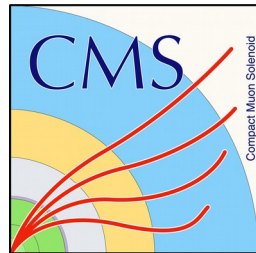


# Dedicated measurements of CP and anomalous Higgs couplings

SM @ LHC 2023  
Workshop July 13<sup>th</sup> 2023

**Savvas Kyriacou (Johns Hopkins University)**

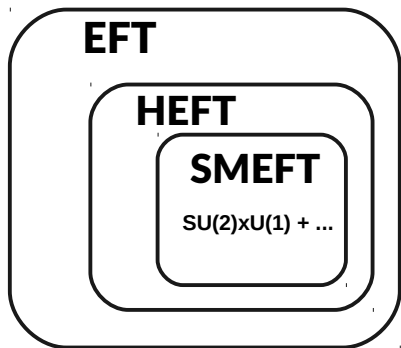
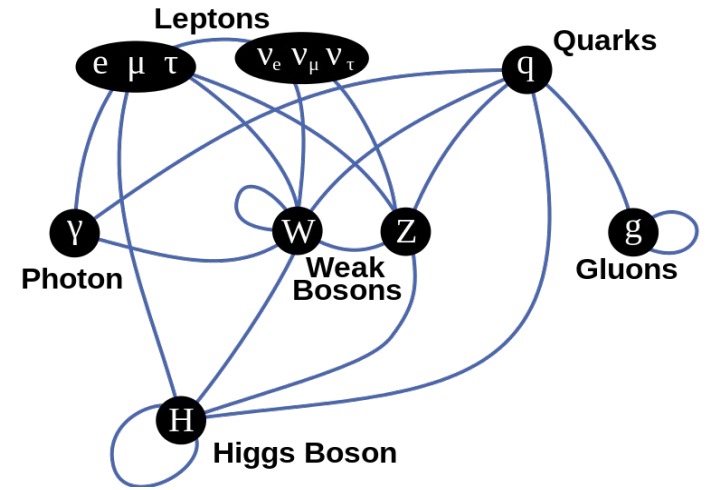
On behalf of the  
**ATLAS** and **CMS** collaborations



**JOHNS HOPKINS**  
UNIVERSITY

# Dedicated AC measurements

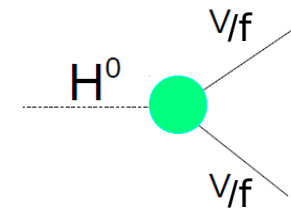
- Study Higgs Couplings to:
  - Uncover CPV in Higgs sector
  - Uncover BSM phenomena
- What consists of a dedicated measurement
  - Targeted analysis
  - Dedicated sensitive observables to specific couplings
  - Gen + Full Detector simulation of AC effects (Interference effects, acceptance effects+ )
- Measurements utilize EFT



$$\mathcal{L}_{SM} = \mathcal{L}_{SM}^{(4)} + \frac{1}{\Lambda} \sum_k C_k^{(5)} Q_k^{(5)} + \frac{1}{\Lambda^2} \sum_k C_k^{(6)} Q_k^{(6)} + \mathcal{O}\left(\frac{1}{\Lambda^3}\right)$$

Mass eigenstate basis + symmetries → **Higgs basis**

Weak eigenstate basis : **Warsaw basis**





# EFT – basis

Mass eigenstate → Higgs basis

$$A(HVV) = \frac{1}{v} \left[ a_1^{VV} + \frac{\kappa_1^{VV} q_{V1}^2 + \kappa_2^{VV} q_{V2}^2}{(\Lambda_1^{VV})^2} + \frac{\kappa_3^{VV} (q_{V1} + q_{V2})^2}{(\Lambda_Q^{VV})^2} \right] m_{V1}^2 \epsilon_{V1}^* \epsilon_{V2}^*$$

$$+ \frac{1}{v} a_2^{VV} f_{\mu\nu}^{*(1)} f^{*(2),\mu\nu} + \frac{1}{v} a_3^{VV} f_{\mu\nu}^{*(1)} \tilde{f}^{*(2),\mu\nu}$$

$$A(Hff) = -\frac{m_f}{v} \bar{\psi}_f (\kappa_f + i \tilde{\kappa}_f \gamma_5) \psi_f$$

V = W,Z,g,γ  
f = leptons + quarks

## Approach 1

$$g_1^{WW} = g_1^{ZZ}$$

$$g_2^{WW} = \frac{2}{c_w} g_2^{ZZ} + \frac{2}{s_w} g_2^{\gamma\gamma} + 2s_w c_w g_2^{Z\gamma}$$

$$g_4^{WW} = \frac{2}{c_w} g_4^{ZZ} + \frac{2}{s_w} g_4^{\gamma\gamma} + 2s_w c_w g_4^{Z\gamma}$$

$$\frac{\kappa_1^{WW}}{(\Lambda_1^{WW})^2} (c_w^2 - s_w^2) = \frac{\kappa_1^{ZZ}}{(\Lambda_1^{ZZ})^2} + 2s_w^2 \frac{g_2^{\gamma\gamma} - g_2^{ZZ}}{M_Z^2} + 2 \frac{s_w}{c_w} (c_w^2 - s_w^2) \frac{g_2^{Z\gamma}}{M_Z^2}$$

$$\frac{\kappa_2^{Z\gamma}}{(\Lambda_1^{Z\gamma})^2} (c_w^2 - s_w^2) = 2s_w c_w \left( \frac{\kappa_1^{ZZ}}{(\Lambda_1^{ZZ})^2} + \frac{g_2^{\gamma\gamma} - g_2^{ZZ}}{M_Z^2} \right) + 2(c_w^2 - s_w^2) \frac{g_2^{Z\gamma}}{M_Z^2}$$

## Approach 2: SMEFT

SU(2)×U(1) + custodial sym.  
set  $c_w^2$  to SM value

$$g_1^{WW} = g_1^{ZZ}$$

$$g_2^{WW} = \frac{2}{c_w} g_2^{ZZ} + \frac{2}{s_w} g_2^{\gamma\gamma} + 2s_w c_w g_2^{Z\gamma}$$

$$g_4^{WW} = \frac{2}{c_w} g_4^{ZZ} + \frac{2}{s_w} g_4^{\gamma\gamma} + 2s_w c_w g_4^{Z\gamma}$$

$$\frac{\kappa_1^{WW}}{(\Lambda_1^{WW})^2} (c_w^2 - s_w^2) = \frac{\kappa_1^{ZZ}}{(\Lambda_1^{ZZ})^2} + 2s_w^2 \frac{g_2^{\gamma\gamma} - g_2^{ZZ}}{M_Z^2} + 2 \frac{s_w}{c_w} (c_w^2 - s_w^2) \frac{g_2^{Z\gamma}}{M_Z^2}$$

$$\frac{\kappa_2^{Z\gamma}}{(\Lambda_1^{Z\gamma})^2} (c_w^2 - s_w^2) = 2s_w c_w \left( \frac{\kappa_1^{ZZ}}{(\Lambda_1^{ZZ})^2} + \frac{g_2^{\gamma\gamma} - g_2^{ZZ}}{M_Z^2} \right) + 2(c_w^2 - s_w^2) \frac{g_2^{Z\gamma}}{M_Z^2}$$

custodial sym.  
set  $c_w^2 = 1$

- Natural choice for Higgs couplings
- Less operators

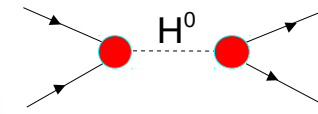
## Weak eigenstate: Warsaw basis

- More general - used in EW TOP and Higgs sector
- SMEFT build in ( SU(2)× U(1) )
- Has many more dim 6 operators

Rotations between basis feasible and demonstrated in measurements !

# HVV: H → 4l CMS

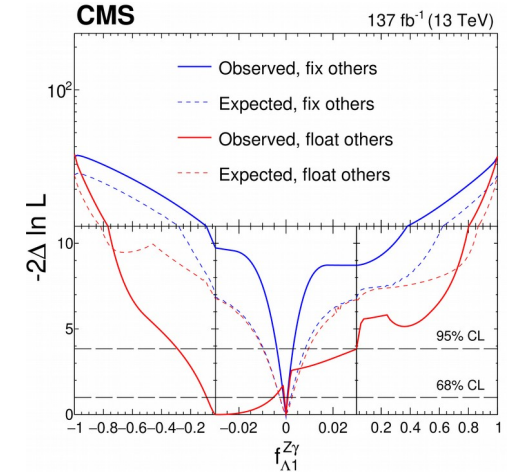
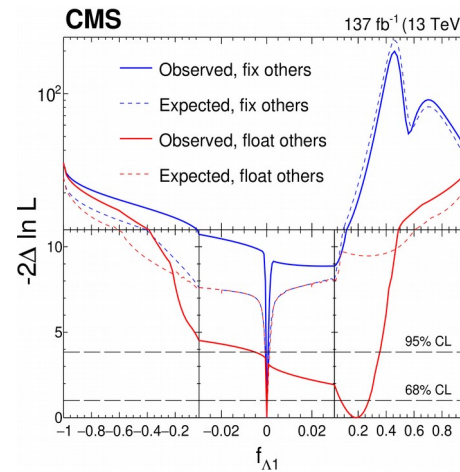
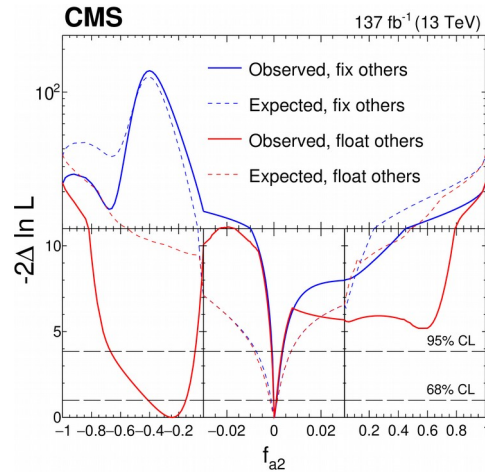
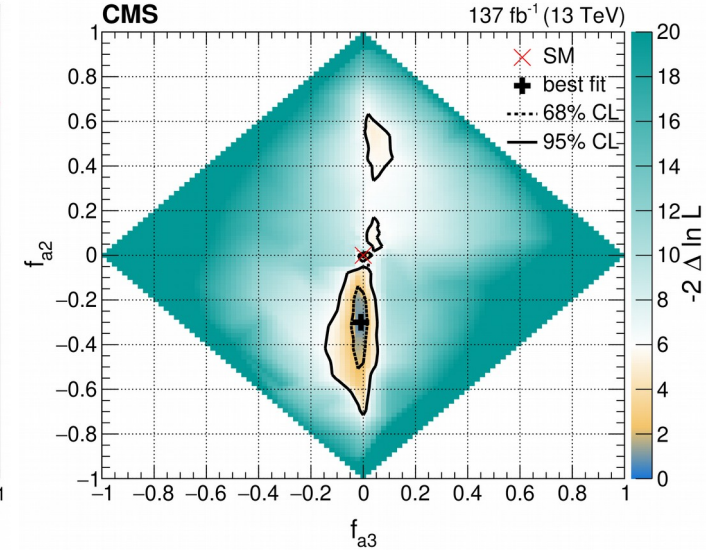
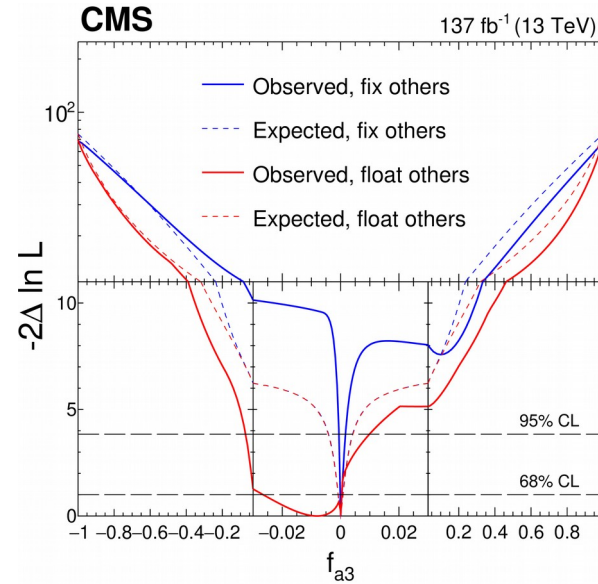
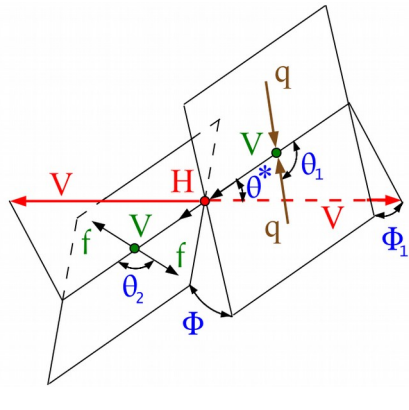
- 2e2μ, 4e, 4μ
- m4l 105 – 140 GeV + 6 categories targeting prod. modes.
- Approach 1 with 4 independent A.C. + SM
- Simultaneous scan of all AC considered
- Non-zero minima
- **SM consistent**



HIG-19-009

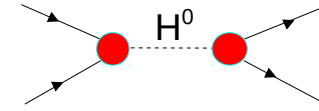
Effective fractional xsec:

$$f_{ai}^{VV} = \frac{|a_i^{VV}|^2 \alpha_{ii}^{(\text{dec})}}{\sum_j |a_j^{VV}|^2 \alpha_{jj}^{(\text{dec})}} \text{sign} \left( \frac{a_i^{VV}}{a_1} \right)$$

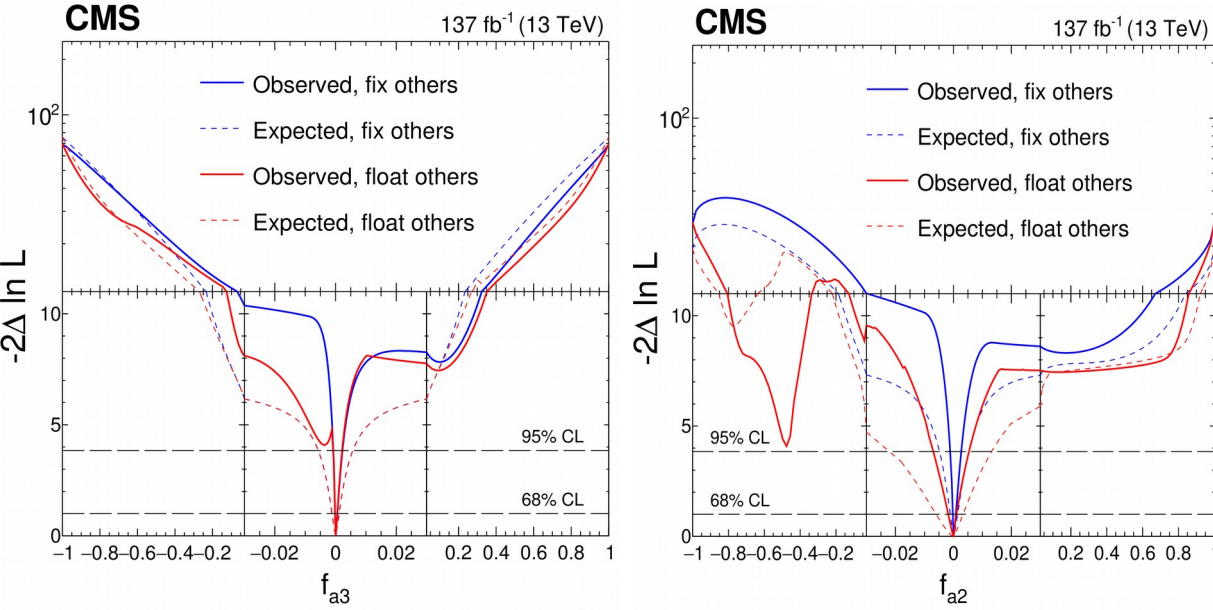




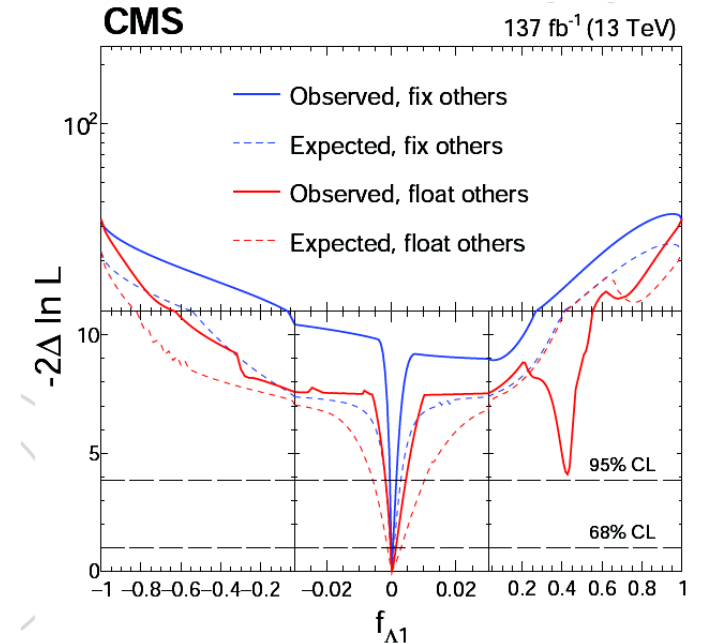
HIG-19-009



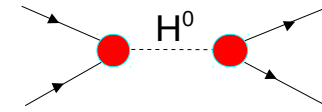
# HVV: $H \rightarrow 4l$ CMS



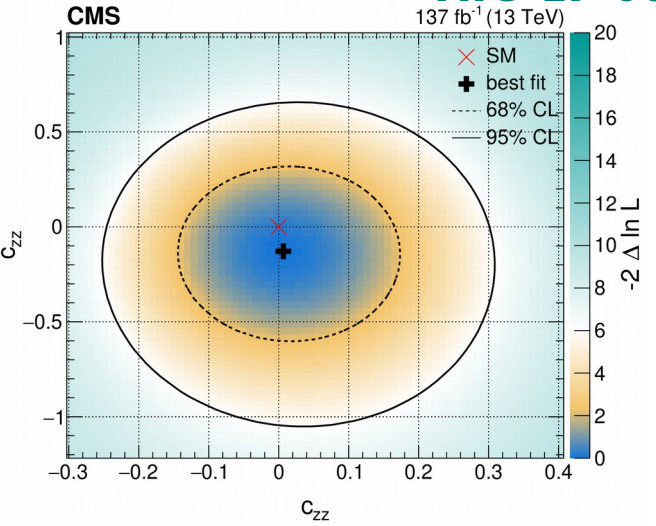
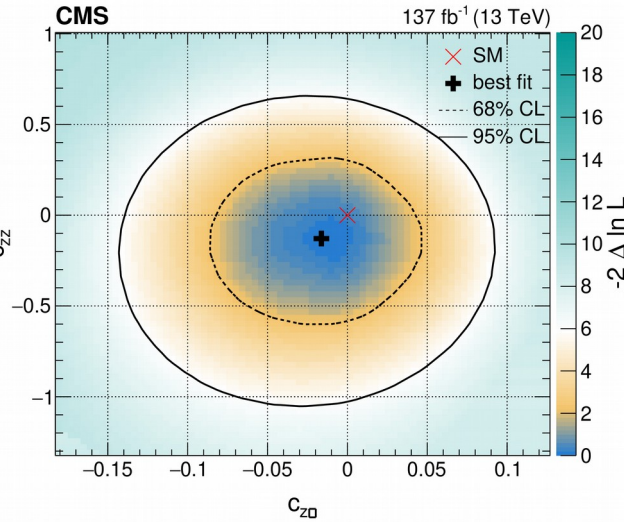
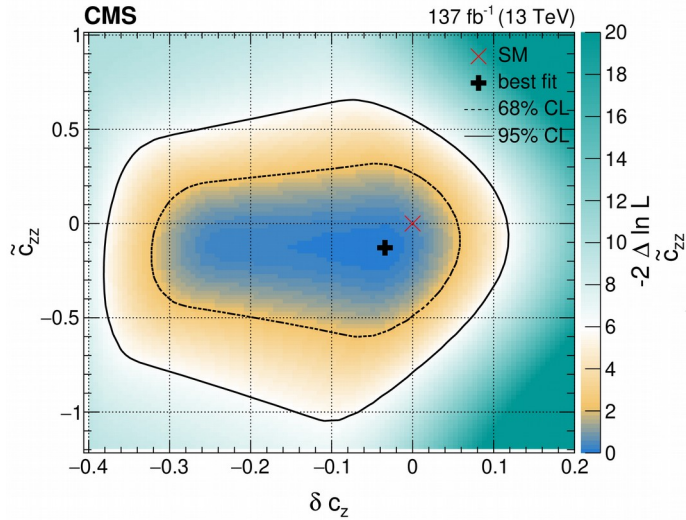
- SU(2)xU(1) sym. (SMEFT) with only 3 independent A.C.
- **Stringent constraints driven by production information**
- **Full Run2**
- **Minima consistent with SM**



Parameter	Scenario	Observed	Expected	
$f_{a3}$	Approach 1	best fit	0.00004	0.00000
	$f_{a2} = f_{\Lambda 1} = f_{\Lambda 1}^{Z\gamma} = 0$	68% CL	$[-0.00007, 0.00044]$	$[-0.00081, 0.00081]$
		95% CL	$[-0.00055, 0.00168]$	$[-0.00412, 0.00412]$
	Approach 1	best fit	-0.00805	0.00000
	float $f_{a2}, f_{\Lambda 1}, f_{\Lambda 1}^{Z\gamma}$	68% CL	$[-0.02656, 0.00034]$	$[-0.00086, 0.00086]$
	95% CL	$[-0.07191, 0.00990]$	$[-0.00423, 0.00422]$	
	Approach 2	best fit	0.00005	0.0000
	float $f_{a2}, f_{\Lambda 1}$	68% CL	$[-0.00010, 0.00061]$	$[-0.0012, 0.0012]$
		95% CL	$[-0.00072, 0.00218]$	$[-0.0057, 0.0057]$

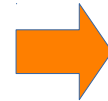


# HVV: $H \rightarrow 4\ell$ CMS



## Translated to Warsaw basis:

Channels	Coupling	Observed
VBF & VH & $H \rightarrow 4\ell$	$\delta c_z$	$-0.03^{+0.06}_{-0.25}$
	$c_{zz}$	$0.01^{+0.11}_{-0.10}$
	$c_{z\Box}$	$-0.02^{+0.04}_{-0.04}$
	$\tilde{c}_{zz}$	$-0.11^{+0.30}_{-0.31}$

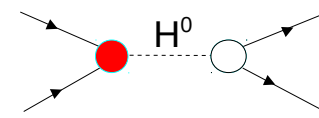


Channels	Coupling	Observed	Expected
VBF & VH & $H \rightarrow 4\ell$	$c_{H\Box}$	$0.04^{+0.43}_{-0.45}$	$0.00^{+0.75}_{-0.93}$
	$c_{HD}$	$-0.73^{+0.97}_{-4.21}$	$0.00^{+1.06}_{-4.60}$
	$c_{HW}$	$0.01^{+0.18}_{-0.17}$	$0.00^{+0.39}_{-0.28}$
	$c_{HWB}$	$0.01^{+0.20}_{-0.18}$	$0.00^{+0.42}_{-0.31}$
	$c_{HB}$	$0.00^{+0.05}_{-0.05}$	$0.00^{+0.03}_{-0.08}$
	$c_{H\bar{W}}$	$-0.23^{+0.51}_{-0.52}$	$0.00^{+1.11}_{-1.11}$
	$c_{H\bar{W}B}$	$-0.25^{+0.56}_{-0.57}$	$0.00^{+1.21}_{-1.21}$
	$c_{H\bar{B}}$	$-0.06^{+0.15}_{-0.16}$	$0.00^{+0.33}_{-0.33}$

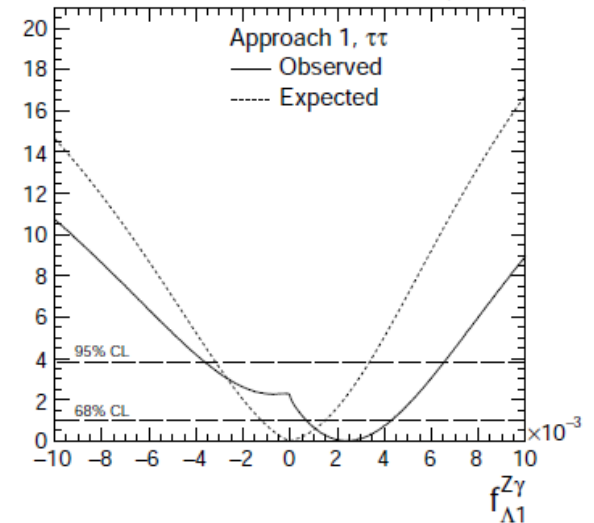
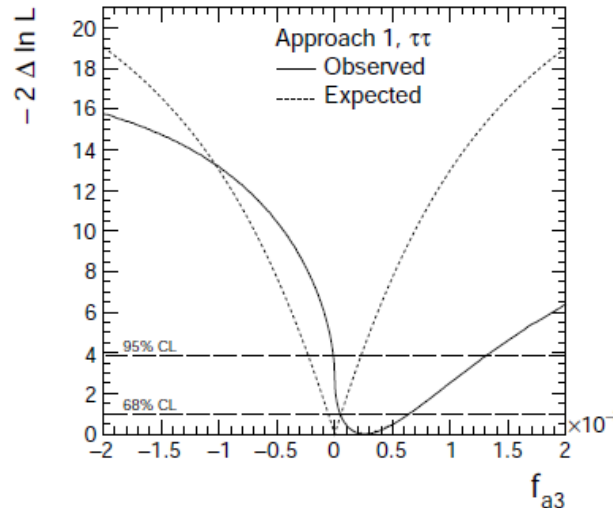
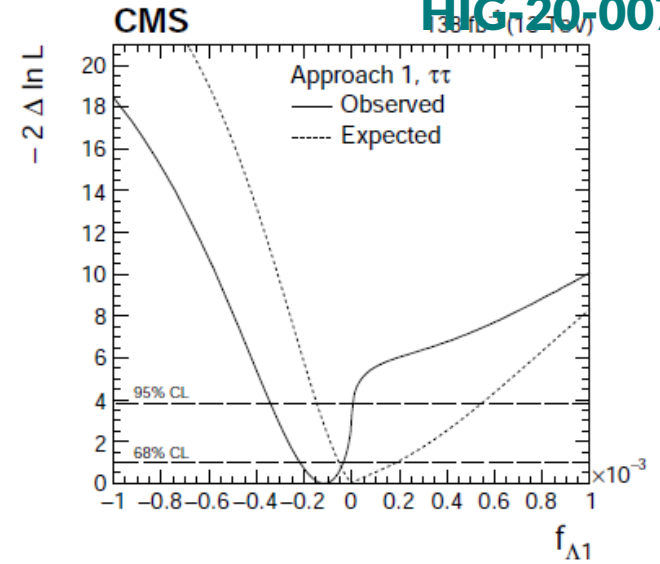
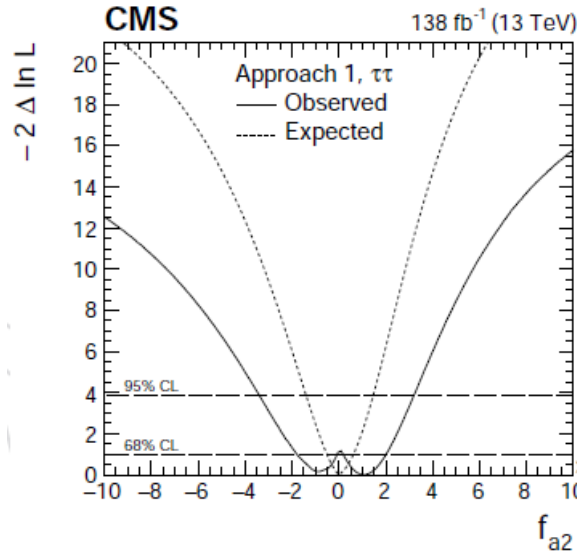
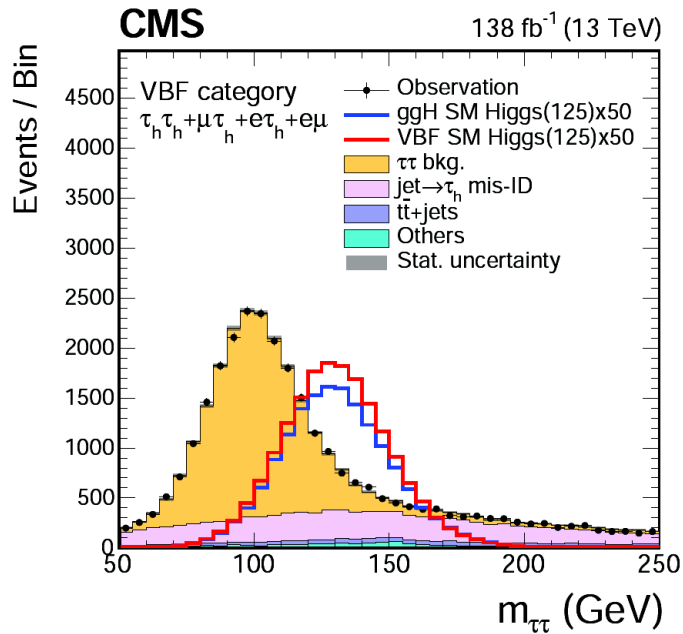


# HVV: $H \rightarrow \tau\tau$ + $H \rightarrow 4l$ CMS

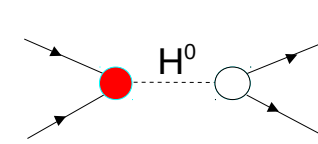
- Single AC scans
- Study production
- Utilize ME discr.
- Combine results with H4l



HIG-20-007

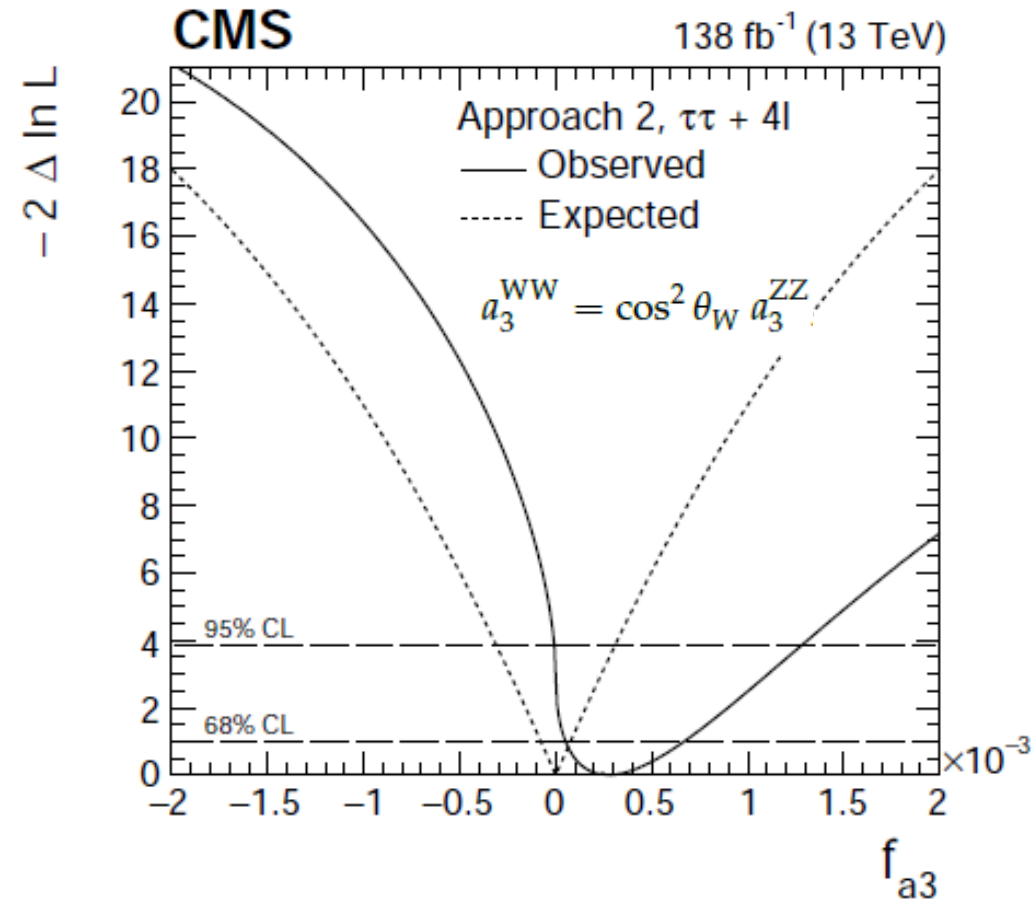
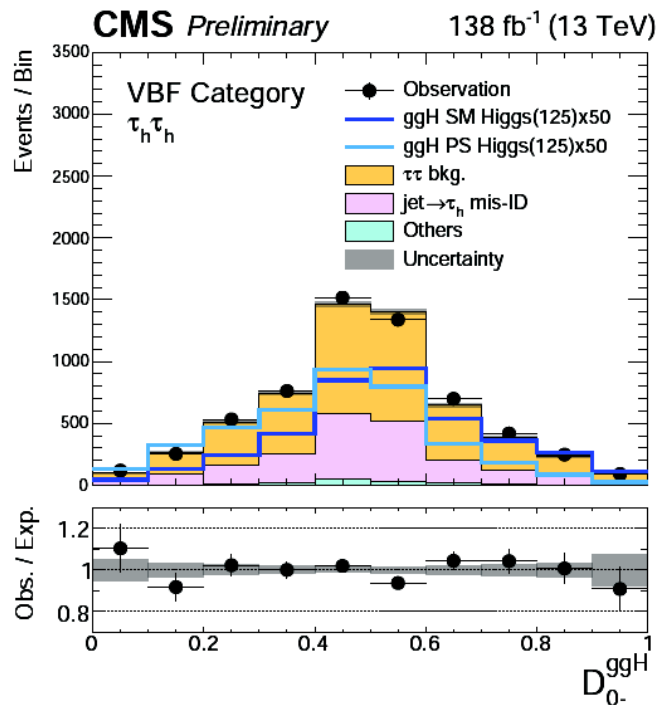


# HVV: $H \rightarrow \tau\tau + H \rightarrow 4l$ CMS



HIG-20-007

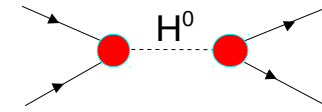
- SMEFT but study only  $a_3$
- High  $H \rightarrow \tau\tau$  BR provides indispensable stat. contribution







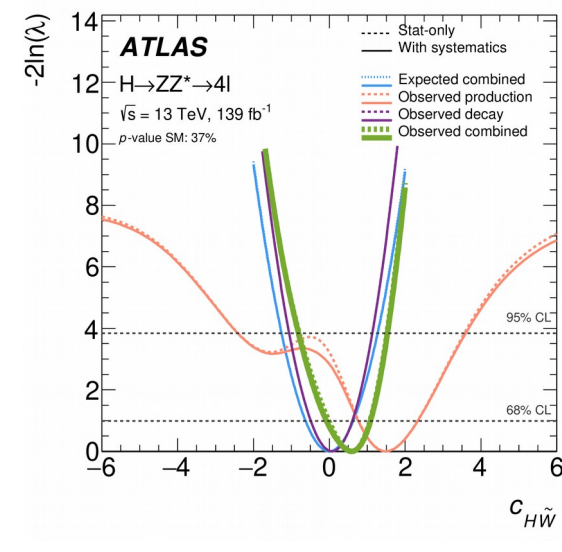
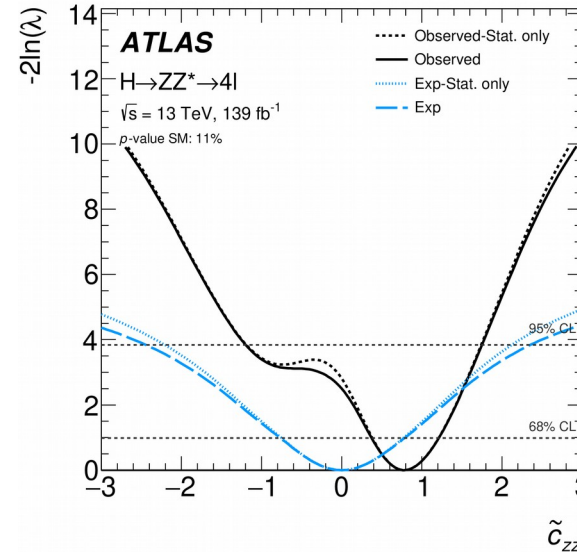
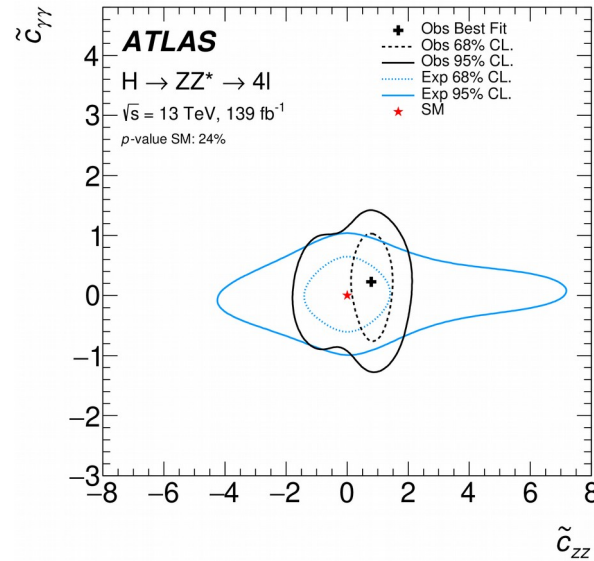
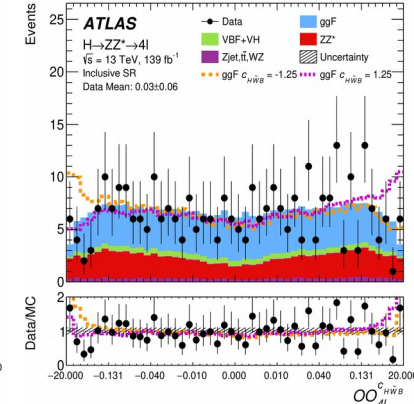
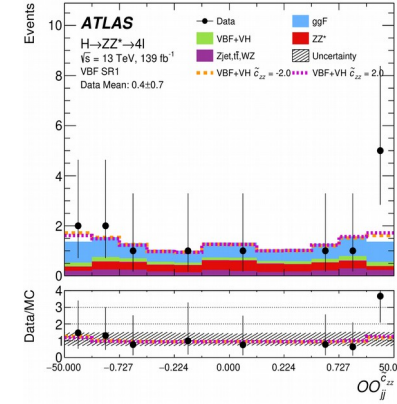
# HVV in $H \rightarrow 4l$ (ATLAS)



HIGG-2018-30

- Use ME based optimal observables
- Single AC fits each time
- Production only or Decay only analysis and then combined
- Warsaw + Higgs basis
- **Full Run2**

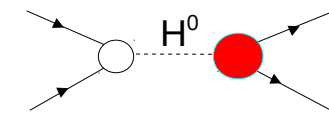
$$OO = \frac{2\Re(\mathcal{M}_{SM}^* \mathcal{M}_{BSM})}{|\mathcal{M}_{SM}|^2}$$





# Yukawa $\tau\tau H$ : $H \rightarrow \tau\tau$ CMS

$$\mathcal{L}_Y = -\frac{m_\tau}{v} H (\kappa_\tau \bar{\tau}\tau + \tilde{\kappa}_\tau \bar{\tau} i \gamma_5 \tau)$$



CERN-EP-2021-189

$$\frac{d\Gamma}{d\phi_{CP}} (H \rightarrow \tau^+\tau^-) \sim 1 - b(E^+)b(E^-) \frac{\pi^2}{16} \cos(\phi_{CP} - 2\alpha^{H\tau\tau})$$

Full Run2 data

Decay vertex probed

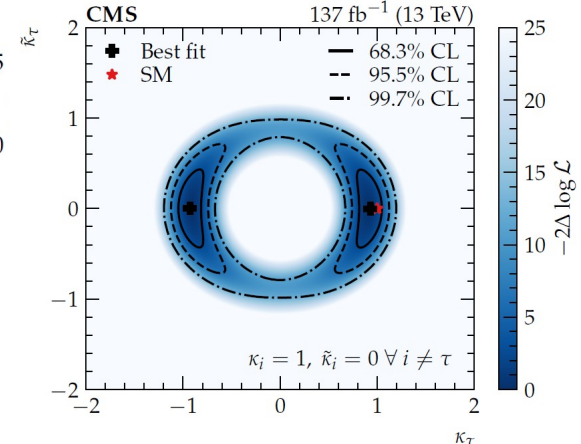
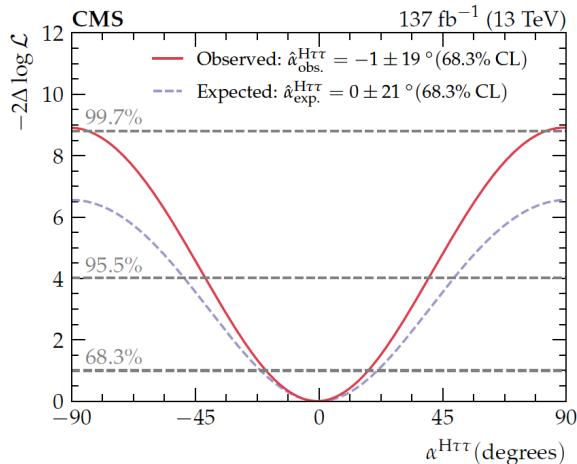
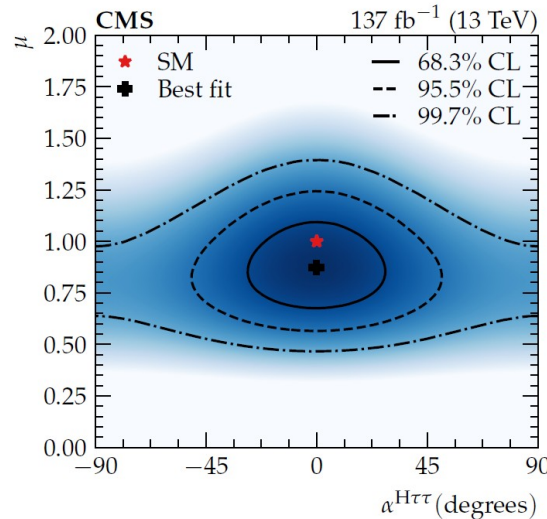
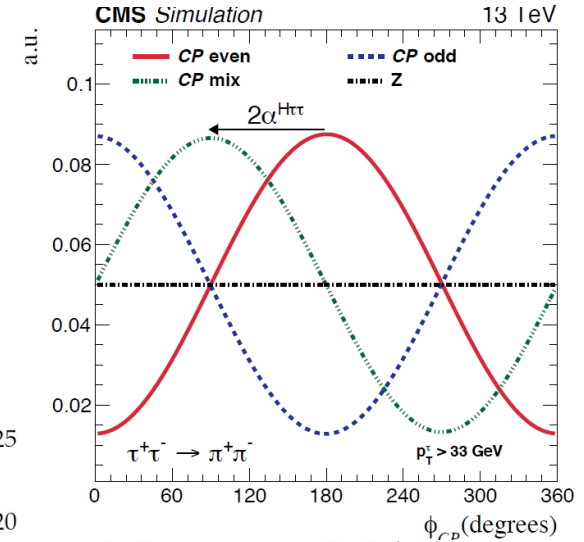
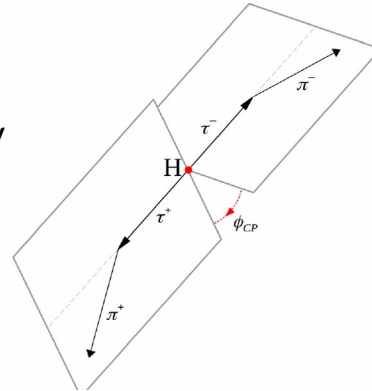
Use decays to  $\tau\tau$  pair to measure CP odd/even mixing in  $H\tau\tau$

Use  $\sim 70\%$  of  $\tau$  BR:

$\tau_h \tau_h, \tau_\mu \tau_h + \tau_e \tau_h$

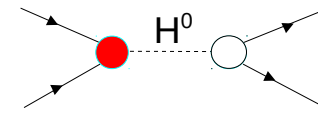
4 reconstruction methods of  $\phi_{CP}$

Pure CP odd excluded at  $3\sigma$



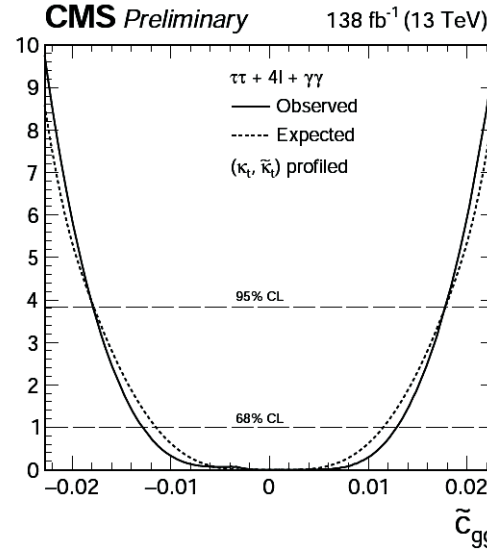
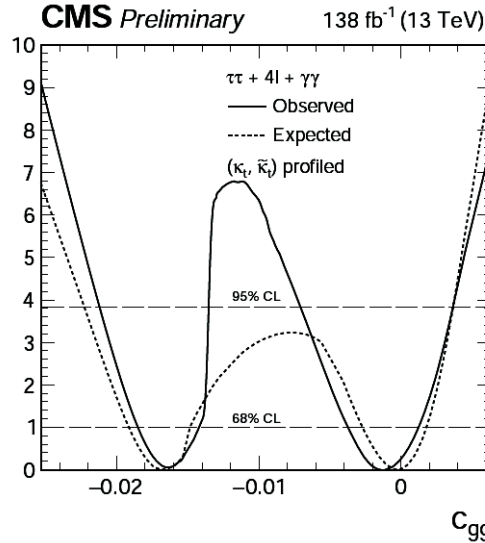
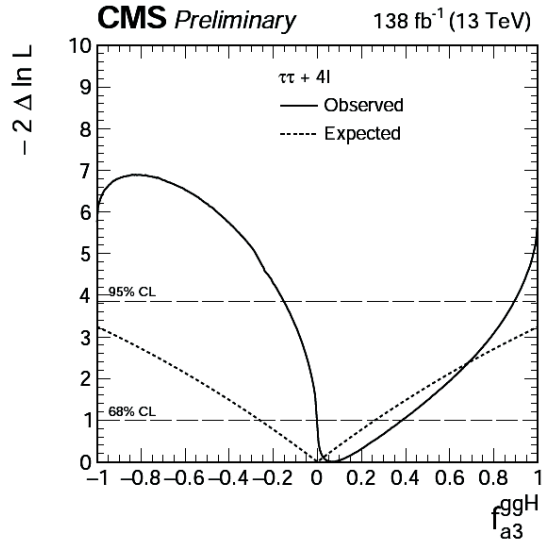


# Hgg : using ggF in 4l, γγ, ττ CMS



HIG-20-007

Phys. Rev D 104 (2021) 052004



Combination of H→4l,  
H→ττ, H→γγ

FULL RUN2

Measure CP sensitive  $fa_{3ggH}$  and  $\mu_{ggH}$

Use events in VBF2J to study ggH category

4 couplings fit simultaneously:

Constrain  $k_{top}$  measured from the tH, ttH process and study  $c_{gg}$

tH & ttH & ggH

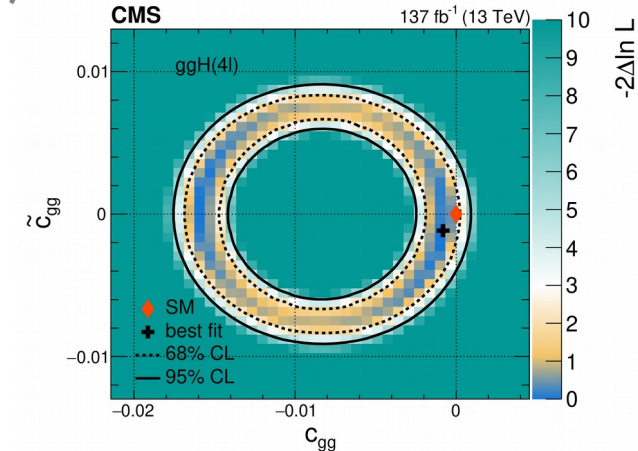
(ttH, tH from: H→4l H→γγ)

Parameter	Scenario		68% CL / (10 <sup>-2</sup> )
$c_{gg}$	Profiled	Observed	$-0.11^{+0.20}_{-0.26} \cup [-1.85, -1.42]$
		Expected	$0.00^{+0.18}_{-0.27} \cup [-1.91, -1.48]$
$\tilde{c}_{gg}$	Profiled	Observed	$0.00 \pm 1.29$
		Expected	$0.00 \pm 1.15$
$c_{gg}$	Fixed	Observed	$-0.08^{+0.07}_{-0.15} \cup [-1.65, -1.54]$
		Expected	$0.00^{+0.06}_{-0.14} \cup [-1.73, -1.50]$
$\tilde{c}_{gg}$	Fixed	Observed	$0.22^{+0.28}_{-0.22} \cup [-0.50, 0.00]$
		Expected	$0.00 \pm 0.45$

$$D_{CP}^{ggH} = \frac{\mathcal{P}_{SM-0-}^{ggH}}{\mathcal{P}_{SM}^{ggH} + \mathcal{P}_{0-}^{ggH}}$$

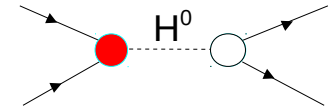
$$c_{gg} = -\frac{1}{2\pi\alpha_S} a_2^{gg}$$

$$\tilde{c}_{gg} = -\frac{1}{2\pi\alpha_S} a_3^{gg}$$





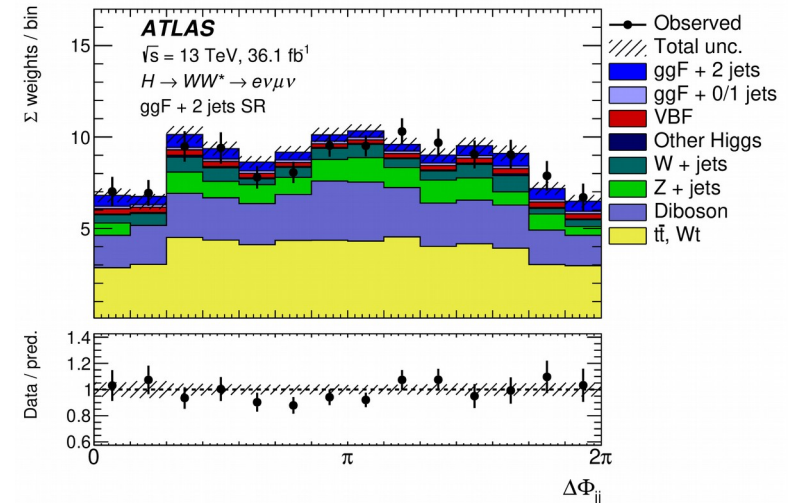
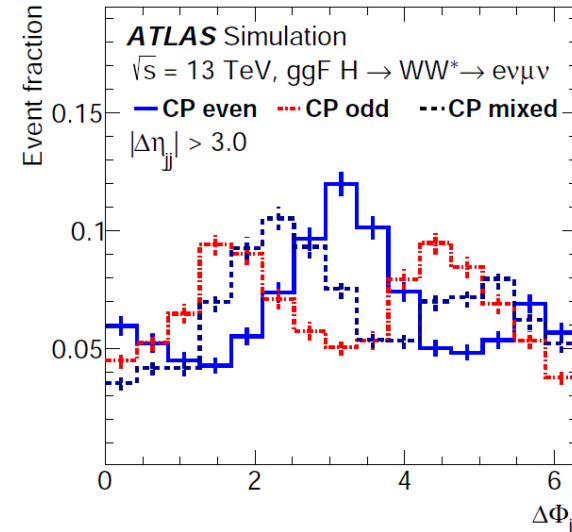
# Hgg in $H \rightarrow WW^* \rightarrow e\nu\mu\nu + jj$ (ATLAS)



- Parameterization in terms of mixing angle and  $\kappa$

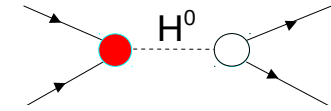
$$\mathcal{L}_0^{\text{loop}} = -\frac{g_{Hgg}}{4} \left( \kappa_{gg} \cos(\alpha) G_{\mu\nu}^a G^{a,\mu\nu} + \kappa_{gg} \sin(\alpha) G_{\mu\nu}^a \tilde{G}^{a,\mu\nu} \right) H$$

- Use **production information**
- Assume HVV -SM like
- BDT to separate signal and background
- 12 categories – BDT and  $\Delta\eta_{jj}$ 
  - CP odd/even separation in  $\Delta\Phi_{jj}$  enhanced in high  $\Delta\eta_{jj}$
- Backgrounds constrained in CR





# Hgg in $H \rightarrow WW^* \rightarrow e\nu\mu\nu + jj$ (ATLAS)



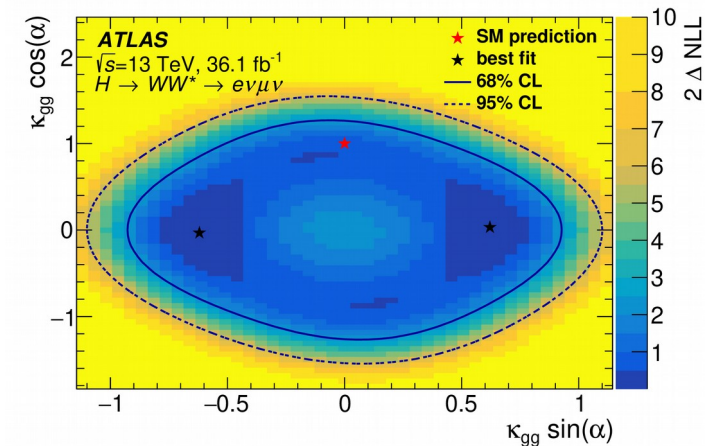
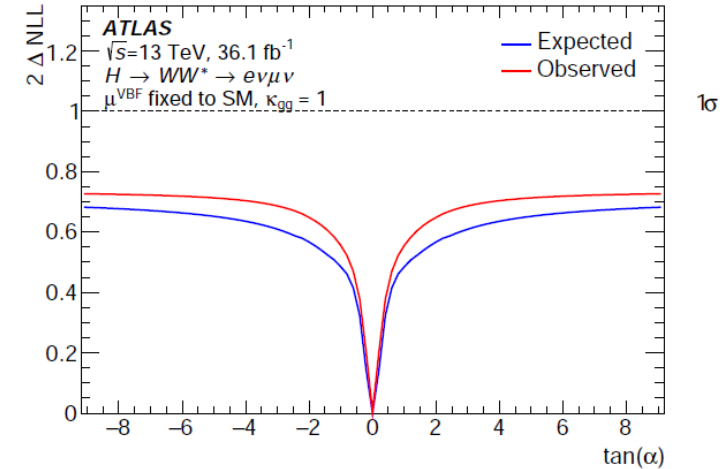
use **BDT** and  $\Delta\Phi_{jj}$  distributions for fitting

Fit  $\Delta\varphi_{jj}$  in SR: 3 BDT X 4  $|\Delta\eta_{jj}|$  regions

2 likelihood fits:

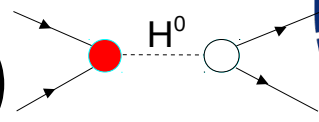
- Shape only considered scan ( BSM rate floated)
- Shape + fix rate to BSM scenario

**1 $\sigma$**  constraints on CP mixing angle

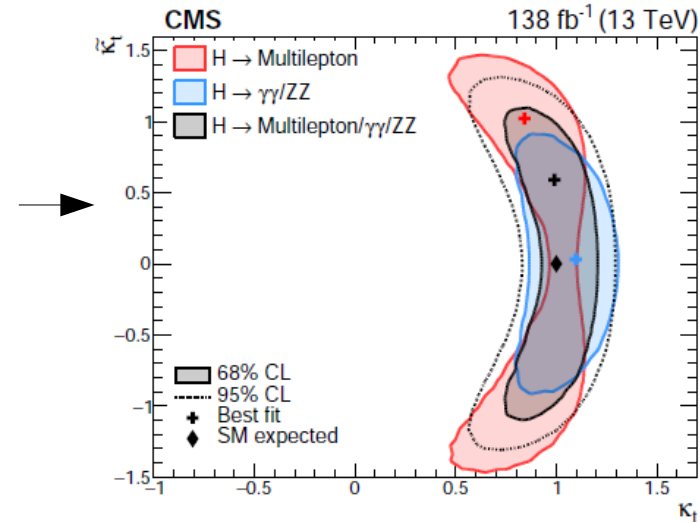
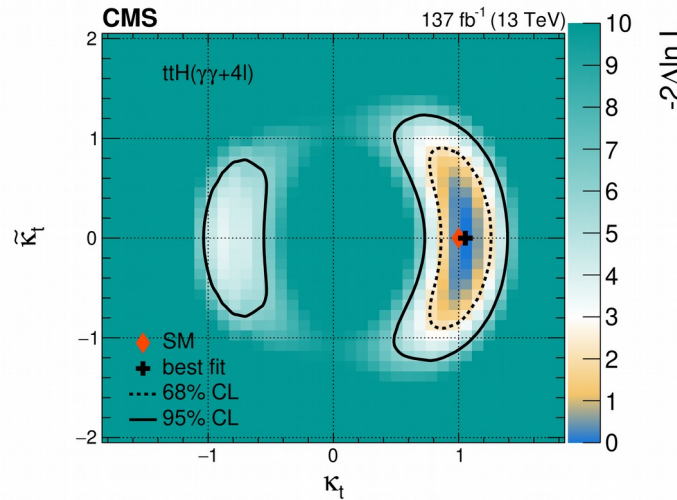
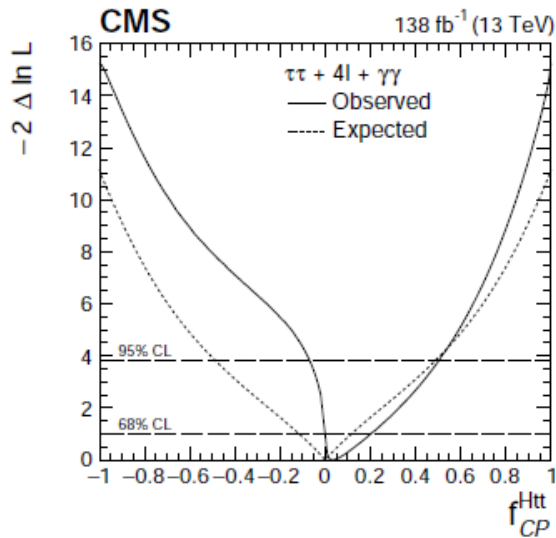




# Yukawa $ttH$ : $ttH(ggH), H \rightarrow 4l/\gamma\gamma/\tau\tau/WW$ (CMS)



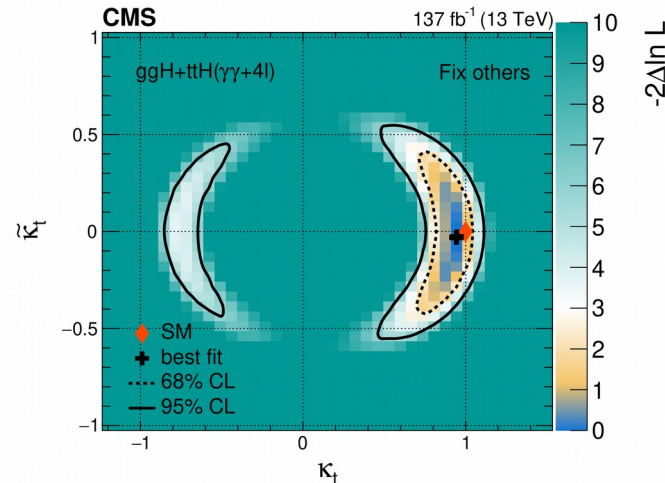
HIG-20-007  
HIG-21-006

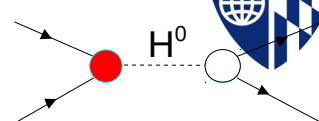


- Measure  $f_{CP}^{Htt}$  in production
  - Combine
    - $H \rightarrow 4l$
    - $H \rightarrow \tau\tau$
    - $H \rightarrow \gamma\gamma$
- + Also multi-lepton ( $\tau\tau/WW$ ) +  $ZZ + \gamma\gamma$

- Combine measurements with uncorrelated  $\mu_s$
- Interpret as top couplings

$$A(Hff) = -\frac{m_f}{v} \bar{\psi}_f (\kappa_f + i \tilde{\kappa}_f \gamma_5) \psi_f$$





# Yukawa ttH with ttH, H → γγ (ATLAS)

Analysis targets **CP mixing angle and  $\kappa_t$**

$$\mathcal{L} = -\frac{m_t}{v} \{ \bar{\psi}_t \kappa_t [\cos(\alpha) + i \sin(\alpha) \gamma_5] \psi_t \} H$$

Classify ttH events in **hadronic** and **leptonic**

- lept: require at least single isolated lepton
- had: at least 2 jets

Use **production information**

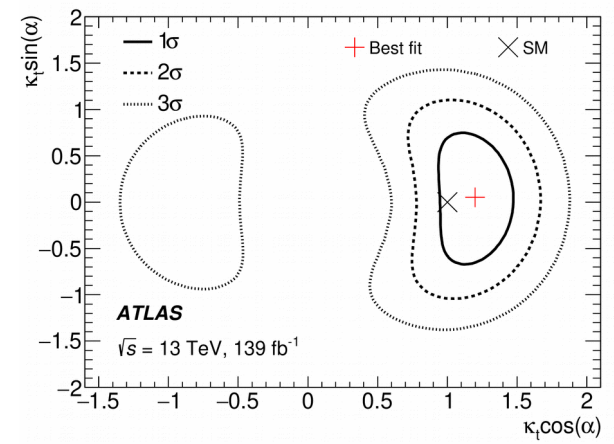
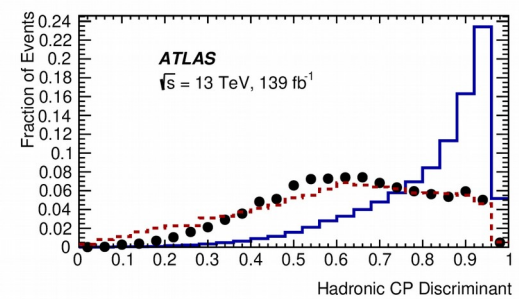
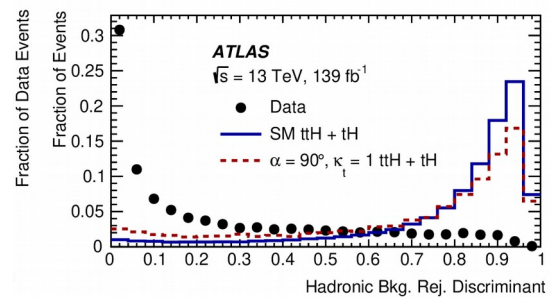
Dedicated signal-background BDT

BDT CP: use top and diphoton system kinematics

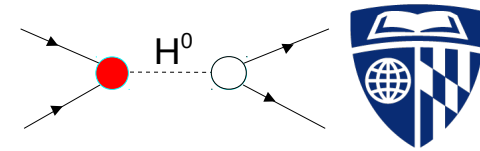
Fit  $m_{\gamma\gamma}$  in overall 20 categories (12Had + 8Lept)

**Tightly constrain pure CP odd ttH coupling**

**Full Run 2**



# Yukawa $t\bar{t}H$ , $H \rightarrow b\bar{b}$ (ATLAS)



HIGG-2020-003

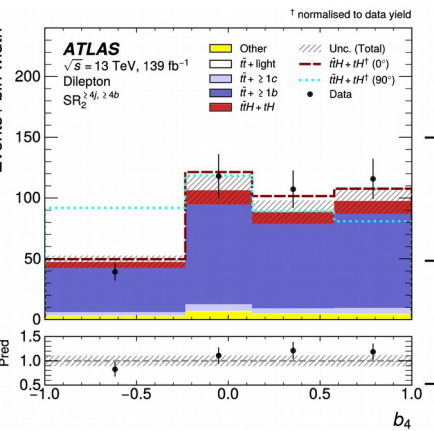
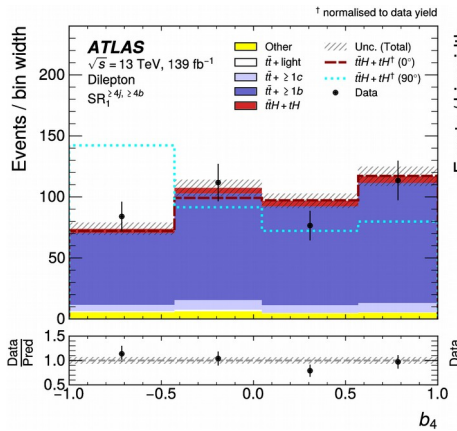
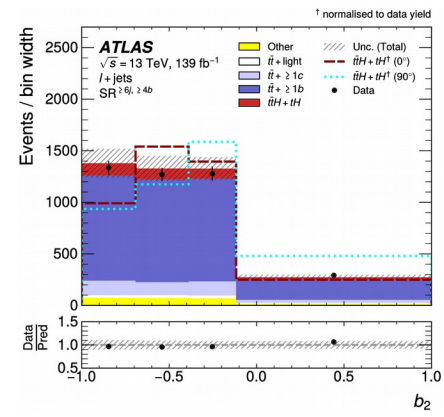
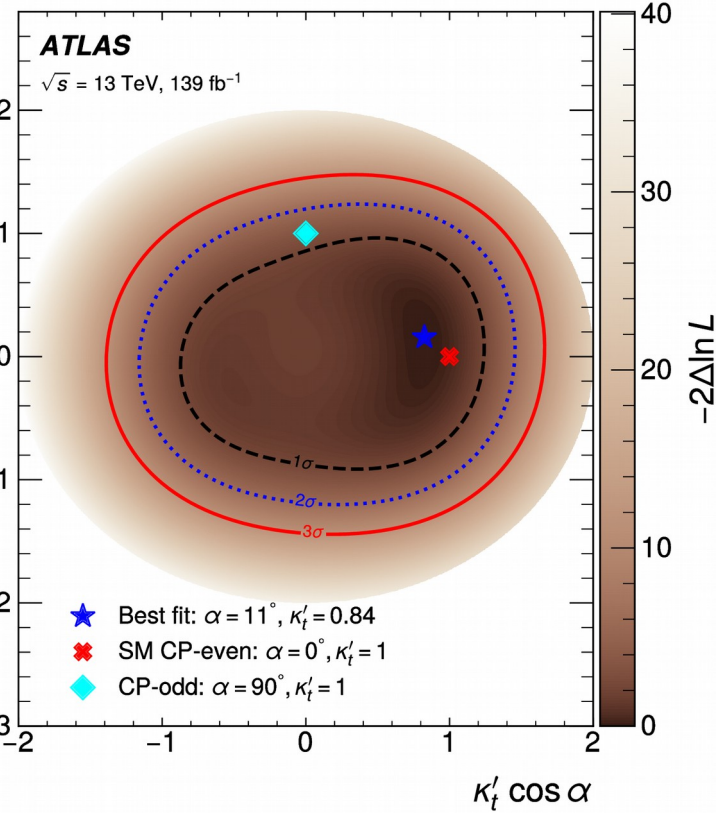
- Dedicated observables
- Requires top 4 vectors
- Background modeling challenging
- **Full Run2**

$$\mathcal{L}_{t\bar{t}H} = -\kappa'_t y_t \phi \bar{\psi}_t (\cos \alpha + i \gamma_5 \sin \alpha) \psi_t$$

$$b_2 = \frac{(\vec{p}_1 \times \hat{z}) \cdot (\vec{p}_2 \times \hat{z})}{|\vec{p}_1| |\vec{p}_2|}$$

$$b_4 = \frac{(\vec{p}_1 \cdot \hat{z})(\vec{p}_2 \cdot \hat{z})}{|\vec{p}_1| |\vec{p}_2|}$$

$\kappa'_t \sin \alpha$

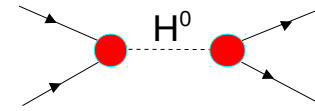






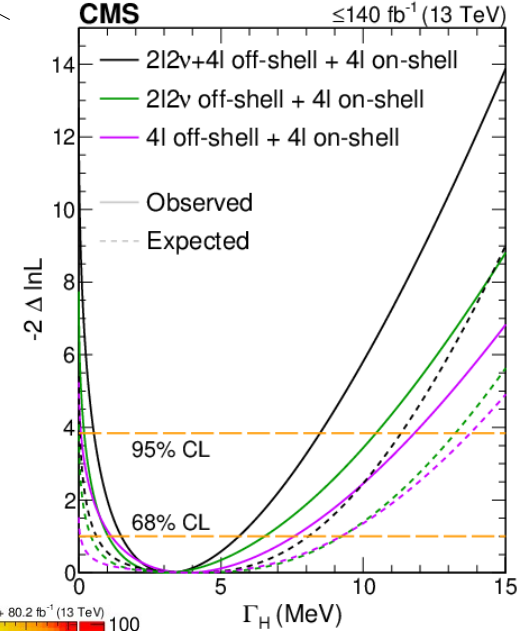
# Off-shell studies in $H \rightarrow 4l + 2l2\nu$ (CMS)

- Use same formalism as on-shell  $H \rightarrow 4l$  AC analysis
- $M_{4l} > 220$  GeV ( $2e2\mu$ ,  $4e$ ,  $4\mu$ )
- Design categories targeting ggF + EW production of the Higgs
- Use ME based observables +  $m_{4l}$
- Consider 1 AC at a time
- Constrain Higgs width + AC
- Combine with  $H \rightarrow ZZ \rightarrow 2l2\nu$

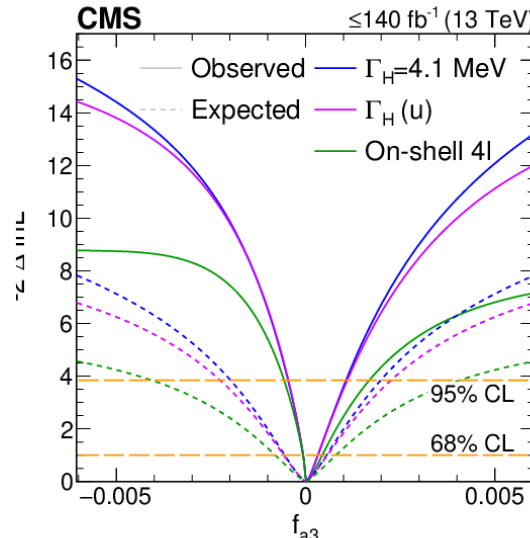
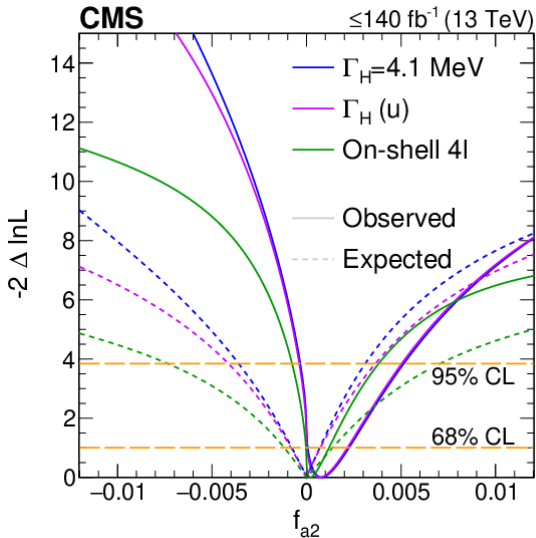


$$\Gamma_H = 3.2^{+2.4}_{-1.7} \text{ MeV}$$

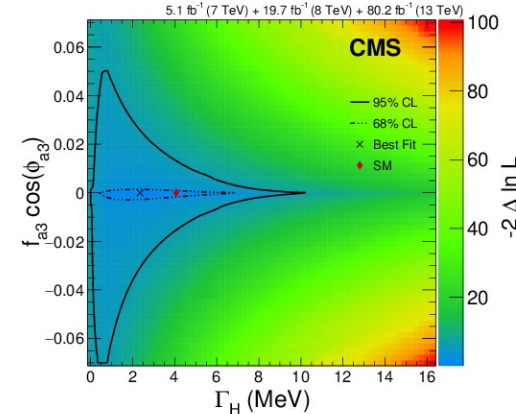
**HIG-21-013**  
**Nat. Phys. 18, 1329–1334 (2022)**



**HIG21-013**



**HIG18-002**



# Summary



- Full on program at the LHC studying Higgs anomalous coupling with dedicated measurements
- Multiple results both from ATLAS and CMS
- Target  $Hff$  /  $HVV$  couplings within EFT framework
- Combinations with multiple final states
- Results consistent with SM
- Full Run2



# Anomalous couplings

- Utilize EFT as a framework for studying the couplings

$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{SM}}^{(4)} + \frac{1}{\Lambda} \sum_k C_k^{(5)} Q_k^{(5)} + \frac{1}{\Lambda^2} \sum_k C_k^{(6)} Q_k^{(6)} + \mathcal{O}\left(\frac{1}{\Lambda^3}\right)$$

- Study low energy approximations of UV complete theory on the SM
- Truncate at certain order + consider cut-off scale
- Study impact of said operator to the couplings



# Effective Lagrangian and couplings (Higgs basis)

(arXiv:2002.09888)

$$\begin{aligned} \mathcal{L}_{\text{hvv}} = & \frac{h}{v} \left[ (1 + \delta c_z) \frac{(g^2 + g'^2)v^2}{4} Z_\mu Z_\mu + c_{zz} \frac{g^2 + g'^2}{4} Z_{\mu\nu} Z_{\mu\nu} + c_{z\Box} g^2 Z_\mu \partial_\nu Z_{\mu\nu} + \tilde{c}_{zz} \frac{g^2 + g'^2}{4} Z_{\mu\nu} \tilde{Z}_{\mu\nu} \right. \\ & + (1 + \delta c_w) \frac{g^2 v^2}{2} W_\mu^+ W_\mu^- + c_{ww} \frac{g^2}{2} W_{\mu\nu}^+ W_{\mu\nu}^- + c_{w\Box} g^2 (W_\mu^- \partial_\nu W_{\mu\nu}^+ + \text{h.c.}) + \tilde{c}_{ww} \frac{g^2}{2} W_{\mu\nu}^+ \tilde{W}_{\mu\nu}^- \\ & + c_{z\gamma} \frac{e\sqrt{g^2 + g'^2}}{2} Z_{\mu\nu} A_{\mu\nu} + \tilde{c}_{z\gamma} \frac{e\sqrt{g^2 + g'^2}}{2} Z_{\mu\nu} \tilde{A}_{\mu\nu} + c_{\gamma\Box} g g' Z_\mu \partial_\nu A_{\mu\nu} \\ & \left. + c_{\gamma\gamma} \frac{e^2}{4} A_{\mu\nu} A_{\mu\nu} + \tilde{c}_{\gamma\gamma} \frac{e^2}{4} A_{\mu\nu} \tilde{A}_{\mu\nu} + c_{gg} \frac{g_s^2}{4} G_{\mu\nu}^a G_{\mu\nu}^a + \tilde{c}_{gg} \frac{g_s^2}{4} G_{\mu\nu}^a \tilde{G}_{\mu\nu}^a \right], \end{aligned}$$

$$\mathcal{L}_{hff} = -\frac{m_f}{v} \bar{\psi}_f (\kappa_f + i \tilde{\kappa}_f \gamma_5) \psi_f h$$



$$\begin{aligned} \delta c_z &= \frac{1}{2} g_1^{ZZ} - 1, & c_{zz} &= -\frac{2s_w^2 c_w^2}{e^2} g_2^{ZZ}, & c_{z\Box} &= \frac{M_Z^2 s_w^2}{e^2} \frac{\kappa_1^{ZZ}}{(\Lambda_1^{ZZ})^2}, & \tilde{c}_{zz} &= -\frac{2s_w^2 c_w^2}{e^2} g_4^{ZZ}, \\ \delta c_w &= \frac{1}{2} g_1^{WW} - 1, & c_{ww} &= -\frac{2s_w^2}{e^2} g_2^{WW}, & c_{w\Box} &= \frac{M_W^2 s_w^2}{e^2} \frac{\kappa_1^{WW}}{(\Lambda_1^{WW})^2}, & \tilde{c}_{ww} &= -\frac{2s_w^2}{e^2} g_4^{WW}, \\ c_{z\gamma} &= -\frac{2s_w c_w}{e^2} g_2^{Z\gamma}, & \tilde{c}_{z\gamma} &= -\frac{2s_w c_w}{e^2} g_4^{Z\gamma}, & c_{\gamma\Box} &= \frac{s_w c_w}{e^2} \frac{M_Z^2}{(\Lambda_1^{Z\gamma})^2} \kappa_2^{Z\gamma}, \\ c_{\gamma\gamma} &= -\frac{2}{e^2} g_2^{\gamma\gamma}, & \tilde{c}_{\gamma\gamma} &= -\frac{2}{e^2} g_4^{\gamma\gamma}, & c_{gg} &= -\frac{2}{g_s^2} g_2^{gg}, & \tilde{c}_{gg} &= -\frac{2}{g_s^2} g_4^{gg}. \end{aligned}$$

**Necessary to consider impact of AC in  $\Gamma$ :**

$$\sigma(i \rightarrow H \rightarrow f) \propto \frac{(\sum \alpha_{jk} g_j g_k) (\sum \alpha_{lm} g_l g_m)}{\Gamma_{\text{tot}}}$$

$$\Gamma_{\text{tot}} = \sum_f \Gamma_f = \Gamma_{\text{tot}}^{\text{SM}} \times \sum_f \left( \frac{\Gamma_f^{\text{SM}}}{\Gamma_{\text{tot}}^{\text{SM}}} \times \frac{\Gamma_f}{\Gamma_f^{\text{SM}}} \right) = \Gamma_{\text{tot}}^{\text{SM}} \times \sum_f (\mathcal{B}_f^{\text{SM}} \times \mathcal{R}_f(\tilde{g}_j))$$