Challenges of entanglement measurement in $t\bar{t}$ final states

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on behalf of the CMS Collaboration

SM@LHC2023: Standard Model at the LHC Workshop 2023

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Overview

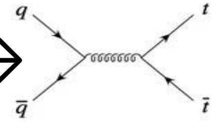
- Overview of top quark physics & spin correlations
- 2016 measurement of spin density matrix
- Entanglement
- Present Challenges for Entanglement

Top Quark Physics

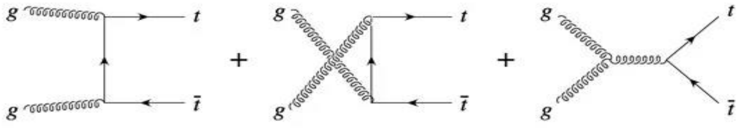
- Top quark is the heaviest fundamental particle discovered thus far: m_t = 173.34 +/- 0.76 GeV [arxiv:1403.4427]
- Unique:

$$\underbrace{\frac{1}{m_t}}_{\substack{\text{production}\\ 10^{-27}\ \text{s}}} < \underbrace{\frac{1}{\Gamma_t}}_{\substack{\text{lifetime}\\ 10^{-25}\ \text{s}}} < \underbrace{\frac{1}{\Lambda_{QCD}}}_{\substack{\text{hadronization}\\ 10^{-24}\ \text{s}}} < \underbrace{\frac{m_t}{\Lambda^2}}_{\substack{\text{spin-flip}\\ 10^{-21}\ \text{s}}}$$

qq →



gg →



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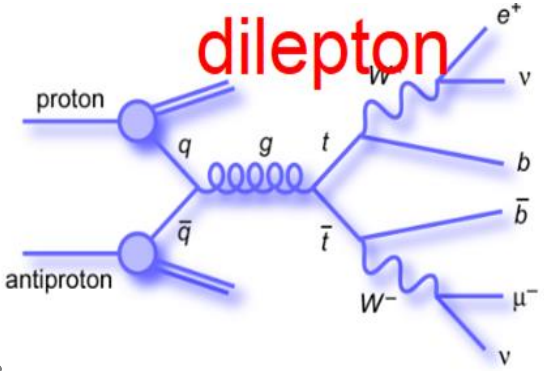
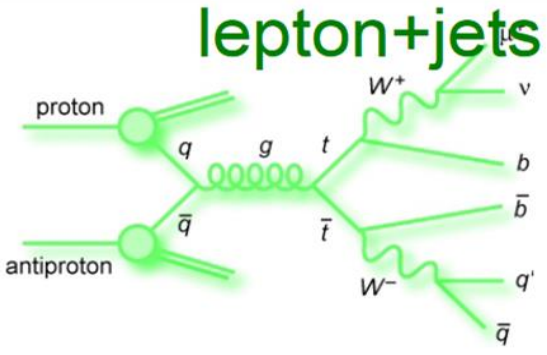
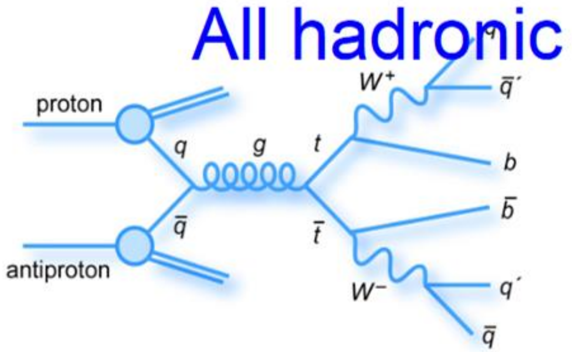
- Allows for probing of bare-quark physics
  - Inaccessible realm of physics except for asymptotic freedom!
- LHC is a top quark factory (100m+ thus far)
- Spin information is accessed "best" in leptonic decays of W



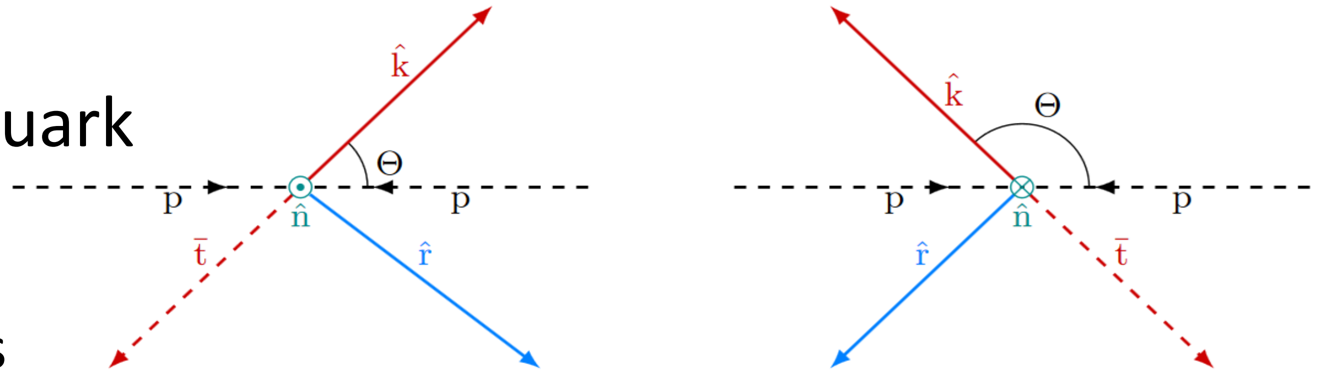




Top Quark Spin Correlations

[Phys. Rev. D 100, 072002]

- Measuring spin directly on top quark is inaccessible
  - Preserved in decay products → measure angle between spin axis and lepton in parent top quark rest frame
  - Measured in the helicity basis for symmetry reasons → sensitive to BSM phenomenon!
- Dependent on:
  - production mode
  - scattering angle of the top quark
  - Invariant mass of the top quark and antiquark system



$$R \propto \tilde{A}\mathbb{1}\otimes\mathbb{1} + \tilde{B}_i^{+}\sigma^i\otimes\mathbb{1} + \tilde{B}_i^{-}\mathbb{1}\otimes\sigma^i + \tilde{C}_{ij}\sigma^i\otimes\sigma^j$$

$$\frac{1}{\Gamma}\frac{\mathrm{d}\Gamma}{\mathrm{d}\cos\chi_a} = \frac{1}{2}(1+\kappa_a\cos\chi_a)$$

$$\frac{1}{\sigma}\frac{\mathrm{d}^4\sigma}{\mathrm{d}\Omega_1\,\mathrm{d}\Omega_2} = \frac{1}{(4\pi)^2}\left(1+\mathbf{B_1}\cdot\hat{\ell}_1+\mathbf{B_2}\cdot\hat{\ell}_2-\hat{\ell}_1\cdot\mathrm{C}\cdot\hat{\ell}_2\right)$$

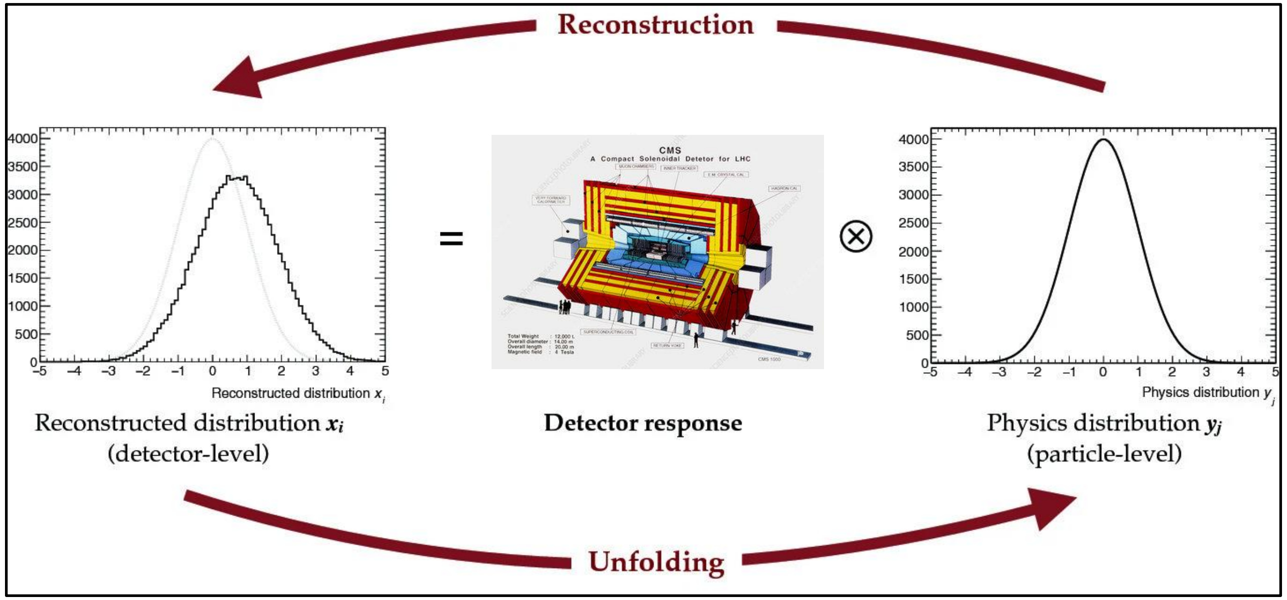
CMS Spin Correlation Measurement using 2016 Data

$L_{int} = 35.9\ fb^{-1}$

Phys. Rev. D 100, 072002

Analysis Strategy

- Unfold to parton level for 10 polarizations, 9 correlations, D, and lab frame observables
  - Polarizations measured along helicity basis and k^*, r^* axes
  - Measured diagonal of spin correlation matrix and off-diagonal sums and differences
  - Measured trace of spin correlation matrix, D, via $\cos\phi$
  - Measure $|\Delta\phi_{l\bar{l}}|$ and $\cos\phi$ in the lab frame – indirectly related to spin correlations



Analysis Strategy

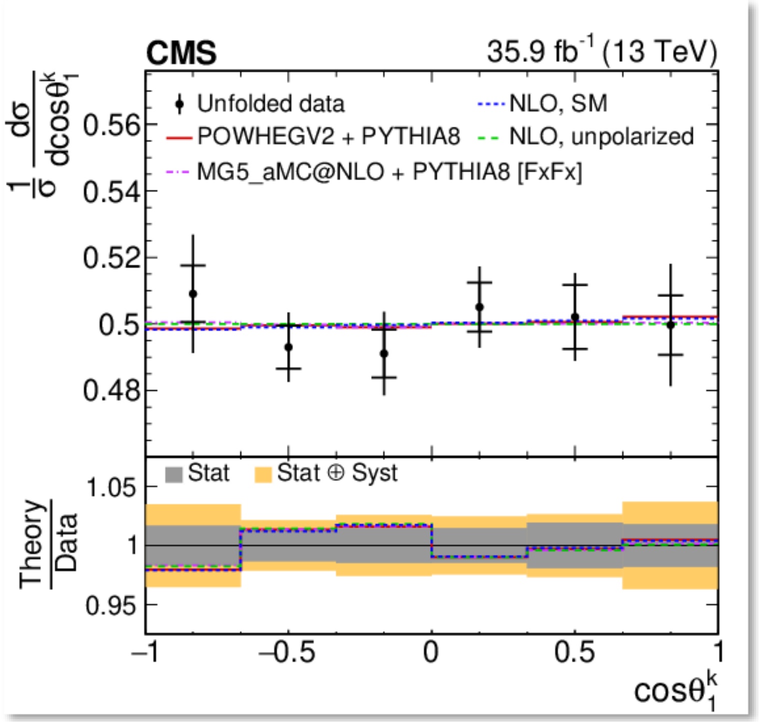
- Compute statistical and systematic correlation matrices between all bins of unfolded distributions
  - Unfolding introduces correlations between bins of the same observable
  - Correlations between observables may exist, e.g., C_{kk}, C_{rr}, C_{nn} and D
  - Bootstrap method is used for computing correlations

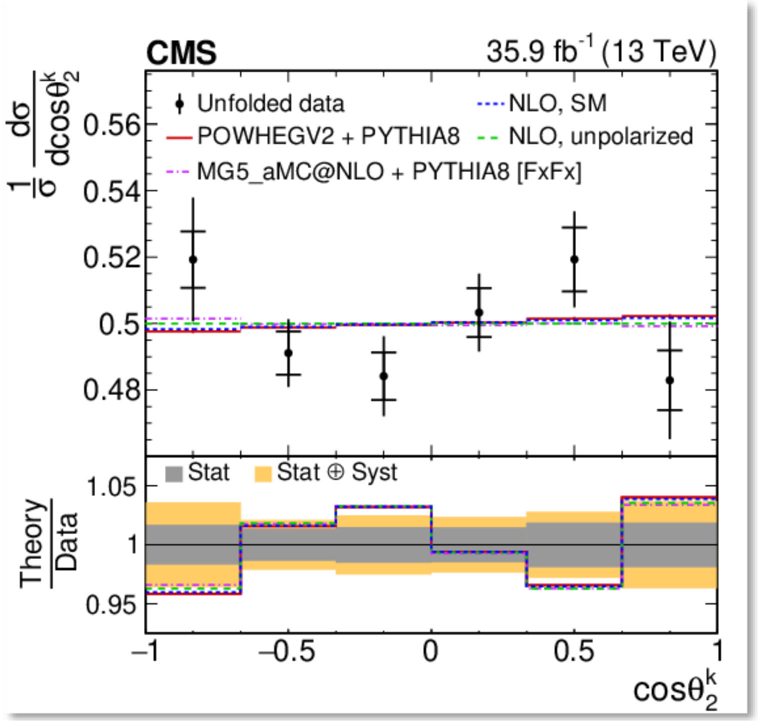
- Correlations are very important when performing a fit/interpretation
  - Uncorrelated is a very poor assumption

Spin Density Matrix Polarizations - 2016 Results

Phys. Rev. D 100, 072002

- All consistent with expectation of SM ≈ 0





Spin Density Matrix Correlations

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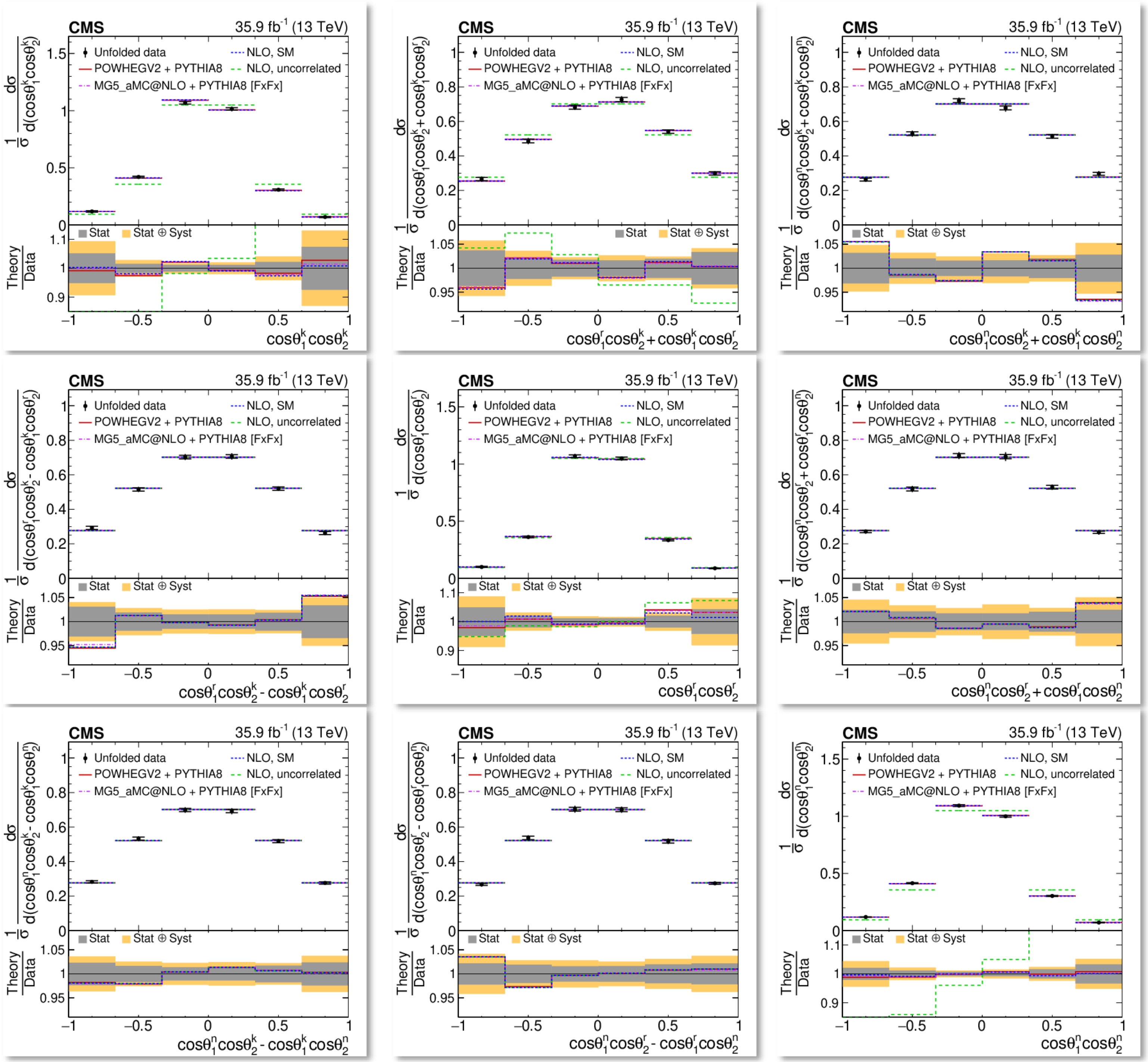
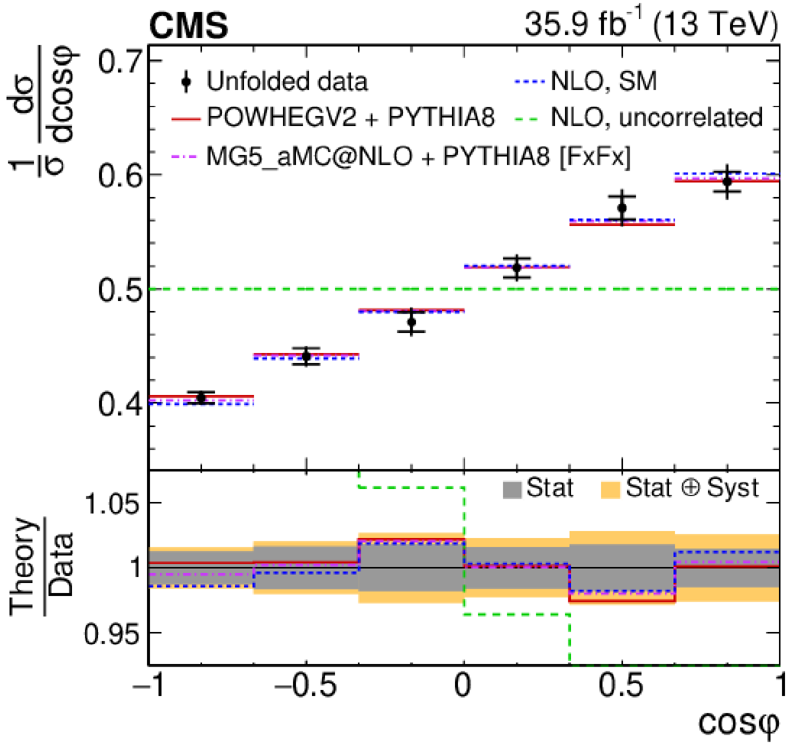
2016 Results

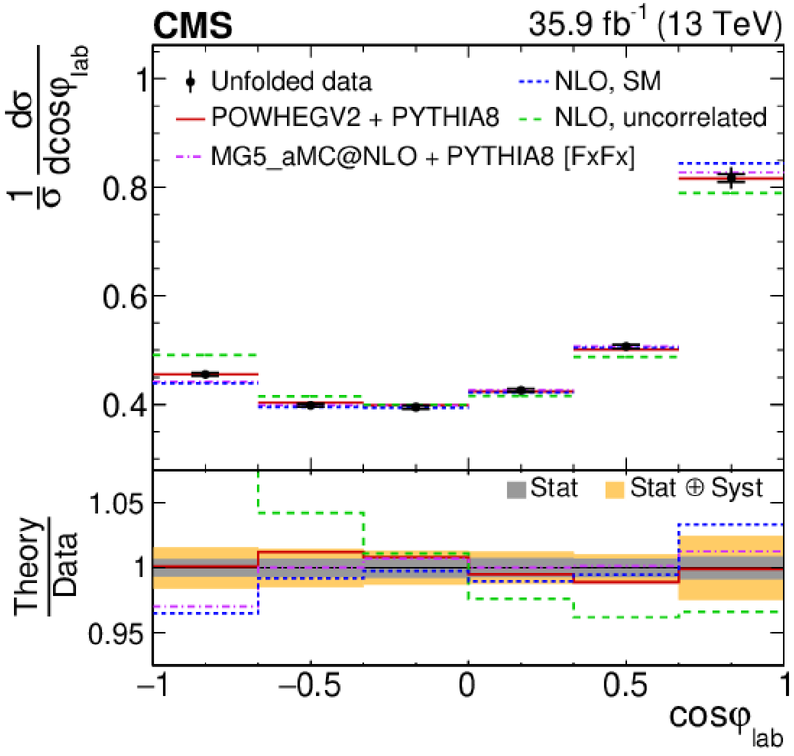


Spin Correlation Matrix Trace & Lab Observables

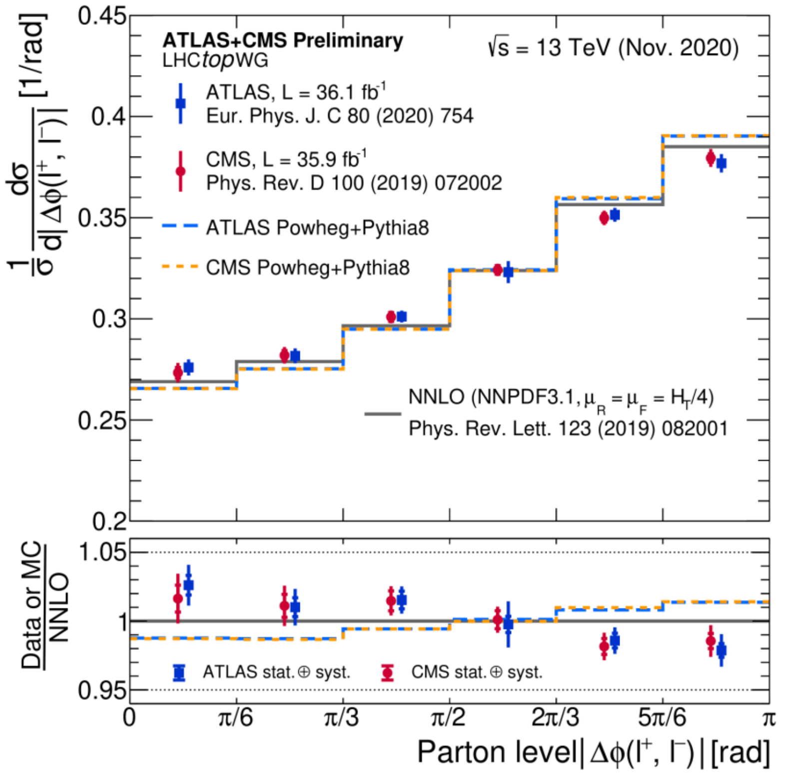
[Phys. Rev. D 100, 072002]

2016 Results



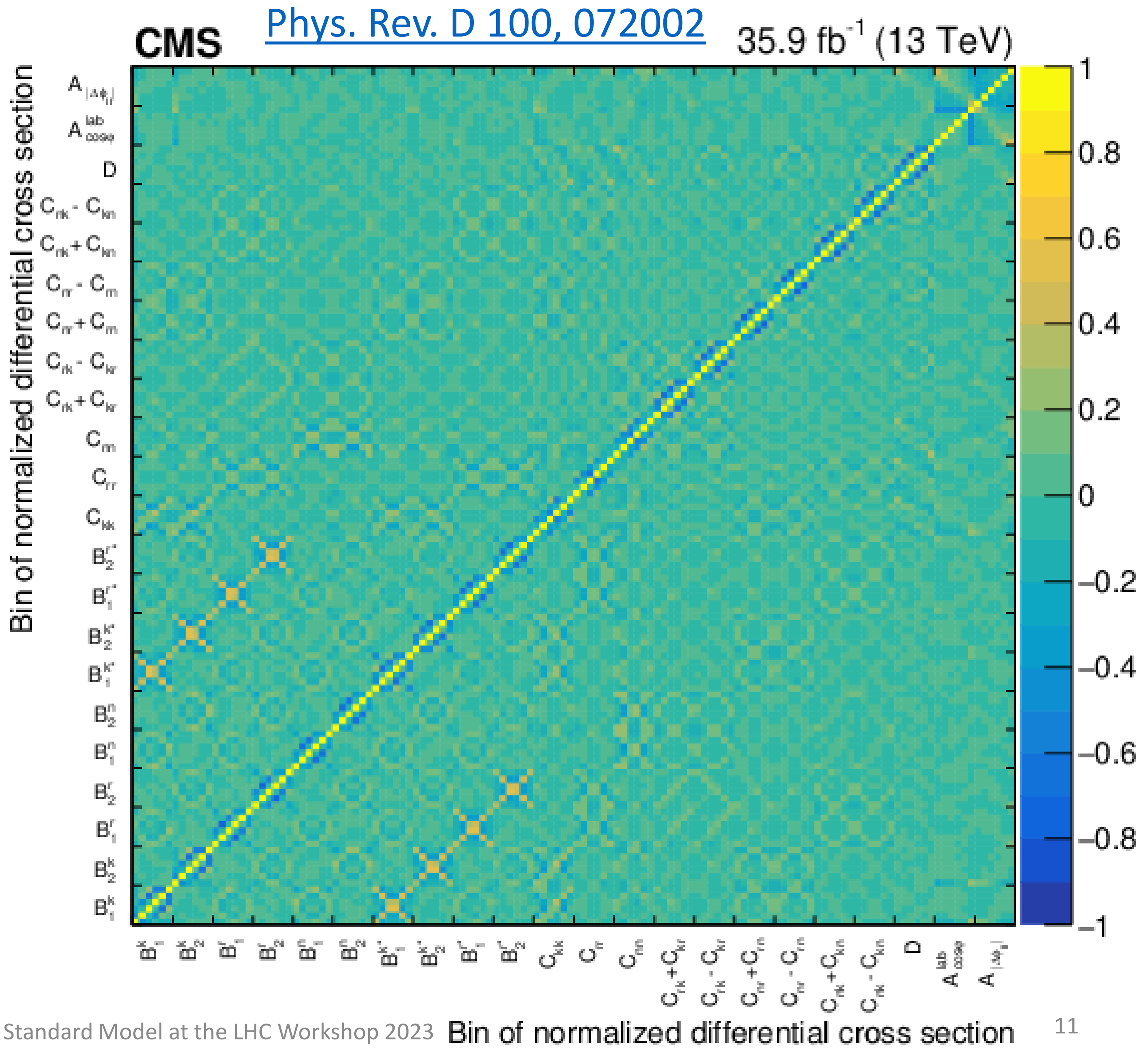


[LHC Top Working Group]



Correlations Between All Observables

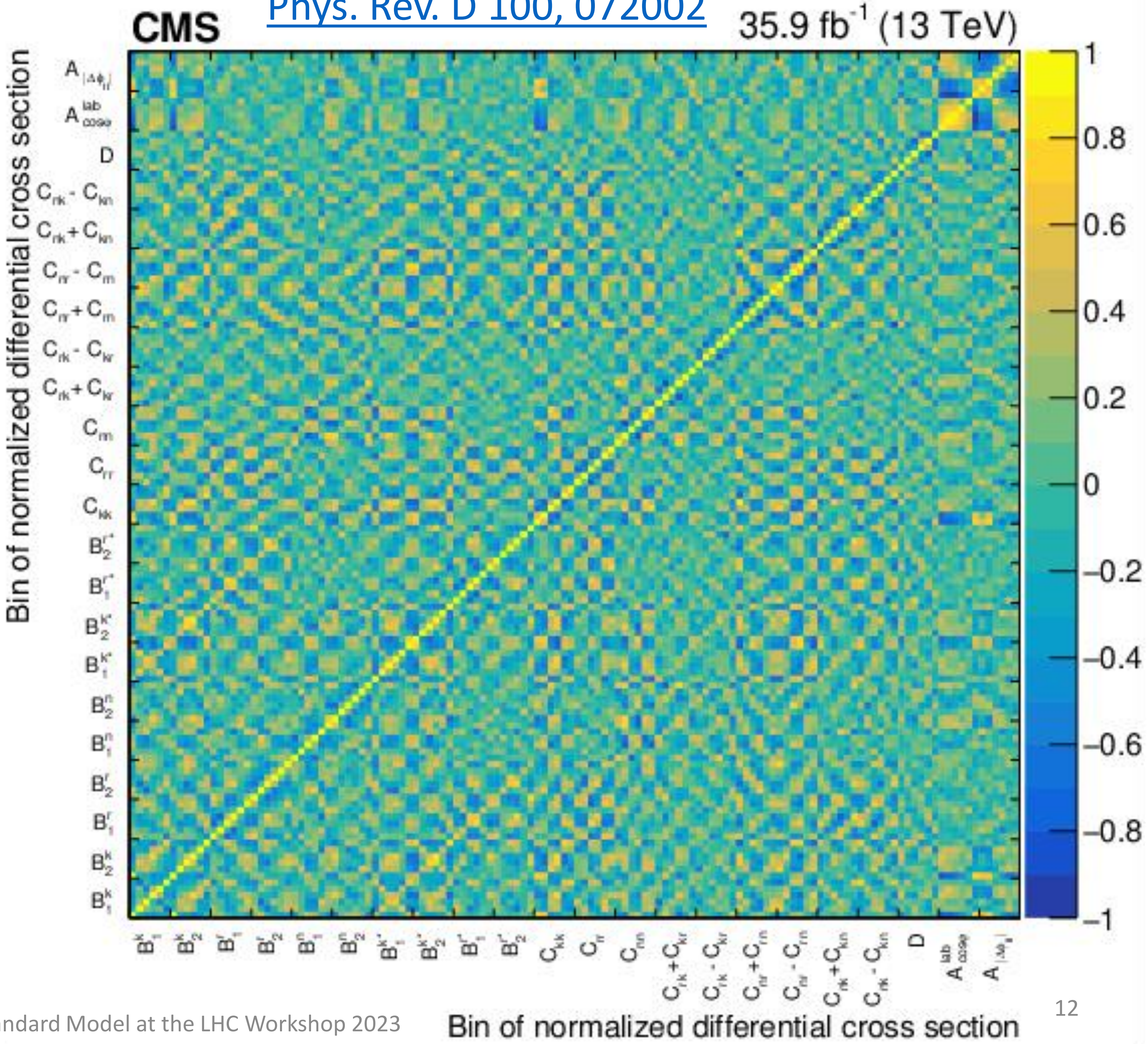
- Statistical correlations between bins of normalized distributions
- Mostly 0 between distributions
- Within distribution can be highly correlated due to unfolding



Correlations Between All Observables

- Systematic correlations between bins of normalized distributions
- Clearly uncorrelated is a poor assumption

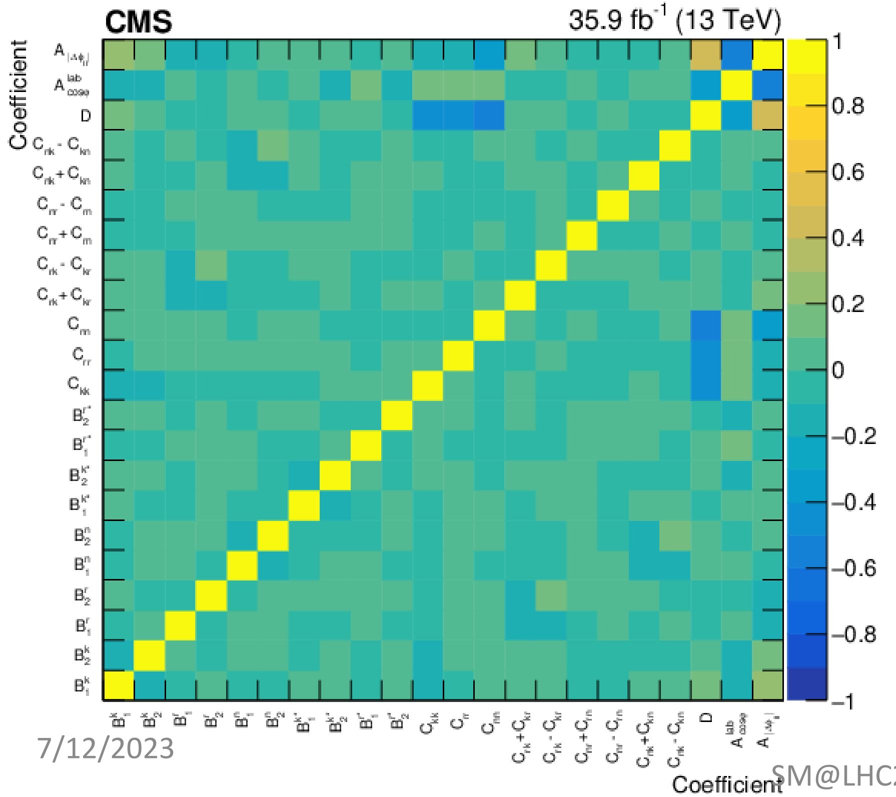
Phys. Rev. D 100, 072002

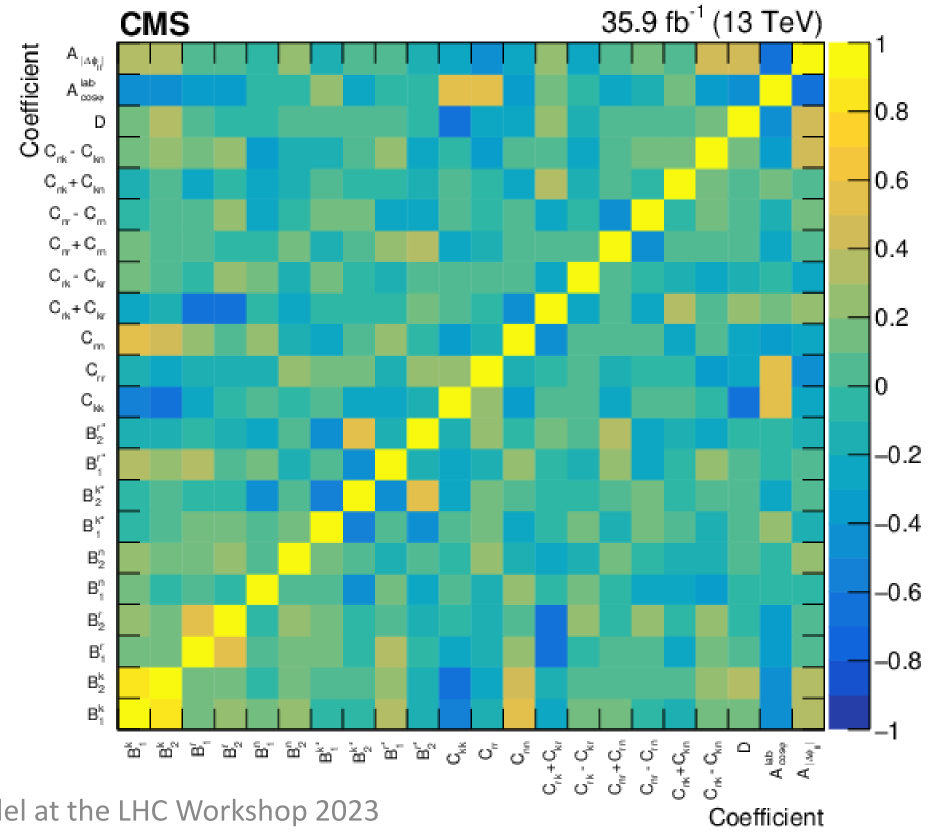


Correlations between all observables

Phys. Rev. D 100, 072002

- Statistical (left) and Systematic (right) correlations between spin coefficients
- Note statistical correlations between diagonal elements of spin correlation matrix and D





Entanglement

Entanglement is an exciting new probe

- RHIC: First-ever observation of quantum interference between dissimilar particles offers new approach for mapping distribution of gluons in atomic nuclei — and potentially more [1]
- BaBar: Used entangled neutral B mesons to measure T reversal violation [2]
- ATLAS & CMS working on entanglement in $t\bar{t}$
- TOP22 & TOP23: lots of discussions...
- LHCtopWG: Multiple talks for entanglement, Bell's inequality, discord & steering
- EFT: Full quantum tomography very sensitive to BSM couplings [3]

[1] DOI: 10.1126/sciadv.abq3903
[2] Phys. Rev. Lett. 109, 211801
[3] arxiv:2203.05619

Quantum Tomography

- Spin correlations are highly phase-space dependent
- Higher dimensional operators are sensitive to this phase-space dependence

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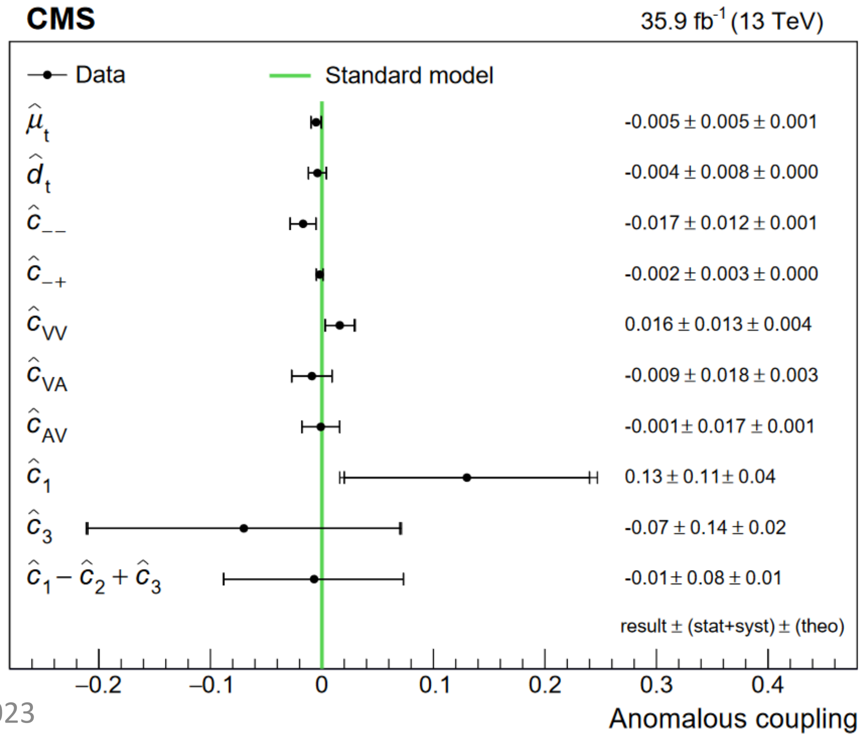
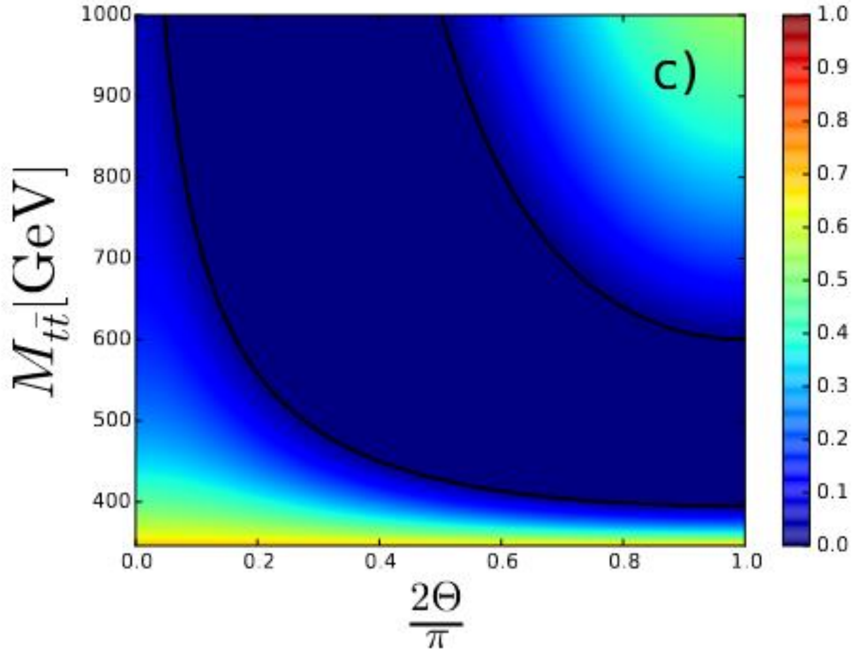
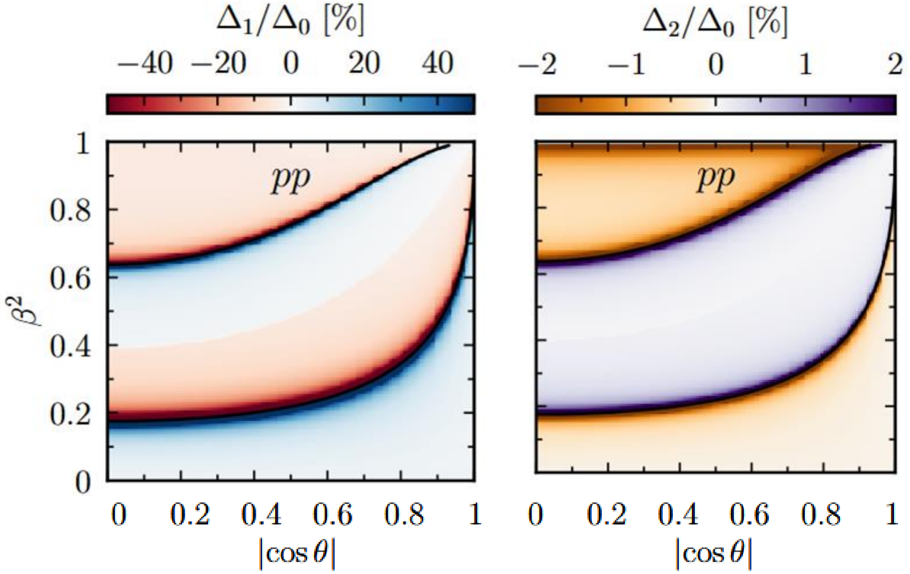




[arxiv:2003.02280]



[arxiv:2203.05619]

How to probe entanglement

- What does it mean to be entangled? Nonseparable!

$$|\psi\rangle = |a\rangle_A \otimes |b\rangle_B$$

- For pure states this is easy → measure entanglement entropy

- At the LHC top quarks are produced in a mixed state and thus can be represented as a density operator

$$\rho = \frac{I_4 + \Sigma_i \left(B_i^+ \sigma^i \otimes I_2 + B_i^- I_2 \otimes \sigma^i \right) + \Sigma_{i,j} C_{ij} \sigma^i \otimes \sigma^j}{4}$$

- Hard to show density operator is separable but you can “easily” show it is non-separable → entangled!

Peres, quant-ph/9604005
Horodecki, quant-ph/9703004

Peres-Horodecki Criterion

- If a state is separable →partial transpose is valid state
    - Unit trace, Hermitian, nonnegative
- State is entangled if the above conditions don't hold
- After some messy algebra a sufficient condition for entanglement is reached: [arxiv:2003.02280]

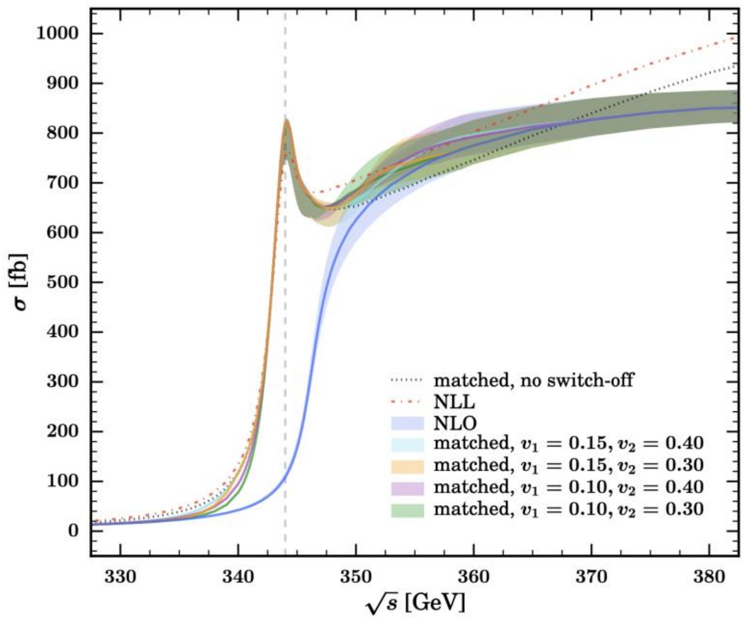
$$\Delta \equiv -C_{33} + |C_{11} + C_{22}| - 1 > 0 \longrightarrow -\,\mathrm{tr}[\mathbf{C}] > 1 \longrightarrow D < -1/3$$

$$D = \frac{\mathrm{tr}[\mathbf{C}]}{3} \qquad \frac{1}{\sigma}\frac{d\sigma}{d\cos\varphi} = \frac{1}{2}(1 - D\cos\varphi)$$

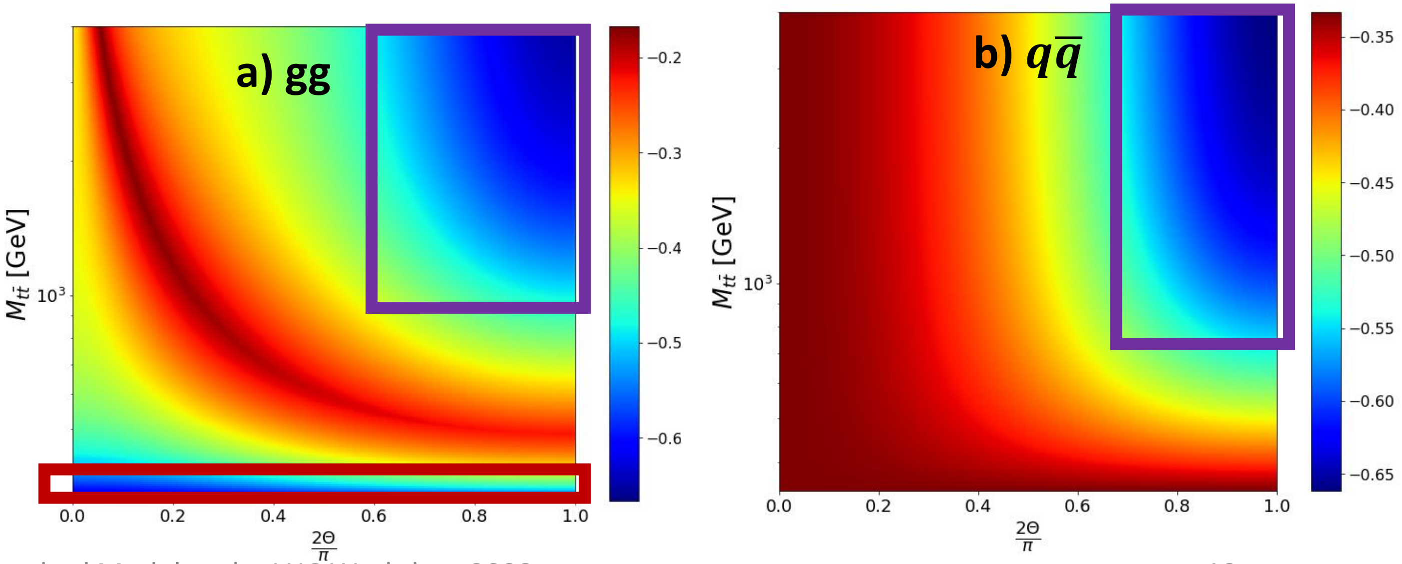
- **Measure D to access entanglement information!**

Challenges Facing Observation of Entanglement in $t\bar{t}$ Final States

- 2 possible regions – **threshold** or **high $m_{t\bar{t}}$ & scattering angle θ**
- **Threshold** – large statistics but dominated by systematics
  - Bound-state effects aren't negligible
  - Limited resolution for unfolding
- **Boosted** – low statistics
  - Cleaner in terms of systematics
  - Need more stat. for observation
  - 3D unfolding



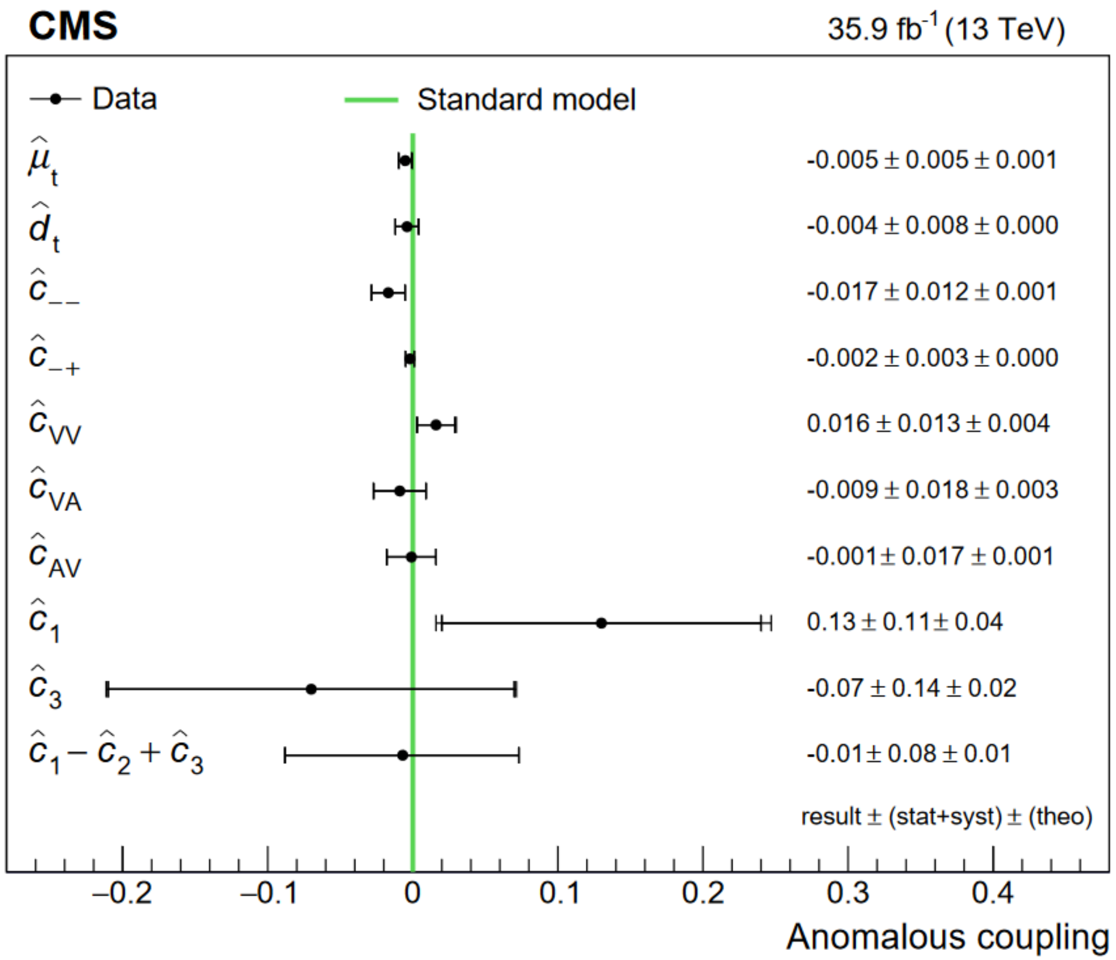
Bach et. al. (e+e- Whizard) 1712.02220



Summary

- Full spin density matrix measurement has been performed
- Agrees thus far with SM expectation
- Spin correlation/information is very useful
  - Constrain EFT operators [Phys. Rev. D 100, 072002]
  - Exclusion limits on stop production [CMS-FTR-18-034]
  - And more…
- Plenty of new exciting ideas
  - Quantum tomography [arxiv:2003.02280]
  - Discord [arxiv:2209.03969]
  - Bell's Inequality[arxiv:2102.11883, arxiv:2110.10112]
  - Entanglement [arxiv:2003.02280, arxiv:2110.10112]

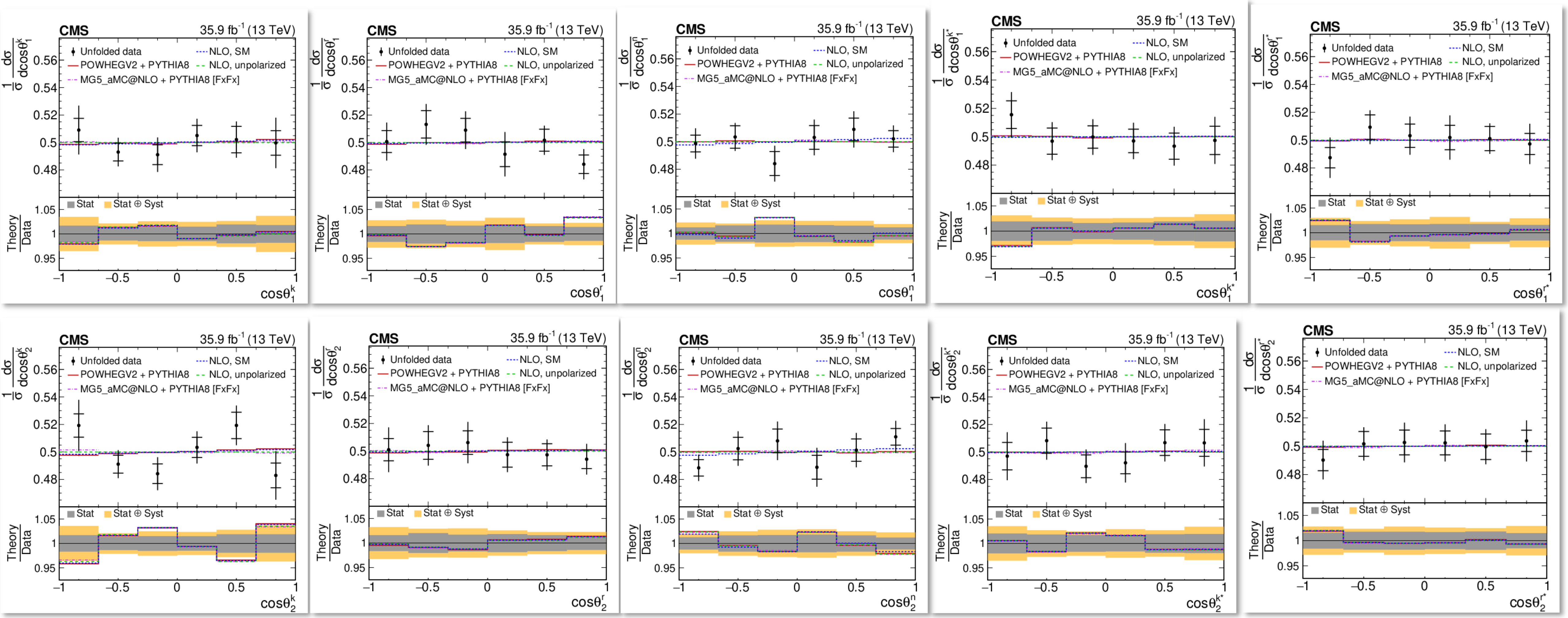
CMS-TOP-18-006



Backup

Spin Density Matrix Polarizations - 2016 Results

Phys. Rev. D 100, 072002

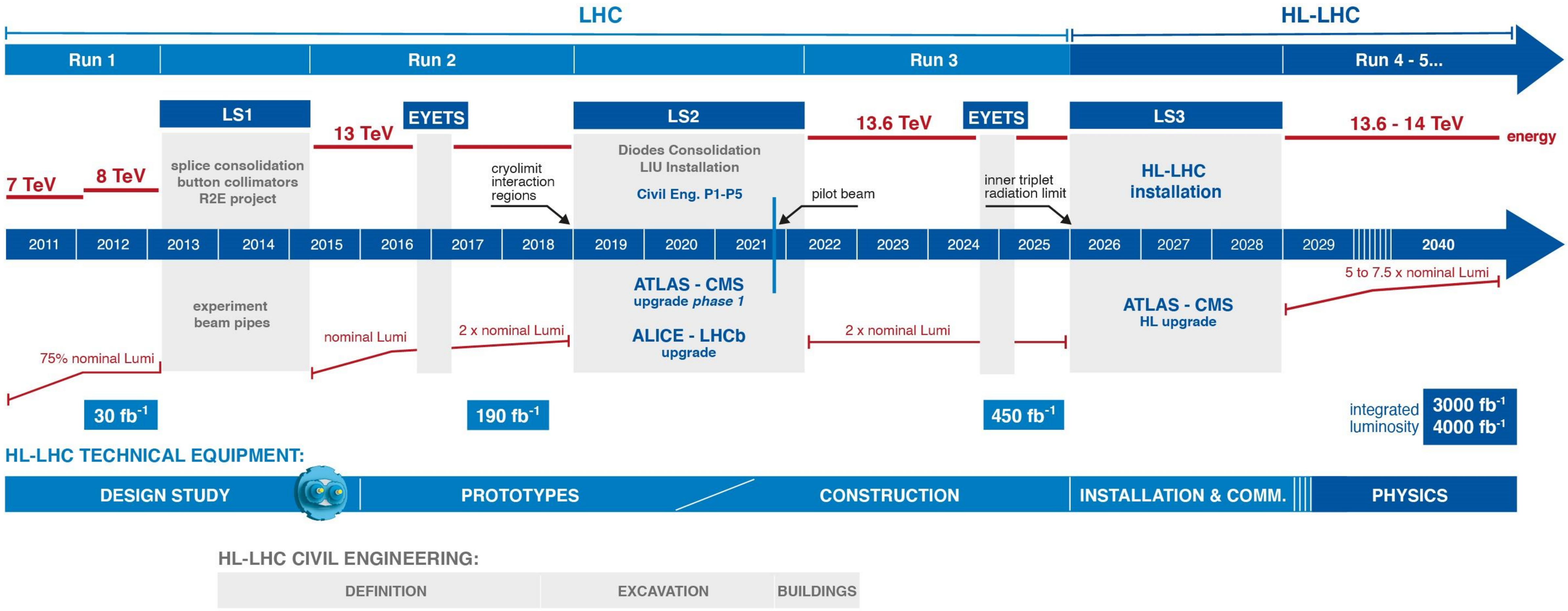


High Luminosity LHC (HL-LHC)

$L_{\text{int}} \cong 3000\ fb^{-1}$

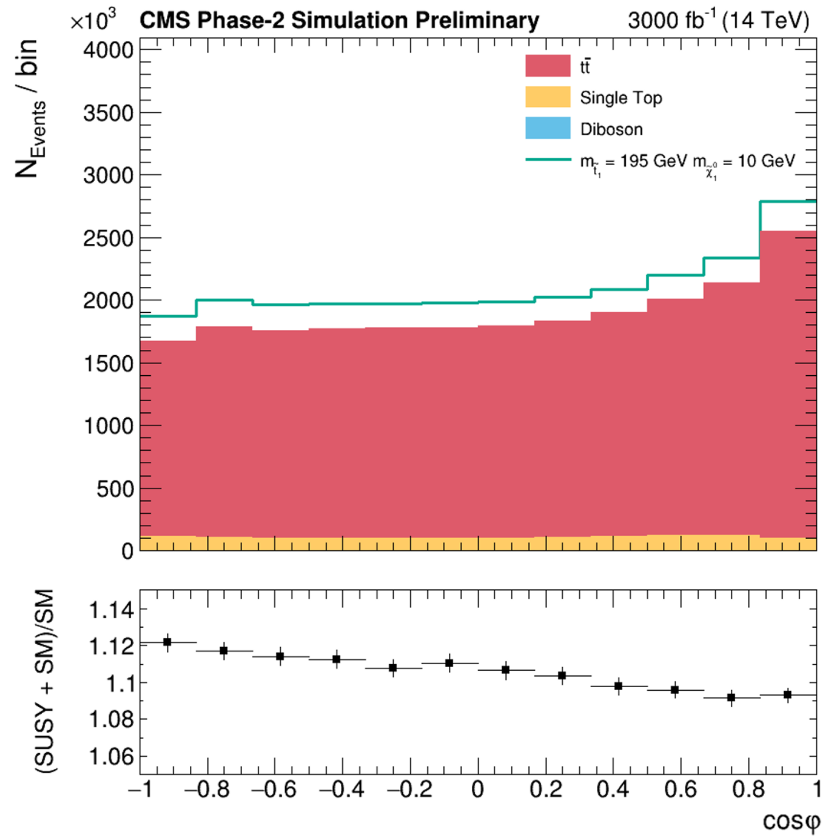
CMS-FTR-18-034

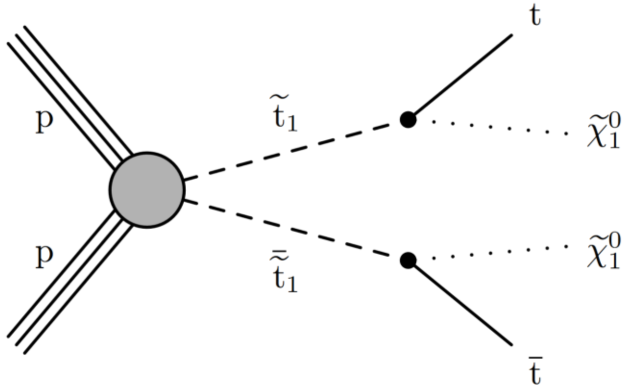
LHC / HL-LHC Plan

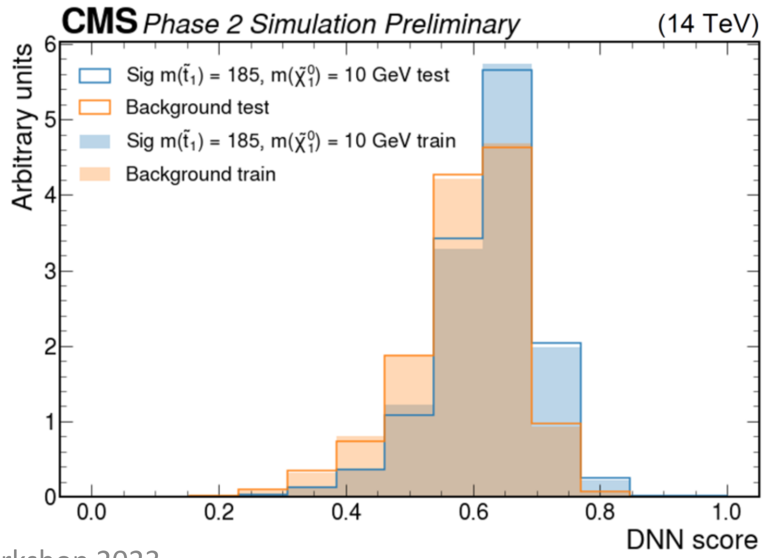


Projection Study [CMS-FTR-18-034]

- Project impact of HL-LHC on spin correlations and limits on stop (supersymmetric top quarks) production
- Uses ellipse reconstruction algorithm [arxiv:1305.1878]
- 14 TeV, 3 ab^{-1}
- DNN trained on spin correlation variables

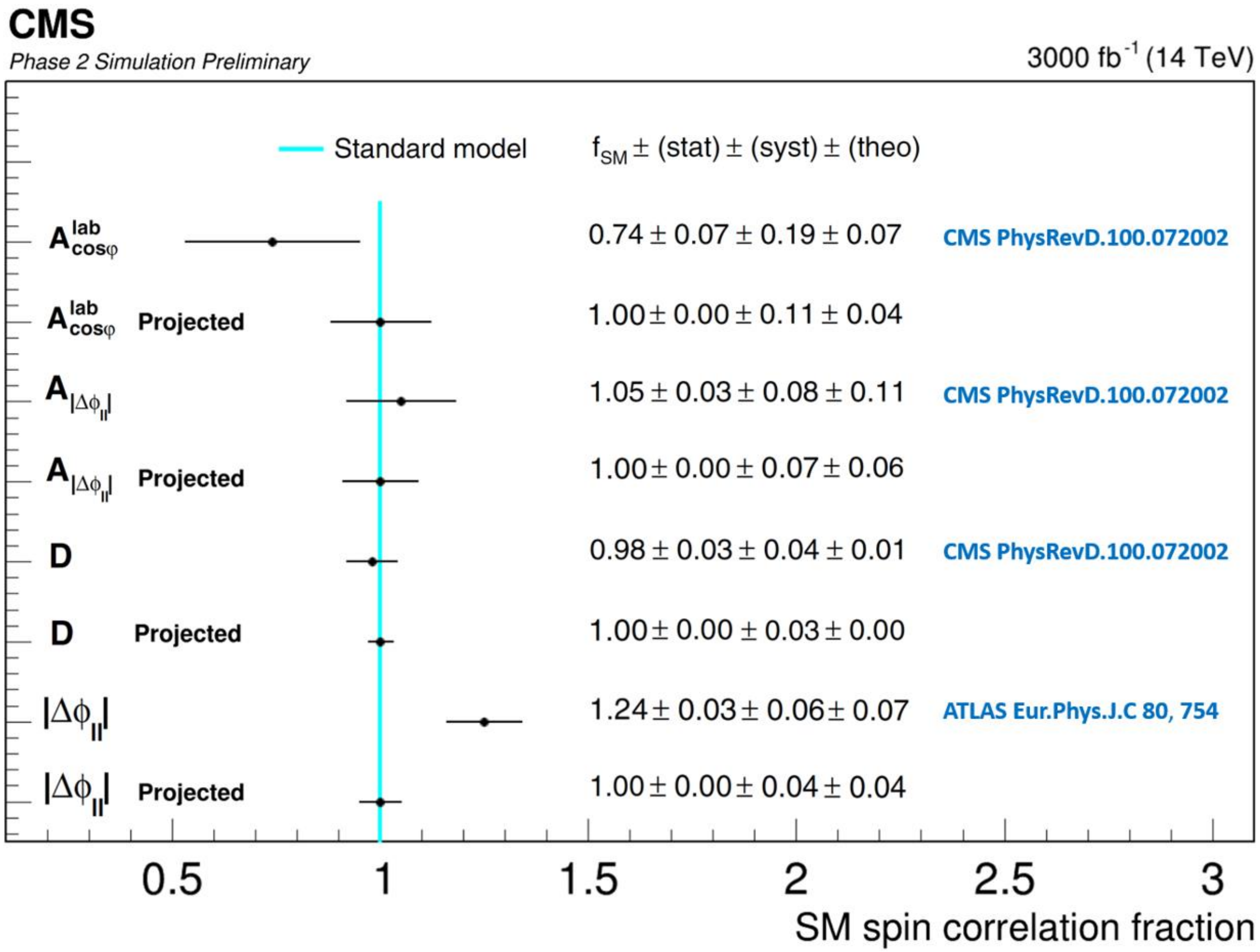






Full LHC Extrapolation [CMS-FTR-18-034]

- Statistical uncertainty becomes negligible
- Systematics are reduced based on yellow paper suggestions
- Improve precision on D by ~60%



SUSY top Production Limit Improvement

- ~4x improvement comes from statistics
- Maybe another ~5x comes from systematic uncertainty reduction
- **Improvements > 10x come from using spin correlations in deep neural network**