Precision predictions for $R(D^{(*)})$ (and exclusive measurements of $|V_{cb}|$)

BERKELEY CENTER FOR THEORETICAL PHYSICS

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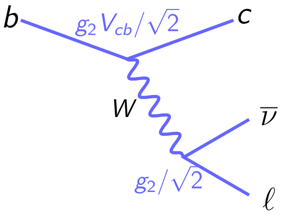
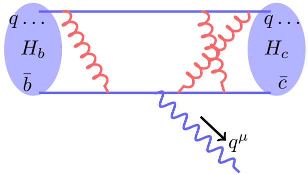
SM@LHC

July 2023

The $b \to c$ semileptonic laboratory



Main theory challenge



Basics:

- Tree level processes: large ($\sim 10\%$) branching fractions, high statistics, and (relatively) theoretically clean
- Lepton gauge coupling universal in the SM: lepton flavor universality violation (LFUV) via lepton mass phase space suppression and couplings
- Can consider $B \to D^{(*)}$, D^{**}, ..., $\Lambda_b \to \Lambda_c^{(*)}$, ...
- Precision description of the hadronic matrix elements required for precision predictions and measurements

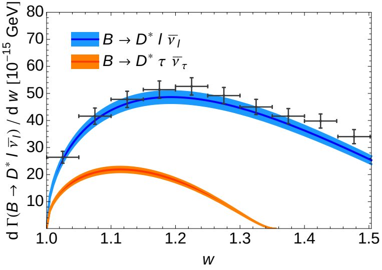
Fits and predictions

Exclusive hadronic matrix elements represented by form factors (FFs). Schematic differential rates

$$\frac{d\Gamma[H_b \to H_c l\nu]}{d\mathrm{PS}} \sim |V_{cb}|^2 \times \Big(\sum_i \mathrm{FF}_i \times \text{helicity ampl}_i\Big)^2 \,.$$

Logic: Precision data for $\ell = e$, μ modes + precision theory (continuum and/or lattice) for FFs $\Rightarrow$ precision fits

- measurement of $|V_{cb}|$
- prediction for τ modes: BFs, τ-ℓ LFUV ratios, τ and D^* polarization fractions



Outline

- New progress/methods to control second-order (NNLO) terms in Heavy Quark Effective Theory (HQET)
  - HQET essential for continuum precision SM predictions for the τ mode decays (and New Physics therein)
  - Provides relations between D and D^* modes, allowing combined fits
  - New, supplemental power counting: "residual chiral expansion"

- Perspectives on systematic truncation of form factor parametrizations
  - Addressing dangers of overfitting in theoretical (and experimental) frameworks

$|V_{cb}|$ status

Long-standing $> 2\sigma$ tension between inclusive and exclusive measurements of $|V_{cb}|$. PDG averages:

$$|V_{cb}|_{\text{incl}} = (42.2 \pm 0.8) \times 10^{-3}\,, \qquad |V_{cb}|_{\text{excl}} = (39.4 \pm 0.8) \times 10^{-3}\,,$$

Exclusive uncertainties now comparable with inclusive!

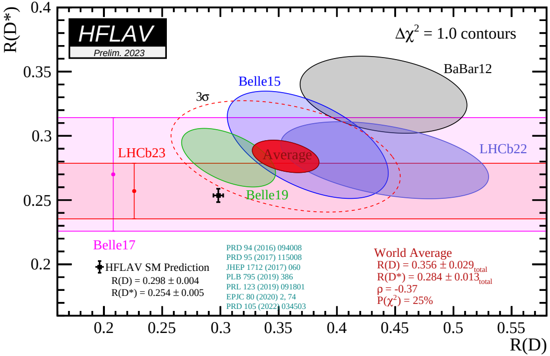
- Origin of the tension remains elusive
- $|V_{cb}|$ is a crucial input to other precision measurements and BSM searches. E.g.
  - $b \to s\ell\ell$ rates (via $|V_{ts}V_{tb}| \simeq |V_{cb}|$).
  - $K_L \to \pi^0\nu\nu$ (tension in $|V_{cb}|$ and $|V_{ub}|$ contributes $\sim$ 50–75% of uncertainties)

LFUV Ratios

In ratios of (exclusive) $b \to c\tau\nu$ vs $b \to c\ell\nu$ leading hadronic uncertainties cancel

(in principle need only fit $\ell = e, \mu$ modes' 'shape' rather than overall normalization)

$$R(H_c) = \mathrm{Br}[H_b \to H_c \tau\nu]/\mathrm{Br}[H_b \to H_c \ell\nu]$$

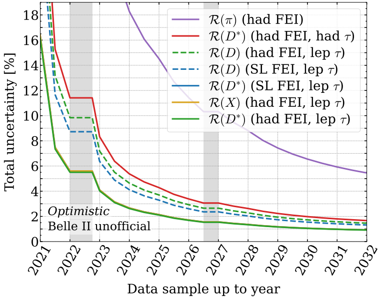


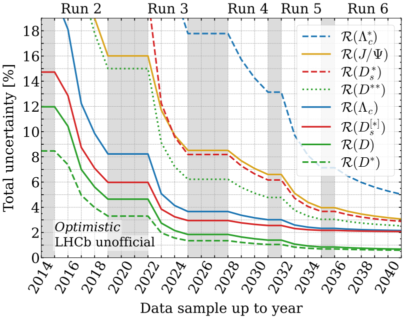
HFLAV av: $R(D) = 0.298(4)$, $R(D^*) = 0.254(5)$

- Comparison with predictions a clean null test of SM
- Expt in $> 3\sigma$ tension with SM (or more; inter-experiment D^{**} correlation model)
- 2% precision on the SM predictions for $R(D)$ and $R(D^*) \ll$ expt stat uncertainties
- Future precision measurements may also become sensitive to theory-based uncertainties!

LFUV Forecast

Long-term forecasts expect few % statistical precision on a multitude of $R(H_c)$ measurements





More details on LHCb forecasts in Manuel's talk on Thurs!

Bernlochner, Franco Sevilla, DR, Wormser [2101.08326]

- For most modes theory uncertainties smaller than mid-term forecast precision
- Eventual (sub)percent precision possible for $R(D)$ and $R(D^*)$!
- Can we control theory-based systematics in description of form factors to this level?

Continuum theory approaches

Various theoretical constructs are available/in use in various combinations.[†]
Precision continuum predictions for $R(D^{(*)})$ use HQET in some capacity.

Approach	Basis	Advantages	Drawbacks
Dispersive Bounds	Unitarity & dispersion relations	model independent	no τ mode predictions, no* NP predictions, truncation sensitivity
HQET	Heavy Quark expansion	model independent, NP/ τ-predictions, correlate D/D^*	second-order power corrections
QCD sum rules (QCDSR)	OPE: 3-pt correlators	calculable constraints	unassessible uncertainties model-dependent
Light-Cone (LCSR)	OPE: LC trans. dist.	calculable constraints	unassessible uncertainties boost outside physical range model-dependent
Quark models	various	predictive, constrained	unassessible uncertainties model-dependent

[†] Plus LQCD (see following talk!)

NB: a very rough summary, not exhaustive!

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HQET short version

idempotent projectors

HQ velocity

- Mass-subtracted field redefinition of quarks $Q^v_\pm(x) = \Pi_\pm\, e^{im_Q v\cdot x} Q(x)$
- Rewrite QCD (exactly) as

$$\mathcal{L}_{\text{QCD}} = \underbrace{\bar{Q}^v_+ iv\cdot D Q^v_+}_{\text{light massless}} + \bar{Q}^v_+ i\not{D}_\perp Q^v_- + \bar{Q}^v_- i\not{D}_\perp Q^v_+ - \underbrace{\bar{Q}^v_-(iv\cdot D + 2m_Q)Q^v_-}_{\text{double heavy}}\,.$$

NB: $D^\mu_\perp = D^\mu - v\cdot D v^\mu$

- Integrate out the double heavy fields to generate HQET. Power expansion in $\sim iv\cdot D/(2m_Q) \sim \Lambda_{\text{QCD}}/(2m_Q)$
  - Lagrangian corrections: $\mathcal{L}_{\text{HQET}} = \bar{Q}^v_+ iv\cdot DQ^v_+ + \sum_{n=1} \mathcal{L}_n/(2m_Q)^n$
  - Current corrections (from field redefn): $\mathcal{J}_{\text{HQET}} = 1 + \sum_{n=1} \mathcal{J}_n/(2m_Q)^n$
  - Perturbative $\mathcal{O}(\alpha_s)$ radiative corrections are fully calculable.

- Obtain EFT of 'light muck' in definite s^P state around a HQ static color source.
  - Hadrons embed into HQ supermultiplets
  - E.g. $s^P = \frac{1}{2}^- \Rightarrow J^P = \frac{1}{2} \times \frac{1}{2}^- = 0^- \oplus 1^-$: The D and D^*!
    HQET relates $B \to D$ and $B \to D^*$ FFs.

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More soon about these terms!

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HQET short version

- Match QCD matrix element onto HQET.
- Each order in $1/m_{c,b}$ matches to HQET matrix elements involving (combinations of)
  - OPE with $\mathcal{L}_n$ or $\mathcal{L}'_n$
  - contact terms involving $\mathcal{J}_n$ or $\mathcal{J}'_n$
  - E.g. NLO $(1/m_Q)$: OPE $\sim \mathcal{L}_1 = -\bar{Q}^v_+\left[D^2 + \frac{g}{2}\sigma_{\alpha\beta}G^{\alpha\beta}\right]Q^v_+$; current $\sim \mathcal{J}_1 = i\not{D}$.
- Each HQET matrix element represented by Isgur-Wise functions (fns of $w = v \cdot v'$)
- $\Rightarrow$ $b \to c$ FFs are expressed in terms of IW functions plus perturbative $\mathcal{O}(\alpha_s)$ radiative corrections. Predictive/constrained when no. of IW fns is few.
- Can also match to derivatives of forward matrix element. Yields a hadron mass expansion

$$m_H = m_Q + \bar{\Lambda} - \frac{\lambda_1 + d_H\lambda_2}{2m_Q}\,.$$

mass parameters

light dof KE in HQ limit

- Schwinger-Dyson relations cause $\bar{\Lambda}$ and $\lambda_{1,2}$ to enter $b \to c$ FFs.

HQET power corrections in $B \to D^{(*)}$

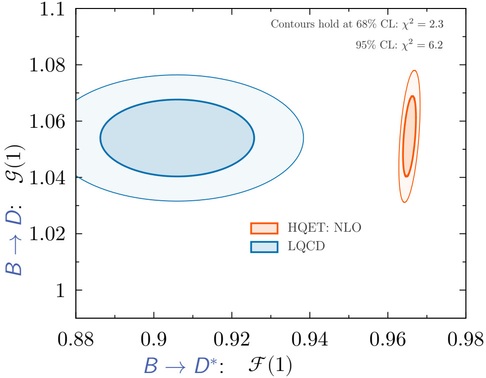
- $B \to D^{(*)}\ell\nu$ process involves $1+3$ of $2+4$ $B \to D^{(*)}$ FFs ($4+8$ with NP).
- NLO in HQET requires 4 IW functions. Predictive HQET constraints!

HQET order	HQET wavefunctions	Isgur-Wise fns
$1/m_{c,b}^0$	1	1
$1/m_{c,b}^1$	6	3
$1/m_c^2$	6	20
$1/m_{c,b}^2$	30	32

- At NNLO $\sim 1/m_c^2$ there are 20 IW functions occuring in six independent combinations aka 'wavefunctions' (often denoted ℓ_i; there are 24 more including $1/m_c m_b$) Lose predictivity/constraints!
- NNLO corrections:
  - Largest $\sim \Lambda_{\text{QCD}}^2/4m_c^2 \sim 4\%$. Larger than current exp precision!
  - Also $\alpha_s/\pi \times \Lambda_{\text{QCD}}/2m_c \sim 2\%$ and $\Lambda_{\text{QCD}}^2/4m_c m_b \sim 0.8\%$. Needed in future!

Zero-recoil difficulties

- Powerful current normalization constraints at zero recoil $(w = 1)$ mean some $1/m_{c,b}$ corrections vanish (Luke's theorem).
- At NLO accidentally precise predictions for $B \to D^*$ FF $F(1)$. Large tensions vs LQCD



- Affects only overall normalization
- For $R(D^*)$ predictions at NLO, need only fit to shape of $B \to D^*$ data.

  Bernlochner, Ligeti, Papucci, DR: 1703.05330

  [aka BLPR. Corrects prior NLO inconsistencies, SM + NP]
- Higher precision, $|V_{cb}|$: requires control of NNLO corrections!

Prior NNLO approaches

- Full HQET expressions for NNLO FFs are long-known Falk and Neubert hep-ph/9209268
- Some recent approaches
  - Guesstimate of $1/m_c^2$ contributions Bigi, Gambino, Schacht 1707.09509
  - $1/m_c^2$ rescaling nuisance parameter Jaiswal, Nandi, Patra 1707.09977, 2002.05726
  - Treat ℓ_i as nuisance parameters Bordone, Jung, van Dyk 1908.09398, 1912.09335
- Large number of nuisance parameters typically requires QCDSR or LCSR constraints for convergent fits

Can we develop a systematically improvable HQET-based approach (and avoid using QCDSR or LCSR)?

Chiral structure

Return to mass-subtracted QCD:

$$\mathcal{L}_{\text{QCD}} = \bar{Q}^v_+ iv\cdot D Q^v_+ + \boxed{\bar{Q}^v_+ i\not{D}_\perp Q^v_- + \bar{Q}^v_- i\not{D}_\perp Q^v_+} - \bar{Q}^v_-(iv\cdot D + 2m_Q)Q^v_- \,.$$

- Kinetic terms have accidental $U(1)\times U(1)$ chiral symmetry broken to $U(1)$ by $\not{D}_\perp$ terms (also break HQ spin symmetry)
- HQET corrections
  - Each Lagrangian correction $\mathcal{L}_n$ generated by $\bar{Q}^v_+ i\not{D}_\perp Q^v_- \bar{Q}^v_- i\not{D}_\perp Q^v_+$: two $\not{D}_\perp$ insertions
  - Each current correction $\mathcal{J}_n$ generated by one insertion of $\not{D}_\perp$

Chiral structure

Return to mass-subtracted QCD:

$$\mathcal{L}_{\mathrm{QCD}} = \bar{Q}^v_+ iv\cdot D Q^v_+ + \theta \boxed{\bar{Q}^v_+ i\not{D}_\perp Q^v_- + \bar{Q}^v_- i\not{D}_\perp Q^v_+} - \bar{Q}^v_-(iv\cdot D + 2m_Q)Q^v_- \,.$$

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Idea: counting $\not{D}_\perp$ insertions provides an additional classification of terms vs $1/m_Q$ power expansion.
Deform QCD by including a $\not{D}_\perp$ power-counting parameter θ

BLPR + Xiong, Prim (BLPRXP) 2206.11281

Residual chiral expansion (RCE)

At any order in HQET, can power count in θ: a symmetry breaking parameter. A well-defined EFT is generated at any order in θ.

Correction or Parameter	Associated HQ order	$\not{D}_\perp$ power counting	operator product (OP) order
LO	$1/m_Q^0$	θ^0	0
$\mathcal{L}_n$	$1/m_Q^n$	θ^2	1
$\mathcal{J}_n$	$1/m_Q^n$	θ	0
$\bar{\Lambda}$, $\mathcal{J}_1$, $\mathcal{L}_1$	$1/m_Q$	θ, θ, θ^2	0, 0, 1
$\mathcal{J}_2$, $\mathcal{L}_2$	$1/m_Q^2$	θ, θ^2	0, 1
$\lambda_{1,2}, \mathcal{J}_1 \times \mathcal{J}_1'$	$1/m_Q^2$, $1/m_Q m_{Q'}$	θ^2	0
$\mathcal{J}_1 \times \mathcal{L}_1^{(\prime)}$	$1/m_Q^2$, $1/m_Q m_{Q'}$	θ^3	1
$\mathcal{L}_1 \times \mathcal{L}_1^{(\prime)}$	$1/m_Q^2$, $1/m_Q m_{Q'}$	θ^4	2

Explicit (N)NLO ↓ pieces

θ is *not* a small parameter. But:

- At $\mathcal{O}(\theta^2)$ capture all NLO plus NNLO with zero OP insertions.
- $\mathcal{O}(\theta^3)$ and higher require more and more OPs!

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RCE conjecture/model: matrix elements involving (many) $\not{D}_\perp$ OP insertions are typically small. Truncate at $\mathcal{O}(\theta^2)$

Residual chiral expansion (RCE)

Number of IW functions tremendously reduced at $\mathcal{O}(\theta^2)$!

HQET order	Isgur-Wise fns All	Isgur-Wise fns RCE
$1/m_{c,b}^0$	1	1
$1/m_{c,b}^1$	3	3
$1/m_c^2$	20	1
$1/m_{c,b}^2$	32	3

IW fn: ξ. Norm $\xi(1) = 1$

IW fns: $\widehat{\chi}_{2,3}$, $\widehat{\eta}$. Norm $\widehat{\chi}_3(1) = 0$.

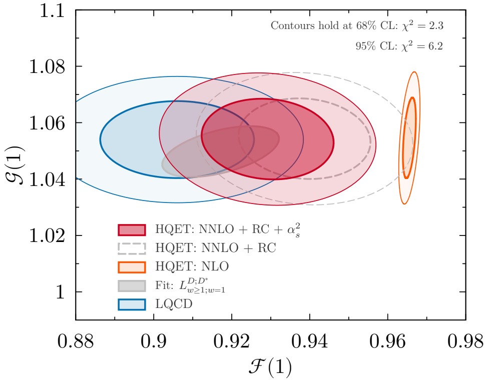
IW fn: $\widehat{\varphi}_1$. Constrained at $w = 1$ in terms of $\lambda_{1,2}$

Obtain form factor expressions to $\mathcal{O}(\theta^2, 1/m_{c,b}^2, \alpha_s/m_{c,b}) \Rightarrow$ Constrained set of HQET-based relations for $B \to D^{(*)} l\nu$ in SM and NP at NNLO.*

* Find two minor errors in prior literature (F & N) wrt $1/m_c m_b$ terms

Zero-recoil $B \to D^{(*)}$ predictions

NNLO correction to $F(1)$ fully determined by mass parameter λ_1.
'Typically-allowed' RCE-based CL at $\mathcal{O}(1/m_{c,b}^2, \alpha_s/m_{c,b})$ in agreement with LQCD calculations $(p = 0.4)$!



Technical ingredients:

- $1S$ mass scheme: m_b^{1S}. Splitting $\delta m_{bc} = m_b - m_c$ from inclusive spectra
- Cancellation of leading renormalon
- Use of inclusive spectra $\Rightarrow$ third-order hadron mass parameter ρ_1
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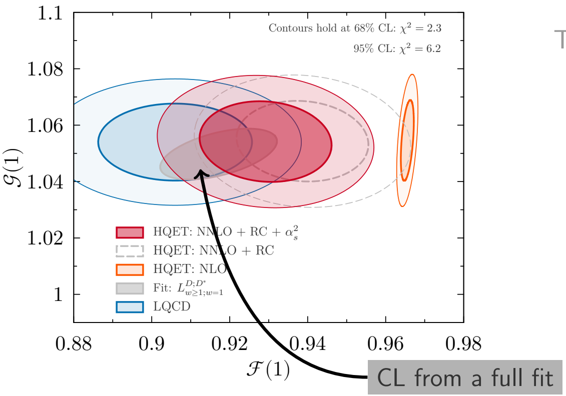

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Parametrization

- For fits, require a parametrization of the IW functions.
- Leading order IW function expressed wrt conformal map $w \mapsto z_*$

slope at w_0

curvature at w_0

$$\frac{\xi(w)}{\xi(w_0)} = 1 - 8a^2\rho_*^2\, z_* + 16(2\,c_*\,a^4 - \rho_*^2 a^2)z_*^2 + \ldots$$

conformal parameter, $z_*(w)$
$z_*(w_0) = 0$

- (Sub)subleading IW functions $(w - 1 \leq 0.59$ in $B \to D^{(*)} l\nu)$

$$X(w) = X(1) + X'(1)(w-1) + \ldots$$

may be constrained
by HQET

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Too many parameters $\Rightarrow$ overfitting/biases

(highly-correlated parameters, runaways)

How does one decide where to truncate?

Nested Hypothesis Tests

Truncation effects are a challenge for any precision parametric fit!

NHT: a test of an N-parameter fit hypothesis versus $N+1$.

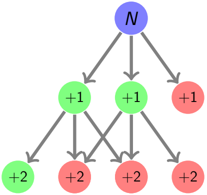
Also can consider Bayesian/Akaike Information Criteria. Others?

- For large N, $\Delta\chi^2 = \chi^2_N - \chi^2_{N+1}$ is approximately a 1-dof χ^2 Wilks' Theorem
- Fit hypothesis graph: an initial space of parameters

$$|V_{cb}|\,; \quad m_b^{1S}\,, \quad \delta m_{bc}\,, \quad \rho_1\,, \quad \lambda_2\,; \quad \rho_*^2\,, \quad c_*\,; \quad \hat{\eta}(1)\,.$$

plus all combinations of five additional candidate parameters

$$\hat{\eta}'(1)\,, \quad \hat{\chi}_2(1)\,, \quad \hat{\chi}_2'(1)\,, \quad \hat{\chi}_3'(1)\,, \quad \hat{\varphi}_1'(1)\,,$$

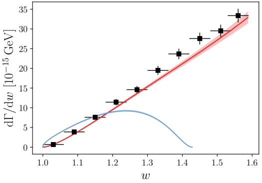


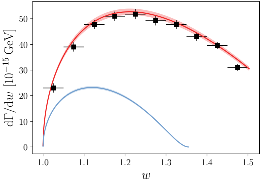
- Traverse graph depth-wise: Set threshold to reject a step $N \to N+1$ at $\Delta\chi^2 = \chi^2_N - \chi^2_{N+1} < 1$ (68% CL)
- Reject any fit hypothesis/graph node with parametric correlations > 0.95, or with runaways.
- Optimal fit: terminating node with fewest parameters and lowest χ^2

Optimal RCE-based Fit

- Fit uses all available $B \to D$ and zero-recoil $B \to D^*$ LQCD data, plus all Belle $B \to D^{(*)}\ell\nu$ data, but no QCDSR or LCSR.
- NHT identifies 8 terminating nodes. Optimal choice:

$\lvert V_{cb}\rvert \times 10^3$	38.70(62)
ρ_*^2	1.10(4)
c_*	2.39(18)
$\widehat{\chi}_2(1)$	$-0.12(2)$
$\widehat{\chi}_2'(1)$	—
$\widehat{\chi}_3'(1)$	—
$\widehat{\eta}(1)$	0.34(4)
$\widehat{\eta}'(1)$	—
m_b^{1S} [GeV]	4.71(5)
δm_{bc} [GeV]	3.41(2)
$\widehat{\varphi}_1'(1)$	0.25(21)
λ_2 [GeV^2]	0.12(2)
ρ_1 [GeV^3]	$-0.36(24)$





- Optimal fit sensitive to NNLO IW parameter
- Goodness of fit $\chi^2/\mathrm{ndf} = 29.8/31$
- $\lvert V_{cb}\rvert$ more precise than exclusive PDG average $39.4(8) \times 10^{-3}$

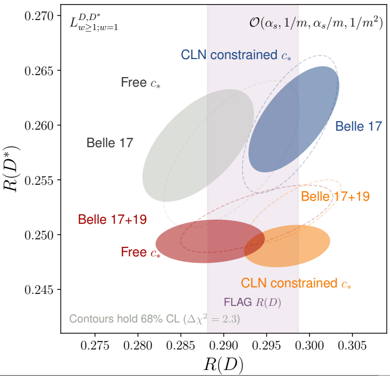
Predicted LFUV Ratios

PDG-style scale factor $S = 2.6$ from tension in Belle 2017 vs 2019 data

Obtain precision result from fit:

$$R(D) = 0.288(4)\,, \quad R(D^*) = 0.249(3)\,, \quad (\rho = 0.121)$$

- Prior NLO $R(D) = 0.298(3)$, $R(D^*) = 0.261(4)$ BLPR 1703.05330. A 2.7σ tension!
  - Relaxation of a heuristic slope-curvature constraint (invented by 'CLN' param)
  - Tension in Belle 2017 vs 2019 data. NEW Belle analysis 2301.07529, fits coming soon! Await more precise Belle II data!
- $R(D^{(*)})$ stable over terminating NHT nodes, but NHT is crucial to avoid biased predictions



Some comparisons

Collaboration/Group	Comment	$R(D)$	$R(D^*)$	corr.	$\lvert V_{cb}\rvert \times 10^3$
2111.09849 [FLAG]	FLAG, LQCD	0.2934(53)	–	–	39.4(7)
2105.14019 [FNAL/MILC]	LQCD	–	0.265(13)	–	38.4(8)
2304.03137 [Harrison, Davies]	LQCD	–	0.279(13)	–	39.3(7)
2306.05657 [JLQCD]	LQCD	–	0.252(22)	–	39.2(9)
1703.05330 [Bernlochner, Ligeti, Papucci, & DR]	HQET NLO Shape	0.298(3)	0.261(4)	0.44	–
1707.09977 [Jaiswal, Nandi, Patra]	$1/m_c^2$ parameter	0.299(4)	0.257(5)	~ 0.1	39.8(9)
1908.09398 [Bordone, Jung, van Dyk]	$1/m_c^2$ nuisance + QCDSR + LCSR	0.298(3)	0.250(3)	–	40.3(8)
2206.07501 HFLAV (2021)	HFLAV Arith. Av.	0.298(4)	0.254(5)	–	
2004.10208 [Iguro, Watanabe]	BJvD + Bayesian IC	0.290(3)	0.248(1)	–	39.3(6)
2206.11281 [BLPRXP]	NNLO RCE + NHT	0.288(4)	0.249(3)	0.121	38.7(6)

- Recent results using systematic truncation methods agree well, percent-level precision.
- Shift in $R(D^*)$ associated with Belle 17 vs 19
- Exclusive $|V_{cb}|$ stable across different approaches

Summary & Outlook

- New approach/model for controlling/constraining NNLO HQET contributions in exclusive $b \to c l \nu$ (and elsewhere): Residual Chiral Expansion
  - Single additional subsubleading IW function at $1/m_c^2$. Agree with LQCD at zero recoil.
  - Coming soon: Updated fits with JLQCD calculations and latest Belle $B \to D^* \ell \nu$ analysis; application to baryonic decays $\Lambda_b \to \Lambda_c l \nu$.

- Systematic approaches to truncation (NHT, etc) are crucial for precision parametric fits and predictions.

- $R(D^{(*)})$ predictions are already near the percent level. Key challenge: assessing biases from sum rules/models/truncation.

- Further LHCb and Belle II measurements are eagerly anticipated (see Manuel's talk on Thurs!)

Thanks!