

Theory predictions for $b \rightarrow s\ell\ell$ decays

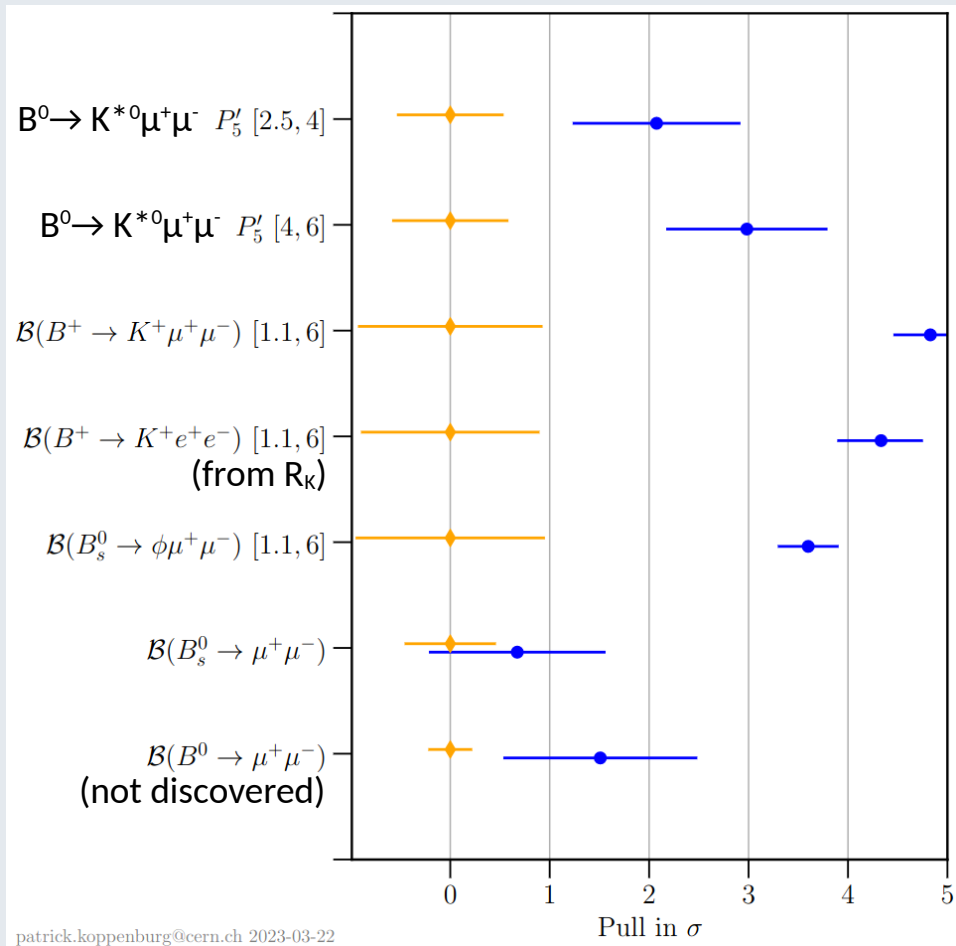
SM@LHC 2023 – Fermilab

Ménil Reboud

Based on:

- Gubernari, van Dyk, Virto 2011.09813
- Gubernari, MR, van Dyk, Virto 2206.03797, [2305.06301](#)

Experimental “Summary plot”

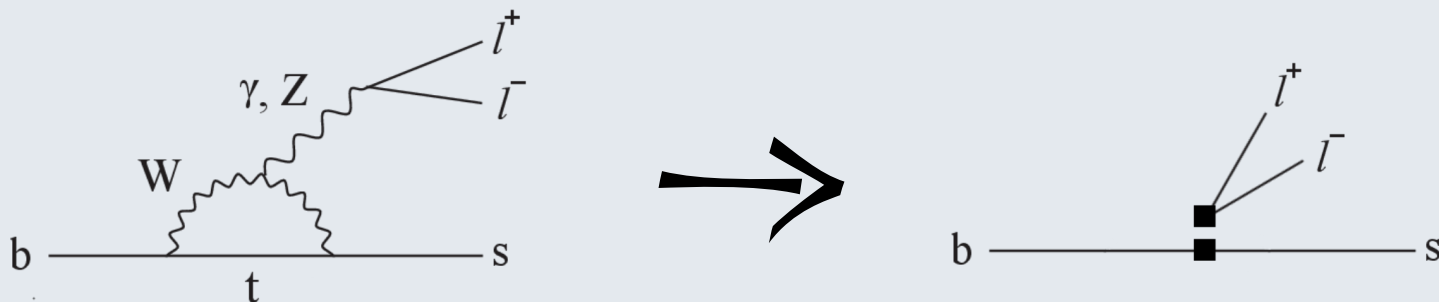


patrick.koppenburg@cern.ch 2023-03-22

- Many **measurements** of $b \rightarrow s \ell \ell$ transitions are in tension with **theory**
- This is a *advertising plot*:
 - ▷ It only shows the observables featuring a tension
 - ▷ Observables and predictions are correlated
- This talk: **theory uncertainties**

Weak Effective Theory

- These processes take place at a scale $m_b < m_W, m_t$



- Allows for a model independent interpretation of the anomalies

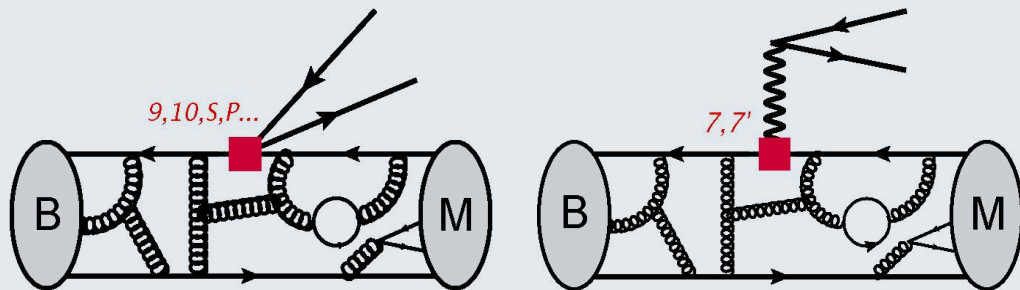
$$\mathcal{H}(b \rightarrow sll) = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu) \mathcal{O}_i(\mu)$$

$$\mathcal{O}_{9(10)} = \frac{e^2}{16\pi^2} (\bar{s}_L \gamma_\mu b_L) (\bar{l} \gamma^\mu (\gamma_5) l)$$

$$\mathcal{O}_7 = \frac{e}{16\pi^2} (\bar{s}_L \sigma_{\mu\nu} b_R) F^{\mu\nu}$$

- Avoids the appearance of large logarithm in the calculations of observables

QCD in $b \rightarrow s \ell \ell$



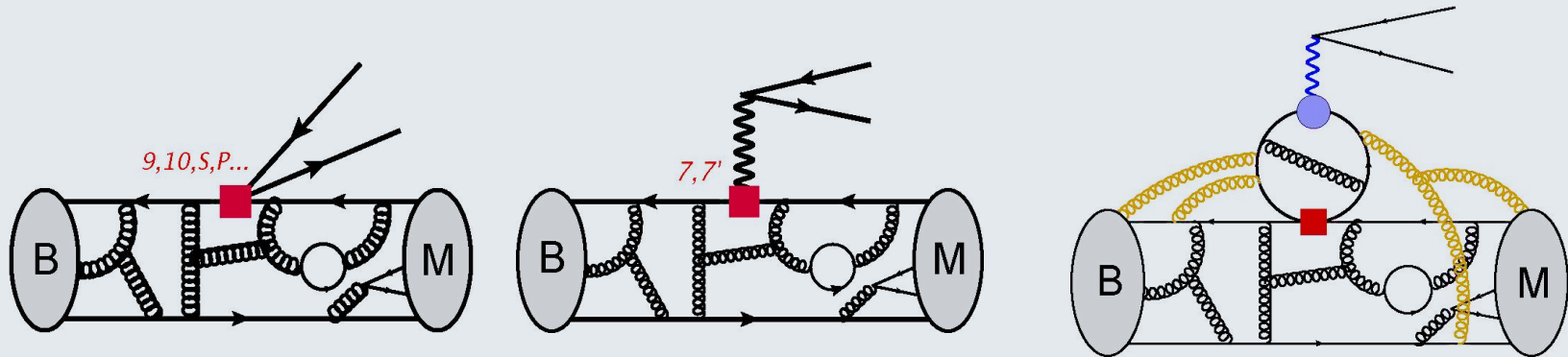
$$\mathcal{A}_\lambda^{L,R}(B \rightarrow M_\lambda \ell \ell) = \mathcal{N}_\lambda \left\{ (C_9 \mp C_{10}) \mathcal{F}_\lambda(q^2) + \frac{2m_b M_B}{q^2} \left[C_7 \mathcal{F}_\lambda^T(q^2) \right] \right\}$$

- $B \rightarrow K^{(*)} \mu \mu$
- $B_s \rightarrow \varphi \mu \mu$
- $\Lambda_b \rightarrow \Lambda^{(*)} \mu \mu$

Local form-factors,
involves e.g.

$$\mathcal{F}_\mu(k, q) = \langle \bar{M}(k) | \bar{s} \gamma_\mu b_L | \bar{B}(q+k) \rangle$$

QCD in $b \rightarrow s \ell \ell$



$$\mathcal{A}_\lambda^{L,R}(B \rightarrow M_\lambda \ell \ell) = \mathcal{N}_\lambda \left\{ (C_9 \mp C_{10}) \mathcal{F}_\lambda(q^2) + \frac{2m_b M_B}{q^2} \left[C_7 \mathcal{F}_\lambda^T(q^2) - 16\pi^2 \frac{M_B}{m_b} \mathcal{H}_\lambda(q^2) \right] \right\}$$

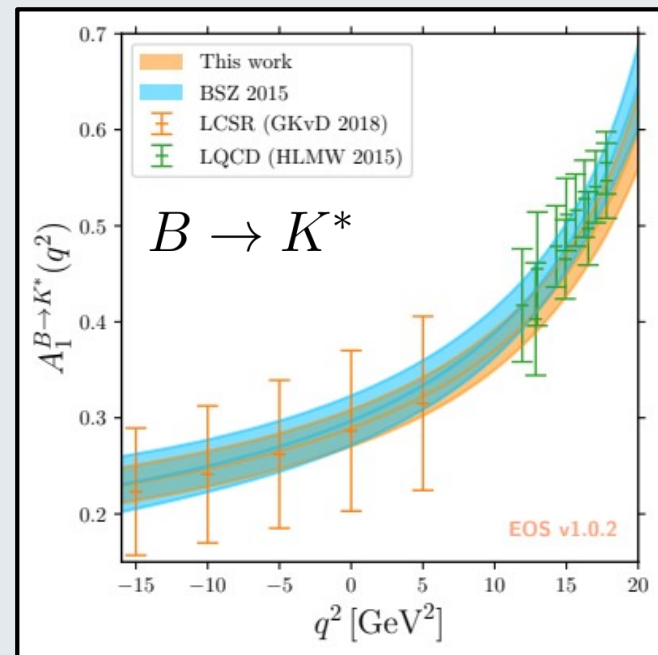
$$\mathcal{H}_\mu(k, q) = i \int d^4x e^{iq \cdot x} \langle \bar{M}(k) | T \{ \mathcal{J}_\mu^{\text{em}}(x), C_i \mathcal{O}_i \} | \bar{B}(q+k) \rangle$$

Non-local form-factors

→ Main contributions: the “charm-loops” $\mathcal{O}_{2(1)}^c = (\bar{s}_L \gamma_\mu (T^a) c_L) (\bar{c}_L \gamma^\mu (T^a) b_L)$

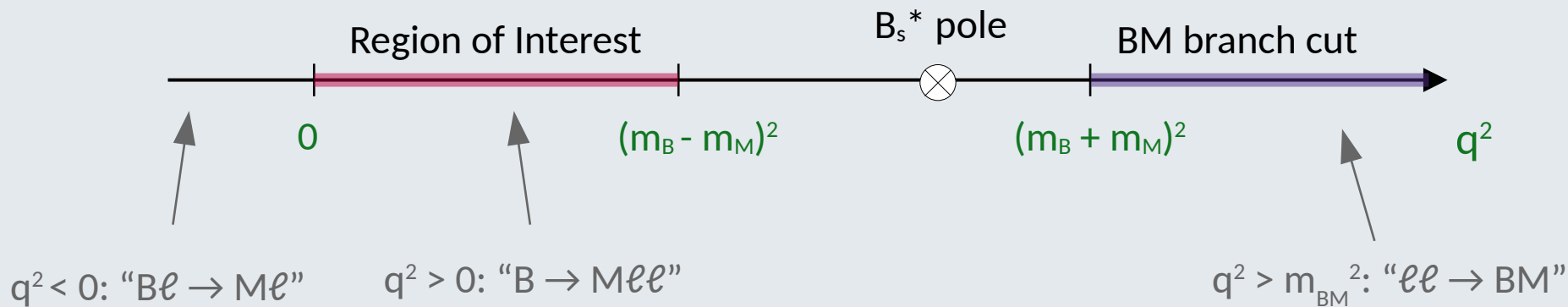
Local form factors

- Conceptually easy, but still the dominant source of uncertainties
 - **2 main approaches**
 - **Lattice QCD** → most feasible at **large q^2**
 - **Light-cone sum rules** → most feasible at **small q^2** , 2 possible **LCSRs**
- **Interpolation/Extrapolation**, depending on the use case
- How to control extrapolation uncertainties?



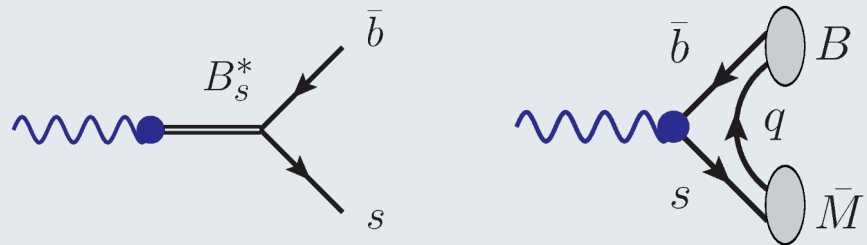
Form Factor Properties

$$\mathcal{F}_\mu(k, q) = \langle \bar{M}(k) | \bar{s} \gamma_\mu b_L | \bar{B}(q+k) \rangle$$



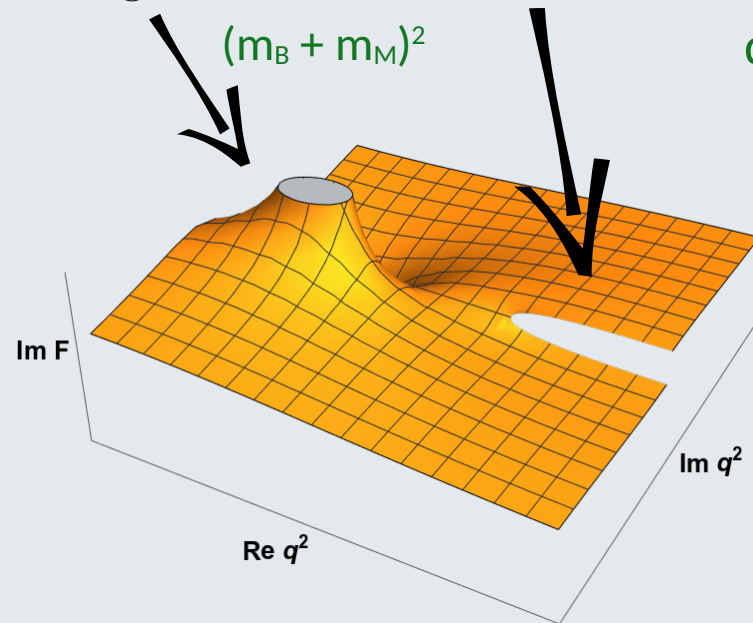
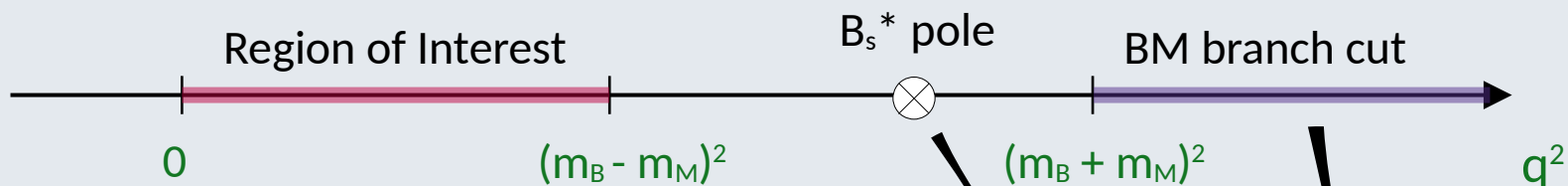
Analytic properties of the form factors:

- Pole due to $\bar{b}s$ bound state
- **Branch cut** due to on-shell BM production



Form Factor Properties

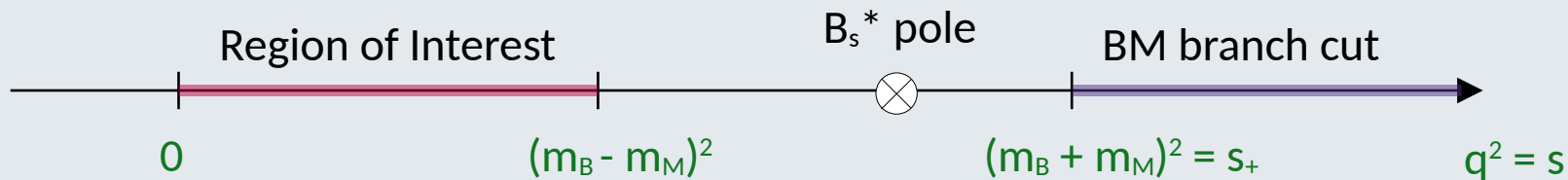
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Analytic properties of the form factors:

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Form Factor Parametrization

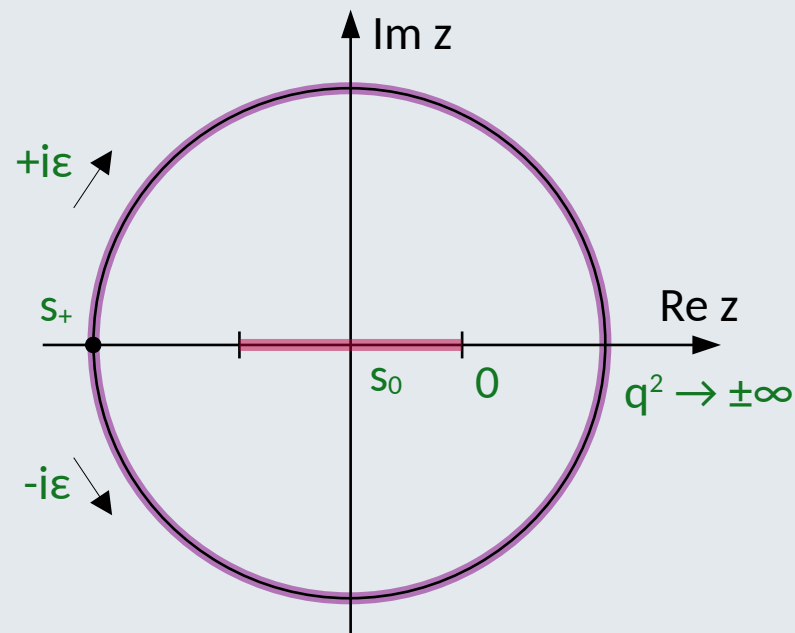


Conformal mapping [Boyd, Grinstein, Lebed '97]

$$z(s) \equiv \frac{\sqrt{s_+ - s} - \sqrt{s_+ - s_0}}{\sqrt{s_+ - s} + \sqrt{s_+ - s_0}}$$

Simplified Series expansion [Bourelly, Caprini, Lellouch, '08; Bharucha, Feldmann, Wick '10]

$$\mathcal{F}_\lambda^{(T)}(q^2) = \frac{1}{q^2 - m_{B_s^*}^2} \sum_{k=0}^N \alpha_{\lambda,k} z^k$$



What is the uncertainty due to the truncation order N?

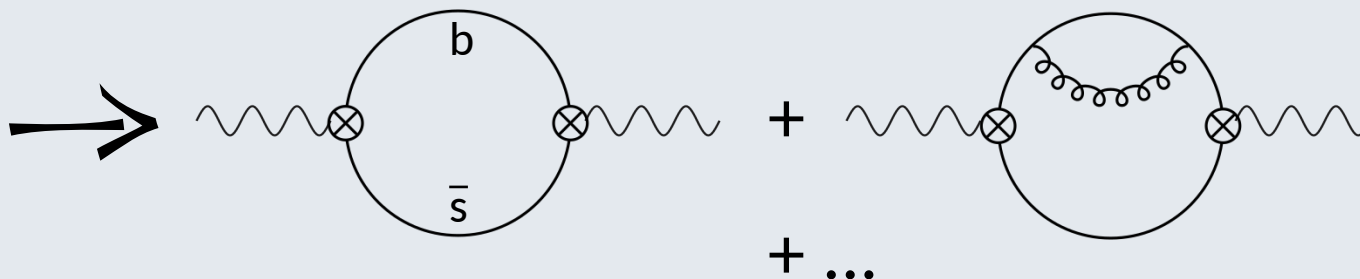
Dispersive bounds

- **Main idea:** Compute the inclusive $e^+e^- \rightarrow \bar{b}s$ cross-section and relate it to the form factors [Bharucha, Feldmann, Wick '10]

$$\Pi_{\Gamma}^{\mu\nu}(q) \equiv i \int d^4x e^{iq \cdot x} \langle 0 | \mathcal{T} \left\{ J_{\Gamma}^{\mu}(x) J_{\Gamma}^{\dagger, \nu}(0) \right\} | 0 \rangle$$

1) Partonic calculation

Insertion of a scalar, vector or tensor current



Dispersive bounds

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$$\Pi_{\Gamma}^{\mu\nu}(q) \equiv i \int d^4x e^{iq \cdot x} \langle 0 | \mathcal{T} \left\{ J_{\Gamma}^{\mu}(x) J_{\Gamma}^{\dagger, \nu}(0) \right\} | 0 \rangle$$

2) Relation to form factors

$$\text{Im } \Pi_I^X(q^2) = \frac{1}{2} \sum_{\Gamma} \int d\rho_{\Gamma} (2\pi)^4 \delta^4(q - p_{\Gamma}) P_I^{\mu\nu} \langle 0 | j_{\mu}^X | \Gamma \rangle \langle \Gamma | j_{\nu}^{\dagger X} | 0 \rangle$$

\uparrow \uparrow

$\sim |\text{form factor}|^2$

Sum over all the $\bar{b}s$ states: $\bar{B}_s, \bar{B}K, \bar{B}K^*, \bar{B}K\pi, \dots$

Dispersive bounds

- **Main idea:** Compute the inclusive $e^+e^- \rightarrow \bar{b}s$ cross-section and relate it to the form factors [Bharucha, Feldmann, Wick '10]
- Assuming global quark-hadron duality we have

$$\chi_{\Gamma}^{(\lambda)} \Big|_{\text{OPE}} = \chi_{\Gamma}^{(\lambda)} \Big|_{\text{1pt}} + \chi_{\Gamma}^{(\lambda)} \Big|_{\bar{B}K} + \chi_{\Gamma}^{(\lambda)} \Big|_{\bar{B}K^*} + \chi_{\Gamma}^{(\lambda)} \Big|_{\bar{B}_s\phi} + \dots$$

Known terms

Sum of positive quantities

Further contributions such as $B \rightarrow K\pi\pi$ or $\Lambda_b \rightarrow \Lambda^{(*)}$.

Any new terms *strengthens* the bound.

Dispersive bounds

- **Main idea:** Compute the inclusive $e^+e^- \rightarrow \bar{b}s$ cross-section and relate it to the form factors [Bharucha, Feldmann, Wick '10]

- Changing the basis of the z -polynomials yields:
$$\sum_{\mathcal{F}} \sum_{n \geq 0} |a_n^{\mathcal{F}}|^2 < 1$$

$$\mathcal{F}_{\Gamma, \lambda}^{B \rightarrow M}(z) = \frac{1}{\mathcal{P}_{\mathcal{F}}(z)\phi_{\mathcal{F}}(z)} \sum_{n \geq 0} a_n^{\mathcal{F}} p_n(z, \alpha_{\Gamma}^{B \rightarrow M})$$

Known functions

Orthnormal polynomials

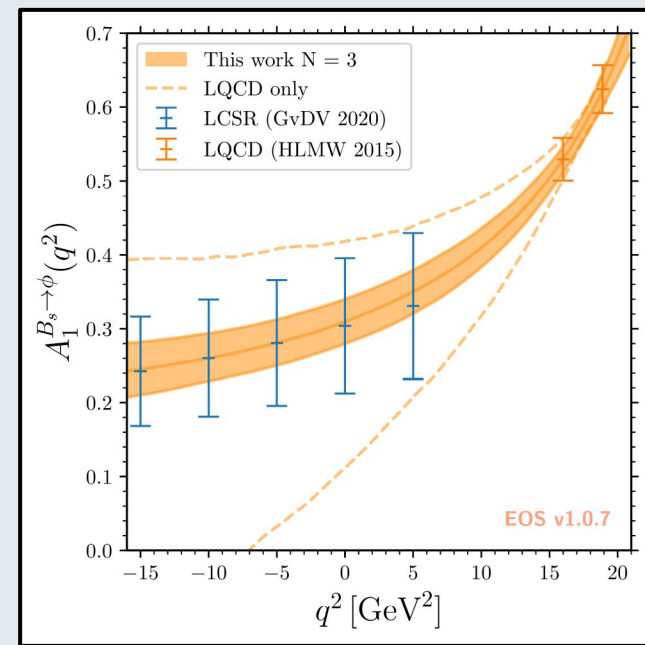
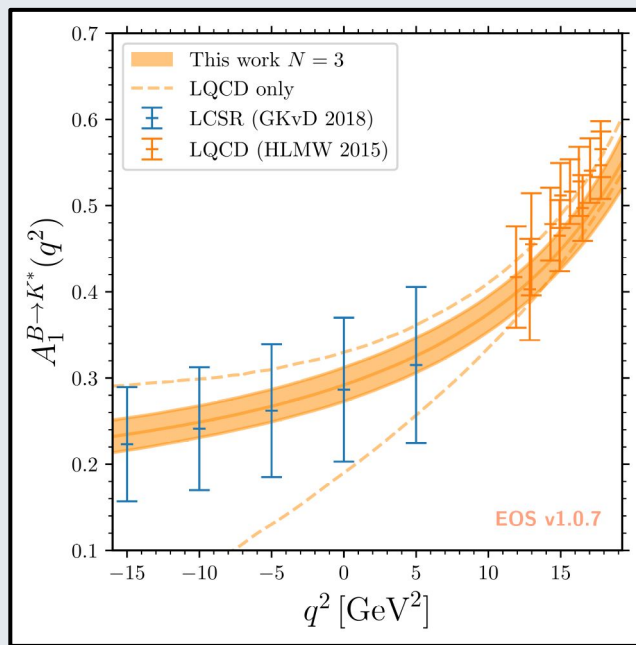
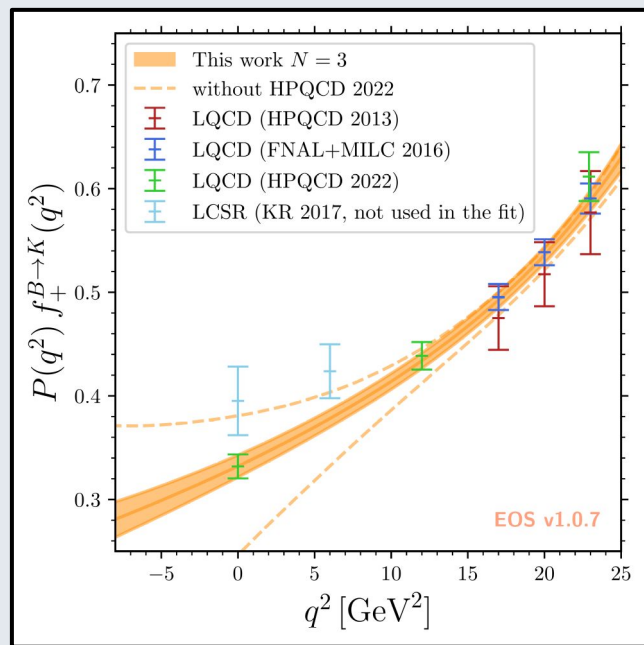
Local form factors fit

- With this framework we perform a **combined fit** of $B \rightarrow K$, $B \rightarrow K^*$ and $B_s \rightarrow \varphi$ LCSR and lattice QCD inputs:
 - $B \rightarrow K$:
 - [HPQCD '13 and '22; FNAL/MILC '17]
 - ([Khodjamiriam, Rusov '17]) \rightarrow large uncertainties, not used in the fit
 - $B \rightarrow K^*$:
 - [Horgan, Liu, Meinel, Wingate '15]
 - [Gubernari, Kokulu, van Dyk '18] (B-meson LCSR)
 - $B_s \rightarrow \varphi$:
 - [Horgan, Liu, Meinel, Wingate '15]
 - [Gubernari, van Dyk, Virto '20] (B-meson LCSR)
- Adding $\Lambda_b \rightarrow \Lambda^{(*)}$ form factors is possible and desirable

Results

Main conclusions:

- Fits are very good already at $N = 2$ (p-values $> 77\%$)
- LCSR and LQCD combine nicely and still dominate the uncertainties
- Progresses in LQCD will gradually replace LCSR

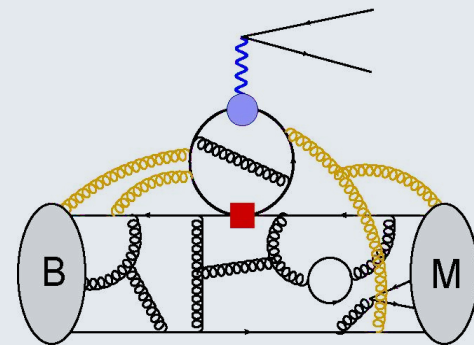


Non-local form factors

$$\mathcal{A}_\lambda^{L,R}(B \rightarrow M_\lambda \ell \ell) = \mathcal{N}_\lambda \left\{ (C_9 \mp C_{10}) \mathcal{F}_\lambda(q^2) + \frac{2m_b M_B}{q^2} \left[C_7 \mathcal{F}_\lambda^T(q^2) - 16\pi^2 \frac{M_B}{m_b} \mathcal{H}_\lambda(q^2) \right] \right\}$$

$$\mathcal{H}_\mu(k, q) = i \int d^4x e^{iq \cdot x} \langle \bar{M}(k) | T \{ \mathcal{J}_\mu^{\text{em}}(x), C_i \mathcal{O}_i \} | \bar{B}(q+k) \rangle$$

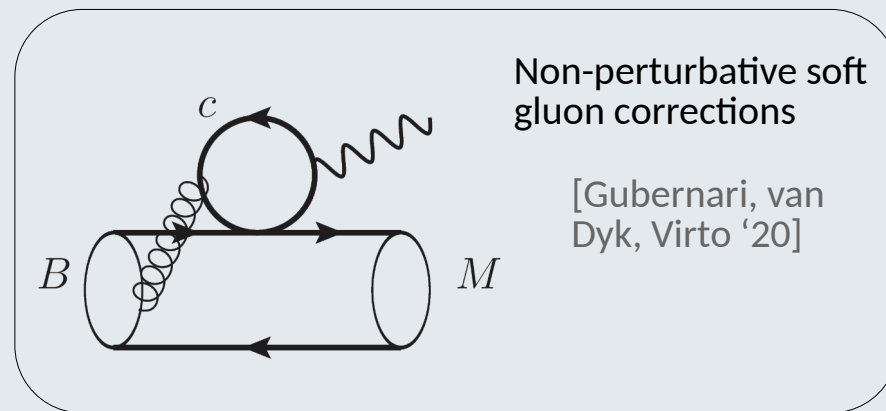
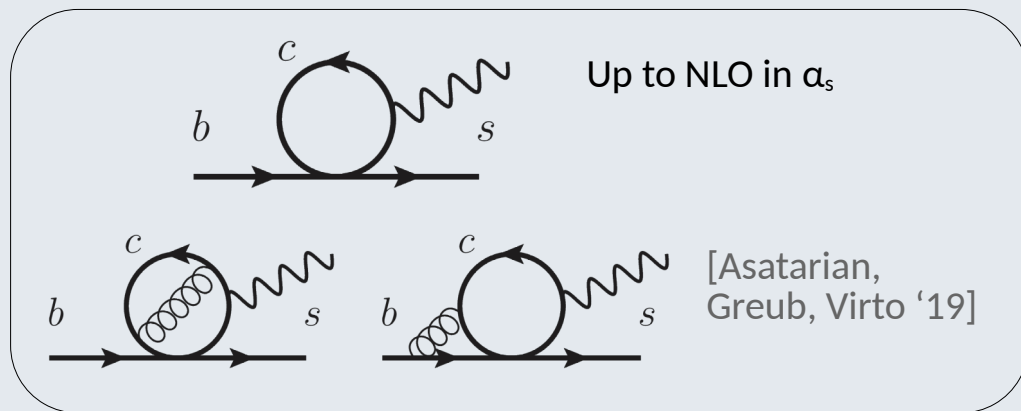
- Problematic because **they can mimic a BSM signal!**
 - \mathcal{H}_λ can be interpreted as a shift to C_9 and C_7
 - This shift is lepton-flavour universal i.e. compatible with the measurements
- Notably **harder to estimate**, no lattice computation so far
- **Different parametrizations** are suggested



Theory inputs

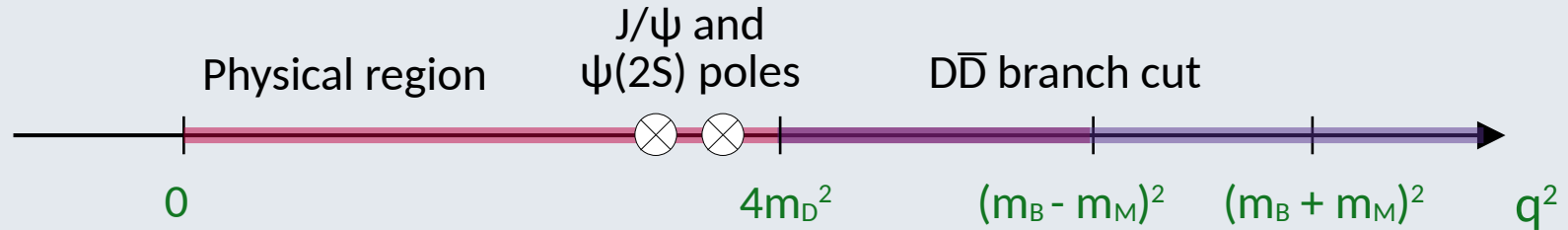
\mathcal{H}_λ can still be calculated in **two kinematics regions**:

- **Local OPE** $|q|^2 \gtrsim m_b^2$ [Grinstein, Piryol '04; Beylich, Buchalla, Feldmann '11]
- **Light Cone OPE** $q^2 \ll 4m_c^2$ [Khodjamirian, Mannel, Pivovarov, Wang '10]



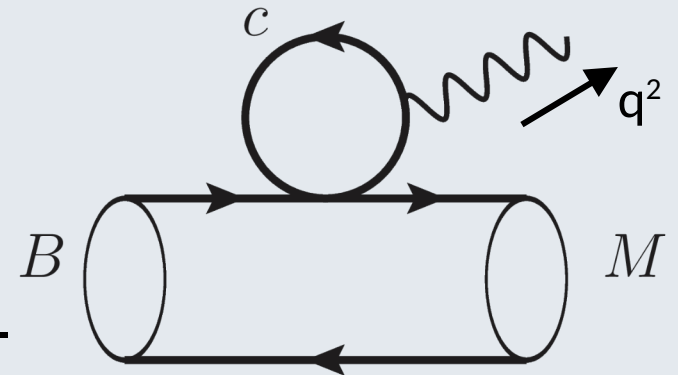
Analyticity properties

$$\mathcal{H}_\mu(k, q) = i \int d^4x e^{iq \cdot x} \langle \bar{M}(k) | T \{ Q_c [\bar{c} \gamma_\mu c](x), \mathcal{C}_i \mathcal{O}_i \} | \bar{B}(q+k) \rangle$$



Analyticity properties:

- Poles due to **charmonium state**
- **Branch cut** in the physical range due to on-shell D meson production: $B \rightarrow M D \bar{D}$
- The coefficients of the z-expansion are now **complex-valued**



- Still focusing on $B \rightarrow K$, $B \rightarrow K^*$ and $B_s \rightarrow \varphi$

Inputs:

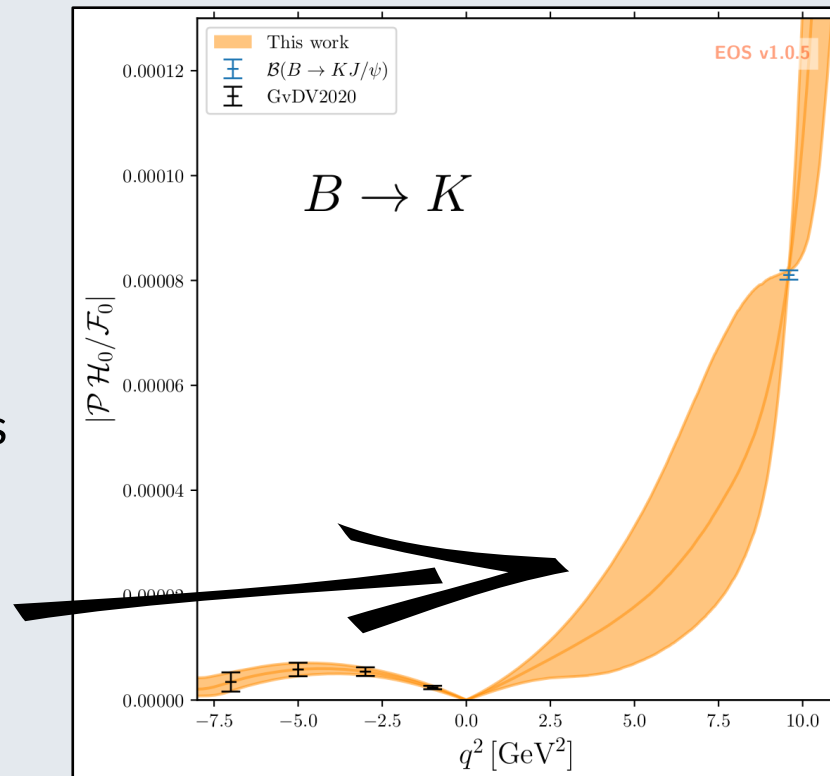
- 4 theory point at negative q^2 from the **light cone OPE**
- Experimental results at the J/ψ (we keep $\psi(2S)$ for future work)

- Use an under-constrained fit ($N = 5$) and allows for saturation of the dispersive bound

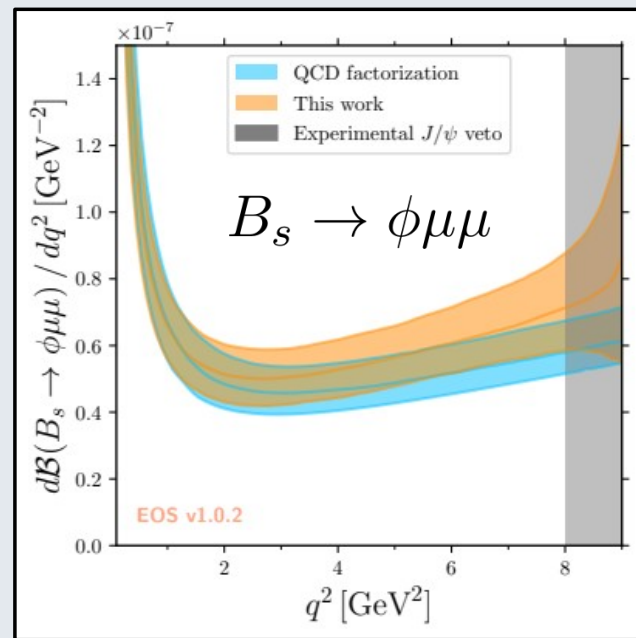
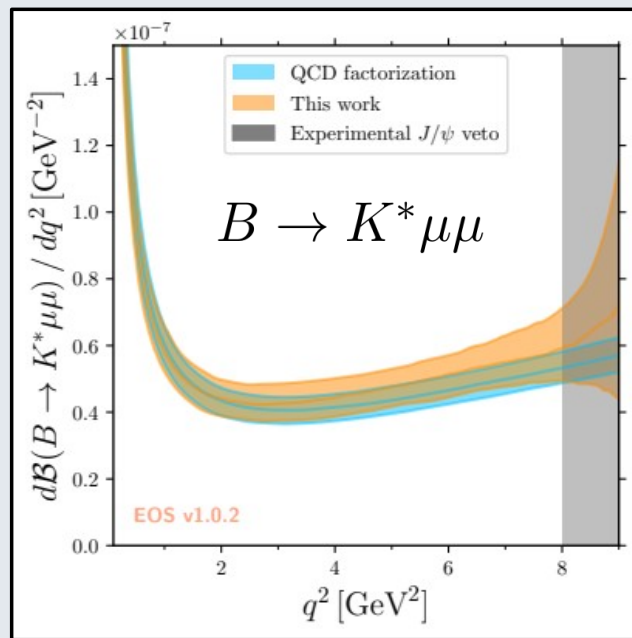
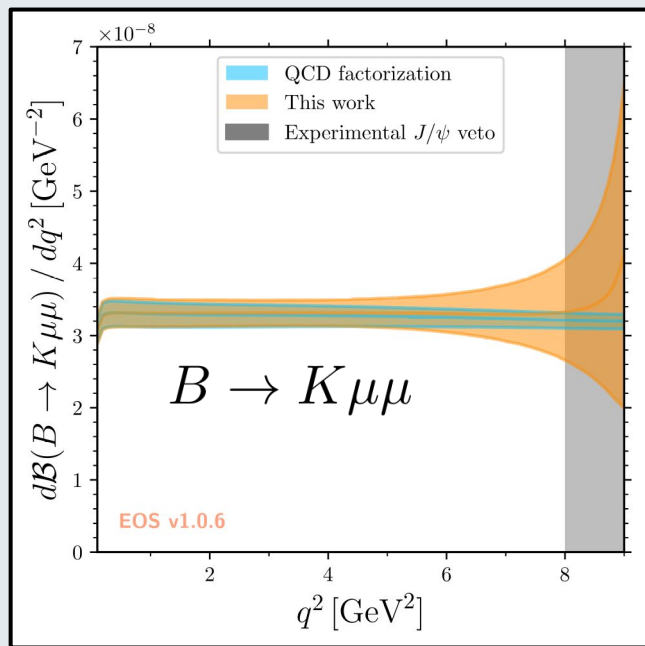
→ The uncertainties are **truncation order independent**, increasing the expansion order does not change their size

→ All p-values are larger than 11%

[Gubernari, MR, van Dyk, Virto '22]

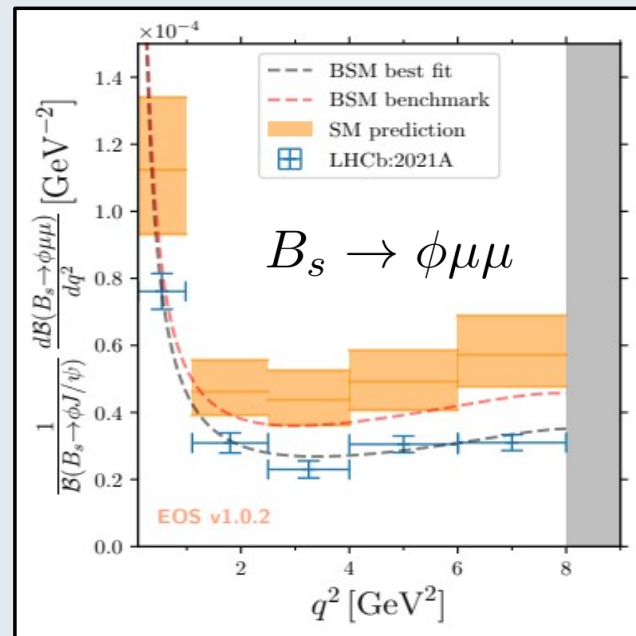
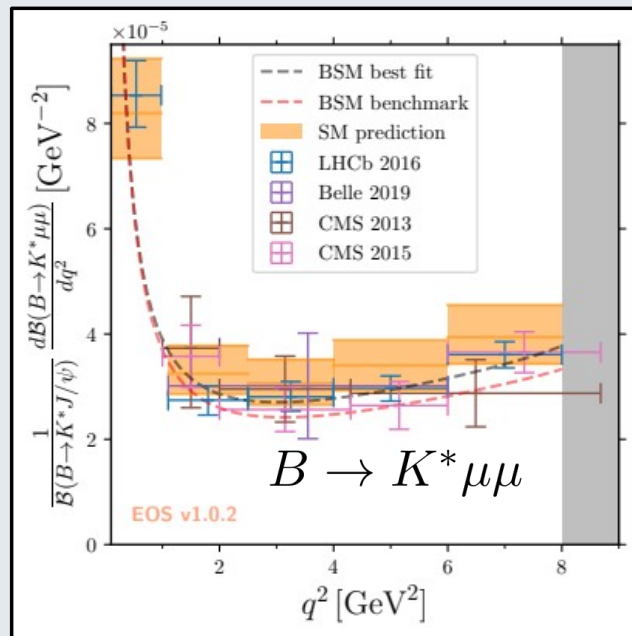
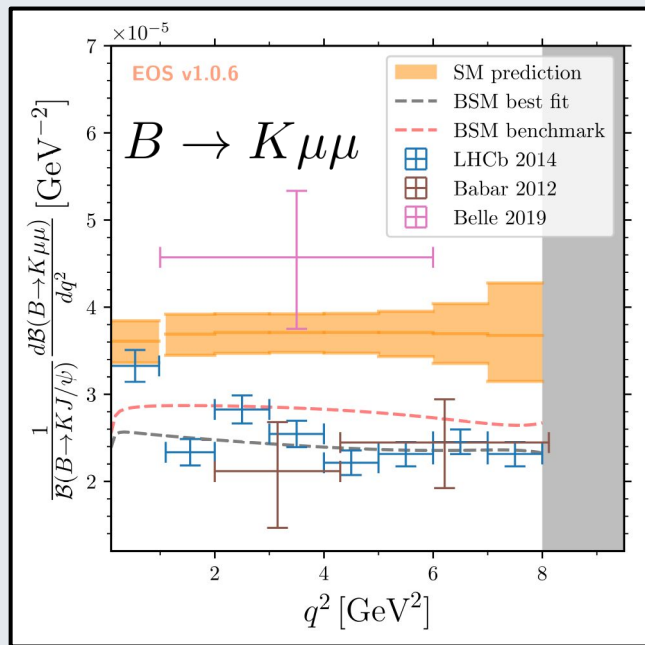


- Good overall agreement with previous theoretical approaches [Beneke, Feldman, Seidel '01 & '04]
 - Small deviation in the slope of $B_s \rightarrow \phi\mu\mu$
- Larger but controlled uncertainties especially near the J/ψ
 - The approach is **systematically improvable** (new channels, $\psi(2S)$ data...)



- Conservatively accounting for the non-local form factors does not solve the $b \rightarrow s\mu\mu$ anomalies
- The largest source of theoretical uncertainty at low q^2 still comes from local form factors

Experimental results:
 [Babar: 1204.3933; Belle: 1908.01848, 1904.02440; ATLAS: 1805.04000, CMS: 1308.3409, 1507.08126, 2010.13968, LHCb: 1403.8044, 2012.13241, 2003.04831, 1606.04731, 2107.13428]



Discussing BSM models requires a solid understanding of the hadronic physics:

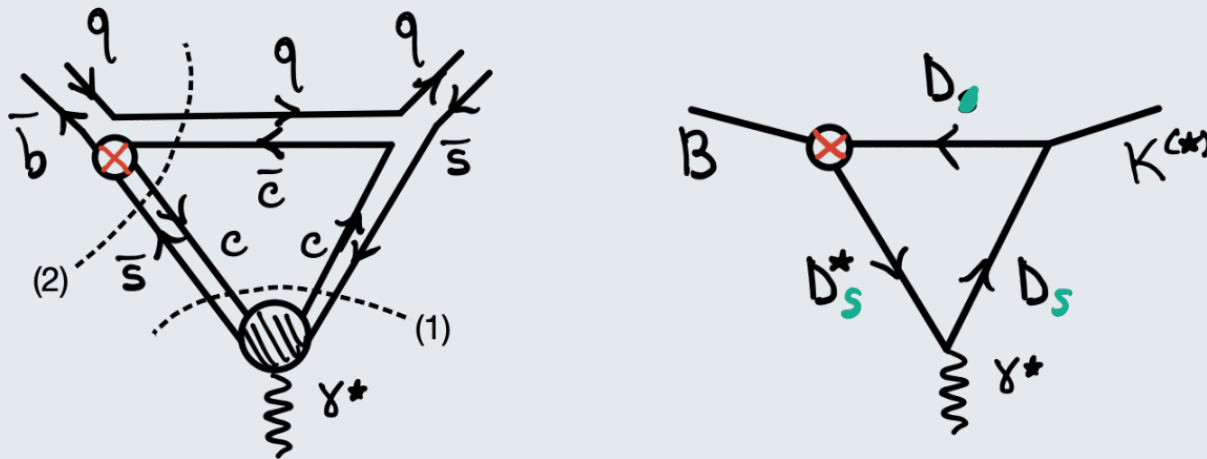
- **Local form factors** uncertainties can be controlled and reduced by using improved dispersive bound and a *appropriate* parametrization
 - This is the first global analysis of $b \rightarrow s$ form factors
 - It is reassuring as it confirms channel-specific analyses...
 - ... and promising as dispersive effects start to be visible
- Understanding of **non-local form factors** is essential to distinguish BSM from SM interpretation of the measurements, but still requires theory inputs.
 - In both cases:
 - Uncertainties are still large, but controlled by dispersive bounds
 - Our approach is systematically improvable

Back-up

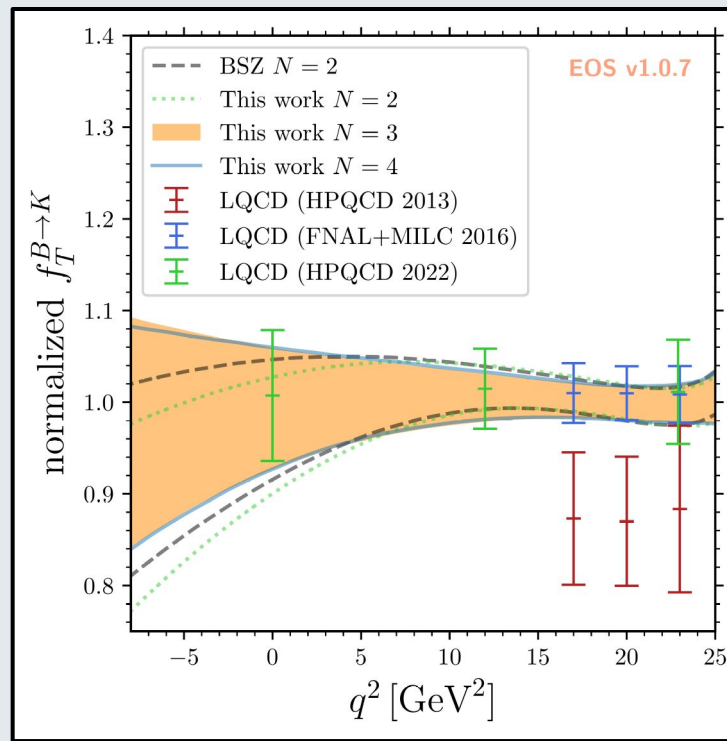
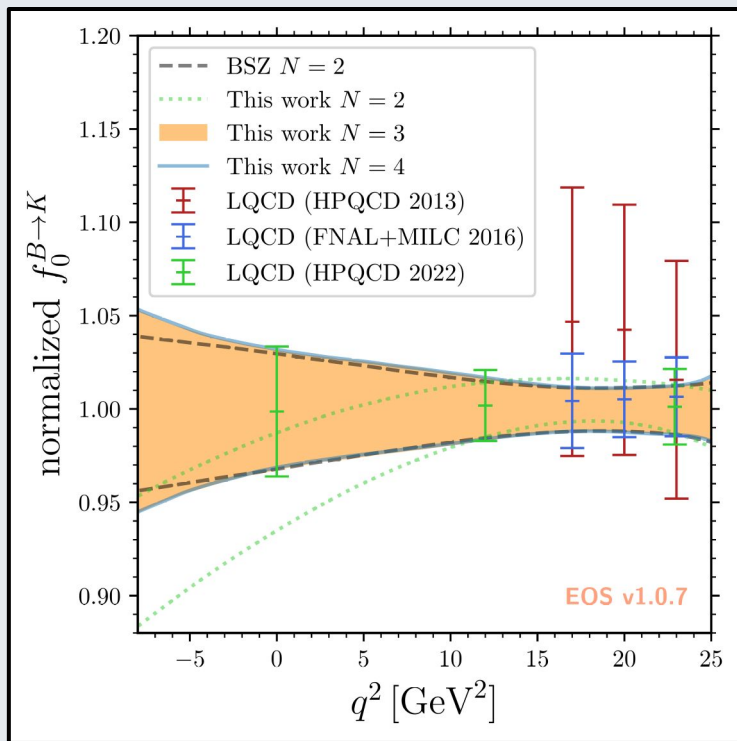
- All the plots are available here: <https://doi.org/10.5281/zenodo.7919635>
- We also added
 - the updated posterior distributions for $N = 2$ in our parametrization and using a SSE as YAML files
 - All the tools/documentation to reproduce our results
- These results are also available in **EOS v1.0.7**:
 - [/eos/constraints/B-to-P-form-factors.yaml](#)
 - [/eos/constraints/B-to-V-form-factors.yaml](#)

Additional effects

- Rescattering of intermediate hadronic states might spoil the analytic structure of the non-local form-factors [Ciuchini, *et al*, '22]



- The effects of the finite width of the K^* amount to a $\sim 10\%$ shift and are accounted for in the fit [Descotes-Genon, Khodjamirian, Virto, '19]



- Normalizing the form factors to the $N = 3$ best fit point allows for a model comparison
- All the plots are available here: <https://doi.org/10.5281/zenodo.7919635>

