X. Zhao, arXiv:1709.08649[hep-ph]

Vh and $t\bar{t}h$ production modes are the most affected

Recent studies include $gg \rightarrow H + j$, $gg \rightarrow h^* \rightarrow ZZ \rightarrow 4l$, $h \rightarrow Z\gamma$

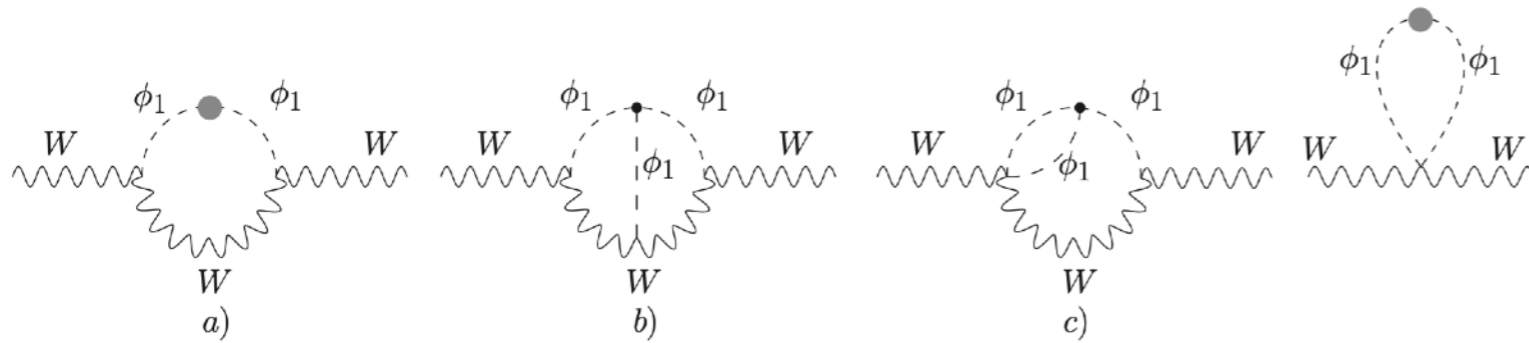
M. Gorbahn, U. Haisch, arXiv:1902.05480[hep-ph]; J. Gao, X.-M. Shen, G. Wang, L. L. Yang, B. Zhou, arXiv:2302.04160[hep-ph];

U. Haisch, G. Koole, arXiv:2111.12589[hep-ph]; G. Degrossi, M. Vitti arXiv:1912.06429[hep-ph]

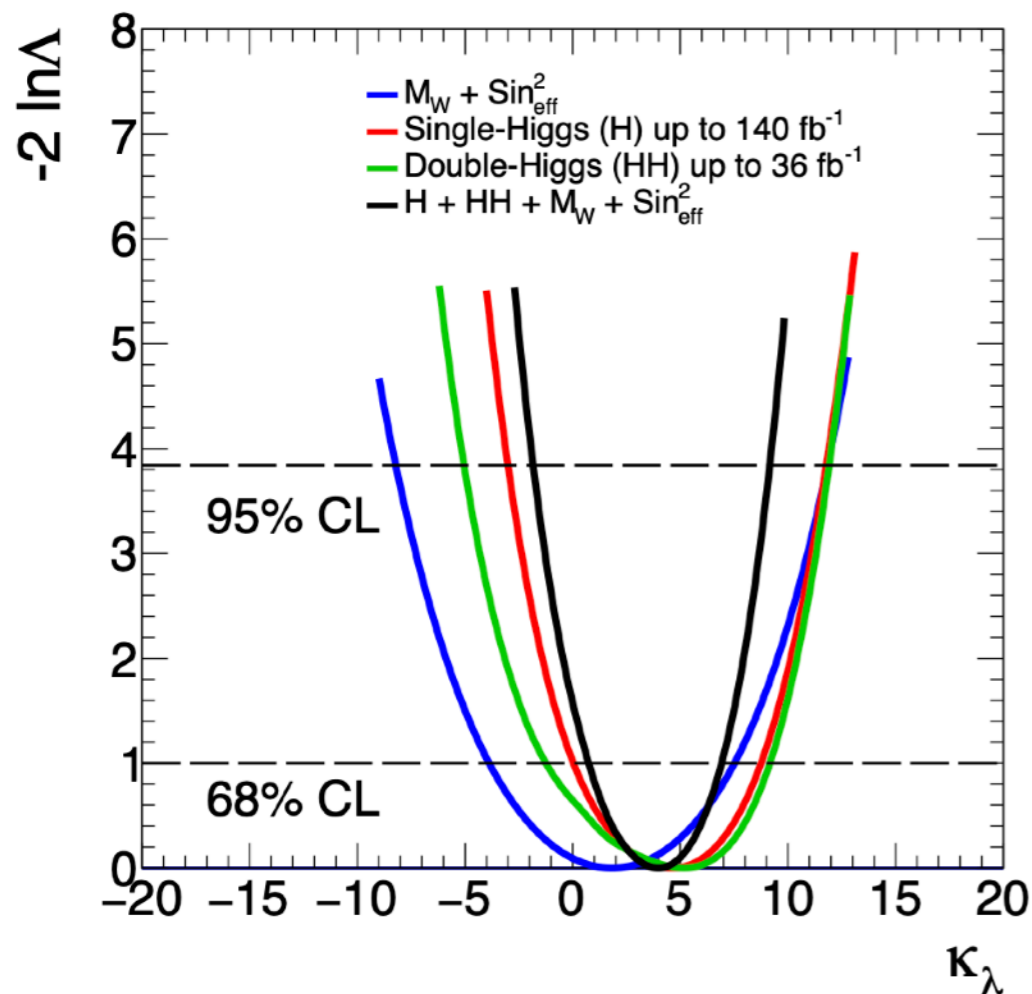


Corrections to the W mass
are also affected by λ_3

G. Degrassi, M. Fedele, PPG, arXiv:1702.01737[hep-ph];
G. D. Kribs, A. Maier, H. Rzehak, M. Spannowsky, P. Waite, arXiv:1702.07678[hep-ph];
Giuseppe Degrassi, Biagio Di Micco, PPG, Eleonora Rossi, arXiv:2102.07651 [hep-ph]



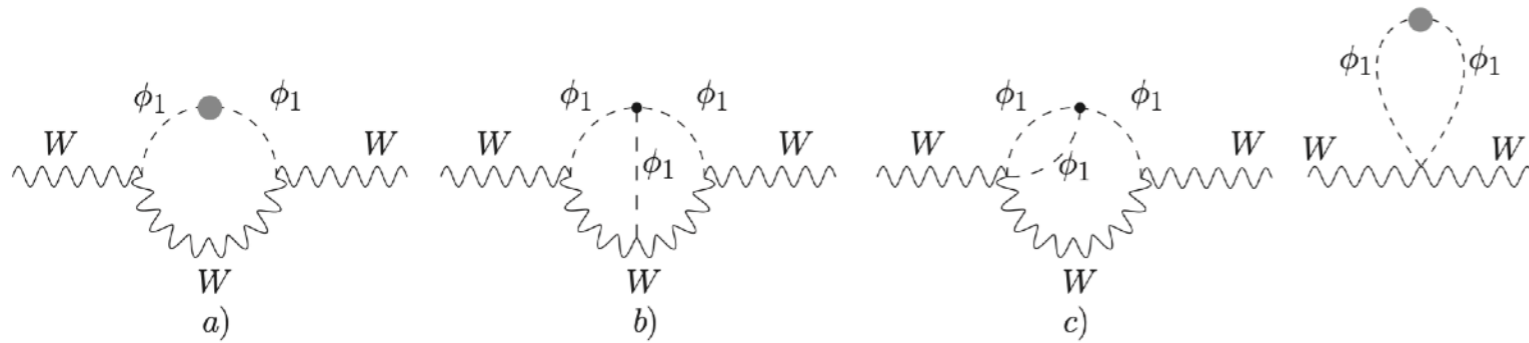
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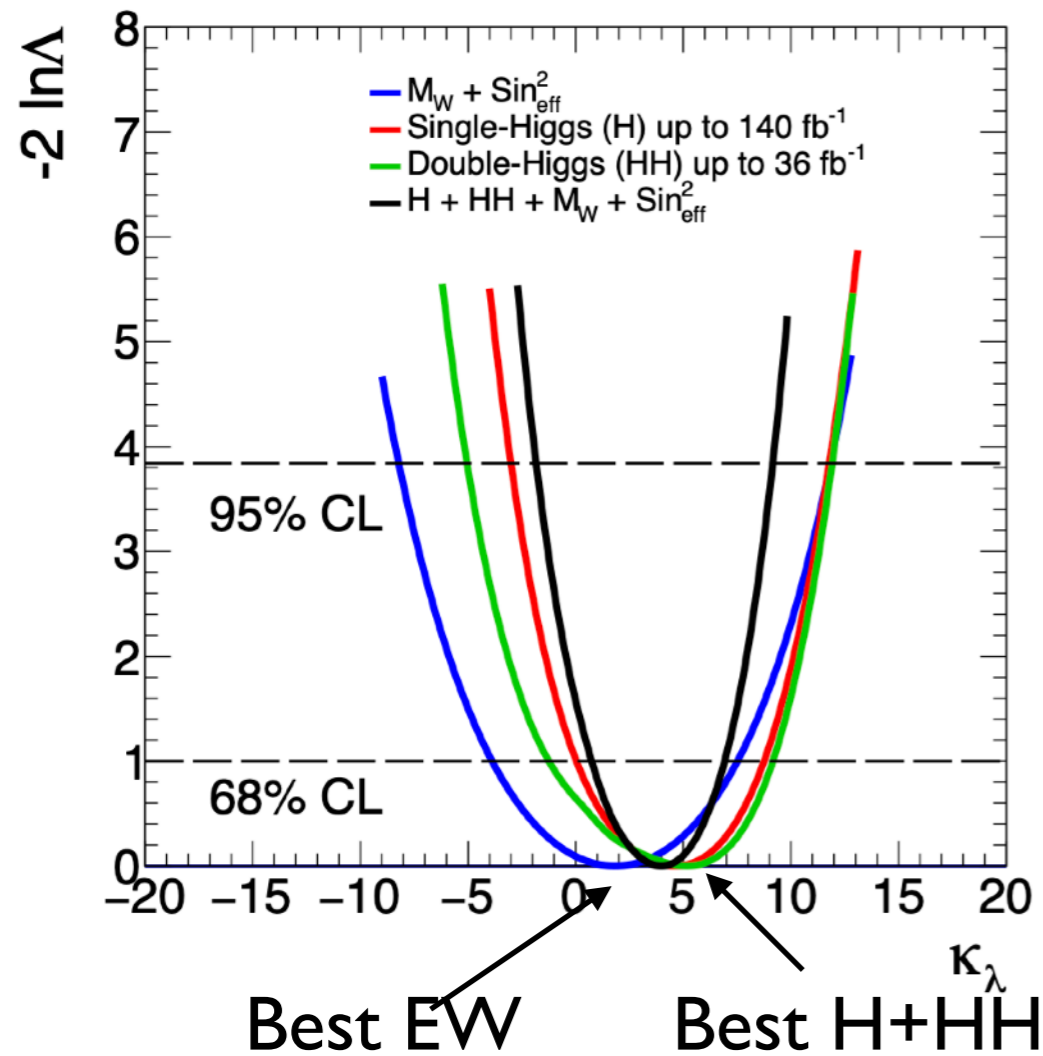
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$\sin^2 \theta_{\text{eff}}^{\text{lep}}$	0.2	-12.8 – 16.2	-18.5 – [> 20]
m_W	1.8	-3.9 – 7.6	-8.4 – 12.1
$m_W + \sin^2 \theta_{\text{eff}}^{\text{lep}}$	1.8	-3.9 – +7.5	-8.2 – 11.8
HH	5.2	-1.2 – +9.2	-5.0 – 11.9
single- H	4.6	+0.05 – +8.8	-3.0 – 11.8
Combination	4.0	0.7 – 6.9	-1.8 – 9.2

Limits from EWPO are less stringent, however the central value is shifted towards SM value

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General bounds from perturbative unitarity and vacuum stability give

$$|\delta_\lambda| \lesssim 5$$

S. Di Vita, C. Grojean, G. Panico, M. Riembau, and T. Vantalon, arXiv:1704.01953 [hep-ph], L. Di Luzio, R. Gröber, and M. Spannowsky, arXiv:1704.02311 [hep-ph], A. Falkowski and R. Rattazzi, arXiv:1902.05936 [hep-ph]

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From general studies of the
EFT structure

G. Durieux, M. McCullough, E. Salvioni arXiv:2209.00666 [hep-ph]

$$\left| \frac{\delta_\lambda}{\delta_{VV}} \right| \lesssim \min \left[\left(\frac{4\pi v}{m_h} \right)^2, \left(\frac{M}{m_h} \right)^2 \right] \lesssim 600$$

Our previous results are based on a BSM where NP induces small modifications to all SM parameters apart from Higgs self-couplings.

What are the theoretical bounds to this theory?

General bounds from perturbative unitarity and vacuum stability give

$$|\delta_\lambda| \lesssim 5$$

S. Di Vita, C. Grojean, G. Panico, M. Riembau, and T. Vantalon, arXiv:1704.01953 [hep-ph], L. Di Luzio, R. Gröber, and M. Spannowsky, arXiv:1704.02311 [hep-ph], A. Falkowski and R. Rattazzi, arXiv:1902.05936 [hep-ph]

From general studies of the EFT structure

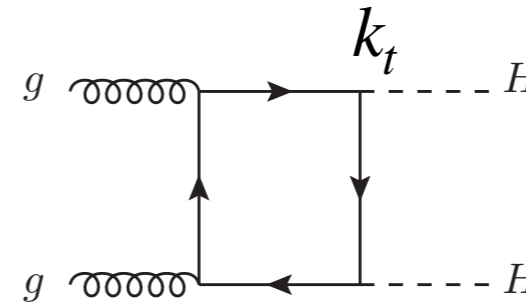
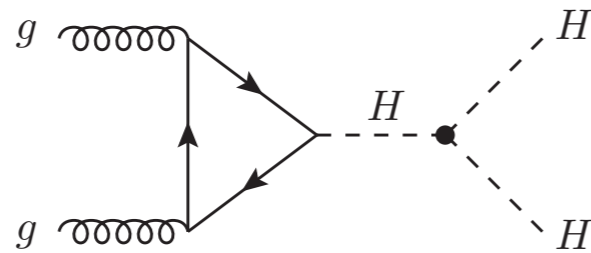
G. Durieux, M. McCullough, E. Salvioni arXiv:2209.00666 [hep-ph]

$$\left| \frac{\delta_\lambda}{\delta_{VV}} \right| \lesssim \min \left[\left(\frac{4\pi v}{m_h} \right)^2, \left(\frac{M}{m_h} \right)^2 \right] \lesssim 600$$

Specific models are more conservative but still seem to allow

$$\delta_\lambda < 200 \delta_{VV}$$

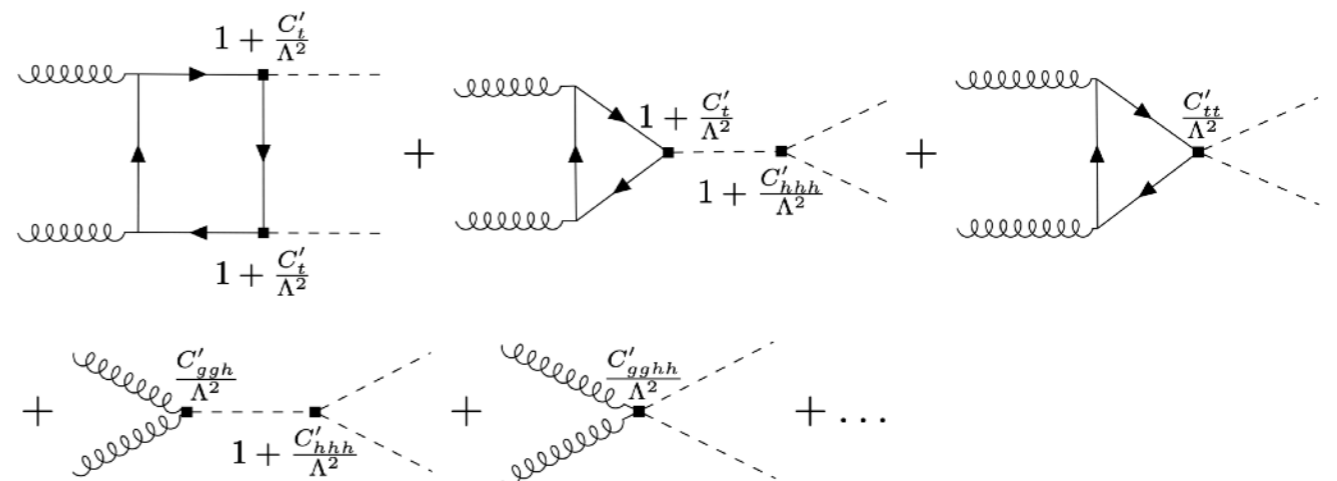
This leads us to studies with general BSM structures.

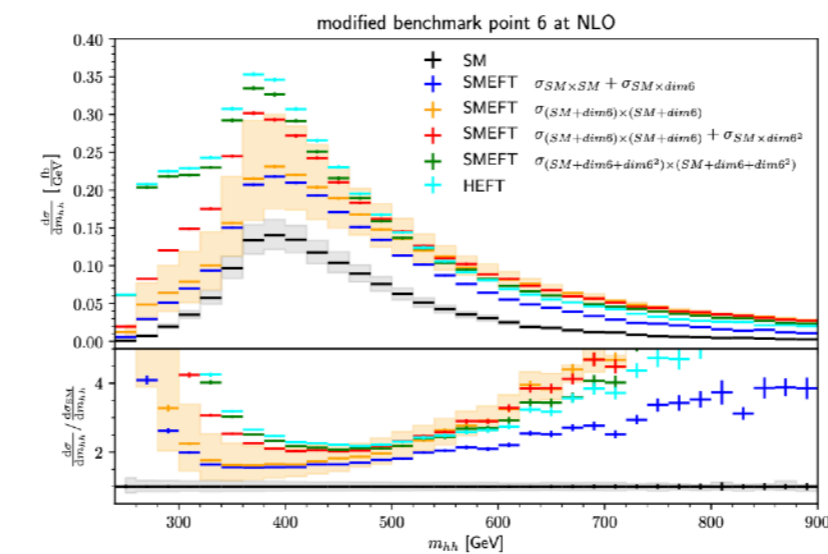
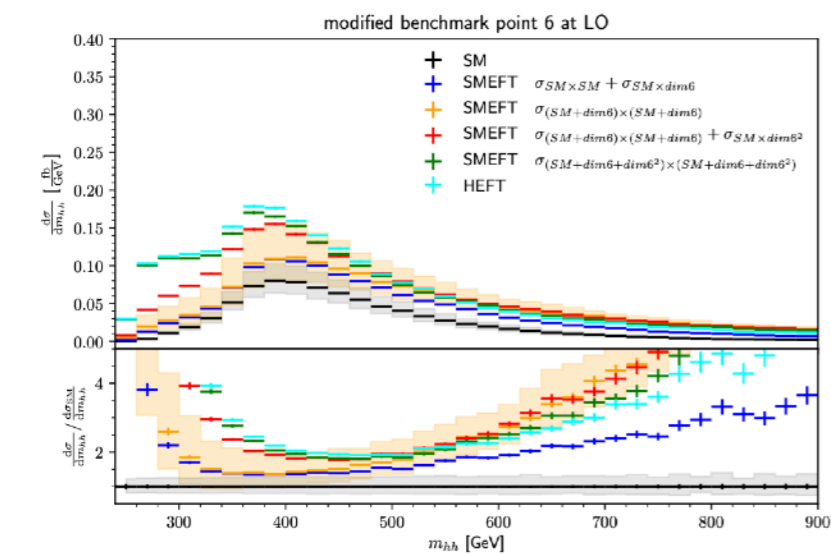
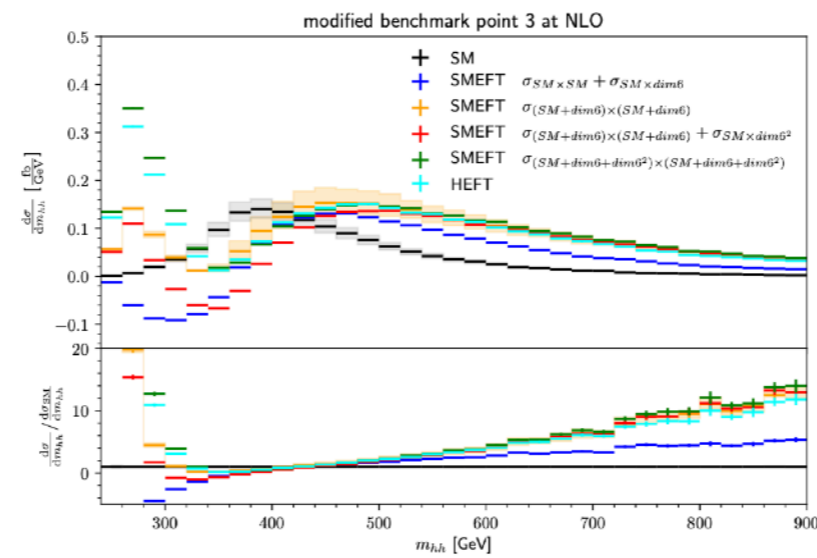
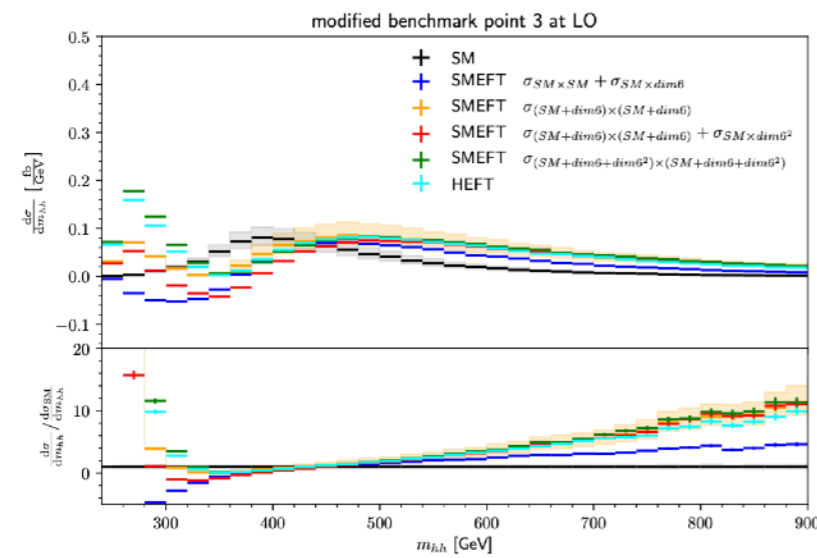
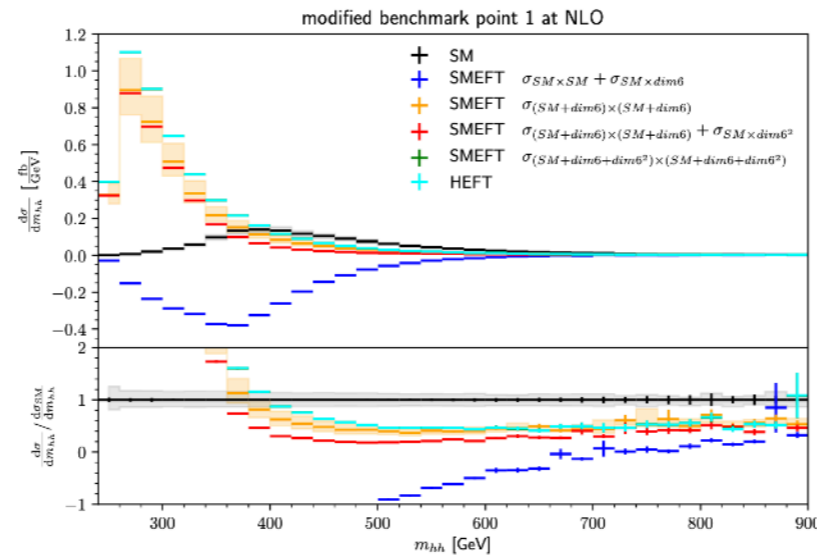
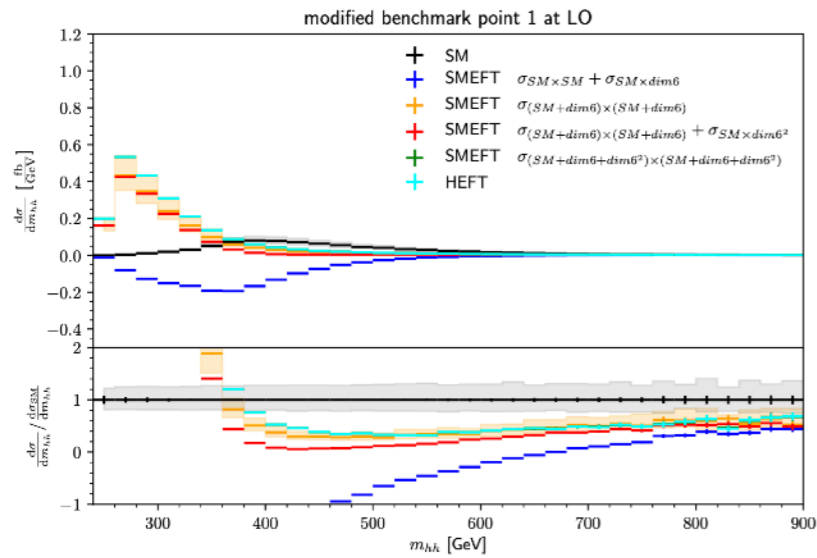


For Higgs Pair production, this requires the inclusion of anomalous top couplings.

However in general we may have to deal with the full EFT description

G. Heinrich, J. Lang, L. Scyboz, arXiv:2204.13045v2 [hep-ph]

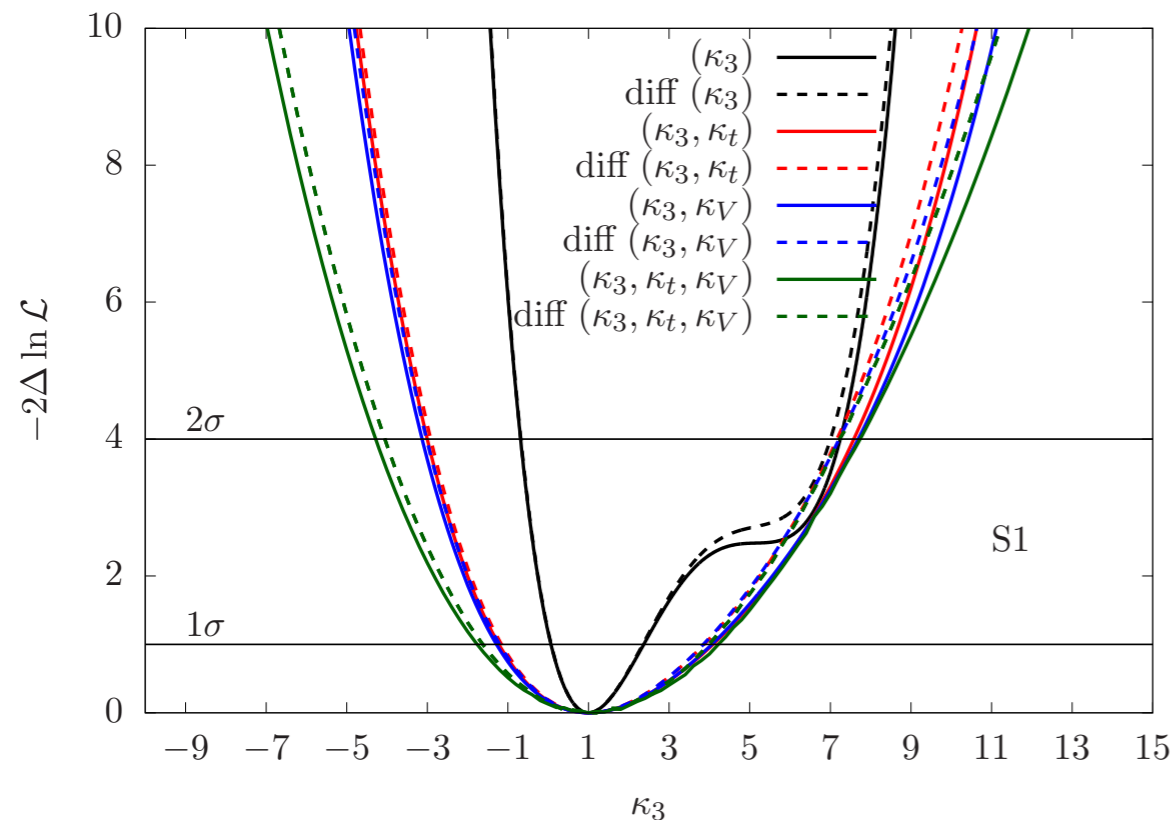




Large differences in differential cross section depending on the EFT benchmark

G. Heinrich, J. Lang, L. Scyboz, arXiv:2204.13045v2 [hep-ph]

For Single Higgs measurements the problem is direr since many operators enter at NLO



Partial studies including LO corrections and subsets of NLO corrections have been performed

F. Maltoni, D. Pagani, A. Shivaji, X. Zhao, arXiv:1709.08649 [hep-ph]
 S. Di Vita, C. Grojean, G. Panico, M. Riembau, T. Vantalona, arXiv:1704.01953 [hep-ph];
 L. Alasfar, J. de Blas, R. Gröber, arXiv:2202.02333 [hep-ph]

More detailed studies have yet to be done

- The study of the Higgs self-couplings are necessary to fully understand the mechanism of EWSB (and its consequences for phenomenology)
- NLO QCD corrections to HPP well under control, still a lot of work to do beyond that
- Indirect measurements (Single Higgs, and EWPO) can be extremely useful tools in improving the bounds coming from HPP.
- A SM-like BSM ($\kappa_\lambda < 3 - 4, \kappa_{VV,T,\dots} \sim 1 + \text{few } \%$) is allowed and could be first seen in measurements of λ_3 .
- General studies are more complicated, and will require the use of global fits (to other Higgs observables, top observables...).