Lattice inputs for CKM determinations

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SM@LHC
Fermilab

Introduction

- The LHC era has produced/is producing an incredible wealth of data
  - New processes
  - Ever increasing levels of precision
- Stringent Standard Model tests request corresponding levels of theoretical precision
  - $\rightarrow$ Tests of the CKM matrix
  - Detailed comparisons with observed data (R-ratios, shape data, ...)

Outline

- Lattice QCD - broad overview
- Leptonic decays (briefly)
- Semileptonic decays
  - $B_{(s)}$ decays
  - $D_{(s)}$ decays
- Summary & Conclusion

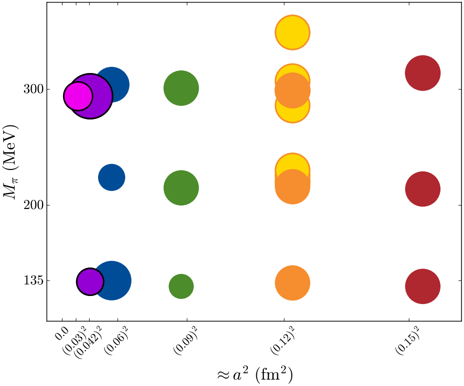
Lattice QCD - Broad overview

- From-first-principles approach to QCD/hadronic physics.
- Discretize the Euclidean time path integral $\rightarrow$ finite lattice spacing, a.
- Calculate hadronic observables using Monte Carlo & supercomputers, take $a \rightarrow 0$.
- Works well for
  - Leptonic decays (two-point functions)
  - Semileptonic decays (three-point functions)

  and is *systematically improvable*.

Lattice QCD - Broad overview

Example: MILC lattice ensembles 1712.09262



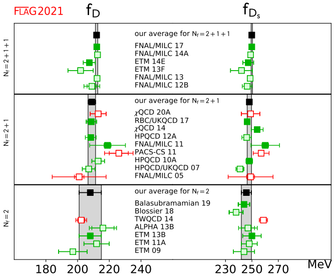
- 6 lattice spacings, finest $a \approx 0.03$ fm
- Physical pion masses at all but finest spacing

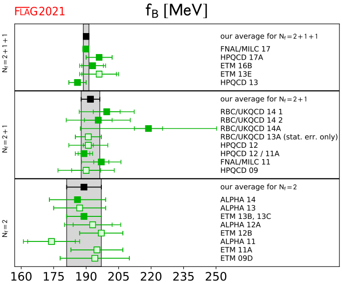
Leptonic decays

Leptonic decays - I

- Relatively straightforward to compute, with meson 2-point functions.
- Allow extraction of CKM matrix elements via leptonic decay measurements.
  - $f_D \to |V_{cd}| = 0.2173(47)_{\rm exp}(28)_{\rm em}(7)_{\rm latt}$
  - $f_{D_s} \to |V_{cs}|$ (sl decays now competitive)
  - $f_B \to |V_{ub}|$
- $f_{B_{(s)}}$ used for SM prediction of rare processes $B_{(s)} \to \mu^+\mu^-$
- In all cases lattice QCD uncertainties well below experimental uncertainties

Leptonic decays - II





- Sub-percent precision obtained, results from different collaborations in good agreement.
- To improve substantially requires including QED and strong-isospin breaking
- Substantial progress including QED corrections to these processes (cf radiative decays)

$B_{(s)} \to \mu^+\mu^-$ and $B \to \ell\nu$

$B_{(s)} \to \mu^+\mu^-$

- Highly suppressed FCNC process can give important constraints on new physics
- $\mathcal{B}(B_s \to \mu\mu)_{\text{SM}} = 3.64(4)_{f_{B_s}}(8)_{\text{CKM}}(7)_{\text{other}} \times 10^{-9}$
  $\mathcal{B}(B_s \to \mu\mu)_{\text{exp}} = 3.52(32) \times 10^{-9}$ 2210.07221
- $\mathcal{B}(B^0 \to \mu\mu)_{\text{SM}} = 1.00(1)_{f_B}(2)_{\text{CKM}}(2)_{\text{other}} \times 10^{-10}$
  $\mathcal{B}(B^0 \to \mu\mu)_{\text{exp}} < 1.6 \times 10^{-10}$

B leptonic decays

- $B \to \tau\nu$ may reach 3-5% precision at Belle II
- $B \to \mu\nu$ may reach $\sim 7\%$ precision at Belle II 1808.10567

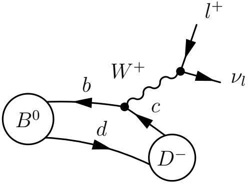
$B_{(s)}$-meson semileptonic decays

Semileptonic decays - I

SL Decay processes critical inputs for heavy flavor studies.

Lattice predictions needed for:

- Extracting CKM matrix elements from expt'l measurements
- Pure SM predictions of R-ratios
- SM predictions $\frac{d\Gamma}{dq^2}$, etc.



Lattice calculations based on 2- & 3-point correlators give matrix elements $\rightarrow f_i(q^2)$

Semileptonic decays - II

Tree level decays:

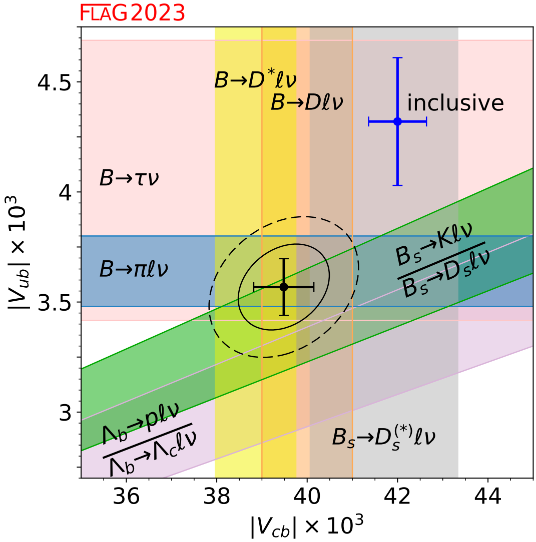
- $B \to D^{(*)} :\to |V_{cb}|$
- $B \to \pi :\to |V_{ub}|$
- $D \to K :\to |V_{cs}|$
- Baryon decays: $\to |V_{ub}|/|V_{cb}|$

Many different processes and results, can only highlight a few illustrative developments!

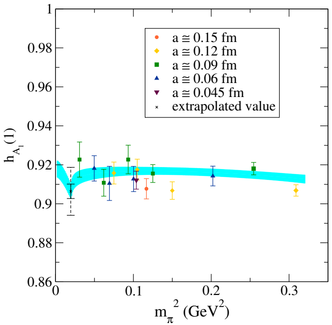
Loop level decays:

- Lattice form factors important for $B \to K\ell^+\ell^-$
- $B \to K^*\ell^+\ell^-$ requires recent theory developments in handling resonances/multi-hadron final states.

Semileptonic decays - III

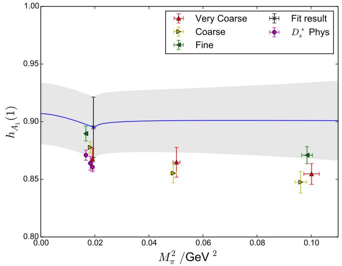


$B \to D^*$ at zero recoil from LQCD



FNAL/MILC 1403.0635

- $n_f = 2+1$ MILC asqtad ensembles
- Clover b with Fermilab interpretation
- $h_{A_1}(1) = 0.906(4)(12)$

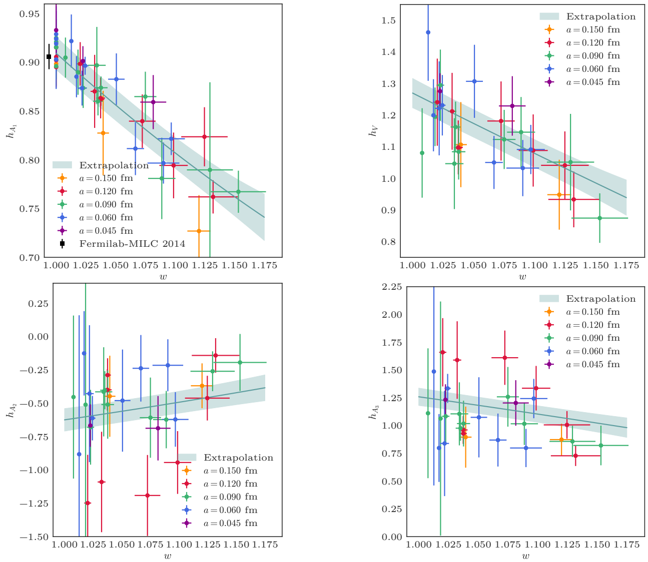


HPQCD 1711.11013

- $n_f = 2+1+1$ MILC HISQ ensembles
- NRQCD b quark
- $h_{A_1}(1) = 0.895(10)(24)$
- $h^s_{A_1}(1) = 0.883(12)(28)$

FNAL/MILC $B \to D^*$

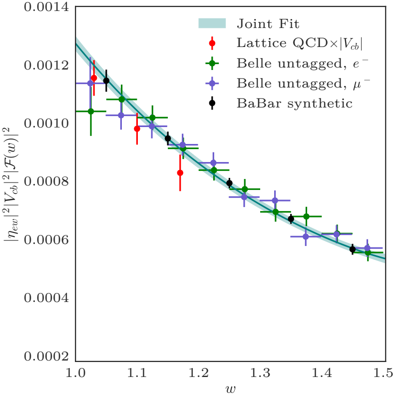
2105.14019

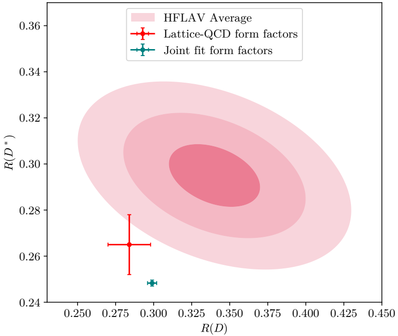


Figs. courtesy A. Vaquero

FNAL/MILC $B \to D^*$

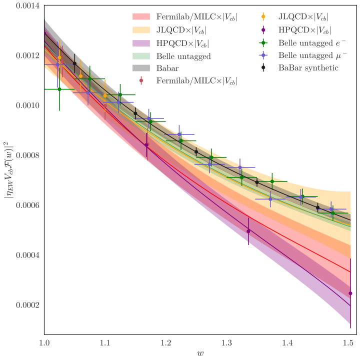
2105.14019

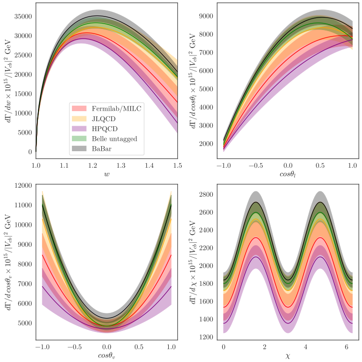




Figs. courtesy A. Vaquero

$B \to D^*$ comparisons





Figs. courtesy A. Vaquero

Lattice refs:

Lattice refs	$\lvert V_{cb}\rvert \times 10^3$
• JLQCD 2306.05657	39.19(90)
• HPQCD 2304.03137	39.31(54)(51)
• FNAL/MILC 2105.14019	38.40(78)

$B_s \to D_s^{(*)} \mu \nu_\mu$ at LHCb [2001.03225]

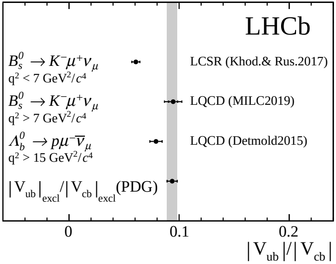
New set of measurements based on LHC Run 1 data.

- CLN: $|V_{cb}| = 41.4(6)_{\rm stat}(9)_{\rm syst}(12)_{\rm ext} \times 10^{-3}$
- BGL: $|V_{cb}| = 42.3(8)_{\rm stat}(9)_{\rm syst}(12)_{\rm ext} \times 10^{-3}$
- In this analysis, LQCD inputs improve statistical precision by 20% and 50% for CLN and BGL, respectively.

See also talk by Mirco Dorigo @LHCP2020
indico.cern.ch/event/856696/contributions/3742179/

$|V_{ub}|/|V_{cb}|$

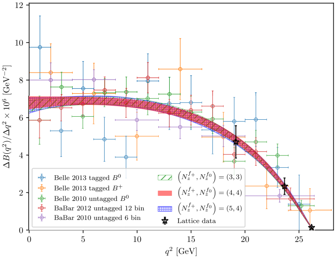
New CKM constraints from LHCb. Based on baryon decays 1504.01568, and first msm't of $B_s \to K\mu\nu$ 2012.05143.

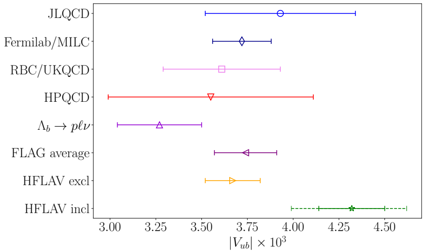


Extractions use lattice form factor calculations of baryon decays ($\Lambda_b \to \Lambda_c$ and $\Lambda_b \to p$ 1503.01421) and $B_s \to K$ 1901.02561 and $B_s \to D_s$ 1906.00710 form factors, respectively.

$|V_{ub}|$ from $B \to \pi$

2203.04938



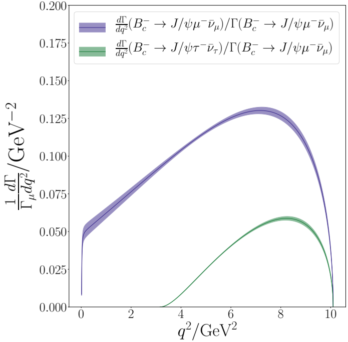


New JLQCD $|V_{ub}|$ determination from $B \to \pi\,\ell\nu$, compared with other lattice results.

$R(B_c \to J/\psi)$

First lattice calculation of $B_c \to J/\psi$ 2007.06956
(All form factors across q^2 range)

- LHCb 1711.05623
  $R(J/\psi) = 0.71(17)_{\text{stat}}(18)_{\text{syst}}$
- w/in 2σ of theory range [0.25, 0.28]
- LQCD result: 0.2582(38)



FNAL-MILC all-HISQ semileptonic decays

Treatment of heavy quarks

Treatment of c and especially b quarks challenging in lattice simulation due to lattice artifacts which grow as $(am_h)^n$

- May use an effective theory framework to handle the b quark.
  - Fermilab method, RHQ, OK, NRQCD
  - Pros: Solves problem w/ am_h artifacts.
  - Cons: Requires matching, can still have ap artifacts.

- Also possible to use relativistic fermion provided a is sufficiently small $am_c \ll 1$, $am_b < 1$.
  - Use improved actions e.g. $\mathcal{O}(a^2) \to \mathcal{O}(\alpha_s a^2)$
  - Pros: Absolutely normalised current, straightforward continuum extrap.
  - Cons: Numerically expensive, extrapolate $m_h \to m_b$.

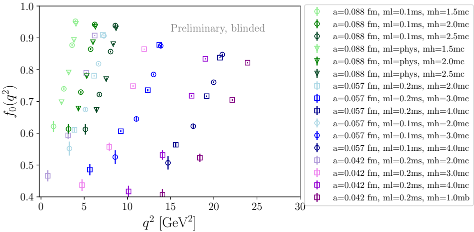
allhisq simulations

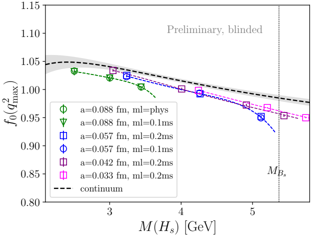
- Here we simulate *all* quarks with the HISQ action.
- Unified treatment for wide range of $B_{(s)}$ (and $D_{(s)}$) to pseudoscalar transitions
  - $B_{(s)} \to D_{(s)}$
  - $B_{(s)}/D_{(s)} \to K$
  - $B/D \to \pi$
- Ensembles with (HISQ) sea quarks down to physical at each lattice spacing.
- Enables correlated studies of ff *ratios*.

See 2022 Lattice proceeding for more details. 2301.09229

B-decays (preliminary) 2301.09229

$B_s \to K$ $B_s \to D_s$





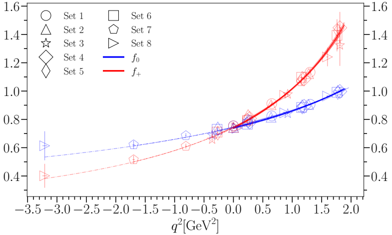
Preliminary results show

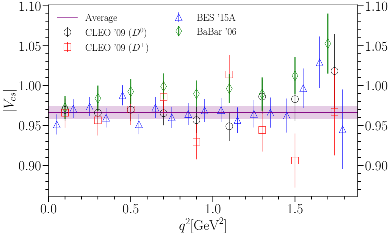
- Good q^2 coverage
- Good statistical precision
- Data at/beyond $M_{B_{(s)}}$ $\to$ *interpolation* in heavy mass m_h

$D_{(s)}$-meson semileptonic decays

$|V_{cs}|$ from $D \to K$

New result from HPQCD 2104.09883. See also ETMC calculations 1706.03017, 1706.03657, 1803.04807.

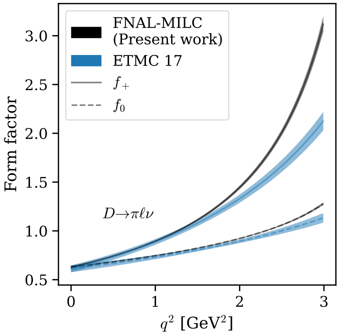


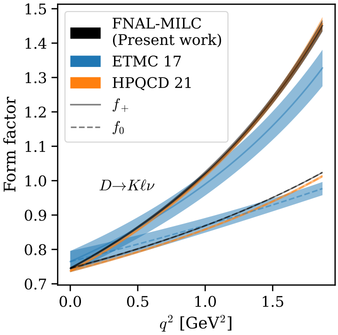


$$|V_{cs}| = 0.9663(53)_{\rm latt}(39)_{\rm exp}(19)_{\eta_{\rm EW}}(40)_{\rm EM}$$

Semileptonic determination of $|V_{cs}|$ now has smaller errors than leptonic determinations.

$D \to \pi, K$

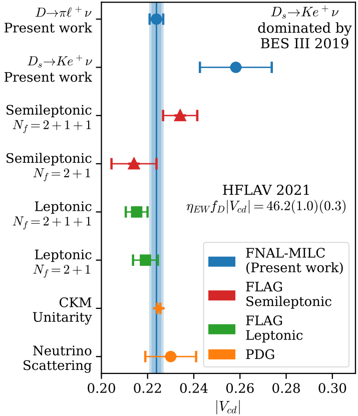


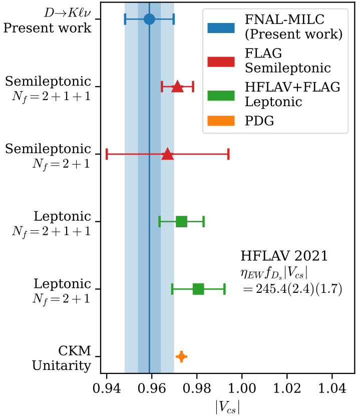


References:

- FNAL-MILC 2212.12648
- HPQCD 21 2104.09883
- ETMC 17 1706.03017

$D_{(s)} \to \pi, K$ $|V_{cd}|$ and $|V_{cs}|$ 2212.12648





Semileptonic decays - Summary

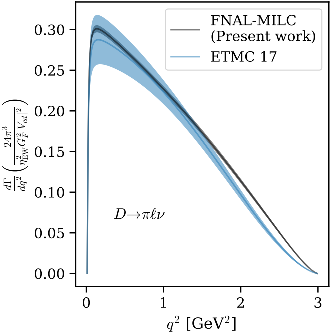
- Thus far, successful collaboration with experiment in the LHC era
  - Theory predictions for new channels
  - Improved kinematic range
  - Improved precision
- Lattice errors roughly commensurate with experimental errors. In the next 5 years or so, these should continue to improve and lattice error may become sub-dominant.
- To go beyond this requires adding EM and strong isospin breaking effects.

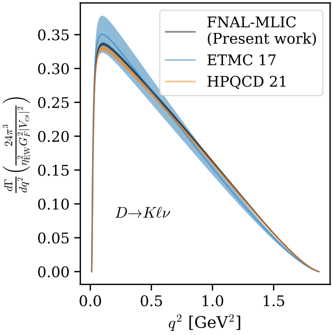
Summary

- We have looked at several places where lattice calculations impact precision SM tests
- Lattice QCD allows stringent comparisons of the Standard Model with experiment
  - Required for precision CKM studies
  - Rare processes and flavor anomalies
- Lattice theorists have kept pace with experimental advances at LHC, Belle II, and BES III
  - Calculations relevant for new msmt's
  - Improving precision

Thank you!

$D \to \pi, K$ rates from LQCD





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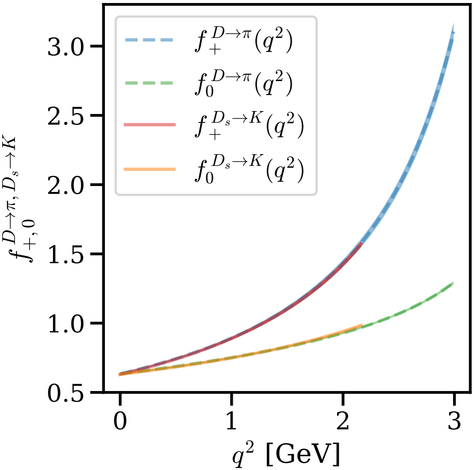
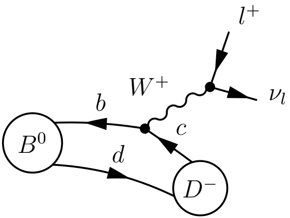
$D_{(s)}$ decay — spectator quark dependence



$B \to D^{(*)} l\nu$



1104.5484

$$\frac{d\Gamma}{dw}(B \to D) = (\text{known})|V_{cb}|^2 (w^2 - 1)^{3/2} |\mathcal{G}(w)|^2$$
$$\frac{d\Gamma}{dw}(B \to D^*) = (\text{known})|V_{cb}|^2 (w^2 - 1)^{1/2} \chi(w) |\mathcal{F}(w)|^2$$

$$w = v_B \cdot v_{D^{(*)}} = \frac{M_B^2 + M_{D^{(*)}}^2 - q^2}{2 M_B M_{D^{(*)}}}$$

At zero recoil $w = 1$, and $\mathcal{F}(1) = h_{A_1}(1)$.

$B \to D^{(*)} l\nu$ matrix elements

The form factors $\mathcal{F}$ and $\mathcal{G}$ can be determined from QCD matrix elements computed on the lattice.

$$\frac{\langle D|V^\mu|B\rangle}{\sqrt{m_B\, m_D}} = (v_B + v_D)^\mu h_+(w) + (v_B - v_D)^\mu h_-(w)$$

$$\frac{\langle D^*_\alpha|V^\mu|B\rangle}{\sqrt{m_B\, m_{D^*}}} = \varepsilon^{\mu\nu\rho\sigma} v_B^\nu v_{D^*}^\rho \epsilon_\alpha^{*\sigma} h_V(w)$$

$$\frac{\langle D^*_\alpha|A^\mu|B\rangle}{\sqrt{m_B\, m_{D^*}}} = i\epsilon_\alpha^{*\nu}\left[h_{A_1}(w)(1+w)g^{\mu\nu} - (h_{A_2}(w)v_B^\mu + h_{A_3}(w)v_{D^*}^\mu)v_B^\nu\right]$$

At zero recoil $w = 1$, and $\mathcal{F}(1) = h_{A_1}(1)$.