Treatment of theory uncertainties in ttbar analyses

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Outline

- review of recent (and less recent) results
- focus on theoretical uncertainties
- (some) comments on what can be done next

- Fixed-order (+ analytic resummation) accuracy: astonishing
- Fully-differential results: less accurate, but ongoing progress
- MC event generators: full assessment of TH uncertainties missing
    - MC gens. used to convert data to parton-level
    - MC gens. play a role in estimating SM backgrounds for BSM,….

Total cross section: QCD corrections (TH)

Collider	σ_{tot} [pb]	scales [pb]	pdf [pb]
Tevatron	7.009	$+0.259(3.7\%)$ $-0.374(5.3\%)$	$+0.169(2.4\%)$ $-0.121(1.7\%)$
LHC 7 TeV	167.0	$+6.7(4.0\%)$ $-10.7(6.4\%)$	$+4.6(2.8\%)$ $-4.7(2.8\%)$
LHC 8 TeV	239.1	$+9.2(3.9\%)$ $-14.8(6.2\%)$	$+6.1(2.5\%)$ $-6.2(2.6\%)$
LHC 14 TeV	933.0	$+31.8(3.4\%)$ $-51.0(5.5\%)$	$+16.1(1.7\%)$ $-17.6(1.9\%)$

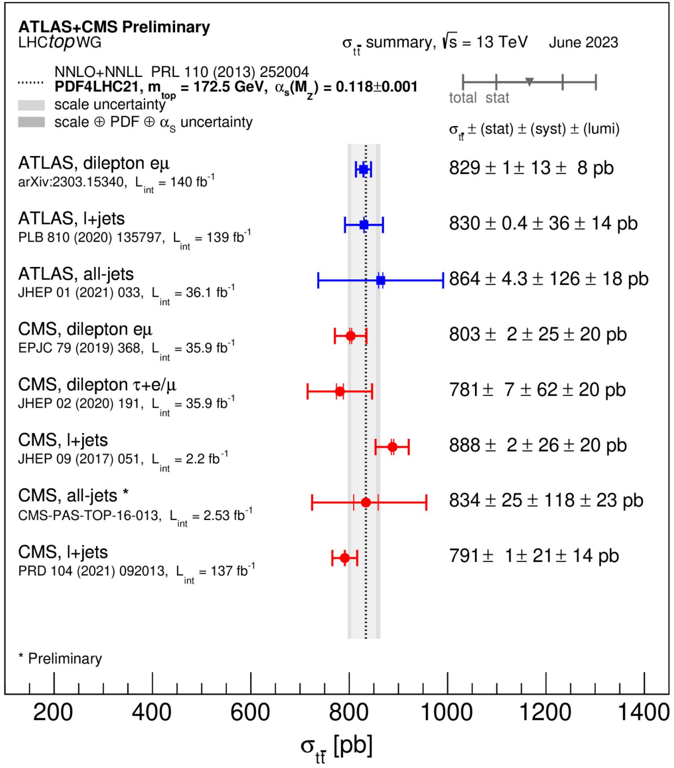
pure NNLO

Collider	σ_{tot} [pb]	scales [pb]	pdf [pb]
Tevatron	7.164	$+0.110(1.5\%)$ $-0.200(2.8\%)$	$+0.169(2.4\%)$ $-0.122(1.7\%)$
LHC 7 TeV	172.0	$+4.4(2.6\%)$ $-5.8(3.4\%)$	$+4.7(2.7\%)$ $-4.8(2.8\%)$
LHC 8 TeV	245.8	$+6.2(2.5\%)$ $-8.4(3.4\%)$	$+6.2(2.5\%)$ $-6.4(2.6\%)$
LHC 14 TeV	953.6	$+22.7(2.4\%)$ $-33.9(3.6\%)$	$+16.2(1.7\%)$ $-17.8(1.9\%)$

NNLO+NNLL

- State of the art: NNLO+NNLL (different approaches possible for resummation)
- Results here: [Czakon,Fiedler,Mitov ‘13], analogous in [Catani et al. ‘19-’20]
- $\mu=m_{top}$ + 7 pts scale variation
- Effect of resummation: +2-3% on σ, reduced TH uncertainty
- Final uncertainty: ± ~4% (@NLO QCD: ± 12%)

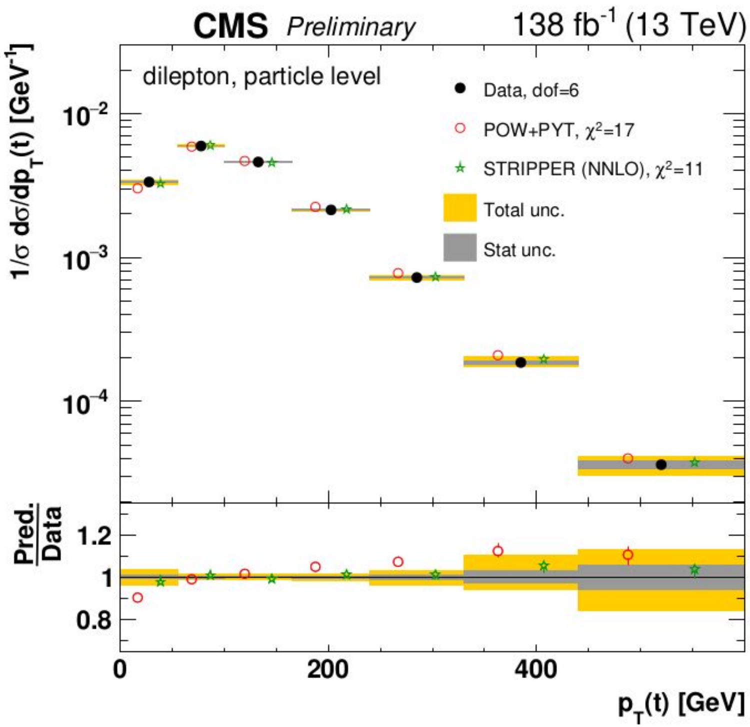
Total cross section: QCD corrections (TH-EXP)



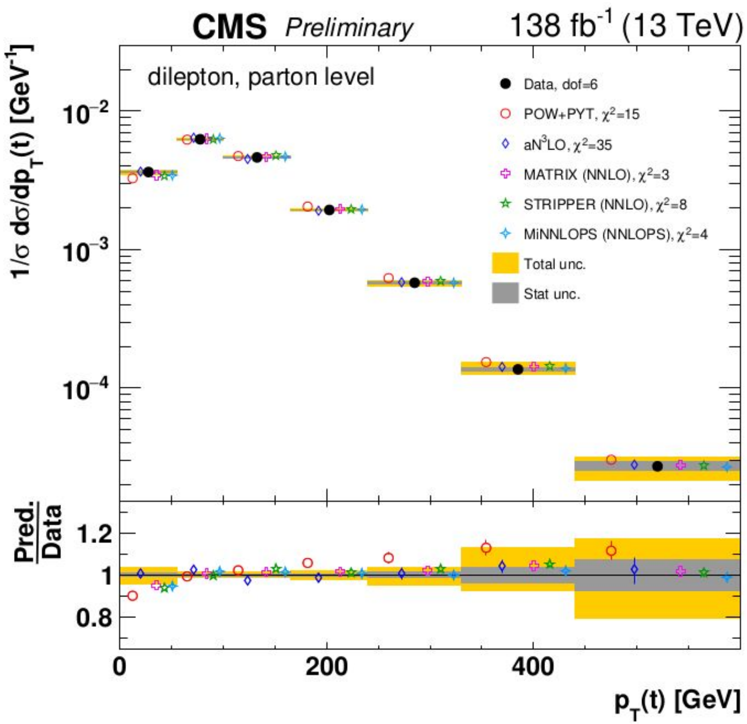
[ATL-PHYS-PUB-2023-014]

- agreement data-theory very good
- central scale choice: $\mu = m_{top}$
- if $\mu \sim H_T$: pQCD uncertainties ~6%
- scale uncertainties $\gtrsim$ PDF uncertainty
- similar trends @ 7 and 8 TeV

Differential distributions: QCD corrections

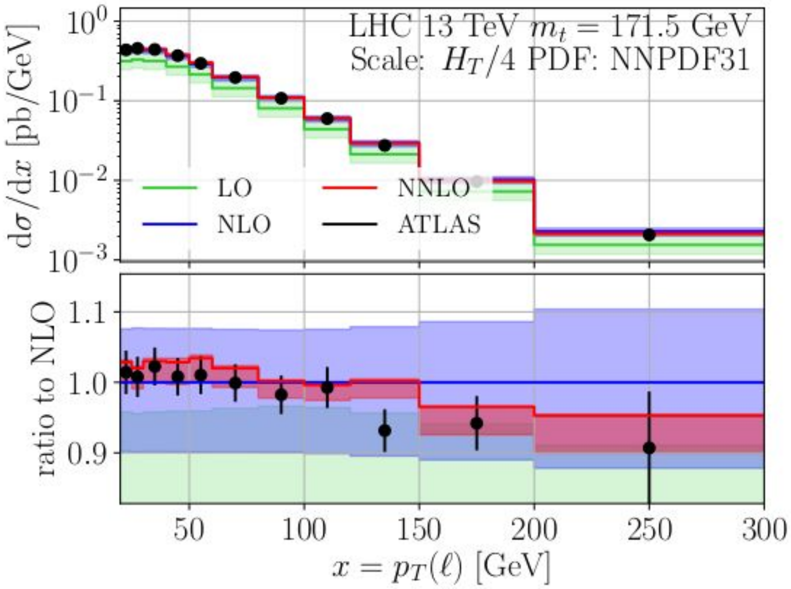


[CMS-PAS-TOP-20-006]

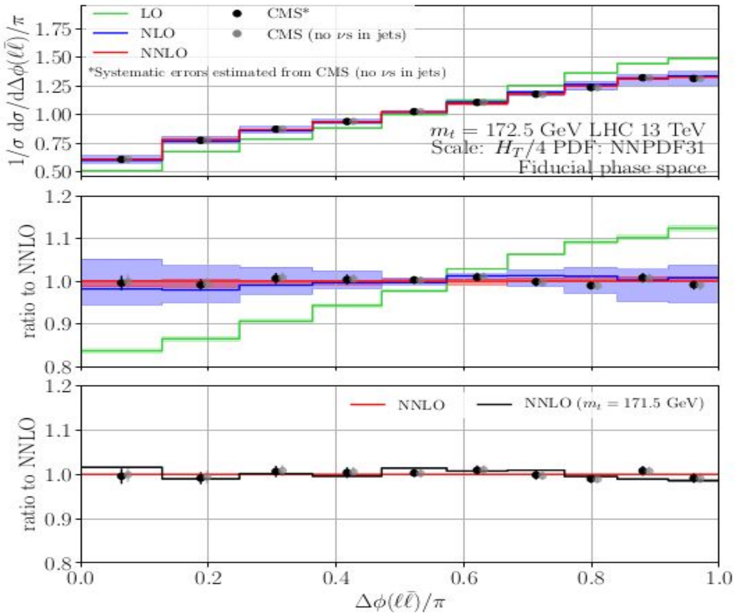


- scale choice: $\mu = H_T/4 = (m_T(t)+m_T(\text{tbar}))/4$ + 7 pts scale variation [studied in 1606.0350]
- NNLO corrections → fix long-standing issue with $p_T(t)$
- recent insights on hardness vs. kinematics in ttbar: [2101.06068]

Differential distributions: QCD corrections (decay)



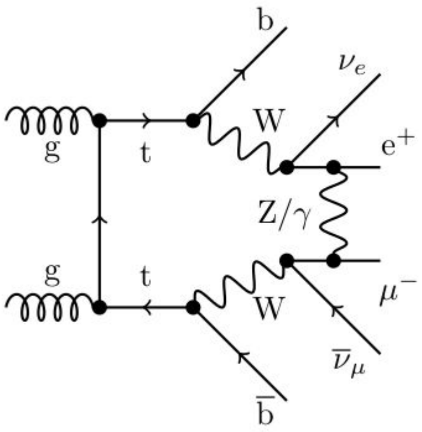
[Czakon,Mitov,Poncelet '20]

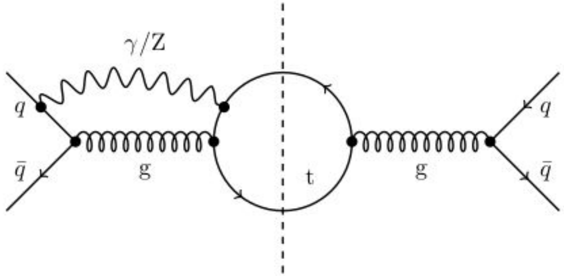


$$d\sigma = d\sigma_{t\bar{t}} \times \frac{d\Gamma_t}{\Gamma_t} \times \frac{d\Gamma_{\bar{t}}}{\Gamma_t}$$

- NWA, keeping spin correlations
- NNLO x NNLO x NNLO
- $\mu = H_T/4$

EW corrections: from total cross sections….



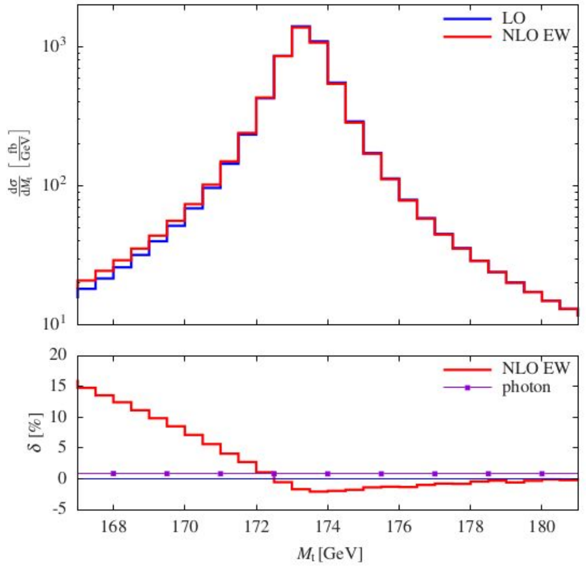


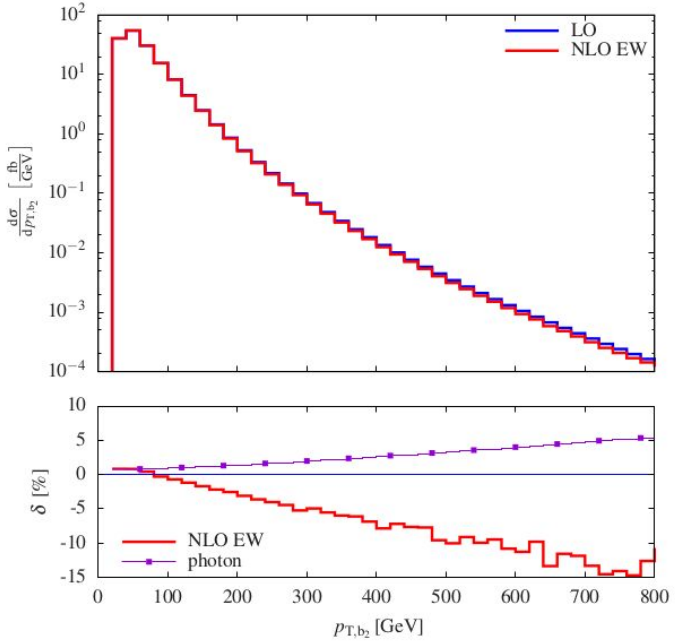
Ch.	$\sigma_{\rm LO}$ [fb]	$\sigma_{\rm NLO\ EW}$ [fb]	δ [%]
gg	2824.2(2)	2834.2(3)	0.35
$q\bar{q}$	375.29(1)	377.18(6)	0.50
$gq(/\bar{q})$		0.259(4)	
γg		27.930(1)	
pp	3199.5(2)	3211.7(3)	0.38

- fully exclusive results (2 lep channel)
- NLO EW = $O(\alpha_s^2\ \alpha^5)$
- total x-section: EW corrections < 1%
- no uncertainty on EW corrections
  (but irrelevant for total x-section)

[Denner,Pellen ‘16]

….to differential distributions

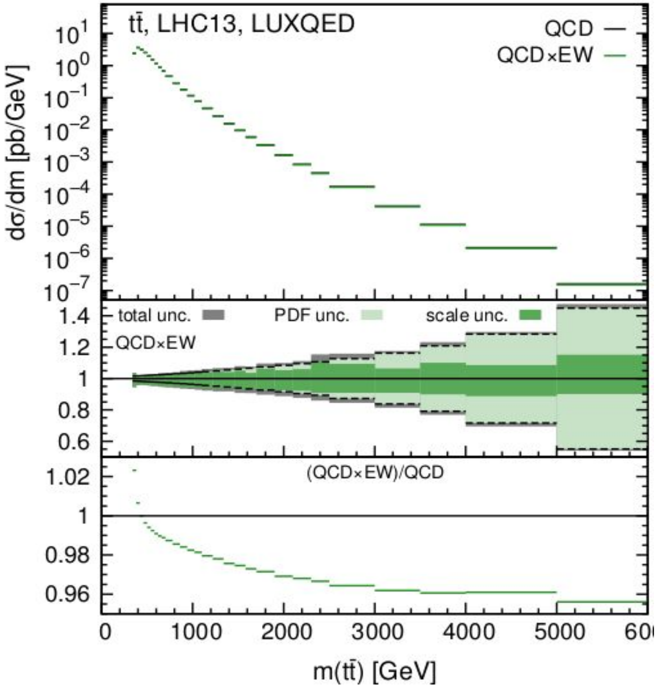




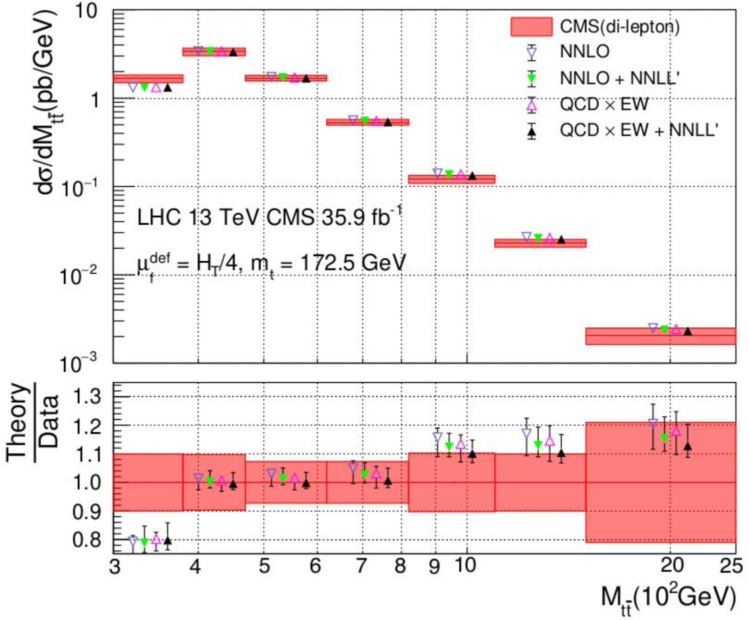
- corrections ~ 10-15 % (radiative tails (left) / Sudakov logs (right))
- here: no approximations. Possible to study “DP approximations” for tops or Ws.
- scales fixed at top mass (enter in α_s & PDFs)
- TH uncertainties: needed? Uncharted territory: change of scheme (?),...

EW and QCD combined results

[1705.04105]



[1901.08281]



$$\Sigma_{\mathrm{QCD}\times\mathrm{EW}} \equiv \Sigma_{\mathrm{QCD+EW}} + (K^{\mathrm{NLO}}_{\mathrm{QCD}} - 1) \times \Sigma_{\mathrm{NLO\ EW}}$$

preferred combination: multiplicative ⇒

- stabilize scale uncertainty of $\Sigma_{\mathrm{NLO\ EW}} = O(\alpha_s^2\,\alpha)$
- $\Sigma_{\mathrm{mixed}} = O(\alpha_s^3\,\alpha) \approx \Sigma_{\mathrm{NLO\ QCD}}\,\Sigma_{\mathrm{NLO\ EW}} / \Sigma_{\mathrm{LO\ QCD}}$
- correct in regime “soft gluon” + “Sudakov log”

Parton-level results: summary

- QCD corrections: clear picture, good perturbative convergence
- different scale choices possible: H_T-like seems to do a good job for distributions

- Full NLO EW available, possible to validate different approximations

- Combination of EW and QCD corrections: done for stable top quarks
- Residual uncertainty (size, PDF vs μ) depends on observables
  - EW+QCD combination: multiplicative vs. additive (the former preferred)
- Possible to supplement it with resummation(s)

MC event generators: status

- Several NLO+PS generators available: **POWHEG BOX**, **MG5_aMC@NLO**, **Sherpa**, **Herwig** (through `Matchbox`).

- Often approximations made to include decays and off-shell effects (e.g. `ttb_NLO_dec`: prod@NLO x decay@NLO)

- Exact MC simulation for decay, offshell effects and interferences: `bb4l` **NLO+PS** generator [Jezo,Lindert et al. ‘16]

- Multijet merging up to 2 jets @ NLO+PS [MEPS@NLO, FxFx, ‘12-’13]

- NNLO+PS results available [Mazzitelli, Monni, Nason, ER, Wiesemann, Zanderighi ‘20-’21]

top pair-production @ NNLO+PS: MiNNLO (I)

- from p_T resummation, differential cross section for $F+X$ production can be written as:

$$\frac{d\sigma}{dp_T d\Phi_F} = \frac{d}{dp_T}\left\{\mathcal{L}(\Phi_F, p_T)\exp(-\tilde{S}(p_T))\right\} + R_{\text{finite}}(p_T)$$

$$\mathcal{L}(\Phi_F, p_T) \ni \{H^{(1)}, H^{(2)}, C^{(1)}, C^{(2)}, (G^{(1)} \cdot G^{(1)})\} \qquad R_{\text{finite}}(p_T) = \frac{d\sigma_{FJ}}{d\Phi_F dp_T} - \frac{d\sigma^{\text{sing}}}{d\Phi_F dp_T}$$

- recast it, to match the POWHEG $\bar{B}^{(FJ)}(\Phi_{FJ})$

$$\frac{d\sigma}{d\Phi_F dp_T} = \exp[-\tilde{S}(p_T)]\left\{D(p_T) + \frac{R_{\text{finite}}(p_T)}{\exp[-\tilde{S}(p_T)]}\right\}$$

$$D(p_T) \equiv -\frac{d\tilde{S}(p_T)}{dp_T}\mathcal{L}(p_T) + \frac{d\mathcal{L}(p_T)}{dp_T} \qquad \tilde{S}(p_T) = \int_{p_T}^{Q}\frac{dq^2}{q^2}\left[A_f(\alpha_S(q))\log\frac{Q^2}{q^2} + B_f(\alpha_S(q))\right]$$

- expand the above integrand in power of $\alpha_S(p_T)$, keep the terms that are needed to get NLO$^{(F)}$ & NNLO$^{(F)}$ accuracy, when integrating over p_T

top pair-production @ NNLO+PS: MiNNLO (II)

$$\frac{\mathrm{d}\bar{B}(\Phi_{\mathrm{FJ}})}{\mathrm{d}\Phi_{\mathrm{FJ}}} = \exp[-\tilde{S}(p_{\mathrm{T}})]\left\{\frac{\alpha_{\mathrm{S}}(p_{\mathrm{T}})}{2\pi}\left[\frac{\mathrm{d}\sigma_{\mathrm{FJ}}}{\mathrm{d}\Phi_{\mathrm{FJ}}}\right]^{(1)}\left(1+\frac{\alpha_{\mathrm{S}}(p_{\mathrm{T}})}{2\pi}[\tilde{S}(p_{\mathrm{T}})]^{(1)}\right) + \left(\frac{\alpha_{\mathrm{S}}(p_{\mathrm{T}})}{2\pi}\right)^2\left[\frac{\mathrm{d}\sigma_{\mathrm{FJ}}}{\mathrm{d}\Phi_{\mathrm{FJ}}}\right]^{(2)} + [D(p_{\mathrm{T}})]^{(\geq 3)} F_\ell^{\mathrm{corr}}(\Phi_{\mathrm{FJ}})\right\}$$

- $[D(p_{\mathrm{T}})]^{(\geq 3)} = \underbrace{-\frac{\mathrm{d}\tilde{S}(p_{\mathrm{T}})}{\mathrm{d}p_{\mathrm{T}}}\mathcal{L}(p_{\mathrm{T}}) + \frac{\mathrm{d}\mathcal{L}(p_{\mathrm{T}})}{\mathrm{d}p_{\mathrm{T}}}}_{D} - \frac{\alpha_{\mathrm{S}}(p_{\mathrm{T}})}{2\pi}[D(p_{\mathrm{T}})]^{(1)} - \left(\frac{\alpha_{\mathrm{S}}(p_{\mathrm{T}})}{2\pi}\right)^2 [D(p_{\mathrm{T}})]^{(2)}$
- $F_\ell^{\mathrm{corr}}(\Phi_{\mathrm{FJ}})$: projection → recover $[D(p_{\mathrm{T}})]^{(\geq 3)}$ when integrating over Φ_{FJ} at fixed $(\Phi_{\mathrm{F}}, p_{\mathrm{T}})$

. The second radiation is generated by the usual POWHEG mechanism.

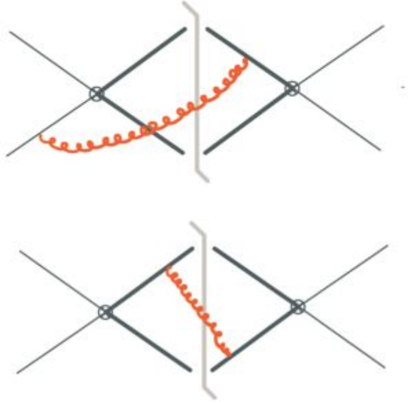
$$\mathrm{d}\sigma = \bar{B}(\Phi_{\mathrm{FJ}})\,\mathrm{d}\Phi_{\mathrm{FJ}}\left\{\Delta_{\mathrm{pwg}}(\Lambda_{\mathrm{pwg}}) + \mathrm{d}\Phi_{\mathrm{rad}}\Delta_{\mathrm{pwg}}(p_{\mathrm{T,rad}})\frac{R(\Phi_{\mathrm{FJ}},\Phi_{\mathrm{rad}})}{B(\Phi_{\mathrm{FJ}})}\right\}$$

. if emissions are strongly ordered, same emission probabilities as in k_t-ordered shower
→ LL shower accuracy preserved

top pair-production @ NNLO+PS: MiNNLO for ttbar

Starting point: resummation formula for $t\bar{t}$ transverse momentum. [Catani,Grazzini,Torre '14]

Very schematically:



$$d\sigma^F_{\rm res} \sim \frac{d}{dp_T}\left\{e^{-S}\,{\rm Tr}(\mathbf{H}\mathbf{\Delta})\,(C\otimes f)\,(C\otimes f)\right\}$$

$$S = -\int\frac{dq^2}{q^2}\left[\frac{\alpha_s(q)}{2\pi}\left(A^{(1)}\log(M/q)+B^{(1)}\right)+\frac{\alpha_s^2(q)}{(2\pi)^2}\left(A^{(2)}\log(M/q)+B^{(2)}\right)+\ldots\right]$$

$${\rm Tr}(\mathbf{H}\mathbf{\Delta}) = \langle M|\mathbf{\Delta}|M\rangle, \quad \mathbf{\Delta}=\mathbf{V}^\dagger\mathbf{D}\,\mathbf{V}, \quad \mathbf{V}=\exp\left\{-\int\frac{dq^2}{q^2}\left[\frac{\alpha_s(q)}{2\pi}\mathbf{\Gamma}_t^{(1)}+\frac{\alpha_s^2(q)}{(2\pi)^2}\mathbf{\Gamma}_t^{(2)}\right]\right\}$$

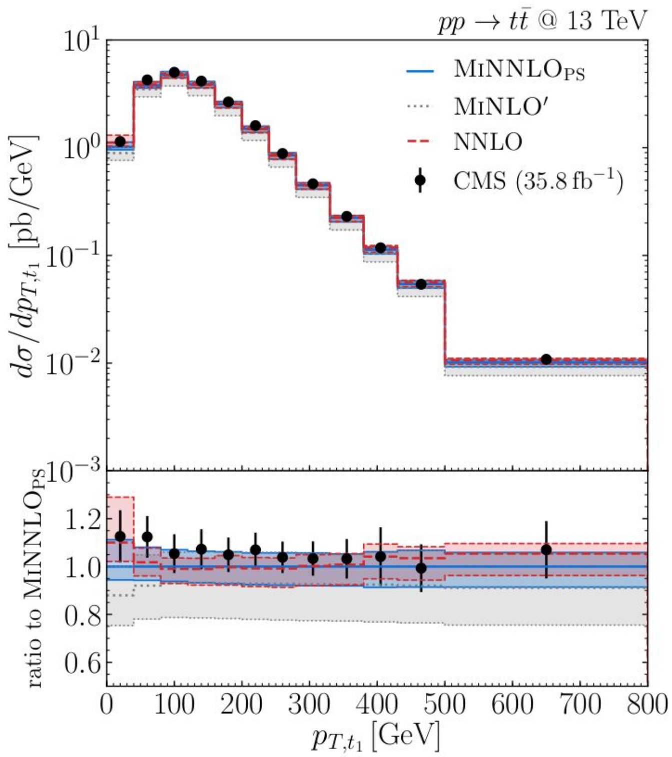
- With some approximations (respecting our goal), terms due to soft interference can be rearranged so that the "resummation" can be eventually recasted as:

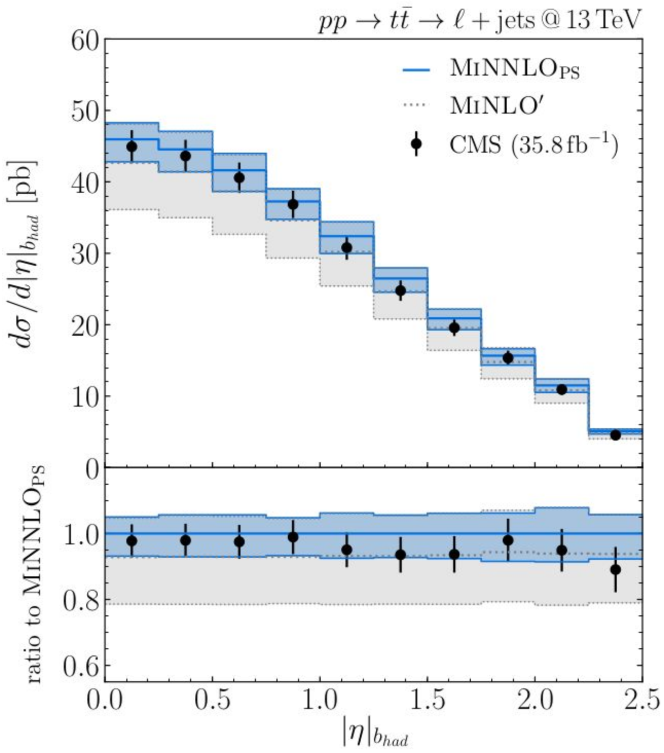
$$d\sigma^F_{\rm res} \sim \frac{d}{dp_T}\left\{\sum_{i\in\text{colours}} e^{-\bar{S}_i}\,\underbrace{c_i\,\overline{H}\,\overline{(C\otimes f)}\,\overline{(C\otimes f)}}_{\equiv\,\overline{\mathscr{L}}_i}\right\}+\mathcal{O}(\alpha_S^5)$$

some inputs derived in [Catani, Devoto, Grazzini, Kallweit, Mazzitelli + Sargsyan '19]

- Each term has the same structure as in the color-singlet case!

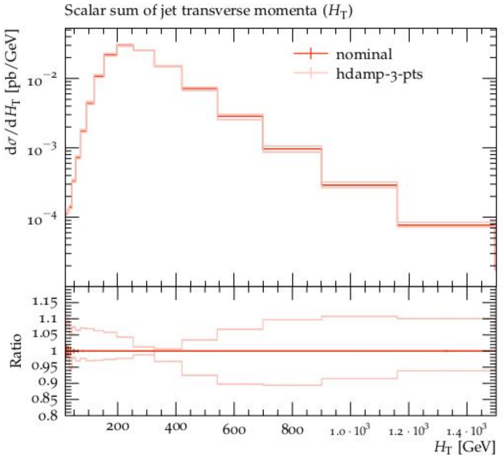
top pair-production @ NNLO+PS: results



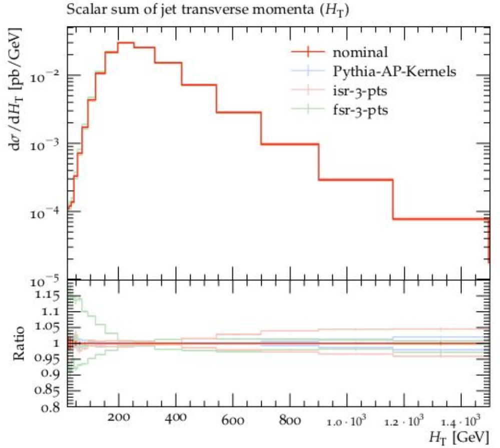


- nice agreement with NNLO (and with data - both ATLAS and CMS). $\mu_{core} = H_T/4$
- implemented top-quark decays @ tree level + approximated off-shell effects
- NB: if analysis probes off-shell/non-resonant regions →`bb4l` **NLO+PS** should be method of choice

TH uncertainties in MC generators



[Amoroso,ER LH19 (2003.01700)]



Possible TH uncertainties:

- scale variation (hard matrix elements)
- matching uncertainties (e.g. `hdamp` in POWHEG, hard veto scale in MC@NLO,...)
- change matching scheme / shower [e.g. Matchbox study, 1810.06493]
- other “shower-related” pQCD uncertainties can be probed
- recoil scheme
- ….
- non-perturbative parameters & tuning

- possible to include such variations (→certainly not the ultimate solution)
- within the current paradigms for matching & merging, rethinking needed for some of the above items, once matching to NLL parton showers will be achieved

recent progress in PS: [talk by F. Herren]

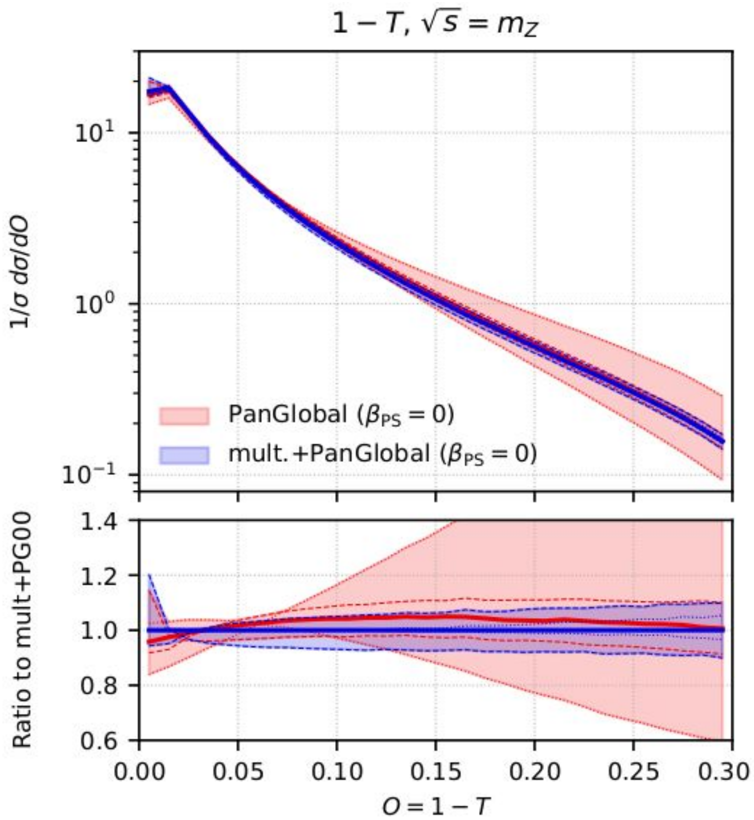
TH uncertainties in MC generators: looking ahead

- plots from: [Hamilton et al. 2301.09645]
- thrust in e^+e^-: NLO+PS multiplicative matching + NLL shower

- dots: modified splitting function in hard region
- dashes: μ_R scale variation (also in hard matrix elements)

- here matching fulfils NNDL accuracy (i.e. the same accuracy of a NLL resummation matched with NLO)

$$\Sigma(O < e^L) = h_1(\alpha_s L^2) + \sqrt{\alpha_s}\, h_2(\alpha_s L^2) + \alpha_s h_3(\alpha_s L^2) + \dots$$



Conclusions

- Assessing TH uncertainties in ttbar production is imperative:

  - “somehow” well-established for QCD fixed-order results

  - combination of EW and QCD corrections: different schemes, with choices made on physics intuition

  - MC generators: many scales, and many parameters: difficult!

  - At least for the perturbative part, recent and ongoing progress will hopefully give insights also for ttbar