The structure of the dimension-8 anomalous dimension matrix

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based on *2106.05291*, *2110.01264*, *2301.09995* *and ongoing work*

Standard Model at the LHC 2023, Fermilab; July 12, 2023

The SMEFT is the SM extended with effective operators

(Probably) the most reasonable model of new physics

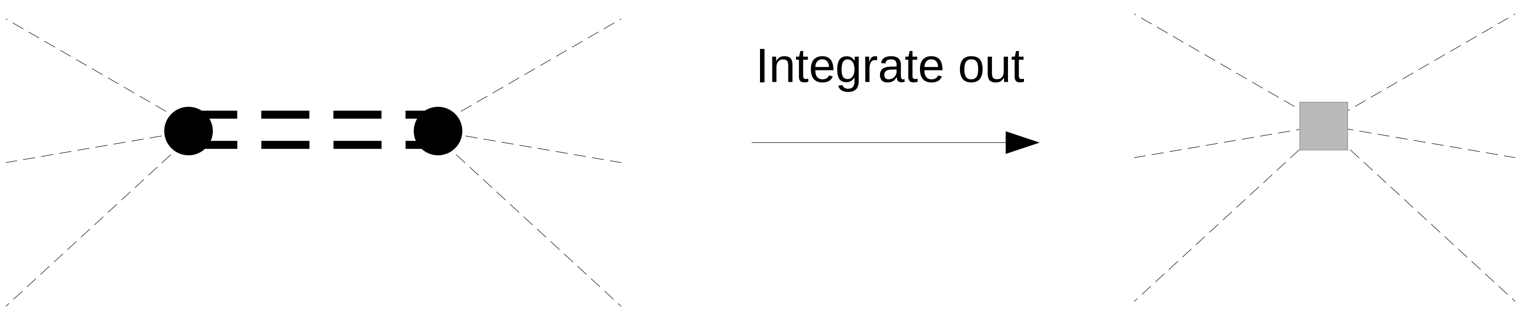
$$\mathcal{L}_{\mathrm{SMEFT}} = \mathcal{L}_{\mathrm{SM}} + \sum_i \frac{c_i^{(6)}}{\Lambda^2}\mathcal{O}_i^{(6)} + \sum_j \frac{c_j^{(8)}}{\Lambda^4}\mathcal{O}_j^{(8)} + \cdots$$

If probed by experiments at very different scales, RGEs of the theory are needed [Jenkins, Manohar, Trott, Alonso '13].

Interesting theoretical aspects at dimension 8 (positivity, tree-loop mixing, test tools, ...)

Besides pure theoretical considerations, anomalous dimensions of dimension-8 operators [Murphy '20; Li, Ren, Shu, Xiao, Yu, Zheng '20] not always phenomenologically irrelevant

Simplest example:



Custodial symmetry violation absent at tree-level dimension-6, one-loop dimension-6 and tree-level dimension-8 [**MC**, Krause, Nardini '18; Durieux, McCullough, Salvioni '22]

Partial results on SMEFT RGEs to dimension 8:

MC, Guedes, Ramos, Santiago; 2106.05291

Accettulli Huber, De Angelis; 2108.03669

Bakshi, **MC**, Diaz-Carmona, Guedes; 2205.03301

Helset, Jenkins, Manohar; 2212.03253

Asteriadis, Dawson, Fontes; 2212.03258

Bakshi, Diaz-Carmona; 2301.07151

Helset, Jenkins, Manohar; 2212.03253

Di Zhang; 2306.03008

. . .

	d_5	d_5^2	d_6	d_5^3	$d_5 \times d_6$	d_7	d_5^4	$d_5^2 \times d_6$	d_6^2	$d_5 \times d_7$	d_8
$d_{\leq 4}$ (bosonic)			✓						✓		✓
$d_{\leq 4}$ (fermionic)			✓						X		X
d_5	✓				✓	✓					
d_6 (bosonic)		✓	✓					X	✓	X	✓
d_6 (fermionic)		✓	✓					X	X	X	X
d_7				✓	✓	✓					
d_8 (bosonic)							✓	✓	✓	✓	✓
d_8 (fermionic)							X	X	X	X	✓

Helset, Jenkins, Manohar; 2212.03253

Asteriadis, Dawson, Fontes; 2212.03258

Bakshi, Diaz-Carmona; 2301.07151

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. . .

	d_5	d_5^2	d_6	d_5^3	$d_5 \times d_6$	d_7	d_5^4	$d_5^2 \times d_6$	d_6^2	$d_5 \times d_7$	d_8
$d_{\leq 4}$ (bosonic)			✓						✓		✓
$d_{\leq 4}$ (fermionic)			✓						X		X
d_5	✓				✓	✓					
d_6 (bosonic)		✓	✓					X	✓	X	✓
d_6 (fermionic)		✓	✓					X	X	X	X
d_7				✓	✓	✓					
d_8 (bosonic)							✓	✓	✓	✓	✓
d_8 (fermionic)							X	X	X	X	✓

More generally, certain aspects of the full anomalous dimension matrix well understood

Craig, Jiang, Li, Sutherland; 2001.00017

$\overline{w}$ \ w	0	2	4	6	8
8	X_L^4	$X_L^3H^2$, $X_L^2\psi^2H$, $X_L\psi^4$	$X_L^2H^4$, $X_L\psi^2H^3$, ψ^4H^2	ψ^2H^5	H^8
6		$X_L^2H^2D^2$, $X_L^2\psi\bar{\psi}D$, $X_L\psi^2HD^2$, ψ^4D^2	$X_LH^4D^2$, $X_L^2\bar{\psi}^2H$, $X_L\psi\bar{\psi}H^2D$, $\psi^2H^3D^2$, $X_L\psi^2\bar{\psi}^2$, $\psi^3\bar{\psi}HD$	H^6D^2, $\psi\bar{\psi}H^4D$, $\psi^2\bar{\psi}^2H^2$	$\bar{\psi}^2H^5$
4			$X_L^2X_R^2$, $X_LX_RH^2D^2$, H^4D^4, $X_LX_R\psi\bar{\psi}D$, $X_R\psi^2HD^2$, $X_L\bar{\psi}^2HD^2$, $\psi\bar{\psi}H^2D^3$, $\psi^2\bar{\psi}^2D^2$	$X_RH^4D^2$, $X_R^2\psi^2H$, $X_R\psi\bar{\psi}H^2D$, $\bar{\psi}^2H^3D^2$, $X_R\psi^2\bar{\psi}^2$, $\psi\bar{\psi}^3HD$	$X_R^2H^4$, $X_R\bar{\psi}^2H^3$, $\bar{\psi}^4H^2$
2				$X_R^2H^2D^2$, $X_R^2\psi\bar{\psi}D$, $X_R\bar{\psi}^2HD^2$, $\bar{\psi}^4D^2$	$X_R^3H^2$, $X_R^2\bar{\psi}^2H$, $X_R\bar{\psi}^4$
0					X_R^4

Murphy ‘20;
based on Craig et al ‘20

$\overline{w}$ \ w	0	2	4	6	8
8	X_L^4	$X_L^3H^2$, $X_L^2\psi^2H$, $X_L\psi^4$	$X_L^2H^4$, $X_L\psi^2H^3$, ψ^4H^2	ψ^2H^5	H^8
6		$X_L^2H^2D^2$, $X_L^2\psi\bar{\psi}D$, $X_L\psi^2HD^2$, ψ^4D^2	$X_LH^4D^2$, $X_L^2\bar{\psi}^2H$, $X_L\psi\bar{\psi}H^2D$, $\psi^2H^3D^2$, $X_L\psi^2\bar{\psi}^2$, $\psi^3\bar{\psi}HD$	H^6D^2, $\psi\bar{\psi}H^4D$, $\psi^2\bar{\psi}^2H^2$	$\bar{\psi}^2H^5$
4			$X_L^2X_R^2$, $X_LX_RH^2D^2$, H^4D^4, $X_LX_R\psi\bar{\psi}D$, $X_R\psi^2HD^2$, $X_L\bar{\psi}^2HD^2$, $\psi\bar{\psi}H^2D^3$, $\psi^2\bar{\psi}^2D^2$	$X_RH^4D^2$, $X_R^2\psi^2H$, $X_R\psi\bar{\psi}H^2D$, $\bar{\psi}^2H^3D^2$, $X_R\psi^2\bar{\psi}^2$, $\psi\bar{\psi}^3HD$	$X_R^2H^4$, $X_R\bar{\psi}^2H^3$, $\bar{\psi}^4H^2$
2				$X_R^2H^2D^2$, $X_R^2\psi\bar{\psi}D$, $X_R\bar{\psi}^2HD^2$, $\bar{\psi}^4D^2$	$X_R^3H^2$, $X_R^2\bar{\psi}^2H$, $X_R\bar{\psi}^4$
0					X_R^4

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$\overline{w}$ \ w	0	2	4	6	8
8	X_L^4	$X_L^3 H^2$, $X_L^2 \psi^2 H$, $X_L \psi^4$	$X_L^2 H^4$, $X_L \psi^2 H^3$, $\psi^4 H^2$	$\psi^2 H^5$	H^8
6		$X_L^2 H^2 D^2$, $X_L^2 \psi \bar{\psi} D$, $X_L \psi^2 H D^2$, $\psi^4 D^2$	$X_L H^4 D^2$, $X_L^2 \bar{\psi}^2 H$, $X_L \psi \bar{\psi} H^2 D$, $\psi^2 H^3 D^2$, $X_L \psi^2 \bar{\psi}^2$, $\psi^3 \bar{\psi} H D$	$H^6 D^2$, $\psi \bar{\psi} H^4 D$, $\psi^2 \bar{\psi}^2 H^2$	$\bar{\psi}^2 H^5$
4			$X_L^2 X_R^2$, $X_L X_R H^2 D^2$, $H^4 D^4$, $X_L X_R \psi \bar{\psi} D$, $X_R \psi^2 H D^2$, $X_L \bar{\psi}^2 H D^2$, $\psi \bar{\psi} H^2 D^3$, $\psi^2 \bar{\psi}^2 D^2$	$X_R H^4 D^2$, $X_R^2 \psi^2 H$, $X_R \psi \bar{\psi} H^2 D$, $\bar{\psi}^2 H^3 D^2$, $X_R \psi^2 \bar{\psi}^2$, $\psi \bar{\psi}^3 H D$	$X_R^2 H^4$, $X_R \bar{\psi}^2 H^3$, $\bar{\psi}^4 H^2$
2				$X_R^2 H^2 D^2$, $X_R^2 \psi \bar{\psi} D$, $X_R \bar{\psi}^2 H D^2$, $\bar{\psi}^4 D^2$	$X_R^3 H^2$, $X_R^2 \bar{\psi}^2 H$, $X_R \bar{\psi}^4$
0					X_R^4

Murphy '20;
based on Craig et al '20

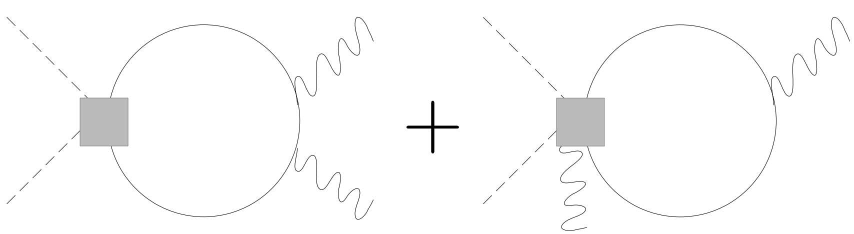
It is obvious that there are zeros in mixing of specific operators of different classes

It is not so clear how to anticipate them, not even with amplitude methods

$$\mathcal{O}^{(1)}_{e^2\phi^2 D^3} = i(\bar{e}\gamma^{\mu} D^{\nu} e)(D_{(\mu} D_{\nu)}\phi^{\dagger}\phi) + \text{h.c.}$$

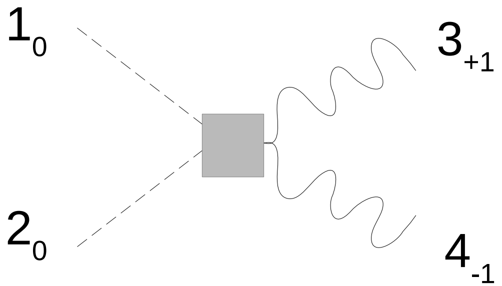
$$\mathcal{O}^{(1)}_{B^2\phi^2 D^2} = (D^{\mu}\phi^{\dagger} D^{\nu}\phi) B_{\mu\rho} B^{\rho}_{\nu}$$

$$\mathcal{O}^{(2)}_{e^2\phi^2 D^3} = i(\bar{e}\gamma^{\mu} D^{\nu} e)(\phi^{\dagger} D_{(\mu} D_{\nu)}\phi) + \text{h.c.}$$

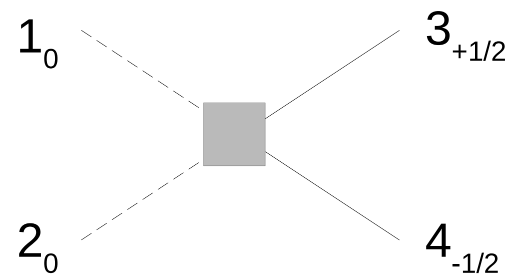


$$+\cdots$$

$$\underbrace{\mathcal{O}^{(1)}_{e^2\phi^2 D^3} - \mathcal{O}^{(2)}_{e^2\phi^2 D^3}}_{\widetilde{\mathcal{O}}^{(1)}_{e^2\phi^2 D^3}} \not\longrightarrow \mathcal{O}^{(1)}_{B^2\phi^2 D^2}$$

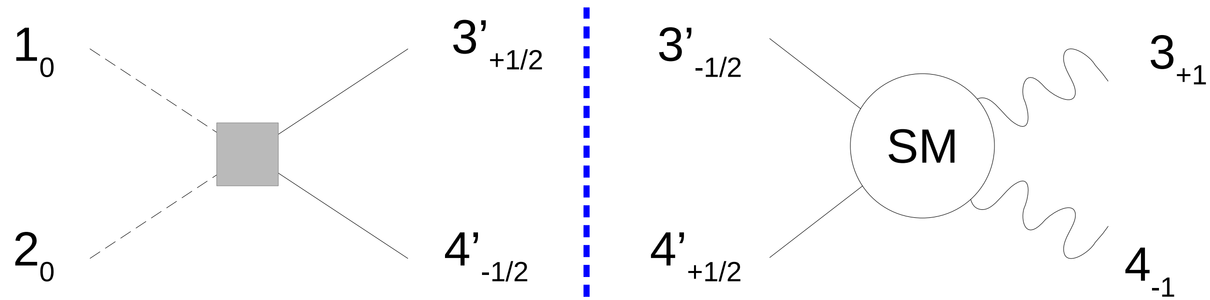


$$= \langle 41\rangle^2 [31]^2$$



$$= \langle 43\rangle \langle 41\rangle [43][31]$$

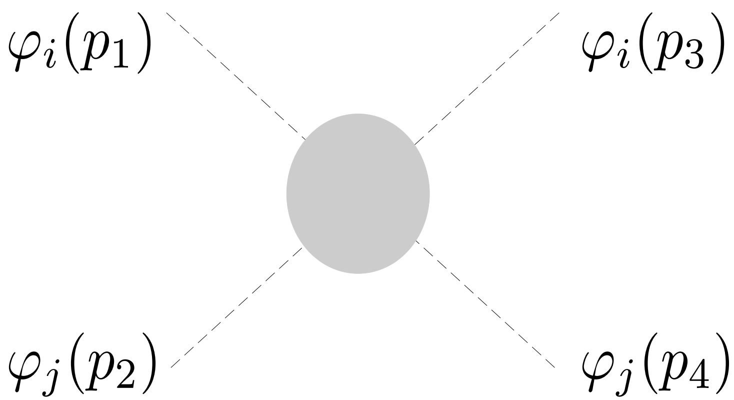
$$\gamma_{\tilde{c}^{(1)}_{e^2\phi^2 D^3} \to c^{(1)}_{B^2\phi^2 D^2}} \propto$$



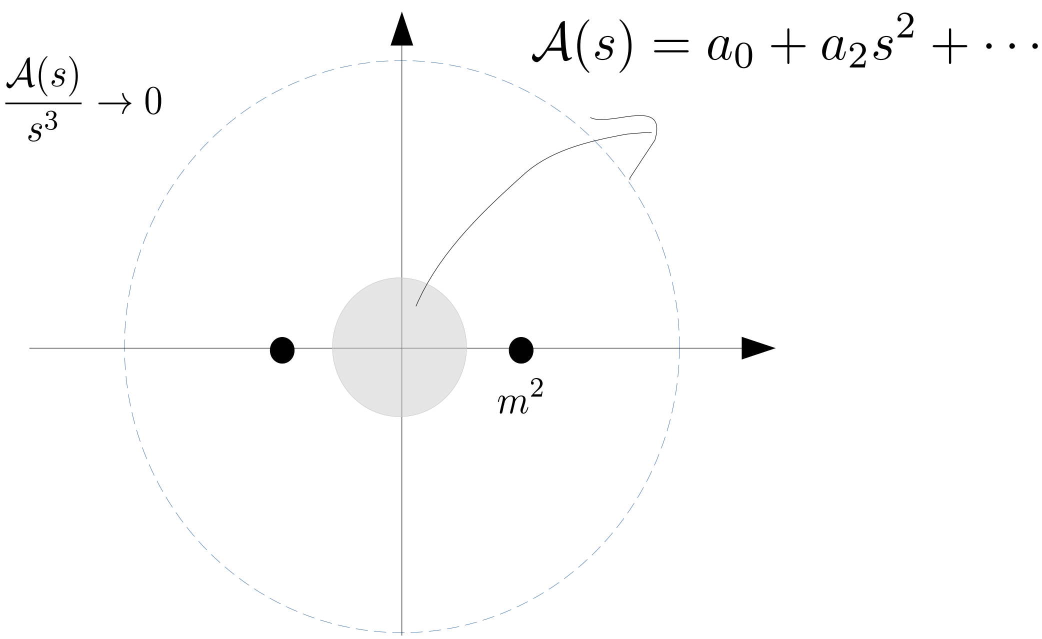
$$= \int d\mathrm{LIPS} \langle 4'3'\rangle \langle 4'1\rangle [4'3'][3'1] \frac{\langle 3'4\rangle^2}{\langle 3'3\rangle \langle 34'\rangle}$$

$$= \int_0^{2\pi} d\phi \int_0^{\pi/2} d\theta s_\theta c_\theta \left[\#_1 e^{i\phi} + \cancelto{0}{\#_2 e^{2i\phi}} + \cdots \right]$$

A different perspective: certain operators are constrained by positivity, from unitarity+locality [Adams, Arkani-Hamed, Nicolis, Rattazzi '06]



$\mathcal{A}(s) \equiv \mathcal{A}(s, t=0)$



$$0 = \sum \mathrm{res} \frac{\mathcal{A}(s)}{s^3} = a_2 - \frac{1}{\pi} \int s \frac{\sigma(s)}{(m^2)^3} \Rightarrow a_2 > 0$$

A different perspective: certain operators are constrained by positivity, from unitarity+locality [Adams, Arkani-Hamed, Nicolis, Rattazzi '06]

$$c^{(1)}_{B^2\phi^2D^2} \leq 0$$

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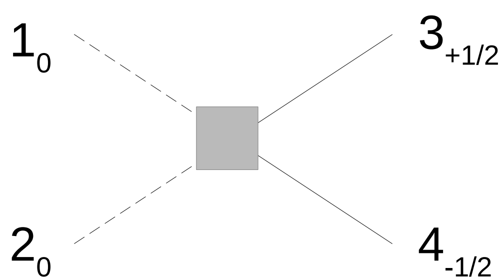
But some others are not:

$$\widetilde{\mathcal{O}}^{(1)}_{e^2\phi^2 D^3} =$$



$$= \langle 43\rangle\langle 41\rangle[43][31]$$

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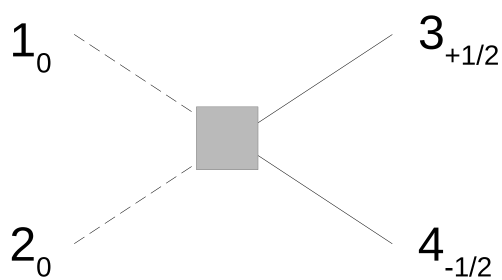
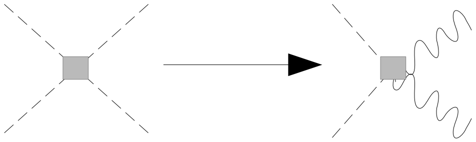


$$= \langle 43\rangle\langle 41\rangle[43][31]$$

"Therefore",

$$\dot{c}^{(1)}_{B^2\phi^2 D^2} = \#_1 \tilde{c}^{(1)}_{e^2\phi^2 D^3} + \cdots \Rightarrow \#_1 = 0$$

Other aspects of anomalous dimensions: signs and inequalities

Let us consider the mixing 

Positivity bounds:

$$c_{\phi^4}^{(2)} \geq 0\,, \quad c_{\phi^4}^{(1)} + c_{\phi^4}^{(2)} \geq 0\,, \quad c_{\phi^4}^{(1)} + c_{\phi^4}^{(2)} + c_{\phi^4}^{(3)} \geq 0$$

$$\dot{c}_{B^2\phi^2D^2}^{(1)} \geq 0$$

From where we obtain:

$$\begin{aligned}\dot{c}_{B^2\phi^2D^2}^{(1)} &= \alpha(c_{\phi^4}^{(1)} + c_{\phi^4}^{(2)} + c_{\phi^4}^{(3)}) + \beta(c_{\phi^4}^{(1)} + c_{\phi^4}^{(2)}) + \gamma c_{\phi^4}^{(2)} + \cdots \\ &= (\alpha+\beta)c_{\phi^4}^{(1)} + (\alpha+\beta+\gamma)c_{\phi^4}^{(2)} + \alpha c_{\phi^4}^{(3)} + \cdots\,,\end{aligned}$$

Other aspects of anomalous dimensions: signs and inequalities

$$\dot{c}^{(1)}_{B^2\phi^2D^2} = \alpha(c^{(1)}_{\phi^4} + c^{(2)}_{\phi^4} + c^{(3)}_{\phi^4}) + \beta(c^{(1)}_{\phi^4} + c^{(2)}_{\phi^4}) + \gamma c^{(2)}_{\phi^4} + \cdots$$
$$= (\alpha+\beta)c^{(1)}_{\phi^4} + (\alpha+\beta+\gamma)c^{(2)}_{\phi^4} + \alpha c^{(3)}_{\phi^4} + \cdots ,$$

1. The anomalous dimensions are positive

	$c^{(1)}_{\phi^4D^4}$	$c^{(2)}_{\phi^4D^4}$	$c^{(3)}_{\phi^4D^4}$
$c^{(1)}_{B^2\phi^2D^2}$	+	+	+

2. They fulfill

$$\gamma_{c^{(1)}_{B^2\phi^2D^2},\,c^{(2)}_{\phi^4D^4}} \geq \gamma_{c^{(1)}_{B^2\phi^2D^2},\,c^{(1)}_{\phi^4D^4}} \geq \gamma_{c^{(1)}_{B^2\phi^2D^2},\,c^{(3)}_{\phi^4D^4}}$$

Full electroweak SMEFT (with no flavour)

	$c^{(1)}_{\phi^4 D^4}$	$c^{(2)}_{\phi^4 D^4}$	$c^{(3)}_{\phi^4 D^4}$	$\tilde{c}^{(1)}_{e^2\phi^2 D^3}$	$\tilde{c}^{(2)}_{e^2\phi^2 D^3}$	$\tilde{c}^{(1)}_{l^2\phi^2 D^3}$	$\tilde{c}^{(2)}_{l^2\phi^2 D^3}$	$\tilde{c}^{(3)}_{l^2\phi^2 D^3}$	$\tilde{c}^{(4)}_{l^2\phi^2 D^3}$	$c_{e^4 D^2}$	$c^{(1)}_{l^4 D^2}$	$c^{(2)}_{l^4 D^2}$	$c^{(1)}_{l^2 e^2 D^2}$	$c^{(2)}_{l^2 e^2 D^2}$
$c^{(1)}_{B^2\phi^2 D^2}$	$+$	$+$	$+$	$\boxed{0}$	$-$	$\boxed{0}$	$-$	$\boxed{0}$	$-$	0	0	0	0	0
$c^{(1)}_{W^2\phi^2 D^2}$	$+$	$+$	$+$	0	0	$\boxed{0}$	$-$	$\boxed{0}$	$-$	0	0	0	0	0
$\tilde{c}^{(2)}_{e^2\phi^2 D^3}$	$+$	$+$	$+$	$\times$	$\times$	$\boxed{0}$	$-$	$\boxed{0}$	$-$	$-$	0	0	$\boxed{0}$	$-$
$\tilde{c}^{(2)}_{l^2\phi^2 D^3}$	$+$	$+$	$+$	$\boxed{0}$	$-$	$\times$	$\times$	$\times$	$\times$	0	$-$	$-$	$\boxed{0}$	$-$
$\tilde{c}^{(4)}_{l^2\phi^2 D^3}$	$+$	$+$	$+$	$\boxed{0}$	$-$	$\times$	$\times$	$\times$	$\times$	0	$-$	$-$	$\boxed{0}$	$-$
$c_{e^2 B^2 D}$	0	0	0	$\boxed{0}$	$-$	0	0	0	0	$-$	0	0	$\boxed{0}$	$-$
$c_{l^2 B^2 D}$	0	0	0	0	0	$\boxed{0}$	$-$	$\boxed{0}$	$-$	0	$-$	$-$	$\boxed{0}$	$-$
$c_{e^2 W^2 D}$	0	0	0	$\boxed{0}$	$-$	0	0	0	0	0	0	0	$\boxed{0}$	$-$
$c^{(1)}_{l^2 W^2 D}$	0	0	0	0	0	$\boxed{0}$	$-$	$\boxed{0}$	$-$	0	$-$	$-$	0	0
$c^{(2)}_{l^2 e^2 D^2}$	0	0	0	$\boxed{0}$	$-$	$\boxed{0}$	$-$	$\boxed{0}$	$-$	$-$	$-$	$-$	$\times$	$\times$

Full electroweak SMEFT (with no flavour)

	$c^{(1)}_{\phi^4 D^4}$	$c^{(2)}_{\phi^4 D^4}$	$c^{(3)}_{\phi^4 D^4}$	$\tilde{c}^{(1)}_{e^2\phi^2 D^3}$	$\tilde{c}^{(2)}_{e^2\phi^2 D^3}$	$\tilde{c}^{(1)}_{l^2\phi^2 D^3}$	$\tilde{c}^{(2)}_{l^2\phi^2 D^3}$	$\tilde{c}^{(3)}_{l^2\phi^2 D^3}$	$\tilde{c}^{(4)}_{l^2\phi^2 D^3}$	$c_{e^4 D^2}$	$c^{(1)}_{l^4 D^2}$	$c^{(2)}_{l^4 D^2}$	$c^{(1)}_{l^2 e^2 D^2}$	$c^{(2)}_{l^2 e^2 D^2}$
$c^{(1)}_{B^2\phi^2 D^2}$	$\boxed{\frac{g^2}{3}}$	$\boxed{\frac{g^2}{2}}$	$\boxed{\frac{g^2}{6}}$	$\boxed{0}$	$-$	$\boxed{0}$	$-$	$\boxed{0}$	$-$	0	0	0	0	0
$c^{(1)}_{W^2\phi^2 D^2}$	$+$	$+$	$+$	0	0	$\boxed{0}$	$-$	$\boxed{0}$	$-$	0	0	0	0	0
$\tilde{c}^{(2)}_{e^2\phi^2 D^3}$	$+$	$+$	$+$	$\times$	$\times$	$\boxed{0}$	$-$	$\boxed{0}$	$-$	$-$	0	0	$\boxed{0}$	$-$
$\tilde{c}^{(2)}_{l^2\phi^2 D^3}$	$+$	$+$	$+$	$\boxed{0}$	$-$	$\times$	$\times$	$\times$	$\times$	0	$-$	$-$	$\boxed{0}$	$-$
$\tilde{c}^{(4)}_{l^2\phi^2 D^3}$	$+$	$+$	$+$	$\boxed{0}$	$-$	$\times$	$\times$	$\times$	$\times$	0	$-$	$-$	$\boxed{0}$	$-$
$c_{e^2 B^2 D}$	0	0	0	$\boxed{0}$	$-$	0	0	0	0	$-$	0	0	$\boxed{0}$	$-$
$c_{l^2 B^2 D}$	0	0	0	0	0	$\boxed{0}$	$-$	$\boxed{0}$	$-$	0	$-$	$-$	$\boxed{0}$	$-$
$c_{e^2 W^2 D}$	0	0	0	$\boxed{0}$	$-$	0	0	0	0	0	0	0	$\boxed{0}$	$-$
$c^{(1)}_{l^2 W^2 D}$	0	0	0	0	0	$\boxed{0}$	$-$	$\boxed{0}$	$-$	0	$-$	$-$	0	0
$c^{(2)}_{l^2 e^2 D^2}$	0	0	0	$\boxed{0}$	$-$	$\boxed{0}$	$-$	$\boxed{0}$	$-$	$-$	$-$	$-$	$\times$	$\times$

Full electroweak SMEFT (with no flavour)

	$c^{(1)}_{\phi^4D^4}$	$c^{(2)}_{\phi^4D^4}$	$c^{(3)}_{\phi^4D^4}$	$\tilde{c}^{(1)}_{e^2\phi^2D^3}$	$\tilde{c}^{(2)}_{e^2\phi^2D^3}$	$\tilde{c}^{(1)}_{l^2\phi^2D^3}$	$\tilde{c}^{(2)}_{l^2\phi^2D^3}$	$\tilde{c}^{(3)}_{l^2\phi^2D^3}$	$\tilde{c}^{(4)}_{l^2\phi^2D^3}$	$c_{e^4D^2}$	$c^{(1)}_{l^4D^2}$	$c^{(2)}_{l^4D^2}$	$c^{(1)}_{l^2e^2D^2}$	$c^{(2)}_{l^2e^2D^2}$
$c^{(1)}_{B^2\phi^2D^2}$	$\frac{g^2}{3}$	$\frac{g^2}{2}$	$\frac{g^2}{6}$	$\boxed{0}$	−	$\boxed{0}$	−	$\boxed{0}$	−	0	0	0	0	0
$c^{(1)}_{W^2\phi^2D^2}$	+	+	+	0	0	$\boxed{0}$	−	$\boxed{0}$	−	0	0	0	0	0
$\tilde{c}^{(2)}_{e^2\phi^2D^3}$	+	+	+	×	×	$\boxed{0}$	$-\frac{4\lvert Y\rvert^2}{3}$	$\boxed{0}$	−	−	0	0	$\boxed{0}$	−
$\tilde{c}^{(2)}_{l^2\phi^2D^3}$	+	+	+	$\boxed{0}$	−	×	×	×	×	0	−	−	$\boxed{0}$	−
$\tilde{c}^{(4)}_{l^2\phi^2D^3}$	+	+	+	$\boxed{0}$	−	×	×	×	×	0	−	−	$\boxed{0}$	−
$c_{e^2B^2D}$	0	0	0	$\boxed{0}$	−	0	0	0	0	−	0	0	$\boxed{0}$	−
$c_{l^2B^2D}$	0	0	0	0	0	$\boxed{0}$	−	$\boxed{0}$	−	0	−	−	$\boxed{0}$	−
$c_{e^2W^2D}$	0	0	0	$\boxed{0}$	−	0	0	0	0	0	0	0	$\boxed{0}$	−
$c^{(1)}_{l^2W^2D}$	0	0	0	0	0	$\boxed{0}$	−	$\boxed{0}$	−	0	−	−	0	0
$c^{(2)}_{l^2e^2D^2}$	0	0	0	$\boxed{0}$	−	$\boxed{0}$	−	$\boxed{0}$	−	−	−	−	×	×

Full electroweak SMEFT (with no flavour)

	$c^{(1)}_{\phi^4D^4}$	$c^{(2)}_{\phi^4D^4}$	$c^{(3)}_{\phi^4D^4}$	$\tilde{c}^{(1)}_{e^2\phi^2D^3}$	$\tilde{c}^{(2)}_{e^2\phi^2D^3}$	$\tilde{c}^{(1)}_{l^2\phi^2D^3}$	$\tilde{c}^{(2)}_{l^2\phi^2D^3}$	$\tilde{c}^{(3)}_{l^2\phi^2D^3}$	$\tilde{c}^{(4)}_{l^2\phi^2D^3}$	$c_{e^4D^2}$	$c^{(1)}_{l^4D^2}$	$c^{(2)}_{l^4D^2}$	$c^{(1)}_{l^2e^2D^2}$	$c^{(2)}_{l^2e^2D^2}$
$c^{(1)}_{B^2\phi^2D^2}$	$\frac{g^2}{3}$	$\frac{g^2}{2}$	$\frac{g^2}{6}$	$\boxed{0}$	$-$	$\boxed{0}$	$-$	$\boxed{0}$	$-$	0	0	0	0	0
$c^{(1)}_{W^2\phi^2D^2}$	$+$	$+$	$+$	0	0	$\boxed{0}$	$-$	$\boxed{0}$	$-$	0	0	0	0	0
$\tilde{c}^{(2)}_{e^2\phi^2D^3}$	$+$	$+$	$+$	$\boxed{g^2 - \lvert Y\rvert^2}$	$\times$	$\boxed{0}$	$-\frac{4\lvert Y\rvert^2}{3}$	$\boxed{0}$	$-$	$-$	0	0	$\boxed{0}$	$-$
$\tilde{c}^{(2)}_{l^2\phi^2D^3}$	$+$	$+$	$+$	$\boxed{0}$	$-$	$\times$	$\times$	$\times$	$\times$	0	$-$	$-$	$\boxed{0}$	$-$
$\tilde{c}^{(4)}_{l^2\phi^2D^3}$	$+$	$+$	$+$	$\boxed{0}$	$-$	$\times$	$\times$	$\times$	$\times$	0	$-$	$-$	$\boxed{0}$	$-$
$c_{e^2B^2D}$	0	0	0	$\boxed{0}$	$-$	0	0	0	0	$-$	0	0	$\boxed{0}$	$-$
$c_{l^2B^2D}$	0	0	0	0	0	$\boxed{0}$	$-$	$\boxed{0}$	$-$	0	$-$	$-$	$\boxed{0}$	$-$
$c_{e^2W^2D}$	0	0	0	$\boxed{0}$	$-$	0	0	0	0	0	0	0	$\boxed{0}$	$-$
$c^{(1)}_{l^2W^2D}$	0	0	0	0	0	$\boxed{0}$	$-$	$\boxed{0}$	$-$	0	$-$	$-$	0	0
$c^{(2)}_{l^2e^2D^2}$	0	0	0	$\boxed{0}$	$-$	$\boxed{0}$	$-$	$\boxed{0}$	$-$	$-$	$-$	$-$	$\times$	$\times$

Can't we just compute all anomalous dimensions in some automated way?

Tools like matchmakereft or matchete not yet fully automatic [Carmona et al ‘21; Fuentes-Martin et al ‘22]

Main obstacles: Green’s and physical bases [MC, Diaz-Carmona, Guedes ‘21; Ren, Yu ‘22; Fonseca]; field redefinitions [MC, Santiago]

EFT in UV

FeynRules
QGRAF
FORM

EFT in IR

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input EFT

EFT in UV

FeynRules

QGRAF

FORM

input Green’s basis

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input EFT

EFT in UV

FeynRules

QGRAF

FORM

input Green's basis

input operator reduction

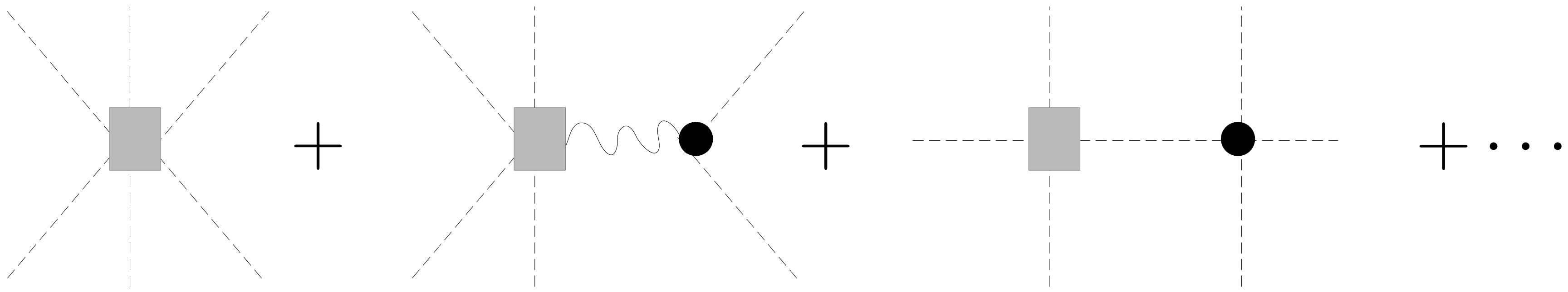
input Physical basis

EFT in IR

Require Lagrangian with redundant operators to provide same S-matrix as that without them

Too many constraints on-shell. Solution: go numerics

Compute the amplitudes in different Montecarlo physical phase-space points. Problem reduced to linear algebra



Application to the purely Higgs sector [to appear in SMEFT-Tools 2022 proceedings]:

$$c_{\phi\Box} \to c_{\phi\Box} + \frac{1}{2} r'_{\phi D} \,, \tag{22}$$

$$c_{\phi^6} \to c_{\phi^6} + 2\lambda r'_{\phi D} \,, \tag{23}$$

$$\begin{aligned} c^{(1)}_{\phi^6 D^2} \to c^{(1)}_{\phi^6 D^2} &+ 2\lambda\big(2 r^{(12)}_{\phi^4 D^4} - 2 r^{(4)}_{\phi^4 D^4} - r^{(6)}_{\phi^4 D^4}\big) \\ &- 4 c_{\phi\Box} r'_{\phi D} - \frac{1}{2} c_{\phi D} r'_{\phi D} - \frac{7}{4} r'^2_{\phi D} + r''^2_{\phi D} \,, \end{aligned} \tag{24}$$

$$c^{(2)}_{\phi^6} \to c^{(2)}_{\phi^6} + 2\lambda\big(r^{(12)}_{\phi^4 D^4} - r^{(6)}_{\phi^4 D^4}\big) - c_{\phi D} r'_{\phi D} \,. \tag{25}$$

Outlook

Positivity bounds on dimension-8 interactions (which are important/interesting by themselves) restrict different aspects of (certain) anomalous dimensions (zeros, signs, inequalities).

Further applications include full SMEFT [MC, Li 'ongoing work], LEFT and other EFTs.

Renormalising fully the SMEFT to dimension 8 is a heavy task, but which can be automatised.

Outlook

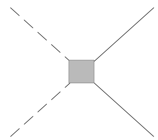
Positivity bounds on dimension-8 interactions (which are important/interesting by themselves) restrict different aspects of (certain) anomalous dimensions (zeros, signs, inequalities).

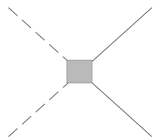
Further applications include full SMEFT [MC, Li 'ongoing work], LEFT and other EFTs.

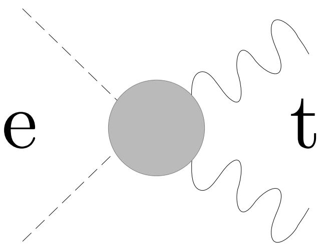
Renormalising fully the SMEFT to dimension 8 is a heavy task, but which can be automatised.

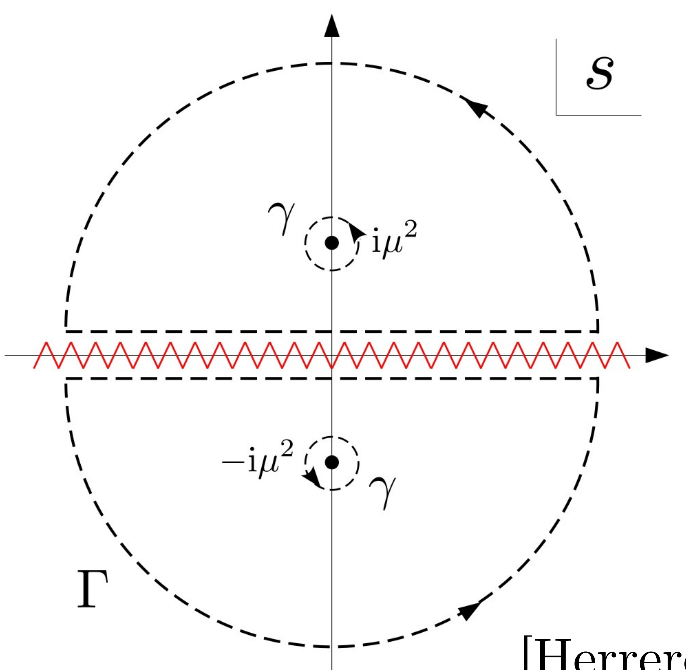
Thank you!

Backup

1. For any $(c^{(1)}_{e^2\phi^2 D^3}, c^{(2)}_{e^2\phi^2 D^3})$ compatible with the positivity bounds ($c^{(1)}_{e^2\phi^2 D^3} + c^{(2)}_{e^2\phi^2 D^3} \leq 0$) , there exists UV such that only $(c^{(1)}_{e^2\phi^2 D^3}, c^{(2)}_{e^2\phi^2 D^3})$ (and lower-dimensional  ones) at tree level.

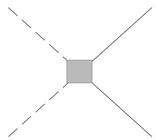
1. For any $(c^{(1)}_{e^2\phi^2 D^3}, c^{(2)}_{e^2\phi^2 D^3})$ compatible with the positivity bounds ($c^{(1)}_{e^2\phi^2 D^3} + c^{(2)}_{e^2\phi^2 D^3} \leq 0$) , there exists UV such that only $(c^{(1)}_{e^2\phi^2 D^3}, c^{(2)}_{e^2\phi^2 D^3})$ (and lower-dimensional  ones) at tree level.

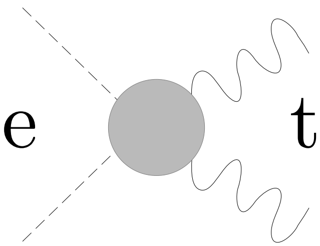
2. Within any such UV, compute  to order $O(g^2)$

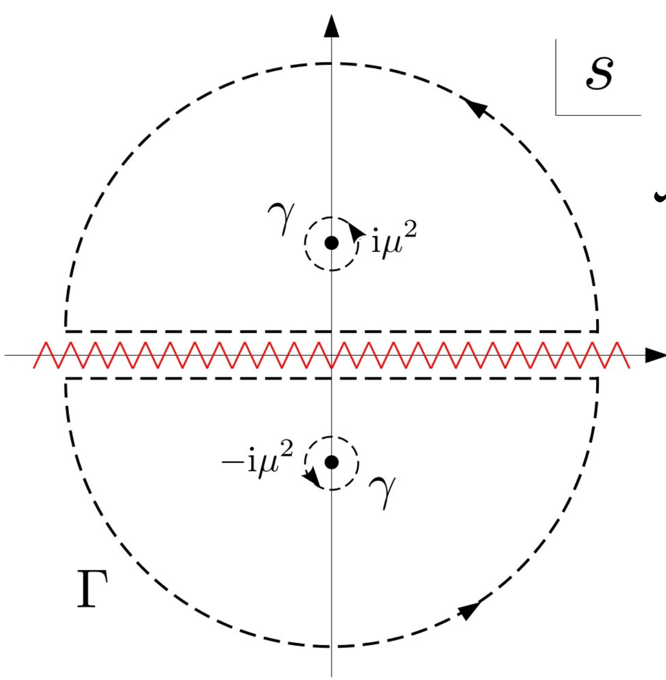


$$\Sigma(\mu) \equiv \frac{1}{2\pi \mathrm{i}} \int_\gamma \frac{\mathcal{A}(s)s^3}{(s^2+\mu^4)^3} \geq 0$$

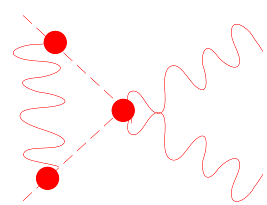
[Herrero-Valea et al ‘20]

1. For any $(c^{(1)}_{e^2\phi^2 D^3}, c^{(2)}_{e^2\phi^2 D^3})$ compatible with the positivity bounds ($c^{(1)}_{e^2\phi^2 D^3} + c^{(2)}_{e^2\phi^2 D^3} \leq 0$) , there exists UV such that only $(c^{(1)}_{e^2\phi^2 D^3}, c^{(2)}_{e^2\phi^2 D^3})$ (and lower-dimensional  ones) at tree level.

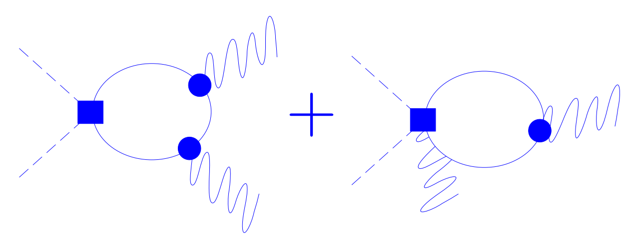
2. Within any such UV, compute  to order $O(g^2)$



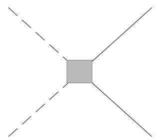
$$\mathcal{A}(s) \sim -(\beta_4 + \beta_8 s^2 + \beta_{12} s^4 + \cdots) \log \frac{s}{\Lambda^2}$$

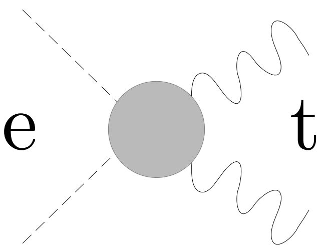


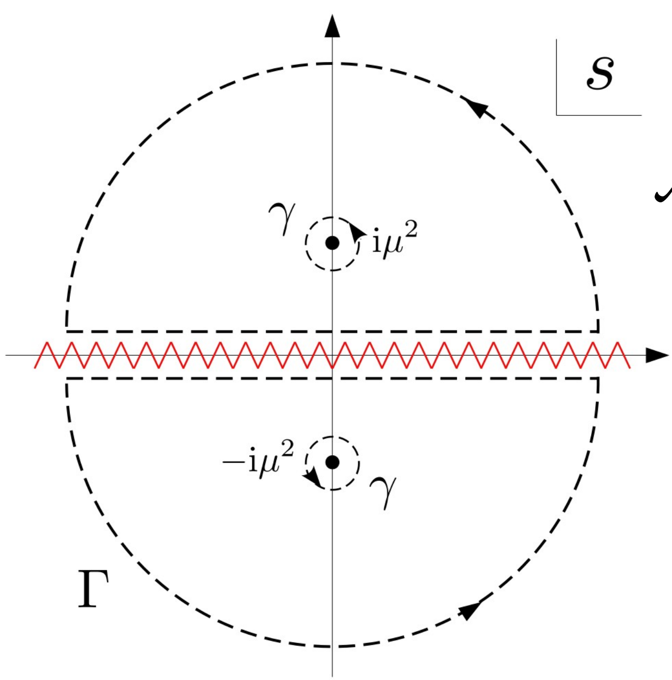
$+ \cdots$



$+ \cdots$

1. For any $(c^{(1)}_{e^2\phi^2 D^3}, c^{(2)}_{e^2\phi^2 D^3})$ compatible with the positivity bounds ($c^{(1)}_{e^2\phi^2 D^3} + c^{(2)}_{e^2\phi^2 D^3} \leq 0$) , there exists UV such that only $(c^{(1)}_{e^2\phi^2 D^3}, c^{(2)}_{e^2\phi^2 D^3})$ (and lower-dimensional  ones) at tree level.

2. Within any such UV, compute  to order $\mathrm{O}(g^2)$



$$\mathcal{A}(s) \sim -(\beta_4 + \beta_8 s^2 + \beta_{12} s^4 + \cdots) \log \frac{s}{\Lambda^2}$$

$$\Sigma(\mu) = -\beta_8 + \beta_{12}\mu^4 + \cdots$$

$$\Rightarrow \lim_{\mu \to 0} \Sigma(\mu) = -\beta_8 \geq 0$$

So $\beta_8 \leq 0$ in any of the aforementioned UV, and therefore for all values of $(c^{(1)}_{e^2\phi^2 D^3}, c^{(2)}_{e^2\phi^2 D^3})$ compatible with $c^{(1)}_{e^2\phi^2 D^3} + c^{(2)}_{e^2\phi^2 D^3} \leq 0$

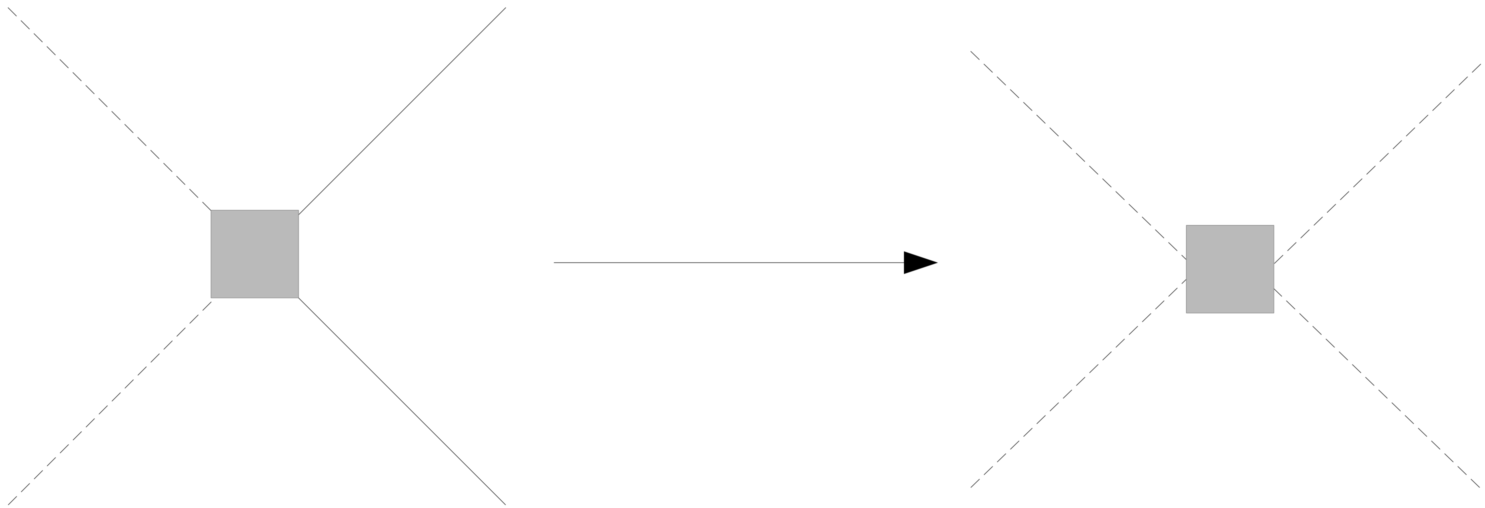
3. The beta function is linear in the Wilson coefficients:

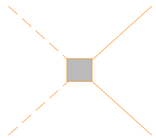
$$\beta_8 = \alpha(c^{(1)}_{e^2\phi^2 D^3} + c^{(2)}_{e^2\phi^2 D^3})\,, \quad \alpha \geq 0$$

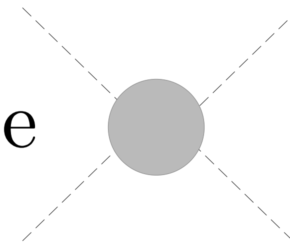
Therefore,

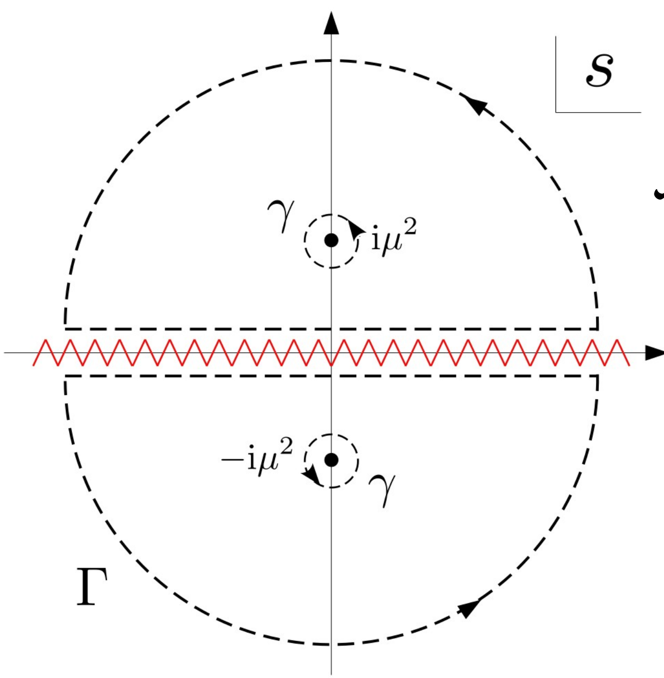
$$\underbrace{\mathcal{O}^{(1)}_{e^2\phi^2 D^3} - \mathcal{O}^{(2)}_{e^2\phi^2 D^3}}_{\widetilde{\mathcal{O}}^{(1)}_{e^2\phi^2 D^3}} \not\longrightarrow \mathcal{O}^{(1)}_{B^2\phi^2 D^2}$$

How do things change if we consider instead...?

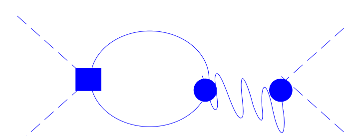


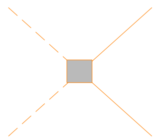
1. For any $(c^{(1)}_{e^2\phi^2 D^3}, c^{(2)}_{e^2\phi^2 D^3})$ compatible with the positivity bounds ($c^{(1)}_{e^2\phi^2 D^3} + c^{(2)}_{e^2\phi^2 D^3} \leq 0$) , there exists UV such that only $(c^{(1)}_{e^2\phi^2 D^3}, c^{(2)}_{e^2\phi^2 D^3})$ (and lower-dimensional  ones) at tree level.

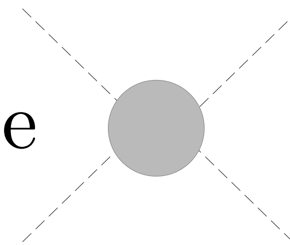
2. Within any such UV, compute  to order $O(g^2)$

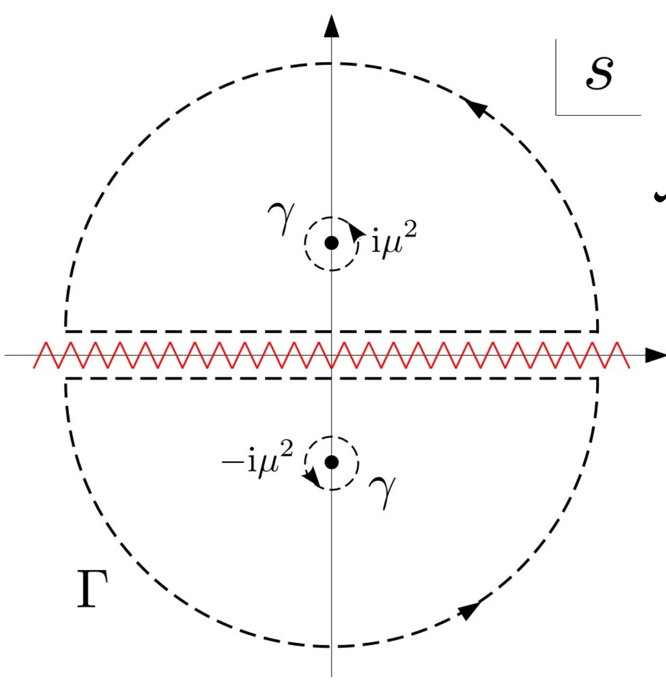


$$\mathcal{A}(s) \sim -(\beta_4 + \beta_8 s^2 + \beta_{12} s^4 + \cdots) \log \frac{s}{\Lambda^2}$$

$\cdots$  $+ \cdots$

1. For any $(c^{(1)}_{e^2\phi^2 D^3}, c^{(2)}_{e^2\phi^2 D^3})$ compatible with the positivity bounds ($c^{(1)}_{e^2\phi^2 D^3} + c^{(2)}_{e^2\phi^2 D^3} \leq 0$) , there exists UV such that only $(c^{(1)}_{e^2\phi^2 D^3}, c^{(2)}_{e^2\phi^2 D^3})$ (and lower-dimensional  ones) at tree level.

2. Within any such UV, compute  to order $\mathrm{O}(g^2)$



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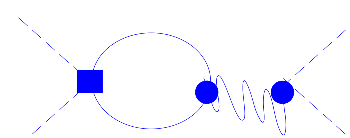
$\cdots$  $+ \cdots$

$+$